

## Session 4A: Post Class Test Solutions

1. A visual check of the normal distribution suggests that it is a good fit, and the same holds true for the Q-Q plot, where the points are close to the line. If you want more confirmation, you can run statistical tests to check to see whether you can reject the normality assumption:

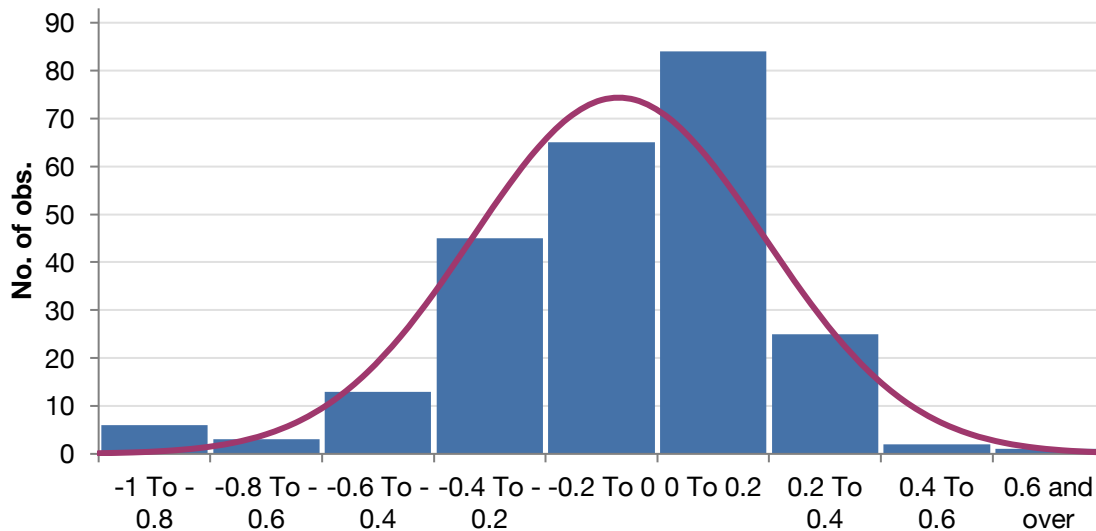
Skewness	0.27289	Fisher's Skewness G1	0.27739
Kurtosis	3.55057		
Kurtosis Excess (-3)	0.55057	Fisher's Kurtosis G2	0.64876
Test	Test Statistic	p-value	H0 (5%)
Shapiro-Wilk W	0.984	0.31516	Cannot reject
Shapiro-Francia	0.98039	0.1535	Cannot reject
Anderson-Darling	0.43722	0.29027	Cannot reject
Cramer-von Mises	0.06375	0.33429	Cannot reject
Kolmogorov-Smirnov (Lilliefors)	0.06166	0.51858	Cannot reject
D'Agostino Skewness	1.13164	0.25779	Cannot reject
D'Agostino Kurtosis	1.29782	0.19435	Cannot reject
D'Agostino Omnibus	2.96495	0.22708	Cannot reject
Jarque-Bera	2.32893	0.31209	Cannot reject

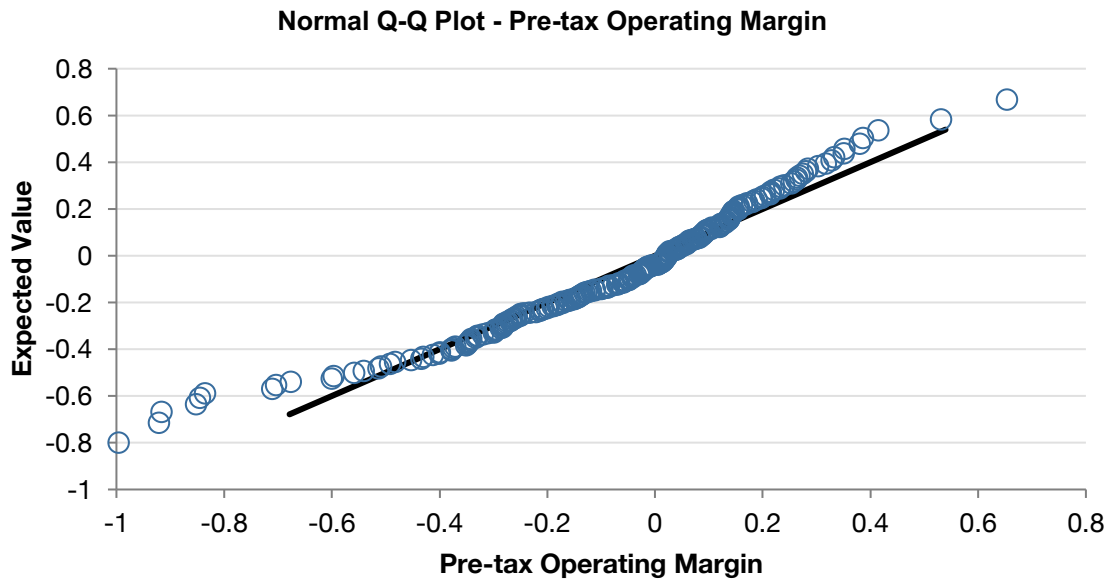
Starting with the simplest tests, the skewness is close to zero and the kurtosis is close to three, conforming to the normal distribution. The other tests range the spectrum, but they all come to the same conclusion, which is that the data is close to normally distributed.

2. Applying the normality tests to software company operating margins, here is what I get:

Skewness	-0.8159	Fisher's Skewness G1	-0.82096
Kurtosis	4.31006		
Kurtosis Excess (-3)	1.31006	Fisher's Kurtosis G2	1.36231
Test	Test Statistic	p-value	H0 (5%)
Shapiro-Wilk W	0.95769	1.39558E-6	Rejected
Shapiro-Francia	0.95645	4.10772E-6	Rejected
Anderson-Darling	2.70102	8.00619E-7	Rejected
Cramer-von Mises	0.45021	8.08769E-6	Rejected
Kolmogorov-Smirnov (Lilliefors)	0.14616	0.00002	Rejected
D'Agostino Skewness	4.7588	1.94747E-6	Rejected
D'Agostino Kurtosis	3.00615	0.00265	Rejected
D'Agostino Omnibus	31.6831	1.31856E-7	Rejected
Jarque-Bera	44.52046	2.15033E-10	Rejected

**Histogram for "Pre-tax Operating Margin"**



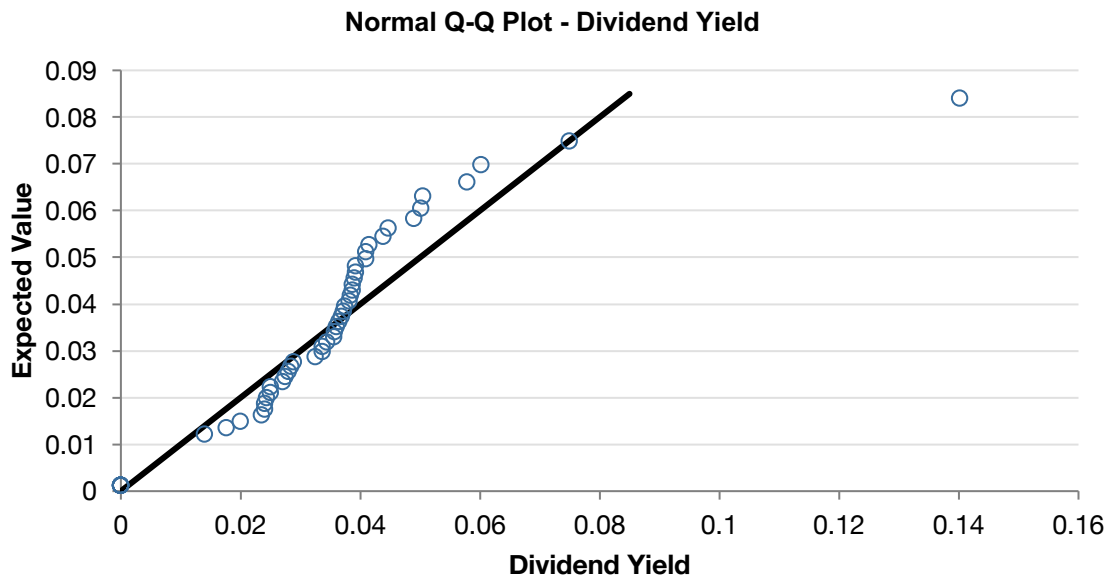
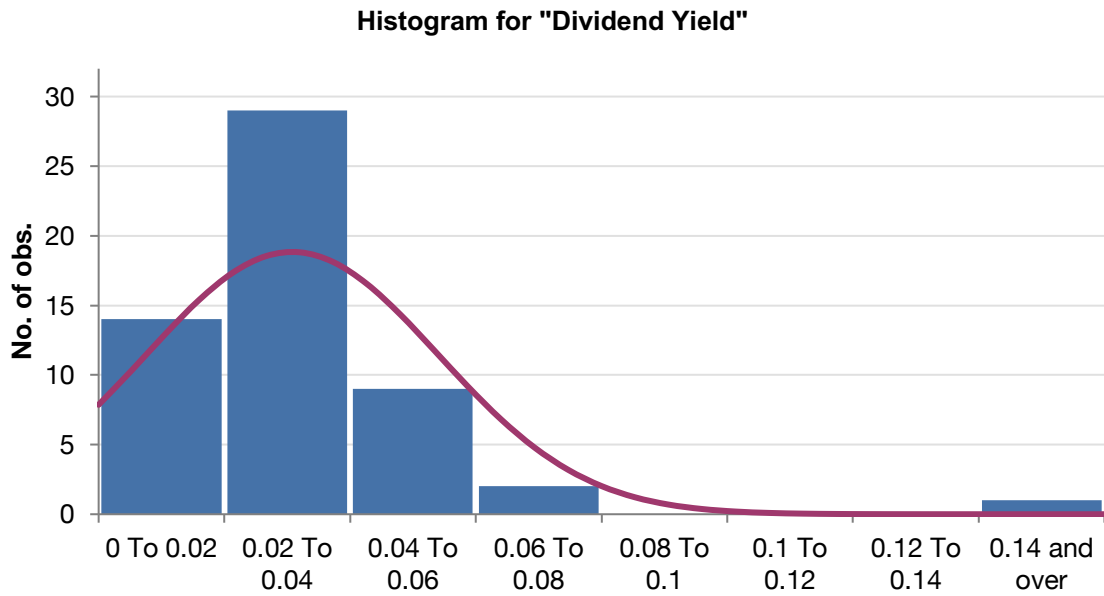


The distribution is too skewed (negatively) and has too many extreme values to fit the normal distribution. I would look for a distribution that allows for both features (negative skewness and fatter tails); the *minimum extreme value distribution*, with a tweaking of the parameters may work.

3. Applying the normality tests to utility company dividend yields, here is what I get:

Skewness	1.69002	Fisher's Skewness G1	1.73778
Kurtosis	10.16989		
Kurtosis Excess (-3)	7.16989	Fisher's Kurtosis G2	7.98467

Test	Test Statistic	p-value	H0 (5%)
Shapiro-Wilk W	0.82099	1.1192E-6	Rejected
Shapiro-Francia	0.80864	3.15251E-6	Rejected
Anderson-Darling	2.06064	0.00003	Rejected
Cramer-von Mises	0.31065	0.00023	Rejected
Kolmogorov-Smirnov (Lilliefors)	0.15918	0.00136	Rejected
D'Agostino Skewness	4.31116	0.00002	Rejected
D'Agostino Kurtosis	4.24321	0.00002	Rejected
D'Agostino Omnibus	36.5909	1.13341E-8	Rejected
Jarque-Bera	143.99008	0	Rejected



The distribution has too much of a positive skew, and much fatter tails than a normal distribution. The lognormal or gamma distributions both can be constructed to have positive skew, but the *log normal distribution* allows for fatter tails.

4. Applying the normality tests to CPI data, here is what I get:

Skewness	0.35153	Fisher's Skewness G1	0.35732
Kurtosis	6.8239		

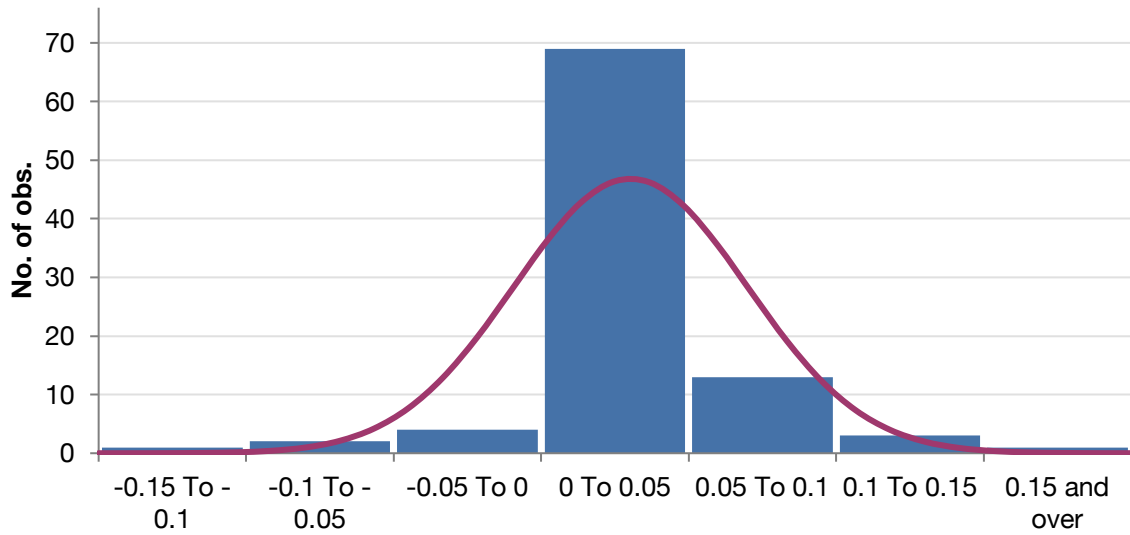
Kurtosis Excess (-3)

3.8239 Fisher's Kurtosis G2

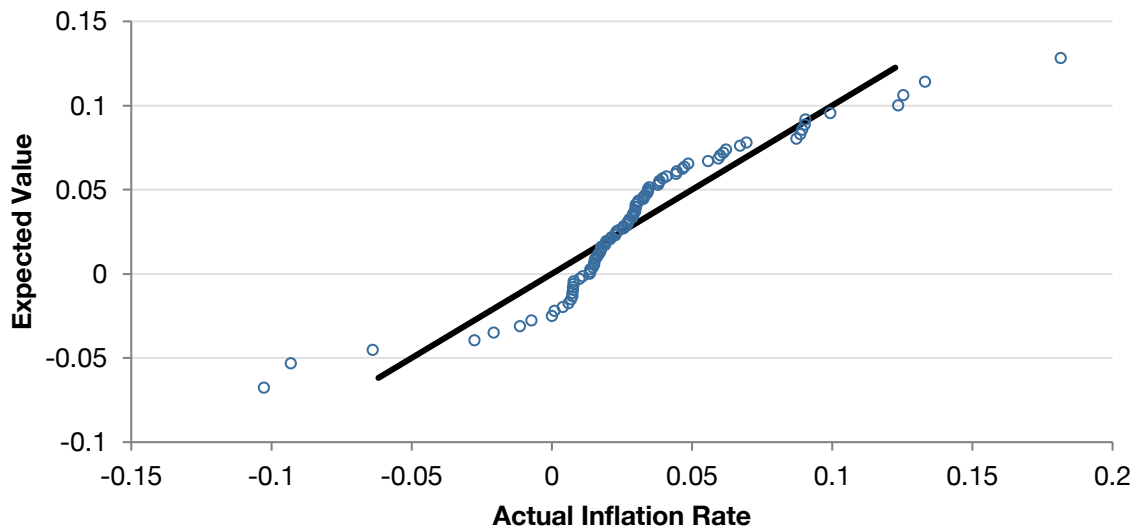
4.10514

Test	Test Statistic	p-value	H0 (5%)
Shapiro-Wilk W	0.87737	3.15312E-7	Rejected
Shapiro-Francia	0.86581	7.72667E-7	Rejected
Anderson-Darling	4.19658	1.64031E-10	Rejected
Cramer-von Mises	0.78171	1.72686E-8	Rejected
Kolmogorov-Smirnov (Lilliefors)	0.16694	9.87804E-7	Rejected
D'Agostino Skewness	1.44462	0.14856	Cannot reject
D'Agostino Kurtosis	3.80121	0.00014	Rejected
D'Agostino Omnibus	16.5361	0.00026	Rejected
Jarque-Bera	58.5765	0.	Rejected

Histogram for "Actual Inflation Rate"



Normal Q-Q Plot - Actual Inflation Rate



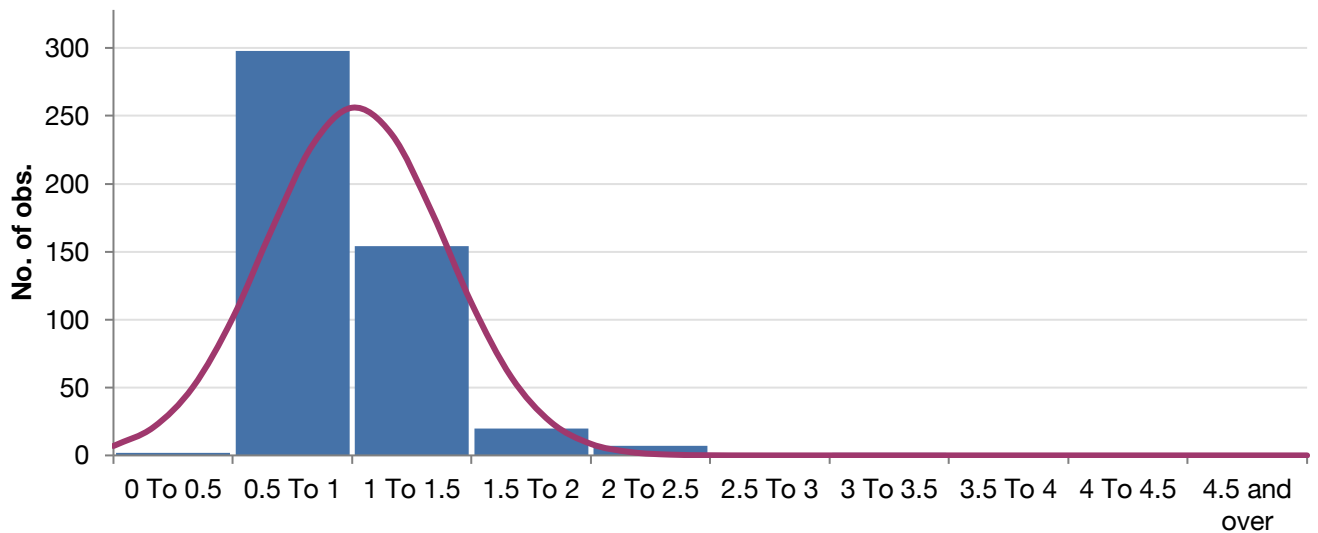
The distribution looks close to symmetric (skewness is close to zero), but it has fatter tails (and a more pronounced peak) than a normal distribution. A Cauchy distribution, which resembles a normal distribution but allows for fatter tails, may be a better fit.

5. Applying the normality tests to bank PBV data, here is what I get:

Skewness	3.94228	Fisher's Skewness G1	3.95452
Kurtosis	31.25424		
Kurtosis Excess (-3)	28.25424	Fisher's Kurtosis G2	28.56017

Test	Test Statistic	p-value	H0 (5%)
Shapiro-Wilk W	0.71032		0. Rejected
Shapiro-Francia	0.70489		0 Rejected
Anderson-Darling	INF		0. Rejected
Cramer-von Mises	5.18549	2.80957E+73	Cannot reject
Kolmogorov-Smirnov (Lilliefors)	0.36853		0. Rejected
D'Agostino Skewness	17.19508		0 Rejected
D'Agostino Kurtosis	12.4563		0 Rejected
D'Agostino Omnibus	450.83024		0 Rejected
Jarque-Bera	17,388.62635		0 Rejected

Histogram for "PBV"



Normal Q-Q Plot - PBV

