

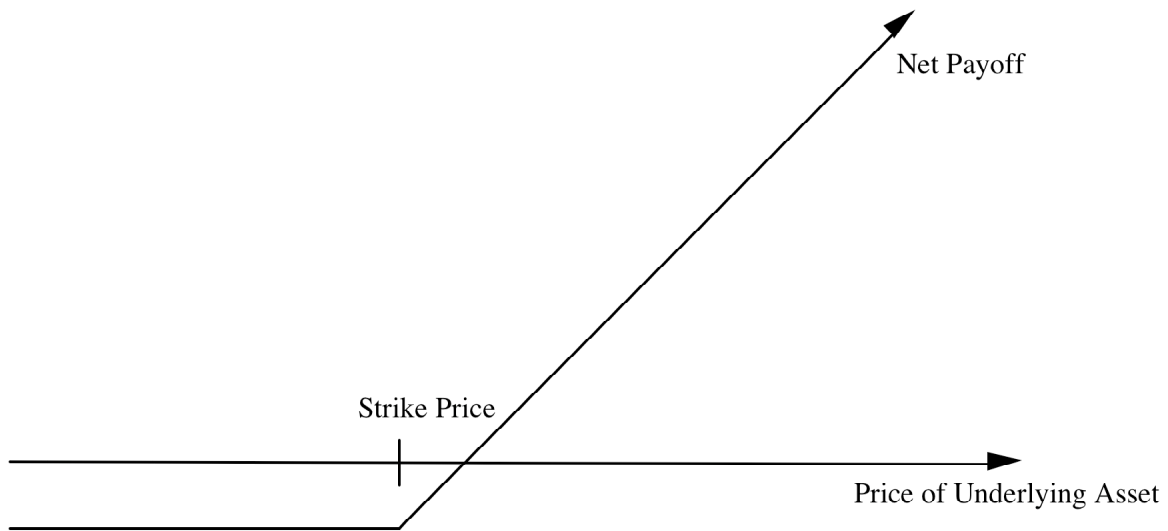
Appendix: Basics of Options and Option Pricing

An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to expire. There are two types of options - call options and put options.

Option Payoffs

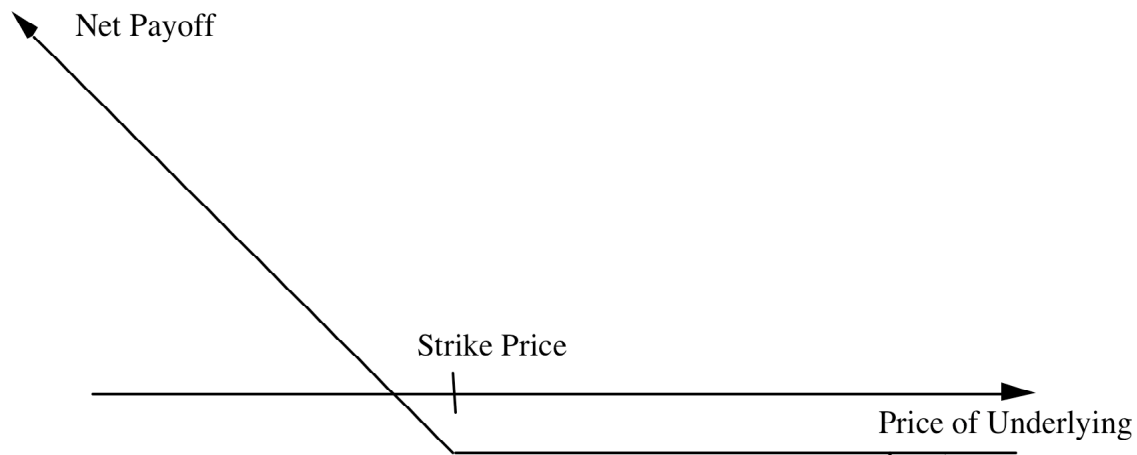
A call option gives the buyer of the option the right to buy the underlying asset at a fixed price, called the strike or the exercise price, at any time prior to the expiration date of the option: the buyer pays a price for this right. If at expiration, the value of the asset is less than the strike price, the option is not exercised and expires worthless. If, on the other hand, the value of the asset is greater than the strike price, the option is exercised - the buyer of the option buys the stock at the exercise price and the difference between the asset value and the exercise price comprises the gross profit on the investment. The net profit on the investment is the difference between the gross profit and the price paid for the call initially. A payoff diagram illustrates the cash payoff on an option at expiration. For a call, the net payoff is negative (and equal to the price paid for the call) if the value of the underlying asset is less than the strike price. If the price of the underlying asset exceeds the strike price, the gross payoff is the difference between the value of the underlying asset and the strike price, and the net payoff is the difference between the gross payoff and the price of the call. This is illustrated in the figure 5A.1:

Figure 5A.1: Payoff on Call Option



A put option gives the buyer of the option the right to sell the underlying asset at a fixed price, again called the strike or exercise price, at any time prior to the expiration date of the option. The buyer pays a price for this right. If the price of the underlying asset is greater than the strike price, the option will not be exercised and will expire worthless. If on the other hand, the price of the underlying asset is less than the strike price, the owner of the put option will exercise the option and sell the stock at the strike price, claiming the difference between the strike price and the market value of the asset as the gross profit. Again, netting out the initial cost paid for the put yields the net profit from the transaction. A put has a negative net payoff if the value of the underlying asset exceeds the strike price, and has a gross payoff equal to the difference between the strike price and the value of the underlying asset if the asset value is less than the strike price. This is summarized in figure 5A.2.

Figure 5A.2: Payoff on Put Option



There is one final distinction that needs to be made. Options are usually categorized as American or European options. A primary distinction between two is that American options can be exercised at any time prior to its expiration, while European options can be exercised only at expiration. The possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. There is one compensating factor that enables the former to be valued using models designed for the latter. In most cases, the time premium associated with the remaining life of an option and transactions costs makes early exercise sub-optimal. In other words, the holders of in-the-money options will generally get much more by selling the option to someone else than by exercising the options.¹

Determinants of Option Value

The value of an option is determined by a number of variables relating to the underlying asset and financial markets.

¹ While early exercise is not optimal generally, there are at least two exceptions to this rule. One is a case where the underlying asset pays large dividends, thus reducing the value of the asset, and any call options on that asset. In this case, call options may be exercised just before an ex-dividend date, if the time premium on the options is less than the expected decline in asset value as a consequence of the dividend payment. The other exception arises when an investor holds both the underlying asset and deep in-the-money puts on that asset at a time when interest rates are high. In this case, the time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

1. Current Value of the Underlying Asset : Options are assets that derive value from an underlying asset. Consequently, changes in the value of the underlying asset affect the value of the options on that asset. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increase.

2. Variance in Value of the Underlying Asset: The buyer of an option acquires the right to buy or sell the underlying asset at a fixed price. The higher the variance in the value of the underlying asset, the greater the value of the option. This is true for both calls and puts. While it may seem counter-intuitive that an increase in a risk measure (variance) should increase value, options are different from other securities since buyers of options can never lose more than the price they pay for them; in fact, they have the potential to earn significant returns from large price movements.

3. Dividends Paid on the Underlying Asset: The value of the underlying asset can be expected to decrease if dividend payments are made on the asset during the life of the option. Consequently, the value of a call on the asset is a decreasing function of the size of expected dividend payments, and the value of a put is an increasing function of expected dividend payments. A more intuitive way of thinking about dividend payments, for call options, is as a cost of delaying exercise on in-the-money options. To see why, consider a option on a traded stock. Once a call option is in the money, i.e, the holder of the option will make a gross payoff by exercising the option, exercising the call option will provide the holder with the stock, and entitle him or her to the dividends on the stock in subsequent periods. Failing to exercise the option will mean that these dividends are foregone.

4. Strike Price of Option: A key characteristic used to describe an option is the strike price. In the case of calls, where the holder acquires the right to buy at a fixed price, the value of the call will decline as the strike price increases. In the case of puts, where the holder has the right to sell at a fixed price, the value will increase as the strike price increases.

5. Time To Expiration On Option: Both calls and puts become more valuable as the time to expiration increases. This is because the longer time to expiration provides more time for the value of the underlying asset to move, increasing the value of both types of

options. Additionally, in the case of a call, where the buyer has to pay a fixed price at expiration, the present value of this fixed price decreases as the life of the option increases, increasing the value of the call.

6. Riskless Interest Rate Corresponding To Life Of Option: Since the buyer of an option pays the price of the option up front, an opportunity cost is involved. This cost will depend upon the level of interest rates and the time to expiration on the option. The riskless interest rate also enters into the valuation of options when the present value of the exercise price is calculated, since the exercise price does not have to be paid (received) until expiration on calls (puts). Increases in the interest rate will increase the value of calls and reduce the value of puts.

Table 5A.1 below summarizes the variables and their predicted effects on call and put prices.

Table 5A.1: Summary of Variables Affecting Call and Put Prices

<i>Factor</i>	<i>Effect on</i>	
	<i>Call Value</i>	<i>Put Value</i>
Increase in underlying asset's value	Increases	Decreases
Increase in Strike Price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interest rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

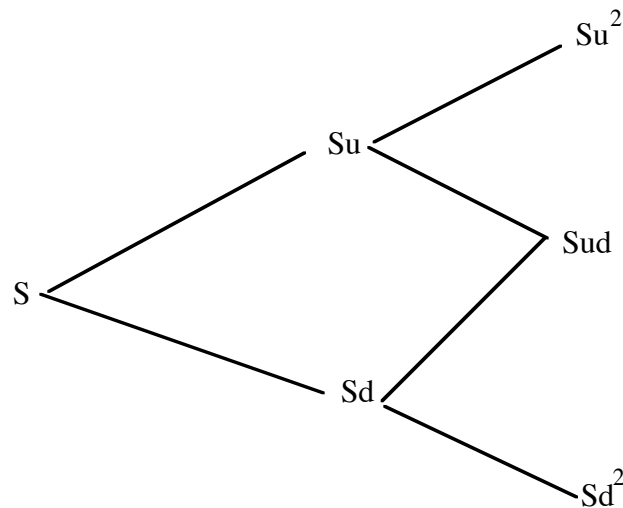
Option Pricing Models

Option pricing theory has made vast strides since 1972, when Black and Scholes published their path-breaking paper providing a model for valuing dividend-protected European options. Black and Scholes used a “replicating portfolio” — a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued— to come up with their final formulation. While their derivation is mathematically complicated, there is a simpler binomial model for valuing options that draws on the same logic.

The Binomial Model

The binomial option pricing model is based upon a simple formulation for the asset price process, in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial is shown in figure 5A.3.

Figure 5A.3: General Formulation for Binomial Price Path



In this figure, S is the current stock price; the price moves up to S_u with probability p and down to S_d with probability $1-p$ in any time period.

The objective in creating a replicating portfolio is to use a combination of risk-free borrowing/lending and the underlying asset to create the same cash flows as the option being valued. The principles of arbitrage apply here, and the value of the option must be equal to the value of the replicating portfolio. In the case of the general formulation above, where stock prices can either move up to S_u or down to S_d in any time period, the replicating portfolio for a call with strike price K will involve borrowing $\$B$ and acquiring Δ of the underlying asset, where:

$$\Delta = \text{Number of units of the underlying asset bought} = (C_u - C_d)/(S_u - S_d)$$

where,

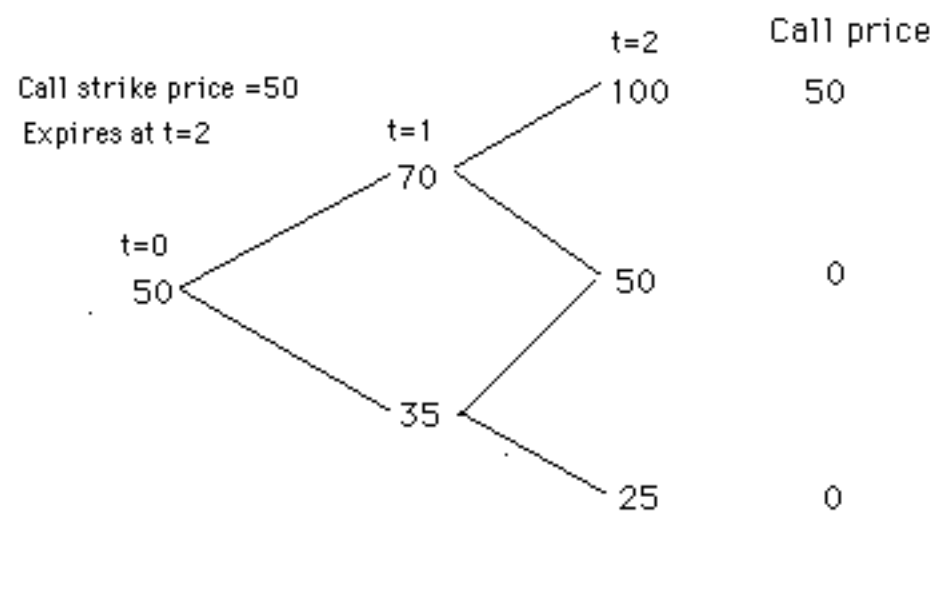
C_u = Value of the call if the stock price is S_u

C_d = Value of the call if the stock price is S_d

In a multi-period binomial process, the valuation has to proceed iteratively; i.e., starting with the last time period and moving backwards in time until the current point in time. The portfolios replicating the option are created at each step and valued, providing the values for the option in that time period. The final output from the binomial option pricing model is a statement of the value of the option in terms of the replicating portfolio, composed of Δ shares (option delta) of the underlying asset and risk-free borrowing/lending.

Value of the call = Current value of underlying asset * Option Delta - Borrowing needed to replicate the option

Consider a simple example. Assume that the objective is to value a call with a strike price of 50, which is expected to expire in two time periods, on an underlying asset whose price currently is 50 and is expected to follow a binomial process:



Now assume that the interest rate is 11%. In addition, define

Δ = Number of shares in the replicating portfolio

B = Dollars of borrowing in replicating portfolio

The objective is to combine Δ shares of stock and B dollars of borrowing to replicate the cash flows from the call with a strike price of \$ 50. This can be done iteratively, starting with the last period and working back through the binomial tree.

Step 1: Start with the end nodes and work backwards:

	t=2	Call Value	Replicating portfolio
t=1 70	100	50	$(100 \times D) - (1.11 \times B) = 50$
	50	0	$(50 \times D) - (1.11 \times B) = 0$
			Solving for D and B
			D = 1 ; B = 45
			Buy 1 share; Borrow \$ 45

Thus, if the stock price is \$70 at t=1, borrowing \$45 and buying one share of the stock will give the same cash flows as buying the call. The value of the call at t=1, if the stock price is \$70, is therefore:

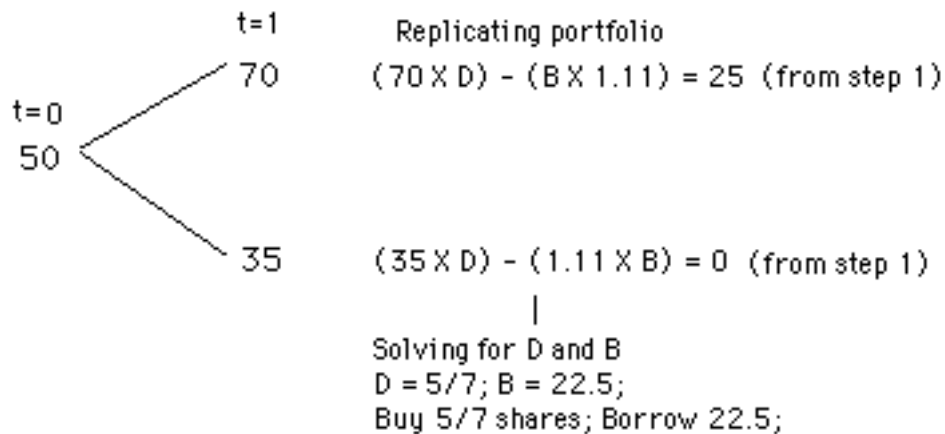
$$\text{Value of Call} = \text{Value of Replicating Position} = 70 \Delta - B = 70 - 45 = 25$$

Considering the other leg of the binomial tree at t=1,

	t=2	Call Value	Replicating portfolio
t=1 35	50	0	$(50 \times D) - (1.11 \times B) = 0$
	25	0	$(25 \times D) - (1.11 \times B) = 0$
			Solving for D and B
			D = 0 ; B = 0

If the stock price is 35 at t=1, then the call is worth nothing.

Step 2: Move backwards to the earlier time period and create a replicating portfolio that will provide the cash flows the option will provide.



In other words, borrowing \$22.5 and buying $5/7$ of a share will provide the same cash flows as a call with a strike price of \$50. The value of the call therefore has to be the same as the value of this position.

$$\text{Value of Call} = \text{Value of replicating position} = 5/7 \times \text{Current stock price} - \$22.5 = \$13.20$$

The binomial model provides insight into the determinants of option value. The value of an option is not determined by the expected price of the asset but by its current price, which, of course, reflects expectations about the future. This is a direct consequence of arbitrage. If the option value deviates from the value of the replicating portfolio, investors can create an arbitrage position, i.e., one that requires no investment, involves no risk, and delivers positive returns. To illustrate, if the portfolio that replicates the call costs more than the call does in the market, an investor could buy the call, sell the replicating portfolio and be guaranteed the difference as a profit. The cash flows on the two positions will offset each other, leading to no cash flows in subsequent periods. The option value also increases as the time to expiration is extended, as the price movements (u and d) increase, and with increases in the interest rate.

The Black-Scholes Model

The binomial model is a discrete-time model for asset price movements, including a time interval (t) between price movements. As the time interval is shortened, the limiting distribution, as t approaches 0, can take one of two forms. If as t approaches 0, price changes become smaller, the limiting distribution is the normal distribution and the

price process is a continuous one. If as t approaches 0, price changes remain large, the limiting distribution is the Poisson distribution, i.e., a distribution that allows for price jumps. The Black-Scholes model applies when the limiting distribution is the normal distribution,² and it explicitly assumes that the price process is continuous.

The Model

The original Black and Scholes model was designed to value European options, which were dividend-protected. Thus, neither the possibility of early exercise nor the payment of dividends affects the value of options in this model. The value of a call option in the Black-Scholes model can be written as a function of the following variables:

S = Current value of the underlying asset

K = Strike price of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the $\ln(\text{value})$ of the underlying asset

The model itself can be written as:

$$\text{Value of call} = S N(d_1) - K e^{-rt} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

The process of valuation of options using the Black-Scholes model involves the following steps:

Step 1: The inputs to the Black-Scholes are used to estimate d_1 and d_2 .

Step 2: The cumulative normal distribution functions, $N(d_1)$ and $N(d_2)$, corresponding to these standardized normal variables are estimated.

² Stock prices cannot drop below zero, because of the limited liability of stockholders in publicly listed firms. Hence, stock prices, by themselves, cannot be normally distributed, since a normal distribution requires some probability of infinitely negative values. The distribution of the natural logs of stock prices is assumed to be log-normal in the Black-Scholes model. This is why the variance used in this model is the variance in the log of stock prices.

Step 3: The present value of the exercise price is estimated, using the continuous time version of the present value formulation:

$$\text{Present value of exercise price} = K e^{-rt}$$

Step 4: The value of the call is estimated from the Black-Scholes model.

The determinants of value in the Black-Scholes are the same as those in the binomial - the current value of the stock price, the variability in stock prices, the time to expiration on the option, the strike price, and the riskless interest rate. The principle of replicating portfolios that is used in binomial valuation also underlies the Black-Scholes model. In fact, embedded in the Black-Scholes model is the replicating portfolio.

$$\text{Value of call} = \underbrace{S N(d_1)}_{\text{Buy } N(d_1) \text{ shares}} - \underbrace{K e^{-rt} N(d_2)}_{\text{Borrow this amount}}$$

$N(d_1)$, which is the number of shares that are needed to create the replicating portfolio is called the option delta. This replicating portfolio is self-financing and has the same value as the call at every stage of the option's life.

Model Limitations and Fixes

The version of the Black-Scholes model presented above does not take into account the possibility of early exercise or the payment of dividends, both of which impact the value of options. Adjustments exist, which while not perfect, provide partial corrections to value.

1. Dividends

The payment of dividends reduces the stock price. Consequently, call options will become less valuable and put options more valuable as dividend payments increase. One approach to dealing with dividends to estimate the present value of expected dividends paid by the underlying asset during the option life and subtract it from the current value of the asset to use as “S” in the model. Since this becomes impractical as the option life becomes longer, we would suggest an alternate approach. If the dividend yield ($y = \text{dividends} / \text{current value of the asset}$) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S e^{-yt} N(d_1) - K e^{-rt} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r - y + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$
$$d_2 = d_1 - \sigma\sqrt{t}$$

From an intuitive standpoint, the adjustments have two effects. First, the value of the asset is discounted back to the present at the dividend yield to take into account the expected drop in value from dividend payments. Second, the interest rate is offset by the dividend yield to reflect the lower carrying cost from holding the stock (in the replicating portfolio). The net effect will be a reduction in the value of calls, with the adjustment, and an increase in the value of puts.

2. Early Exercise

The Black-Scholes model is designed to value European options, whereas most options that we consider are American options, which can be exercised anytime before expiration. Without working through the mechanics of valuation models, an American option should always be worth at least as much and generally more than a European option because of the early exercise option. There are three basic approaches for dealing with the possibility of early exercise. The first is to continue to use the unadjusted Black-Scholes, and regard the resulting value as a floor or conservative estimate of the true value. The second approach is to value the option to each potential exercise date. With options on stocks, this basically requires that we value options to each ex-dividend day and chooses the maximum of the estimated call values. The third approach is to use a modified version of the binomial model to consider the possibility of early exercise.

While it is difficult to estimate the prices for each node of a binomial, there is a way in which variances estimated from historical data can be used to compute the expected up and down movements in the binomial. To illustrate, if σ^2 is the variance in $\ln(\text{stock prices})$, the up and down movements in the binomial can be estimated as follows:

$$u = \text{Exp} [(r - \sigma^2/2)(T/m) + \sqrt{(\sigma^2 T/m)}]$$

$$d = \text{Exp} [(r - \sigma^2/2)(T/m) - \sqrt{(\sigma^2 T/m)}]$$

where u and d are the up and down movements per unit time for the binomial, T is the life of the option and m is the number of periods within that lifetime. Multiplying the stock price at each stage by u and d will yield the up and the down prices. These can then be used to value the asset.

3. *The Impact Of Exercise On The Value Of The Underlying Asset*

The derivation of the Black-Scholes model is based upon the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for some types of options. For instance, the exercise of warrants increases the number of shares outstanding and brings fresh cash into the firm, both of which will affect the stock price.³ The expected negative impact (dilution) of exercise will decrease the value of warrants compared to otherwise similar call options. The adjustment for dilution in the Black-Scholes to the stock price is fairly simple. The stock price is adjusted for the expected dilution from the exercise of the options. In the case of warrants, for instance:

$$\text{Dilution-adjusted } S = (S n_s + W n_w) / (n_s + n_w)$$

where

S = Current value of the stock
 n_w = Number of warrants outstanding

W = Market value of warrants outstanding n_s = Number of shares outstanding

When the warrants are exercised, the number of shares outstanding will increase, reducing the stock price. The numerator reflects the market value of equity, including both stocks and warrants outstanding. The reduction in S will reduce the value of the call option.

There is an element of circularity in this analysis, since the value of the warrant is needed to estimate the dilution-adjusted S and the dilution-adjusted S is needed to estimate the value of the warrant. This problem can be resolved by starting the process off with an estimated value of the warrant (say, the exercise value), and then iterating with the new estimated value for the warrant until there is convergence.

³ Warrants are call options issued by firms, either as part of management compensation contracts or to raise equity.

Valuing Puts

The value of a put can be derived from the value of a call with the same strike price and the same expiration date through an arbitrage relationship that specifies that:

$$C - P = S - K e^{-rt}$$

where C is the value of the call and P is the value of the put (with the same life and exercise price).

This arbitrage relationship can be derived fairly easily and is called put-call parity. To see why put-call parity holds, consider creating the following portfolio:

- (a) Sell a call and buy a put with exercise price K and the same expiration date "t"
- (b) Buy the stock at current stock price S

The payoff from this position is riskless and always yields K at expiration (t). To see this, assume that the stock price at expiration is S*:

<i>Position</i>	<i>Payoffs at t if $S^* > K$</i>	<i>Payoffs at t if $S^* < K$</i>
Sell call	-(S*-K)	0
Buy put	0	K-S*
Buy stock	S*	S*
<i>Total</i>	<i>K</i>	<i>K</i>

Since this position yields K with certainty, its value must be equal to the present value of K at the riskless rate ($K e^{-rt}$).

$$S + P - C = K e^{-rt}$$

$$C - P = S - K e^{-rt}$$

This relationship can be used to value puts. Substituting the Black-Scholes formulation for the value of an equivalent call,

$$\text{Value of put} = S e^{-yt} (N(d_1) - 1) - K e^{-rt} (N(d_2) - 1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$