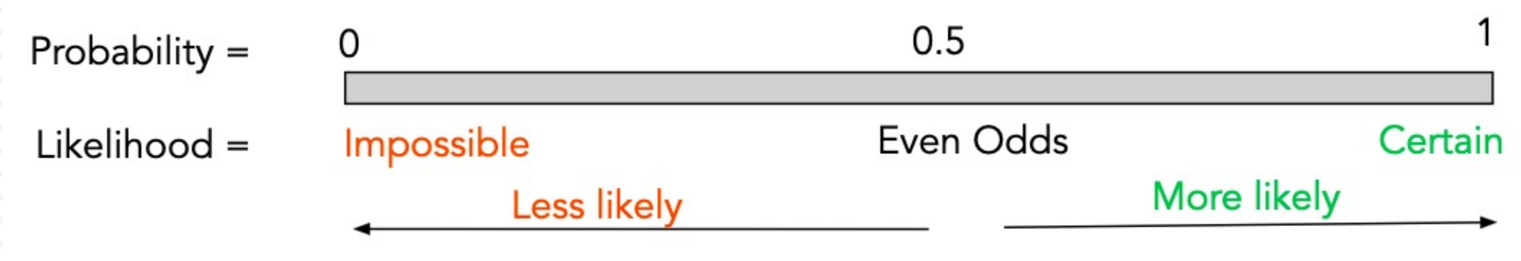
SESSION 6: PROBABILITIES & PROBABILISTIC TOOLS

Session 6

There is a chance!

What is a probability?

- When you cannot predict outcomes with certainty, probabilities allow you to measure the likelihood of an outcome occurring.
- The probability of an outcome will range from zero to one, with zero representing no chance and one representing total certainty.



- You can measure the probabilities of discrete events (an acquisition, bankruptcy etc.) as well as the probabilities of a continuous variable taking a value greater or less than a specified value (earnings will exceed zero, market cap will exceed a billion).

Two views of probability

- Frequentist view: In the frequentist view, probability measures the frequency with which an event will occur, if it is repeated many times.
  - In theory, you want to measure the frequency of an outcome if an experiment is repeated an infinite number of times.
  - While it is objective, it requires that an event be repeatable, and is thus narrow.
- Subjectivist (Bayesian) view: In the subjectivist view, probability is the value that a rational agent would assign to the likelihood of an event happening.
  - You can assign probabilities to events, even if they are not repeatable.
  - Unlike the frequentist view, rational individuals can disagree on the probability of an event occurring.

Cumulative and Conditional Probabilities

- When you have sequential events, the *cumulative probability* measures the likelihood of a specific series of outcomes. To compute cumulative probabilities, each event in the sequence has to be independent of the others in that sequence.
  - Thus, you can estimate the probability that an investor will beat the market ten years in a row, even if he or she is picking stocks randomly.
- A conditional probability measures the *likelihood of an event or outcome*, based upon the occurrence of a *prior event or outcome*.
  - For instance, you can estimate the probability that the market will be up tomorrow, given that it was up today.

Probability Rules

1. *It is bounded*: For any event X, $0 \le P(X) \le 1$
2. *It covers the universe of outcomes*: The sum of the probabilities of all possible outcomes is one.
3. *The Complement Rule* states that P(not X) = 1 – P(X)
4. *The General Addition Rule* states that for any two events, P(X or Y) = P(X) + P(Y) – P(X and Y), where P(X and Y) is the probability of both X and Y occurring at the same time. If they are mutually exclusive, the rule simplifies to P(X or Y) = P(X) + P(Y).
5. *The Multiplication Rule* states that for two independent events to occur together, P(X and Y) = P(X) * P(Y). If they are not independent, P(X and Y) = P(X) * P(Y/X), where P(Y/X) is the conditional probability of Y happening, given that X has happened.

Bayesian Statistics

- **Bayes' theorem** describes the probability of an event occurring, based upon prior knowledge of other variables related to that event.
  - In effect, it is a conditional probability, with the probability of an event conditioned on the information/knowledge that you have.
  - Since the information/knowledge that different individuals can have about an event can vary, Bayes' theorem allows for differences in probability estimates for the same event across individuals.
- In **Bayesian Inference**, you update the probability of an event happening as you receive new evidence or information.
  - The probability that you assign to an event before you receive the new information represent your priors.
  - The probability that you assign to that same event after receiving and processing new information represent your *posterior* estimate.

1. Probit & Logit Models: Techniques for estimating probabilities

- A *probit model* is a variant on the standard regression approach, with the key difference being that the dependent variable can take on only one of two values (binary) and a secondary one being a transformation of the linear model to ensure that the predicted value from the model is a number between 0 and 1 (limits for a probability estimate).
  - Example: You can estimate the probability of a hostile acquisition of a company based upon its size, past stock price performance and insider holdings.
- A *logit model* is a close relative to a probit, with the difference revolving around the function that they use to transform the standard linear model, with the logit model using a logistic distribution and the probit a normal distribution.

A Logit Model: Predicting Bankruptcy

- In investing and valuation, one of the concerns that you face is that the business that you are investing in or valuing may go out of business. Predicting the probability of bankruptcy becomes a part of the challenge.
- There are logit models that have been used to predict bankruptcy, where
  - You start with all firms at the start of the period
  - The dependent variable becomes the stand-in for whether a firm survives or goes bankrupt during the study period
  - The independent variables reflect what you believe are key drivers of bankruptcy (earnings level and volatility, debt level and payments due, market access to capital).
  - You build a logit model that will yield as output an equation that resembles a regression, but will yield a probability of bankruptcy.

A Probit Model: Hostile Acquisitions

- While there are no easy pathways to making money, it seems clear that investors in companies that are targeted in acquisitions (especially hostile ones) earn high returns, but only if they invest before the event.
- There are probit models for predicting companies that will be targeted, and they involve:
  - You start with all firms that publicly traded at the start of a period
  - The dependent variable becomes the stand-in for whether a firm is targeted in a hostile acquisition
  - The independent variables reflect what you believe are key drivers of hostile acquisitions, including poor stock price performance, lagging accounting returns and managers with little or no shareholdings.
  - You build a probit model that will yield as output an equation that resembles a regression, but will yield a probability of a hostile acquisition.

2. Decision Trees

- A decision tree is a flow chart used to capture the probabilities and outcomes of random events, where the likelihood of an event is affected by prior events occurring or not occurring.
- Within decision trees, you can build not only the outcomes of chance but also the decisions that follow from these outcomes and the end outcomes for each branch.
  - Event or Change nodes represent when uncertainty is resolved, with event branches representing mutually exclusive and exhaustive, with probability estimates.
  - Decision nodes point where you decide to go on or stop. The branches represent mutually exclusive paths to follow.
  - Terminal or End node is the final outcome or payoff from each branch.

And follow through..

- *Folding (Rolling) back a decision tree:* If you have a complete decision tree, with probabilities attached to event branches, and outcomes following from the end of each decision branch, you can roll back the decision tree, by
  - Estimating the expected value of each branch, starting with the right and working towards the left, using the probabilities and payoffs.
  - Compute an expected value for the entire tree, across branches.
- If there is a time lag across the branches, and/or risk involved in the process, you can incorporate those elements into a decision tree as well in the form of a discount rate that you use discount payoffs at each leg.

3. Scenario Analysis

- Scenario analysis is best employed when the outcomes of a project are a function of the macro economic environment and/or competitive responses.
- There are a couple of ways in which you can structure scenario analysis
  - Best-case, Worst-case analyses, where you set all the inputs at their most optimistic and most pessimistic levels. In general, this leaves you with findings that are not of much practical use.
  - Plausible scenarios: Here, you define what you feel are the most plausible scenarios (allowing for the interaction across variables) and come up with estimates for each scenario.
- For scenario analysis to be most useful, you have to be able to
  - List scenarios to cover all possible outcomes
  - Estimate probabilities for the scenarios
  - Complete with an expected value.