

## CHAPTER 3

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**PROBABILISTIC VALUATION: SCENARIO ANALYSIS,  
DECISION TREES AND SIMULATIONS**

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In the last chapter, we examined ways in which we can adjust the value of a business for its risk. Notwithstanding their popularity, all of the approaches that we described share a common theme. The riskiness of an asset or business is encapsulated in one number – a higher discount rate, or lower cash flows – and the computation almost always requires us to make assumptions (often unrealistic) about the nature of risk.

In this chapter, we consider a different and potentially more informative way of assessing and presenting the risk in an investment. Rather than compute an expected value for an asset or firm that tries to capture the expected value across different possible outcomes, we could provide information on what the value of the asset will be under each outcome or at least a subset of outcomes. We will begin this section by looking at the simplest version, which is an analysis of an asset's value under three scenarios – a best case, most likely case and worse case – and then extend the discussion to look at scenario analysis more generally. We will move on to examine the use of decision trees, a more complete approach to dealing with discrete risk. We will close the chapter by evaluating Monte Carlo simulations, the most complete approach to assessing risk across spectrum.

**Scenario Analysis**

The expected cash flows that we use to value risky assets can be estimated in one or two ways. They can represent a probability-weighted average of cash flows under all possible scenarios or they can be the cash flows under the most likely scenario. While the former is the more precise measure, it is seldom used simply because it requires far more information to compile. In both cases, there are other scenarios where the cash flows will be different from expectations; higher than expected in some and lower than expected in others. In scenario analysis, we estimate expected cash flows and asset value under various scenarios, with the intent of getting a better sense of the effect of risk on value. In this section, we first consider an extreme version of scenario analysis where we consider

the value in the best and the worst case scenarios and then a more generalized version of scenario analysis.

### **Best Case/ Worse Case**

With risky assets, the actual cash flows can be very different from expectations. At the minimum, we can estimate the cash flows if everything works to perfection – a best case scenario – and if nothing does – a worse case scenario. In practice, there are two ways in which this analysis can be structured. In the first, each input into asset value is set to its best (or worst) possible outcome and the cash flows estimated with those values. Thus, when valuing a firm, you may set the revenue growth rate and operating margin at the highest possible level while setting the discount rate at its lowest level, and compute the value as the best-case scenario. The problem with this approach is that it may not be feasible; after all, to get the high revenue growth, the firm may have to lower prices and accept lower margins. In the second, the best possible scenario is defined in terms of what is feasible while allowing for the relationship between the inputs. Thus, instead of assuming that revenue growth and margins will both be maximized, we will choose that combination of growth and margin that is feasible and yields the maximum value. While this approach is more realistic, it does require more work to put into practice.

How useful is best case/worse case analysis? There are two ways in which the results from this analysis can be useful to decision makers. First, the difference between the best case and worst case values can be used as a measure of risk on an asset; the range in value (scaled to size) should be higher for riskier investments. Second, firms that are concerned about the potential spill over effects on their operations of an investment going bad may be able to gauge the effects by looking at the worst case outcome. Thus, a firm that has significant debt obligations may use the worst case outcome to make a judgment as to whether an investment has the potential to push them into default.

In general, though, best case/worse case analyses are not very informative. After all, there should be no surprise in knowing that an asset will be worth a lot in the best case and not very much in the worst case. Thus, an equity research analyst who uses this approach to value a stock, priced at \$ 50, may arrive at values of \$ 80 for the best case

and \$ 10 for the worst case; with a range that large, it will be difficult to make a judgment on a whether the stock is a good investment or not.

### **Multiple scenario analysis**

Scenario analysis does not have to be restricted to the best and worst cases. In its most general form, the value of a risky asset can be computed under a number of different scenarios, varying the assumptions about both macro economic and asset-specific variables. While the concept of sensitivity analysis is a simple one, it has four critical components:

- The first is the determination of which factors the scenarios will be built around. These factors can range from the state of the economy for an automobile firm, to the response of competitors for a consumer product firm, to the behavior of regulatory authorities for a phone company. In general, analysts should focus on the two or three most critical factors that will determine the value of the asset and build scenarios around these factors.
- The second component is determining the number of scenarios to analyze for each factor. While more scenarios may be more realistic than fewer, it becomes more difficult to collect information and differentiate between the scenarios in terms of asset cash flows. Thus, estimating cash flows under each scenario will be easier if we lays out five scenarios, for instance, than if we specify 15 scenarios. The question of how many scenarios to consider will depend then upon how different the scenarios are, and how well the analyst can forecast cash flows under each scenario.
- The third component is the estimation of asset cash flows under each scenario. It is to ease the estimation at this step that we focus on only two or three critical factors and build relatively few scenarios for each factor.
- The final component is the assignment of probabilities to each scenario. For some scenarios, involving macro-economic factors such as exchange rates, interest rates and overall economic growth, we can draw on the expertise of services that forecast these variables. For other scenarios, involving either the sector or competitors, we have to draw on our knowledge about the industry. Note, though, that this makes sense only if the scenarios cover the full spectrum of possibilities. If the scenarios

represent only a sub-set of the possible outcomes on an investment, the probabilities will not add up to one.

The output from a scenario analysis can be presented as values under each scenario and as an expected value across scenarios (if the probabilities can be estimated in the fourth step).

Multiple scenario analysis provides more information than a best case/worst case analysis by providing asset values under each of the specified scenarios. It does, however, have its own set of problems:

1. Garbage in, garbage out: It goes without saying that the key to doing scenario analysis well is the setting up of the scenarios and the estimation of cash flows under each one. Not only do the outlined scenarios have to be realistic but they also have to try to cover the spectrum of possibilities. Once the scenarios have been laid out, the cash flows have to be estimated under each one; this trade off has to be considered when determining how many scenarios will be run.
2. Continuous Risk: Scenario analysis is best suited for dealing with risk that takes the form of discrete outcomes than for continuous risk. An example of the former would be a shift in regulatory rules, whereas changes in margins or market share would be an example of the latter.
3. Double counting of risk: As with the best case/ worst case analysis, there is the danger that decision makers will double count risk when they do scenario analysis. Thus, an investor, looking at the output from a scenario analysis, may decide not to invest in an undervalued stock, because its value under some scenarios is lower than the market price. Since the expected value is already risk adjusted, this would represent a double counting of potentially the same risk or risk that should not be a factor in the decision in the first place (because it is diversifiable).

*Illustration 3.1: Valuing a company with scenario analysis*

To illustrate scenario analysis, consider a simple example. Assume that you are valuing TechSmart, a manufacturing company that gets 20% of its revenues and half its operating profits from Walmart, and that the contract with Walmart is up for renewal at the start of the next year. Assume that there are three scenarios – the first and most likely

one is that the contract will be renewed with the existing terms, the second is that the contract will be renewed but with more restrictive (and less profitable) terms for TechSmart and the third is that the contract will not be renewed.

Your valuation of the company will be heavily dependent upon whether the contract is renewed, and you therefore estimate three sets of numbers for expected revenues and after-tax operating income next year, depending upon what happens with the contract. Table 3.1 below summarizes the estimates for each scenario:

*Table 3.1: Expected Revenues and Operating Earnings next year – Contract Scenarios*

<i>Scenario</i>	<i>Revenues</i>	<i>After-tax Operating Income</i>
Contract renewed	\$1,500	\$240
Contract renewed with restrictions	1500	180
Contract not renewed	1200	120

Under every scenario, the firm is expected to be in stable growth, with a growth rate of 3% and a cost of capital of 8%. However, the returns on capital will vary under each scenario, leading to differences in reinvestment needed to sustain the expected growth rate.

*Table 3.2: Return on Capital (ROC) and Reinvestment Rate- Contract Scenarios*

<i>Scenario</i>	<i>Growth Rate</i>	<i>ROC</i>	<i>Reinvestment Rate (g/ ROC)</i>
Contract renewed	3%	12%	25.00%
Contract renewed with restrictions	3%	9%	33.33%
Contract not renewed	3%	6%	50.00%

Using the estimates of expected after-tax operating income from table 3.1 and the reinvestment rate from table 3.2, we estimate the value of the firm under each scenario as follows:

$$\text{Value of firm} = \frac{\text{Expected After - tax Operating Income (1 - Reinvestment Rate)}}{(\text{Cost of capital} - \text{Expected Growth Rate})}$$

*Table 3.3: Value of the firm – Contract Scenarios*

<i>Scenario</i>	<i>Operating Income</i>	<i>Reinvestment Rate</i>	<i>Value of firm</i>
Contract renewed	\$240	25.00%	3600
Contract renewed with restrictions	180	33.33%	2400

Contract not renewed	120	50.00%	1200
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Finally, let us assume that the probabilities for the three scenarios are as follows: contract renewal without restrictions is 50%, contract renewal with restricting is 30% and contract cancellation is 20%. The expected value of the firm across the scenarios is:

$$\text{Value of firm} = (.50) (3600) + (.30) (2400) + .20 (1200) = \$2,760 \text{ million}$$

Note that we could have arrived at precisely the same value, using expected values for the operating income and return on capital in a single discounted cash flow valuation.

## Decision Trees

In some valuations, risk is not only discrete but is sequential. In other words, for the asset to have value, it has to pass through a series of tests, with failure at any point potentially translating into a complete loss of value. This is the case, for instance, with a pharmaceutical drug that is just being tested on human beings. The three-stage FDA approval process lays out the hurdles that have to be passed for this drug to be commercially sold, and failure at any of the three stages dooms the drug's chances. Decision trees allow us to not only consider the risk in stages but also to devise the right response to outcomes at each stage.

### Steps in Decision Tree Analysis

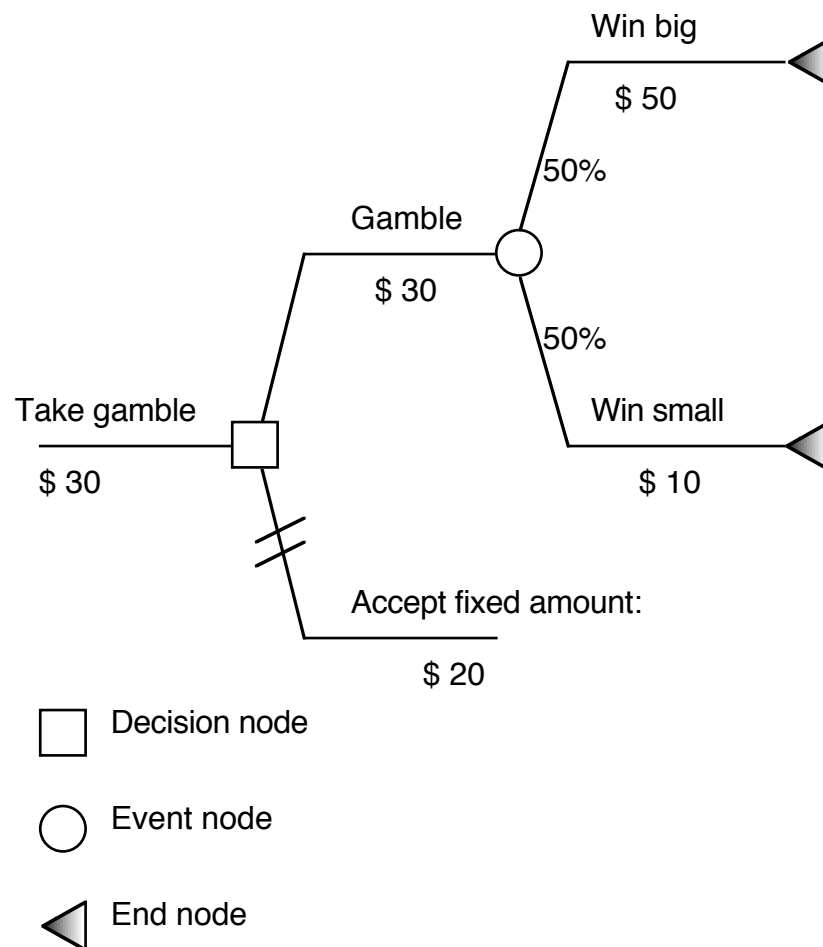
The first step in understanding decision trees is to distinguish between root nodes, decision nodes, event nodes and end nodes.

- The *root node* represents the start of the decision tree, where a decision maker can be faced with a decision choice or an uncertain outcome. The objective of the exercise is to evaluate what a risky investment is worth at this node.
- *Event nodes* represent the possible outcomes on a risky gamble; whether a drug passes the first stage of the FDA approval process or not is a good example. We have to figure out the possible outcomes and the probabilities of the outcomes occurring, based upon the information we have available today.
- *Decision nodes* represent choices that can be made by the decision maker –to expand from a test market to a national market, after a test market's outcome is known.

- *End nodes* usually represent the final outcomes of earlier risky outcomes and decisions made in response.

Consider a very simple example. You are offered a choice where you can take a certain amount of \$ 20 or partake in a gamble, where you can win \$ 50 with probability 50% and \$10 with probability 50%. The decision tree for this offered gamble is shown in figure 3.1:

*Figure 3.1: Simple Decision Tree*



Note the key elements in the decision tree. First, only the event nodes represent uncertain outcomes and have probabilities attached to them. Second, the decision node represents a choice. On a pure expected value basis, the gamble is better (with an expected value of \$ 30) than the guaranteed amount of \$20; the double slash on the latter branch indicates that it would not be selected. While this example may be simplistic, the elements of building a decision tree are in it.

Step 1: Divide analysis into risk phases: The key to developing a decision tree is outlining the phases of risk that you will be exposed to in the future. In some cases, such as the FDA approval process, this will be easy to do since there are only two outcomes – the drug gets approved to move on to the next phase or it does not. In other cases, it will be more difficult. For instance, a test market of a new consumer product can yield hundreds of potential outcomes; here, you will have to create discrete categories for the success of the test market.

Step 2: In each phase, estimate the probabilities of the outcomes: Once the phases of risk have been put down and the outcomes at each phase are defined, the probabilities of the outcomes have to be computed. In addition to the obvious requirement that the probabilities across outcomes has to sum up to one, the analyst will also have to consider whether the probabilities of outcomes in one phase can be affected by outcomes in earlier phases. For example, how does the probability of a successful national product introduction change when the test market outcome is only average?

Step 3: Define decision points: Embedded in the decision tree will be decision points where you will get to determine, based upon observing the outcomes at earlier stages, and expectations of what will occur in the future, what your best course of action will be. With the test market example, for instance, you will get to determine, at the end of the test market, whether you want to conduct a second test market, abandon the product or move directly to a national product introduction.

Step 4: Compute cash flows/value at end nodes: The next step in the decision tree process is estimating what the final cash flow and value outcomes will be at each end node. In some cases, such as abandonment of a test market product, this will be easy to do and will represent the money spent on the test marketing of the product. In other cases, such as a national launch of the same product, this will be more difficult to do since you will have to estimate expected cash flows over the life of the product and discount these cash flows to arrive at value.

Step 5: Folding back the tree: The last step in a decision tree analysis is termed “folding back’ the tree, where the expected values are computed working backwards through the tree. If the node is a chance node, the expected value is computed as the probability weighted average of all of the possible outcomes. If it is a decision node, the expected



value is computed for each branch, and the highest value is chosen (as the optimal decision). The process culminates in an expected value for the asset or investment today.<sup>1</sup>

There are two key pieces of output that emerge from a decision tree. The first is the expected value today of going through the entire decision tree. This expected value will incorporate the potential upside and downside from risk and the actions that you will take along the way in response to this risk. In effect, this is analogous to the risk adjusted value that we talked about in the last chapter. The second is the range of values at the end nodes, which should encapsulate the potential risk in the investment.

*Illustration 3.2: Valuing a biotech company with a decision tree*

To illustrate the steps involved in developing a decision tree, we will value a small biotechnology company that has only product: a pharmaceutical drug for treating Type 1 diabetes that has gone through preclinical testing and is about to enter phase 1 of the FDA approval process.<sup>2</sup> Assume that you are provided with the additional information on each of the three phases:

1. Phase 1 is expected to cost \$ 50 million and will involve 100 volunteers to determine safety and dosage; it is expected to last 1 year. There is a 70% chance that the drug will successfully complete the first phase.
2. In phase 2, the drug will be tested on 250 volunteers for effectiveness in treating diabetes over a two-year period. This phase will cost \$ 100 million and the drug will have to show a statistically significant impact on the disease to move on to the next phase. There is only a 30% chance that the drug will prove successful in treating type 1 diabetes but there is a 10% chance that it will be successful in treating both type 1 and type 2 diabetes and a 10% chance that it will succeed only in treating type 2 diabetes.
3. In phase 3, the testing will expand to 4,000 volunteers to determine the long-term consequences of taking the drug. If the drug is tested on only type 1 or type 2 diabetes

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<sup>1</sup> There is a significant body of literature examining the assumptions that have to hold for this folding back process to yield consistent values. In particular, if a decision tree is used to portray concurrent risks, the risks should be independent of each other. See Sarin, R. and P.Wakker, 1994, Folding Back in Decision Tree Analysis, *Management Science*, v40, pg 625-628.

<sup>2</sup> In type 1 diabetes, the pancreas do not produce insulin. The patients are often young children and the disease is unrelated to diet and activity; they have to receive insulin to survive. In type 2 diabetes, the pancreas produce insufficient insulin. The disease manifests itself in older people and can be sometimes controlled by changing lifestyle and diet.

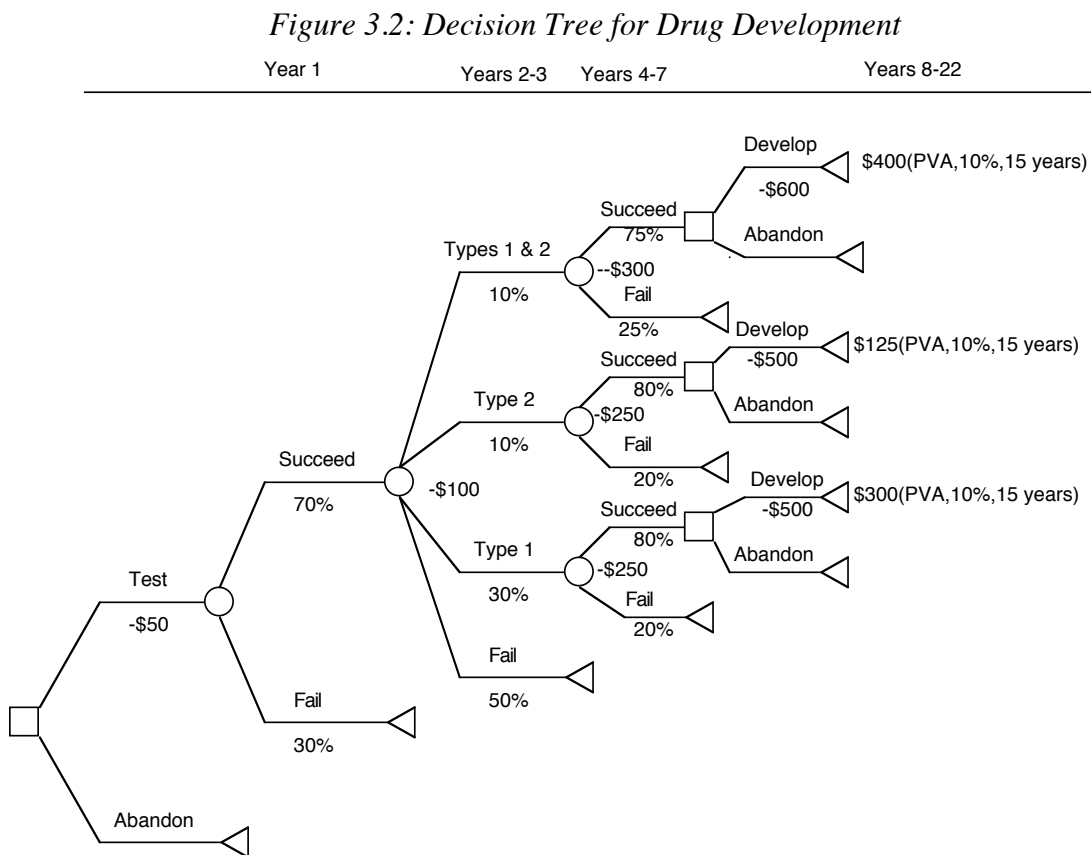
patients, this phase will last 4 years and cost \$ 250 million; there is an 80% chance of success. If it is tested on both types, the phase will last 4 years and cost \$ 300 million; there is a 75% chance of success.

If the drug passes through all 3 phases, the costs of developing the drug and the annual cash flows are provided below:

Disease treatment	Cost of Development	Annual Cash Flow
Type 1 diabetes only	\$ 500 million	\$ 300 million for 15 years
Type 2 diabetes only	\$ 500 million	\$ 125 million for 15 years
Type 1 and 2 diabetes	\$ 600 million	\$ 400 million for 15 years

Assume that the cost of capital for the firm is 10%.

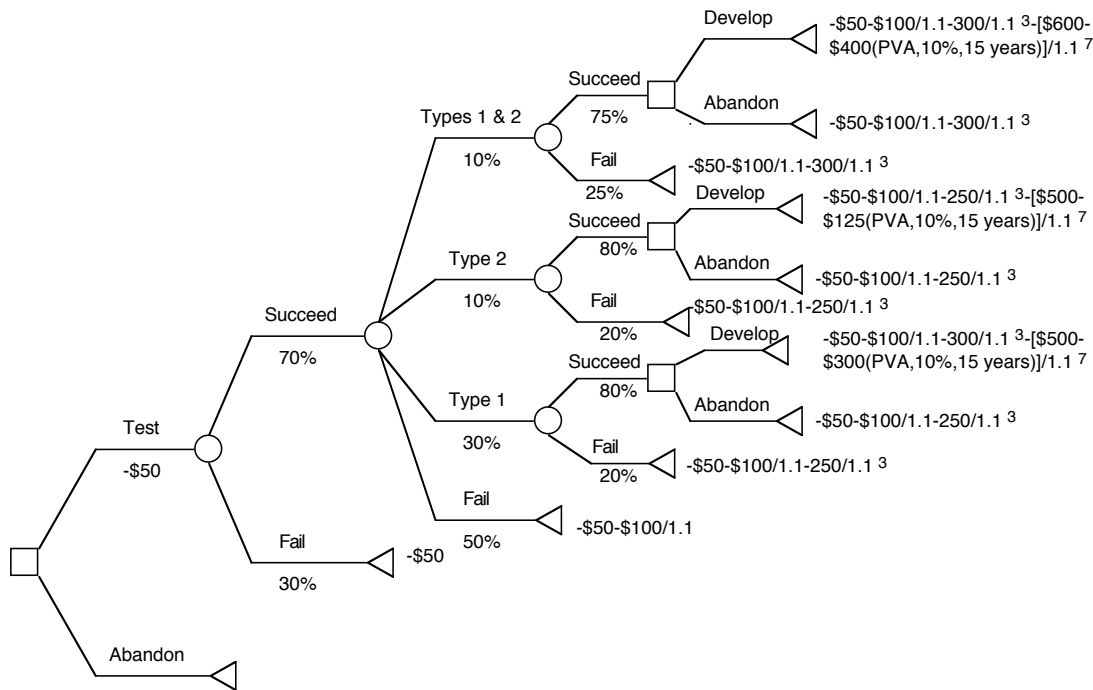
We now have the information to draw the decision tree for this drug. We will first draw the tree in figure 3.2, specifying the phases, the cash flows at each phase and the probabilities:



The decision tree shows the probabilities of success at each phase and the additional cash flow or marginal cash flow associated with each step. Since it takes time to go through

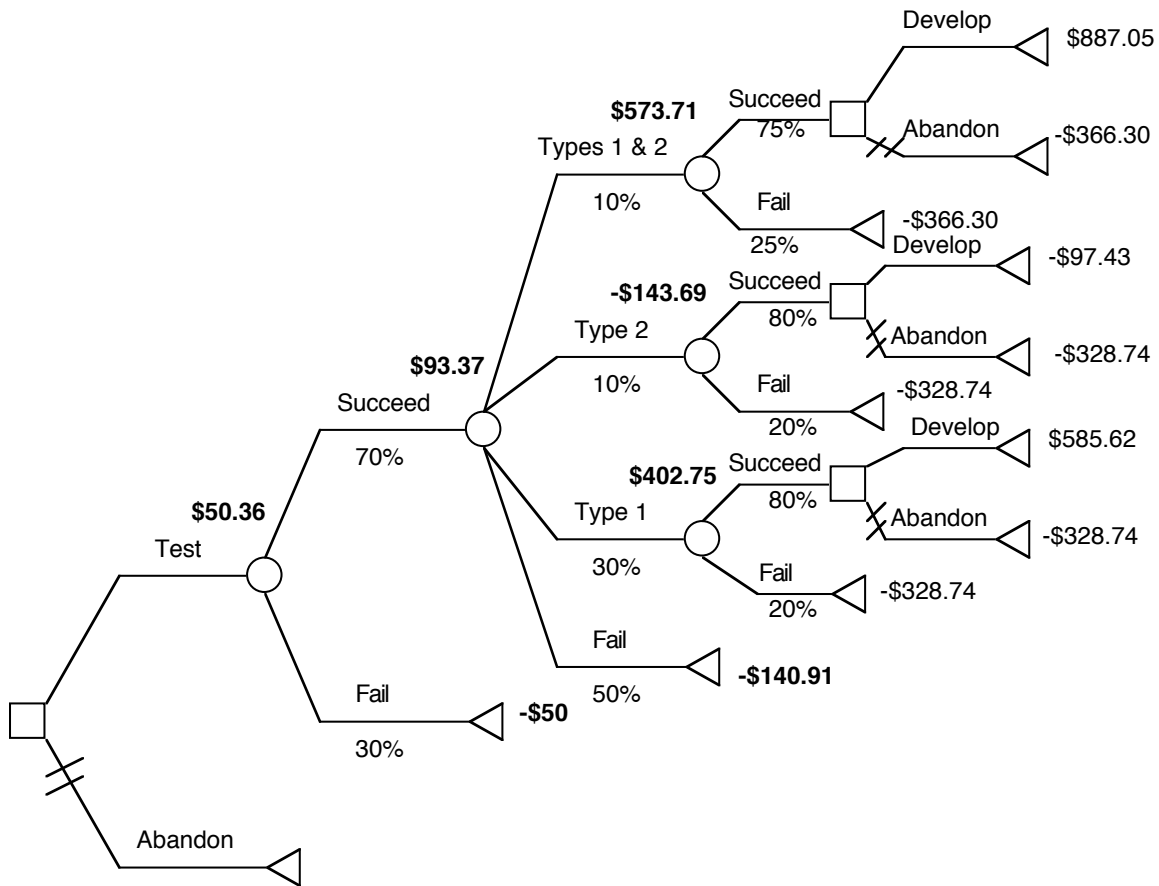
the phases, there is a time value effect that has to be built into the expected cash flows for each path. We introduce the time value effect and compute the cumulative present value (today) of cash flows from each path, using the 10% cost of capital as the discount rate, in figure 3.3:

*Figure 3.3: Present Value of Cash Flows at End Nodes: Drug Development Tree*



Note that the present value of the cash flows from development after the third phase gets discounted back an additional seven years (to reflect the time it takes to get through three phases). In the last step in the process, we compute the expected values by working backwards through the tree and estimating the optimal action in each decision phase in figure 3.4:

Figure 3.4: Drug Decision Tree Folded Back



The expected value of the drug today, given the uncertainty over its success, is \$50.36 million. This value reflects all of the possibilities that can unfold over time and shows the choices at each decision branch that are sub-optimal and thus should be rejected. For example, once the drug passes phase 3, developing the drug beats abandoning it in all three cases – as a treatment for type 1, type 2 or both types. The decision tree also provides a range of outcomes, with the worst case outcome being failure in phase 3 of the drug as a treatment for both phase 1 and 2 diabetes (-\$366.30 million in today's dollars) to the best case outcome of approval and development of the drug as treatment for both types of diabetes (\$887.05 million in today's dollars).

There may one element in the last set of branches that may seem puzzling. Note that the present value of developing the drug as a treatment for just type 2 diabetes negative (-\$97.43 million). Why would the company still develop the drug? Because the alternative of abandoning the drug at the late stage in the process has an even more

negative net present value (-\$328.74 million). Another way to see this is to look at the marginal effect of developing the drug just for type 2 diabetes. Once the firm has expended the resources to take the firm through all three phases of testing, the testing cost becomes a sunk cost and is not a factor in the decision.<sup>3</sup> The marginal cash flows from developing the drug after phase 3 yield a positive net present value of \$451 million (in year 7 cash flows):

$$\text{Present value of developing drug to treat Type 2 diabetes in year 7} = -500 + 125(\text{PV of annuity, 10\%, 15 years}) = \$451 \text{ million}$$

At each stage in the decision tree, you make your judgments based upon the marginal cash flows at that juncture. Rolling back the decision tree allows you to see what the value of the drug is at each phase in the process.

In summary, this decision tree would lead us to assess a value of \$50.36 million for the diabetes drug, and by extension, the firm that owns the rights to the drug. The decision tree allows provides information on the value that we should attach to the firm as it moves through the phases. If the initial test succeeds, for instance, the value of the firm will jump to \$93.37 million. In the follow up test, if the drug has promise for treating both type 1 and 2 diabetes, the value of the firm will jump to \$573.71 million.

### **Estimation Issues**

There are some types of risk that decision trees are capable of handling and others that they are not. In particular, decision trees are best suited for risk that is sequential; the FDA process where approval occurs in phases is a good example. Risks that affect an asset concurrently cannot be easily modeled in a decision tree.<sup>4</sup> As with scenario analysis, decision trees generally look at risk in terms of discrete outcomes. Again, this is not a problem with the FDA approval process where there are only two outcomes – success or failure. There is a much wider range of outcomes with most other risks and we have to create discrete categories for the outcomes to stay within the decision tree framework.

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<sup>3</sup> It would be more accurate to consider only the costs of the first two phases as sunk, since by the end of phase 2, the firm knows that the drug is effective only against type 2 diabetes. Even if we consider only the costs of the first 2 phases as sunk, it still makes sense on an expected value basis to continue to phase 3.

<sup>4</sup> If we choose to model such risks in a decision tree, they have to be independent of each other. In other words, the sequencing should not matter.

Assuming risk is sequential and can be categorized into discrete boxes, we are faced with estimation questions to which there may be no easy answers. In particular, we have to estimate the cash flow under each outcome and the associated probability. With the drug development example, we had to estimate the cost and the probability of success of each phase. The advantage that we have when it comes to these estimates is that we can draw on empirical data on how frequently drugs that enter each phase make it to the next one and historical costs associated with drug testing. To the extent that there may be wide differences across different phase 1 drugs in terms of success – some may be longer shots than others – there can still be errors that creep into decision trees.

The expected value of a decision tree is heavily dependent upon the assumption that we will stay disciplined at the decision points in the tree. In other words, if the optimal decision is to abandon if a test market fails and the expected value is computed, based on this assumption, the integrity of the process and the expected value will quickly fall apart, if managers decide to overlook the market testing failure and go with a full launch of the product anyway.

Finally, decision trees are most useful when valuing companies that are entirely dependent upon a single product or asset for their value. In the biotechnology company valuation in illustration 3.2, we assumed that the entire value was derived from the single diabetes drug working its way through the pipeline. The valuations that we obtain, though, may not be realistic if we assume that the firm has the research potential to develop other drugs in the future, in which case there might be additional value generated from these new drugs.

### **Risk Adjusted Value and Decision Trees**

Are decision trees an alternative or an addendum to discounted cash flow valuation? The question is an interesting one because there are some analysts who believe that decision trees, by factoring in the possibility of good and bad outcomes, are already risk adjusted. In fact, they go on to make the claim that the right discount rate to use estimating present value in decision trees is the riskfree rate; using a risk adjusted discount rate, they argue, would be double counting the risk. Barring a few exceptional circumstance, they are incorrect in their reasoning.

a. Expected values are not risk adjusted: Consider decision trees, where we estimate expected cash flows by looking at the possible outcomes and their probabilities of occurrence. The probability-weighted expected value that we obtain is not risk adjusted. The only rationale that can be offered for using a risk free rate is that the risk embedded in the uncertain outcomes is asset-specific and will be diversified away, in which case the risk adjusted discount rate would be the riskfree rate. In the FDA drug development example, for instance, this may be offered as the rationale for why we would use the risk free rate to discount cash flows for the first seven years, when the only the risk we face is drug approval risk. After year 7, though, the risk is likely to contain a market element and the risk-adjusted rate will be higher than the risk free rate.

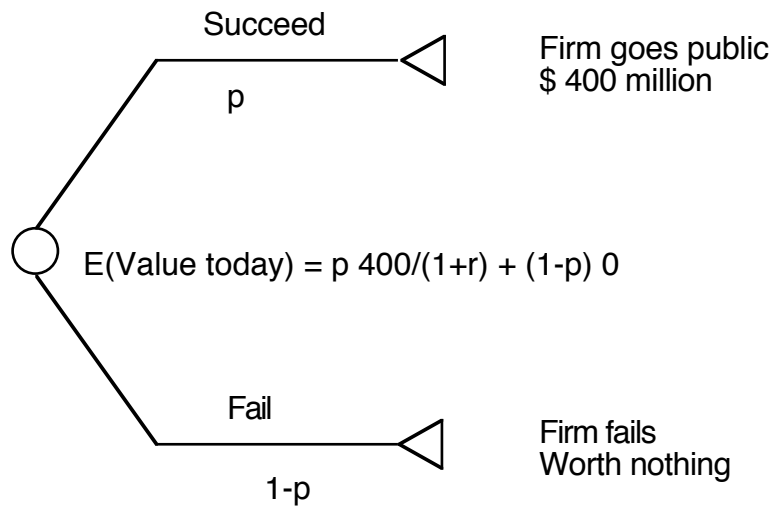
b. Double Counting of Risk: We do have to be careful about making sure that we don't double count for risk in decision trees by using risk-adjusted discount rates that are set high to reflect the possibility of failure at the earlier phases. One common example of this phenomenon is in venture capital valuation. A conventional approach that venture capitalists have used to value young start-up companies is to estimate an exit value, based on projected earnings and a multiple of that earnings in the future, and to then discount the exit value at a target rate. Using this approach, for instance, the value today for a firm that is losing money currently but is expected to make \$ 10 million in 5 years (when the earnings multiple at which it will be taken public is estimated to be 40) can be computed as follows (if the target rate is 35%):

$$\text{Value of the firm in 5 years} = \text{Earnings in year 5} * \text{PE} = 10 * 40 = \$ 400 \text{ million}$$

$$\text{Value of firm today} = \$ 400 / 1.35^5 = \$89.20 \text{ million}$$

Note, however, that the target rate is set at a high level (35%) because of the probability that this young firm will not make it to a public offering. In fact, we could frame this as a simple decision tree in figure 3.5:

Figure 3.5: Decision Tree for start-up firm



Assume that  $r$  is the correct discount rate, based upon the risk that the venture capitalist faces on this venture. Going back to the numeric example, assume that this discount rate would have been 15% for this venture. We can solve for the implied probability of failure, embedded in the venture capitalist's estimate of value of \$89.20 million:

$$\text{Estimated Value} = \$89.20 = \frac{\$400}{1.15^5} (p)$$

Solving for  $p$ , we estimate the probability of success at 44.85%. With this estimate of probability in the decision tree, we would have arrived at the same value as the venture capitalist, assuming that we use the right discount rate. Using the target rate of 35% as the discount rate in a decision tree would lead to a drastically lower value, because risk would have been counted twice. Using the same reasoning, we can see why using a high discount rate in assessing the value of a bio-technology drug in a decision tree will under value the drug, especially if the discount rate already reflects the probability that the drug will not make it to commercial production. If the risk of the approval process is drug-specific and thus diversifiable, this would suggest that discount rates should be moderate in decision tree analysis, even for drugs with very high likelihoods of not making it through the approval process.

c. The Right Discount Rate: If the right discount rate to use in a decision tree should reflect the non-diversifiable risk looking forward, it is not only possible but likely that discount rates will be different at different points in the tree. For instance, extraordinary success at the test market stage may yield more predictable cash flows than an average



test market outcome; this would lead us to use a lower discount rate to value the former and a higher discount rate to value the latter. In the drug development example, it is possible that the expected cash flows, if the drug works for both types of diabetes, will be more stable than if it is a treatment for only one type. It would follow that a discount rate of 8% may be the right one for the first set of cash flows, whereas a 12% discount rate may be more appropriate for the second.

Reviewing the discussion, decision trees are not alternatives to risk adjusted valuation. Instead, they can be viewed as a different way of adjusting for discrete risk that may be difficult to bring into expected cash flows or into risk adjusted discount rates.

## **Simulations**

If scenario analysis and decision trees are techniques that help us to assess the effects of discrete risk, simulations provide a way of examining the consequences of continuous risk. To the extent that most risks that we face in the real world can generate hundreds of possible outcomes, a simulation will give us a fuller picture of the risk in an asset or investment.

### **Steps in simulation**

Unlike scenario analysis, where we look at the values under discrete scenarios, simulations allow for more flexibility in how we deal with uncertainty. In its classic form, distributions of values are estimated for each parameter in the valuation (growth, market share, operating margin, beta etc.). In each simulation, we draw one outcome from each distribution to generate a unique set of cashflows and value. Across a large number of simulations, we can derive a distribution for the value of investment or an asset that will reflect the underlying uncertainty we face in estimating the inputs to the valuation. The steps associated with running a simulation are as follows:

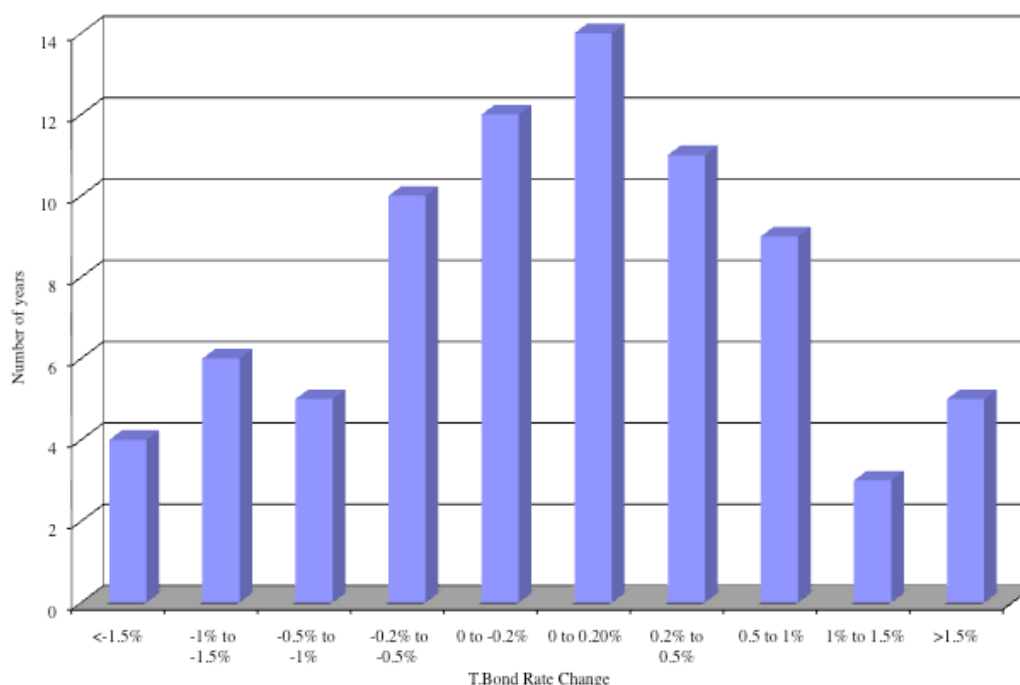
1. Determine “probabilistic” variables: In any analysis, there are potentially dozens of inputs, some of which are predictable and some of which are not. Unlike scenario analysis and decision trees, where the number of variables that are changed and the potential outcomes have to be few in number, there is no constraint on how many variables can be allowed to vary in a simulation. At least in theory, we can define probability distributions for each and every input in a valuation. The reality, though, is

that this will be time consuming and may not provide much of a payoff, especially for inputs that have only marginal impact on value. Consequently, it makes sense to focus attention on a few variables that have a significant impact on value.

2. Define probability distributions for these variables: This is a key and the most difficult step in the analysis. Generically, there are three ways in which we can go about defining probability distributions:

a. *Historical data:* For variables that have a long history and reliable data over that history, it is possible to use the historical data to develop distributions. Assume, for instance, that you are trying to develop a distribution of expected changes in the long-term Treasury bond rate (to use as an input in investment analysis). You could use the histogram in figure 3.6, based upon the annual changes in Treasury bond rates every year from 1928 to 2005, as the distribution for future changes.

Figure 3.6: Change in T.Bond Rate - 1928 -2005

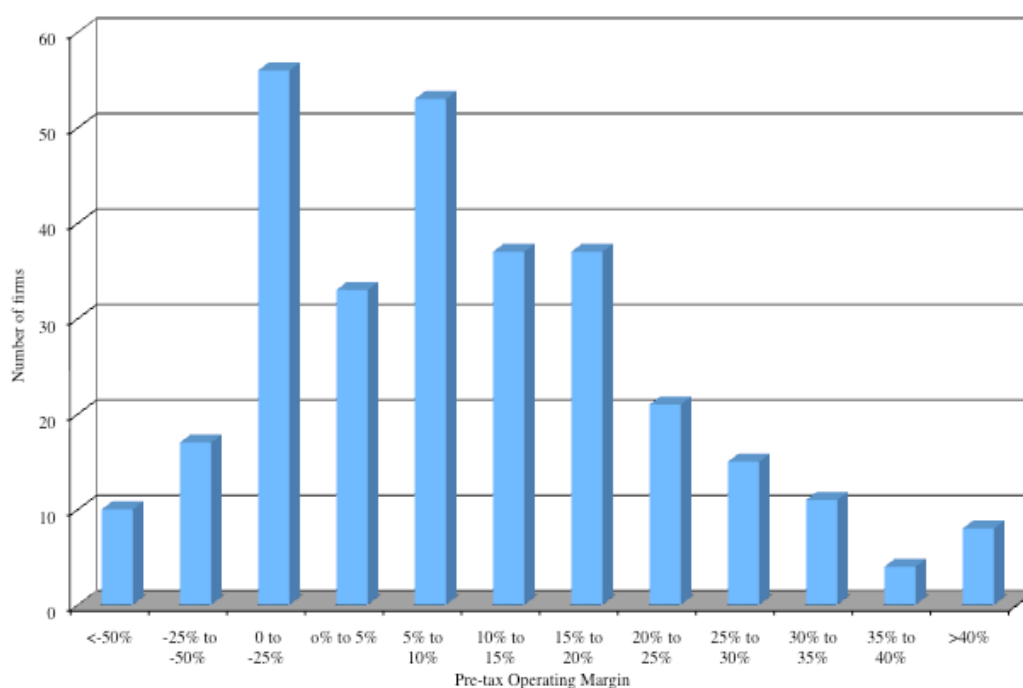


approach is the assumption that there have been no structural shifts in the market that will render the historical data unreliable.

b. *Cross sectional data:* In some cases, you may be able to substitute data on differences in a specific variable across existing investments that are similar to the

investment being analyzed. Consider two examples. Assume that you are valuing a software firm and are concerned about the volatility in operating margins. Figure 3.7 provides a distribution of pre-tax operating margins across software companies in 2006:

*Figure 3.7: Pre-tax Operating Margin across Software Companies (US) - January 2006*



distribution, we are in effect assuming that the underlying distribution of margins is the same across software firms. In a second example, assume that you work for Target, the retailer, and that you are trying to estimate the sales per square foot for a new store investment. Target could use the distribution on this variable across existing stores as the basis for its simulation of sales at the new store.

*c. Statistical Distribution and parameters:* For most variables that you are trying to forecast, the historical and cross sectional data will be insufficient or unreliable. In these cases, we have to pick a statistical distribution that best captures the variability in the input and estimate the parameters for that distribution. Thus, we may conclude that operating margins will be distributed uniformly, with a minimum of 4% and a maximum of 8% and that revenue growth is normally distributed with an expected value of 8% and a standard deviation of 6%. Many simulation packages available for personal computers now provide a rich array of

distributions, but picking the right distribution and the parameters for the distribution remains difficult for two reasons. The first is that few inputs that we see in practice meet the stringent requirements that statistical distributions demand; revenue growth, for instance, cannot really be normally distributed because the lowest value it can take on is -100%. Consequently, we have to settle for statistical distributions that are close enough to the real distribution that the resulting errors will not wreak havoc on our conclusion. The second is that the parameters still need to be estimated, once the distribution is picked. For this, we can draw on historical or cross sectional data; for the revenue growth input, we can look at revenue growth in prior years or revenue growth rate differences across peer group companies. The caveats about structural shifts that make historical data unreliable and peer group companies not being continue to apply.

The probability distributions for discrete for some inputs and continuous for other inputs and should be based upon historical data for some and statistical distributions for others.

3. Check for correlation across variables: While it is tempting to jump to running simulations right after the distributions have been specified, it is important that we check for correlations across variables. Assume, for instance, that you are developing probability distributions for both interest rates and inflation. While both inputs may be critical in determining value, they are likely to be correlated with each other; high inflation is usually accompanied by high interest rates. When there is strong correlation, positive or negative, across inputs, you have two choices. One is to pick only one of the two inputs to vary; it makes sense to focus on the input that has the bigger impact on value. The other is to build the correlation explicitly into the simulation; this does require more sophisticated simulation packages and adds more detail to the estimation process.

4. Run the simulation: For the first simulation, you draw one outcome from each distribution and compute the value based upon those outcomes. This process can be repeated as many times as desired, though the marginal contribution of each simulation drops off as the number of simulations increases. The number of simulations you run should be determined by the following:

- a. Number of probabilistic inputs: The larger the number of inputs that have probability distributions attached to them, the greater will be the required number of simulations.
- b. Characteristics of probability distributions: The greater the diversity of distributions in an analysis, the larger will be the number of required simulations. Thus, the number of required simulations will be smaller in a simulation where all of the inputs have normal distributions than in one where some have normal distributions, some are based upon historical data distributions and some are discrete.
- c. Range of outcomes: The greater the potential range of outcomes on each input, the greater will be the number of simulations.

Most simulation packages allow users to run thousands of simulations, with little or no cost attached to increasing that number. Given that reality, it is better to err on the side of too many simulations rather than too few.

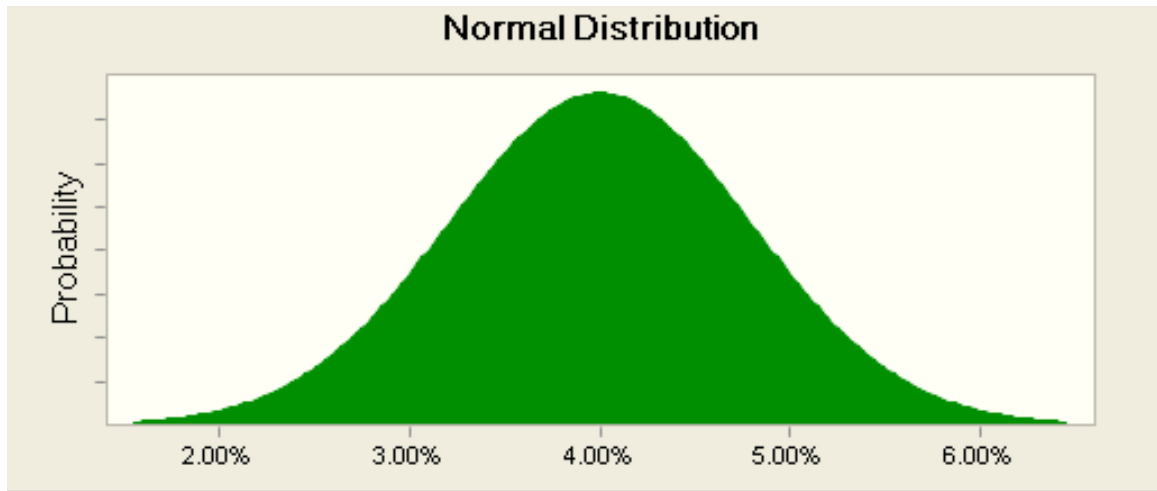
There have generally been two impediments to good simulations. The first is informational: estimating distributions of values for each input into a valuation is difficult to do. In other words, it is far easier to estimate an expected growth rate of 8% in revenues for the next 5 years than it is to specify the distribution of expected growth rates – the type of distribution, parameters of that distribution – for revenues. The second is computational; until the advent of personal computers, simulations tended to be too time and resource intensive for the typical analyst. Both these constraints have eased in recent years and simulations have become more feasible.

### *Illustration 3.3: Valuing 3M – Monte Carlo Simulation*

In chapter 3, we valued 3M, using a conventional discounted cash flow model, where we discounted expected cash flows at a risk adjusted rate to arrive at an estimate of value of \$ 86.95 per share. In the process, though, we did make a number of assumptions about not only how the company would evolve over time, but about riskfree rates and risk premiums in the future. To run a simulation on 3M's value, we will make the following assumptions:

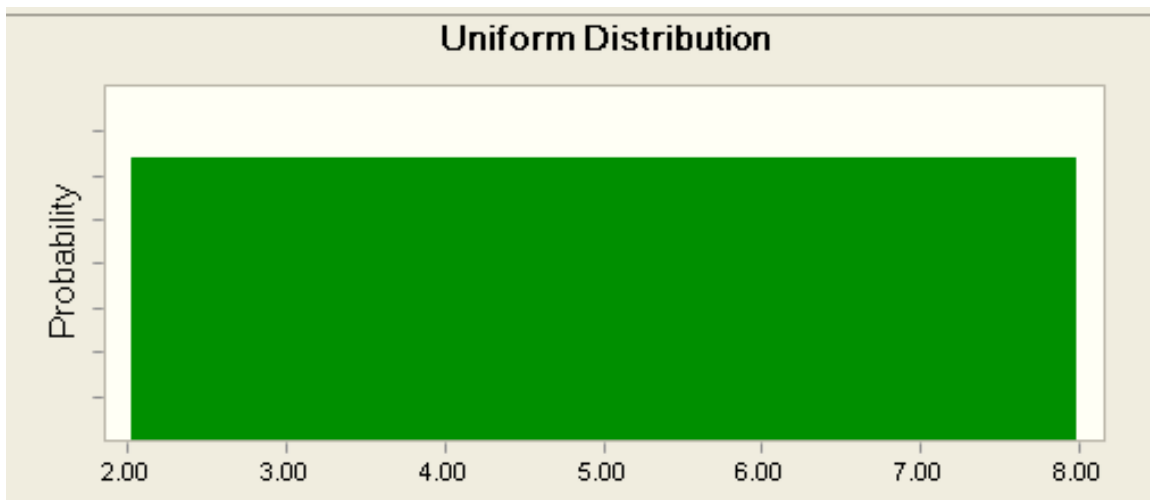
- Equity Risk Premium: In the base case valuation, we used an equity risk premium of 4%, reflecting the historical average of implied premiums in the S&P 500 from 1960 to 2007. Given that this estimate has some error associated with it, we will assume that the equity risk premium is normally distributed with an expected value of 4% and a standard deviation of 0.80% (see figure 3.8)

*Figure 3.8: Equity Risk Premium - Distribution*



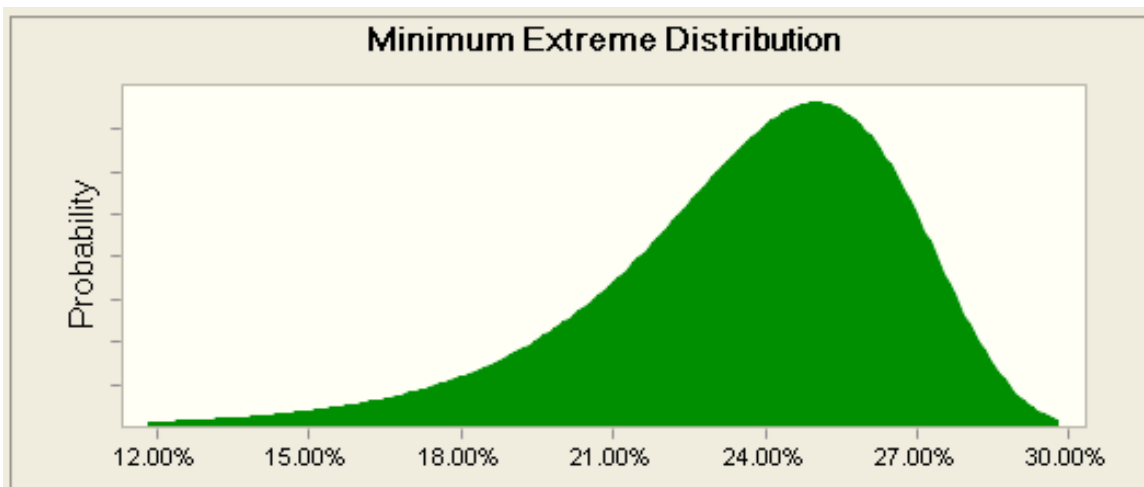
- Length of the growth period: We assumed that 3M would be able to continue to grow at rates higher than the economy for the next 5 years. To reflect the uncertainty in the estimate, we allowed the length of the growth period to vary from 2 to 8 years, with equal probabilities attached to each time period (see figure 3.9)

*Figure 3.9: Length of the Growth Period: Distribution*



- Return on capital: When valuing 3M, a key component determining value was the assumption that the firm would be able to maintain its existing return on capital (approximately 25%) for the next 5 years. Since returns on capital can shift over time, as competition increases, we assumed the following distribution (see figure 3.10):

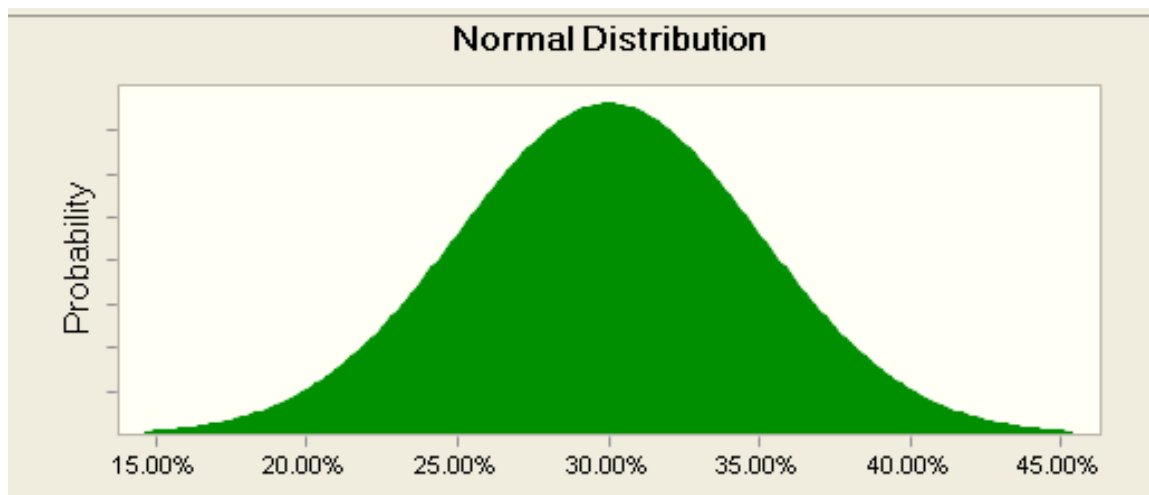
*Figure 3.10: Return on Invested Capital - Distribution*



Note that we are assuming that while the expected return on capital is 25%, it is unlikely that the return will exceed 30% but that there is the possibility of much lower returns in future years. We obtained this distribution by looking at the distribution of returns on capital across companies in this sector.

- Reinvestment Rate: In the base case valuation, we assumed that 3M would maintain a reinvestment rate of 30% for the next 5 years, based upon past history. However, there is the possibility that the firm may ramp up or lower this reinvestment rate. Using the standard deviation in past reinvestment rates at 3M as guidance, we assumed that the reinvestment rate would be normally distributed, with an expected value of 30% and a standard deviation of 5% (see figure 3.11)

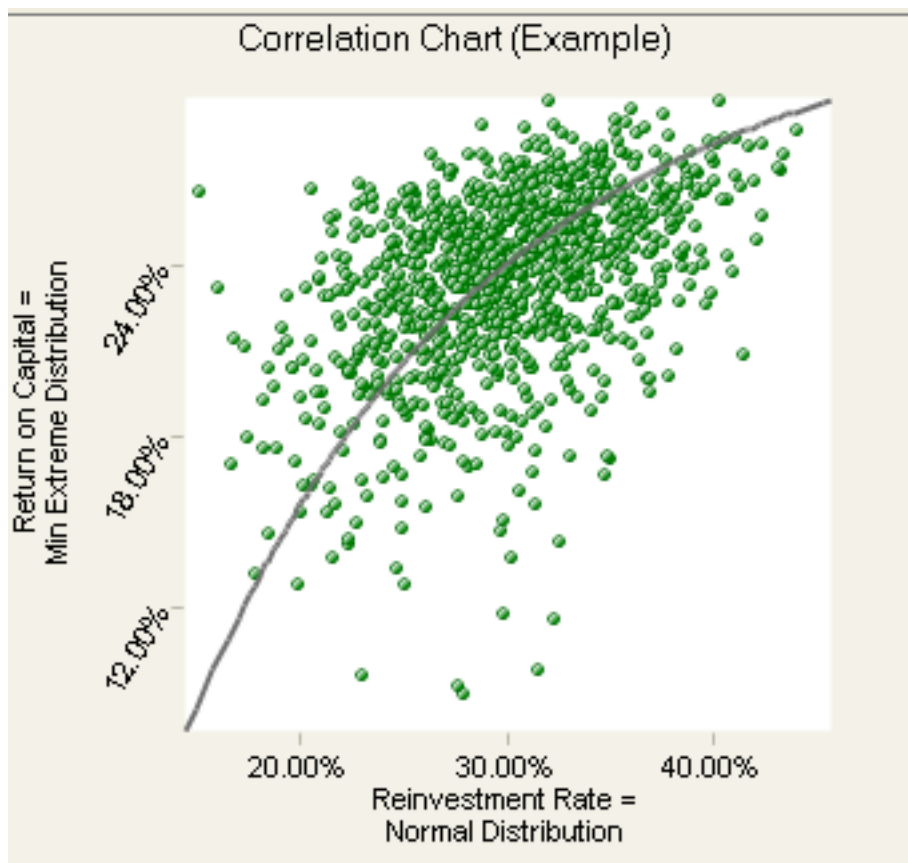
*Figure 3.11: Reinvestment Rate - Distribution*



However, it is very likely that the reinvestment rate will be a function of the return on capital, with high reinvestment rates occurring if the returns on capital are high. We capture this co-movement between return on capital and the reinvestment rate by assuming a correlation of 0.40 between the two, resulting in the scatter plot in figure 3.12:



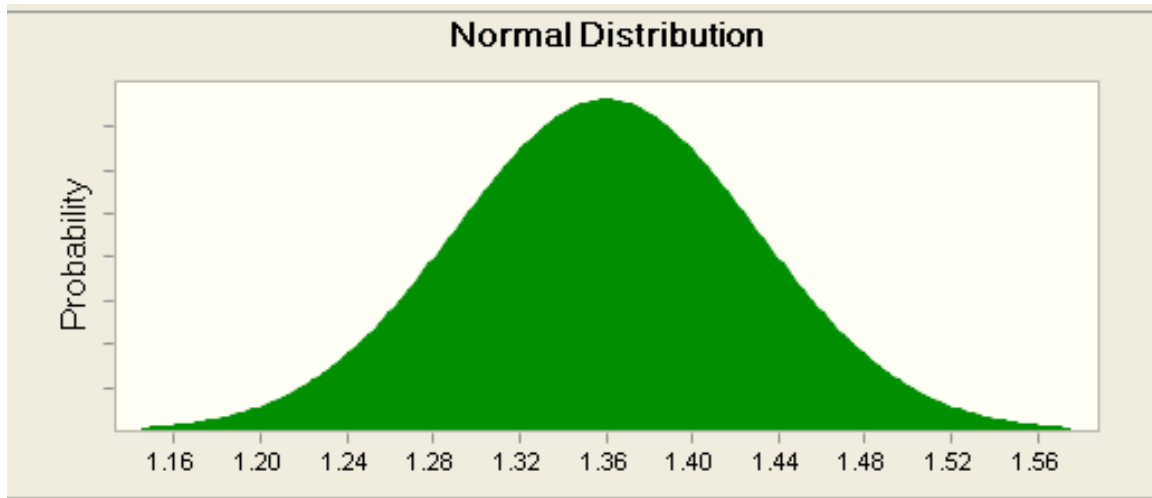
Figure 3.12: Return on Capital and Reinvestment Rate - Comovement



Thus, if the return on capital is close to 30%, the reinvestment rate will be approximately 40%; if the return on capital drops to 12%, the reinvestment rate will decline to 20%.

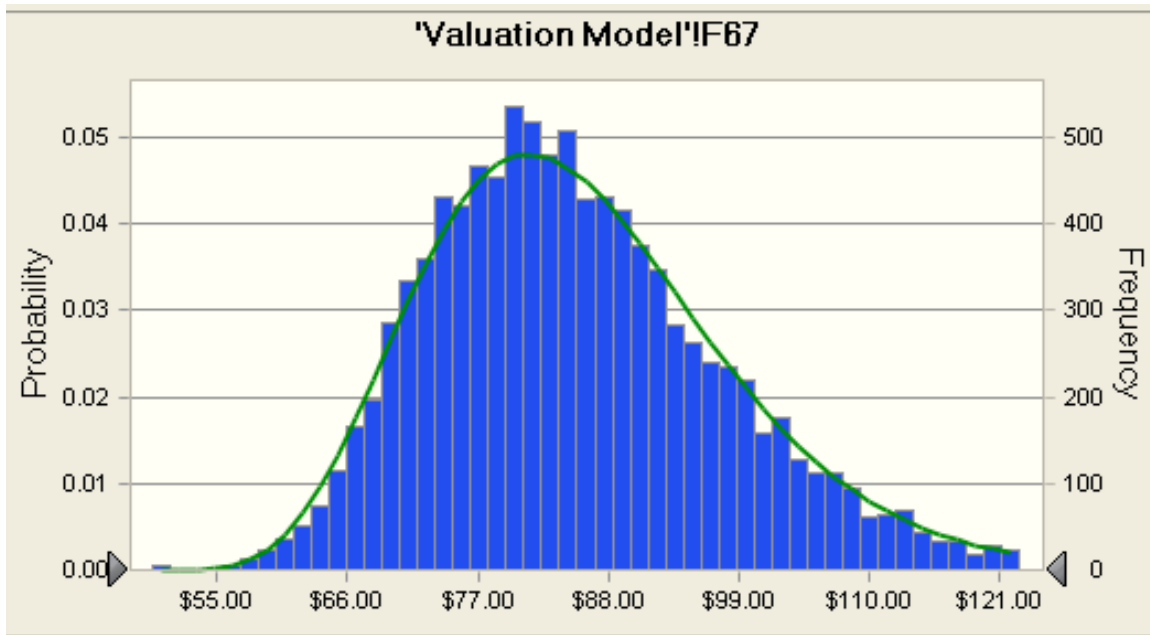
- Beta: In the base case valuation, we estimated a beta, based upon the businesses that 3M operated in, of 1.36 and used this beta for the high growth period. It is possible that this beta estimate could be incorrect and to reflect the imprecision, we assume that the beta is normally distributed with a mean of 1.36 and a standard error of 0.07 (see figure 3.13)

*Figure 3.13: Beta Distribution*



With these inputs in place, we can estimate the value per share for 3M, allowing the parameters specified above to vary across simulations. The results of the simulation are captured in the distribution that we obtain (across 10000 simulations) in figure 3.14:

*Figure 3.14: Value of Equity per share at 3M – Simulation Results*



The key statistics on the values obtained across the 10,000 runs are summarized below:

- The average value across the simulations was \$87.35 a share, a trifle higher the risk adjusted value of \$86.95 a share; the median value was \$ 87.10 a share.

- There was substantial variation in values, with the lowest value across all runs of - \$55.22 a share and the highest value of \$121 a share; the standard deviation in values per share was \$16.15.

### **Use in decision making**

A well-done simulation provides us with more than just an expected value for an asset or business.

- Better input estimation: In an ideal simulation, analysts will examine both the historical and cross sectional data on each input variable before making a judgment on what distribution to use and the parameters of the distribution. In the process, they may be able to avoid the sloppiness that is associated with the use of point estimates; many discounted cash flow valuations are based upon expected growth rates that are obtained from services such as Zack's or IBES, which report analysts' consensus estimates.
- It yields a distribution for expected value rather than a point estimate: Consider the valuation example that we completed in the last section. In addition to reporting an expected value of \$87.35 a share, we also estimated a standard deviation of \$16.15 million in that value and a breakdown of the values, by percentile. The distribution reinforces the obvious but important point that valuation models yield estimates of value for risky assets that are imprecise and explains why different analysts valuing the same asset may arrive at different estimates of value.

Note that there are two claims about simulations that we are unwilling to make. The first is that simulations yield better estimates of expected value than conventional risk adjusted value models. In fact, the expected values from simulations should be fairly close to the expected value that we would obtain using the expected values for each of the inputs (rather than the entire distribution). The second is that simulations, by providing estimates of the expected value and the distribution in that value, lead to better investment decisions. This may not always be the case since the benefits that decision-makers get by getting a fuller picture of the uncertainty in value in a risky asset may be more than offset by misuse of that risk measure. As we will argue later in this chapter, it

is all too common for risk to be double counted in simulations and for decisions to be based upon the wrong type of risk.

### **Simulations with Constraints**

To use simulations as a tool in valuation, we have to introduce a constraint, which, if violated, creates very large costs for the firm and perhaps even causes its demise. We can then evaluate the effectiveness of risk hedging tools by examining the likelihood that the constraint will be violated with each one and weighing that off against the cost of the tool.

### ***Book Value Constraints***

The book value of equity is an accounting construct and, by itself, means little. Firms like Microsoft and Intel trade at market values that are several times their book values. At the other extreme, there are firms that trade at half their book value or less. In fact, there are several hundred firms in the United States, some with significant market values that have negative book values for equity. There are two types of restrictions on book value of equity that may call for risk hedging.

- a. Regulatory Capital Restrictions: Financial service firms such as banks and insurance companies are required to maintain book equity as a fraction of loans or other assets at or above a floor ratio specified by the authorities. Firms that violate these capital constraints can be taken over by the regulatory authorities with the equity investors losing everything if that occurs. Not surprisingly, financial service firms not only keep a close eye on their book value equity (and the related ratios) but they are also conscious of the possibility that the risk in their investments or positions can manifest itself as a drop in book equity. In fact, value at risk (VAR) represents the efforts by financial service firms to understand the potential risks in their investments and to be ready for the possibility of a catastrophic outcome, though the probability of it occurring might be very small. By simulating the values of their investments under a variety of scenarios, they can identify not only the possibility of falling below the regulatory ratios but also look for ways of hedging against this occurring. When valuing a bank using a discounted cash flow model, a simulation might provide an indicator of the risk

that an investor is exposed to that the bank may run into regulatory capital constraints.

- b. Negative Book Value for Equity: As noted, there are hundreds of firms in the United States with negative book values of equity that survive its occurrence and have high market values for equity. There are some countries where a negative book value of equity can create substantial costs for the firm and its investors. For instance, companies with negative book values of equity in parts of Europe are required to raise fresh equity capital to bring their book values above zero. In some countries in Asia, companies that have negative book values of equity are barred from paying dividends. Even in the United States, lenders to firms can have loan covenants that allow them to gain at least partial control of a firm if its book value of equity turns negative. As with regulatory capital restrictions, we can use simulations to assess the probability of a negative book value for equity and to adjust the value for that possibility.

### ***Earnings and Cash flow Constraints***

Earnings and cash flow constraints can be either internally or externally imposed and both can affect the value of firms. In some firms managers of firms may decide that the consequences of reporting a loss or not meeting analysis estimates of earnings are so dire, including perhaps the loss of their jobs, that they are willing to expend the resources on risk hedging products to prevent this form happening. The cost of hedging risk can reduce earnings, cash flows and value. In other firms, the constraints on earnings and cashflows can be externally imposed. For instance, loan covenants can be related to earnings outcomes. Not only can the interest rate on the loan be tied to whether a company makes money or not, but the control of the firm can itself shift to lenders in some cases if the firm loses money. In either case, we can use simulations to both assess the likelihood that these constraints will be violated and to examine the effect on value of this likelihood.

### ***Market Value Constraints***

In discounted cash flow valuation, the value of the firm is computed as a going concern, by discounting expected cashflows at a risk-adjusted discount rate. Deducting

debt from this estimate yields equity value. The possibility and potential costs of not being able to meet debt payments is considered only peripherally in the discount rate. In reality, the costs of not meeting contractual obligations can be substantial. In fact, these costs are generally categorized as indirect bankruptcy costs and could include the loss of customers, tighter supplier credit and higher employee turnover. The perception that a firm is in trouble can lead to further trouble. By allowing us to compare the value of a business to its outstanding claims in all possible scenarios (rather than just the most likely one), simulations allow us to not only quantify the likelihood of distress but also build in the cost of indirect bankruptcy costs into valuation. In effect, we can explicitly model the effect of distress on expected cash flows and discount rates.

### **Issues in using Simulations**

The use of simulations in investment analysis was first suggested in an article by David Hertz in the Harvard Business Review.<sup>5</sup> He argued that using probability distributions for input variables, rather than single best estimates, would yield more informative output. In the example that he provided in the paper, he used simulations to compare the distributions of returns of two investments; the investment with the higher expected return also had a higher chance of losing money (which was viewed as an indicator of its riskiness). In the aftermath, there were several analysts who jumped on the simulation bandwagon, with mixed results. In recent years, there has been a resurgence in interest in simulations as a tool for risk assessment, especially in the context of derivatives. There are several key issues, though, that we have to deal with in the context of using simulations in risk assessment:

a. Garbage in, garbage out: For simulations to have value, the distributions chosen for the inputs should be based upon analysis and data, rather than guesswork. It is worth noting that simulations yield great-looking output, even when the inputs are random. Unsuspecting decision makers may therefore be getting meaningless pictures of the risk in an investment. It is also worth noting that simulations require more than a passing knowledge of statistical distributions and their characteristics; analysts who cannot assess

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<sup>5</sup> Hertz, D., 1964, Risk Analysis in Capital Investment, Harvard Business Review.

the difference between normal and lognormal distributions should not be doing simulations.

b. Real data may not fit distributions: The problem with the real world is that the data seldom fits the stringent requirements of statistical distributions. Using probability distributions that bear little resemblance to the true distribution underlying an input variable will yield misleading results.

c. Non-stationary distributions: Even when the data fits a statistical distribution or where historical data distributions are available, shifts in the market structure can lead to shifts in the distribution as well. In some cases, this can change the form of the distribution and in others, it can change the parameters of the distribution. Thus, the mean and variance estimated from historical data for an input that is normally distributed may change for the next period. What we would really like to use in simulations, but seldom can assess, are forward looking probability distributions.

d. Changing correlation across inputs: Earlier in this chapter, we noted that correlation across input variables can be modeled into simulations. However, this works only if the correlations remain stable and predictable. To the extent that correlations between input variables change over time, it becomes far more difficult to model them.

### **Risk Adjusted Value and Simulations**

In our discussion of decision trees, we referred to the common misconception that decision trees are risk adjusted because they consider the likelihood of adverse events. The same misconception is prevalent in simulations, where the argument is that the cash flows from simulations are somehow risk adjusted because of the use of probability distributions and that the riskfree rate should be used in discounting these cash flows. With one exception, this argument does not make sense. Looking across simulations, the cash flows that we obtain are expected cash flows and are not risk adjusted. Consequently, we should be discounting these cash flows at a risk-adjusted rate.

The exception occurs when you use the standard deviation in values from a simulation as a measure of investment or asset risk and make decisions based upon that. In this case, using a risk-adjusted discount rate will result in a double counting of risk. Consider a simple example. Assume that you are trying to choose between two

investments, both of which you have valued using simulations and risk adjusted discount rates. Table 3.4 summarizes your findings:

*Table 3.4: Simulation Results on two investments*

Asset	Risk-adjusted Discount Rate	Simulation Expected Value	Simulation Std deviation
A	12%	\$ 100	15%
B	15%	\$ 100	21%

Note that you view asset B to be riskier and have used a higher discount rate to compute value. If you now proceed to reject asset B, because the standard deviation is higher across the simulated values, you would be penalizing it twice. You can redo the simulations using the riskfree rate as the discount rate for both assets, but a note of caution needs to be introduced. If we then base our choice between these assets on the standard deviation in simulated values, we are assuming that all risk matters in investment choice, rather than only the risk that cannot be diversified away. Put another way, we may end up rejecting an asset because it has a high standard deviation in simulated values, even though adding that asset to a portfolio may result in little additional risk (because much of its risk can be diversified away).

This is not to suggest that simulations are not useful to us in understanding risk. Looking at the variance of the simulated values around the expected value provides a visual reminder that we are estimating value in an uncertain environment. It is also conceivable that we can use it as a decision tool in portfolio management in choosing between two stocks that are equally undervalued but have different value distributions. The stock with the less volatile value distribution may be considered a better investment than another stock with a more volatile value distribution.

### **An Overall Assessment of Probabilistic Risk Assessment Approaches**

Now that we have looked at scenario analysis, decision trees and simulations, we can consider not only when each one is appropriate but also how these approaches complement or replace risk adjusted value approaches.



## Comparing the approaches

Assuming that we decide to use a probabilistic approach to assess risk and could choose between scenario analysis, decision trees and simulations, which one should we pick? The answer will depend upon how you plan to use the output and what types of risk you are facing:

1. Selective versus Full Risk Analysis: In the best-case/worst-case scenario analysis, we look at only three scenarios (the best case, the most likely case and the worst case) and ignore all other scenarios. Even when we consider multiple scenarios, we will not have a complete assessment of all possible outcomes from risky investments or assets. With decision trees and simulations, we attempt to consider all possible outcomes. In decision trees, we try to accomplish this by converting continuous risk into a manageable set of possible outcomes. With simulations, we can use distributions to capture all possible outcomes. Put in terms of probability, the sum of the probabilities of the scenarios we examine in scenario analysis can be less than one, whereas the sum of the probabilities of outcomes in decision trees and simulations has to equal one. As a consequence, we can compute expected values across outcomes in the latter, using the probabilities as weights, and these expected values are comparable to the single estimate risk adjusted values that we talked about in the last chapter.

2. Discrete versus Continuous Risk: As noted above, scenario analysis and decision trees are generally built around discrete outcomes in risky events whereas simulations are better suited for continuous risks. Focusing on just scenario analysis and decision trees, the latter are better suited for sequential risks, since risk is considered in phases, whereas the former is easier to use when risks occur concurrently.

3. Correlation across risks: If the various risks that an investment is exposed to are correlated, simulations allow for explicitly modeling these correlations (assuming that you can estimate and forecast them). In scenario analysis, we can deal with correlations subjectively by creating scenarios that allow for them; the high (low) interest rate scenario will also include slower (higher) economic growth. Correlated risks are difficult to model in decision trees.

Table 3.5 summarizes the relationship between risk type and the probabilistic approach used:

Table 3.5: Risk Type and Probabilistic Approaches

<i>Discrete/Continuous</i>	<i>Correlated/Independent</i>	<i>Sequential/Concurrent</i>	<i>Risk Approach</i>
Discrete	Independent	Sequential	Decision Tree
Discrete	Correlated	Concurrent	Scenario Analysis
Continuous	Either	Either	Simulations

Finally, the quality of the information will be a factor in your choice of approach. Since simulations are heavily dependent upon being able to assess probability distributions and parameters, they work best in cases where there is substantial historical and cross sectional data available that can be used to make these assessments. With decision trees, you need estimates of the probabilities of the outcomes at each chance node, making them best suited for risks where these risks can be assessed either using past data or population characteristics. Thus, it should come as no surprise that when confronted with new and unpredictable risks, analysts continue to fall back on scenario analysis, notwithstanding its slapdash and subjective ways of dealing with risk.

### **Complement or Replacement for Risk Adjusted Value**

As we noted in our discussion of both decision trees and simulations, these approaches can be used as either complements to or substitutes for risk-adjusted value. Scenario analysis, on the other hand, will always be a complement to risk adjusted value, since they do not look at the full spectrum of possible outcomes.

When any of these approaches are used as complements to risk adjusted value, the caveats that we offered earlier in the chapter continue to apply and bear repeating. All of these approaches use expected rather than risk adjusted cash flows and the discount rate that is used should be a risk-adjusted discount rate; the riskfree rate cannot be used to discount expected cash flows. In all three approaches, though, we still preserve the flexibility to change the risk adjusted discount rate for different outcomes. Since all of these approaches will also provide a range for estimated value and a measure of variability (in terms of value at the end nodes in a decision tree or as a standard deviation in value in a simulation), it is important that we do not double count for risk. In other words, it is patently unfair to risky investments to discount their cash flows back at a risk-

adjusted rate (in simulations and decision trees) and to then reject them because the variability in value is high.

Both simulations and decision trees can be used as alternatives to risk adjusted valuation, but there are constraints on the process. The first is that the cash flows will be discounted back at a riskfree rate to arrive at value. The second is that we now use the measure of variability in values that we obtain in both these approaches as a measure of risk in the investment. Comparing two assets with the same expected value (obtained with riskless rates as discount rates) from a simulation, we will pick the one with the lower variability in simulated values as the better investment. If we do this, we are assuming that all of the risks that we have built into the simulation are relevant for the investment decision. In effect, we are ignoring the line drawn between risks that could have been diversified away in a portfolio and asset-specific risk on which much of modern finance is built. For an investor considering investing all of his or her wealth in one asset, this should be reasonable. For a portfolio manager comparing two risky stocks that he or she is considering adding to a diversified portfolio, it can yield misleading results; the rejected stock with the higher variance in simulated values may be uncorrelated with the other investments in the portfolio and thus have little marginal risk.

## Conclusion

Estimating the risk adjusted value for a risky asset or investment may seem like an exercise in futility. After all, the value is a function of the assumptions that we make about how the risk will unfold in the future. With probabilistic approaches to risk assessment, we estimate not only an expected value but also get a sense of the range of possible outcomes for value, across good and bad scenarios.

- In the most extreme form of scenario analysis, you look at the value in the best case and worst case scenarios and contrast them with the expected value. In its more general form, you estimate the value under a small number of likely scenarios, ranging from optimistic to pessimistic.
- Decision trees are designed for sequential and discrete risks, where the risk in an investment is considered into phases and the risk in each phase is captured in the possible outcomes and the probabilities that they will occur. A decision tree

provides a complete assessment of risk and can be used to determine the optimal courses of action at each phase and an expected value for an asset today.

- Simulations provide the most complete assessments of risk since they are based upon probability distributions for each input (rather than just discrete outcomes). The output from a simulation takes the form of an expected value across simulations and a distribution for the simulated values.

With all three approaches, the keys are to avoid double counting risk (by using a risk-adjusted discount rate and considering the variability in estimated value as a risk measure) or making decisions based upon the wrong types of risk.