CHAPTER 4

RISK MEASUREMENT AND HURDLE RATES

In the last chapter, we presented the argument that the expected return on an equity investment should be a function of the market or non-diversifiable embedded in that investment. In this chapter, we turn our attention to how best to estimate the parameters of market risk in each of the models described in the previous chapter - the capital asset pricing model, the arbitrage pricing model and the mutli-factor model. We will present three alternative approaches for measuring the market risk in an investment; the first is to use historical data on market prices for the firm considering the project, the second is to use the market risk parameters estimated for other firms that are in the same business as the project being analyzed and the third is to use accounting earnings or revenues to estimate the parameters.

In addition to estimating market risk, we will also discuss how best to estimate a riskless rate and a risk premium (in the CAPM) or risk premiums (in the APM and multi-factor models) to convert the risk measures into expected returns. We will present a similar argument for converting default risk into a cost of debt, and then bring the discussion to fruition by combining both the cost of equity and debt to estimate a cost of capital, which will become the minimum acceptable hurdle rate for an investment.

Cost of Equity

The cost of equity is the rate of return that investors require to make an equity investment in a firm. All of the risk and return models described in the previous chapter need a riskfree rate and a risk premium (in the CAPM) or premiums (in the APM and multi-factor models). We will begin by discussing those common inputs before we turn our attention to the estimation of risk parameters.

I. Riskfree Rate

Most risk and return models in finance start off with an asset that is defined as risk free and use the expected return on that asset as the risk free rate. The expected returns on

risky investments are then measured relative to the risk free rate, with the risk creating an expected risk premium that is added on to the risk free rate.

Requirements for an asset to be riskfree

We defined a riskfree asset as one where the investor knows the expected returns with certainty. Consequently, for an investment to be riskfree, i.e., to have an actual return be equal to the expected return, two conditions have to be met –

- There has to be *no default risk*, which generally implies that the security has to be issued by a government. Note, though, that not all governments are default free and the presence of government or sovereign default risk can make it very difficult to estimate riskfree rates in some currencies.
- There can be *no uncertainty about reinvestment rates*, which implies that there are no intermediate cash flows. To illustrate this point, assume that you are trying to estimate the expected return over a five-year period and that you want a risk free rate. A six-month treasury bill rate, while default free, will not be risk free, because there is the reinvestment risk of not knowing what the treasury bill rate will be in six months. Even a 5-year treasury bond is not risk free, since the coupons on the bond will be reinvested at rates that cannot be predicted today. The risk free rate for a five-year time horizon has to be the expected return on a default-free (government) five-year zero coupon bond.

This clearly has painful implications for anyone doing corporate financial analysis, where expected returns often have to be estimated for periods ranging from multiple years. A purist's view of risk free rates would then require different risk free rates for each period and different expected returns. As a practical compromise, however, it is worth noting that the present value effect of using risk free rates that vary from year to year tends to be small for most well behaved[1] term structures. In these cases, we could use a duration matching strategy, where the duration of the default-free security used as the risk free asset is matched up to the duration[2] of the cash flows in the analysis. If, however, there

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[1] By well behaved term structures, I would include a normal upwardly sloping yield curve, where long term rates are at most 2-3% higher than short term rates.

[2] In investment analysis, where we look at projects, these durations are usually between 3 and 10 years. In valuation, the durations tend to be much longer, since firms are assumed to have infinite lives. The duration

are very large differences, in either direction, between short term and long term rates, it does pay to stick with year-specific risk free rates in computing expected returns.

Cash Flows and Risk free Rates: The Consistency Principle

The risk free rate used to come up with expected returns should be measured consistently with how the cash flows are measured. If the cashflows are nominal, the riskfree rate should be in the same currency in which the cashflows are estimated. This also implies that it is not where a project or firm is domiciled that determines the choice of a risk free rate, but the currency in which the cash flows on the project or firm are estimated. Thus, Disney can analyze a proposed project in Mexico in dollars, using a dollar discount rate, or in pesos, using a peso discount rate. For the former, it would use the US treasury bond rate as the riskfree rate but for the latter, it would need a peso riskfree rate.

Under conditions of high and unstable inflation, valuation is often done in real terms. Effectively, this means that cash flows are estimated using real growth rates and without allowing for the growth that comes from price inflation. To be consistent, the discount rates used in these cases have to be real discount rates. To get a real expected rate of return, we need to start with a real risk free rate. While government bills and bonds offer returns that are risk free in nominal terms, they are not risk free in real terms, since expected inflation can be volatile. The standard approach of subtracting an expected inflation rate from the nominal interest rate to arrive at a real risk free rate provides at best an estimate of the real risk free rate. Until recently, there were few traded default-free securities that could be used to estimate real risk free rates; but the introduction of inflation-indexed treasuries has filled this void. An inflation-indexed treasury security does not offer a guaranteed nominal return to buyers, but instead provides a guaranteed real return. In early 2004, for example, the inflation indexed US 10-year treasury bond rate was only 1.6%, much lower than the nominal 10-year bond rate of 4%.

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in these cases is often well in excess of ten years and increases with the expected growth potential of the firm.

4.1. ☞: **What is the right riskfree rate?**

The correct risk free rate to use in the capital asset pricing model

a. is the short term government security rate

b. is the long term government security rate

c. can be either, depending upon whether the prediction is short term or long term.

In Practice: What if there is no default-free rate?

Our discussion, hitherto, has been predicated on the assumption that governments do not default, at least on local borrowing. There are many emerging market economies where this assumption might not be viewed as reasonable. Governments in these markets are perceived as capable of defaulting even on local borrowing. When this is coupled with the fact that many governments do not borrow long term locally, there are scenarios where obtaining a l risk free rate in the local currency, especially for the long term, becomes difficult. In these cases, there are compromises that give us reasonable estimates of the risk free rate.

- Look at the largest and safest firms in that market and use the rate that they pay on their long-term borrowings in the local currency as a base. Given that these firms, in spite of their size and stability, still have default risk, you would use a rate that is marginally lower[3] than the corporate borrowing rate.
- If there are long term dollar-denominated forward contracts on the currency, you can use interest rate parity and the treasury bond rate (or riskless rate in any other base currency) to arrive at an estimate of the local borrowing rate.[4]
- You could adjust the local currency government borrowing rate by the estimated default spread on the bond to arrive at a riskless local currency rate. The default

[3] Reducing the corporate borrowing rate by 1% (which is the typical default spread on highly rated corporate bonds in the U.S) to get a riskless rate yields reasonable estimates.

[4] For instance, if the current spot rate is 38.10 Thai Baht per US dollar, the ten-year forward rate is 61.36 Baht per dollar and the current ten-year US treasury bond rate is 5%, the ten-year Thai risk free rate (in nominal Baht) can be estimated as follows.

$$61.36 = (38.1)\left(\frac{1 + \text{Interest Rate}_{\text{Thai Baht}}}{1 + 0.05}\right)^{10}$$

Solving for the Thai interest rate yields a ten-year risk free rate of 10.12%.

spread on the government bond can be estimated using the local currency ratings[5] that are available for many countries. For instance, assume that the Brazilian government bond rate (in nominal Brazilian Reals (BR)) is 14% and that the local currency rating assigned to the Brazilian government is BB+. If the default spread for BB+ rated bonds is 5%, the riskless Brazilian real rate would be 9%.

Riskless BR rate = Brazil Government Bond rate – Default Spread = 14% -5% = 9%

II. Risk premium

The risk premium(s) is clearly a significant input in all of the asset pricing models. In the following section, we will begin by examining the fundamental determinants of risk premiums and then look at practical approaches to estimating these premiums.

What is the risk premium supposed to measure?

The risk premium in the capital asset pricing model measures the extra return that would be demanded by investors for shifting their money from a riskless investment to an average risk investment. It should be a function of two variables:

1. *Risk Aversion of Investors*: As investors become more risk averse, they should demand a larger premium for shifting from the riskless asset. While of some of this risk aversion may be inborn, some of it is also a function of economic prosperity (when the economy is doing well, investors tend to be much more willing to take risk) and recent experiences in the market (risk premiums tend to surge after large market drops).
2. *Riskiness of the Average Risk Investment*: As the riskiness of the average risk investment increases, so should the premium. This will depend upon what firms are actually traded in the market, their economic fundamentals and how good they are at managing risk. For instance, the premium should be lower in markets where only the largest and most stable firms trade in the market.

[5] Ratings agencies generally assign different ratings for local currency borrowings and dollar borrowing, with higher ratings for the former and lower ratings for the latter.

Since each investor in a market is likely to have a different assessment of an acceptable premium, the premium will be a weighted average of these individual premiums, where the weights will be based upon the wealth the investor brings to the market. Put more directly, what Warren Buffett, with his substantial wealth, thinks is an acceptable premium will be weighted in far more into market prices than what you or I might think about the same measure.

In the arbitrage pricing model and the multi-factor models, the risk premiums used for individual factors are similar wealth-weighted averages of the premiums that individual investors would demand for each factor separately.

☞ 4.2: What is your risk premium?

Assume that stocks are the only risky assets and that you are offered two investment options:

- A riskless investment (say a Government Security), on which you can make 4%
- A mutual fund of all stocks, on which the returns are uncertain

How much of an expected return would you demand to shift your money from the riskless asset to the mutual fund?

a. Less than 4%
b. Between 4-6%
c. Between 6-8%
d. Between 8-10%
e. Between 10-12%
f. More than 12%

Your answer to this question should provide you with a measure of your risk premium. (For instance, if your answer is 6%, your premium is 2%.)

Estimating Risk Premiums

There are three ways of estimating the risk premium in the capital asset pricing model - large investors can be surveyed about their expectations for the future, the actual premiums earned over a past period can be obtained from historical data and the implied premium can be extracted from current market data. The premium can be estimated only from historical data in the arbitrage pricing model and the multi-factor models.

1. Survey Premiums

Since the premium is a weighted average of the premiums demanded by individual investors, one approach to estimating this premium is to survey investors about their expectations for the future. It is clearly impractical to survey all investors; therefore, most surveys focus on portfolio managers who carry the most weight in the process. Morningstar regularly survey individual investors about the return they expect to earn, investing in stocks. Merrill Lynch does the same with equity portfolio managers and reports the results on its web site. While numbers do emerge from these surveys, very few practitioners actually use these survey premiums. There are three reasons for this reticence:

- There are no constraints on reasonability; individual money managers could provide expected returns that are lower than the riskfree rate, for instance.
- Survey premiums are extremely volatile; the survey premiums can change dramatically, largely as a function of recent market movements.
- Survey premiums tend to be short term; even the longest surveys do not go beyond one year.

☞ 4.3: Do risk premiums change?

In the previous question, you were asked how much of a premium you would demand for investing in a portfolio of stocks as opposed to a riskless asset. Assume that the market dropped by 20% last week, and you were asked the same question today. Would your premium

a. be higher?
b. be lower?
c. be unchanged?

2. Historical Premiums

The most common approach to estimating the risk premium(s) used in financial asset pricing models is to base it on historical data. In the arbitrage pricing model and multi- factor models, the raw data on which the premiums are based is historical data on asset prices over very long time periods. In the CAPM, the premium is defined as the

difference between average returns on stocks and average returns on risk-free securities over an extended period of history.

Basics

In most cases, this approach is composed of the following steps. It begins by defining a time period for the estimation, which can range to as far back as 1871 for U.S. data. It then requires the calculation of the average returns on a stock index and average returns on a riskless security over the period. Finally, it calculates the difference between the returns on stocks and the riskless return and uses it as a risk premium looking forward. In doing so, we implicitly assume that

1. The risk aversion of investors has not changed in a systematic way across time. (The risk aversion may change from year to year, but it reverts back to historical averages.)
2. It assumes that the average riskiness of the "risky" portfolio (stock index) has not changed in a systematic way across time.

Estimation Issues

While users of risk and return models may have developed a consensus that historical premium is, in fact, the best estimate of the risk premium looking forward, there are surprisingly large differences in the actual premiums we observe being used in practice. For instance, the risk premium estimated in the US markets by different investment banks, consultants and corporations range from 4% at the lower end to 12% at the upper end. Given that we almost all use the same database of historical returns, provided by Ibbotson Associates[6], summarizing data from 1926, these differences may seem surprising. There are, however, three reasons for the divergence in risk premiums.

- Time Period Used: While there are many who use all the data going back to 1926, there are almost as many using data over shorter time periods, such as fifty, twenty or even ten years to come up with historical risk premiums. The rationale presented by those who use shorter periods is that the risk aversion of the average investor is likely to change over time and that using a shorter and more recent time period provides a

[6] See "Stocks, Bonds, Bills and Inflation", an annual edition that reports on the annual returns on stocks, treasury bonds and bills, as well as inflation rates from 1926 to the present. (http://www.ibbotson.com)

more updated estimate. This has to be offset against a cost associated with using shorter time periods, which is the greater noise in the risk premium estimate. In fact, given the annual standard deviation in stock prices[7] between 1928 and 2002 of 20%, the standard error[8] associated with the risk premium estimate can be estimated as follows for different estimation periods in Table 4.1.

Table 4.1: Standard Errors in Risk Premium Estimates

Estimation Period	Standard Error of Risk Premium Estimate
5 years	$\frac{20}{\sqrt{5}} = 8.94\%$
10 years	$\frac{20}{\sqrt{10}} = 6.32\%$
25 years	$\frac{20}{\sqrt{25}} = 4.00\%$
50 years	$\frac{20}{\sqrt{50}} = 2.83\%$

Note that to get reasonable standard errors, we need very long time periods of historical returns. Conversely, the standard errors from ten-year and twenty-year estimates are likely to be almost as large or larger than the actual risk premium estimated. This cost of using shorter time periods seems, in our view, to overwhelm any advantages associated with getting a more updated premium.

- Choice of Riskfree Security: The Ibbotson database reports returns on both treasury bills and treasury bonds and the risk premium for stocks can be estimated relative to each. Given that the yield curve in the United States has been upward sloping for most of the last seven decades, the risk premium is larger when estimated relative to shorter term government securities (such as treasury bills). The riskfree rate chosen in computing the premium has to be consistent with the riskfree rate used to compute expected returns. For the most part, in corporate finance and valuation, the riskfree rate will be a long term default-free (government) bond rate and not a treasury bill

[7] For the historical data on stock returns, bond returns and bill returns, check under "updated data" in www.stern.nyu.edu/~adamodar.

[8] These estimates of the standard error are probably understated because they are based upon the assumption that annual returns are uncorrelated over time. There is substantial empirical evidence that returns are correlated over time, which would make this standard error estimate much larger.

rate. Thus, the risk premium used should be the premium earned by stocks over treasury bonds.

- Arithmetic and Geometric Averages: The final sticking point when it comes to estimating historical premiums relates to how the average returns on stocks, treasury bonds and bills are computed. The arithmetic average return measures the simple mean of the series of annual returns, whereas the geometric average looks at the compounded return[9]. Conventional wisdom argues for the use of the arithmetic average. In fact, if annual returns are uncorrelated over time and our objectives were to estimate the risk premium for the next year, the arithmetic average is the best unbiased estimate of the premium. In reality, however, there are strong arguments that can be made for the use of geometric averages. First, empirical studies seem to indicate that returns on stocks are negatively correlated[10] over time. Consequently, the arithmetic average return is likely to over state the premium. Second, while asset pricing models may be single period models, the use of these models to get expected returns over long periods (such as five or ten years) suggests that the single period may be much longer than a year. In this context, the argument for geometric average premiums becomes even stronger.

In summary, the risk premium estimates vary across users because of differences in time periods used, the choice of treasury bills or bonds as the riskfree rate and the use of arithmetic as opposed to geometric averages. The effect of these choices is summarized in table 4.2, which uses returns from 1928 to 2003. [11]

Table 4.2: Historical Risk Premia for the United States – 1928- 2003

	Stocks – Treasury Bills	*Stocks – Treasury Bonds*

[9] The compounded return is computed by taking the value of the investment at the start of the period ($Value_0$) and the value at the end ($Value_N$) and then computing the following:

$$\text{Geometric Average} = \left(\frac{\text{Value}_N}{\text{Value}_0}\right)^{1/N} - 1$$

[10] In other words, good years are more likely to be followed by poor years and vice versa. The evidence on negative serial correlation in stock returns over time is extensive and can be found in Fama and French (1988). While they find that the one-year correlations are low, the five-year serial correlations are strongly negative for all size classes.

[11] The raw data on treasury bill rates, treasury bond rates and stock returns was obtained from the Federal Reserve data archives maintained by the Fed in St. Louis.

	Arithmetic	*Geometric*	*Arithmetic*	*Geometric*
1928 – 2003	7.92%	5.99%	6.54%	4.82%
1962 – 2003	6.09%	4.85%	4.70%	3.82%
1992 – 2003	8.43%	6.68%	4.87%	3.57%

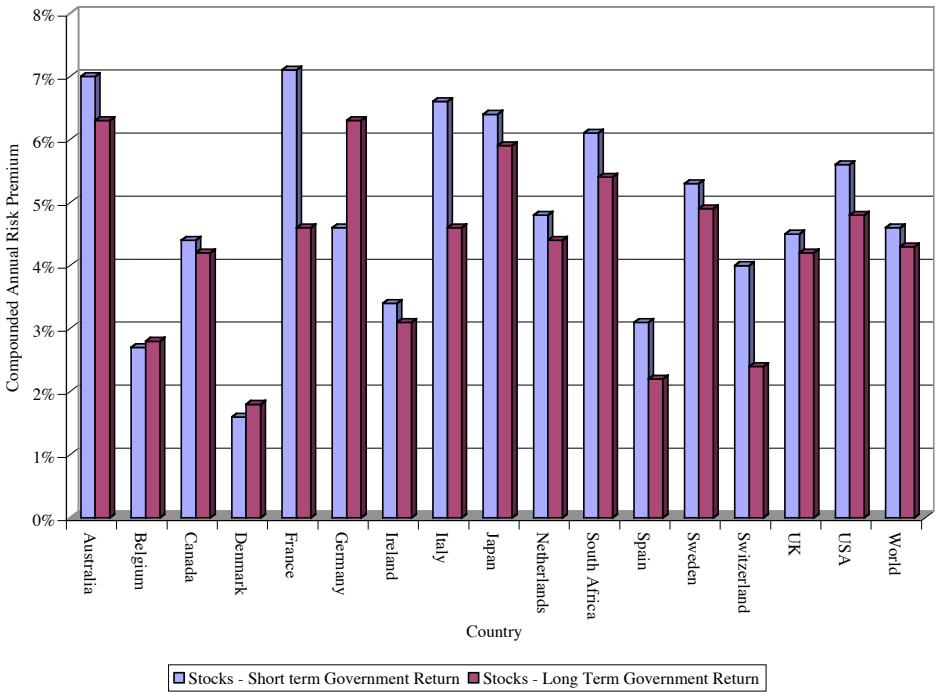
Note that the premiums can range from 3.57% to 8.43%, depending upon the choices made. In fact, these differences are exacerbated by the fact that many risk premiums that are in use today were estimated using historical data three, four or even ten years ago. If we follow the propositions about picking a long-term geometric average premium over the long term treasury bond rate, the historical risk premium that makes the most sense is 4.82%.

Historical Premiums in other markets

While historical data on stock returns is easily available and accessible in the United States, it is much more difficult to get this data for foreign markets. The most detailed look at these returns estimated the returns you would have earned on 14 equity markets between 1900 and 2001 and compared these returns with those you would have earned investing in bonds.[12] Figure 4.1 presents the risk premiums – i.e., the additional returns - earned by investing in equity over treasury bills and bonds over that period in each of the 14 markets:

[12] Dimson, E., P. March and M. Staunton, 2002, *Triumph of the Optimists*, Princeton University Prsss.

Figure 4.1: Equity Risk Premiums - By Country



Data from Dimson et al. The differences in compounded annual returns between stocks and short term governments/ long term governments is reported for each country.

While equity returns were higher than what you would have earned investing in government bonds or bills in each of the countries examined, there are wide differences across countries. If you had invested in Spain, for instance, you would have earned only 3% over government bills and 2% over government bonds on an annual basis by investing in equities. In France, in contrast, the corresponding numbers would have been 7.1% and 4.6%. Looking at 40-year or 50-year periods, therefore, it is entirely possible that equity returns can lag bond or bill returns, at least in some equity markets. In other words, the notion that stocks always win in the long term is not only dangerous but does not make sense. If stocks always beat riskless investments in the long term, stocks should be riskless to an investor with a long time horizon.

histretSP.xls: This data set has yearly data on treasury bill rates, treasury bond rates and returns and stock returns going back to 1928.

A Modified Historical Risk Premium

In many emerging markets, there is very little historical data and the data that exists is too volatile to yield a meaningful estimate of the risk premium. To estimate the risk premium in these countries, let us start with the basic proposition that the risk premium in any equity market can be written as:

Equity Risk Premium = Base Premium for Mature Equity Market + Country Premium

The country premium could reflect the extra risk in a specific market. This boils down our estimation to answering two questions:

- What should the base premium for a mature equity market be?
- How do we estimate the additional risk premium for individual countries?

To answer the first question, we will make the argument that the US equity market is a mature market and that there is sufficient historical data in the United States to make a reasonable estimate of the risk premium. In fact, reverting back to our discussion of historical premiums in the US market, we will use the geometric average premium earned by stocks over treasury bonds of 4.82% between 1928 and 2003. We chose the long time period to reduce standard error, the treasury bond to be consistent with our choice of a riskfree rate and geometric averages to reflect our desire for a risk premium that we can use for longer term expected returns. There are three approaches that we can use to estimate the country risk premium.

1. *Country bond default spreads*: While there are several measures of country risk, one of the simplest and most easily accessible is the rating assigned to a country's debt by a ratings agency (S&P, Moody's and IBCA all rate countries). These ratings measure default risk (rather than equity risk), but they are affected by many of the factors that drive equity risk – the stability of a country's currency, its budget and trade balances and its political stability, for instance[13]. The other advantage of ratings is that they come with default spreads over the US treasury bond. For instance, Brazil was rated B2 in early 2004 by Moody's and the 10-year Brazilian C-Bond, which is a dollar denominated bond was priced to yield 10.01%, 6.01% more than the interest rate

[13] The process by which country ratings are obtained is explained on the S&P web site at http://www.ratings.standardpoor.com/criteria/index.htm.

(4%) on a 10-year treasury bond at the same time.[14] Analysts who use default spreads as measures of country risk typically add them on to both the cost of equity and debt of every company traded in that country. For instance, the cost of equity for a Brazilian company, estimated in U.S. dollars, will be 6.01% higher than the cost of equity of an otherwise similar U.S. company. If we assume that the risk premium for the United States and other mature equity markets is 4.82%, the cost of equity for a Brazilian company can be estimated as follows (with a U.S. Treasury bond rate of 4% and a beta of 1.2).

Cost of equity = Riskfree rate + Beta *(U.S. Risk premium) + Country Bond Default Spread

$$= 4\% + 1.2\,(4.82\%) + 6.01\% = 15.79\%$$

In some cases, analysts add the default spread to the U.S. risk premium and multiply it by the beta. This increases the cost of equity for high beta companies and lowers them for low beta firms.

2. *Relative Standard Deviation*: There are some analysts who believe that the equity risk premiums of markets should reflect the differences in equity risk, as measured by the volatilities of these markets. A conventional measure of equity risk is the standard deviation in stock prices; higher standard deviations are generally associated with more risk. If you scale the standard deviation of one market against another, you obtain a measure of relative risk.

$$\text{Relative Standard Deviation}_{\text{Country X}} = \frac{\text{Standard Deviation}_{\text{Country X}}}{\text{Standard Deviation}_{\text{US}}}$$

This relative standard deviation when multiplied by the premium used for U.S. stocks should yield a measure of the total risk premium for any market.

$$\text{Equity risk premium}_{\text{Country X}} = \text{Risk Premum}_{\text{US}} * \text{Relative Standard Deviation}_{\text{Country X}}$$

Assume, for the moment, that you are using a mature market premium for the United States of 4.82% and that the annual standard deviation of U.S. stocks is 20%. The

[14] These yields were as of January 1, 2004. While this is a market rate and reflects current expectations, country bond spreads are extremely volatile and can shift significantly from day to day. To counter this volatility, the default spread can be normalized by averaging the spread over time or by using the average default spread for all countries with the same rating as Brazil in early 2003.

annualized standard deviation[15] in the Brazilian equity index was 36%, yielding a total risk premium for Brazil:

$$\text{Equity Risk Premium}_{\text{Brazil}} = 4.82\% * \frac{36\%}{20\%} = 8.67\%$$

The country risk premium can be isolated as follows:

$$\text{Country Risk Premium}_{\text{Brazil}} = 8.67\% - 4.82\% = 3.85\%$$

While this approach has intuitive appeal, there are problems with using standard deviations computed in markets with widely different market structures and liquidity. There are very risky emerging markets that have low standard deviations for their equity markets because the markets are illiquid. This approach will understate the equity risk premiums in those markets.

3. *Default Spreads + Relative Standard Deviations*: The country default spreads that come with country ratings provide an important first step, but still only measure the premium for default risk. Intuitively, we would expect the country equity risk premium to be larger than the country default risk spread. To address the issue of how much higher, we look at the volatility of the equity market in a country relative to the volatility of the bond market used to estimate the spread. This yields the following estimate for the country equity risk premium.

$$\text{Country Risk Premium} = \text{Country Default Spread} * \left(\frac{\sigma_{\text{Equity}}}{\sigma_{\text{Country Bond}}} \right)$$

To illustrate, consider the case of Brazil. As noted earlier, the dollar denominated bonds issued by the Brazilian government trade with a default spread of 6.01% over the US treasury bond rate. The annualized standard deviation in the Brazilian equity index over the previous year was 36%, while the annualized standard deviation in the Brazilian dollar denominated C-bond was 27%[16]. The resulting additional country equity risk premium for Brazil is as follows:

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[15] Both the US and Brazilian standard deviations were computed using weekly returns for two years from the beginning of 2002 to the end of 2003. While you could use daily standard deviations to make the same judgments, they tend to have much more noise in them.

[16] The standard deviation in C-Bond returns was computed using weekly returns over 2 years as well. Since there returns are in dollars and the returns on the Brazilian equity index are in real, there is an inconsistency

$$\text{Brazils Country Risk Premium} = 6.01\% \left(\frac{36\%}{27\%}\right) = 8.01\%$$

Note that this country risk premium will increase if the country rating drops or if the relative volatility of the equity market increases. It is also in addition to the equity risk premium for a mature market. Thus, the total equity risk premium for a Brazilian company using the approach and a 4.82% premium for the United States would b2 12.83%.

Why should equity risk premiums have any relationship to country bond spreads? A simple explanation is that an investor who can make 11% on a dollar-denominated Brazilian government bond would not settle for an expected return of 10.5% (in dollar terms) on Brazilian equity. Both this approach and the previous one use the standard deviation in equity of a market to make a judgment about country risk premium, but they measure it relative to different bases. This approach uses the country bond as a base, whereas the previous one uses the standard deviation in the U.S. market. This approach assumes that investors are more likely to choose between Brazilian government bonds and Brazilian equity, whereas the previous one approach assumes that the choice is across equity markets.

The three approaches to estimating country risk premiums will generally give you different estimates, with the bond default spread and relative equity standard deviation approaches yielding lower country risk premiums than the melded approach that uses both the country bond default spread and the equity and bond standard deviations. In the case of Brazil, for instance, the country risk premiums range from 3.85% using the relative equity standard deviation approach to 6.01% for the country bond approach to We believe that the larger country risk premiums that emerge from the last approach are the most realistic for the immediate future, but that country risk premiums may decline over time. Just as companies mature and become less risky over time, countries can mature and become less risky as well.

In Practice: Should there be a country risk premium?

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here. We did estimate the standard deviation on the Brazilian equity index in dollars but it made little difference to the overall calculation since the dollar standard deviation was close to 36%.

Is there more risk in investing in a Malaysian or Brazilian stock than there is in investing in the United States? The answer, to most, seems to be obviously affirmative. That, however, does not answer the question of whether there should be an additional risk premium charged when investing in those markets. Note that the only risk that is relevant for the purpose of estimating a cost of equity is market risk or risk that cannot be diversified away. The key question then becomes whether the risk in an emerging market is diversifiable or non-diversifiable risk. If, in fact, the additional risk of investing in Malaysia or Brazil can be diversified away, then there should be no additional risk premium charged. If it cannot, then it makes sense to think about estimating a country risk premium.

For purposes of analyzing country risk, we look at the marginal investor – the investor most likely to be trading on the equity. If that marginal investor is globally diversified, there is at least the potential for global diversification. If the marginal investor does not have a global portfolio, the likelihood of diversifying away country risk declines substantially. Even if the marginal investor is globally diversified, there is a second test that has to be met for country risk to not matter. All or much of country risk should be country specific. In other words, there should be low correlation across markets. Only then will the risk be diversifiable in a globally diversified portfolio. If, on the other hand, the returns across countries have significant positive correlation, country risk has a market risk component and is not diversifiable and can command a premium. Whether returns across countries are positively correlated is an empirical question. Studies from the 1970s and 1980s suggested that the correlation was low and this was an impetus for global diversification. Partly because of the success of that sales pitch and partly because economies around the world have become increasingly intertwined over the last decade, more recent studies indicate that the correlation across markets has risen. This is borne out by the speed at which troubles in one market, say Russia, can spread to a market with which it has little or no obvious relationship, say Brazil.

So where do we stand? We believe that while the barriers to trading across markets have dropped, investors still have a home bias in their portfolios and that markets remain partially segmented. While globally diversified investors are playing an increasing role in the pricing of equities around the world, the resulting increase in

correlation across markets has resulted in a portion of country risk becoming non-diversifiable or market risk..

 There is a data set on the website that contains the updated ratings for countries and the risk premiums associated with each.

3. Implied Equity Premiums

There is an alternative to estimating risk premiums that does not require historical data or corrections for country risk, but does assume that the overall stock market is correctly priced. Consider, for instance, a very simple valuation model for stocks.

$$\text{Value} = \frac{\text{Expected Dividends Next Period}}{(\text{Required Return on Equity} - \text{Expected Growth Rate in Dividends})}$$

This is essentially the present value of dividends growing at a constant rate. Three of the four variables in this model can be obtained externally – the current level of the market (i.e., value), the expected dividends next period and the expected growth rate in earnings and dividends in the long term. The only "unknown" is then the required return on equity; when we solve for it, we get an implied expected return on stocks. Subtracting out the riskfree rate will yield an implied equity risk premium.

To illustrate, assume that the current level of the S&P 500 Index is 900, the expected dividend yield on the index for the next period is 2% and the expected growth rate in earnings and dividends in the long term is 7%. Solving for the required return on equity yields the following:

$$900 = \frac{900(0.02)}{r - 0.07}$$

Solving for r,

$$r - 0.07 = 0.02$$

$$r = 0.09 = 9\%$$

If the current riskfree rate is 6%, this will yield a premium of 3%.

This approach can be generalized to allow for high growth for a period and extended to cover cash flow based, rather than dividend based, models. To illustrate this, consider the S&P 500 Index on January 1, 2004. The index was at 1111.91 and the dividend yield on the index in 2003 was roughly 2.81%.[17] In addition, the consensus estimate[18] of growth in earnings for companies in the index was approximately 9.5% for the next 5 years and the 10-year treasury bond rate on that day was 4.25%. Since a growth rate of 9.5% cannot be sustained forever, we employ a two-stage valuation model, where we allow dividends to grow at 9.5% for 5 years and then lower the growth rate to the treasury bond rate of 4.25% after the 5 year period.[19] Table 4.3 summarizes the expected cash flows for the next 5 years of high growth and the first year of stable growth thereafter.

Table 4.3: Expected Cashflows on S&P 500

Year	*Cash Flow on Index*
1	34.26
2	37.52
3	41.08
4	44.98
5	49.26
6	51.35

[a]Cash flow in the first year = 2.81% of 1111.91 (1.095)

If we assume that these are reasonable estimates of the cash flows and that the index is correctly priced, then

$$\text{Index level} = 1111.91 = \frac{34.26}{(1+r)} + \frac{37.52}{(1+r)^2} + \frac{41.08}{(1+r)^3} + \frac{44.98}{(1+r)^4} + \frac{49.26}{(1+r)^5} + \frac{49.26(1.0425)}{(r-.0425)(1+r)^5}$$

Note that the last term of the equation is the terminal value of the index, based upon the stable growth rate of 4.25%, discounted back to the present. Solving for r in this equation yields us the required return on equity of 7.94%. Subtracting out the treasury bond rate of 4.25% yields an implied equity premium of 3.69%.

The advantage of this approach is that it is market-driven and current and it does not require any historical data. Thus, it can be used to estimate implied equity premiums

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[17] Stock buybacks during the year were added to the dividends to obtain a consolidated yield.

[18] We used the average of the analyst estimates for individual firms (bottom-up). Alternatively, we could have used the top-down estimate for the S&P 500 earnings.

in any market. It is, however, bounded by whether the model used for the valuation is the right one and the availability and reliability of the inputs to that model. For instance, the equity risk premium for the Brazilian market in January 2004 was estimated from the following inputs. The index (Bovespa) was at 21050 and the current dividend yield on the index was 4%. Earnings in companies in the index are expected to grow 14% (in US dollar terms) over the next 5 years and 4.5% thereafter. These inputs yield a required return on equity of 10.70%, which when compared to the treasury bond rate of 4% on that day results in an implied equity premium of 6.70%. For simplicity, we have used nominal dollar expected growth rates[20] and treasury bond rates, but this analysis could have been done entirely in the local currency.

The implied equity premiums change over time much more than historical risk premiums. In fact, the contrast between these premiums and the historical premiums is best illustrated by graphing out the implied premiums in the S&P 500 going back to 1960 in Figure 4.2.

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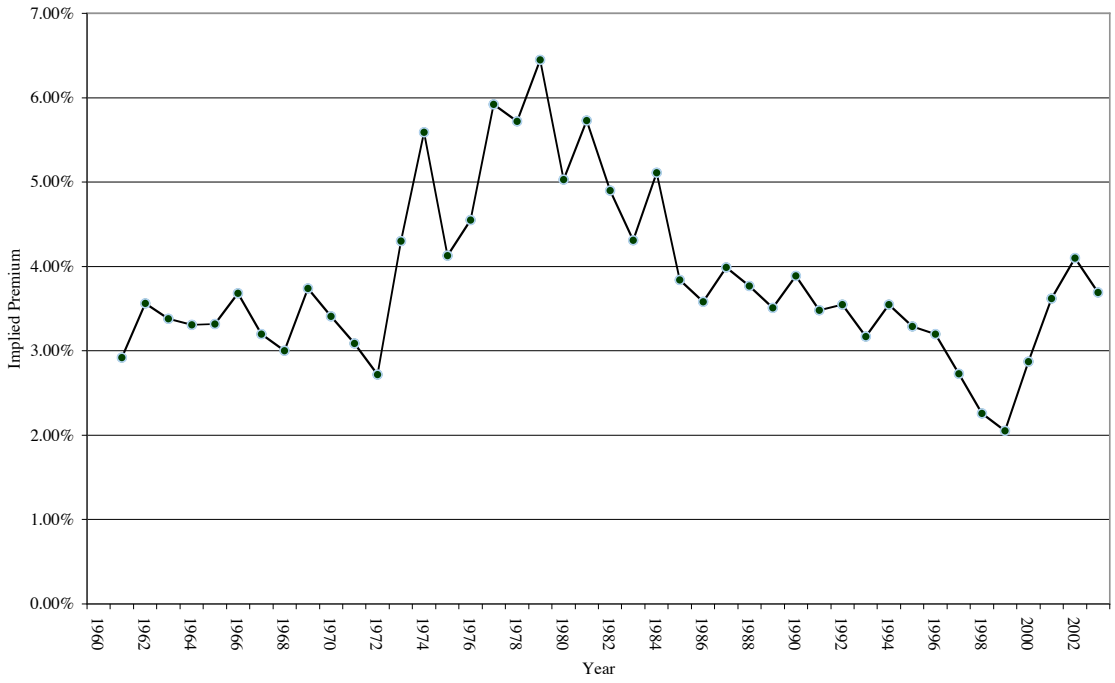
[19] The treasury bond rate is the sum of expected inflation and the expected real rate. If we assume that real growth is equal to the real rate, the long term stable growth rate should be equal to the treasury bond rate.

[20] The input that is most difficult to estimate for emerging markets is a long term expected growth rate. For Brazilian stocks, I used the average consensus estimate of growth in earnings for the largest Brazilian companies which have listed ADRs . This estimate may be biased, as a consequence.

Figure 4.2: Implied Premium for US Equity Market



In terms of mechanics, we used smoothed historical growth rates in earnings and dividends as our projected growth rates and a two-stage dividend discount model. Looking at these numbers, we would draw the following conclusions.

- The implied equity premium has seldom been as high as the historical risk premium. Even in 1978, when the implied equity premium peaked, the estimate of 6.50% is well below what many practitioners use as the risk premium in their risk and return models. In fact, the average implied equity risk premium has been between about 4% over the last 40 years.
- The implied equity premium did increase during the seventies, as inflation increased. This does have interesting implications for risk premium estimation. Instead of assuming that the risk premium is a constant and unaffected by the level of inflation and interest rates, which is what we do with historical risk premiums, it may be more realistic to increase the risk premium as expected inflation and interest rates increase.


histimpl.xls: This data set on the web shows the inputs used to calculate the premium in each year for the U.S. market.

 implprem.xls: This spreadsheet allows you to estimate the implied equity premium in a market.

☞ **4.4: Implied and Historical Premiums**

Assume that the implied premium in the market is 3%, and that you are using a historical premium of 7.5%. If you valued stocks using this historical premium, you are likely

a. to find more under valued stocks than over valued ones
b. to find more over valued stocks than under valued ones
c. to find about as many undervalued as overvalued stocks

III. Risk Parameters

The final set of inputs we need to put risk and return models into practice are the risk parameters for individual assets and projects. In the CAPM, the beta of the asset has to be estimated relative to the market portfolio. In the APM and Multi-factor model, the betas of the asset relative to each factor have to be measured. There are three approaches available for estimating these parameters; one is to use historical data on market prices for individual assets; the second is to estimate the betas from fundamentals and the third is to use accounting data. We will use all three approaches in this section.

A. Historical Market Betas

This is the conventional approach for estimating betas used by most services and analysts. For firms that have been publicly traded for a length of time, it is relatively straightforward to estimate returns that a investor would have made on the assets in intervals (such as a week or a month) over that period. These returns can then be related to a proxy for the market portfolio to get a beta in the capital asset pricing model, or to multiple macro economic factors to get betas in the multi factor models, or put through a factor analysis to yield betas for the arbitrage pricing model.

Standard Procedures for Estimating CAPM Parameters - Betas and Alphas

The standard procedure for estimating betas is to regress[21] stock returns (R_j) against market returns (R_m) -

$$R_j = a + b\ R_m$$

where

a = Intercept from the regression

b = Slope of the regression = Covariance (R_j, R_m) / σ^2_m

The slope of the regression corresponds to the beta of the stock and measures the riskiness of the stock.

Jensen's Alpha: This is the difference between the actual returns on an asset and the return you would have expected it to make during a past period, given what the market did, and the asset's beta.

The intercept of the regression provides a simple measure of performance during the period of the regression, relative to the capital asset pricing model.

$R_j = R_f + \beta\ (R_m - R_f)$

$= R_f\ (1-\beta) + \beta\ R_m$ ..... Capital Asset Pricing Model

$R_j = a + b\ R_m$ ..... Regression Equation

Thus, a comparison of the intercept (a) to $R_f\ (1-\beta)$ should provide a measure of the stock's performance, at least relative to the capital asset pricing model.[22]

If $a > R_f\ (1-\beta)$ .... Stock did better than expected during regression period

$a = R_f\ (1-\beta)$ .... Stock did as well as expected during regression period

$a < R_f\ (1-\beta)$ .... Stock did worse than expected during regression period

The difference between a and $R_f\ (1-\beta)$ is called Jensen's alpha, and provides a measure of whether the asset in question under or out performed the market, after adjusting for risk, during the period of the regression.

R Squared: The R squared measures the proportion of the variability of a dependent variable that is explained by an independent variable or variables.

The third statistic that emerges from the regression is the R squared (R^2) of the regression.

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[21] The appendix to this chapter provides a brief overview of ordinary least squares regressions.

[22] The regression can be run using returns in excess of the risk-free rate, for both the stock and the market. In that case, the intercept of the regression should be zero if the actual returns equal the expected returns from the CAPM, greater than zero if the stock does better than expected and less than zero if it does worse than expected.

While the statistical explanation of the R squared is that it provides a measure of the goodness of fit of the regression, the financial rationale for the R squared is that it provides an estimate of the proportion of the risk (variance) of a firm that can be attributed to market risk; the balance ($1 - R^2$) can then be attributed to firm-specific risk.

The final statistic worth noting is the standard error of the beta estimate. The slope of the regression, like any statistical estimate, is made with noise, and the standard error reveals just how noisy the estimate is. The standard error can also be used to arrive at confidence intervals for the "true" beta value from the slope estimate.

Estimation Issues

There are three decisions the analyst must make in setting up the regression described above. The first concerns the length of the estimation period. The trade-off is simple: A longer estimation period provides more data, but the firm itself might have changed in its risk characteristics over the time period. Disney and Deutsche Bank have changed substantially in terms of both business mix and financial leverage over the last few years and any regression that we run using historical data will be affected by these changes.

The second estimation issue relates to the return interval. Returns on stocks are available on an annual, monthly, weekly, daily and even on an intra-day basis. Using daily or intra-day returns will increase the number of observations in the regression, but it exposes the estimation process to a significant bias in beta estimates related to non-trading.[23] For instance, the betas estimated for small firms, which are more likely to suffer from non-trading, are biased downwards when daily returns are used. Using weekly or monthly returns can reduce the non-trading bias significantly.[24]

The third estimation issue relates to the choice of a market index to be used in the regression. The standard practice used by most beta estimation services is to estimate the betas of a company relative to the index of the market in which its stock trades. Thus, the betas of German stocks are estimated relative to the Frankfurt DAX, British stocks

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[23] The non-trading bias arises because the returns in non-trading periods is zero (even though the market may have moved up or down significantly in those periods). Using these non-trading period returns in the regression will reduce the correlation between stock returns and market returns and the beta of the stock.

[24] The bias can also be reduced using statistical techniques suggested by Dimson and Scholes-Williams.

relative to the FTSE, Japanese stocks relative to the Nikkei, and U.S. stocks relative to the S&P 500. While this practice may yield an estimate that is a reasonable measure of risk for the parochial investor, it may not be the best approach for an international or cross-border investor, who would be better served with a beta estimated relative to an international index.

Illustration 4.1: Estimating CAPM risk parameters for Disney

In assessing risk parameters for Disney, the returns on the stock and the market index are computed as follows –

(1) The returns to a stockholder in Dsiney are computed month by month from January 1999 to December 2003. These returns include both dividends and price appreciation and are defined as follows –

$$\text{Stock Return}_{\text{intel, j}} = (\text{Price}_{\text{Intel, j}} - \text{Price}_{\text{Intel, j-1}} + \text{Dividends}_{\text{j}}) / \text{Price}_{\text{Intel,j-1}}$$

where $\text{Stock Return}_{\text{Intel,j}}$ = Returns to a stockholder in Disney in month j

$\text{Price}_{\text{Intel, j}}$ = Price of Disney stock at the end of month j

$\text{Dividends}_{\text{j}}$ = Dividends on Disney stock in month j

Dividends are added to the returns of the month in which the stock went ex-dividend.[25] If there was a stock split[26] during the month, the returns have to take into account the split factor, since stock prices will be affected.[27]

(2) The returns on the S&P 500 market index are computed for each month of the period, using the level of the index at the end of each month, and the monthly dividend yield on stocks in the index. –

$$\text{Market Return}_{\text{intel, j}} = (\text{Index}_{\text{j}} - \text{Index}_{\text{j-1}} + \text{Dividends}_{\text{t}}) / \text{Index}_{\text{j-1}}$$

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[25] The ex-dividend day is the day by which the stock has to be bought for an investor to be entitled to the dividends on the stock.

[26] A split changes the number of shares outstanding in a company without affecting any of its fundamentals. Thus, in a three-for-two split, there will be 50% more shares outstanding after the split. Since the overall value of equity has not changed, the stock price will drop by an equivalent amount (1 - 100/150 = 33.33%)

[27] While there were no stock splits in the time period of the regression, Disney did have a 3 for 1 stock split in July 1998. The stock price dropped significantly, and if not factored in will result in very negative returns in that month. Splits can be accounted for as follows –

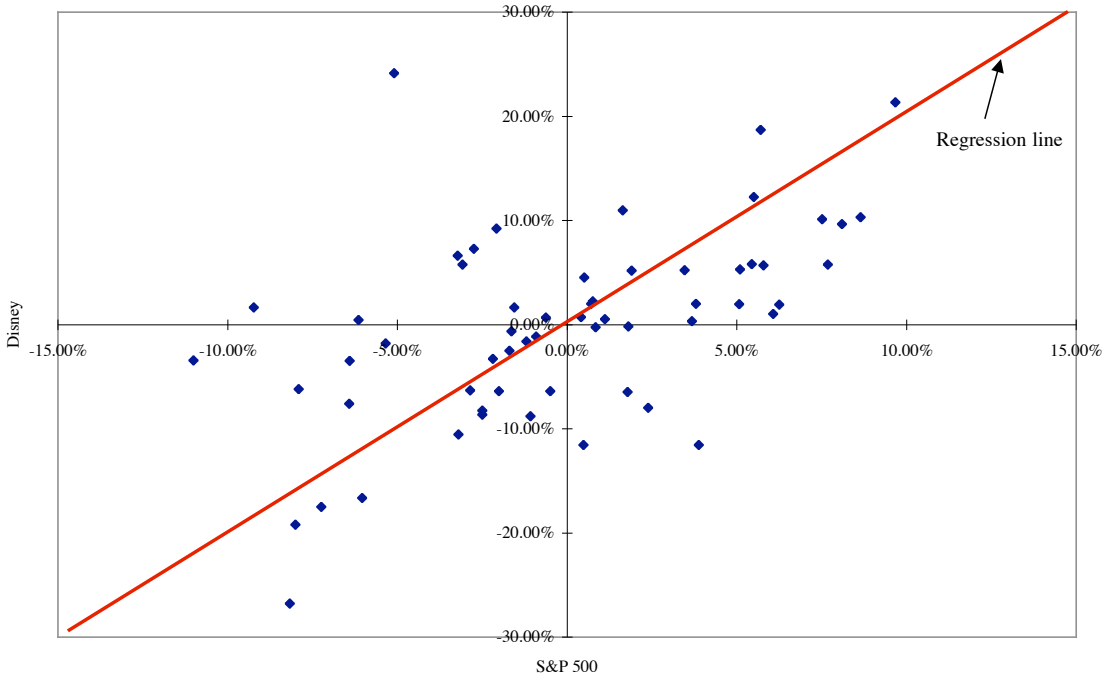
$$\text{Return}_{\text{intel, j}} = (\text{Factor}_{\text{j}} * \text{Price}_{\text{Intel, j}} - \text{Price}_{\text{Intel, j-1}} + \text{Factor} * \text{Dividends}_{\text{j}}) / \text{Price}_{\text{Intel,j-1}}$$

The factor would be set to 3 for July 1998 and the ending price would be multiplied by 3, as would the dividends per share, if they were paid after the split.

where $Index_j$ is the level of the index at the end of month j and $Dividend_j$ is the dividends paid on the index in month j. While the S&P 500 and the NYSE Composite are the most widely used indices for U.S. stocks, they are, at best, imperfect proxies for the market portfolio in the CAPM, which is supposed to include all assets.

Figure 4.3 graphs monthly returns on Disney against returns on the S&P 500 index from January 1999 to December 2003.

Figure 4.3: Disney versus S&P 500: 1999 - 2003



The regression statistics for Disney are as follows:[28]

(a) Slope of the regression = 1.01. This is Disney's beta, based on returns from 1999 to 2003. Using a different time period for the regression or different return intervals (weekly or daily) for the same period can result in a different beta.

(b) Intercept of the regression = 0.0467%. This is a measure of Disney's performance, when it is compared with R_f (1-β).[29] The monthly risk-free rate (since the returns used in

[28] The regression statistics are computed in the conventional way. The appendix explains the process in more detail.

[29] In practice, the intercept of the regression is often called the alpha and compared to zero. Thus, a positive intercept is viewed as a sign that the stock did better than expected and a negative intercept as a sign that the stock did worse than expected. In truth, this can be done only if the regression is run in terms of excess returns, i.e., returns over and above the riskfree rate in each month for both the stock and the market index.

the regression are monthly returns) between 1999 and 2003 averaged 0.313%, resulting in the following estimate for the performance:

$R_f (1-\beta) = 0.313\% (1-1.01) = -.0032\%$

Intercept - $R_f (1-\beta) = 0.0467\% - (-0.0032\%) = 0.05\%$

This analysis suggests that Disney's stock performed 0.05% better than expected, when expectations are based on the CAPM, on a monthly basis between January 1999 and December 2003. This results in an annualized excess return of approximately 0.60%.

Annualized Excess Return $= (1 + \text{Monthly Excess Return})^{12} - 1$

$= (1+0.0005)^{12} -1 = .0060$ or 0.60%

By this measure of performance, Disney did slightly better than expected during the period of the regression, given its beta and the market's performance over the period. Note, however, that this does not imply that Disney would be a good investment looking forward. It also does not provide a breakdown of how much of this excess return can be attributed to 'industry-wide' effects, and how much is specific to the firm. To make that breakdown, the excess returns would have to be computed over the same period for other firms in the entertainment industry and compared with Disney's excess return. The difference would be then attributable to firm-specific actions. In this case, for instance, the average annualized excess return on other entertainment firms between 1999 and 2003 was 1.33%. This would imply that Disney stock underperformed it's peer group by 0.73% between 1999 and 2003, after adjusting for risk. (Firm-specific Jensen's alpha = 0.60% - 1.33% = - 0.73%)

(c) R squared of the regression = 29%. This statistic suggests that 29% of the risk (variance) in Disney comes from market sources (interest rate risk, inflation risk etc.), and that the balance of 71% of the risk comes from firm-specific components. The latter risk should be diversifiable, and therefore unrewarded. Disney's R squared is slightly higher than the median R squared of companies listed on the New York Stock Exchange, which was approximately 21% in 2003.

(d) Standard Error of Beta Estimate = 0.20. This statistic implies that the true beta for Disney could range from 0.81 to 1.21 (subtracting adding one standard error to beta estimate of 1.01) with 67% confidence and from 0.61 to 1.41 (subtracting adding two standard error to beta estimate of 1.01) with 95% confidence. While these ranges may

seem large, they are not unusual for most U.S. companies. This suggests that we should consider regression estimates of betas from regressions with caution.

☞ 4.5: The Relevance of R-squared to an Investor

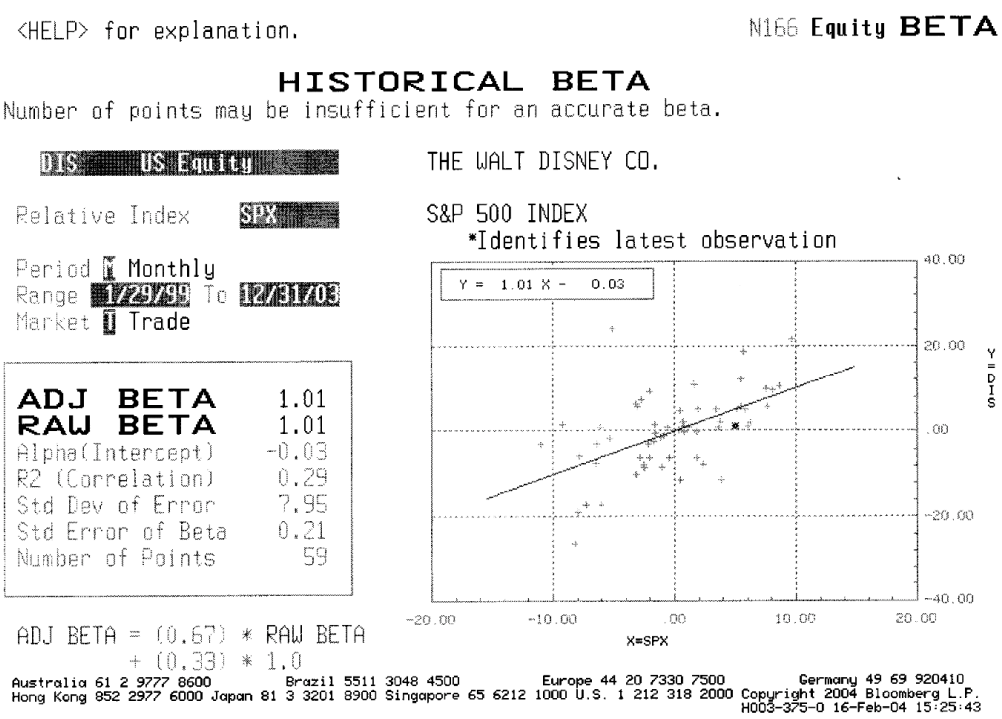
Assume that, having done the regression analysis, both Disney and Amgen, a biotechnology company, have betas of 1.01. Disney, however, has an R-squared of 31%, while Amgen has an R-squared of only 15%. If you had to pick between these investments, which one would you choose?

a. Disney, because it's higher R-squared suggests that it is less risky
b. Amgen, because it's lower R-squared suggests a greater potential for high returns
c. I would be indifferent, because they both have the same beta

Would your answer be any different if you were running a well-diversified fund?

In Practice: Using a Service beta

Most analysts who use betas obtain them from an estimation service; Merrill Lynch, Barra, Value Line, Standard and Poor's, Morningstar and Bloomberg are some of the well known services. All these services begin with regression betas and make what they feel are necessary changes to make them better estimates for the future. While most of these services do not reveal the internal details of this estimation, Bloomberg is an honorable exception. The following is the beta calculation page from Bloomberg for Disney, using the same period as our regression (January 1999 to December 2003):



While the time period used is identical to the one used in our earlier regression, there are subtle differences between this regression and the earlier one in Figure 4.1. First, Bloomberg uses price appreciation in the stock and the market index in estimating betas and ignores dividends.[30] This does not make much of a difference for a Disney, but it could make a difference for a company that either pays no dividends or pays significantly higher dividends than the market. Second, Bloomberg also computes what they call an adjusted beta, which is estimated as follows:

$$\text{Adjusted Beta} = \text{Raw Beta } (0.67) + 1\ (0.33)$$

These weights do not vary across stocks, and this process pushes all estimated betas towards one. Most services employ similar procedures to adjust betas towards one. In doing so, they are drawing on empirical evidence that suggests that the betas for most companies, over time, tend to move towards the average beta, which is one. This may be explained by the fact that firms get more diversified in their product mix and client base as they get larger.

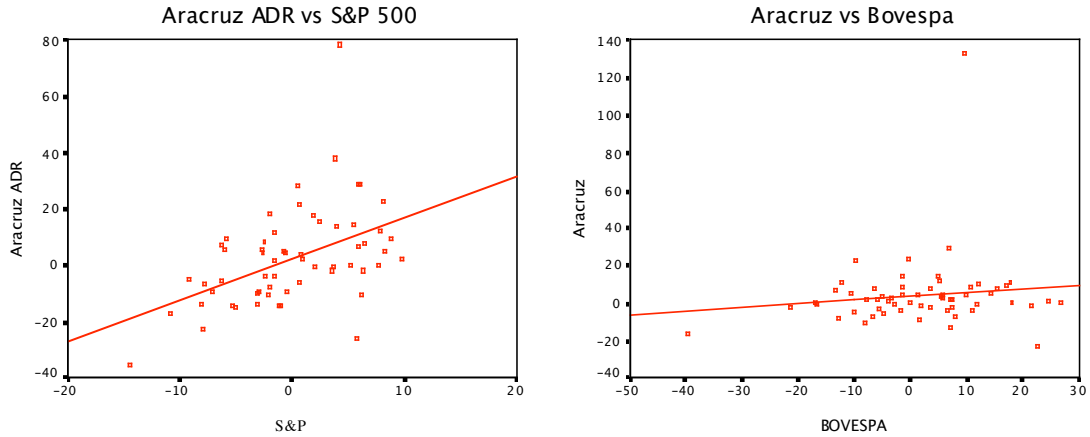
[30] This is why the intercept in the Bloomberg print out (0.03%) is slightly different from the intercept estimated earlier in the chapter (0.05%). The beta and R-squared are identical.

In general, betas reported by different services for the same firm can be very different because they use different time periods (some use 2 years and others 5 years), different return intervals (daily, weekly or monthly), different market indices and different post-regression estimates. While these beta differences may be troubling, the beta estimates delivered by each of these services comes with a standard error, and it is very likely that all of the betas reported for a firm fall within the range of the standard errors from the regressions.

Illustration 4.2: Estimating Historical Betas for Aracruz and Deutsche Bank

Aracruz is a Brazilian company and we can regress returns on the stock against a Brazilian index to obtain risk parameters. The stock also had an ADR listed on the U.S. exchanges and we can regress returns on the ADR against a U.S. index to obtain parameters. Figure 4.4 presents both graphs for the January 1999- December 2003 time period:

Figure 4.4: Estimating Aracruz's Beta: Choice of Indices



Aracruz ADR = 2.80% + 1.00 S&P *Aracruz = 2.62% + 0.22 Bovespa*

How different are the risk parameters that emerge from the two regressions? Aracruz has a beta of 1.00, when the ADR is regressed against the S&P 500, and a beta of only 0.22, when the local listing is regressed against the Bovespa.[31] Each regression has its own problems. The Bovespa is a narrow index dominated by a few liquid stocks and does not

[31] The biggest source of the difference is one month (January 1999). In that month, Aracruz had a return of 133% in the Sao Paulo exchange while the ADR dropped by 9.67% in the same month. The disparity in returns can be attributed to a steep devaluation in the Brazilian Real in that month.

represent the broad spectrum of Brazilian equities. While the S&P 500 is a broader index, the returns on the ADR have little relevance to a large number of non-US investors who bought the local listing.

Deutsche Bank does not have an ADR listed in the United States but we can regress return on Deutsche against a multitude of indices. Table 4.4 presents comparisons of the results of the regressions of returns on Deutsche against three indices – a German equity index (DAX), an index of large European companies (FTSE Euro 300) and a global equity index (Morgan Stanley Capital Index (MSCI)).

Table 4.4: Deutsche Bank Risk Parameters: Index Effect

	DAX	*FTSE Euro 300*	*MSCI*
Intercept	1.24%	1.54%	1.37%
Beta	1.05	1.52	1.23
Std Error of Beta	0.11	0.19	0.25
R Squared	62%	52%	30%

Here again, the risk parameters estimated for Deutsche Bank are a function of the index used in the regression. The standard error is lowest (and the R squared is highest) for the regression against the DAX; this is not surprising since Deutsche is a large component of the DAX. The standard error gets larger and the R squared gets lower as the index is broadened to initially include other European stocks and then to global stocks.

In Practice: Which index should we use to estimate betas?

In most cases, analysts are faced with a mind-boggling array of choices among indices when it comes to estimating betas; there are more than 20 broad equity indices ranging from the Dow 30 to the Wilshire 5000 in the United States alone. One common practice is to use the index that is most appropriate for the investor who is looking at the stock. Thus, if the analysis is being done for a U.S. investor, the S&P 500 index is used. This is generally not appropriate. By this rationale, an investor who owns only two stocks should use an index composed of only those stocks to estimate betas.

The right index to use in analysis should be determined by the holdings of the marginal investor in the company being analyzed. Consider Aracruz and Deutsche Bank in the earlier illustration. If the marginal investors in.these companies are investors who

holds only domestic stocks – just Brazilian stocks in the case of Aracruz or German stocks in the case of Deutsche – we can use the regressions against the local indices. If the marginal investor is a global investor, a more relevant measure of risk may emerge by using the global index. Over time, you would expect global investors to displace local investors as the marginal investors, because they will perceive far less of the risk as market risk and thus pay a higher price for the same security. Thus, one of the ironies of our notion of risk is that Aracruz will be less risky to an overseas investor who has a global portfolio than to a Brazilian investor with all of his or her wealth in Brazilian assets.

Standard Procedures for Estimating Risk Parameters in the Arbitrage Pricing and Multi-factor Models

Like the CAPM, the arbitrage pricing model defines risk to be non-diversifiable risk, but, unlike the CAPM, the APM allows for multiple economic factors in measuring this risk. While the process of estimation of risk parameters is different for the arbitrage pricing model, many of the issues raised relating to the determinants of risk in the CAPM continue to have relevance for the arbitrage pricing model.

The parameters of the arbitrage pricing model are estimated from a factor analysis on historical stock returns, which yields the number of common economic factors determining these returns, the risk premium for each factor and the factor-specific betas for each firm.

Factor Analysis: This is a statistical technique, where past data is analyzed with the intent of extracting common factors that might have affected the data.

Once the factor-specific betas are estimated for each firm, and the factor premia are measured, the arbitrage pricing model can be used to estimated expected returns on a stock.

$$\text{Cost of Equity} = R_f + \sum_{j=1}^{j=k} \beta_j \, (E(R_j) - R_f)$$

where,

R_f = Risk-free rate

β_j = Beta specific to factor j

$E(R_j) - R_f =$ Risk premium per unit of factor j risk

k = Number of factors

In a multi-factor model, the betas are estimated relative to the specified factors, using historical data for each firm.

B. Fundamental Betas

The beta for a firm may be estimated from a regression but it is determined by fundamental decisions that the firm has made on what business to be in, how much operating leverage to use in the business and the degree to which the firm uses financial leverage. In this section, we will examine an alternative way of estimating betas, where we are less reliant on historical betas and more cognizant of the intuitive underpinnings of betas.

Determinants of Betas

The beta of a firm is determined by three variables -(1) the type of business or businesses the firm is in, (2) the degree of operating leverage in the firm and (3) the firm's financial leverage. While much of the discussion in this section will be couched in terms of CAPM betas, the same analysis can be applied to the betas estimated in the APM and the multi-factor model as well.

Type of Business Since betas measure the risk of a firm relative to a market index, the more sensitive a business is to market conditions, the higher is its beta. Thus, other things remaining equal, cyclical firms can be expected to have higher betas than non-cyclical firms. Other things remaining equal, then, companies involved in housing and automobiles, two sectors of the economy which are very sensitive to economic conditions, will have higher betas than companies which are in food processing and tobacco, which are relatively insensitive to business cycles.

Cyclical Firm: A cyclical firm has revenues and operating income that tend to move strongly with the economy - up when the economy is doing well, and down during recessions.

Building on this point, we would also argue that the degree to which a product's purchase is discretionary will affect the beta of the firm manufacturing the product. Thus,

the betas of food processing firms, such as General Foods and Kellogg's, should be lower than the betas of specialty retailers, since consumers can defer the purchase of the latter's products during bad economic times.

It is true that firms have only limited control over how discretionary the product or service that they provide is to their customers. There are firms, however, that have used this limited control to maximum effect to make their products less discretionary to buyers, and by extension, lowered their business risk. One approach is to make the product or service a much more integral and necessary part of everyday life, thus making its purchase more of a requirement. A second approach is to effectively use advertising and marketing to build brand loyalty. The objective in good advertising, as we see it, is to make discretionary products or services seem like necessities to the target audience. Thus, corporate strategy, advertising and marketing acumen can, at the margin, alter the business risk and betas over time.

☞ 4.7: Betas and Business Risk

Polo Ralph Lauren, the upscale fashion designer, went public in 1997. Assume that you were asked to estimate its beta. Based upon what you know about the firm's products, would you expect the beta to be

a. greater than one
b. about one
c. less than one

Why?

Degree of Operating Leverage The degree of operating leverage is a function of the cost structure of a firm, and is usually defined in terms of the relationship between fixed costs and total costs. A firm that has high operating leverage (i.e., high fixed costs relative to total costs) will also have higher variability in operating income than would a firm producing a similar product with low operating leverage.[32] Other

Operating Leverage: This is a measure of the proportion of the operating expenses of a company which are fixed costs.

[32] To see why, compare two firms with revenues of $ 100 million and operating income of $ 10 million, but assume that the first firm's costs are all fixed whereas only half of the second firm's costs are fixed. If revenues increase at both firms by $ 10 million, the first firm will report a doubling of operating income

things remaining equal, the higher variance in operating income will lead to a higher beta for the firm with high operating leverage.

While operating leverage affects betas, it is difficult to measure the operating leverage of a firm, at least from the outside, since fixed and variable costs are often aggregated in income statements. It is possible to get an approximate measure of the operating leverage of a firm by looking at changes in operating income as a function of changes in sales.

Degree of Operating leverage = % Change in Operating Profit / % Change in Sales

For firms with high operating leverage, operating income should change more than proportionately, when sales change.

Can firms change their operating leverage? While some of a firm's cost structure is determined by the business it is in (an energy utility has to build expensive power plants, and airlines have to lease expensive planes), firms in the United States have become increasingly inventive in lowering the fixed cost component in their total costs. Labor contracts that emphasize flexibility and allow the firm to make its labor costs more sensitive to its financial success, joint venture agreements, where the fixed costs are borne by someone else, and sub-contracting of manufacturing, which reduce the need for expensive plant and equipment, are only some of the manifestations of this phenomenon. While the arguments for such actions may be couched in terms of competitive advantage and flexibility, they do reduce the operating leverage of the firm and its exposure to "market" risk.

Illustration 4.3: Measuring Operating Leverage for Disney Corporation

In table 4.5, we estimate the degree of operating leverage for Disney from 1987 to 2003:

Table 4.5: Degree of Operating Leverage: Disney

Year	Net Sales	% Change in Sales	EBIT	% Change in EBIT
1987	2877		756	

(from $ 10 to $ 20 million) whereas the second firm will report a rise of 55% in its operating income (since costs will rise by $ 4.5 million, 45% of the revenue increment).

1988	3438	19.50%	848	12.17%
1989	4594	33.62%	1177	38.80%
1990	5844	27.21%	1368	16.23%
1991	6182	5.78%	1124	-17.84%
1992	7504	21.38%	1287	14.50%
1993	8529	13.66%	1560	21.21%
1994	10055	17.89%	1804	15.64%
1995	12112	20.46%	2262	25.39%
1996	18739	54.71%	3024	33.69%
1997	22473	19.93%	3945	30.46%
1998	22976	2.24%	3843	-2.59%
1999	23435	2.00%	3580	-6.84%
2000	25418	8.46%	2525	-29.47%
2001	25172	-0.97%	2832	12.16%
2002	25329	0.62%	2384	-15.82%
2003	27061	6.84%	2713	13.80%
1987-2003		15.83%		10.09%
1996-2003		11.73%		4.42%

The degree of operating leverage changes dramatically from year to year, because of year-to-year swings in operating income. Using the average changes in sales and operating income over the period, we can compute the operating leverage at Disney:

Operating Leverage = % Change in EBIT/ % Change in Sales

= 10.09% / 15.83% = 0.64

There are two important observations that can be made about Disney over the period, though. First, the operating leverage for Disney is lower than the operating leverage for other entertainment firms, which we computed to be 1.12.[33] This would suggest that Disney has lower fixed costs than its competitors. Second, the acquisition of Capital Cities by Disney in 1996 may be affecting the operating leverage. Looking at the numbers since 1996, we get an even lower estimate of operating leverage:

$$\text{Operating Leverage}_{1996\text{-}03} = 4.42\%/11.73\% = 0.38$$

We would not read too much into these numbers because Disney has such a wide range of businesses. We would hypothesize that Disney's theme part business has higher fixed costs (and operating leverage) than it's movie business.

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[33] To compute this statistic, we looked at the aggregate revenues and operating income of entertainment companies each year from 1987 to 2003.

☞ 4.8: Social Policy and Operating Leverage

Assume that you are comparing a European automobile manufacturing firm with a U.S. automobile firm. European firms are generally much more constrained in terms of laying off employees, if they get into financial trouble. What implications does this have for betas, if they are estimated relative to a common index?

a. The European firm will have much a higher beta than the U.S. firms
b. The European firms will have a similar betas to the U.S. firm
c. The European firms will have a much lower beta than the U.S. firms

Should small or high growth firms have higher betas than larger and more mature firms?

Though the answer may seem obvious at first sight – that smaller, higher growth firms should are riskier than larger firms – it is not an easy question to answer. If the question were posed in terms of total risk, smaller and higher growth firms will tend to be riskier simply because they have more volatile earnings streams (and their market prices reflect that). When it is framed in terms of betas or market risk, smaller and higher growth firms should have higher betas only if the products and services they offer are more discretionary to their customers or if they have higher operating leverage. It is possible that smaller firms operate in niche markets and sell products which customers can delay or defer buying and that the absence of economies of scales lead to higher fixed costs for these firms. These firms should have higher betas than their larger counterparts. It is also possible that neither condition holds for a particular small firm. The answer will therefore depend both on the company in question and the industry in which it operates.

In practice, analysts often add what is called a small firm premium to the cost of equity for smaller firms. This small firm premium is usually estimated from historical data to be the difference between the average annual returns on small market cap stocks and the rest of the market – about 3 to 3.5% when we look at the 1926-2003 period. This practice can be dangerous for three reasons. The first is that the small firm premium has been volatile and disappeared for an extended period in the 1980s. The second is that the definition of a small market cap stock varies across time and that the historical small cap premium is largely attributable to the smallest (among the small cap) stocks. The third is

that using a constant small stock premium adjustment removes any incentive that the analyst may have to examine the product characteristics and operating leverage of individual small market cap companies more closely.

Degree of Financial Leverage Other things remaining equal, an increase in financial leverage will increase the equity beta of a firm. Intuitively, we would expect that the fixed interest payments on debt to increase earnings per share in good times and to push it down in bad times.[34] Higher leverage increases the variance in earnings per share and makes equity investment in the firm riskier. If all of the firm's risk is borne by the stockholders (i.e., the beta of debt is zero)[35], and debt creates a tax benefit to the firm, then,

$$\beta_L = \beta_u \ (1 + (1-t) \ (D/E))$$

where

β_L = Levered Beta for equity in the firm

β_u = Unlevered beta of the firm (i.e., the beta of the firm without any debt)

t = Marginal tax rate for the firm

D/E = Debt/Equity Ratio

The marginal tax rate is the tax rate on the last dollar of income earned by the firm generally will not be equal to the effective or average rates, and it is used because interest expenses save taxes on the marginal income. Intuitively, we expect that as leverage increases (as measured by the debt to equity ratio), equity investors bear increasing amounts of market risk in the firm, leading to higher betas. The tax factor in the equation captures the benefit created by the tax deductibility of interest payments.

The unlevered beta of a firm is determined by the types of the businesses in which it operates and its operating leverage. This unlevered beta is often also referred to as the **asset beta** since its value is determined by the assets (or businesses) owned by the firm.

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[34] Interest expenses always lower net income, but the fact that the firm uses debt instead of equity implies that the number of shares will also be lower. Thus, the benefit of debt shows up in earnings per share.

[35] to ignore the tax effects and compute the levered beta as

$$\beta_L = \beta_u \ (1+ D/E)$$

If debt has market risk (i.e., its beta is greater than zero), the original formula can be modified to take it into account. If the beta of debt is β_D , the beta of equity can be written as:

$$\beta_L = \beta_u \ (1+(1-t)(D/E)) - \beta_D \ (1-t)D/E$$

Thus, the equity beta of a company is determined both by the riskiness of the business it operates in, as well as the amount of financial leverage risk it has taken on. Since financial leverage multiplies the underlying business risk, it stands to reason that firms that have high business risk should be reluctant to take on financial leverage. It also stands to reason that firms which operate in relatively stable businesses should be much more willing to take on financial leverage. Utilities, for instance, have historically had high debt ratios, but have not had high betas, mostly because their underlying businesses have been stable and fairly predictable.

Breaking risk down into business and financial leverage components also provides some insight into why companies have high betas, since they can end up with high betas in one of two ways - they can operate in a risky business, or they can use very high financial leverage in a relatively stable business.

Illustration 4.4: Effects of Financial Leverage on betas: Disney

From the regression for the period from 1999 to 2003, Disney had a beta of 1.01. To estimate the effects of leverage on Disney, we began by estimating the average debt/equity ratio between 1999 and 2003, using market values for debt and equity.

Average Market Debt/Equity Ratio between 1999 and 2003 = 27.5%

The unlevered beta is estimated using a marginal corporate tax rate of 37.3%:[36]

Unlevered Beta = Current Beta / (1 + (1 - tax rate) (Average Debt/Equity))

= 1.01 / (1 + (1 - 0.373)) (0.275) = 0.8615

The levered beta at different levels of debt can then be estimated:

Levered Beta = Unlevered Beta * [1 + (1 - tax rate) (Debt/ Equity)]

For instance, if Disney were to increase its debt equity ratio to 10%, its equity beta will be

Levered Beta (@10% D/E) = 0.8615*(1+ (1 - 0.373) (0.10)) = 0.9155

If the debt equity ratio were raised to 25%, the equity beta would be

Levered Beta (@25% D/E) = 0.8615 *(1+ (1 - 0.373) (0.25)) = 1.00

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[36] The marginal corporate tax rate in the United States in 2003 was 35%. The marginal state and local tax rates, corrected for federal tax savings, is estimated by Disney in its annual report to be 2.3%. Disney did report some offsetting tax benefits that reduced their effective tax rate to 35%. We assumed that these offsetting tax benefits were temporary.

Table 4.6 summarizes the beta estimates for different levels of financial leverage ranging from 0 to 90% debt.

Table 4.6: Financial Leverage and Betas

Debt to Capital	*Debt/Equity Ratio*	*Beta*	*Effect of Leverage*
0.00%	0.00%	0.86	0.00
10.00%	11.11%	0.92	0.06
20.00%	25.00%	1.00	0.14
30.00%	42.86%	1.09	0.23
40.00%	66.67%	1.22	0.36
50.00%	100.00%	1.40	0.54
60.00%	150.00%	1.67	0.81
70.00%	233.33%	2.12	1.26
80.00%	400.00%	3.02	2.16
90.00%	900.00%	5.72	4.86

As Disney's financial leverage increases, the beta increases concurrently.


levbeta.xls: This spreadsheet allows you to estimate the unlevered beta for a firm and compute the betas as a function of the leverage of the firm.


marginaltaxrate.xls: This data set on the web has marginal tax rates for different countries.

In Practice: Dueling Tax Rates

The marginal tax rate, which is the tax rate on marginal income (or the last dollar of income) is a key input not only for the levered beta calculation but also for the after-tax cost of debt that we will be estimating later in this chapter. Estimating it can be problematic because firms seldom report it in their financials. Most firms report an effective tax rate on taxable income in their annual reports and filings with the SEC. This rate is computed by dividing the taxes paid by the net taxable income, reported in the financial statement. The effective tax rate can be different from the marginal tax rate for several reasons:

- If it is a small firm and the tax rate is higher for higher income brackets, the average tax rate across all income will be lower than the tax rate on the last dollar of income. For larger firms, where most of the income is at the highest tax bracket, this is less of an issue.
- Publicly traded firms, at least in the United States, often maintain two sets of books, one for tax purposes and one for reporting purposes. They generally use different accounting rules for the two and report lower income to tax authorities and higher income in their annual reports. Since taxes paid are based upon the tax books, the effective tax rate will usually be lower than the marginal tax rate.
- Actions that defer or delay the payment of taxes can also cause deviations between marginal and effective tax rates. In the period when taxes are deferred, the effective tax rate will lag the marginal tax rate. In the period when the deferred taxes are paid, the effective tax rate can be much higher than the marginal tax rate.

The best source of the marginal tax is the tax code of the country where the firm earns its operating income. If there are state and local taxes, they should be incorporated into the marginal tax rate as well. For companies in multiple tax locales, the marginal tax rate used should be the average of the different marginal tax rates, weighted by operating income by locale.

Bottom Up Betas

Breaking down betas into their business, operating leverage and financial leverage components provides us with an alternative way of estimating betas, where we do not need past prices on an individual firm or asset to estimate its beta.

To develop this alternative approach, we need to introduce an additional feature that betas possess that proves invaluable. The beta of two assets put together is a weighted average of the individual asset betas, with the weights based upon market value. Consequently, the beta for a firm is a weighted average of the betas of all of different businesses it is in. Thus, the bottom-up beta for a firm, asset or project can be estimated as follows.

1. Identify the business or businesses that make up the firm, whose beta we are trying to estimate. Most firms provide a breakdown of their revenues and operating income by business in their annual reports and financial filings.
2. Estimate the average unlevered betas of other publicly traded firms that are primarily or only in each of these businesses. In making this estimate, we have to consider the following estimation issues:
   - *Comparable firms*: In most businesses, there are at least a few comparable firms and in some businesses, there can be hundreds. Begin with a narrow definition of comparable firms, and widen it if the number of comparable firms is too small.
   - *Beta Estimation*: Once a list of comparable firms has been put together, we need to estimate the betas of each of these firms. Optimally, the beta for each firm will be estimated against a common index. If that proves impractical, we can use betas estimated against different indices.
   - *Unlever first or last*: We can compute an unlevered beta for each firm in the comparable firm list, using the debt to equity ratio and tax rate for that firm, or we can compute the average beta, debt to equity ratio and tax rate for the sector and unlever using the averages. Given the standard errors of the individual regression betas, we would suggest the latter approach.
   - *Averaging approach*: The average beta across the comparable firms can be either a simple average or a weighted average, with the weights based upon market capitalization. Statistically, the savings in standard error are larger if a simple averaging process is used.
   - *Adjustment for Cash*: Investments in cash and marketable securities have betas close to zero. Consequently, the unlevered beta that we obtain for a business by looking at comparable firms may be affected by the cash holdings of these firms. To obtain an unlevered beta cleansed of cash:

$$\text{Unlevered Beta corrected for Cash} = \frac{\text{Unlevered Beta}}{(1 - \text{Cash/ Firm Value})}$$

3. To calculate the unlevered beta for the firm, we take a weighted average of the unlevered betas, using the proportion of firm value derived from each business as the weights. These firm values will have to be estimated since divisions of a firm usually

do not have market values available.[37] If these values cannot be estimated, we use operating income or revenues as weights. This is also take into account the cash holdings of the firm by computing it as a percent of firm value and attaching a beta of zero if the cash is invested in riskless securities (like commercial paper or treasury bills) or a higher beta if it is invested in riskier securities (like corporate bonds). This weighted average is called the bottom-up unlevered beta.

4. Calculate the current debt to equity ratio for the firm, using market values if available. If not, use the target leverage specified by the management of the firm or industry-typical debt ratios.
5. Estimate the levered beta for the firm (and each of its businesses) using the unlevered beta from step 3 and the leverage from step 4.

Clearly, this process rests on being able to identify the unlevered betas of individual businesses.

There are three advantages associated with using bottom-up betas and they are significant:

- We can estimate betas for firms that have no price history since all we need is an identification of the business they operate in. In other words, we can estimate bottom up betas for initial public offerings, private businesses and divisions of companies.
- Since the beta for the business is obtained by averaging across a large number of regression betas, it will be more precise than any individual firm's regression beta estimate. The standard error of the average beta estimate will be a function of the number of comparable firms used in step 2 above and can be approximated as follows:

$$\sigma_{\text{Average Beta}} = \frac{\text{Average } \sigma_{\text{Beta}}}{\sqrt{\text{Number of firms}}}$$

  Thus, the standard error of the average of the betas of 100 firms, each of which has a standard error of 0.25, will be only 0.025. ($0.25/\sqrt{100}$).
- The bottom-up beta can reflect recent and even forthcoming changes to a firm's business mix and financial leverage, since we can change the mix of businesses and the weight on each business in making the estimate.

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[37] The exception is when you have stock tracking each division traded separately in financial markets.

 This data set on the web has updated betas and unlevered betas by business sector in the United States.

Illustration 4.5: Bottom Up Beta for Disney

Disney is an entertainment firm with diverse holdings. In addition to its theme parks, it has significant investments in broadcasting and movies. To estimate Disney's beta today, we broke their business into four major components -

1. *Studio Entertainment*, which is the production and acquisition of motion pictures for distribution in theatrical, television and home video markets as well as television programming for network and syndication markets. Disney produces movies under five imprints – Walt Disney Pictures, Touchstone Pictures, Hollywood Pictures, Miramax and Dimension.
2. *Media Networks,* which includes the ABC Television and Radio networks, and reflects the acquisition made in 1995. In addition, Disney has an extensive exposure in the cable market through the Disney channel, A & E and ESPN among others.
3. *Park Resorts*, which include Disney World (in Orlando, Florida) and Disney Land (in Anaheim, California), as well as royalty holdings in Tokyo Disneyland and Disneyland Paris. The hotels and villas at each of these theme parks are considered part of the theme parks, since they derive their revenue almost exclusively from visitors to these parks.
4. *Consumer Products*, which includes a grab bag of businesses including Disney's retail outlets, its licensing revenues, software, interactive products and publishing.

This breakdown reflects Disney's reporting in its annual report. In reality, there are a number of smaller businesses that Disney is in that are embedded in these four businesses including:

- *Cruise lines*: Disney operates two ships – Disney Magic and Disney Wonder – that operate out of Florida and visit Caribbean ports.
- *Internet operations*: Disney made extensive investments in the GO network and other online operations. While much of this investment was written off by 2002, they still represent a potential source of future revenues.

- *Sports franchises*: Disney owns the National Hockey League franchise, the Mighty Ducks of Anaheim; in 2002 it sold it's stake in the Anaheim Angels, a Major League Baseball team.

Absent detailed information on the operations of these businesses, we will assume that they represent too small a portion of Disney's overall revenues to make a significant difference in the risk calculation. For the four businesses for which we have detailed information, we estimated the unlevered beta by looking at comparable firms in each business. Table 4.7 summarizes the comparables used and the unlevered beta for each of the businesses.

Table 4.7: Estimating Unlevered Betas for Disney's Business Areas

Business	*Comparable firms*	*Number of firms*	*Average levered beta*	*Median D/E*	*Unlevered beta*	*Cash/Firm Value*	*Unlevered beta corrected for cash*
Media Networks	Radia and TV broadcasting companies	24	1.22	20.45%	1.0768	0.75%	1.0850
Parks and Resorts	Theme park & Entertainment firms	9	1.58	120.76%	0.8853	2.77%	0.9105
Studio Entertainment	Movie companies	11	1.16	27.96%	0.9824	14.08%	1.1435
Consumer Products	Toy and apparel retailers; Entertainment software	77	1.06	9.18%	0.9981	12.08%	1.1353

To obtain the beta for Disney, we have to estimate the weight that each business is of Disney as a company. The value for each of the divisions was estimated by applying the typical revenue multiple at which comparable firm trade at to the revenue reported by Disney for that segment in 2003.[38] The unlevered beta for Disney as a company in 2003

[38] We first estimated the enterprise value for each firm by adding the market value of equity to the book value of debt and subtracting out cash. We divided the aggregate enterprise value by revenues for all of the comparable firms to obtain the multiples. We did not use the averages of the revenue multiples of the individual firms because a few outliers skewed the results. While Disney has about $1.2 billion in cash, it represents about 1.71% of firm value and will have a negligible impact on the beta. We have ignored it in computing the beta for Disney's equity.

is a value-weighted average of the betas of each of the different business areas. Table 4.8 summarizes this calculation.

Table 4.8: Estimating Disney's Unlevered Beta

Business	*Revenues in 2002*	*EV/Sales*	*Estimated Value*	*Firm Value Proportion*	*Unlevered beta*
Media Networks	$10,941	3.41	$37,278.62	49.25%	1.0850
Parks and Resorts	$6,412	2.37	$15,208.37	20.09%	0.9105
Studio Entertainment	$7,364	2.63	$19,390.14	25.62%	1.1435
Consumer Products	$2,344	1.63	$3,814.38	5.04%	1.1353
Disney	$27,061		$75,691.51	100.00%	1.0674

The equity beta can then be calculated using the current financial leverage for Disney as a firm. Combining the market value of equity of $ 55,101 million an estimated market value of debt of $14,668 million[39], we arrive at the current beta for Disney:

Equity Beta for Disney = 1.0674 (1+(1-.373)(14, 668/55,101) = 1.2456

This contrasts with the beta of 1.01 that we obtained from the regression, and is, in our view, a much truer reflection of the risk in Disney.

In Practice: Can't find comparable firms?

A problem faced by analysts using the bottom up approach for some firms is a paucity of comparable firms, either because the firm is unique in terms of the product it offers or because the bulk of the firms in the sector are private businesses. Rather than fall back on the regression approach, which is likely to yield a very wide range for the beta, we would suggest the following to expand the comparable firm sample:

- *Geographic expansion*: When analyzing firms from smaller markets, such as Brazil or Greece, the number of comparable firms will be small if we restrict ourselves only to firms in the market. One way to increase sample size is to consider firms in the same business that are listed and traded in other markets – European markets for Greece and Latin American markets for Brazil. With commodity companies that trade in global markets, like paper and oil companies, we can consider a global sample.

[39] The details of this calculation will be explored later in this chapter.

- *Production Chain*: Another way to expand the sample is to look for firms that either provide supplies to the firm that you are analyzing or firms that feed off your firm. For instance, when analyzing book retailers, we can consider book publishers as part of the sample since the fortunes of the two are entwined. It is unlikely that one of these groups can have a good year without the other partaking in the success
- *Customer specialization*: Using the same rationale, the betas of firms that derive the bulk of their revenues from a sector may be best estimated using firms in the sector. Thus, the beta of a law firm that derives all of its revenues from investment banks may be best estimated by looking at the betas of investment banks.

Illustration 4.6: Bottom-up Beta for Bookscape Books

We cannot estimate a regression beta for Bookscape Books, the private firm, since it does not have a history of past prices. We can, however, estimate the beta for Bookscape Books, using the bottom up approach. Since we were able to find only three publicly traded book retailers in the United States, we expanded the sample to include book publishers. We list the betas of these firms as well as debt, cash and equity values in Table 4.9:

Table 4.9: Betas and Leverage of Publicly Traded Book Retailers and Publishers

Firm	*Beta*	*Debt*	*Equity*	*Cash*
Books-A-Million	0.532	$45	$45	$5
Borders Group	0.844	$182	$1,430	$269
Barnes & Noble	0.885	$300	$1,606	$268
Courier Corp	0.815	$1	$285	$6
Info Holdings	0.883	$2	$371	$54
John Wiley &Son-A	0.636	$235	$1,662	$33
Scholastic Corp	0.744	$549	$1,063	$11
	0.7627	$1,314	$6,462	$645

While the firms in this sample are very different in terms of market capitalization, the betas are consistent. To estimate the unlevered beta for the sector, we use the average beta across the firms in conjunction with the aggregate values of debt and market value of equity (with a marginal tax rate of 35%):

Debt to Equity Ratio for industry = 1314/6462 = 20.33%

Unlevered Beta = 0.7627/(1+(1-.35)(.2033)) = 0.6737

To correct for cash, we use the aggregate cash balance across the firms:

Unlevered Beta corrected for cash = 0.6737 (1 – 645/(1314+6462)) = 0.7346

Since Bookscape has a negligible cash balance, the unlevered beta for book retailing is also the unlevered beta for the firm.

Since the debt/equity ratios used are market debt equity ratios, and the only debt equity ratio we can compute for Bookscape is a book value debt equity ratio, we have assumed that Bookscape is *close to the industry average* debt to equity ratio of 20.33%. Using a marginal tax rate of 40% (based upon personal income tax rates) for Bookscape, we get a levered beta of 0.82.

Levered beta for Bookscape = 0.7346 (1 +(1-.40) (.2033)) = 0.82

Illustration 4.7: Bottom up Beta for Aracruz

The bottom up beta for Aracruz is difficult to estimate if we remain within its home market, which is Brazil, for two reasons. First, there are only three publicly traded firms within the market that are in the same line of business as Aracruz (i.e. paper and pulp production). Second, the betas for all Brazilian firms are unreliable because the index used to estimate these betas, the Bovespa, is a narrow one, dominated by a few large companies.

There are three groups of comparable firms that we can use as comparable firms in the bottom-up beta estimate:

- *Emerging Market Paper and Pulp companies*: This is a much larger sample of firms. While the individual firm betas may skewed by the limitations of the local indices, the errors should average out over the sample.
- *U.S Paper and Pulp companies*: The advantage gained is not just in terms of the number of firms but also in terms of reliable betas. The peril in this approach is that the risk in U.S. companies can be different from the risk in Brazilian because of regulatory differences.[40]
- *Global Paper and Pulp companies*: This is the largest group and includes a diverse group of companies in both emerging and developed markets. Since betas

are measures of relative risk, we would argue that barring significant differences in regulation and monopoly power across markets, it is reasonable to compare betas across markets.

	Number of firms	*Average Beta*	*D/E*	*Unlevered Beta*	*Cash/Value*	*Unlevered Beta corrected for cash*
Emerging Markets	111	0.6895	38.33%	0.5469	6.58%	0.5855
US	34	0.7927	83.57%	0.5137	2.09%	0.5246
Global	288	0.6333	38.88%	0.5024	6.54%	0.5375

The tax rates used were 32% for emerging market companies, 35% for U.S. companies and 33% for Global companies, based upon averaging the marginal tax rates in each group. The unlevered beta of emerging market companies is slightly higher than the U.S. and global groupings. While the average beta for U.S. companies is higher than the rest of the sample, the difference is entirely due to the higher debt to equity ratios of these companies. We will use an emerging market unlevered beta of 0.59 as the beta for the paper and pulp business that Aracruz is involved in.

We can estimate the unlevered beta for Aracruz in two steps. First, we consider the asset composition for Aracruz. In addition to being in the paper business, Aracruz has a cash balance of 1,018 million BR, which is roughly 7.07% of the firm value. Since this is much larger than the typical cash balances of the companies on our comparable firm list, and the beta of cash is zero, the unlevered beta for Aracruz can be estimated as follows:

Unlevered Beta for Aracruz = (0.9293) (0.585) + (0.0707) (0) = 0.5440

Aracruz had gross debt outstanding of 4.093 million BR at the end of 2003 and a market value of equity of 9,189 million BR leading to a debt/equity ratio of 44.59%. Allowing for a tax rate of 34% (the Brazilian marginal tax rate), the levered beta for Aracruz can then be estimated as follows:

Levered Beta for Aracruz = 0.5440 (1+ (1-.34) (.4459)) = 0.7040

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[40] As a counterpoint, paper and pulp companies are commodity companies and are governed by the vagaries of the price of paper and pulp. In other words, there is a reasonable argument to be made that paper and pulp companies globally are governed by the same primary risk factors.

If we wanted a levered beta for just the paper business of Aracruz, we would use the levered beta for the paper and pulp business and the gross debt to equity ratio for the firm:

Levered Beta for paper business = 0.585 (1+ (1-.34) (.4459))) = 0.7576

In Practice: Gross Debt or Net Debt

Many analysts in Europe and Latin America prefer to subtract the cash from the gross debt to arrive at a net debt figure. While there is no conceptual problem with this approach, they should remain consistent. Consider, the calculation of unlevered and levered betas in illustration 4.7. First, the computation of unlevered beta for the emerging market paper and pulp companies would have been based upon the net debt to equity ratio for firms in the sector rather than the debt to equity ratio:

Net Debt/Equity = (Gross Debt – Cash)/ Equity = 29.22%

Unlevered Beta = Levered Beta / (1 + (1- tax rate) (Net D/E))

= 0.6895/ (1 + (1-.32)(.2922)) = 0.5751

This unlevered beta is already corrected for cash and no further adjustments are needed. To make the levered beta calculation for Aracruz, we would use the net debt to equity ratio for the company. The net debt is computed by subtracting Aracruz's cash balance of 1,018 million BR from its gross debt of 4,093 million BR yielding a net debt to equity ratio of 33.47%.

Levered Beta for Aracruz = Unlevered Beta (1 + (1 – tax rate) (Net D./E))

= 0.5751 (1 + (1-.34)(.3347)) = 0.7022

Again, we can dispense with the adjustment for cash since the net debt to equity ratio captures the cash holdings.

Notice that the levered beta of 0.7040 computed for Aracruz in illustration 4.7 does not exactly match the computation using the net debt to equity ratio. The reason lies in an implicit assumption that we make when we net cash against debt. We assume that both debt and cash are riskless and that the tax benefit from debt is exactly offset by the tax paid on interest earned on cash. It is generally not a good idea to net debt if the debt is very risky or if the interest rate earned on cash is substantially lower than the interest rate paid on debt.

With a net debt to equity ratio, there is one more potential complication. Any firm that has a cash balance that exceeds its debt will have negative net debt and using this negative net D/E ratio will yield an unlevered beta that exceeds the levered beta. While this may trouble some, it makes sense because the unlevered beta reflects the beta of the business that the firm operates in. Firms that have vast cash balances that exceed their borrowing can have levered betas that are lower than the unlevered betas of the businesses they operate in.

Illustration 4.8: Bottom Up Beta for Deutsche Bank

There are a few banks in Germany that can be viewed as competitors to Deutsche Bank, though none of them are as large as it is, or have as large of a stake in investment banking. Since the rules and regulatory constraints governing banking in the United States are different from the rules governing banks in much of the Eurozone, we will look at the betas of European banks with market capitalizations exceeding $ 5 billion to estimate the beta for the commercial banking arm of Deutsche Bank. To estimate the beta of Deutsche Bank's investment banking arm, which includes Morgan Grenfell and Banker's Trust, we use the betas of investment banking firms in the United States. The results are presented below:

Comparable Firms	*Number of firms*	*Average Beta*
Large commercial Banks in Europe	58	0.7345
U.S. investment banks	9	1.5167

Note that we do not adjust for differences in leverage, since regulatory constraints and the needs of the business keep the leverage of most commercial banks at similar levels.[41] The beta for Deutsche Bank as a firm can be estimated as a weighted average of these two betas. Using estimating market value weights of 69% for the commercial banking and

[41] Regulators often specify capital ratios, specified in terms of book values of debt and equity that banks must meet to stay in business. Most banks stay close to these ratios though some tend to be better capitalized than others.

31% for the investment banking arms (based upon the revenues that Deutsche Bank made from each in the most recent year), we arrive at a beta for Deutsche Bank's equity:[42]

Deutsche Bank's beta = 0.7345 (0.69) + 1.5167 (0.31) = 0.9767

This beta will change over time as the weights on the businesses change.

Calculating Betas after A Major Restructuring

The bottom-up process of estimating betas provides a solution when firms go through a major restructuring, where they change both their financial mix and leverage. In these cases, the regression betas are misleading because they do not reflect fully the effects of these changes. Disney's beta, estimated from the bottom up approach, is likely to provide a more precise estimate than the beta from a regression, given Disney's acquisition of Capital Cities and its increase in leverage. In fact, a firm's beta can be estimated even before the restructuring becomes effective using this approach. In the illustration that follows, for instance, we estimate Disney's beta just before and after its acquisition of Capital Cities/ABC, allowing for the changes in both the business mix and the leverage.

Illustration 4.9: Beta of a Firm After an Acquisition: Disney / Capital Cities

In 1995, Disney announced that it was acquiring Capital Cities, the owner of the ABC television and radio network, for approximately $ 120 per share, and that it would finance the acquisition partly through the issue of $ 10 billion in debt. At the time of the acquisition, Disney had a market value of equity of $31.1 billion, debt outstanding of $3.186 billion and a levered beta of 1.15. Capital Cities, based upon the $120 offering price, had a market value of equity of $18.5 billion, debt outstanding of $ 615 million and a levered beta of 0.95.

In order to evaluate the effects of the acquisition on Disney's beta, we do the analysis in two parts. First, we examine the effects of the merger on the business risk of the combined firm, by estimating the unlevered betas of the two companies, and calculating the combined firm's unlevered beta.

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[42] Deutsche Bank does not explicitly break down income into commercial banking and investment banking components. The firm reported 5,470 million in Euros in trading revenues (investment banking), $ 15,179 million in net interest revenues and fiduciary commissions (commercial banking).

Disney's unlevered beta = 1.15/(1+0.64*0.10) = 1.08

Capital Cities unlevered beta = 0.95/(1+0.64*0.03) = 0.93

The unlevered beta for the combined firm can be calculated as the weighted average of the two unlevered betas, with the weights being based upon the market values of the two firms.[43]

Value of Disney = 31,100 + 3,186 = $ 34, 286 million

Value of Capital Cities = 18,500 + 615 = $ 19, 115 million

Unlevered Beta for combined firm = 1.08 (34286/53401) + 0.93 (19115/53401)

= 1.026

Then, we examine the effects of the financing of the merger on the betas, by calculating the debt/equity ratio for the combined firm after the acquisition. Since Disney is assuming the old debt of Capital Cities, we add that debt to Disney's existing debt and add the additional $ 10 billion in debt used to fund this acquisition:[44]

Debt = Capital Cities Old Debt + Disney's Old Debt + New Debt

= $ 615 + $ 3,186 + $ 10,000 = $ 13,801 million

Equity = Disney's Old Equity + New Equity used for Acquisition

= $ 31,100 + $ 8,500 = $ 39,600 million

where New Equity = Total Cost of Acquisition - New Debt Issued

= $ 18,500 - $ 10,000 = $ 8,500 million

Notice that the equity in Capital Cities of $18,500 million disappears after the acquisition and is replaced with new debt of $ 10,000 million and new Disney equity of $ 8,500 million. The debt/equity ratio can then be computed as follows –

D/E Ratio = 13,801/39600 = 34.82%

This debt/equity ratio in conjunction with the new unlevered beta for the combined firm yields a new beta of

New Beta = 1.026 (1 + 0.64 (.3482)) = 1.25

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[43] Unlevered betas should always be weighted based upon firm values. With levered (equity) betas, the values of equity can be used as weights.

[44] If Disney had paid off Capital Cities' old debt instead of assuming it, we could have ignored it in the debt calculation. However, Disney would then have had to raise an extra $615 million in financing to fund this acquisition.

C. Accounting Betas

A third approach is to estimate the market risk parameters from accounting earnings rather than from traded prices. Thus, changes in earnings at a division or a firm, on a quarterly or annual basis, can be regressed against changes in earnings for the market, in the same periods, to arrive at an estimate of a "market beta" to use in the CAPM. While the approach has some intuitive appeal, it suffers from three potential pitfalls. First, accounting earnings tend to be smoothed out relative to the underlying value of the company, resulting in betas that are "biased down", especially for risky firms, or "biased up", for safer firms. In other words, betas are likely to be closer to one for all firms using accounting data. Second, accounting earnings can be influenced by non-operating factors, such as changes in depreciation or inventory methods, and by allocations of corporate expenses at the divisional level. Finally, accounting earnings are measured, at most, once every quarter, and often only once every year, resulting in regressions with few observations and not much power.

Illustration 4.10: Estimating Accounting Betas — Bookscape Books

Bookscape Books, even though it is a private business, has been in existence since 1980 and has accounting earnings going back to that year. Table 4.10 summarizes accounting earnings changes at Bookscape and for the S&P 500 for each year since 1980.

Table 4.10: Earnings for Bookscape versus S&P 500

Year	S&P 500	Bookscape
1980	3.01%	3.55%
1981	1.31%	4.05%
1982	-8.95%	-14.33%
1983	-3.84%	47.55%
1984	26.69%	65.00%
1985	-6.91%	5.05%
1986	-7.93%	8.50%
1987	11.10%	37.00%
1988	42.02%	45.17%
1989	5.52%	3.50%
1990	-9.58%	-10.50%
1991	-12.08%	-32.00%
1992	-5.12%	55.00%
1993	9.37%	31.00%
1994	36.45%	21.06%

1995	30.70%	11.55%
1996	1.20%	19.88%
1997	10.57%	16.55%
1998	-3.35%	7.10%
1999	18.13%	14.40%
2000	15.13%	10.50%
2001	-14.94%	-8.15%
2002	6.81%	4.05%
2003	14.63%	12.56%

Regressing the changes in profits at Bookscape against changes in profits for the S&P 500 yields the following:

Bookscape Earnings Change = 0.1003 + 0.7329 (S & P 500 Earnings Change)

Based upon this regression, the beta for Bookscape is 0.73. In calculating this beta, we used net income to arrive at an equity beta. Using operating earnings for both the firm and the S&P 500 should yield the equivalent of an unlevered beta.

Technically, there is no reason why we cannot estimate accounting betas for Disney, Aracruz Cellulose and Deutsche Bank. In fact, for Disney, we could get net income numbers every quarter, which increases the data that we have in the regression. We could even estimate accounting betas by division, since the divisional income is reported. We do not attempt to estimate accounting betas for the following reasons:

1. To get a sufficient number of observations in our regression, we would need to go back in time at least 10 years and perhaps more. The changes that many large companies undergo over time make this a hazardous exercise.
2. Publicly traded firms smooth out accounting earnings changes even more than private firms do. This will bias the beta estimates downwards.

 spearn.xls: This data set on the web has earnings changes, by year, for the S&P 500 going back to 1960.

Market, Fundamental and Accounting Betas: Which one do we use?

For most publicly traded firms, betas can be estimated using accounting data, market data or from fundamentals. Since the betas will almost never be the same, the question then becomes one of choosing between these betas. We would almost never use

accounting betas, for all of the reasons specified above. We are almost as reluctant to use historical market betas for individual firms because of the standard errors in beta estimates, the failures of the local indices and the inability of these regressions to reflect the effects of major changes in the business and financial risk at the firm. Fundamental betas, in our view, provide us with the best beta estimates because they are not only more precise (because of the averaging) but also because they allow us to reflect changes in business and financial mix. In summary, we will use the fundamental estimates of equity betas of 1.25 for Disney, 0.82 for Bookscape, 0.70 for Aracruz and 0.98 for Deutsche Bank.

IV. Estimating the Cost of Equity

Having estimated the riskfree rate, the risk premium(s) and the beta(s), we can now estimate the expected return from investing in equity at any firm. In the CAPM, this expected return can be written as:

Expected Return = Riskfree Rate + Beta * Expected Risk Premium

where the riskfree rate would be the rate on a long term government bond, the beta would be either the historical, fundamental or accounting betas described above and the risk premium would be either the historical premium or an implied premium.

In the arbitrage pricing and multi-factor model, the expected return would be written as follows:

$$\text{Expected Return} = \text{Riskfree Rate} + \sum_{j-1}^{j=n} \beta_j * \text{Risk Premium}_j$$

where the riskfree rate is the long term government bond rate, β_j is the beta relative to factor j, estimated using historical data or fundamentals, and Risk Premium_j is the risk premium relative to factor j, estimated using historical data.

The expected return on an equity investment in a firm, given its risk, has key implications for both equity investors in the firm and the managers of the firm. For equity investors, it is the *rate that they need to make* to be compensated for the risk that they have taken on investing in the firm. If after analyzing an investment, they conclude that they cannot make this return, they would not buy this investment; alternatively, if they decide they can make a higher return, they would make the investment. For managers in

the firm, the return that investors need to make to break even on their equity investments becomes the return that they have to try and deliver to keep these investors from becoming restive and rebellious. Thus, it becomes the rate that they have to beat in terms of returns on their equity investments in individual project. In other words, this is the *cost of equity* to the firm.

Illustration 4.11: Estimating the Cost of Equity

In illustration 4.5, we estimated a bottom-up levered beta for Disney and each of its divisions. Using the prevailing treasury bond rate of 4% and the historical risk premium of 4.82% from table 4.2, we estimate the cost of equity for Disney as a company and for each of its divisions:

Table 4.11: Levered Beta and Cost of Equity: Disney

Business	*Unlevered Beta*	*D/E Ratio*	*Levered Beta*	*Cost of Equity*
Media Networks	1.0850	26.62%	1.2661	10.10%
Parks and Resorts	0.9105	26.62%	1.0625	9.12%
Studio Entertainment	1.1435	26.62%	1.3344	10.43%
Consumer Products	1.1353	26.62%	1.3248	10.39%
Disney	1.0674	26.62%	1.2456	10.00%

Note that since none of the divisions carry their own debt, we have assumed that they are all funded using the same mix of debt and equity as Disney as a company.[45] The costs of equity vary across the remaining divisions, with studio entertainment having the highest beta and parks and resorts the lowest.[46]

To estimate the cost of equity for Deutsche Bank, we will use the same risk premium (4.82%) that we have used for the U.S, since Deutsche's business is still primarily in mature markets in Europe and the United States. Using the 10-year German Euro bond rate of 4.05% as the Euro riskfree rate[47] and Deutsche Bank's bottom up beta of 0.98, the cost of equity for Deutsche Bank is:

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[45] Disney provides no breakdown of debt by division. If it did, we could use division specific debt to equity ratios.

[46] If we consider cash as a division, the cost of equity is the riskfree rate because cash is invested in commercial paper and treasuries.

[47] There are about 8 countries that issue 10-year Euro denominated bonds. We used the German Euro bond rate as the riskfree rate, not because Deutsche Bank was a German company, but because the German Euro bond rate was the lowest of the government bond rates. The Greek and Spanish 10-year Euro bond rates

Table 4.12: Cost of Equity for Deutsche Bank

Business	*Beta*	*Cost of Equity*	*Weights*
Commercial Banking	0.7345	7.59%	69.03%
Investment Banking	1.5167	11.36%	30.97%
Deutsche Bank		8.76%	

Note that the cost of equity for investment banking is significantly higher than the cost of equity for commercial banking, reflecting the higher risks.

For Aracruz, we will add the country risk premium estimated for Brazil of 7.67%, estimated earlier in the chapter, to the mature market premium, estimated from the U.S, of 4.82% to arrive at a total risk premium of 12.49%. The cost of equity in U.S. dollars for Aracruz as a company can then be computed using the bottom up beta estimated in illustration 4.7:

Cost of Equity = Riskfree Rate in US $ + Beta * Risk Premium

= 4% + 0.7040 (12.49%) = 12.79%

As an emerging market company, Aracruz clearly faces a much higher cost of equity than its competitors in developed markets. We can also compute a cost of equity for Aracruz in real terms, by using a real riskfree rate in this calculation. Using the 10-year inflation-index U.S. treasury bond of 2% as the real riskfree rate, Aracruz's real cost of equity is:

Cost of Equity = 2% + 0.7040 (13.70%) = 10.79%

If we want to compute the cost of equity in nominal BR terms, the adjustment is more complicated and requires estimates of expected inflation rates in Brazil and the United States. If we assume that the expected inflation in BR is 8% and in U.S. dollars is 2%, the cost of equity in BR terms is:

$$\text{Cost of Equity in BR} = (1+\text{Cost of Equity in \$})\frac{(1+\text{Inflation Rate}_{\text{Brazil}})}{(1+\text{Inflation Rate}_{\text{US}})} - 1$$

$$= (1.1279)\ \frac{(1.08)}{(1.02)} - 1 = .1943 \text{ or } 19.43\%$$

Note that these estimates of cost of equity are affected by the cash holdings of Aracruz. We can estimate the cost of equity for the paper and pulp business of Aracruz

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were about 0.20% higher, reflecting the perception of default risk in those countries. We would continue to

(independent of the cash holdings) by using the levered beta of 0.7576 for the business estimated in illustration 4.7:

Real Cost of Equity (paper business) = 2.00% + 0.7576 (12.49%) = 11.46%

US $ Cost of Equity (paper business) = 4.00% + 0.7576 (12.49%) = 13.46%

Nominal BR Cost of Equity (paper business) $1.1346 \frac{(1.08)}{(1.02)} - 1 = 20.14\%$

Finally, for Bookscape, we will use the beta of 0.82 estimated from illustration 4.6 in conjunction with the riskfree rate and risk premium for the US:

Cost of Equity = 4% + 0.82 (4.82%) = 7.73%

This cost of equity may seem incongruously low for a small, privately held business but it is legitimate if we assume that the only risk that matters is non-diversifiable risk.

In Practice: Risk, Cost of Equity and Private Firms

Implicit in the use of beta as a measure of risk is the assumption that the marginal investor in equity is a well diversified investor. While this is a defensible assumption when analyzing publicly traded firms, it becomes much more difficult to sustain for private firms. The owner of a private firm generally has the bulk of his or her wealth invested in the business. Consequently, he or she cares about the total risk in the business rather than just the market risk. Thus, for a business like Bookscape, the beta that we have estimated of 0.82 (leading to a cost of equity of 7.73%) will understate the risk perceived by the owner of Bookscape. There are two solutions to this problem:

1. Assume that the business is run with the near-term objective of sale to a large publicly traded firm. In such a case, it is reasonable to use the market beta and cost of equity that comes from it.
2. Add a premium to the cost of equity to reflect the higher risk created by the owner's inability to diversify. This may help explain the high returns that some venture capitalists demand on their equity investments in fledgling businesses.

Adjust the beta to reflect total risk rather than market risk. This adjustment is a relatively simple one, since the R squared of the regression measures the proportion of the risk that is market risk. Dividing the market beta by the square root of the R squared (which is the correlation coefficient) yields a total beta. In the Bookscape example,

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use the German Euro bond rate to value Greek and Spanish companies in Euros.

the regressions for the comparable firms against the market index have an average R squared of about 16%. The total beta for Bookscape can then be computed as follows:

$$\text{Total Beta} = \frac{\text{Market Beta}}{\sqrt{\text{R squared}}} = \frac{0.82}{\sqrt{.16}} = 2.06$$

Using this total beta would yield a much higher and more realistic estimate of the cost of equity.

$$\text{Cost of Equity} = 4\% + 2.06\ (4.82\%) = 13.93\%$$

Thus, private businesses will generally have much higher costs of equity than their publicly traded counterparts, with diversified investors. While many of them ultimately capitulate by selling to publicly traded competitors or going public, some firms choose to remain private and thrive. To do so, they have to diversify on their own (as many family run businesses in Asia and Latin America did) or accept the lower value as a price paid for maintaining total control.

From Cost of Equity to Cost of Capital

While equity is undoubtedly an important and indispensable ingredient of the financing mix for every business, it is but one ingredient. Most businesses finance some or much of their operations using debt or some hybrid of equity and debt. The costs of these sources of financing are generally very different from the cost of equity, and the minimum acceptable hurdle rate for a project will reflect their costs as well, in proportion to their use in the financing mix. Intuitively, the *cost of capital* is the weighted average of the costs of the different components of financing -- including debt, equity and hybrid securities -- used by a firm to fund its financial requirements.

☞ 4.9: Interest Rates and the Relative Costs of Debt and Equity

It is often argued that debt becomes a more attractive mode of financing than equity as interest rates go down and a less attractive mode when interest rates go up. Is this true?

a. Yes

b. No

Why or why not?

The Costs of Non-Equity Financing

To estimate the cost of the funding that a firm raises, we have to estimate the costs of all of the non-equity components. In this section, we will consider the cost of debt first and then extend the analysis to consider hybrids such as preferred stock and convertible bonds.

The Cost of Debt

The cost of debt measures the current cost to the firm of borrowing funds to finance projects. In general terms, it is determined by the following variables:

(1) The current level of interest rates: As interest rates rise, the cost of debt for firms will also increase.

(2) The default risk of the company: As the default risk of a firm increases, the cost of borrowing money will also increase.

Default Risk: This is the risk that a firm will fail to make obligated debt payments, such as interest expenses or principal payments.

(3) The tax advantage associated with debt: Since interest is tax deductible, the after-tax cost of debt is a function of the tax rate. The tax benefit that accrues from paying interest makes the after-tax cost of debt lower than the pre-tax cost. Furthermore, this benefit increases as the tax rate increases.

After-tax cost of debt = Pre-tax cost of debt (1 – marginal tax rate)

☞ **4.10: Costs of Debt and Equity**

Can the cost of equity ever be lower than the cost of debt for any firm at any stage in its life cycle?

a. Yes

b. No

Estimating the Default Risk and Default Spread of a firm

The simplest scenario for estimating the cost of debt occurs when a firm has long-term bonds outstanding that are widely traded. The market price of the bond, in conjunction with its coupon and maturity can serve to compute a yield we use as the cost of debt. For instance, this approach works for firms that have dozens of outstanding bonds that are liquid and trade frequently.

Many firms have bonds outstanding that do not trade on a regular basis. Since these firms are usually rated, we can estimate their costs of debt by using their ratings and associated default spreads. Thus, Disney with a BBB+ rating can be expected to have a cost of debt approximately 1.25% higher than the treasury bond rate, since this is the spread typically paid by BBB+ rated firms.

Some companies choose not to get rated. Many smaller firms and most private businesses fall into this category. While ratings agencies have sprung up in many emerging markets, there are still a number of markets where companies are not rated on the basis of default risk. When there is no rating available to estimate the cost of debt, there are two alternatives:

- *Recent Borrowing History:* Many firms that are not rated still borrow money from banks and other financial institutions. By looking at the most recent borrowings made by a firm, we can get a sense of the types of default spreads being charged the firm and use these spreads to come up with a cost of debt.
- *Estimate a synthetic rating and default spread*: An alternative is to play the role of a ratings agency and assign a rating to a firm based upon its financial ratios; this rating is called a synthetic rating. To make this assessment, we begin with rated firms and examine the financial characteristics shared by firms within each ratings class. Consider a very simpler version, where the ratio of operating income to interest expense, i.e., the interest coverage ratio, is computed for each rated firm. In table 4.12, we list the range of interest coverage ratios for small manufacturing firms in each S&P ratings class[48]. We also report the typical default spreads for bonds in each ratings class.[49]

Table 4.12: Interest Coverage Ratios and Ratings

Interest Coverage Ratio	*Rating*	*Typical default spread*
> 12.5	AAA	0.35%

[48] This table was developed in early 2000, by listing out all rated firms, with market capitalization lower than $ 2 billion, and their interest coverage ratios, and then sorting firms based upon their bond ratings. The ranges were adjusted to eliminate outliers and to prevent overlapping ranges.

[49] These default spreads are obtained from an online site: http://www.bondsonline.com. You can find default spreads for industrial and financial service firms; these spreads are for industrial firms.

9.50 - 12.50	AA	0.50%
7.50 – 9.50	A+	0.70%
6.00 – 7.50	A	0.85%
4.50 – 6.00	A-	1.00%
4.00 – 4.50	BBB	1.50%
3.50 - 4.00	BB+	2.00%
3.00 – 3.50	BB	2.50%
2.50 – 3.00	B+	3.25%
2.00 - 2.50	B	4.00%
1.50 – 2.00	B-	6.00%
1.25 – 1.50	CCC	8.00%
0.80 – 1.25	CC	10.00%
0.50 – 0.80	C	12.00%
< 0.65	D	20.00%

Source: Compustat and Bondsonline.com

Now consider a private firm with $ 10 million in earnings before interest and taxes and $3 million in interest expenses; it has an interest coverage ratio of 3.33. Based on this ratio, we would assess a "synthetic rating" of BB for the firm and attach a default spread of 2.50% to the riskfree rate to come up with a pre-tax cost of debt.

By basing the synthetic rating on the interest coverage ratio alone, we run the risk of missing the information that is available in the other financial ratios used by ratings agencies. The approach described above can be extended to incorporate other ratios. The first step would be to develop a score based upon multiple ratios. For instance, the Altman Z score, which is used as a proxy for default risk, is a function of five financial ratios, which are weighted to generate a Z score. The ratios used and their relative weights are usually based upon past history on defaulted firms. The second step is to relate the level of the score to a bond rating, much as we have done in table 4.12 with interest coverage ratios. In making this extension, though, note that complexity comes at a cost. While credit or Z scores may, in fact, yield better estimates of synthetic ratings than those based only upon interest coverage ratios, changes in ratings arising from these scores are much more difficult to explain than those based upon interest coverage ratios. That is the reason we prefer the flawed but simpler ratings that we get from interest coverage ratios.

Short Term and Long Term Debt

Most publicly traded firms have multiple borrowings – short term and long term bonds and bank debt with different terms and interest rates. While there are some analysts who create separate categories for each type of debt and attach a different cost to each category, this approach is both tedious and dangerous. Using it, we can conclude that short-term debt is cheaper than long term debt and that secured debt is cheaper than unsecured debt, even though neither of these conclusions is justified.

The solution is simple. Combine all debt – short and long term, bank debt and bonds- and attach the long term cost of debt to it. In other words, add the default spread to the long term riskfree rate and use that rate as the pre-tax cost of debt. Firms will undoubtedly complain, arguing that their effective cost of debt can be lowered by using short-term debt. This is technically true, largely because short-term rates tend to be lower than long-term rates in most developed markets, but it misses the point of computing the cost of debt and capital. If this is the hurdle rate we want our long-term investments to beat, we want the rate to reflect the cost of long-term borrowing and not short-term borrowing. After all, a firm that funds long term projects with short-term debt will have to return to the market to roll over this debt.

Operating Leases and Other Fixed Commitments

The essential characteristic of debt is that it gives rise to a tax-deductible *obligation that firms have to meet in both good times and bad and the failure to meet this obligation can result in bankruptcy or loss of equity control over the firm.* If we use this definition of debt, it is quite clear that what we see reported on the balance sheet as debt may not reflect the true borrowings of the firm. In particular, a firm that leases substantial assets and categorizes them as operating leases owes substantially more than is reported in the financial statements.[50] After all, a firm that signs a lease commits to making the

[50] In an operating lease, the lessor (or owner) transfers only the right to use the property to the lessee. At the end of the lease period, the lessee returns the property to the lessor. Since the lessee does not assume the risk of ownership, the lease expense is treated as an operating expense in the income statement and the lease does not affect the balance sheet. In a capital lease, the lessee assumes some of the risks of ownership and enjoys some of the benefits. Consequently, the lease, when signed, is recognized both as an asset and as a liability (for the lease payments) on the balance sheet. The firm gets to claim depreciation each year on the asset and also deducts the interest expense component of the lease payment each year. In general, capital leases recognize expenses sooner than equivalent operating leases.

lease payment in future periods and risks the loss of assets if it fails to make the commitment.

For corporate financial analysis, we should treat all lease payments as financial expenses and convert future lease commitments into debt by discounting them back the present, using the current pre-tax cost of borrowing for the firm as the discount rate. The resulting present value can be considered the debt value of operating leases and can be added on to the value of conventional debt to arrive at a total debt figure. To complete the adjustment, the operating income of the firm will also have to be restated:

Adjusted Operating income = Stated Operating income + Operating lease expense for the current year – Depreciation on leased asset

In fact, this process can be used to convert any set of financial commitments into debt.

Book and Market Interest Rates

When firms borrow money, they do so often at fixed rates. When they issue bonds to investors, this rate that is fixed at the time of the issue is called the coupon rate. The cost of debt is not the coupon rate on outstanding bonds nor is it the rate at which the company was able to borrow at in the past. While these factors may help determine the interest cost the company will have to pay in the current year, they do not determine the pre-tax cost of debt in the cost of capital calculations. Thus, a company that has debt that it took on when interest rates were low, on the books cannot contend that it has a low cost of debt.

To see why, consider a firm that has $ 2 billion of debt on its books and assume that the interest expense on this debt is $ 80 million. The book interest rate on the debt is 4%. Assume also that the current riskfree rate is 6%. If we use the book interest rate of 4% in our cost of capital calculations, we are requiring the projects we fund with the capital to earn more than 4% to be considered good investments. Since we can invest that money in treasury bonds and earn 6%, without taking any risk, this is clearly not a high enough hurdle. To ensure that projects earn more than what we can make on alternative investments of equivalent risk today, the cost of debt has to be based upon market interest rates today rather than book interest rates.

Assessing the Tax Advantage of Debt

Interest is tax deductible and the resulting tax savings reduce the cost of borrowing to firms. In assessing this tax advantage, we should keep in mind that:

- Interest expenses offset the marginal dollar of income and the tax advantage has to be therefore calculated using the marginal tax rate.

  After-tax cost of debt = Pre-tax cost of debt (1 – Marginal Tax Rate)

- To obtain the tax advantages of borrowing, firms have to be profitable. In other words, there is no tax advantage from interest expenses to a firm that has operating losses. It is true that firms can carry losses forward and can offset them against profits in future periods. The most prudent assessment of the tax effects of debt will therefore provide for no tax advantages in the years of operating losses and will begin adjusting for tax benefits only in future years when the firm is expected to have operating profits.

  After-tax cost of debt = Pre-tax cost of debt If operating income < 0
  Pre-tax cost of debt (1-t) If operating income>0

Illustration 4.12: Estimating the Costs of Debt for Disney et al.

Disney, Deutsche Bank and Aracruz are all rated companies and we will estimate their pre-tax costs of debt based upon their rating. To provide a contrast, we will also estimate synthetic ratings for Disney and Aracruz. For Bookscape, we will use the synthetic rating of BBB, estimated from the interest coverage ratio to assess the pre-tax cost of debt.

- Bond Ratings: While S&P, Moody's and Fitch rate all three companies, the ratings are consistent and we will use the S&P ratings and the associated default spreads (from table 3.4 in chapter 3) to estimate the costs of debt in table 4.13:

Table 4.13: Cost of Debt

	S&P Rating	Riskfree Rate	Default Spread	Cost of debt	Tax Rate	After-tax Cost of Debt

Disney	BBB+	4% ($)	1.25%	5.25%	37.3%	3.29%
Deutsche Bank	AA-	4.05% (Eu)[51]	1.00%	5.05%	38%	3.13%
Aracruz[52]	B+	4% ($)	3.25%	7.25%	34%	4.79%

The marginal tax rates of the US (Disney), Brazil (Aracruz) and Germany (Deutsche Bank) are used to compute the after-tax cost of debt. We will assume that all of Disney's divisions have the same cost of debt and marginal tax rate as the parent company.

- Synthetic Ratings: For Bookscape, there are no recent borrowings on the books, thus making the synthetic rating for the firm our only choice. In 2003, Bookscape had no interest expenses and reported operating income of $ 2 million after operating lease expenses of $ 600,000. If we consider the current year's operating operating lease expenses to be the equivalent of interest expenses, the resulting interest coverage ratio is 4.33, yielding a synthetic rating of A- for the firm.[53] Adding the default spread of 1.5% associated with that rating to the riskfree rate results in a pre-tax cost of debt for 5.50%. The after-tax cost of debt is computed using a 40% marginal tax rate:

After-tax cost of debt = 5.5% (1- .40) = 3.30%

Actual and Synthetic Ratings

It is usually easy to estimate the cost of debt for firms that have bond ratings available for them. There are, however, a few potential problems that sometimes arise in practice:

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[51] The default spreads for bonds issued by banks can be very different from the spreads for industrial companies. The default spread for an AA- rated financial service company was much higher at the time of this analysis than the default spread for an AA- rated manufacturing company.

[52] With Araacruz, one troublesome aspect of the pre-tax cost of debt is that it is lower than the rate at which the Brazilian government can borrow. While there are some cases where we would add the default spread of the country to that of the firm to get to a pre-tax cost of debt, Aracruz may be in a stronger position to borrow in U.S. dollars than the Brazilian government because it sells its products in a global market and gets paid in dollars.

[53] To estimate the interest coverage ratio here, we added the operating lease expense back to both the numerator and the denominator:
Interest coverage ratio = (EBIT + Operating lease expense)/ (Interest expense + Operating lease expense)
This is a conservative estimate of the rating. In reality, only a portion of the operating lease expense should be considered as interest expense. This, in turn, will increase the rating and improve the rating. In fact, the synthetic rating with this approach will be A.

- *Disagreement between ratings agencies*: While the ratings are consistent across ratings agencies for many firms, there are a few firms where the ratings agencies disagree with one agency assigning a much higher or lower rating to the firm than the others.
- *Multiple bond ratings for same firm*: Since ratings agencies rate bonds, rather than firms, the same firm can have many bond issues with different ratings depending upon how the bond is structured and secured.
- *Lags or Errors in the Rating Process*: Ratings agencies make mistakes and there is evidence that ratings changes occur after the bond market has already recognized the change in the default risk.

It is a good idea to estimate synthetic ratings even for firms that have actual ratings. If there is disagreement between ratings agencies or a firm has multiple bond ratings, the synthetic rating can operate as a tie-breaker. If there is a significant difference between actual and synthetic ratings and there is no fundamental reason that can be pinpointed for the difference, the synthetic rating may be providing an early signal of a ratings agency mistake.

We computed the synthetic ratings for Disney and Aracruz using the interest coverage ratios:

Disney: Interest coverage ratio = 2,805/758 = 3.70 Synthetic rating = A-

Aracruz: Interest coverage ratio = 888/339= 2.62 Synthetic rating = BBB

While Disney's synthetic rating is close to it's actual rating of BBB+, the synthetic rating for Aracruz is much higher than it's rating of B-. The reason for the discrepancy lies in the fact that Aracruz has two ratings – one for its local currency borrowings of BBB- and one for its dollar borrowings of B+. We used the latter to estimate the cost of debt because almost all of Aracruz's debt is dollar debt. You can also consider the difference to be a reflection of the riskiness of Brazil as a country and the penalty that Aracruz pays for being a Brazilian company. In fact, we can quantify this difference by measuring the difference in interest rates (in US dollar terms) of Aracruz with the synthetic and actual ratings:

Cost of debt with actual rating of B- : 4% + 3.25% = 7.25%

Cost of debt with synthetic rating of BBB: 4% + 1.50% = 5.50%

Country default penalty attached to Aracruz debt = 7.25% - 5.50% = 1.75%

Calculating the Cost of Preferred Stock

Preferred stock shares some of the characteristics of debt - the preferred dividend is pre-specified at the time of the issue and is paid out before common dividend -- and some of the characteristics of equity - the payments of preferred dividend are not tax deductible. If preferred stock is viewed as perpetual, the cost of preferred stock can be written as follows:

k_{ps} = Preferred Dividend per share/ Market Price per preferred share

This approach assumes that the dividend is constant in dollar terms forever and that the preferred stock has no special features (convertibility, callability etc.). If such special features exist, they will have to be valued separately to come up with a good estimate of the cost of preferred stock. In terms of risk, preferred stock is safer than common equity but riskier than debt. Consequently, it should, on a pre-tax basis, command a higher cost than debt and a lower cost than equity.

Illustration 4.13: Calculating the Cost Of Preferred Stock: Disney and Deutsche Bank

Both Disney and Deutsche Bank have preferred stock outstanding. THe preferred dividend yields on the issues are computed in March 2004 in table 4.14:

Table 4.14: Cost of Preferred Stock

Company	*Preferred Stock Price*	*Annual Dividends/share*	*Dividend Yield*
Disney	$ 26.74	$ 1.75	1.75/26.74 = 6.54%
Deutsche Bank	103.75 Euros	6.60 Euros	6.6/103.75 = 6.36%

Notice that the cost of preferred stock for Disney is higher than its pre-tax cost of debt of 5.25% and is lower than its cost of equity of 10%. For Deutsche Bank as well, the cost of preferred stock is higher than its pre-tax cost of debt (5.05%) and is lower than its cost of equity of 8.76%. For both firms, the market value of preferred stock is so small relative to the market values of debt and equity that it makes almost no impact on the overall cost of capital.

☞ **4.11: Why do companies issue preferred stock?**

Which of the following are "good" reasons for a company issuing preferred stock?

a. Preferred stock is cheaper than equity

b. Preferred stock is treated as equity by the ratings agencies and regulators

c. Preferred stock is cheaper than debt

d. Other:

Explain.

Calculating the Cost of Other Hybrid Securities

In general terms, hybrid securities share some of the characteristics of debt and some of the characteristics of equity. A good example is a convertible bond, which can be viewed as a combination of a straight bond (debt) and a conversion option (equity). Instead of trying to calculate the cost of these hybrid securities individually, they can be broken down into their debt and equity components and treated separately.

In general, it is not difficult to decompose a hybrid security that is publicly traded (and has a market price) into debt and equity components. In the case of a convertible bond, this can be accomplished in two ways:

- An option pricing model can be used to value the conversion option and the remaining value of the bond can be attributed to debt.
- The convertible bond can be valued as if it were a straight bond, using the rate at which the firm can borrow in the market, given its default risk (pre-tax cost of debt) as the interest rate on the bond. The difference between the price of the convertible bond and the value of the straight bond can be viewed as the value of the conversion option.

If the convertible security is not traded, we have to value both the straight bond and the conversion options separately.

Illustration 4.14: Breaking down a convertible bond into debt and equity components: Disney

Convertible Debt: This is debt that can be converted into stock at a specified rate, called the conversion ratio.

In March 2004, Disney had convertible bonds outstanding with 19 years left to maturity and a coupon rate of 2.125%, trading at $1,064 a bond. Holders of this bond have the right to convert the bond into 33.9444 shares of

stock anytime over the bond's remaining life.[54] To break the convertible bond into straight bond and conversion option components, we will value the bond using Disney's pre-tax cost of debt of 5.25%:[55]

Straight Bond component

= Value of a 2.125% coupon bond due in 19 years with a market interest rate of 5.25%

= PV of $21.25 in coupons each year for 19 years[56] + PV of $1000 at end of year 19

$$= 21.25\left[\frac{1-(1.0525)^{-19}}{.0525}\right] + \frac{1000}{(1.0525)^{19}} = \$629.91$$

Conversion Option = Market value of convertible – Value of straight bond

= 1064 - $629.91 = $434.09

The straight bond component of $630 is treated as debt, while the conversion option of $434 is treated as equity.

☞ 4.12: Increases in Stock Prices and Convertible Bonds

As stock prices go up, which of the following is likely to happen to the convertible bond (you can choose more than one)

a. The convertible bond will increase in value
b. The straight bond component of the convertible bond will decrease in value
c. The equity component of the convertible bond will increase as a percentage of the total value
d. The straight bond component of the convertible bond will increase as a percentage of the total value

Explain.

Calculating the Weights of Debt and Equity Components

Once we have costs for each of the different components of financing, all we need are weights on each component to arrive at a cost of capital. In this section, we will

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[54] At this conversion ratio, the price that investors would be paying for Disney shares would be $29.46, much higher than the stock price of $20.46 prevailing at the time of the analysis.

[55] This rate was based upon a 10-year treasury bond rate. If the 5-year treasury bond rate had been substantially different, we would have recomputed a pre-tax cost of debt by adding the default spread to the 5-year rate.

[56] The coupons are assumed to be annual. With semi-annual coupons, you would divide the coupon by 2 and apply a semi-annual rate to calculate the present value.

consider the choices for weighting, the argument for using market value weights and whether the weights can change over time.

Choices for Weighting

In computing weights for debt, equity and preferred stock, we have two choices. We can take the accounting estimates of the value of each funding source from the balance sheet and compute book value weights. Alternatively, we can use or estimate market values for each component and compute weights based upon relative market value. *As a general rule, the weights used in the cost of capital computation should be based upon market values*. This is because the cost of capital is a forward-looking measure and captures the cost of raising new funds to finance projects. Since new debt and equity has to be raised in the market at prevailing prices, the market value weights are more relevant.

There are some analysts who continue to use book value weights and justify them using four arguments, none of which are convincing:

- *Book value is more reliable than market value because it is not as volatile*: While it is true that book value does not change as much as market value, this is more a reflection of weakness than strength, since the true value of the firm changes over time as new information comes out about the firm and the overall economy. We would argue that market value, with its volatility, is a much better reflection of true value than is book value.[57]
- *Using book value rather than market value is a more conservative approach to estimating debt ratios*. The book value of equity in most firms in developed markets is well below the value attached by the market, whereas the book value of debt is usually close to the market value of debt. Since the cost of equity is much higher than the cost of debt, the cost of capital calculated using book value ratios

[57] There are some who argue that stock prices are much more volatile than the underlying true value. Even if this argument is justified (and it has not conclusively been shown to be so), the difference between market value and true value is likely to be much smaller than the difference between book value and true value.

will be lower than those calculated using market value ratios, making them less conservative estimates, not more so.[58]

- *Since accounting returns are computed based upon book value, consistency requires the use of book value in computing cost of capital:* While it may seem consistent to use book values for both accounting return and cost of capital calculations, it does not make economic sense. The funds invested in these projects can be invested elsewhere, earning market rates, and the costs should therefore be computed at market rates and using market value weights.

Estimating Market Values

In a world where all funding was raised in financial markets and are securities were continuously traded, the market values of debt and equity should be easy to get. In practice, there are some financing components with no market values available, even for large publicly traded firms, and none of the financing components are traded in private firms.

The Market Value of Equity

The market value of equity is generally the number of shares outstanding times the current stock price. Since it measures the cost of raising funds today, it is not good practice to use average stock prices over time or some other normalized version of the price.

- *Multiple Classes of Shares*: If there is more than one class of shares outstanding, the market values of all of these securities should be aggregated and treated as equity. Even if some of the classes of shares are not traded, market values have to be estimated for non-traded shares and added to the aggregate equity value.
- *Equity Options*: If there other equity claims in the firm - warrants and conversion options in other securities - these should also be valued and added on to the value of the equity in the firm. In the last decade, the use of options as management

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[58] To illustrate this point, assume that the market value debt ratio is 10%, while the book value debt ratio is 30%, for a firm with a cost of equity of 15% and an after-tax cost of debt of 5%. The cost of capital can be calculated as follows –

With market value debt ratios: 15% (.9) + 5% (.1) = 14%
With book value debt ratios: 15% (.7) + 5% (.3) = 12%

compensation has created complications, since the value of these options has to be estimated.

How do we estimate the value of equity for private businesses? We have two choices. One is to estimate the market value of equity by looking at the multiples of revenues and net income at which publicly traded firms trade. The other is to bypass the estimation process and use the market debt ratio of publicly traded firms as the debt ratio for private firms in the same business. This is the assumption we made for Bookscape, where we used the industry average debt to equity ratio for the book/publishing business as the debt to equity ratio for Bookscape.

The Market Value of Debt

The market value of debt is usually more difficult to obtain directly since very few firms have all of their debt in the form of bonds outstanding trading in the market. Many firms have non-traded debt, such as bank debt, which is specified in book value terms but not market value terms. To get around the problem, many analysts make the simplifying assumptions that the book value of debt is equal to its market value. While this is not a bad assumption for mature companies in developed markets, it can be a mistake when interest rates and default spreads are volatile.

A simple way to convert book value debt into market value debt is to treat the entire debt on the books as a coupon bond, with a coupon set equal to the interest expenses on all of the debt and the maturity set equal to the face-value weighted average maturity of the debt, and to then value this coupon bond at the current cost of debt for the company. Thus, the market value of $ 1billion in debt, with interest expenses of $ 60 million and a maturity of 6 years, when the current cost of debt is 7.5% can be estimated as follows:

$$\text{Estimated Market Value of Debt} = 60\left[\frac{(1-\frac{1}{(1.075)^6}}{.075}\right] + \frac{1{,}000}{(1.075)^6} = \$930$$

This is an approximation and that a more accurate computation would require valuing each debt issue separately, using this process. As a final point, we should add the present

value of operating lease commitments to this market value of debt to arrive at an aggregate value for debt in computing the cost of capital.

Can financing weights change over time?

Using the current market values to obtain weights will yield a cost of capital for the current year. But can the weights attached to debt and equity, and the resulting cost of capital, change from year to year? Absolutely, and especially in the following scenarios:

- *Young firms*: Young firms often are all equity funded largely because they do not have the cash flows (or earnings) to sustain debt. As they become larger, increasing earnings and cashflow usually allow for more borrowing. When analyzing firms early in the life cycle, we should allow for the fact that the debt ratio of the firm will probably increase over time towards the industry average.
- *Target Debt Ratios and Changing financing mix*: Mature firms sometimes decide to change their financing strategies, pushing towards target debt ratios that are much higher or lower than current levels. When analyzing these firms, we should consider the expected changes as the firm moves from the current to the target debt ratio.

As a general rule, we should view the cost of capital as a year-specific number, and change the inputs each year. Not only will the weights attached to debt and equity change over time, but so will the estimates of beta and the cost of debt. In fact, one of the advantages of using bottom-up betas is that the beta each year can be estimated as a function of the expected debt to equity ratio that year.

Illustration 4.15: Market value and book value debt ratios: Disney and Aracruz

Disney has a number of debt issues on its books, with varying coupon rates and maturities. Table 4.15 summarizes Disney's outstanding debt:

Table 4.15: Debt at Disney: September 2003

Debt	*Face Value*	*Stated Interest rate*	*Maturity*	*Wtd Maturity*
Commercial Paper	$0	2.00%	0.5	0.0000
Medium term paper	$8,114	6.10%	15	9.2908
Senior Convertibles	$1,323	2.13%	10	1.0099
Other U.S. dollar denominated debt	$597	4.80%	15	0.6836
Privately Placed Debt	$343	7.00%	4	0.1047
Euro medium-term debt	$1,519	3.30%	2	0.2319

Preferred Stock[59]	$485	7.40%	1	0.0370
Cap Cities Debt	$191	9.30%	9	0.1312
Other	$528	3.00%	1	0.0403
Total	$13,100	5.60%		11.5295

To convert the book value of debt to market value, we use the current pre-tax cost of debt for Disney of 5.25% as the discount rate, $13,100 as the book value of debt and the current year's interest expenses of $ 666 million as the coupon:

$$\text{Estimated MV of Disney Debt} = 666\left[\frac{(1-\frac{1}{(1.0525)^{11.53}}}{.0525}\right] + \frac{13{,}100}{(1.0525)^{11.53}} = \$12{,}915 \text{ million}$$

To this amount, we add the present value of Disney's operating lease commitments. This present value is computed by discounting the lease commitment each year at the pre-tax cost of debt for Disney (5.25%):[60]

Year	Commitment	Present Value
1	$ 271.00	$ 257.48
2	$ 242.00	$ 218.46
3	$ 221.00	$ 189.55
4	$ 208.00	$ 169.50
5	$ 275.00	$ 212.92
6 –9	$ 258.25	$ 704.93
Debt Value of leases =		$ 1,752.85

Adding the debt value of operating leases to the market value of debt of $12,915 million yields a total market value for debt of $14,668 million at Disney.

Aracruz has debt with a book value of 3,946 million BR, interest expenses of 339 million BR in the current year and an average maturity for the debt of 3.20 years. Since most of the debt is dollar debt, we used the nominal dollar pre-tax cost of debt for the firm of 7.25% (from illustration 4.12)[61]. The market value of Aracruz debt is:

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[59] Preferred stock should really not be treated as debt. In this case, though, the amount of preferred stock is small that we have included it as part of debt for Disney.

[60] Disney reports total commitments of $715 million beyond year 6. Using the average commitment from year one through five as an indicator, we assumed that this total commitment would take the form of an annuity of $178.75 million a year for four years.

[61] If the debt had been predominantly nominal BR debt, we would have used a nominal BR cost of debt.

$$\text{MV of Aracruz Debt} = 339\left[\frac{(1-\frac{1}{(1.0725)^{3.20}}}{.0725}\right] + \frac{3,946}{(1.0725)^{3.20}} = 4,094 \text{ million BR}$$

There are no lease commitments reported in Aracruz's financial statements.[62]

In table 4.16 we contrast the book values of debt and equity with the market values for Disney and Aracruz. The market value of equity is estimated using the current market price and the number of shares outstanding.

Table 4.16: Book value versus Market Value: Debt Ratios

	BV: Debt	BV: Equity	BV: D/(D+E)	MV: Debt	MV: Equity	MV: D/E
Disney	$13,100	$24,219	35.10%	$14,668	$55,101	21.02%
Aracruz	$3,946	$5,205	43.12%	$4,094	$9,189	30.82%

For Disney, the market value debt ratio of 21.02% is much lower than the book value debt ratio of 35.10%. For Aracruz, the market debt ratio is 30.82%, lower than the book debt ratio of 43.12%.

Bookscape's only debt takes the form of operating lease commitments. The bookstore has a 25 years remaining on a real estate leases, requiring the payment of $500,000 a year. The present value of these operating lease commitments, using a 5.50% pre-tax cost of borrowing, is:

Present value of operating lease commitments = 500 (PV of annuity, 5.5%, 25 years)

= $6.707 million

Bookscape does not have a market value of equity, since it is a private firm. The book value of equity for the firm at the end of 2003 was $ 5 million.

Estimating and using the cost of capital

With the estimates of the costs of the individual components – debt, equity and preferred stock (if any) – and the market value weights of each of the components, the cost of capital can be computed. Thus if E, D and PS are the market values of equity, debt and preferred stock respectively, the cost of capital can be written as follows:

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[62] While many companies outside the United States do no provide details on lease commitments in future years, Aracruz publishes financial statements that use US accounting standards for its ADR listing.

$$\text{Cost of Capital} = k_e \left(E/ (D+E+PS)\right) + k_d \left(D/ (D+E+PS)\right) + k_{ps} \left(PS/ (D+E+PS)\right)$$

The cost of capital is a measure of the composite cost of raising money that a firm faces. It will generally be lower than the cost of equity, which is the cost of just equity funding.

It is a source of confusion to many analysts that both the cost of equity and the cost of capital are used as hurdle rates in investment analysis. The way to resolve this confusion is to recognize when it is appropriate to use each one.

- If we want to adopt the perspective of just the equity investors in a business or a project and measure the returns earned just by these investors on their investment, the cost of equity is the correct hurdle rate to use. In measuring the returns to equity investors then, we have to consider only the income or cashflows left over after all other claimholders needs (interest payments on debt and preferred dividends, for instance) have been met.
- If the returns that we are measuring are composite returns to all claimholders, based upon earnings before payments to debt and preferred stockholders, the comparison should be to the cost of capital.

While these principles are abstract, we will consider them in more detail in the next chapter when we look at examples of projects.

Illustration 4.16: Estimating Cost of Capital

Culminating the analysis in this chapter, we first estimate the costs of capital for each of Disney's divisions, In making these estimates, we use the costs of equity that we obtained for the divisions in illustration 4.11 and Disney's cost of debt from illustration 4.12. We also assume that all of the divisions are funded with the same mix of debt and equity as the parent company. Table 4.17 provides estimates of the costs of capital for the divisions:

Table 4.17: Cost of capital for Disney's divisions

Business	*Cost of Equity*	*After-tax cost of debt*	*E/(D+E)*	*D/(D+E)*	*Cost of capital*
Media Networks	10.10%	3.29%	78.98%	21.02%	8.67%
Parks and Resorts	9.12%	3.29%	78.98%	21.02%	7.90%
Studio Entertainment	10.43%	3.29%	78.98%	21.02%	8.93%
Consumer Products	10.39%	3.29%	78.98%	21.02%	8.89%
Disney	10.00%	3.29%	78.98%	21.02%	8.59%

The cost of capital for Disney as a company is 8.59% but the costs of capitals vary across divisions with a low of 7.90% for the parks and resorts division to a high or 8.93% for studio entertainment.

To estimate the cost of capital in both real and nominal US dollar terms for Aracruz in real terms, we use the cost of equity (from illustration 4.11) and the after-tax cost of debt (from illustration 4.12) and the estimates are reported in table 4.18:

Table 4.18: Cost of Capital for Aracruz: Real and US Dollars

	Levered Beta	*Cost of Equity*	*After-tax Cost of Debt*	*D/(D+E)*	*Cost of Capital*
In Real Terms					
Paper & Pulp	0.7576	11.46%	3.47%	30.82%	9.00%
Cash	0	2.00%			2.00%
Aracruz	0.7040	10.79%	3.47%	30.82%	8.53%
In US Dollar Terms					
Paper & Pulp	0.7576	13.46%	4.79%	30.82%	10.79%
Cash	0	4.00%			4.00%
Aracruz	0.7040	12.79%	4.79%	30.82%	10.33%

The nominal dollar costs of capital can be converted into nominal BR costs of capital using the differential inflation rates in the two countries, just as the cost of equity was earlier in the chapter.

When estimating the cost of equity for Bookscape, we assumed that the company would be funded using the same market debt to equity ratio as the book/publishing industry. Staying consistent, we will use the market debt to capital ratio to compute the cost of capital for the firm. We will also present two estimates of the cost of capital – one using the market beta and the other using the total beta:

	Beta	*Cost of Equity*	*After-tax Cost of debt*	*D/(D+E)*	*Cost of Capital*
Market Beta	0.82	7.97%	3.30%	16.90%	7.18%
Total Beta	2.06	13.93%	3.30%	16.90%	12.14%

The cost of capital estimated using the total beta is a more realistic estimate, given that this is a private company, and we will use it as the cost of capital for Bookscape in the coming chapters.

Equity, Debt and Cost of Capital for Banks

Note that we did not estimate a cost of capital for Deutsche Bank even though we have estimates of the costs of equity and debt for the firm. The reason is simple and goes to the heart of how firms view debt. For non-financial service firms, debt is a source of capital and is used to fund real projects – building a factory or making a movie. For banks, debt is raw material that is used to generate profits. Boiled down to its simplest elements, it is a bank's job to borrow money (debt) at a low rate and lend it out at a higher rate. It should come as no surprise that when banks (and their regulators) talk about capital, they mean equity capital.[63]

There is also a practical problem in computing the cost of capital for a bank. If we define debt as any fixed commitment where failure to meet the commitment can lead to loss of equity control, the deposits made by customers at bank branches would qualify and the debt ratio of a bank will very quickly converge on 100%. If we define it more narrowly, we still are faced with a problem of where to draw the line. A pragmatic compromise is to view only long term bonds issued by a bank as debt, but it is an artificial one. Deutsche Bank, for instance, had long-term debt in December 2003 was 82 billion Euros, common equity with a market value of 40.96 billion Euros and preferred stock with a market value of 4.1 billion Euros. Using the cost of equity of 8.76% (from illustration 4.11), the after-tax cost of debt of 3.13% from illustration 4.12 and the cost of preferred stock (6.36%) from illustration 4.13:

Cost of capital = 3.13% (82/127.06) + 8.47% (40.96/127.06) + 6.36%(4.1/127.06)

= 5.05%

With Deutsche Bank, we will do almost all of our analyses using the cost of equity rather than the cost of capital.

Conclusion

This chapter explains the process of estimating discount rates, by relating them to the risk and return models described in the previous chapter –

- The cost of equity can be estimated using risk and return models -- the capital asset pricing model, where risk is measured relative to a single market factor, the arbitrage

[63] All of the capital ratios that govern banks are stated in terms of book value of equity, though equity is defined broadly to include preferred stock.

pricing model, where the cost of equity is determined by the sensitivity to multiple unspecified economic factors or a multiple factor model, where sensitivity to macro-economic variables is used to measure risk.

    - In both these models, the key inputs are the riskfree rate, the risk premiums and the beta (in the CAPM) or betas (in the APM). The last of these inputs is usually estimated using historical data on prices; in the case of private firms, they might have to be estimated using comparable publicly traded firms.
    - While the betas are estimated using historical data, they are determined by the fundamental decisions that a firm makes on its business mix, its operating and financial leverage.

- The cost of capital is a weighted average of the costs of the different components of financing, with the weights based on the market values of each component. The cost of debt is the market rate at which the firm can borrow, adjusted for any tax advantages of borrowing. The cost of preferred stock, on the other hand, is the preferred dividend.
- The cost of capital is the minimum acceptable hurdle rate that will be used to determine whether to invest in a project.

Live Case Study

Risk and Return: Analysis for the Firm

Objective: To develop a risk profile for your company, estimate its risk parameters and use these parameters to estimate costs of equity and capital for the firm.

Key Questions:

- What is the risk profile of your company? (How much overall risk is there in this firm? Where is this risk coming from (market, firm, industry or currency)? How is the risk profile changing?)
- What is the performance profile of an investment in this company? What return would you have earned investing in this company's stock? Would you have under or out performed the market? How much of the performance can be attributed to management?
- How risky is this company's equity? Why? What is its cost of equity?
- How risky is this company's debt? What is its cost of debt?
- What is this company's current cost of capital?

Framework for Analysis:

1. *Estimating Historical Risk Parameters (Top Down Betas)*

   Run a regression of returns on your firm's stock against returns on a market index, preferably using monthly data and 5 years of observations (or)

If you have access to Bloomberg, go into the beta calculation page and print of the page (after setting return intervals to monthly and using 5 years of data)

- What is the intercept of the regression? What does it tell you about the performance of this company's stock during the period of the regression?
- What is the slope of the regression?
  - What does it tell you about the risk of the stock?
  - How precise is this estimate of risk? (Provide a range for the estimate.)
- What portion of this firm's risk can be attributed to market factors? What portion to firm-specific factors? Why is this important?
- How much of the "risk" for this firm is due to business factors? How much of it is due to financial leverage?

2. *Comparing to Sector Betas (Bottom up Betas)*
    - Break down your firm by business components, and estimate a business beta for each component
    - Attach reasonable weights to each component and estimate a unlevered beta for the business.
    - Using the current leverage of the company, estimate a levered beta for each component.
3. *Choosing Between Betas*
    - Which of the betas that you have estimated for the firm (top down or bottom up) would you view as more reliable? Why?
    - Using the beta that you have chosen, estimate the expected return on an equity investment in this company to
        - a short term investor
        - a long term investor
    - As a manager in this firm, how would you use this expected return?
4. *Estimating Default Risk and Cost of Debt*
    - If your company is rated,
        - What is the most recent rating for the firm?
        - What is the default spread and interest rate associated with this rating?
        - If your company has bonds outstanding, estimate the yield to maturity on a long term bond? Why might this be different from the rate estimated in the last step?
        - What is the company's marginal tax rate?
    - If your company is not rated,
        - Does it have any recent borrowings? If yes, what interest rate did the company pay on these borrowing?
        - Can you estimate a "synthetic" rating? If yes, what interest rate would correspond to this rating?)
5. *Estimating Cost of Capital*
    - Weights for Debt and Equity
        - What is the market value of equity?

- Estimate a market value for debt. (To do this you might have to collect information on the average maturity of the debt, the interest expenses in the most recent period and the book value of the debt)
- What are the weights of debt and equity?

- *Cost of Capital*
  - What is the cost of capital for the firm?

Getting Information on Risk and Return

If you want to run a regression of stock returns against a market index to estimate a beta, you will need to estimate past returns for both the stock and index. Several services including Bloomberg and S&P provide access to the data. If you want a beta estimate for your firm, you can find it online or look it up in Value Line. If you want to estimate bottom-up betas, based upon comparable firms, you will first have to identify the businesses that your firm operates in (which should be available in the firm's 10-K), find comparable firms in each business and then estimate the average beta and debt to equity ratio for these firms.

You can find the rating for your company from the S&P and Moody publications that list all traded bonds and their ratings. Alternatively, you can estimate an interest coverage ratio and a synthetic rating.

Online sources of information:

http://www.stern.nyu.edu/~adamodar/cfin2E/project/data.htm

Problems and Questions

1. In December 1995, Boise Cascade's stock had a beta of 0.95. The treasury bill rate at the time was 5.8%, and the treasury bond rate was 6.4%. The firm had debt outstanding of $ 1.7 billion and a market value of equity of $ 1.5 billion; the corporate marginal tax rate was 36%.
a. Estimate the expected return on the stock for a short term investor in the company.
b. Estimate the expected return on the stock for a long term investor in the company.
c. Estimate the cost of equity for the company.

2. Boise Cascade also had debt outstanding of $ 1.7 billion and a market value of equity of $ 1.5 billion; the corporate marginal tax rate was 36%.
a. Assuming that the current beta of 0.95 for the stock is a reasonable one, estimate the unlevered beta for the company.
b. How much of the risk in the company can be attributed to business risk and how much to financial leverage risk?

3. Biogen Inc., as biotechnology firm, had a beta of 1.70 in 1995. It had no debt outstanding at the end of that year.
a. Estimate the cost of equity for Biogen, if the treasury bond rate is 6.4%.
b. What effect will an increase in long term bond rates to 7.5% have on Biogen's cost of equity?
c. How much of Biogen's risk can be attributed to business risk?

4. Genting Berhad is a Malaysian conglomerate, with holding in plantations and tourist resorts. The beta estimated for the firm, relative to the Malaysian stock exchange, is 1.15, and the long term government borrowing rate in Malaysia is 11.5%.
a. Estimate the expected return on the stock.
b. If you were an international investor, what concerns, if any, would you have about using the beta estimated relative to the Malaysian Index? If you do, how would you modify the beta?

5. You have just done a regression of monthly stock returns of HeavyTech Inc., a manufacturer of heavy machinery, on monthly market returns over the last five years and come up with the following regression:

$$R_{HeavyTech} = 0.5\% + 1.2\, R_M$$

The variance of the stock is 50% and the variance of the market is 20%. The current T.Bill rate is 3% (It was 5% one year ago). The stock is currently selling for $50, down $4 over the last year, and has paid a dividend of $2 during the last year and expects to pay a dividend of $2.50 over the next year. The NYSE composite has gone down 8% over the last year, with a dividend yield of 3%. HeavyTech Inc. has a tax rate of 40%.

a. What is the expected return on HeavyTech over the next year?

b. What would you expect HeavyTech's price to be one year from today?

c. What would you have expected HeavyTech's stock returns to be over the last year?

d. What were the actual returns on HeavyTech over the last year?

e. HeavyTech has $100 million in equity and $ 5 million in debt. It plans to issue $50 million in new equity and retire $50 million in debt. Estimate the new beta.

6. Safecorp, which owns and operates grocery stores across the United States, currently has $50 million in debt and $100 million in equity outstanding. Its stock has a beta of 1.2. It is planning a leveraged buyout , where it will increase its debt/equity ratio of 8. If the tax rate is 40%, what will the beta of the equity in the firm be after the LBO?

7. Novell, which had a market value of equity of $2 billion and a beta of 1.50, announced that it was acquiring WordPerfect, which had a market value of equity of $ 1 billion, and a beta of 1.30. Neither firm had any debt in its financial structure at the time of the acquisition, and the corporate tax rate was 40%.

a. Estimate the beta for Novell after the acquisition, assuming that the entire acquisition was financed with equity.

b. Assume that Novell had to borrow the $ 1 billion to acquire WordPerfect. Estimate the beta after the acquisition.

8. You are analyzing the beta for Hewlett Packard and have broken down the company into four broad business groups, with market values and betas for each group.

Business Group	Market Value of Equity	Beta
Mainframes	$ 2.0 billion	1.10
Personal Computers	$ 2.0 billion	1.50
Software	$ 1.0 billion	2.00
Printers	$ 3.0 billion	1.00

a. Estimate the beta for Hewlett Packard as a company. Is this beta going to be equal to the beta estimated by regressing past returns on HP stock against a market index. Why or Why not?
b. If the treasury bond rate is 7.5%, estimate the cost of equity for Hewlett Packard. Estimate the cost of equity for each division. Which cost of equity would you use to value the printer division?
c. Assume that HP divests itself of the mainframe business and pays the cash out as a dividend. Estimate the beta for HP after the divestiture. (HP had $ 1 billion in debt outstanding.)

9. The following table summarizes the percentage changes in operating income, percentage changes in revenue and betas for four pharmaceutical firms.

Firm	% Change in Revenue	% Change in Operating Income	Beta
PharmaCorp	27%	25%	1.00
SynerCorp	25%	32%	1.15
BioMed	23%	36%	1.30
Safemed	21%	40%	1.40

a. Calculate the degree of operating leverage for each of these firms.
b. Use the operating leverage to explain why these firms have different betas.

10. A prominent beta estimation service reports the beta of Comcast Corporation, a major cable TV operator, to be 1.45. The service claims to use weekly returns on the stock over the prior five years and the NYSE composite as the market index to estimate betas. You replicate the regression using weekly returns over the same period and arrive at a beta estimate of 1.60. How would you reconcile the two estimates?

11. Battle Mountain is a mining company, which mines gold, silver and copper in mines in South America, Africa and Australia. The beta for the stock is estimated to be 0.30. Given the volatility in commodity prices, how would you explain the low beta?
12. You have collected returns on AnaDone Corporation (AD Corp.), a large diversified manufacturing firm, and the NYSE index for five years:

Year	AD Corp	NYSE
1981	10%	5%
1982	5%	15%

1983	-5%	8%
1984	20%	12%
1985	-5%	-5%

a. Estimate the intercept (alpha) and slope (beta) of the regression.

b. If you bought stock in AD Corp. today how much would you expect to make as a return over the next year? [The six-month T.Bill rate is 6%]

c. Looking back over the last five years, how would you evaluate AD's performance relative to the market?

d. Assume now that you are an undiversified investor and that you have all of your money invested in AD Corporation. What would be a good measure of the risk that you are taking on? How much of this risk would you be able to eliminate if you diversify?

e. AD is planning to sell off one of its divisions. The division under consideration has assets which comprise half of the book value of AD Corporation, and 20% of the market value. Its beta is twice the average beta for AD Corp (before divestment). What will the beta of AD Corporation be after divesting this division?

13. You run a regression of monthly returns of Mapco Inc, an oil and gas producing firm, on the S&P 500 index and come up with the following output for the period 1991 to 1995.

Intercept of the regression = 0.06%

X-coefficient of the regression = 0.46

Standard error of X-coefficient = 0.20

R squared = 5%

There are 29.5 million shares outstanding, and the current market price is $ 53. The firm has $ 753 million in debt outstanding. (The firm has a tax rate of 36%)

a. What would an investor in Mapco's stock require as a return, if the T.Bond rate is 6%?

b. What proportion of this firm's risk is diversifiable?

c. Assume now that Mapco has three divisions, of equal size (in market value terms). It plans to divest itself of one of the divisions for $ 20 million in cash and acquire another for $ 50 million (It will borrow $ 30 million to complete this acquisition). The division it is divesting is in a business line where the average unlevered beta is 0.20, and the division it is acquiring is in a

business line where the average unlevered beta is 0.80. What will the beta of Mapco be after this acquisition?

14. You have just run a regression of monthly returns of American Airlines (AMR) against the S&P 500 over the last five years. You have misplaced some of the output and are trying to derive it from what you have.

a. You know the R squared of the regression is 0.36, and that your stock has a variance of 67%. The market variance is 12%. What is the beta of AMR?

b. You also remember that AMR was not a very good investment during the period of the regression and that it did worse than expected (after adjusting for risk) by 0.39 % a month for the five years of the regression. During this period, the average riskfree rate was 4.84%. What was the intercept on the regression?

c. You are comparing AMR Inc. to another firm which also has an R squared of 0.48. Will the two firms have the same beta? If not, why not?

15. You have run a regression of monthly returns on Amgen, a large biotechnology firm, against monthly returns on the S&P 500 index, and come up with the following output –

$$R_{stock} = 3.28\% + 1.65\ R_{Market} \qquad R^2 = 0.20$$

The current one-year treasury bill rate is 4.8% and the current thirty-year bond rate is 6.4%. The firm has 265 million shares outstanding, selling for $ 30 per share.

i. What is the expected return on this stock over the next year?

ii. Would your expected return estimate change if the purpose was to get a discount rate to analyze a thirty-year capital budgeting project?

iii. An analyst has estimated, correctly, that the stock did 51.10% better than expected, annually, during the period of the regression. Can you estimate the annualized riskfree rate that she used for her estimate?

iv. The firm has a debt/equity ratio of 3%, and faces a tax rate of 40%. It is planning to issue $2 billion in new debt and acquire a new business for that amount, with the same risk level as the firm's existing business. What will the beta be after the acquisition?

16. You have just run a regression of monthly returns on MAD Inc., a newspaper and magazine publisher, against returns on the S&P 500, and arrived at the following result –

$$R_{MAD} = -0.05\% + 1.20\ R_{S\&P}$$

The regression has an R-squared of 22%. The current T.Bill rate is 5.5% and the current T.Bond rate is 6.5%. The riskfree rate during the period of the regression was 6%.. Answer the following questions relating to the regression –

a. Based upon the intercept, you can conclude that the stock did

A. 0.05% worse than expected on a monthly basis, during the regression.

B. 0.05% better than expected on a monthly basis during the period of the regression

C. 1.25% better than expected on a monthly basis during the period of the regression.

D. 1.25% worse than expected on a monthly basis during the period of the regression.

E. None of the above. (1 point)

b. You now realize that MAD Inc went through a major restructuring at the end of last month (which was the last month of your regression), and made the following changes –

- The firm sold off its magazine division, which had an unlevered beta of 0.6, for $ 20 million.
- It borrowed an additional $ 20 million, and bought back stock worth $ 40 million.

After the sale of the division and the share repurchase, MAD Inc. had $ 40 million in debt and $ 120 million in equity outstanding.

If the firm's tax rate is 40%, re-estimate the beta, after these changes.

17. Time Warner Inc., the entertainment conglomerate, has a beta of 1.61. Part of the reason for the high beta is the debt left over from the leveraged buyout of Time by Warner in 1989, which amounted to $10 billion in 1995. The market value of equity at Time Warner in 1995 was also $ 10 billion. The marginal tax rate was 40%.

a. Estimate the unlevered beta for Time Warner.

b. Estimate the effect of reducing the debt ratio by 10% each year for the next two years on the beta of the stock.

18. Chrysler, the automotive manufacturer, had a beta of 1.05 in 1995. It had $ 13 billion in debt outstanding in that year, and 355 million shares trading at $ 50 per share. The firm had a cash balance of $ 8 billion at the end of 1995. The marginal tax rate was 36%.

a. Estimate the unlevered beta of the firm.

b. Estimate the effect of paying out a special dividend of $ 5 billion on this unlevered beta.

c. Estimate the beta for Chrysler after the special dividend.

19. You are trying to estimate the beta of a private firm that manufactures home appliances. You have managed to obtain betas for publicly traded firms that also manufacture home appliances.

Firm	*Beta*	*Debt*	*MV of Equity*
Black & Decker	1.40	$ 2,500	$ 3,000
Fedders Corp.	1.20	$ 5	$ 200
Maytag Corp.	1.20	$ 540	$ 2250
National Presto	0.70	$ 8	$ 300
Whirlpool	1.50	$ 2900	$ 4000

The private firm has a debt equity ratio of 25%, and faces a tax rate of 40%. The publicly traded firms all have marginal tax rates of 40%, as well.

a. Estimate the beta for the private firm.

b. What concerns, if any, would you have about using betas of comparable firms?

20. As the result of stockholder pressure, RJR Nabisco is considering spinning off its food division. You have been asked to estimate the beta for the division, and decide to do so by obtaining the beta of comparable publicly traded firms. The average beta of comparable publicly traded firms is 0.95, and the average debt/equity ratio of these firms is 35%. The division is expected to have a debt ratio of 25%. The marginal corporate tax rate is 36%.

a. What is the beta for the division?

b. Would it make any difference if you knew that RJR Nabisco had a much higher fixed cost structure than the comparable firms used here?

21. Southwestern Bell, a phone company, is considering expanding its operations into the media business. The beta for the company at the end of 1995 was 0.90, and the debt/equity ratio was 1. The media business is expected to be 30% of the overall firm value in 1999, and the average beta

of comparable firms is 1.20; the average debt/equity ratio for these firms is 50%. The marginal corporate tax rate is 36%.

a. Estimate the beta for Southwestern Bell in 1999, assuming that it maintains its current debt/equity ratio.

b. Estimate the beta for Southwestern Bell in 1999, assuming that it decides to finance its media operations with a debt/equity ratio of 50%.

22. The chief financial officer of Adobe Systems, a growing software manufacturing firm, has approached you for some advice regarding the beta of his company. He subscribes to a service which estimates Adobe System's beta each year, and he has noticed that the beta estimates have gone down every year since 1991 - 2.35 in 1991 to 1.40 in 1995. He would like the answers to the following questions –

a. Is this decline in beta unusual for a growing firm?

b. Why would the beta decline over time?

c. Is the beta likely to keep decreasing over time?

23. You are analyzing Tiffany's, an upscale retailer, and find that the regression estimate of the firm's beta is 0.75; the standard error for the beta estimate is 0.50. You also note that the average unlevered beta of comparable specialty retailing firms is 1.15.

a. If Tiffany's has a debt/equity ratio of 20%, estimate the beta for the company based upon comparable firms. (The tax rate is 40%)

b. Estimate a range for the beta from the regression.

c. How would you reconcile the two estimates? Which one would you use in your analysis?