

CHAPTER 12

VALUATION: PRINCIPLES AND PRACTICE

In this chapter, we look at how to value a firm and its equity, given what we now know about investment, financing, and dividend decisions. We will consider three approaches to valuation. The first and most fundamental approach to valuing a firm is *discounted cash flow valuation*, which extends the present value principles that we developed to analyze projects to value a firm. The value of any firm is determined by four factors—its capacity to generate cash flows from assets in place, the expected growth rate of these cash flows, the length of time it will take for the firm to reach stable growth, and the cost of capital. Consequently, to increase the value of a firm, we have to change one or more of these variables.

The second way of valuing a firm or its equity is to base the value on how the market is valuing similar or comparable firms; this approach is called *relative valuation*. This approach can yield values that are different from a discounted cash flow valuation, and we will look at some of the reasons these differences occur.

The third approach to valuing a firm applies for highly levered firms, where the equity acquires the characteristics of a *call option*. In this special case, equity becomes more valuable, as debt maturity increases and the volatility in asset value goes up. Equity investors, in effect, derive their value from the expectation (or hope) that asset value will increase over time.

In a departure from previous chapters, we will take the perspective of investors in financial markets in estimating value. Investors assess the value of a firm's stock to decide whether to buy the stock or, if they already own it, whether to continue holding it.

Discounted Cash Flow Valuation

In discounted cash flow valuation, we estimate the value of any asset by discounting the expected cash flows on that asset at a rate that reflects their riskiness. In a sense, we measure the intrinsic value of an asset. The value of any asset is a function of the cash flows generated by that asset, the life of the asset, the expected growth in the cash flows, and the riskiness associated with these cashflows. In other words, it is the present value of the expected cash flows on that asset.

$$\text{Value of Asset} = \sum_{t=1}^{t=N} \frac{E(\text{Cash Flow}_t)}{(1+r)^t}$$

where the asset has a life of N years and r is the discount rate that reflects both the riskiness of the cash flows and financing mix used to acquire the asset. If we view a firm as a portfolio of assets, this equation can be extended to value a firm, using cash flows to the firm over its life and a discount rate that reflects the collective risk of the firm's assets.

This process is complicated by the fact that although some of the assets of a firm have already been created, and thus are assets-in-place, a significant component of firm value reflects expectations about future investments. Thus we not only need to measure the cash flows from current investments but also must estimate the expected value from future investments. In the sections that follow, we will introduce the discounted cash flow model in steps. We begin by discussing two different ways of approaching valuation—equity and firm valuation—and then move on to consider how best to estimate the inputs into valuation models. We then consider how to go from the value of a firm to the value of equity per share.

Equity Valuation versus Firm Valuation

There are two paths to discounted cash flow valuation—the first is to value just the equity stake in the business; the second is to value the entire firm, including equity and any other claims in the firm (from bondholders, preferred stockholders, etc.). Although both approaches discount expected cash flows, the relevant cash flows and discount rates are different for each.

Value of Equity: The value of the equity stake in a business; in the context of a publicly traded firm, the value of the common stock in the firm.

The **value of equity** is obtained by discounting expected cash flows to equity—that is, the residual cash flows after meeting all operating expenses, tax obligations, and interest and principal payments—at the cost of equity—that is, the rate of return required by equity investors in the firm.

$$\text{Value of Equity} = \sum_{t=1}^{t=n} \frac{\text{CF to Equity}_t}{(1+k_e)^t}$$

where $CF \text{ to Equity}_t$ = expected cash flow to equity in period t , and k_e = cost of equity. The dividend discount model is a special case of equity valuation, where the value of a stock is the present value of expected future dividends.

The **value of the firm** is obtained by discounting expected cash flows to the firm, that is, residual cash flows after meeting all operating expenses, taxes and reinvestment needs, but prior to debt payments—at the weighted average cost of capital (WACC)—that is, the cost of the different components of financing used by the firm, weighted by their market value proportions.

$$\text{Value of Firm} = \sum_{t=1}^{t=n} \frac{CF \text{ to Firm}_t}{(1 + WACC)^t}$$

Value of Firm: The value of all investors who have claims on the firm; thus, it includes lenders and debt holders, who have fixed claims, and equity investors, who have residual claims.

where $CF \text{ to Firm}_t$ = expected cash flow to firm in period t , and WACC = weighted average cost of capital. Although the two approaches use different definitions of cash flow and discount rates, they will yield consistent estimates of the value of equity as long as the same set of assumptions is applied for both. It is important to avoid mismatching cash flows and discount rates, because discounting cash flows to equity at the weighted average cost of capital will lead to an upwardly biased estimate of the value of equity, whereas discounting cash flows to the firm at the cost of equity will yield a downwardly biased estimate of the value of the firm.



12.1. Firm Valuation and Leverage

It is often argued that equity valuation requires more assumptions than firm valuation, because cash flows to equity require explicit assumptions about changes in leverage, whereas cash flows to the firm are predebt cash flows and do not require assumptions about leverage. Is this true?

a. Yes

b. No

Explain.

Choosing the Right Valuation Model

All discounted cash flow models ultimately boil down to estimating four inputs—current cash flows, an expected growth rate in these cash flows, a point in time when the firm will be growing a rate it can sustain forever, and a discount rate to use in discounting these cash flows. In this section, we will examine the choices available in terms of each of these inputs.

In terms of cash flows, there are three choices—dividends or free cash flows to equity (FCFE) for equity valuation models, and free cash flows to the firm (FCFF) for firm valuation models. Discounting dividends usually provides the most conservative estimate of value for the equity in any firm, because most firms pay less in dividends than they can afford to. In the dividend policy section, we noted that the FCFE, that is, the cash flow left over after meeting all investment needs and making debt payments, is the amount that a firm can pay in dividends. The value of equity, based on the FCFE, will therefore yield a more realistic estimate of value for equity, especially in the context of a takeover, since the acquirer can lay claim to the entire FCFE rather than just the dividends. Even if a firm is not the target of a takeover, it can be argued that the value of equity has to reflect the possibility of a takeover, and hence the expected FCFE. The choice between FCFE and FCFF is really a choice between equity and firm valuation. Done consistently, both approaches should yield the same values for the equity in a business. As a practical concern, however, cash flows to equity are after net debt issues or payments and become much more difficult to estimate when financial leverage is changing over time, whereas cash flows to the firm are predebt cash flows and are unaffected by changes in financial leverage. Ease of use dictates that firm valuation will be more straightforward under this scenario.

Although we can estimate cash flows from the most recent financial statements, the challenge in valuation is in estimating them in future years. In most valuations, this takes the form of an *expected growth rate in earnings* that is then used to forecast earnings and cash flows in future periods. The growth rates estimated should be consistent with our definition of cash flows. When forecasting cash flows to equity, we will generally forecast growth in net income or earnings per share that are measures of

equity earnings. When forecasting cash flows to the firm, the growth rate that matters is the growth rate in operating earnings.¹

The choice of *discount rates* will be dictated by the choice in cash flows. If the cash flow being discounted is dividends or FCFE, the appropriate discount rate is the cost of equity. If the cash flow being discounted is the cash flow to the firm, the discount rate has to be the cost of capital.

The final choice that all discounted cash flow models have to make relates to *expected growth patterns*. Because firms have infinite lives, the way we apply closure is to estimate a terminal value at a point in time and dispense with estimating cash flows beyond that point. To do this in the context of discounted cash flow valuation, we have to assume that the growth rate in cash flows beyond this point in time are constant forever, an assumption that we refer to as stable growth. If we do this, the present value of these cash flows can be estimated as the present value of a growing perpetuity. There are three questions that every valuation then has to answer:

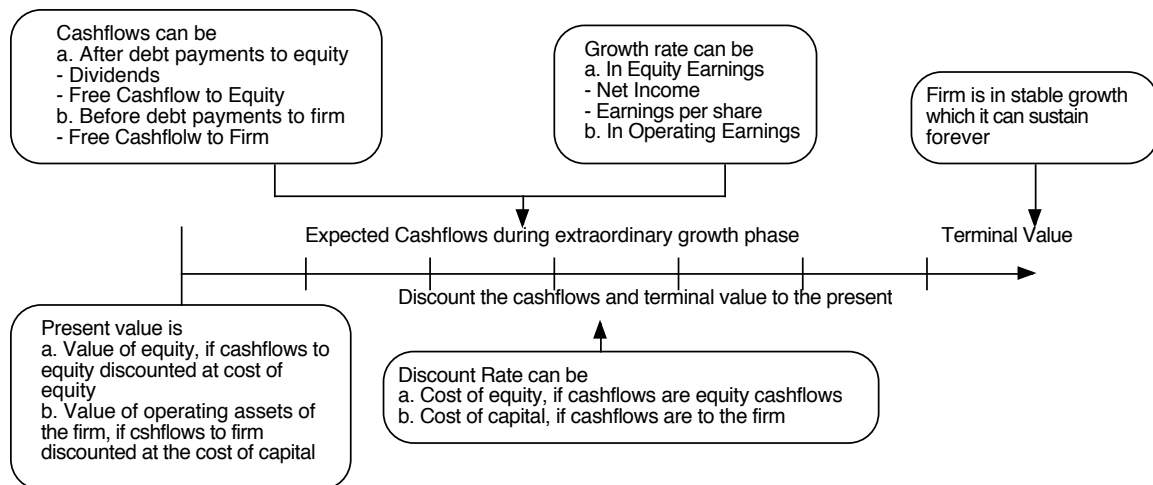
1. How long into the future will a company be able to grow at a rate higher than the stable growth rate?
2. How high will the growth rate be during the high-growth period, and what pattern will it follow?
3. What will happen to the firm's fundamentals (risk, cash flow patterns, etc.) as the expected growth rate changes?

At the risk of being simplistic, we can broadly classify growth patterns into three categories—firms that are in stable growth already, firms that expect to maintain a constant high growth rate for a period and then drop abruptly to stable growth, and firms that will have high growth for a specified period and then grow through a transition phase to reach stable growth at some point in the future. As a practical point, it is important that as the growth rate changes, the firm's risk and cash flow characteristics change as well. In general, as expected growth declines toward stable growth, firms should see their risk

¹We should generally become much more conservative in our growth estimates as we move up the income statements. Generally, growth in earnings per share will be lower than the growth in net income, and growth in net income will be lower than the growth in operating income.

approach the average and reinvestment needs decline. These choices are summarized in Figure 12.1.

Figure 12.1 The Ingredients in a Valuation



We will examine each of these valuation models in more detail in the next section.



model.xls: This spreadsheet allows you to pick the right discounted cash flow valuation model for your needs, given the characteristics of the business you are valuing.

In Practice: What Is a Stable Growth Rate?

Determining when your firm will be in stable growth is difficult to do without first defining what we mean by a *stable growth rate*. There are two insights to keep in mind when estimating a stable growth rate. First, because the growth rate in the firm's cash flows is expected to last forever, the firm's other measures of performance (including revenues, earnings, and reinvestment) can be expected to grow at the same rate. Consider the long-term consequences of a firm whose earnings grow 6 percent a year forever while its dividends grow at 8 percent. Over time, the dividends will exceed earnings. Similarly, if a firm's earnings grow at a faster rate than its dividends in the long run, the payout ratio will converge toward zero, which is also not a steady state. The second issue relates to what growth rate is reasonable as a stable growth rate. Again, the assumption that this growth rate will last forever establishes rigorous constraints on reasonableness. In the long run, a firm cannot grow at a rate significantly greater than the growth rate in the economy in which it operates. Thus, a firm that grows at 8 percent

forever in an economy growing at 4 percent will eventually become larger than the economy. In practical terms, if the valuation is done in nominal (real) terms, the stable growth rate cannot be larger than the nominal (real) growth rate in the economy in which the firm operates.

Can a stable growth rate be much lower than the growth rate in the economy? There are no logical or mathematical limits on the downside. Firms that have stable growth rates much lower than the growth rate in the economy will become smaller in proportion to the economy over time. Because there is no economic basis for arguing that this cannot happen, there is no reason to prevent analysts from using a stable growth rate much lower than the nominal growth rate in the economy. In fact, the stable growth rate can be a negative number. Using a negative stable growth rate will ensure that your firm peaks in the last year of high growth and becomes smaller each year after that.

There is one rule of thumb that works well in setting a cap on the stable growth rate. The stable growth rate should not exceed the risk-free rate used in a valuation. Why should the two be related? The risk-free rate can be decomposed into an expected inflation rate and an expected real interest rate. If we assume that the real growth rate of an economy will be equal to the real interest rate in the long run, the risk-free rate becomes a proxy for the nominal growth rate in the economy.



12.2 Cyclical Firms and Constant Growth Rates

Models built on the assumption of an expected constant growth rate over time cannot be used for cyclical firms, whose earnings growth is likely to be very volatile over time—high during economic booms, and very low or negative during recessions.

a. True

b. False

Explain.

Estimation in Discounted Cash Flow Models

Although all discounted cash flow models require the same four ingredients—cash flows, a discount rate, a period of high growth, and a growth rate during the

period—there are different estimation challenges we face with each model. In this section, we will begin by estimating these inputs to the simplest of the three models, the dividend discount model, and then extend the discussion to cash flow to equity and firm valuation models.

1. Dividend Discount Models

When an investor buys stock, he or she generally expects to get two types of cash flows—dividends during the holding period and an expected price at the end of the holding period. Because this expected price is itself determined by future dividends, the value of a stock is the present value of just expected dividends. The dividend discount model is therefore the most direct and most conservative way of valuing a stock because it counts only those cash flows that are actually paid out to stockholders.

Setting Up the Model

In its most general form, the value of a stock in the dividend discount model is the present value of the expected dividends on the stock in perpetuity.

$$\text{Value per share of stock} = \sum_{t=1}^{t=\infty} \frac{\text{Expected Dividends in period } t}{(1 + \text{Cost of Equity})^t}$$

Because we cannot estimate dividends in perpetuity, we generally allow for a period where dividends can grow at extraordinary rates, but we allow for closure in the model by assuming that the growth rate will decline to a stable rate that can be sustained forever at some point in the

Terminal Value: The expected price of a stock (or equity) at the end of a specified holding period.

future. By assuming stable growth at some point in the future, we can stop estimating annual dividends and estimate what we think the stock will be worth at the end of the extraordinary growth period.

$$\text{Value}_0 = \sum_{t=1}^{t=n} \frac{E(\text{Dividends})_t}{(1+r)^t} + \frac{\text{Terminal Value}_n}{(1+r)^n} \text{ where } \text{Terminal Value}_n = \frac{E(\text{Dividends})_{n+1}}{(r_n - g_n)}$$

where r is the cost of equity and g_n is the expected growth rate in dividends in perpetuity after year n .² Note that it is possible for a firm to already be in stable growth, in which case this model collapses into its simplest form:

$$\text{Value of a Stock in Stable Growth} = \text{Expected Dividends Next Year} / (r_n - g_n)$$

This model is called the Gordon growth model and is a special case of the dividend discount model. It can be used only for firms that are already in stable growth.³

Estimating Model Inputs

By breaking down the general version of the dividend discount model, we find four basic components. The first is the length of the high-growth period, during which the firm can sustain extraordinary growth. The second is the expected dividends each year during the high-growth period. The third is the cost of equity that stockholders will demand for holding the stock, based on their assessments of risk. The final input is the expected price at the end of the high-growth period—the **terminal value**. In this section, we will consider the challenges associated with estimating each of these components.

a. Length of High-Growth Period

The question of how long a firm will be able to sustain high growth is perhaps the most difficult to answer in a valuation, but two points are worth keeping in mind. One is that it is not a question of whether but when; all firms will ultimately

High-Growth Period: A period during which a company's earnings or cash flows are expected to grow at a rate much higher than the overall growth rate of the economy.

become stable-growth firms, because high growth makes firms larger, and the firm's size will eventually become a barrier to further growth. The second is that high growth in valuation, at least high growth that creates value, comes from firms earning high returns on their marginal investments. Using the terminology that we have used before in investment analysis, it comes from firms having a return on equity (capital) that is well in excess of the cost of equity (capital). Thus, when we assume that a firm will experience

²The cost of equity can be different for the high-growth and stable growth periods. Hence, r_n is the cost of equity for the stable growth period.

³When the Gordon growth model is used to value high-growth companies, it is entirely possible that $g > r$ and the model will yield a negative value. If this occurs, the problem is not with the model but in its misapplication to a high-growth firm.

high growth for the next five or ten years, we also implicitly assume that it will earn excess returns (over and above the cost of equity or capital) during that period. In a competitive market, these excess returns will eventually draw in new competitors, and the excess returns will disappear.

We should look at three factors when considering how long a firm will be able to maintain high growth.

1. ***Size of the Firm in relation to the market:*** Smaller firms are much more likely to earn excess returns and maintain them than otherwise similar larger firms. This is so because they have more room to grow and a larger potential market. When looking at the size of the firm, we should look not only at its current market share but also the potential growth in the total market for its products or services. Thus, Microsoft may have a large market share of the computer software market, but it may be able to grow in spite of it because the entire software market is growing. On the other hand, Boeing dominates the market for commercial aircraft, but we do not expect the overall market for aircraft to increase substantially. Boeing, therefore, is far more constrained in terms of future growth.
2. ***Existing Growth Rate and Excess Returns:*** Although the returns we would like to estimate are the marginal returns on new investments, there is a high correlation between the returns on current investments and these marginal returns. Thus, a firm earning excess returns of 20 percent on its current investments is far more likely to have large positive excess returns on its marginal investments and a long growth period than a firm currently earning excess returns of 2 percent. There are cases where this rule will not work, such as in industries going through major restructuring.
3. ***Magnitude and Sustainability of Competitive Advantages:*** This is perhaps the most critical determinant of the length of the high-growth period. If there are significant barriers to entry and sustainable competitive advantages, firms can maintain high growth for longer periods. On the other hand, if there are no or only minor barriers to entry, or if the firm's existing competitive advantages are fading, we should be far more conservative about allowing for long growth periods. The quality of existing

management also influences growth. Some top managers have the capacity to make the strategic choices that increase competitive advantages and create new ones.⁴

Again, the sensitivity of value to changes in the length of the high-growth period can always be estimated. Some analysts use growth periods greater than ten years, but the combination of high growth rates and long periods creates a potent mix in terms of increasing the size of the firm, in many cases well beyond the realm of what is reasonable. Looking back, there are very few firms that have been able to grow at high rates for more than ten years.

Illustration 12.1 Length of High-Growth Period

To assess how long high growth will last at Disney, Aracruz, and Tata Chemicals, we assessed their standings on each of the above characteristics in Table 12.1.

Table 12.1 Assessment of Length of High-Growth Period

	<i>Disney</i>	<i>Aracruz</i>	<i>Tata Chemicals</i>
Firm size/market size	Firm is one of the largest players in the entertainment and theme park business, but the businesses are being redefined and are expanding.	Firm has a small market share of the paper/pulp business, but the business is mature.	Firm has a large market share of Indian (domestic) market, but is small by global standards. Domestic market is also growing.
Current excess returns	Firm is earning less than its cost of capital and has done so for past few years.	Returns on capital are largely a function of paper/pulp prices, but on average have been less than the cost of capital.	Firm has a return on capital that is roughly equal to its cost of capital.
Competitive advantages	Has some of the most recognized brand names in the world. Knows more about operating theme parks than any other firm in the world. Has skilled	Cost advantages because of access to Brazilian rain forests. Has invested in newer, updated plants and has skilled workforce.	Has cost advantages, because of lower labor and production costs in India.

⁴Jack Welch at GE and Robert Goizueta at Coca-Cola are good examples of CEOs who made a profound difference in the growth of their firms and the market assessment of their values.

	animation studio staff.		
Length of high-growth period	Ten years, entirely because of its strong competitive advantages (which have been wasted over the past few years), but the excess returns are likely to be small.	Five years, largely due to access to cheap raw material.	Five years, primarily because of high real growth in India.

Using the same template for Deutsche Bank, its size and maturity work against high growth in its asset base, but in the current banking turmoil, Deutsche Bank's biggest competitive advantage is its safety and stability, as competitors are forced to raise fresh capital to meet regulatory requirements. As a consequence, we expect Deutsche Bank's income to rebound from current levels over the next five years. What about Bookscape? The single biggest competitive advantage possessed by this firm is its long-term lease at favorable terms in a superb location in New York City. It is unlikely that the firm will be able to replicate this advantage elsewhere. In addition, this is a private firm, which leads us to conclude that there will be no high-growth period.



12.3. Length of High-Growth Period and Barriers to Entry

Assume that you are analyzing two firms, both of which are enjoying high growth. The first firm is Earthlink Network, an Internet service provider, which operates in an environment with few barriers to entry and extraordinary competition. The second firm is Biogen, a biotechnology firm that is enjoying growth from two drugs for which it owns patents for the next decade. Assuming that both firms are well managed, which of the two firms would you expect to have a longer high-growth period?

- a. Earthlink Network
- b. Biogen

Both are well managed and should have the same high-growth period

b. Expected Dividends during High-Growth Period

The first step in estimating expected dividends during the high-growth period is to estimate the expected earnings for each year. This can be done in one of two ways—you

can apply an expected growth rate to current earnings, or you can begin by estimating future revenues first and then estimate net profit margins in each year. The first approach is easier, but the second provides for more flexibility because margins can change over time. The resulting expected earnings are paired with estimated dividend payout ratios in each period, which may change over the high-growth period. This may seem like an awkward procedure, because expected dividends could well be estimated using the current dividends and applying a dividend growth rate, but it is used for two reasons. First, most analyst projections for growth are stated in terms of revenues and earnings rather than dividends. Second, separating earnings forecasts from dividend payout provides more flexibility in terms of changing dividend payout ratios as earnings growth rates change. In particular, it allows us to raise dividend payout ratios as earnings growth rates decline.

The growth rate in earnings can be estimated using one of three approaches. The first is to look at the past and measure the **historical growth rate in earnings** over previous years. When measuring earnings growth, we have to consider how far back to go in time and whether to use arithmetic average or geometric average growth rates.⁵ In general, geometric growth rates yield more meaningful values than arithmetic average growth rates. The second is to look at estimates made by others following the same stock. In fact, growth estimates made by equity research analysts following a stock are public information and are easily accessible.⁶ The third is to consider the fundamentals and to estimate a growth rate based on a firm's investment policy. In particular, the growth in earnings per share of a firm can be written as the product of two variables—the percentage of the net income retained in the firm to generate future growth (retention ratio) and the return earned on equity in these new investments:

Historical Growth Rate (in Earnings): The growth rate over the past few periods in earnings; it can be calculated either by averaging the year-specific growth rates (arithmetic average) or by estimating at the compounded growth rate over the whole period.

⁵ Arithmetic average growth rates represent simple averages of growth rates over multiple years. The geometric average growth rate is a compounded growth rate.

$$\text{Expected Growth Rate} = \text{Retention Ratio} * \text{Return on Equity}$$

Thus, a firm with a return on equity of 20 percent and a retention ratio of 70 percent should have earnings growth of 14 percent a year. Reverting back to the discussion of dividend policy in Chapter 10, note that the retention ratio and the payout ratio are two sides of the same coin:

$$\text{Retention Ratio} = 1 - \text{Payout Ratio}$$

Because the retention ratio cannot exceed 100 percent, the expected growth in earnings per share in the long run for a firm cannot exceed its return on equity.

Assuming that we can obtain all three estimates of the growth rate in earnings for a firm, which one should we use in valuing a company? Past growth should be weighted least, because earnings are volatile and past growth has generally not been highly correlated with future growth.⁷ Analyst estimates are useful signposts of what the investment community thinks about a company and could include information that is not in the financial statements. In particular, it could reflect changes in both the company's management and strategic plans. However, trusting analysts, no matter how well informed they may be, to come up with the most important input in a valuation is not prudent. Ultimately, the fundamental growth equation offers the most promise because it relates growth back to what the firm does and also constrains us to pay for growth (by requiring firms to reinvest) as we estimate value.



12.4. Differences in Growth Rates

The growth rates from historical earnings, analyst projections, and fundamentals can often be very different. These differences can be best explained by which of the following statements?

- a. The past is not always a good indicator of the future
- b. Analysts are biased toward making optimistic estimates of growth.

⁶I/B/E/S, First Call, and Zacks are services that track equity research analyst forecasts continuously, and the consensus estimate across all analysts is publicly available.

⁷One of the most famous studies of growth was titled "Higgledy Piggledy Growth" (I. M. D. Little, 1962, *Higgledy Piggledy Growth*, Oxford: Institute of Statistics) precisely because earnings growth was so difficult to predict based on history.

- c. The inputs used to estimate fundamental growth reflect what happened last year rather than what we expect will happen in the future.
- d. All of the above.

Illustration 12.2 Growth in Earnings per Share: Deutsche Bank in early 2008

In January 2008, in calmer times, we estimated the earnings growth for Deutsche Bank, using fundamentals. In 2007, Deutsche Bank reported net income of 6.51 billion Euros on a book value of equity of 33.475 billion Euros at the start of the year (end of 2006). The resulting return on equity is 19.45%:

$$\text{Return on Equity} = \frac{\text{Net Income}_{2007}}{\text{Book Value of Equity}_{2006}} = \frac{6,510}{33,475} = 19.45\%$$

In 2007, Deutsche Bank paid out 2.146 billion Euros to equity investors. The resulting retention ratio is 67.03%.

$$\text{Retention Ratio} = 1 - \frac{\text{Dividends}}{\text{Net Income}} = 1 - \frac{2,146}{6,510} = 67.03\%$$

If Deutsche Bank maintains the return on equity (ROE) and retention ratio that it delivered in 2007 for the long run, its expected growth rate in earnings will be strong.

$$\text{Expected Growth Rate}_{\text{Existing Fundamentals}} = \text{Retention Ratio} * \text{ROE} = 0.6703 * 0.1945 = 13.04\%$$

The danger with this estimate is that it is based upon 2007, a very profitable year for Deutsche Bank. If we replace the net income in 2007 with average net income from 2003 to 2007, we arrive at lower estimates of ROE and expected growth rate:

$$\text{Normalized Return on Equity} = \frac{\text{Average Net Income}_{2003-07}}{\text{Book Value of Equity}_{2006}} = \frac{3,954}{33,475} = 11.81\%$$

$$\text{Normalized Retention Ratio} = 1 - \frac{\text{Dividends}}{\text{Net Income}} = 1 - \frac{2,146}{3,954} = 45.72\%$$

$$\text{Expected Growth Rate}_{\text{Normalized Fundamentals}} = \text{Retention Ratio} * \text{ROE} = 0.4572 * 0.1181 = 5.40\%$$

How does this contrast and compare to the historical growth in net income at Deutsche Bank? Deutsche Bank's net income grew from 1.365 billion Euros in 2003 to 6.510 billion Euros in 2007, resulting in a compounded earnings growth rate of 47.78%.

$$\text{Compounded Earnings Growth Rate} = \left(\frac{\text{Net Income}_{2007}}{\text{Net Income}_{2003}} \right)^{1/4} - 1 = \left(\frac{6,510}{1,365} \right)^{1/4} - 1 = 47.78\%$$

This high growth rate, however, reflects the fact that the net income at Deutsche Bank was depressed between 2001 and 2003 and that much of this growth reflect a recovery back to more normal earnings levels.

In hindsight, all of these estimates of earnings growth would have been wrong, since the financial crisis in 2008 caused billions of dollars in write-offs at Deutsche Bank and the firm reported a loss of 3.896 billion Euros for the year. In the first quarter of 2009, Deutsche Bank reported a return to profitability and net income in excess of 1 billion Euros.

c. Cost of Equity

The dividends and terminal price should be discounted back at a rate that reflects the risk in the investment to stockholders to arrive at the current value. In Chapter 4, we argued that the only risk that diversified investors see in a stock is market risk, and this risk can be measured with a beta (in the capital asset pricing model) or multiple betas (in the arbitrage pricing or multifactor models). The same reasoning applies here. In fact, the costs of equity that we estimated for Disney, Deutsche Bank, and Aracruz in Chapter 4 will be the costs of equity that will be used if we were valuing stock in these companies using a dividend discount model. The only point that relates specifically to valuation is that high-growth firms tend to have higher betas than low-growth firms. Building on this premise, it is important that as we change growth rates over time, we also adjust risk accordingly. Thus, when a firm goes from high growth to low growth, its beta should be moved toward one to reflect the lower growth.

d. Terminal Value

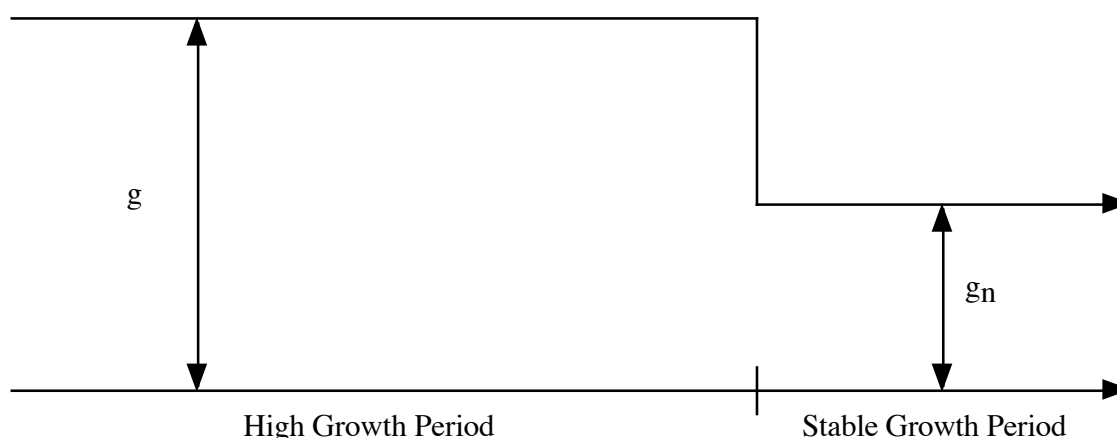
The last component of the model is the value attached to the equity at the end of a period of high growth. This value is estimated from expected dividends in the first time period following the high-growth period, the cost of equity in the stable phase, and the expected stable growth rate in dividends as follows:

$$\text{Value of Equity in year } n = \frac{\text{Expected Dividends}_{n+1}}{r_n - g_n}$$

where r_n is the cost of equity in the stable growth period and g_n is the expected growth rate in dividends beyond year n (forever).

Before you estimate terminal value, you need to map out a path for the earnings growth during the high growth phase to move toward the stable growth rate. The simplest assumption to make is that your earnings growth rate is constant for the high-growth period, after which the growth rate drops to the stable level, as shown in Figure 12.2.

Figure 12.2: Two-Stage Growth Model



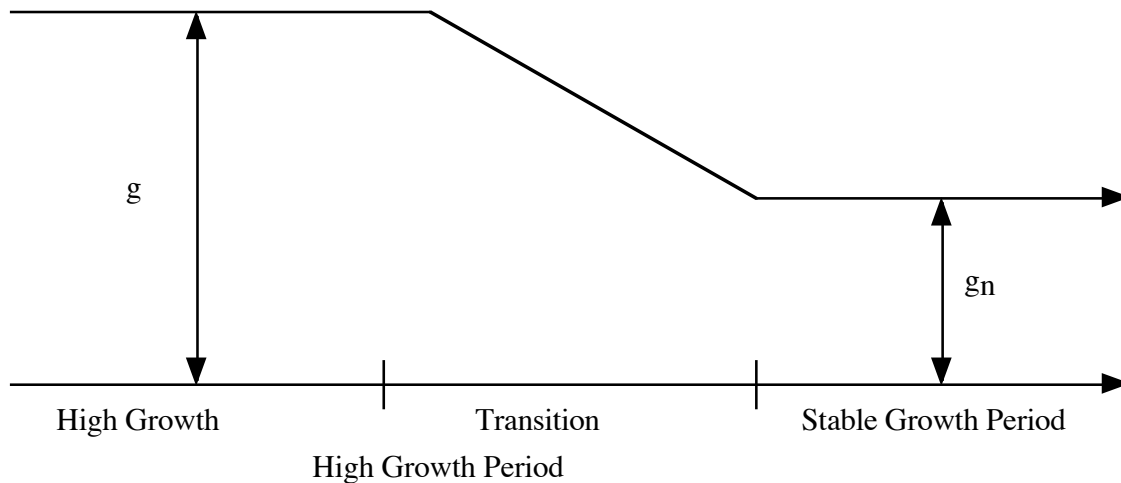
This is a two-stage model, and its limitation is obvious. It assumes that the growth rate is high during the initial period and is transformed overnight to a lower, stable rate at the end of the period. Although these sudden transformations in growth can happen, it is much more realistic to assume that the shift from high growth to stable growth happens gradually over time. The assumption that the growth rate drops precipitously from its level in the initial phase to a stable rate also implies that this model is more appropriate for firms with modest growth rates in the initial phase. For instance, it is more reasonable to assume that a firm growing at 8 percent in the high-growth period will see its growth rate drop to 4 percent, than it is for a firm growing at 40 percent in the high-growth period. If we assume that the growth rate and payout ratio are fixed for the high-growth period, the present value of the dividends during the high-growth period can be estimated as follows:⁸

⁸Unlike the stable growth model equation, this one can be used even if the expected growth rate exceeds the discount rate. Although this makes the denominator negative, it will also result in a negative numerator, and the net effect will be positive. The only condition when it will not work is if $g = r$, but the PV of

$$\text{PV of High-growth dividends}_0 = \frac{\text{Dividends}_0 * (1+g) * \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)}{r - g}$$

A more general formulation would allow for growth during the high-growth period, followed by a gradual reduction to stable growth over a transition period, as illustrated in Figure 12.3.

Figure 12.3: High Growth followed by transition



This model allows for growth rates and payout ratios to change gradually during the transition period.

Whatever path you devise to get your firm to stable growth, it is not just the growth rate that should change in stable growth. The other characteristics of the firm should also change to reflect the stable growth rates.

- The cost of equity should be more reflective of that of a mature firm. If it is being estimated using a beta, that beta should be closer to one in stable growth even though it can take on very high or very low values in high growth.
- The dividend payout ratio, which is usually low or zero for high-growth firms, should increase as the firm becomes a stable-growth firm. In fact, drawing on the fundamental growth equation from the last section, we can estimate the payout ratio in stable growth:

dividends in that case will just be the product of the number of years of growth and dividends today because the growth and the discounting effects each year will cancel out.

$$\text{Dividend Payout Ratio} = 1 - \text{Retention Ratio} = 1 - \frac{g_{\text{Stable}}}{\text{ROE}_{\text{Stable}}}$$

If we expect the stable growth rate to be 4 percent and the return on equity in stable growth to be 12 percent, the payout ratio in stable growth will be 66.67 percent ($1 - 4/12$).

- The return on equity in stable growth, if used to estimate the payout ratio, should be also reflective of a stable-growth firm. The most conservative estimate to make in stable growth is that the return on equity will be equal to the cost of equity, thus denying the firm the possibility of excess returns in perpetuity. If this is too rigid a framework, you can assume that the return on equity will converge on an industry average in the stable-growth phase.

If there is a transition period for growth, as in Figure 12.3, the betas and payout ratios should adjust in the transition period, as the growth rate changes.



12.5. Terminal Value and Present Value

The bulk of the present value in most discounted cash flow valuations comes from the terminal value. Therefore it is reasonable to conclude that the assumptions about growth during the high-growth period do not affect value as much as assumptions about the terminal value.

- True
- False

Explain.

Closing Thoughts on the Dividend Discount Model

Many analysts view the dividend discount model as outmoded, but it is a useful starting point in valuing all companies and may be the only choice in valuing companies where estimating cash flows is not feasible. As noted in Chapter 11, estimating free cash flows for financial service companies is often difficult both because the line between operating and capital expenses is fuzzy and because working capital, defined broadly, could include just about all of the balance sheet. Although we can arrive at approximations of cash flows by making assumptions about capital expenditures, we are

often left in the uncomfortable position of assuming that dividends represent FCFE for these firms. Even for firms where we can estimate FCFE with reasonable precision, the dividend discount model allows us to estimate a “floor value” in most cases because firms tend to pay out less in dividends than they have available in FCFE.

It is often argued that the dividend discount model cannot be used to value high-growth companies that pay little in dividends. That is true only if we use the inflexible version of the model whereby future dividends are estimated by growing current dividends. In the more flexible version, where both payout ratios and earnings growth can change over time, the dividend discount model can be extended to cover all types of firms.

There is one final point worth making in this section. We can estimate the value of equity on a per-share basis by using dividends per share, or we can obtain the aggregate value of equity using total dividends paid. The two approaches will yield the same results if there are no management options, warrants, or convertible bonds outstanding. If there are equity options, issued by the firm, that are outstanding, it is safest to value the equity on an aggregate basis. We will consider how best to deal with equity options in arriving at a value per share later in this chapter.



12.6. Payout Ratios and Expected Growth

The dividend discount model cannot be used to value stock in a company with high growth that does not pay dividends.

- a. True
- b. False

Explain.

Valndata.xls: This file online contains the industry averages by sector for returns on capital, retention ratios, debt equity ratios, and interest rates.

Illustration 12.3 Valuing Equity Using the Dividend Discount Model: Deutsche Bank in January 2008

In Illustration 12.2, we estimated the annual growth rate of 5.40% for the next five years at Deutsche Bank at the start of 2008, using normalized earnings from 2003 to 2007 to compute the return on equity, retention ratio and expected growth rate.

Normalized growth rate in net income = 5.40%

Normalized dividend payout ratio = 54.28%

In the analysis that follows, we will value Deutsche Bank at the start of 2008, using this growth rate. In 2007, Deutsche Bank paid out dividends of 2,146 million Euros on normalized net income of 3,954 million Euros. In chapter 4, we estimated a beta of 1.162 for Deutsche Bank, which used in conjunction with the Euro risk-free rate of 4% (in January 2008) and a risk premium of 4.50% (the mature market risk premium in early 2008), yielded a cost of equity of 9.23%.⁹

$$\begin{aligned}\text{Cost of Equity}_{\text{January 2008}} &= \text{Riskfree Rate}_{\text{January 2008}} + \text{Beta} * \text{Mature Market Risk Premium} \\ &= 4.00\% + 1.162 (4.5\%) = 9.23\%\end{aligned}$$

Based on these inputs, we estimate the expected net income and dividends for the next five years and the present value of these dividends in Table 12.2.

Table 12.2 Present Value of Expected Dividends for High-Growth Period

<i>Year</i>	<i>Net Income</i>	<i>Payout Ratio</i>	<i>Dividends</i>	<i>PV @ 9.23%</i>
2008	4,167 €	54.28%	2,262 €	2,071 €
2009	4,392 €	54.28%	2,384 €	1,998 €
2010	4,629 €	54.28%	2,384 €	1,928 €
2011	4,879 €	54.28%	2,513 €	1,861 €
2012	5,143 €	54.28%	2,648 €	1,795 €
				9,653 €

⁹ In truth, we should be estimating a beta at the start of 2008, instead of using the beta that we estimated at the start of 2009. However, the difference should be small enough to not affect value by much.

Note that we could have arrived at the same present value using the shortcut described earlier (because the payout ratio and the cost of equity remain unchanged for the high-growth period):

$$\text{PV of High-growth dividends}_0 = \frac{2,146 * (1.054) * \left(1 - \frac{(1.054)^5}{(1.0923)^5}\right)}{.0923 - .054} = 9,653 \text{ million Euros}$$

At the end of year five, we will assume that Deutsche Bank's earnings growth will drop to 3 percent and stay at that level in perpetuity. In keeping with the assumption of stable growth, we will also assume that

- The beta will drop marginally to 1, resulting in a slightly lower cost of equity of 8.50%.

$$\text{Cost of Equity} = \text{Risk-Free Rate} + \text{Beta} * \text{Risk Premium} = 4\% + 4.50\% = 8.50\%$$

- The return on equity will drop to the cost of equity of 8.50 percent, thus preventing excess returns from being earned in perpetuity.
- The payout ratio will adjust to reflect the stable period growth rate and return on equity.

$$\text{Stable Period Payout Ratio} = 1 - g/\text{ROE} = 1 - 0.03/0.085 = 0.6471 \text{ or } 64.71\%$$

The expected dividends in year six is calculated using this payout ratio:

$$\begin{aligned} \text{Expected Dividends in Year 6} &= \text{Expected Net Income}_5 * (1 + g_{\text{Stable}}) * \text{Stable Payout Ratio} \\ &= \text{€}5,143 (1.03) * 0.6471 = \text{€}3,427 \text{ million} \end{aligned}$$

The value of equity at the end of the fifth year can be estimated using these inputs:

$$\text{Terminal Value} = \frac{\text{Expected Dividends}_6}{(\text{Cost of Equity} - g)} = \frac{3,247}{(.085 - .03)} = 62,318 \text{ million Euros}$$

The present value of the terminal value is computed using the high-growth period cost of equity:

$$\text{PV of Terminal Value} = \frac{\text{Terminal Value}_n}{(1 + \text{Cost of Equity}_{\text{High growth}})^n} = \frac{62,318}{(1.0923)^5} = 40,079 \text{ mil Euros}$$

The total value of equity is the sum of this value and the present value of the expected dividends in the high growth period:

$$\begin{aligned} \text{Value per Share} &= \text{PV of Expected Dividends in High Growth} + \text{PV of Terminal Value} \\ &= \text{€}9,653 + \text{€}40,079 = \text{€}49,732 \text{ million Euros} \end{aligned}$$

Dividing this value by the number of shares outstanding at the start of 2008 yields the value of equity per share:

$$\text{Value of equity per share} = \frac{\text{Value of Equity}}{\# \text{ Shares}} = \frac{49,732}{474.2} = 104.88 \text{ Euros/share}$$

The market price of Deutsche Bank at the time of this valuation was 89 Euros per share. Based on our assumptions, Deutsche Bank looked under valued at the start of 2008.

Illustration 12.4 Valuing Equity in more unsettled times: Deutsche Bank in 2009

In the last illustration, we estimated a value of 105 Euros/share for Deutsche Bank at the beginning of 2008, and concluded that it was under valued at its then prevailing stock price of 89 Euros/share. During 2008, the landscape for financial service firms changed, as banks entered crisis mode and financial markets collapsed. After taking billions of dollars of write offs, Deutsche Bank reported a loss of 3,835 million Euros for 2008 and cut dividends to 285 million Euros. While neither of these numbers represents a stable starting point, we made the following assumptions to value Deutsche Bank:

- a. Net Income bounce back: We will assume that net income will bounce back to 3.147 billion Euros in 2009, and base this assumption on the improved earnings for the first quarter of 2009 reported by Deutsche Bank (1.12 billion Euros in quarterly profits) and the average net income between 2003 and 2007 (approximately 3.95 billion Euros).
- b. Asset Base and Target ROE: We will assume that the current asset base for the firm (312,882 million Euros) will grow 4% a year for the next five years and that the return on equity will improve to 10% over this period.
- c. Potential dividends: Rather than focus on current dividends, which have been cut drastically, we estimate the potential dividends, based upon the assumption that the firm will move towards a target regulatory capital ratio of 10%. (We are replicating the analysis we did in chapter 11, to estimate FCFE)
- d. Cost of Equity: To arrive at the cost of equity, we use the beta of 1.162 that we estimated in chapter 4, in conjunction with the Euro riskfree rate of 3.6% at the start of 2009 and the updated equity risk premium of 6% for mature markets:

$$\begin{aligned} \text{Cost of equity} &= \text{Riskfree Rate} + \text{Beta (Equity Risk Premium)} \\ &= 3.6\% + 1.162 (6\%) = 10.572\% \end{aligned}$$

Table 12.3 summarizes the estimates of net income, potential dividends and the present value of these dividends over the next 5 years:

Table 12.3: Expected Potential Dividends over next 5 years: Deutsche Bank in 2009

	Current	1	2	3	4	5	Sum
Asset Base	312,882 €	325,398 €	338,414 €	351,950 €	366,028 €	380,669 €	
Capital ratio	10.20%	10.16%	10.12%	10.08%	10.04%	10.00%	
Regulatory Capital	31,914 €	33,060 €	34,247 €	35,477 €	36,749 €	38,067 €	
Change in Regulatory Capital		1,146 €	1,187 €	1,229 €	1,273 €	1,318 €	
ROE	9.40%	9.52%	9.64%	9.76%	9.88%	10.00%	
Net Income	3,000 €	3,147 €	3,302 €	3,463 €	3,631 €	3,807 €	
- Investment in Regulatory Capital		1,146 €	1,187 €	1,229 €	1,273 €	1,318 €	
FCFE (Potential Dividend)		2,001 €	2,114 €	2,233 €	2,358 €	2,489 €	
Present value @ 10.572%		1,810 €	1,729 €	1,652 €	1,578 €	1,506 €	8,275 €

At the end of year 5, we assume that the firm will be in stable growth, growing 3% a year in perpetuity. In addition, we will also assume that

- The beta will decrease to 1, resulting in a drop of cost of equity to 9.60%.

$$\begin{aligned}\text{Cost of equity} &= \text{Riskfree Rate} + \text{Beta} * \text{Equity Risk Premium} \\ &= 3.6\% + 1 (6\%) = 9.60\%\end{aligned}$$

- The return on equity after year 5 will be equal to the stable period cost of equity of 9.60%.
- Given the expected growth rate of 3% after year 5 and the stable ROE of 9.60%, the payout ratio in stable growth is 68.75%

$$\text{Stable Payout Ratio} = 1 - \frac{\text{Stable growth Rate}}{\text{Stable ROE}} = 1 - \frac{.03}{.096} = 68.75\%$$

The value of equity at the end of year 5 can be estimated as follows:

$$\text{Terminal Value} = \frac{\text{Expected Dividends}_6}{(\text{Cost of Equity} - g)} = \frac{3,807(1.03)(.6875)}{(.096 - .03)} = 39,728 \text{ million Euros}$$

Discounting the terminal value back at the cost of equity for the high growth period:

$$\text{PV of Terminal Value} = \frac{\text{Terminal Value}_n}{(1 + \text{Cost of Equity}_{\text{High growth}})^n} = \frac{39,728}{(1.10572)^5} = 24,036 \text{ mil Euros}$$

Adding the present value of dividends to this number yields the value of equity for Deutsche Bank in early 2009:

$$\text{Value of equity} = 8,275 \text{ million €} + 24,036 \text{ million €} = 32,311 \text{ million €}$$

Dividing by the number of shares outstanding at the start of 2009 (581.85 million), we can obtain the value of equity per share:

$$\text{Value of equity per share} = \frac{\text{Value of Equity}}{\# \text{ Shares}} = \frac{32,311}{581.85} = 55.53 \text{ Euros/share}$$

In June 2009, Deutsche Bank was trading at 48.06 Euros per share and thus remains under valued.



Valuation Biases: A Behavioral Perspective

In theory, we start with the financial fundamentals and move “objectively” from the numbers to the value of the firm, making reasonable assumptions along the way. In practice, though, valuations are not just subjective but are contaminated by biases that analysts bring to the process. In fact, there are at least three sources of bias.

- a. Anchoring bias: When valuing a company, we generally look for a number to use as a basis or comparison and that number then affects the valuation. With publicly traded companies, for instance, the market price becomes a logical anchor to compare our estimates of value to. In fact, it is not uncommon to see analysts change their assumptions to move their valuations closer to the stock price.
- b. Recency bias: There is evidence that when data is presented sequentially, the most recent data is weighted too much (relative to its importance) and less recent data too little. In the context of valuing companies, this often manifests itself as too great a dependence on how these companies have done in the most recent year and too little attention paid to historical data. As a consequence, we tend to overvalue companies after good years and under value companies after bad years.
- c. Confirmation bias: There is some evidence that analysts who form a perception of what the fair value is early in the process tend to then model the data to confirm that perception.

As a result of these biases, we would argue that in many valuations, the value gets set first and the valuation follows.

As a confession, the valuations of Deutsche Bank in 2008 and 2009 in this chapter reflect some of these biases. While some of the drop in value per share (from 105 Euros/share to 56 Euros/share can be attributed to changing fundamentals, some of it also reflects the effect of not only the market crisis but also seeing the drop in Deutsche Bank's stock price from 89 Euros to 48 Euros between the two valuations. Put another way, it is entirely possible that I am over reacting to recent events (by raising the equity risk premium from 4.5% to 6%) and under valuing Deutsche Bank as a consequence.

II. FCFE Models

In Chapter 11, while developing a framework for analyzing dividend policy, we estimated the free cash flow to equity as the cash flow that the firm can afford to pay out as dividends and contrasted it with the actual dividends. We noted that many firms do not pay out their FCFE as dividends; thus, the dividend discount model may not capture their true capacity to generate cash flows for stockholders. A more appropriate model is the *FCFE model*.

Setting Up the Model

The FCFE is the residual cash flow left over after meeting interest and principal payments and providing for reinvestment to maintain existing assets and create new assets for future growth. The FCFE is measured as follows:

$$\text{FCFE} = \text{Net Income} + \text{Depreciation} - \text{Capital Expenditures} - \Delta \text{Working Capital} - \text{Principal Repayments} + \text{New Debt Issues}$$

where $\Delta \text{Working Capital}$ is the change in noncash working capital.

In the special case where the capital expenditures and the working capital are financed at the target debt ratio δ and principal repayments are made from new debt issues, the FCFE is measured as follows:

$$\text{FCFE} = \text{Net Income} - (1-\delta)(\text{Capital Expenditures} - \text{Depreciation}) - (1-\delta)\Delta \text{Working Capital}$$

There is one more way in which we can present the FCFE. If we define the portion of the net income that equity investors reinvest back into the firm as the equity reinvestment rate, we can state the FCFE as a function of this rate.

$$\text{Equity Reinvestment Rate} = \frac{(\text{Capital Expenditures} - \text{Depreciation} + \Delta \text{ Working Capital}) (1 - \delta)}{\text{Net Income}}$$

$$\text{FCFE} = \text{Net Income} (1 - \text{Equity Reinvestment Rate})$$

Once we estimate the FCFE, the general version of the FCFE model resembles the dividend discount model, with FCFE replacing dividends in the equation:

$$\text{Value of the Stock} = \text{PV of FCFE during High Growth} + \text{PV of Terminal Price}$$

$$\text{Value}_0 = \sum_{t=1}^{t=n} \frac{E(\text{FCFE})_t}{(1+r)^t} + \frac{\text{Terminal Value}_n}{(1+r)^n} \text{ where } \text{Terminal Value}_n = \frac{E(\text{FCFE})_{n+1}}{(r_n - g_n)}$$

where the expected FCFEs are estimated each year for the high growth period, r is the cost of equity, and g_n is the stable growth rate.

There is one key difference between the two models, though. Although the dividends can never be less than zero, the FCFE can be negative. This can occur even if earnings are positive, if the firm has substantial working capital and capital expenditure needs. In fact, the expected FCFE for many small, high-growth firms will be negative at least in the early years, when reinvestment needs are high, but will become positive as the growth rates and reinvestment needs decrease.

In Practice: Estimating Capital Expenditure and Working Capital Needs

Two components go into estimating reinvestments. The first is net capital expenditures, which is the difference between capital expenditures and depreciation. Although these numbers can easily be obtained for the current year for any firm in the United States, they should be used with the following caveats:¹⁰

1. Firms seldom have smooth capital expenditure streams. They can go through periods when capital expenditures are very high, followed by periods of relatively light expenditures. Consequently, when estimating the capital expenditures to use for forecasting future cash flows, we should look at capital expenditures over time and normalize them by taking an average, or we should look at industry norms.

¹⁰It is surprisingly difficult to obtain the capital expenditure numbers even for large, publicly traded firms in some markets outside the United States. Accounting standards in these markets often allow firms to lump investments together and report them in the aggregate.

2. If we define capital expenditures as expenses designed to generate benefits over many years, research and development (R&D) expenses are really capital expenditures. Consequently, R&D expenses need to be treated as capital expenditures, and the research asset that is created as a consequence needs to be amortized, with the amortization showing up as part of depreciation.¹¹
3. Finally, when estimating capital expenditures, we should not distinguish between internal investments (which are usually categorized as capital expenditures in cash flow statements) and external investments (which are acquisitions). The capital expenditures of a firm therefore need to include acquisitions, whether they are funded with stock or cash. Because firms seldom make acquisitions every year, and each acquisition has a different price tag, the point about normalizing capital expenditures applies even more strongly to this item.

The second component of reinvestment is the cash that needs to be set aside for working capital needs. As in the chapters on investment analysis, we define working capital needs as noncash working capital, and the cash flow effect is the period-to-period change in this number. Again, although we can estimate this change for any year using financial statements, it has to be used with caution. Changes in noncash working capital are volatile, with big increases in some years followed by big decreases in the following years. To ensure that the projections are not the result of an unusual base year, we tie the changes in working capital to expected changes in revenues or costs of goods sold at the firm over time. For instance, we use the noncash working capital as a percent of revenues, in conjunction with expected revenue changes each period, to estimate projected changes in noncash working capital. As a final point, noncash working capital can be negative, which can translate into positive cash flows from working capital as revenue increases. It is prudent, when this occurs, to set noncash working capital needs to zero.¹²

¹¹Capitalizing R&D is a three-step process. First, you need to specify, on average, how long it takes for research to pay off (amortizable life). Second, you have to collect R&D expenses from the past for an equivalent period. Third, the past R&D expenses have to be written off (straight line) over the amortizable life.

¹²Although it is entirely possible that firms can generate positive cash flows from working capital decreasing for short periods, it is dangerous to assume that this can occur forever.

Estimating Model Inputs

Just as in the dividend discount model, there are four basic inputs needed for this model to be usable. First, the *length of the high-growth period* is defined. Second, the *FCFE* each period during the growth period is computed; this means that net capital expenditures, working capital needs, and the debt financing mix are all estimated for the high-growth period. Third, the *rate of return* stockholders will demand for holding the stock is estimated. Finally, the *terminal value of equity at the end of the high-growth period* is calculated, based on the estimates of stable growth, the FCFE, and required return after the high-growth ends. Of the four inputs, the length of the high-growth period and the rate of return required by stockholders are the same for the dividend discount and FCFE valuation models. On the other two, the differences in the other two inputs are minor but still worth emphasizing.

a. Estimating FCFE during High-Growth Period

As in the dividend discount model, we start with the earnings per share and estimate expected growth in earnings. Thus the entire discussion about earnings growth in the dividend discount model applies here as well. The only difference is in the estimation of fundamental growth. When estimating fundamental growth in the dividend discount model, we used the retention ratio and the return on equity to estimate the expected growth in earnings. When estimating fundamental growth in the FCFE valuation model, it is more consistent to use the equity reinvestment rate defined in the last section and the return on equity to estimate expected growth:

$$\text{Expected Growth in Net Income} = \text{Equity Reinvestment Rate} * \text{Return on Equity}$$

Unlike the retention ratio, which cannot exceed 100 percent or be less than 0 percent, the equity reinvestment rate can be negative (if capital expenditures drop below depreciation) or greater than 100 percent. If the equity reinvestment rate is negative and is expected to remain so for the foreseeable future, the expected growth in earnings will be negative. If the equity reinvestment rate is greater than 100 percent, the net income can grow at a rate

higher than the return on equity, though the firm will have to issue new stock to fund the reinvestment.¹³

Once the earnings are estimated, the net capital expenditures, working capital needs, and debt financing needs have to be specified to arrive at the FCFE. Just as the dividend payout ratio was adjusted to reflect changes in expected growth, the net capital expenditure and working capital needs should change as the growth rate changes. In particular, high growth companies will have relatively higher net capital expenditures and working capital needs. In other words, the equity reinvestment rate will generally be high in high growth and decline as the growth rate declines. A similar point can be made about leverage. High-growth, high-risk firms generally do not use much leverage to finance investment needs; as the growth tapers off, however, the firm will be much more willing to use debt, suggesting that debt ratios will increase as growth rates drop.

There is one final point worth making about equity valuations. Because the net income includes both income from operations and income from cash and marketable securities, we have two choices when it comes to equity valuations. The first and easier (albeit less precise) option is to discount the total FCFE (including the income from cash holdings) at a cost of equity that is adjusted to reflect the cash holdings.¹⁴ The present value of equity will then incorporate the cash holdings of the company. The second and more precise way is to discount the net income, without including the interest income from cash, at a cost of equity that reflects only the operations of the firm and then to add the cash and marketable securities on to this present value at the end.

Capex.xls: This file online contains the industry averages by sector for net capital expenditures and working capital as a percent of revenues.

¹³If the equity reinvestment rate exceeds 100 percent, the net income of the firm is insufficient to cover the equity reinvestment needs of the firm. Fresh equity will have to be issued to fund the difference. This will increase the number of shares outstanding.

¹⁴The beta for equity will be based on an unlevered beta, adjusted for the cash holdings of the company. In other words, if the company is 20 percent cash and 80 percent operations, the unlevered beta will be estimated attaching a 20 percent weight to cash and a beta of zero for cash.

Illustration 12.5 Estimating Growth Rate in Net Income

Like many manufacturing firms, Tata Chemicals has volatile reinvestment outlays and the cash flows from debt swing wildly from year to year. In Table 12.4, we report net income and equity reinvestment (capital expenditures – depreciation + change in noncash working capital – net cash flow from debt) each year from 2004 to 2008.

Table 12.4 Equity Reinvestment and Net Income at Tata Chemicals: 2004-08

Year	Net Income	Cap Ex	Depreciation	Change in WC	Change in Debt	Equity Reinvestment	Equity Reinvestment Rate
2003-04	\$3,418	\$357	\$1,442	-\$557	-\$2,771	\$1,129	33.04%
2004-05	\$4,550	\$692	\$1,377	-\$493	\$5,448	-\$6,626	-145.64%
2005-06	\$5,156	\$11,730	\$1,389	\$2,823	\$867	\$12,297	238.51%
2006-07	\$6,338	\$1,196	\$1,504	-\$1,662	-\$4,411	\$2,442	38.53%
2007-08	\$11,571	\$28,956	\$1,488	\$88	\$17,054	\$10,502	90.76%
Aggregate	\$31,033	\$42,930	\$7,199	\$200	\$16,187	\$19,744	63.62%

Rather than base the equity reinvestment rate on the most recent year's numbers, we will use the aggregate values for each of the variables over the entire period to compute a normalized equity reinvestment rate:

$$\text{Equity Reinvestment Rate} = \frac{\text{Equity Reinvestment}_{\text{Total 2004-08}}}{\text{Net Income}_{\text{Total 2004-08}}} = \frac{19,744}{31,033} = 63.62\%$$

To estimate the return on equity, we look at the same time period and look at the net income and the book value of equity each year from 2004 to 2008 in table 12.5:

Table 12.5: Net Income and ROE – 2003-2008

Year	Net Income	BV of Equity	ROE
2003-04	\$3,418	\$20,353	16.80%
2004-05	\$4,550	\$19,978	22.78%
2005-06	\$5,156	\$39,451	13.07%
2006-07	\$6,338	\$37,258	17.01%
2007-08	\$11,571	\$61,952	18.68%
Aggregate	\$31,033	\$178,992	17.34%

The normalized return on equity over the period is computed using the aggregated values of net income and book value of equity:

$$\text{Return on Equity} = \frac{\text{Net Income}_{\text{Total 2004-08}}}{\text{Book Value of Equity}_{\text{Total 2004-08}}} = \frac{31,033}{178,992} = 17.34\%$$

The expected growth in net income can be computed as the product of the ROE and the equity reinvestment rate.

$$\begin{aligned}\text{Expected Growth in Net Income} &= \text{Equity Reinvestment Rate} * \text{ROE} \\ &= 63.62\% * 17.34\% = 11.03\%\end{aligned}$$

Based on fundamentals, we would expect Tata Chemical's net income to grow 11.03% a year.

In Practice: Paths to a Higher ROE

The expected growth rate in earnings per share and net income are dependent on the return on equity that a firm makes on its new investments. The higher the return on equity, the higher the expected growth rate in earnings. But how do firms generate higher returns on equity? Algebraically, the return on equity can be decomposed into a return on capital and a leverage effect:

$$\text{ROE} = \text{Return on Capital} + \frac{\text{Debt}}{\text{Equity}}(\text{Return on Capital} - \text{After-tax Cost of Debt})$$

The second term in the equation reflects the influence of debt. To the extent that a firm can earn a return on capital that exceeds the after-tax cost of debt, its return on equity will increase as it uses more debt. A firm with a return on capital of 12 percent, a debt to equity ratio of 0.5, and an after-tax cost of debt of 4 percent will have a return on equity of 16 percent. Lest firms view this as a free lunch, we hasten to point out that using more debt will also increase the firm's beta and cost of equity and the value of equity may very well decrease with higher borrowing, even though the return on equity and expected growth rate may be higher.

b. Estimating Terminal Value

As with the dividend discount model, the terminal value in the FCFE model is determined by the stable growth rate and cost of equity. The difference between this model and the dividend discount model lies primarily in the cash flow used to calculate the terminal price: The latter uses expected dividends in the period after high growth, whereas the former uses the FCFE in that period:

$$\text{Terminal value of Equity}_n = \frac{\text{FCFE}_{n+1}}{r - g_n}$$

In estimating that cash flow, the net capital expenditures and working capital needs should be consistent with the definition of stability. The simplest way to ensure this is to estimate an equity reinvestment rate from the stable period return on equity:

$$\text{Equity Reinvestment Rate in Stable Growth} = 1 - \frac{g_{\text{Stable}}}{\text{ROE}_{\text{Stable}}}$$

This is exactly the same equation we used to compute the retention ratio in stable growth in the dividend discount model.

Many analysts assume that stable-growth firms have capital expenditures that offset depreciation and no working capital requirements. This will yield a equity reinvestment rate of zero, which is consistent only with a stable growth rate of zero. Using a stable growth rate of 3 or 4 percent while allowing for no reinvestment essentially allows your firm to grow without paying for the growth and will yield too high a value for the firm.

Reconciling FCFE and Dividend Discount Model Valuations

The FCFE discounted cash flow model can be viewed as an alternative to the dividend discount model. Because the two approaches sometimes provide different estimates of value, however, it is worth examining why this occurs.

There are two conditions under which the value obtained from using the FCFE in discounted cash flow valuation will be the same as the value obtained from using the dividend discount model. The first is obvious: When the dividends are equal to the FCFE, the value will be the same. The second is more subtle: When the FCFE is greater than dividends, but the excess cash (FCFE – Dividends) is invested in projects with a net present value of zero, the values will also be similar. For instance, investing in financial assets that are fairly priced should yield an NPV of zero.¹⁵

More often, the two models will provide different estimates of value. First, when the FCFE is greater than the dividend and the excess cash either earns below-market returns or is invested in negative NPV projects, the value from the FCFE model will be greater than the value from the dividend discount model. This is not uncommon. There

¹⁵Mechanically, this will work out only if you keep track of the cash build-up in the dividend discount model and add it to the terminal value. If you do not do this, you will under value your firm with the dividend discount model.

are numerous case studies of firms having accumulated large cash balances by paying out low dividends relative to FCFE that have chosen to use this cash to finance unwise takeovers (the price paid is greater than the value received). Second, the payment of smaller dividends than the firm can afford lowers debt-equity ratios; accordingly, the firm may become underleveraged, reducing its value.

In those cases where dividends are greater than FCFE, the firm will have to issue new shares or borrow money to pay these dividends, leading to at least one of three possible negative consequences. One is the flotation cost on these security issues, which can be substantial for equity issues. Second, if the firm borrows the money to pay the dividends, the firm may become overleveraged (relative to the optimal), leading to a loss in value. Finally, paying too much in dividends can lead to capital rationing constraints, whereby good projects are rejected, resulting in a loss of wealth.

When the two models yield different values, two questions remain: (1) What does the difference between the two models tell us? (2) Which of the two models is appropriate to use in evaluating the market price? In most cases, the value from the FCFE model will exceed the value from the dividend discount model. The difference between the value obtained from the FCFE model and that obtained from the dividend discount model can be considered one component of the value of controlling a firm—that is, it measures the value of controlling dividend policy. In a hostile takeover, the bidder can expect to control the firm and change the dividend policy (to reflect FCFE), thus capturing the higher FCFE value. In the more infrequent case—the value from the dividend discount model exceeds the value from the FCFE—the difference has less economic meaning but can be considered a warning on the sustainability of expected dividends.

As for which of the two values is more appropriate for evaluating the market price, the answer lies in the openness of the market for corporate control. If there is a significant probability that a firm can be taken over or its management changed, the market price will reflect that likelihood; in that case, the value from the FCFE model would be a more appropriate benchmark. As changes in corporate control become more difficult, either because of a firm's size and/or legal or market restrictions on takeovers,

the value from the dividend discount model will provide a more appropriate benchmark for comparison.

12.7. FCFE and Discount Dividend Value

Most firms can be valued using FCFE and discount dividend valuation models. Which of the following statements would you most agree with on the relationship between these two values?

- The FCFE value will always be higher than the discount dividend value.
- The FCFE value will usually be higher than the discount dividend value.
- The discount dividend value will usually be higher than the FCFE value.
- The discount dividend value will generally be equal to the FCFE value.

Illustration 12.6 FCFE Valuation: Tata Chemicals

To value Tata Chemicals using the FCFE model, we will use the expected growth in net income that we estimated in Illustration 12.4 and value the equity in operating assets first and then add on the value of cash and other non-operating assets. Summarizing the basic information that we will be using:

- Rather than use the net income from 2007-08 as the base year income, we used the normalized return on equity of 17.34% (from illustration 11.5) and the current book value of equity (Rs 35,717 million) to estimate the base year net income:

$$\begin{aligned}\text{Normalized Net Income} &= \text{Current Book Value of Equity} * \text{Normalized ROE} \\ &= 35,717 * .1734 = \text{Rs, 6,193 million}\end{aligned}$$

- We will use the average equity reinvestment rate of 63.62 percent, based on the average values from 2004-08, that we computed in Illustration 12.5 as the equity reinvestment rate for the next five years. In conjunction with the normalized return on equity of 17.34% that we computed in that illustration, we estimate an expected growth rate of 11.03 percent a year for the next five years.

- In Illustration 4.9, we estimated a beta for equity of 0.945 for Tata Chemical's operating assets.¹⁶ With a nominal rupee risk-free rate of 4 percent and an equity risk premium of 10.51% for India (also estimated in Chapter 4), we arrive at a cost of equity of 13.93%.

$$\text{Cost of Equity} = 4\% + 0.945 (10.51\%) = 13.93\%$$

After year five, we will assume that the beta will increase to 1 and that the equity risk premium will decline to 7.5 percent.¹⁷ The resulting cost of equity is 11.5 percent.

$$\text{Cost of Equity in Stable Growth} = 4\% + 1(7.5\%) = 11.5\%$$

- After year five, we will assume that the growth in net income will drop to 4% and that the return on equity will rise to 11.5% (which is also the cost of equity). The equity reinvestment rate in stable growth can then be estimated as follows:

$$\begin{aligned}\text{Equity Reinvestment Rate}_{\text{Stable Growth}} &= \text{Expected Growth Rate/Return on Equity} \\ &= 4\%/11.5\% = 34.78\%\end{aligned}$$

To value the equity in Tata Chemicals, we begin by estimating the FCFE from operations in Table 12.6.

Table 12.6 Expected FCFE at Tata Chemicals, 2009-2013

	2009	2010	2011	2012	2013	Sum
Net Income	Rs 6,876	Rs 7,634	Rs 8,476	Rs 9,411	Rs 10,449	
Equity Reinvestment Rate	63.62%	63.62%	63.62%	63.62%	63.62%	
FCFE	Rs 2,501	Rs 2,777	Rs 3,084	Rs 3,423	Rs 3,801	
Cost of Equity	13.93%	13.93%	13.93%	13.93%	13.93%	
Present Value	Rs 2,195	Rs 2,160	Rs 2,085	Rs 2,032	Rs 1,980	Rs 10,433

FCFE = Net Income (1 – Reinvestment Rate).

To estimate the terminal value of equity, we first estimate the FCFE in year six:

$$\begin{aligned}\text{FCFE in Year 6} &= \text{Net Income in Year 6} (1 - \text{Equity Reinvestment Rate}_{\text{Stable Growth}}) \\ &= 10,449(1.04)(1 - 0.3478) = \text{Rs } 7,087 \text{ million}\end{aligned}$$

¹⁶We used the equity beta of just the operating assets in this valuation. If we had chosen to include the cash from financial holdings as part of net income, we would have adjusted the beta for Tata Chemical's cross holdings.

¹⁷We halved the country risk premium from 7.67 percent to 3.84 percent. We assume that as Brazil grows, it will become a less risky country in which to invest.

The terminal value is then computed using the stable period cost of equity of 10.56 percent:

$$\text{Terminal Value of Equity} = 7,087 / (0.115 - 0.04) = \text{Rs } 94,497 \text{ million}$$

The current value of equity is the sum of the present values of the expected cash flows in Table 12.3, the present value of the terminal value of equity, and the value of cash and non-operating assets today:

Present Value of FCFEs in High-Growth Phase	= Rs 10,433
+ Present Value of Terminal Equity Value = $94,497 / 1.1393^5$	= Rs 49,231
= Value of Equity in Operating Assets	= Rs 59,664
+ Value of Cash and Marketable Securities	= Rs 1,759
Value of Equity in Firm	= Rs 61,423

Dividing by the 235.17 million shares outstanding yields a value per share of Rs 261, about 20% higher than the stock price of Rs 222 at the time of the valuation (June 2009).

In Practice: Reconciling Value with the Market Price

When you value a company and arrive at a number very different from the market price, there are three possible explanations. The first is that we are mistaken in our assumptions and that our valuations are wrong while the market is right. Without resorting to the dogma of efficient markets, this is a reasonable place to start because this is the most likely scenario. The second is that the market is wrong and we are right, in which case we have to decide whether we have enough confidence in our valuations to act on them. If we find a company to be undervalued, this would require buying and holding the stock. If the stock is overvalued, we would have to sell short. The problem, though, is that there is no guarantee that markets, even if they are wrong, will correct their mistakes in the near future. In other words, a stock that is overvalued can become even more overvalued, and a stock that is undervalued may stay that way for years, wreaking havoc on our portfolio. This also makes selling short a much riskier strategy because we generally can do so only for a few months.

One way to measure market expectations is to solve for a growth rate that will yield the market price. In the Tata Chemicals valuation, for instance, we would need an expected growth rate of 7% in earnings over the next five years to justify the current

market price. This is called an implied growth rate and can be compared to the estimate of growth we used in the valuation of 11.03%.

III. Free Cash Flow to the Firm Models

The dividend discount and FCFE models are models for valuing the equity in a firm directly. The alternative is to value the entire business and then to use this value to arrive at a value for the equity. That is precisely what we try and do in firm valuation models, where we focus on the operating assets of the firm and the cash flows they generate.

Setting Up the Model

The cash flow to the firm can be measured in two ways. One is to add up the cash flows to all of the different claim holders in the firm. Thus, the cash flows to equity investors (which take the form of dividends or stock buybacks) are added to the cash flows to debt holders (interest and net debt payments) to arrive at the cash flow to the firm. The other approach to estimating cash flow to the firm, which should yield equivalent results, is to estimate the cash flows to the firm prior to debt payments but after reinvestment needs have been met:

$$\begin{aligned}
 & \text{EBIT} (1 - \text{Tax Rate}) \\
 & - (\text{Capital Expenditures} - \text{Depreciation}) \\
 & - \text{Change in Noncash Working Capital} \\
 & = \text{Free Cash Flow to the Firm}
 \end{aligned}$$

The difference between capital expenditures and depreciation (net capital expenditures) and the increase in noncash working capital represents the reinvestment made by the firm to generate future growth. Another way of presenting the same equation is to add the net capital expenditures and the change in working capital and state that value as a percentage of the after-tax operating income. This ratio of reinvestment to after-tax operating income is called the *reinvestment rate*, and the FCFF can be written as:

$$\text{Reinvestment Rate} = \frac{(\text{Capital Expenditures} - \text{Depreciation} + \Delta \text{Working Capital})}{\text{EBIT} (1 - \text{tax rate})}$$

$$\text{Free Cash Flow to the Firm} = \text{EBIT} (1 - t)(1 - \text{Reinvestment Rate})$$

Note that the reinvestment rate can exceed 100 percent if the firm has substantial reinvestment needs.¹⁸ If that occurs, the FCFF will be negative even though after-tax operating income is positive. The cash flow to the firm is often called an unlevered cash flow, because it is unaffected by debt payments or the tax benefits flowing from these payments.¹⁹

As with the dividends and the FCFE, the value of the operating assets of a firm can be written as the present value of the expected cash flows during the high-growth period and a terminal value at the end of the period:

$$\text{Value}_0 = \sum_{t=1}^{t=n} \frac{E(\text{FCFF})_t}{(1+r)^t} + \frac{\text{Terminal Value}_n}{(1+r)^n} \text{ where } \text{Terminal Value}_n = \frac{E(\text{FCFF})_{n+1}}{(r_n - g_n)}$$

where r is the cost of capital and g_n is the expected growth rate in perpetuity.

Estimating Model Inputs

As with the dividend discount and the FCFE discount models, there are four basic components that go into the value of the operating assets of the firm—a period of high growth, the FCFF during that period, the cost of capital to use as a discount rate, and the terminal value for the operating assets of the firm. We have additional steps to take to get to the value of equity per share. In particular, we have to incorporate the value of nonoperating assets, subtract out debt, and then consider the effect of options outstanding on the equity of the firm.

a. Estimating FCFF during High-Growth Period

We base our estimate of a firm's value on expected future cash flows, not current cash flows. The forecasts of earnings, net capital expenditures, and working capital will yield these expected cash flows. One of the most significant inputs into any valuation is the expected growth rate in operating income. As with the growth rates we estimated for dividends and net income, the variables that determine expected growth are simple. The

¹⁸In practical terms, this firm will need external financing, either from debt or equity or both, to cover the excess reinvestment.

¹⁹The tax benefits from interest payments, which are real cash benefits, show up in the discount rate, when we compute the after-tax cost of debt. If we add this tax benefit as a cash flow to the FCFF, we double count the tax benefit.

expected growth in operating income is a product of a firm's reinvestment rate, that is, the proportion of the after-tax operating income that is invested in net capital expenditures and noncash working capital, and the quality of these reinvestments, measured as the after-tax return on the capital invested.

$$\text{Expected Growth}_{\text{EBIT}} = \text{Reinvestment Rate} * \text{Return on Capital}$$

where

$$\text{Reinvestment Rate} = \frac{\text{Capital Expenditure} - \text{Depreciation} + \Delta \text{ Non-cash WC}}{\text{EBIT} (1 - \text{tax rate})}$$

$$\text{Return on Capital} = \frac{\text{EBIT}(1 - t)}{(\text{BV of Equity} + \text{BV of Debt} - \text{Cash})}$$

Both measures should be forward-looking, and the return on capital should represent the expected return on capital on future investments. In the rest of this section, we consider how best to estimate the reinvestment rate and the return on capital.

The reinvestment rate is often measured using a firm's past history on reinvestment. Although this is a good place to start, it is not necessarily the best estimate of the future reinvestment rate. A firm's reinvestment rate can ebb and flow, especially in firms that invest in relatively few large projects or acquisitions. For these firms, looking at an average reinvestment rate over time may be a better measure of the future. In addition, as firms grow and mature, their reinvestment needs (and rates) tend to decrease. For firms that have expanded significantly over the past few years, the historical reinvestment rate is likely to be higher than the expected future reinvestment rate. For these firms, industry averages for reinvestment rates may provide a better indication of the future than using numbers from the past. Finally, it is important that we continue treating R&D expenses and operating lease expenses consistently. The R&D expenses in particular need to be categorized as part of capital expenditures for purposes of measuring the reinvestment rate.

The return on capital is often based on the firm's return on capital on existing investments, where the book value of capital is assumed to measure the capital invested in these investments. Implicitly, we assume that the current accounting return on capital is a good measure of the true returns earned on existing investments, and that this return is a good proxy for returns that will be made on future investments. This assumption, of course, is open to question if the book value of capital is not a good measure of the

capital invested in existing projects and/or if the operating income is mismeasured or volatile. Given these concerns, we should consider not only a firm's current return on capital but also any trends in this return as well as the industry average return on capital. If the current return on capital for a firm is significantly higher than the industry average, the forecasted return on capital should be set lower than the current return to reflect the erosion that is likely to occur as competition responds.

Finally, any firm that earns a return on capital greater than its cost of capital is earning an excess return. These excess returns are the result of a firm's competitive advantages or barriers to entry into the industry. High excess returns locked in for very long periods imply that a firm has a permanent competitive advantage.

In Practice: After-Tax Operating Income

The income statement for a firm provides a measure of the operating income of the firm in the form of the EBIT and a tax rate in the form of an effective tax rate. Because the operating income we would like to estimate is before capital and financing expenses, we have to make at least two adjustments to the accounting operating income:

- The first adjustment is for financing expenses that accountants treat as operating expenses. The most significant example is operating leases. Because these lease payments constitute firm commitments into the future, they are tax-deductible, and the failure to make lease payments can result in bankruptcy, so we treat these expenses as financial expenses. The adjustment, which we describe in detail in Chapter 4, results in an increase in both the operating income and the debt outstanding at the firm.
- The second adjustment is to correct for the incidence of one-time or irregular income and expenses. Any expense (or income) that is truly a one-time expense (or income) should be removed from the operating income and should not be used in forecasting future operating income. Although this would seem to indicate that all extraordinary charges should be expunged from operating income, there are some extraordinary charges that seem to occur at regular intervals—say, once every four or five years. Such expenses should be viewed as irregular rather than extraordinary expenses and should be built into forecasts. The easiest way to do this is to annualize the expense.

Put simply, this would mean taking one-fifth of any expense that occurs once every five years, and computing the income based on this apportioned expense.

As for the tax rate, the effective tax rates reported by most firms are much lower than the marginal tax rates. As with the operating income, we should look at the reasons for the difference and see if these firms can maintain their lower tax rates. If they cannot, it is prudent to shift to marginal tax rates in computing future after-tax operating income.

Illustration 12.7 Estimating Growth Rate in Operating Income: Disney

We begin by estimating the reinvestment rate and return on capital for Disney in 2008 using the numbers from the latest financial statements. We converted operating leases into debt and adjusted the operating income and capital expenditure accordingly.²⁰

$$\begin{aligned}\text{Reinvestment Rate}_{2008} &= \frac{(\text{Cap Ex} - \text{Depreciation} + \text{Chg in WC})}{\text{EBIT} (1 - t)} \\ &= \frac{(2,752 - 1,839 + 241)}{7,030 (1 - .38)} = 26.48\%\end{aligned}$$

We include \$516 million in acquisitions made during 2008 in capital expenditures, but this is a volatile item. Disney does not make large acquisitions every year, but it does so infrequently - \$ 7.5 billion to buy Pixar in 2006 and \$ 11.5 billion to buy Capital Cities in 1996. Averaging out acquisitions from 1994-2008, we estimate an average annual value of \$1,761 million for acquisitions over this period. Replacing the current year's acquisition with this normalized value yields a higher reinvestment rate:

$$\text{Reinvestment Rate}_{2008} = \frac{(3,939 - 1,839 + 241)}{7,030 (1 - .38)} = 53.72\%$$

We compute the return on capital, using operating income in 2008 and capital invested at the start of 2008 (end of 2007):

$$\begin{aligned}\text{Return on Capital}_{2008} &= \frac{\text{EBIT} (1 - t)}{(\text{BV of Equity} + \text{BV of Debt} - \text{Cash})} = \frac{7,030 (1 - .38)}{(30,753 + 16,892 - 3,670)} = 9.91\%\end{aligned}$$

²⁰The book value of debt is augmented by the \$1,720 million in present value of operating lease commitments. The unadjusted operating income for Disney was \$6,726 million. The operating lease adjustment adds the current year's operating lease expense to capital expenditures (\$550 million), and subtracts out the depreciation on the leased asset to depreciation (\$246 million) to arrive at an adjusted operating income of \$7,030 million.

If Disney maintains its 2008 reinvestment rate and return on capital for the next few years, its growth rate will be only 2.35 percent.

$$\text{Expected Growth Rate from Existing Fundamentals} = 53.72\% * 9.91\% = 5.32\%$$



Valuing Growth Companies: A Behavioral Perspective

In theory, we should expect to see larger valuation errors with growth companies than with mature companies, because there is more firm-specific uncertainty that we face in valuing growth companies, insofar as we have to estimate how long growth will last and how high growth will be during the period. In practice, we generally find support for this hypothesis but we also find that there is more bias in the valuation of growth companies. In particular, there is evidence to suggest that high growth (and high PE) stocks tend to earn returns that are too low and are thus priced too high, relative to low growth stocks. There are three reasons why this may occur:

- a. Over confidence: Through this book, we have chronicled the effects of over confidence on corporate finance decisions. Over confident managers tend to take too many investments, over pay on acquisitions and borrow too much. Over confident investors tend to under estimate the likelihood that firms will fail and over estimate future potential. While all valuations are affected by this over confidence, the effects on value are much greater with growth companies, where failure is much more likely and future potential accounts for a much larger proportion of value.
- b. Scaling biases: For better or worse, analysts tend to look at growth rates in recent periods and extrapolate that growth into future periods. While this practice again may affect all valuations, it has a much bigger effect when valuing small companies that have been able to post very high growth rates (reflecting their small size) in recent time periods.

There is a final factor that may be at play here. The analysts, managers and appraisers who are attracted to the sectors with high growth (technology, for instance) may represent the most over optimistic individuals in the overall population and their valuations will reflect that selection bias.



fundgrEB.xls: There is a data set online that summarizes reinvestment rates and return on capital by industry group in the United States for the most recent quarter.

b. Estimating Cost of Capital

Unlike equity valuation models, where the cost of equity is used to discount cash flows to equity, the cost of capital is used to discount cash flows to the firm. The cost of capital is a composite cost of financing that includes the costs of both debt and equity and their relative weights in the financing structure:

$$\text{Cost of Capital} = k_{\text{equity}}(\text{Equity}/[\text{Debt} + \text{Equity}]) + k_{\text{debt}}(\text{Debt}/[\text{Debt} + \text{Equity}])$$

where the cost of equity represents the rate of return required by equity investors in the firm and the cost of debt measures the current cost of borrowing, adjusted for the tax benefits of borrowing. The weights on debt and equity have to be market value weights. We discussed the cost of capital estimation extensively earlier in this book, in the context of both investment analysis and capital structure. We will consider each of the inputs in the model in the context of valuing a firm.

The cost of equity, as we have defined it through this book, is a function of the non-diversifiable risk in an investment, which in turn is measured by a beta (in the single factor model) or betas (in the multiple factor models). We argued that the beta(s) are better measured by looking at the average beta(s) of other firms in the business, that is, bottom-up estimates, and reflecting a firm's current business mix and leverage. This argument is augmented when we value companies by the fact that a firm's expected business mix and financial leverage can change over time, and its beta will change with both. As the beta changes, the cost of equity will also change from year to year.

Just as the cost of equity can change over time as a firm's exposure to market risk changes, so can the cost of debt as its exposure to default risk changes. The default risk of a firm can be expected to change for two reasons. One is that the firm's size will change as we project earnings further into the future; the volatility in these earnings is also likely to change over time. The second reason is that changes occur in financial leverage. If we expect a firm's financial leverage to change over time, it will affect its capacity to service debt and hence its cost of borrowing. The after-tax cost of debt can also change as a consequence of expected changes in the tax rate over time.

As a firm changes its leverage, the weights attached to equity and debt in the cost of capital computation will change. Should a firm's leverage be changed over the forecast period? The answer to this depends on two factors. The first is whether the firm is initially under- or overlevered. If it is at its appropriate leverage, there is a far smaller need to change leverage in the future. The second is the views of the firm's management and the degree to which they are responsive to the firm's stockholders. Thus, if the management of a firm is firmly entrenched and steadfast in its opposition to debt, an underlevered firm will stay that way over time. In an environment where stockholders have more power, there will eventually be pressure on this firm to increase its leverage toward its optimal level.

Illustration 12.8 Cost of Capital: Disney

Recapping the inputs we used to estimate the cost of capital in Disney, we will make the following assumptions:

- The beta for the first five years will be the bottom-up beta of 0.9011 that we estimated in Illustration 4.7. In conjunction with a risk-free rate of 3.5 percent and market risk premium of 6%, this yields a cost of equity of 8.91 percent.

$$\text{Cost of Equity} = \text{Risk-Free Rate} + \text{Beta} * \text{Risk Premium} = 3.5\% + 0.9011 (6\%) = 8.91\%$$

- The cost of debt for Disney for the first five years, based on its rating of A, is 6%. Using Disney's tax rate of 38 percent gives an after-tax cost of debt of 3.29 percent:

$$\text{After-Tax Cost of Debt} = 6\% (1 - 0.38) = 3.72\%$$

- The current market debt ratio of 26.7% debt will be used as the debt ratio for the first five years of the valuation. Keep in mind that this debt ratio is computed using the market value of debt (inclusive of operating leases) of \$16,682 million and a market value of equity of \$46,045 million.

The cost of capital for Disney, at least for the first five years of the valuation, is 7.52%.

$$\begin{aligned} \text{Cost of Capital} &= \text{Cost of Equity } (E/[D + E]) + \text{After-Tax Cost of Debt } (D/[D + E]) \\ &= 8.91\%(0.763) + 3.72\%(0.267) = 7.52\% \end{aligned}$$



12.8. Firm Valuation and Leverage

A standard critique of the use of cost of capital in firm valuation is that it assumes that leverage stays stable over time (through the weights in the cost of capital). Is this true?

- a. Yes
- b. No



wacc.xls: There is a data set online that summarizes the costs of capital for firms in the United States by industry group.

c. *Estimating Terminal Value*

The approach most consistent with a discounted cash flow model assumes that cash flows beyond the terminal year will grow at a constant rate forever, in which case the terminal value can be estimated as follows:

$$\text{Terminal Value}_n = \text{Free Cash Flow to Firm}_{n+1} / (\text{Cost of Capital}_{n+1} - g_n)$$

where the cost of capital and the growth rate in the model are sustainable forever. We can use the relationship between growth and reinvestment rates that we noted earlier to estimate the reinvestment rate in stable growth:

$$\text{Reinvestment Rate in Stable Growth} = \text{Stable Growth Rate} / \text{ROC}_n$$

where the ROC_n is the return on capital that the firm can sustain in stable growth. This reinvestment rate can then be used to generate the FCFF in the first year of stable growth:

$$\text{Terminal Value} = \frac{\text{EBIT}_{n+1}(1-t) \left(1 - \frac{g_n}{\text{ROC}_n}\right)}{(\text{Cost of Capital}_n - g_n)}$$

In the special case where ROC is equal to the cost of capital, this estimate simplifies to become the following:

$$\text{Terminal Value}_{\text{ROC=WACC}} = \frac{\text{EBIT}_{n+1}(1-t)}{\text{Cost of Capital}_n}$$

Thus, in every discounted cash flow valuation, there are two critical assumptions we need to make on stable growth. The first relates to when the firm we are valuing will become a stable-growth firm, if it is not one already. The second relates to what the characteristics of the firm will be in stable growth, in terms of return on capital and cost of capital. We

examined the first question earlier in this chapter when we looked at the dividend discount model. Let us consider the second question now.

As firms move from high growth to stable growth, we need to give them the characteristics of stable-growth firms. A firm in stable growth will be different from that same firm in high growth on a number of dimensions. For instance,

- As we noted with equity valuation models, high-growth firms tend to be *more exposed to market risk* (and have higher betas) than stable-growth firms. Thus, although it might be reasonable to assume a beta of 1.8 in high growth, it is important that the beta be lowered, if not to one at least toward one in stable growth.²¹
- High-growth firms tend to have *high returns on capital and earn excess returns*. In stable growth, it becomes more difficult to sustain excess returns. There are some who believe that the only assumption sustainable in stable growth is a zero excess return assumption; the return on capital is set equal to the cost of capital. Although we agree in principle, with this view it is difficult in practice to assume that all investments, including those in existing assets, will suddenly lose the capacity to earn excess returns. Because it is possible for entire industries to earn excess returns over long periods, we believe that assuming a firm's return on capital will move toward its industry average sometimes yields more reasonable estimates of value.
- Finally, high-growth firms tend to *use less debt* than stable-growth firms. As firms mature, their debt capacity increases. The question of whether the debt ratio for a firm should be moved toward its optimal cannot be answered without looking at the incumbent managers' power relative to their stockholders and their views about debt. If managers are willing to change their debt ratios and stockholders retain some power, it is reasonable to assume that the debt ratio will move to the optimal level in stable growth; if not, it is safer to leave the debt ratio at existing levels.



12.9. Net Capital Expenditures, FCFE, and Stable Growth

Assume that you are valuing a high-growth firm with high risk (beta) and large reinvestment needs (high reinvestment rate). You assume the firm will be in stable

growth after five years, but you leave the risk and reinvestment rate at high-growth levels. Will you undervalue or overvalue this firm?

- a. Undervalue the firm
- b. Overvalue the firm

Illustration 12.9 Stable Growth Inputs and Transition Period: Disney

We will assume that Disney will be in stable growth after year ten. In its stable growth phase, we will assume the following:

- The beta for the stock will drop to one, reflecting Disney's status as a mature company. This will lower the cost of equity for the firm to 9.50 percent.

$$\text{Cost of Equity} = \text{Risk-Free Rate} + \text{Beta} * \text{Risk Premium} = 3.5\% + 6\% = 9.50\%$$

- The debt ratio for Disney will rise to 30 percent. This is at the lower end of the optimal we computed for Disney in Chapter 8, and we are assuming that investor pressure will be the impetus for this change. Because we assume that the cost of debt remains unchanged at 6%, this will result in a cost of capital of 7.96 percent

$$\text{Cost of Capital} = 9.5\%(0.70) + 6\%(1 - 0.38)(0.30) = 7.96\%$$

- The return on capital for Disney will drop from its high-growth period level of 9.91 percent to a stable growth return of 9 percent. This is still higher than the cost of capital of 7.91%, but Disney's competitive advantages are unlikely to dissipate completely by the end of the tenth year.
- The expected growth rate in stable growth will be 3 percent. In conjunction with the return on capital of 9%, this yields a stable period reinvestment rate of 33.33%:

$$\text{Reinvestment Rate} = \text{Growth Rate} / \text{Return on Capital} = 3\% / 9\% = 33.33\%$$

The values of all of these inputs adjust gradually during the transition period, from years six to ten, from high-growth levels to stable-growth values.

a. From Operating Asset Value to Firm Value

The operating income is the income from operating assets, and the cost of capital measures the cost of financing these assets. When the operating cash flows are discounted to the present, we value the operating assets of the firm. Firms, however, often have

²¹As a rule of thumb, betas above 1.2 or below 0.8 are inconsistent with stable-growth firms. Two-thirds of

significant amounts of cash and marketable securities on their books. The value of these assets should be added to the value of the operating assets to arrive at firm value.

Cash and marketable securities can easily be incorporated into firm value, whereas other non operating assets are more difficult to value. Consider, for instance, minority holdings in other firms and subsidiaries, where income statements are not consolidated.²² If we consider only the reported income from these holdings, we will miss a significant portion of the value of the holdings.²³ The most accurate way to incorporate these holdings into firm value is to value each subsidiary or firm in which there are holdings and assign a proportional share of this value to the firm. If a firm owns more than 50 percent of a subsidiary, accounting standards in the United States require that the firm fully consolidate the income and assets of the subsidiary into its own. The portion of the equity that does not belong to the firm is shown as minority interest on the balance sheet and should be subtracted out to get to the value of the equity in the firm.²⁴

There is one final asset to consider. Firms with defined pension liabilities sometimes accumulate pension fund assets in excess of these liabilities. Although the excess does belong to the owners of the firm, they face a tax liability if they claim it. The conservative rule would be to assume that the social and tax costs of reclaiming the excess pension funds are so large that few firms would ever even attempt to do so.

Illustration 12.10 Value of Non-operating Assets at Disney

At the end of 2008, Disney reported holding \$3,795 million in cash and marketable securities. In addition, Disney reported a book value of \$1.763 billion for minority investments in other companies, primarily in foreign Disney theme parks.²⁵ In the absence of detailed financial statements for these investments, we will assume that the

all U.S. firms have betas that fall within this range.

²²When income statements are consolidated, the entire operating income of the subsidiary is shown in the income statement of the parent firm. Firms do not have to consolidate financial statements if they hold minority stakes in firms and take a passive role in their management.

²³When firms hold minority, passive interests in other firms, they report only the portion of the dividends they receive from these investments. With minority, active holdings, they report the portion of the net income that is attributable to them, but not as part of operating income.

²⁴Optimally, we would like to subtract out the market value of the minority interests rather than the book value, which is reported in the balance sheet.

²⁵Disney owns 39 percent of Euro Disney and 43 percent of the Hong Kong Disney park. It also owns 37.5 percent of the A&E network and 39.6 percent of E! Television.

book value is roughly equal to the market value. Note that we consider the rest of the assets on Disney's balance sheet including the \$5.4 billion it shows in capitalized TV and film costs and \$22.2 billion it shows in goodwill and intangibles to be operating assets that we have already captured in the cash flows.²⁶

Finally, Disney consolidates its holdings in a few subsidiaries in which it owns less than 100 percent. The portion of the equity in these subsidiaries that does not belong to Disney is shown on the balance sheet as a liability (minority interests) of \$1,344 million. As with its holdings in other companies, we assume that this is also the estimated market value and subtract it from firm value to arrive at the value of equity in Disney.



cash.xls: There is a data set online that summarizes the value of cash and marketable securities by industry group in the United States for the most recent quarter.

b. From Firm Value to Equity Value

The general rule that you should use is the debt you subtract from the value of the firm should be at least equal to the debt that you use to compute the cost of capital. Thus, if you decide to convert operating leases to debt to compute the cost of capital, you should subtract out the debt value of operating leases from the value of operating assets to estimate the value of equity. If the firm you are valuing has preferred stock, you would use the market value of the stock (if it is traded) or estimate a market value (if it is not) and deduct it from firm value to get to the value of common equity.²⁷

There may be other claims on the firm that do not show up in debt for purposes of computing cost of capital but should be subtracted out from firm value to get to the value of equity.

- *Expected Liabilities on Lawsuits*: You could be analyzing a firm that is the defendant in a lawsuit, where it potentially could have to pay tens of millions of dollars in damages. You should estimate the probability that this will occur and use this probability to estimate the expected liability. Thus, if there is a 10 percent chance that

²⁶Adding these on to the present value of the cash flows would represent double counting.

²⁷Estimating market value for preferred stock is relatively simple. Preferred stock generally is perpetual and the estimated market value of the preferred stock is therefore

Cost of preferred stock = Preferred Dividend/Cost of Preferred Stock

The cost of preferred stock should be higher than the pretax cost of debt, because debt has a prior claim on the cash flows and assets of the firm.

you could lose a case that you are defending and the expected damage award is \$1 billion, you would reduce the value of the equity in the firm by \$100 million (Probability * Expected Damages). If the expected liability is not expected to occur until several years from now, you would compute the present value of the payment.

- *Unfunded Pension and Health Care Obligations:* If a firm has significantly underfunded a pension or a health plan, it will need to set aside cash in future years to meet these obligations. Although it would not be considered debt for cost of capital purposes, it should be subtracted from firm value to arrive at equity value.
- *Deferred Tax Liability:* The deferred tax liability that shows up on the financial statements of many firms reflects the fact that firms often use strategies that reduce their taxes in the current year while increasing their taxes in future years. Of the three items listed here, this one is the least clearly defined, because it is not clear when or even whether the obligation will come due. Ignoring it, though, may be foolhardy, because the firm could find itself making these tax payments in the future. The most sensible way of dealing with this item is to consider it an obligation, but one that will come due only when the firm's growth rate moderates. Thus, if you expect your firm to be in stable growth in ten years, you would discount the deferred tax liability back ten years and deduct this amount from the firm value to get to equity value.

e. From Equity Value to Equity Value per Share

Once the value of the firm, inclusive of nonoperating assets, has been estimated, we generally subtract the value of the outstanding debt to arrive at the value of equity and then divide the value of equity by the number of shares outstanding to estimate the value per share. This approach works only when common stock is the only equity outstanding. When there are warrants and employee options outstanding, the estimated value of these options has to be subtracted from the value of the equity before we divide by the number of shares outstanding. The same procedure applies when the firm has convertible bonds outstanding, because these conversion options represent claims on equity as well.

For those unwilling to use option pricing models, there are two shortcuts available. One is to divide the value of equity by the fully diluted number of shares

outstanding rather than by the actual number.²⁸ This approach will underestimate the value of the equity because it fails to consider the cash proceeds from option exercise. The other shortcut, which is called the *treasury stock approach*, adds the expected proceeds from the exercise of the options (exercise price multiplied by the number of options outstanding) to the numerator before dividing by the number of shares outstanding. Although this approach will yield a more reasonable estimate than the first one, it does not include the time premium of the options outstanding. Thus, it tends to overstate the value of the common stock.



warrants.xls: This spreadsheet allows you to value the options outstanding in a firm, allowing for the dilution effect.

Illustration 12.11 Value of Equity Options

Disney has granted considerable numbers of options to its managers. At the end of 2008, there were 171 million options outstanding, with a weighted average exercise price of \$28.37 and weighted average life of six years. Using the current stock price of \$24.34, an estimated standard deviation of 29 percent,²⁹ a dividend yield of 1.54%, and an option pricing model, we estimate the value of these equity options to \$851 million.³⁰ The value we have estimated for the options above are probably too high, because we assume that all the options are exercisable. In fact, a significant proportion of these options (about 30%) are not vested yet,³¹ and this fact will reduce their estimated value. We will also assume that these options, when exercised, will generate a tax benefit to the firm equal to 38% of their value:

$$\text{After-Tax Value of Equity Options} = 851(1 - 0.38) = \$528 \text{ million}$$

To get to the value of equity in common stock, we will reduce the overall value of equity by the after-tax value of options granted by the firm. Dividing the value of equity in

²⁸We assume that all options will be exercised and compute the number of shares that will be outstanding in that event.

²⁹We used the historical standard deviation in Disney's stock price to estimate this number.

³⁰The option pricing model used is the Black-Scholes model, adjusted for potential dilution. It is explained in appendix 4.

³¹When options are not vested, they cannot be exercised. When providing options to their employees, firms often require that they continue as employees for a set period (vesting period) before they can exercise these options.

common stock by the actual number of shares outstanding should yield a value of equity value per share.

Reconciling Equity and Firm Valuations

This model, unlike the dividend discount model or the FCFE model, values the firm rather than equity. The value of equity, however, can be extracted from the value of the firm by subtracting out the market value of outstanding debt. Because this model can be viewed as an alternative way of valuing equity, two questions arise: Why value the firm rather than equity? Will the values for equity obtained from the firm valuation approach be consistent with the values obtained from the equity valuation approaches described in the previous section?

The advantage of using the firm valuation approach is that cash flows relating to debt do not have to be considered explicitly, because the FCFF is a pre-debt cash flow, whereas they do have to be taken into account in estimating FCFE. In cases where the leverage is expected to change significantly over time, this is a significant saving, because estimating new debt issues and debt repayments when leverage is changing can become increasingly messy the further into the future you go. The firm valuation approach does, however, require information about debt ratios and interest rates to estimate the WACC.

The value for equity obtained from the firm valuation and equity valuation approaches will be the same if you make consistent assumptions about financial leverage. Getting them to converge in practice is much more difficult. Let us begin with the simplest case—a no-growth, perpetual firm. Assume that the firm has \$166.67 million in EBIT and a tax rate of 40 percent. Assume that the firm has equity with a market value of \$600 million, with a cost of equity of 13.87 percent, and debt of \$400 million, with a pretax cost of debt of 7 percent. The firm's cost of capital can be estimated:

$$\text{Cost of Capital} = (13.87\%)\left(\frac{600}{1000}\right) + (7\%)(1 - 0.4)\left(\frac{400}{1000}\right) = 10\%$$

$$\text{Value of the Firm} = \frac{\text{EBIT}(1 - t)}{\text{Cost of capital}} = \frac{166.67(1 - 0.4)}{0.10} = \$1,000$$

Note that the firm has no reinvestment and no growth. We can value equity in this firm by subtracting out the value of debt.

$$\text{Value of Equity} = \text{Value of Firm} - \text{Value of Debt} = \$1,000 - \$400 = \$600 \text{ million}$$

Now let us value the equity directly by estimating the net income:

$$\text{Net Income} = (\text{EBIT} - \text{Pretax Cost of Debt} * \text{Debt})(1 - t) = (166.67 - 0.07 * 400)(1 - 0.4) = \$83.202 \text{ million}$$

The value of equity can be obtained by discounting this net income at the cost of equity:

$$\text{Value of Equity} = \frac{\text{Net Income}}{\text{Cost of equity}} = \frac{83.202}{0.1387} = \$ 600 \text{ million}$$

Even this simple example works because of the following assumptions that we made implicitly or explicitly during the valuation.

1. The values for debt and equity used to compute the cost of capital were equal to the values that we obtained in the valuation. Notwithstanding the circularity in reasoning—you need the cost of capital to obtain the values in the first place—it indicates that a cost of capital based on market value weights will not yield the same value for equity as an equity valuation model if the firm is not fairly priced in the first place.
2. There are no extraordinary or nonoperating items that affect net income but not operating income. Thus, to get from operating to net income, all we do is subtract out interest expenses and taxes.
3. The interest expenses are equal to the pretax cost of debt multiplied by the market value of debt. If a firm has old debt on its books, with interest expenses that are different from this value, the two approaches will diverge.

If there is expected growth, the potential for inconsistency multiplies. You have to ensure that you borrow enough money to fund new investments to keep your debt ratio at a level consistent with what you are assuming when you compute the cost of capital.

 *fcffvsfcfe.xls*: This spreadsheet allows you to compare the equity values obtained using FCFF and FCFE models.

Illustration 12.12 FCFF Valuation: Disney

To value Disney, we will consider all of the numbers that we have estimated already in this section. Recapping those estimates:

- The operating income in 2008, before taxes and adjusted for operating leases, is \$7,030 million. Based upon the capital invested at the start of 2008, we estimate a return on capital is 9.91%.

- For years one through five, we will assume that Disney will maintain its return on capital on new investments at 9.91% and that the reinvestment rate will be 53.72% (see Illustration 12.7). This will result in an expected growth rate of 5.32% a year.
- For years one through five, we will assume that Disney will maintain its existing debt ratio of 26.73% and its current cost of capital of 7.51% (see Illustration 12.8).
- The assumptions for stable growth (after year ten) and for the transition period are listed in Illustration 12.9.

In Table 12.7, we estimate the after-tax operating income, reinvestment, and free cash flow to the firm each year for the next ten years.

Table 12.7 Estimated FCFF, Disney

Year	Expected growth rate	EBIT (1-t)	Reinvestment rate	Reinvestment	FCFF
2009	5.32%	\$4,591	53.72%	\$2,466	\$2,125
2010	5.32%	\$4,835	53.72%	\$2,598	\$2,238
2011	5.32%	\$5,093	53.72%	\$2,736	\$2,357
2012	5.32%	\$5,364	53.72%	\$2,882	\$2,482
2013	5.32%	\$5,650	53.72%	\$3,035	\$2,615
2014	4.86%	\$5,924	49.64%	\$2,941	\$2,983
2015	4.39%	\$6,185	45.57%	\$2,818	\$3,366
2016	3.93%	\$6,428	41.49%	\$2,667	\$3,761
2017	3.46%	\$6,650	37.41%	\$2,488	\$4,162
2018	3.00%	\$6,850	33.33%	\$2,283	\$4,567

In Table 12.8, we estimate the present value of the FCFF using the cost of capital. Because the beta and debt ratio change each year from year six to ten, the cost of capital also changes each year.

Table 12.8 Present Value of Free Cash Flows to Firm, Disney

Year	FCFF	Cost of capital	Cumulated Cost of capital	PV of cash flow
2009	\$2,125	7.52%	1.0752	\$1,976
2010	\$2,238	7.52%	1.1561	\$1,936
2011	\$2,357	7.52%	1.2430	\$1,896
2012	\$2,482	7.52%	1.3365	\$1,857
2013	\$2,615	7.52%	1.4370	\$1,819
2014	\$2,983	7.61%	1.5463	\$1,929

2015	\$3,366	7.69%	1.6653	\$2,022
2016	\$3,761	7.78%	1.7948	\$2,095
2017	\$4,162	7.87%	1.9361	\$2,150
2018	\$4,567	7.95%	2.0901	\$2,185

To compute the present value of the cash flows in years six through ten, we have to use the compounded cost of capital over the previous years. To illustrate, the present value of \$3,761 million in cash flows in 2016 is:

$$\text{PV of Cash Flow in 2009} = \frac{3,761}{(1.0752)^5(1.0761)(1.0769)(1.0778)} = \$2,095 \text{ million}$$

The final piece of the valuation is the terminal value. To estimate the terminal value, at the end of year ten, we estimate the free cash flow to the firm in year 11, using the reinvestment rate of 33.33% that we estimated in illustration 12.9:

$$\begin{aligned} \text{FCFF}_{11} &= \text{EBIT}_{10}(1 - t)(1 + g_n)(1 - \text{Reinvestment Rate}_{\text{Stable Growth}}) \\ &= 6,850(1.03)(1 - 0.333) = \$4,704 \text{ million} \end{aligned}$$

$$\begin{aligned} \text{Terminal Value} &= \text{FCFF}_{11} / (\text{Cost of Capital}_{\text{Stable Growth}} - g) \\ &= 4704 / (0.0795 - 0.03) = \$94,928 \text{ million} \end{aligned}$$

The value of the firm is the sum of the present values of the cash flows during the high-growth period, the present value of the terminal value, and the value of the nonoperating assets that we estimated in Illustration 12.10.

PV of cash flows during the high growth phase = \$19,865 million

$$\text{PV of Terminal Value} = \frac{\$94,928}{(1.0752)^5(1.0761)(1.0769)(1.0778)(1.0787)(1.0795)} = \$45,419$$

+ Cash and Marketable Securities = \$ 3,795

+ Nonoperating Assets (Holdings in Other Companies) = \$ 1,763

Value of the Firm = \$70,842

Subtracting out the market value of debt (including operating leases) of \$16,682 million the value of minority interests (\$1,344 million) and the value of the equity options (estimated to be worth \$528 million in Illustration 12.11) yields the value of the common stock:

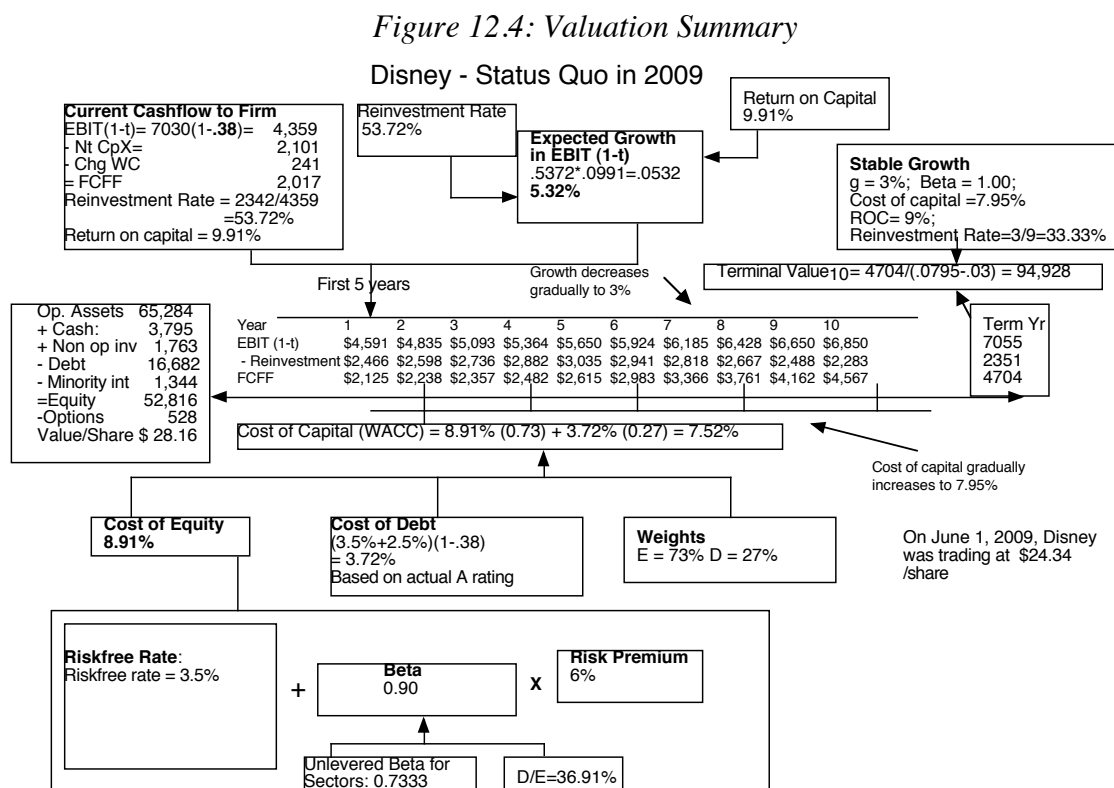
Value of Equity in Common Stock

$$= \text{Value of Firm} - \text{Debt} - \text{Minority interests} - \text{Equity Options}$$

$$= \$70,842 - \$16,682 - \$1,344 - \$528 = \$52,288$$

Dividing by the number of shares outstanding (1856.75 million), we arrive at a value per share of \$28.16, about 17% above the market price of \$24.34 at the time of this valuation.

Figure 12.4 summarizes the valuation:



12.10. Net Capital Expenditures and Value

In the valuation above, we assumed that the reinvestment rate would be 33.33 percent in perpetuity to sustain the 3% stable growth rate. What would the terminal value have been if instead we had assumed that the reinvestment rate was zero while continuing to use a stable growth rate of 3%?

- Higher than \$94.9 billion (estimated in illustration 12.12)
- Lower than \$94.9 billion

In Practice: Adjusted Present Value

In Chapter 8, we presented the adjusted present value (APV) approach to estimate the optimal debt ratio for a firm. In that approach, we begin with the value of the firm

without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. To do this, we assume that the primary advantage of borrowing is a tax benefit and the most significant cost is the added risk of bankruptcy.

The first step in this approach is the estimation of the value of the unlevered firm. This can be accomplished by valuing the firm as if it had no debt, that is, by discounting the expected FCFF at the unlevered cost of equity. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

$$\text{Value of Unlevered Firm} = \frac{\text{FCFF}_0(1+g)}{\rho_u - g}$$

where FCFF_0 is the current after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity, and g is the expected growth rate. In the more general case, you can value the firm using any set of growth assumptions you believe are reasonable for the firm.

The second step is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

$$\begin{aligned} &= \frac{(\text{Tax Rate})(\text{Cost of Debt})(\text{Debt})}{\text{Cost of Debt}} \\ \text{Value of Tax Benefits} &= (\text{Tax Rate})(\text{Debt}) \\ &= t_c D \end{aligned}$$

The tax rate used here is the firm's marginal tax rate, and it is assumed to stay constant over time. If we anticipate the tax rate changing over time, we can still compute the present value of tax benefits over time, but we cannot use the perpetual growth equation.

The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. In theory at least, this requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

$$\begin{aligned} \text{PV of Expected Bankruptcy Cost} &= (\text{Probability of Bankruptcy})(\text{PV of Bankruptcy Cost}) \\ &= \pi_a BC \end{aligned}$$

This step of the APV approach poses the most significant estimation problems, because neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly.

In theory, both the APV and the cost of capital approach will yield the same values for a firm if consistent assumptions are made about financial leverage. The difficulties associated with estimating the expected bankruptcy cost, though, often lead many to use an abbreviated version of the APV model, where the tax benefits are added to the unlevered firm value and bankruptcy costs are ignored. This approach will overvalue firms.

Valuing Private Businesses

All of the principles that we have developed for valuation apply to private companies as well. In other words, the value of a private company is the present value of the expected cash flows that you would expect that company to generate over time, discounted back at a rate that reflects the riskiness of the cash flows. The differences that exist are primarily in the estimation of the cash flows and the discount rates:

- When estimating cash flows, keep in mind that although accounting standards may not be adhered to consistently in publicly traded firms, they can diverge dramatically in private firms. In small, private businesses, we should reconstruct financial statements rather than trust the earnings numbers that are reported. There are also two common problems that arise in private firm accounting that we have to correct for. The first is the failure on the part of many owners to attach a cost to the time that they spend running their businesses. Thus the owner of a store who spends most of every day stocking the store shelves, staffing the cash register, and completing the accounting will often not show a salary associated with these activities in his or her income statement, resulting in overstated earnings. The second is the intermingling of personal and business expenses that is endemic in many private businesses. When reestimating earnings, we have to strip the personal expenses out of the analysis.
- When estimating discount rates for publicly traded firms, we hewed to two basic principles. With equity, we argued that the only risk that matters is the risk that cannot be diversified away by marginal investors, who we assumed were well diversified. With debt, the cost of debt was based on a bond rating and the default spread associated with that rating. With private firms, both these assumptions will

come under assault. First, the owner of a private business is almost never diversified and often has his or her entire wealth tied up in the firm's assets. That is why we developed the concept of a total beta for private firms in Chapter 4, where we scaled the beta of the firm up to reflect all risk and not just nondiversifiable risk. Second, private businesses usually have to borrow from the local bank and do not have the luxury of accessing the bond market. Consequently, they may well find themselves facing a higher cost of debt than otherwise similar publicly traded firms.

- The final issue relates back to terminal value. With publicly traded firms, we assume that firms have infinite lives and use this assumption, in conjunction with stable growth, to estimate a terminal value. Private businesses, especially smaller ones, often have finite lives since they are much more dependent on the owner/founder for their existence.

With more conservative estimates of cash flows, higher discount rates to reflect the exposure to total risk and finite life assumptions, it should come as no surprise that the values we attach to private firms are lower than those that we would attach to otherwise similar publicly traded firms. This also suggests that private firms that have the option of becoming publicly traded will generally opt to do so even though the owners might not like the oversight and loss of control that comes with this transition.

Illustration 12.13 Valuing a Private Business: Bookscape

To value Bookscape, we will use the pre-tax operating income of \$3 million that the firm had in its most recent year as a starting point. Adjusting for the operating lease commitments that the firm has, we arrive at an adjusted pre-tax operating income of \$3,575 million.³² To estimate the cost of capital, we draw on the estimates of total beta and the assumption that the firm's debt to capital ratio would resemble the industry average of 34.84% that we made in Chapter 4 :

$$\begin{aligned}\text{Cost of Capital} &= \text{Cost of Equity}(D/[D + E]) + \text{After-Tax Cost of Debt}(D/[D + E]) \\ &= 20.94\%(0.6516) + 6\%(1 - 0.4)(0.3484) = 0.149 \text{ or } 14.9\%\end{aligned}$$

³²In Illustration 4.15, we estimated the present value of the operating lease commitments at Bookscape to be \$9.588 million. To adjust the operating income, we add back the imputed interest expense on this debt, obtained by multiplying the pretax cost of borrowing by the present value of the operating leases (6% of \$9.588 million).

The total beta for Bookscape is 2.91, and we will continue to use the 40 percent tax rate for the firm, as long as the firm has taxable income.

In Chapter 6, we estimated a return on capital for Bookscape of 13.76% and we will assume that the firm will continue to generate this return on capital for the foreseeable future, while growing its earnings at 2 percent a year. The resulting reinvestment rate is 14.53 percent:

$$\text{Reinvestment Rate} = \text{Growth Rate} / \text{Return on Capital} = 2\% / 13.76\% = 14.53\%$$

The present value of the cash flows, assuming perpetual growth, can be computed as follows:

$$\begin{aligned} \text{Value of Operating Assets} &= \frac{\text{EBIT}(1-t)(1-\text{Reinvestment Rate})(1+g)}{(\text{Cost of capital} - g)} \\ &= \frac{3.575(1-.4)(1-0.1453)(1.02)}{(.149 - .02)} = \$14.497 \text{ mil} \end{aligned}$$

To get to the value of equity, we add back the cash holdings (\$500,000) and subtract out the debt (\$9.588 million).

$$\begin{aligned} \text{Value of Equity} &= \text{Value of Operating Assets} + \text{Cash} - \text{Debt} \\ &= 14.497 + 0.5 - 9.588 = \$5.409 \text{ million} \end{aligned}$$

Note that this valuation of equity is conditioned on two assumptions, that the firm will continue operating in perpetuity and that the buyer is an undiversified individual.

To see the effect on value of altering the assumption of perpetual life, we assumed instead that the business would continue for only as long as the lease (25 years), with cash flows growing at 2% a year for that period, and that there is no residual value at the end of 25 years. With this assumption, the value of the business drops to \$13.576 million and the value of equity to \$4.67 million:

$$\begin{aligned} \text{Value of Operating Assets} &= \text{EBIT}(1-t)(1-\text{Reinvestment Rate}) \frac{1 - \frac{(1+g)^n}{(1+r)^n}}{(r-g)} \\ &= 3.575(1-.4)(1-.1453) \frac{1 - \frac{(1.02)^{25}}{(1.149)^{25}}}{(.149 - .02)} = 13.576 \text{ mil} \end{aligned}$$

$$\begin{aligned} \text{Value of Equity} &= \text{Value of Operating Assets} + \text{Cash} - \text{Debt} \\ &= 13.576 + 0.5 - 9.588 = \$4.67 \text{ million} \end{aligned}$$

Finally, we also consider the value of the firm to a diversified investor or a publicly traded company by reverting back to a perpetual life and using the cost of capital of 8.81% that we estimated for Bookscape, using a market beta (see illustration 4.18):

$$\begin{aligned}\text{Value of Operating Assets} &= \frac{\text{EBIT}(1-t)(1-\text{Reinvestment Rate})(1+g)}{(\text{Cost of capital} - g)} \\ &= \frac{3.575(1-.4)(1-0.1453)(1.02)}{(.0881 - .02)} = \$27.442 \text{ mil}\end{aligned}$$

$$\begin{aligned}\text{Value of Equity} &= \text{Value of Operating Assets} + \text{Cash} - \text{Debt} \\ &= 27.4442 + 0.5 - 9.588 = \$18.35 \text{ million}\end{aligned}$$

The gap between the value of equity to a private buyer (\$5.4 million) and to a public buyer (\$18.35 million) yields some interesting implications:

- a. Diversification discount: The only reason for the difference in values lies in the fact that the private owner is not diversified and thus sees more risk (and demands a higher return to compensate) than a public buyer, looking at the same business.
- b. A rationale for acquisitions: The different perspectives on risk and value on the part of private and public buyers also offers a rationale for acquisitions of private businesses by publicly traded companies, where both sides see themselves as winners. Thus, if a public company (say, Barnes and Noble) offers \$8 million for the equity in Bookscape, the owner of the company is being offered more than what he thinks the business is worth (\$5.4 million) and the public company gets a bargain (since the equity is worth \$18.35 million to them).
- c. Intermediate Solutions: Venture capital and private equity investors fall between the two extremes, since they are more diversified than the private owner but less so than public investors. Consequently, they will arrive at values between \$5.4 million and \$18.35 million and derive their payoff from nurturing the business for an initial public offering or sale or public company.

The fact that some public companies go private is often viewed as inconsistent with our analyses here. After all, why would investors in a firm accept a huge drop in value by taking a company off the market? Note that when private equity investors such as KKR or Blackstone take a company private, their intent is not to keep them private, but to fix what they see as potential problems and take the company back public sooner rather than

later. Since the endgame remains the public market, they continue to run these businesses as if they were publicly held.

In Practice: Illiquidity Discounts in Private Firm Valuation

If you buy stock in a publicly traded firm and then change your mind and decide to sell, you face modest transaction costs. If you buy a private business and change your mind, it is far more difficult to reverse your decision. As a consequence, many analysts valuing private businesses apply an illiquidity discount that ranges from 20 to 40 percent of the value to arrive at a final value. Although the size of the discount is large, there is surprisingly little thought that goes into the magnitude of the discount. In fact, it is almost entirely based on studies of restricted stock issued by publicly traded firms. These stock are placed with investors who are restricted from trading on the stock for two years after the issue, and the price on the issue can be compared to the market price of the traded shares of the company to get a sense of the discount that investors demand for the enforced illiquidity. Because there are relatively few restricted stock issues, the sample sizes tend to be small and involve companies that may have other problems raising new funds.

Although we concede the necessity of illiquidity discounts in the valuation of private businesses, the discount should be adjusted to reflect the characteristics of the firm in question. Other things remaining equal, we would expect smaller firms with less liquid assets and in poorer financial health to have much larger illiquidity discounts attached to their values. One way to make this adjustment is to take a deeper look at the restricted stock issues for which we have data and look at reasons for the differences in discounts across stocks.³³ Another way is to view the bid-ask spread as the illiquidity discount on publicly traded companies and extend an analysis of the determinants of these spreads to come up with a reasonable measure of it or illiquidity discount of a private business.³⁴

³³Silber did this in a 1989 study, where he found that the discount tended to be larger for companies with smaller revenues and negative earnings. See Silber, W.L., 1991, *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*, Financial Analysts Journal, 60-64.

³⁴See *Investment Valuation (Second Edition)* by Aswath Damodaran. (John Wiley and Sons, 2001) for more details.

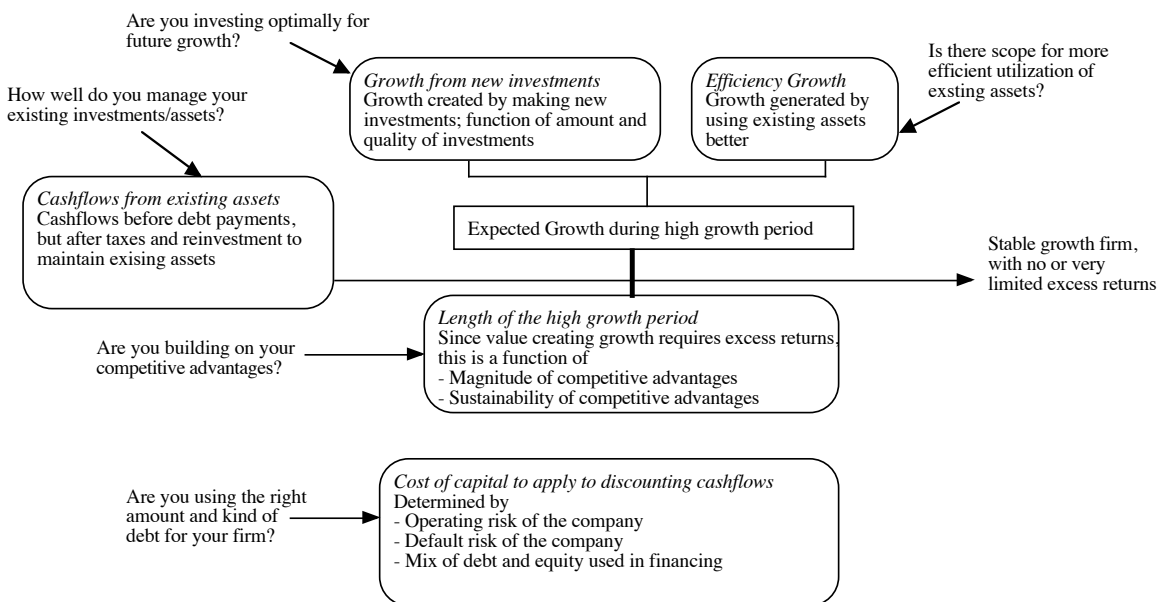
Value Enhancement

In a discounted cash flow valuation, the value of a firm is the function of four key inputs—the cash flows from existing investments, the expected growth rate in these cash flows for the high-growth period, the length of time before the company becomes a stable-growth company, and the cost of capital. Put simply, to enhance the value of a firm, we have to change one or more of these inputs:

- a. *Increase cash flows from existing assets:* There are a number of ways we can increase cash flows from assets. First, we can use assets more efficiently, cutting costs and improving productivity. If we succeed, we should see higher operating margins and profits. Second, we can, within the bounds of the law, reduce the taxes we pay on operating income through good tax planning. Third, we can reduce maintenance capital expenditures and investments in working capital—inventory and accounts receivable—thus increasing the cash left over after these outflows.
- b. *Increase the growth rate during the high-growth period:* Within the structure that we used in the last section, there are only two ways of increasing growth. We can reinvest more in internal investments and acquisitions, or we can try to earn higher returns on the capital that we invest in new investments. To the extent that we can do both, we can increase the expected growth rate. One point to keep in mind, though, is that increasing the reinvestment rate will almost always increase the growth rate, but it will not increase value if the return on capital on new investments lags the cost of capital.
- c. *Increase the length of the high-growth period:* It is not growth per se that creates value but excess returns. Because excess returns and the capacity to continue earning them comes from the competitive advantages possessed by a firm, a firm has to either create new competitive advantages—brand name, economies of scale, and legal restrictions on competition all come to mind—or augment existing ones.
- d. *Reduce the cost of capital:* In Chapter 8, we considered how changing the mix of debt and equity may reduce the cost of capital, and in Chapter 9, we considered how matching your debt to your assets can reduce your default risk and reduce your overall cost of financing. Holding all else constant, reducing the cost of capital will increase firm value.

Figure 12.5 summarizes the ways in which value can be enhanced at a public company.

Figure 12.5: Ways of Enhancing Value



Which one of these four approaches you choose will depend on where the firm you are analyzing or advising is in its growth cycle. For large mature firms, with little or no growth potential, cash flows from existing assets and the cost of capital offer the most promise for value enhancement. For smaller, risky, high-growth firms, it is likely to be changing the growth rate and the growth period that generate the biggest increases in value.

Illustration 12.14 Value Enhancement at Disney

In Illustration 12.12, we valued Disney at \$28.16 a share. In the process, though, we assumed that there would be no significant improvement in the return on capital that Disney earns on its existing assets, which at 9.91% is still well below the return on capital that Disney earned until 1996 and that the debt ratio would remain unchanged at the existing level of 27%. To examine how much the value per share could be enhanced at Disney if it were run differently, we made the following changes:

- We assumed that there is little scope left for operating efficiencies on existing investments and that the return on capital on these investments will remain at its existing level of 9.91%.

- We assumed that the return on capital on new investments would increase to 12%, higher than the 9.91% that we used in the status quo valuation. This is closer to the return that Disney used to make prior to its acquisition of Capital Cities. We kept the reinvestment rate unchanged at 53.72 percent. The resulting growth rate in operating income (for the first five years) is 7.98 percent a year.
- We assumed that the firm would increase its debt ratio immediately to 40 percent, which is its current optimal debt ratio (from Chapter 8). Though the beta will increase to 1.04 as a consequence, the cost of capital will drop to 7.33 percent. Keeping this debt ratio in stable growth, assuming that the beta moves to 1, results in a cost of capital in stable growth of 7.19%.

Keeping the assumptions about stable growth unchanged, we estimate significantly higher cash flows for the firm for the high-growth period in Table 12.9.

Table 12.9 Expected FCFF, Disney

Year	Expected growth rate	EBIT (1-t)	Reinvestment rate	Reinvestment	FCFF	Cost of capital	Cumulated Cost of capital	PV
2009	6.45%	\$4,640	53.72%	\$2,492	\$2,147	7.33%	1.0733	\$2,001
2010	6.45%	\$4,939	53.72%	\$2,653	\$2,286	7.33%	1.1520	\$1,984
2011	6.45%	\$5,257	53.72%	\$2,824	\$2,433	7.33%	1.2365	\$1,968
2012	6.45%	\$5,596	53.72%	\$3,006	\$2,590	7.33%	1.3271	\$1,951
2013	6.45%	\$5,957	53.72%	\$3,200	\$2,757	7.33%	1.4244	\$1,935
2014	5.76%	\$6,300	49.64%	\$3,127	\$3,172	7.30%	1.5285	\$2,076
2015	5.07%	\$6,619	45.57%	\$3,016	\$3,603	7.27%	1.6397	\$2,197
2016	4.38%	\$6,909	41.49%	\$2,866	\$4,043	7.25%	1.7585	\$2,299
2017	3.69%	\$7,164	37.41%	\$2,680	\$4,484	7.22%	1.8854	\$2,378
2018	3.00%	\$7,379	33.33%	\$2,460	\$4,919	7.19%	2.0209	\$2,434

The terminal value is also pushed up, as a result of the higher growth in the high growth period:

$$\text{Terminal Value} = \text{FCFF}_{11} / (\text{Cost of Capital} - g) = 4,919 (1.03) / (0.0719 - 0.03) = \\ \$120,982 \text{ mil}$$

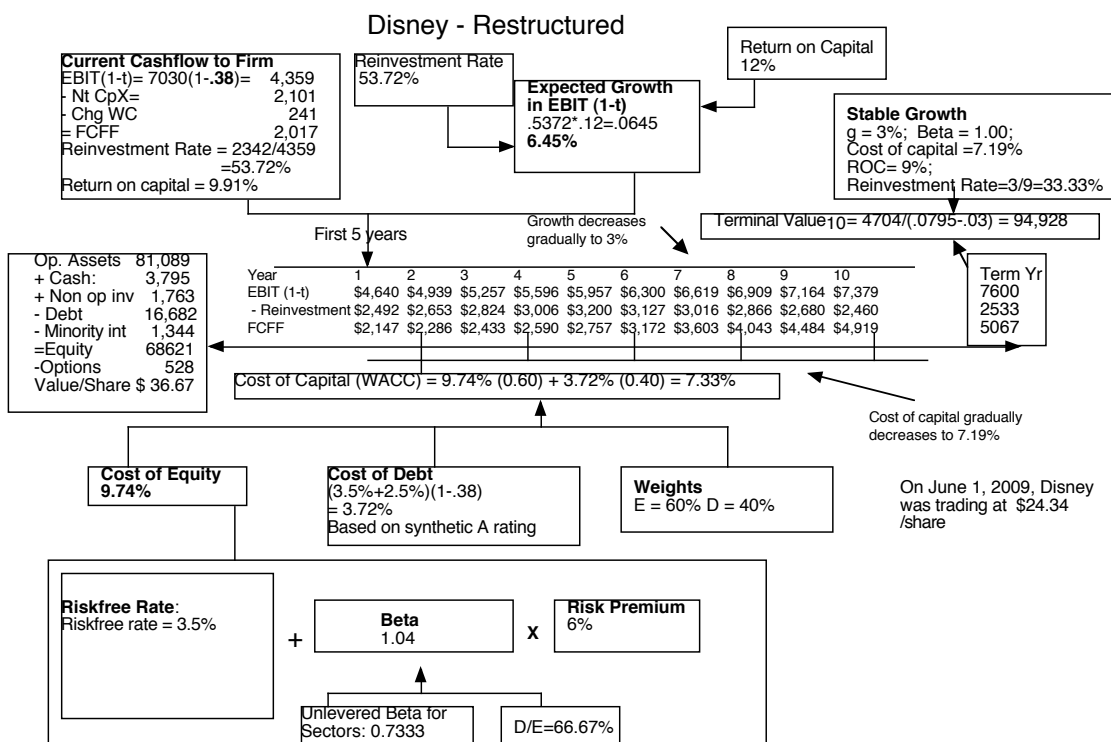
The value of the firm and the value per share can now be estimated:

Present Value of FCFF in High-Growth Phase =	\$21,233
+ Present Value of Terminal Value of Firm =	\$59,866
+ Value of Cash & Marketable Securities =	\$ 3,795
+ Value of Minority Holdings in other companies =	\$ 1,763
Value of Firm =	\$86,647
– Market Value of Outstanding Debt =	\$16,682
- Minority Interests =	\$ 1,344
– Value of Equity in Options =	\$ 528
Value of Equity in Common Stock =	\$68,093
Market Value of Equity/Share =	\$36.67

Disney's value per share increases from \$28.16 per share in Illustration 12.12 to \$36.67 a share when we make the changes to the way it is managed.³⁵ Figure 12.6 presents the restructured valuation:

³⁵You may wonder why the dollar debt does not change even though the firm is moving to a 30 percent debt ratio. In reality, it will increase, but the number of shares will decrease when Disney recapitalizes. The net effect is that the value per share will be close to our estimated value.

Figure 12.6: Value of Control



In Practice: The Value of Control

The notion that control is worth 15 percent or 20 percent or some fixed percent of every firm's value is deeply embedded in valuation practice, and it is not true. The value of control is the difference between two values—the value of the firm run by its existing management (status quo) and the value of the same firm run optimally.

$$\text{Value of Control} = \text{Optimal Value for Firm} - \text{Status Quo Value}$$

Thus, a firm that takes poor investments and funds them with a suboptimal mix of debt and equity will be worth more if it takes better investments and funds them with the right mix of debt and equity. In general, the worse managed a firm is the greater the value of control. This view of the world has wide ramifications in corporate finance and valuation:

- In a hostile acquisition, which is usually motivated by the desire to change the way that a firm is run, you should be willing to pay a premium that at best is equal to the value of control. You would prefer to pay less to preserve some of the benefits for yourself (rather than give them to target company stockholders).
- In companies with voting and nonvoting shares, the difference in value between the two classes should be a function of the value of control. If the value of control is high,

and there is a high likelihood of control changing, the value of the voting shares will increase relative to nonvoting shares.

In the Disney valuation, the value of control can be estimated by comparing the value of Disney run optimally with the status quo valuation done earlier in the chapter.

Value of Control_{Disney} = Optimal Value – Status Quo Value = \$36.67 – \$28.16 = \$8.51
 Since the stock trades at \$24.34, we could pay a premium of up to \$12.33 to acquire the firm.

Relative Valuation

In discounted cash flow valuation, the objective is to find the value of assets, given their cash flow, growth, and risk characteristics. In relative valuation, the objective is to value assets, based on how similar assets are currently priced in the market. In this section, we consider why and how asset prices have to be standardized before being compared to similar assets, and how to control for differences across comparable firms.

Standardized Values and Multiples

To compare the values of similar assets in the market, we need to standardize the values in some way. They can be standardized relative to the earnings they generate, to the book value or replacement value of the assets themselves, or to the revenues that they generate. We discuss each method next.

1. Earnings Multiples

One of the more intuitive ways to think of the value of any asset is as a multiple of the earnings it generates. When buying a stock, it is common to look at the price paid as a multiple of the earnings per share generated by the company. This *price/earnings ratio* can be estimated using earnings per share over the last four quarters, which is called a *trailing PE*, or an expected earnings per share in the next financial year, called a *forward PE*. When buying a business, as opposed to just the equity in the business, it is common to examine the value of the firm, usually net of cash (enterprise value), as a multiple of the operating income or the earnings before interest, taxes, depreciation, and amortization (EBITDA). To a buyer of the equity or the firm, a lower multiple is better

than a higher one, but these multiples will be affected by the growth potential and risk of the business being acquired.

2. Book Value or Replacement Value Multiples

Although markets provide one estimate of the value of a business, accountants often provide a very different estimate. The accounting estimate of book value is determined by accounting rules and is heavily influenced by the original price paid for the asset and any accounting adjustments (such as depreciation) made since that time. Investors often look at the relationship between the market's assessment of the value of equity and the book value of equity (or net worth) as a measure of how over- or undervalued a stock is; the *price/book value* ratio that emerges can vary widely across industries, depending again on the growth potential and the quality of the investments in each. When valuing businesses, we estimate this ratio using the value of the firm and the book value of all capital (rather than just the equity). For those who believe that book value is not a good measure of the true value of the assets, an alternative is to use the replacement cost of the assets; the ratio of the value of the firm to replacement cost is called the Q Ratio.

3. Revenue Multiples

Both earnings and book value are accounting measures and are determined by accounting rules and principles. An alternative, which is far less affected by these factors, is to use the ratio of the value of an asset to the revenues it generates. For equity investors, this ratio is the *price/sales ratio (PS)*, where the market value of equity is divided by the revenues generated by the firm. For firm value, this ratio can be modified as the *value/sales ratio (VS)*, where the numerator becomes the value of the firm. This ratio again varies widely across sectors, largely as a function of the profit margins in each. The advantage of using revenue multiples, however, is that it becomes far easier to compare firms in different markets, with different accounting systems at work, than it is to compare earnings or book value multiples.

Determinants of Multiples

One reason commonly given for the use of these multiples to value equity and firms is that they require far fewer assumptions than discounted cash flow valuation does.

We believe this is a misconception. The difference between discounted cash flow valuation and relative valuation is that the assumptions we make are explicit in the former and remain implicit in the latter. It is important that we know what the variables are that cause multiples to change, because these are the variables we have to control for when comparing these multiples across firms.

To look under the hood, so to speak, of equity and firm value multiples, we will go back to fairly simple discounted cash flow models for equity and firm value and use them to derive our multiples. Thus, the simplest discounted cash flow model for equity, which is a stable growth dividend discount model, would suggest that the value of equity is:

$$\text{Value of Equity} = P_0 = \frac{DPS_1}{k_e - g_n}$$

where DPS_1 is the expected dividend in the next year, k_e is the cost of equity, and g_n is the expected stable growth rate. Dividing both sides by the earnings, we obtain the discounted cash flow equation specifying the PE ratio for a stable growth firm:

$$\frac{P_0}{EPS_0} = PE = \frac{\text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

Dividing both sides by the book value of equity, we can estimate the price/book value ratio for a stable-growth firm:

$$\frac{P_0}{BV_0} = PBV = \frac{ROE * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

Dividing by the sales per share, the price/sales ratio for a stable-growth firm can be estimated as a function of its profit margin, payout ratio, profit margin, and expected growth.

$$\frac{P_0}{Sales_0} = PS = \frac{\text{Net Profit Margin} * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

We can do a similar analysis from the perspective of firm valuation.³⁶ The value of a firm in stable growth can be written as:

$$\text{Value of Firm} = V_0 = \frac{\text{FCFF}_1}{k_c - g_n}$$

where k_c is the cost of capital. Dividing both sides by the expected FCFF yields the value/FCFF multiple for a stable growth firm:

$$\frac{V_0}{\text{FCFF}_1} = \frac{1}{k_c - g_n}$$

Because the FCFF is the after-tax operating income netted against the net capital expenditures and working capital needs of the firm, the multiples of EBIT, after-tax EBIT, and EBITDA can also be estimated similarly. The value/EBITDA multiple, for instance, can be written as follows:

$$\frac{\text{Value}}{\text{EBITDA}} = \frac{(1 - t)}{k_c - g} + \frac{\text{Depr (t)/EBITDA}}{k_c - g} - \frac{\text{CEx/EBITDA}}{k_c - g} - \frac{\Delta \text{ Working Capital/EBITDA}}{k_c - g}$$

The point of this analysis is not to suggest that we go back to using discounted cash flow valuation but to understand the variables that may cause these multiples to vary across firms in the same sector. If we ignore these variables, we might conclude that a stock with a PE of 8 is cheaper than one with a PE of 12, when the true reason may be that the latter has higher expected growth or we might decide that a stock with a P/BV ratio of 0.7 is cheaper than one with a P/BV ratio of 1.5, when the true reason may be that the latter has a much higher return on equity. Table 12.10 lists the multiples that are widely used and the variables that determine each; the variable that (in our view) is the most significant determinant is highlighted for each multiple. This variable is what we would call the *companion variable* for this multiple, that is, the one variable we need to know to use this multiple to find under or over valued assets.

³⁶In practice, cash and marketable securities are subtracted from firm value to arrive at what is called enterprise value. All the multiples in the following section can be written in terms of enterprise value, and the determinants remain unchanged.

Table 12.10 Multiples and Companion Variables (*in Italics*)

<i>Multiple</i>	<i>Determining Variables</i>
Price/earnings ratio	<i>Growth</i> , payout, risk
Price/book value ratio	Growth, payout, risk, <i>ROE</i>
Price/sales ratio	Growth, payout, risk, <i>net margin</i>
Value/EBIT Value/EBIT (1 – <i>t</i>) Value/EBITDA	Growth, <i>reinvestment needs</i> , leverage, risk
Value/sales	Growth, reinvestment needs, leverage, risk, <i>operating margin</i>
Value/book capital	Growth, leverage, risk, <i>ROC</i>



eqmult.xls: This spreadsheet allows you to estimate the equity multiples for a firm, given its fundamentals.



firmmult.xls: This spreadsheet allows you to estimate the firm value multiples for a firm, given its fundamentals.

The Use of Comparable Firms

When we use multiples, we tend to use them in conjunction with comparable firms to determine the value of a firm or its equity. This analysis begins with two choices—the multiple that will be used in the analysis and the group of firms that will make up the comparable firms. The multiple is computed for each of the comparable firms, and the average is computed. To evaluate an individual firm, we then compare its multiple to the average computed; if it is significantly different, we make a subjective judgment about whether the firm's individual characteristics (growth, risk, or cash flows) may explain the difference. Thus, a firm may have a PE ratio of 22 in a sector where the average PE is only 15, but the analyst may conclude that this difference can be justified because the firm has higher growth potential than the average firm in the industry. In the analysts' judgment, if the difference on the multiple cannot be explained by the variables listed in Table 12.10, the firm will be viewed as overvalued (if its multiple is higher than the average) or undervalued (if its multiple is lower than the average). Choosing

comparable firms and adequately controlling for differences across these comparable firms, then become critical steps in this process. In this section, we consider both decisions.

1. Choosing Comparables

The first step in relative valuation is usually the selection of comparable firms. A comparable firm is one with cash flows, growth potential, and risk similar to the firm being valued. It would be ideal if we could value a firm by looking at how an exactly identical firm—in terms of risk, growth, and cash flows—is priced. In most analyses, however, analysts define comparable firms to be other firms in the same business or businesses. If there are enough firms in the industry to allow for it, this list is pruned further using other criteria; for instance, only firms of similar size may be considered. The implicit assumption being made here is that firms in the same sector have similar risk, growth, and cash flow profiles and therefore can be compared with much more legitimacy.

This approach becomes more difficult to apply when there are relatively few firms in a sector. In most markets outside the United States, the number of publicly traded firms in a particular sector is small, especially if it is defined narrowly. It is also difficult to find comparable firms if differences in risk, growth, and cash flow profiles across firms within a sector are large. Thus, there may be hundreds of computer software companies listed in the United States, but the differences across these firms are also large. The trade-off is therefore simple. Defining a industry more broadly increases the number of comparable firms, but it also results in a more diverse group.

2. Controlling for Differences across Firms

In Table 12.10, we listed the variables that determined each multiple. Because it is impossible to find firms identical to the one being valued, we have to find ways of controlling for differences across firms on these variables. The process of controlling for the variables can range from very simple approaches, which modify the multiples to take into account differences on one key variable, to more complex approaches that allow for differences on more than one variable.

a. Simple Adjustments

Let's start with the simple approaches. In this case, we modify the multiple to take into account the most important variable determining it. Thus, the PE ratio is divided by the expected growth rate in earnings per share (EPS) for a company to determine a growth-adjusted PE ratio or the *PEG ratio*. Similarly, the PBV ratio is divided by the ROE to find a *value ratio*. These modified ratios are then compared across companies in a sector. The implicit assumption we make is that these firms are comparable on all the other measures of value, besides the one being controlled for.

Illustration 12.15 Comparing PE Ratios and Growth Rates across Firms: Entertainment Companies

To value Disney, we look at the PE ratios and expected growth rates in EPS over the next five years, based on consensus estimates from analysts, for all entertainment companies where data is available on PE ratios and analyst estimates of expected growth in earnings over the next five years. Table 12.11 lists the firms and PE ratios.

Table 12.11 Entertainment Firm PE Ratios and Growth Rates, 2009

<i>Company Name</i>	<i>Current PE</i>	<i>Trailing PE</i>	<i>Forward PE</i>	<i>Expected Growth in EPS: next 5 years</i>	<i>PEG</i>
Belo Corp. 'A'	1.56	2.60	1.88	1.50%	1.04
CBS Corp. 'B'	3.76	4.23	5.93	4.50%	0.84
Central European Media Enterps	9.10	5.69	5.58	9.78%	0.93
CTC Media Inc	5.26	4.18	4.44	6.55%	0.80
Cumulus Media Inc	20.41	4.89	5.53	12.00%	1.70
Disney (Walt)	10.24	9.40	9.45	14.50%	0.71
DreamWorks Animation	9.37	10.17	12.26	14.50%	0.65
Global Traffic Network Inc	62.00	39.04	32.44	21.40%	2.90
Lin TV Corp.	4.22		1.43	8.00%	0.53
News Corp.	6.74	7.12	11.98	14.00%	0.48
Playboy Enterprises 'B'	15.81		30.86	42.50%	0.37
RC2 Corp	7.23	22.00	6.93	10.50%	0.69
Regal Entertainment Group	13.87		11.74	8.00%	1.73
Rentrak Corp	25.35	29.15	35.73	50.00%	0.51
Saga Commun. 'A'	2.21	2.29	2.76	8.00%	0.28
Sinclair Broadcast	12.36	4.84	9.39	15.00%	0.82
Time Warner	8.42	9.35	8.98	6.00%	1.40
Viacom Inc. 'B'	6.69	6.69	7.72	12.00%	0.56
World Wrestling Ent.	14.39	14.06	12.18	15.00%	0.96

Median	9.10	6.90	8.98	12.00%	0.80
Average	12.58	10.98	11.43	14.41%	0.94

Source: Value Line.

At 9.4 times forward earnings and 9.45 times current earnings, Disney looks overvalued relative to the median values for the sector. It is true that it looks reasonably priced, using the averages for the two values, but those numbers are skewed by a few outliers.

In this valuation, we assume that Disney has a growth rate similar to the average for the sector. One way of bringing growth into the comparison is to compute the PEG ratio, which is reported in the last column. On this measure, Disney looks more under valued, with a PEG ratio of 0.71, below both the median (0.8) and the average (0.94) for the sector. Although this may seem like an easy adjustment to resolve the problem of differences across firms, the conclusion holds only if these firms are of equivalent risk. Implicitly, this approach assumes a linear relationship between growth rates and PE.³⁷



12.11. Underlying Assumptions in Comparable Valuation

Assume that you are reading an equity research report where a buy recommendation for a company is being based on the fact that its PE ratio is lower than the average for the industry. Implicitly, what is the underlying assumption or assumptions being made by this analyst?

- The sector itself is, on average, fairly priced.
- The earnings of the firms in the group are being measured consistently.
- The firms in the group are all of equivalent risk.
- The firms in the group are all at the same stage in the growth cycle.
- The firms in the group have similar cash flow patterns.

All of the above.



pe.xls: There is a data set online that summarizes PE ratios and PEG ratios by industry group in the United States for the most recent quarter.

³⁷Put another way, we are assuming that as growth doubles, the PE ratio will also double.

b. Adjusting for More than One Variable

When firms differ on more than one variable, it becomes difficult to modify the multiples to account for the differences across firms. We can run regressions of the multiples against the variables and then use these regressions to find predicted values for each firm. This approach works reasonably well when the number of comparable firms is large and the relationship between the multiple and the variables is stable. When these conditions do not hold, a few outliers can cause the coefficients to change dramatically and make the predictions much less reliable.

Illustration 12.16 Price to Book Value Ratios and Return on Equity: European Banks

Table 12.12 lists the price/book value ratios of European banks and reports on their returns on equity and risk levels (measured using the stock beta over the previous five years).

Table 12.12 European Banks: Price to Book Value Ratio, 2009

Company Name	P/Book Equity	Beta	ROE
Royal Bank of Scotland Group plc	0.24	1.77	-15.37%
Deutsche Bank AG	0.41	1.61	5.48%
UniCredito Italiano S.p.A. (CM:UCG)	0.44	1.38	7.15%
Credit Agricole SA (ENXTPA:ACA)	0.44	1.16	1.13%
Barclays plc (LSE:BARC)	0.49	1.32	15.71%
Lloyds TSB Group plc (LSE:LLOY)	0.52	1.02	21.53%
KBC Group NV (ENXTBR:KBC)	0.54	1.53	5.96%
Banca Monte dei Paschi di Siena SpA (CM:BMPS)	0.58	1.37	9.59%

Unione di Banche Italiane Scpa (CM:UBI)	0.62	1.12	8.54%
Intesa Sanpaolo SpA (CM:ISP)	0.67	0.99	8.43%
Nordea Bank AB (OM:NDA SEK)	0.81	1.17	16.25%
Credit Suisse Group (VIRTX:CSGN)	0.84	1.07	14.24%
HSBC Holdings plc (LSE:HSBA)	0.9	0.56	12.14%
UBS AG (VIRTX:UBSN)	0.99	1.31	12.93%
Svenska Handelsbanken AB (OM:SHB A)	1.02	0.72	19.58%
National Bank of Greece SA (ATSE:ETE)	1.02	1.09	21.62%
Banco Santander, S.A. (CATS:SAN)	1.03	1.23	17.52%
Banco Popular Espanol SA (CATS:POP)	1.14	0.5	19.26%
Banco Bilbao Vizcaya Argentaria (CATS:BBVA)	1.24	0.85	22.30%
Standard Chartered PLC (LSE:STAN)	1.31	0.85	16.18%
Median	0.74	1.14	0.13585
Average	0.7625	1.131	0.120085

Source: Capital IQ.

Trading at 0.41 times book equity, Deutsche looks cheap, relative to the rest of the sector. However, part of the reason for this may be its low return on equity in 2008 (5.48%) and high beta (1.61). Because these firms differ on both risk and return on equity, we run a regression of PBV ratios on both variables:

$$\text{PBV} = 1.03 + 1.54 \text{ ROE} - 0.40 \text{ Beta} \quad R^2 = 58.31\%$$

(3.79) (2.26) (2.18)

Firms with higher return on equity and lower standard deviations trade at much higher price to book ratios. The numbers in parentheses are t -statistics and suggest that the relationships between PBV ratios and both variables in the regression are statistically significant. The R^2 indicates the percentage of the differences in PBV ratios that is explained by the independent variables. Finally, the regression itself can be used to get predicted PBV ratios for the companies in the list.³⁸ Thus, the predicted PBV ratio for Deutsche Bank, based on its return on equity of 5.48 percent and its beta of 1.61, would be 0.47.

$$\text{Predicted PBV}_{\text{Deutsche Bank}} = 1.03 + 1.54 (0.0548) - 0.40 (1.61) = 0.47$$

Because the actual PBV ratio for Deutsche Bank at the time of the analysis was 0.41, this would suggest that the stock is trading close to its fundamentals.



pbv.xls: There is a data set online that summarizes price to book ratios and returns on equity by industry group in the United States for the most recent quarter.



ps.xls: There is a data set online that summarizes price to sales ratios and margins by industry group in the United States for the most recent quarter.

3. Expanding the Range of Comparable Firms

Searching for comparable firms within the sector in which a firm operates is fairly restrictive, especially when there are relatively few firms in the sector or when a firm operates in more than one sector. Because the definition of a comparable firm is not one that is in the same business but one that has the same growth, risk, and cash flow characteristics as the firm being analyzed, we need not restrict our choice of comparable firms to those in the same industry. A software firm should be comparable to an automobile firm if we can control for differences in the fundamentals.

The regression introduced in the previous section allows us to control for differences on those variables that we believe cause multiples to vary across firms. Based

on the variables listed in Table 12.10, we should be able to regress multiples against the variables that should affect them. It is, however, possible that the proxies that we use for risk (beta), growth (expected growth rate), and cash flow (payout) are imperfect and that the relationship may not be linear. To deal with these limitations, we can add more variables to the regression—for example, the size of the firm may operate as a good proxy for risk—and use transformations of the variables to allow for nonlinear relationships.

We ran these regressions for multiples across publicly listed firms in the United States in January 2009 against analyst estimates of expected growth in earnings per share and other financial indicators from the most recent year.³⁹ The sample, which had about 7,000 firms in it, yielded the regressions reported in table 12.13:

Table 12.13: Market-wide Regressions of Multiples: US Companies in January 2009
T statistics in brackets below coefficients

<i>Regression</i>	<i>R²</i>
PE = 7.62 + 77.98 g _{EPS} + 7.67 Payout -5.37 Beta (8.77) (26.71) (13.09) (7.21)	28.6%
PBV= 1.28 + 6.72 g _{EPS} + 0.33 Payout -1.65 Beta + 8.67 ROE (10.09) (15.85) (4.95) (11.70) (38.48)	68.3%
PS= 0.29 + 4.32 g _{EPS} + 0.31 Payout – 0.86 Beta + 11.42 Net Margin (2.48) (9.52) (4.58) (8.60) (35.72)	62.3%
EV/Invested Capital= 1.10 + 3.99 g + 5.06 ROIC – 1.35 (Debt/Capital) (10.23) (6.60) (20.59) (10.1)	50.1%
EV/Sales = 1.72 + 1.94 g+ 5.58 Operating Margin – 4.87 Tax Rate (16.46) (3.32) (29.00) (18.80)	50.3%
EV/EBITDA= 6.68 +25.34 g- 7.99 Tax rate -1.59 (Debt/Capital) -1.837 RIR (18.58) (12.35) (9.78) (3.84) (1.94)	19.3%
g _{EPS} = Expected growth rate in EPS for next 5 years (analyst estimates) g = Expected growth rate in revenues for next 5 years (if not available, use g _{EPS}) Payout = Dividends/Earnings ROIC = Return on capital = EBIT (1- tax rate)/ Invested Capital Invested Capital = Book value of equity + Book value of debt - Cash ROE = Net Income/ Book value of Equity Tax Rate = Effective tax rate Debt/Capital = Debt/ (Market value of Equity + Debt)	

³⁸Both approaches described assume that the relationship between a multiple and the variables driving value are linear. Because this is not always true, we might have to run nonlinear versions of these regressions.

³⁹We ran the regression both with intercepts and without intercepts. If the intercept is negative, we report the regression without the intercept.

$$\text{RIR} = \text{Reinvestment Rate} = (\text{Cap Ex} - \text{Depreciation} + \text{Chg in WC}) / \text{EBIT} (1-t)$$

The first advantage of this approach over the “subjective” comparison across firms in the same sector is that it does quantify, based on actual market data, the degree to which higher growth or risk should affect the multiples. It is true that these estimates can have error in them, but the error is a reflection of the reality that many analysts choose not to face when they make subjective judgments. Second, by looking at all firms in the market, this approach allows us to make more meaningful comparisons of firms that operate in industries with relatively few firms. Third, it allows us to examine whether all firms in an industry are under- or overvalued by estimating their values relative to other firms in the market.

Illustration 12.17 Applying Market Regression to Estimate Multiples: Disney

We will use the results of the market regression just summarized to estimate the appropriate value for Disney. Consider the regression for the PE ratio:

$$\text{PE} = 7.62 + 77.98 g_{\text{EPS}} + 7.67 \text{ Payout} - 5.37 \text{ Beta}$$

The corresponding values for Disney are as follows:

Expected Growth Rate = 14.5% (analyst consensus estimate for EPS growth)

Payout Ratio = 15.35%

Beta = 0.9011

The estimated price earnings ratio for Disney is:

$$\text{PE} = 7.62 + 77.98 (.145) + 7.67 (.1535) - 5.37 (0.90) = 15.27$$

Because Disney trades at an actual PE ratio of 9.45, it looks significantly undervalued (by almost 40%), relative to the market.



multreg.xls: This data set summarizes the latest regression of multiples against fundamentals for the United States for the most recent quarter.

Equity as an Option

In most publicly traded firms, equity has two features. The first is that the equity investors run the firm and can choose to liquidate its assets and pay off other claim holders at any time. The second is that the liability of equity investors in some private

firms and almost all publicly traded firms is restricted to their equity investments in these firms. This combination of the option to liquidate and limited liability allows equity to have the features of a call option. In firms with substantial debt and a significant potential for bankruptcy, the option value of equity may be in excess of the discounted cash flow value of equity.

The Payoff on Equity as an Option

The equity in a firm is a residual claim, that is, equity holders lay claim to all cash flows left after other financial claimholders (debt, preferred stock, etc.) have been satisfied. If a firm is liquidated, the same principle applies; equity investors receive the cash that is left in the firm after all outstanding debt and other financial claims have been paid off. With limited liability, if the value of the firm is less than the value of the outstanding debt, equity investors cannot lose more than their investment in the firm. The payoff to equity investors on liquidation can therefore be written as:

$$\begin{aligned} \text{Payoff to equity on liquidation} &= V - D && \text{if } V > D \\ &= 0 && \text{if } V \leq D \end{aligned}$$

where

V = Liquidation Value of the firm

D = Face Value of the outstanding debt and other external claims

Equity can thus be viewed as a call option on the firm, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercise price) be paid off. The firm is the underlying asset and the option expires when the debt comes due. The payoffs are shown in Figure 12.7.

Figure 12.7: Payoff on Equity as Option on a Firm

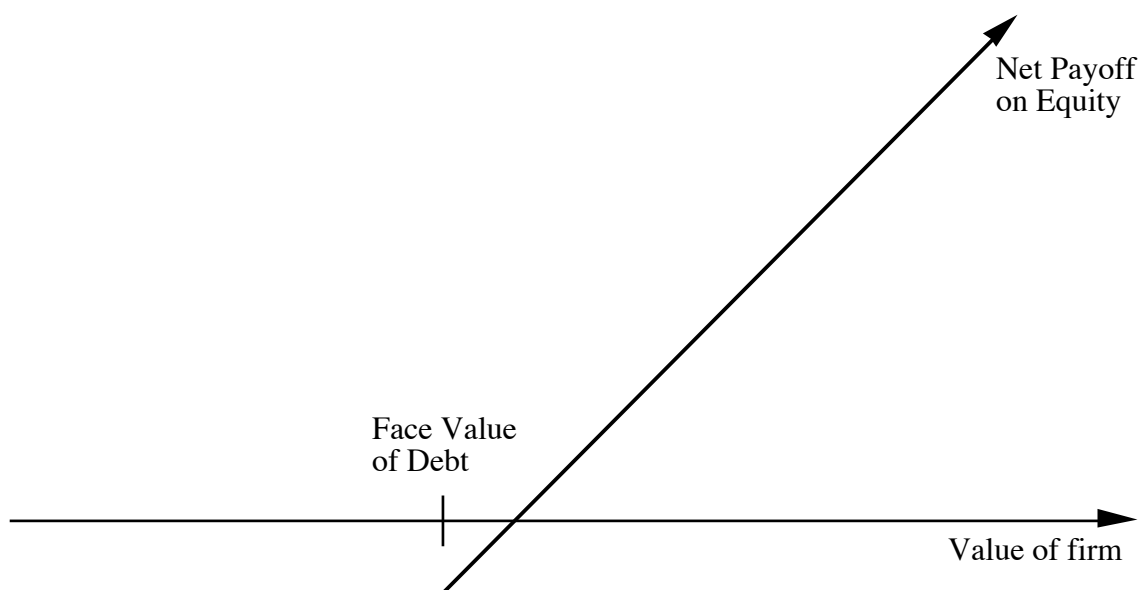


Illustration 12.18: Valuing Equity as an Option

Assume that we are valuing the equity in a firm whose assets are currently valued at \$100 million; the standard deviation in this asset value is 40%. The face value of debt is \$80 million (it is zero coupon debt with 10 years left to maturity). The 10-year treasury bond rate is 10%. We can value equity as a call option on the firm, using the following inputs for the option pricing model.

Value of the underlying asset = S = Value of the firm = \$ 100 million

Exercise price = K = Face Value of outstanding debt = \$ 80 million

Life of the option = t = Life of zero-coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based upon these inputs, the Black-Scholes model provides the following value for the call.

$$d_1 = 1.5994$$

$$N(d_1) = 0.9451$$

$$d_2 = 0.3345$$

$$N(d_2) = 0.6310$$

$$\text{Value of the call} = 100 (.9451) - 80 e^{-(.10)(10)} (.6310) = \$75.94 \text{ million}$$

Since the call value represents the value of equity and the firm value is \$100 million, the estimated value of the outstanding debt can be calculated.

Value of the outstanding debt = \$100 - \$75.94 = \$24.06 million

Since the debt is a 10-year zero coupon bond, the market interest rate on the bond can be calculated.

$$\text{Interest rate on debt} = \left(\frac{\$80}{\$24.06} \right)^{\frac{1}{10}} - 1 = 12.77\%$$

Thus, the default spread on this bond should be 2.77%.

Implications of viewing Equity as an Option

When the equity in a firm takes on the characteristics of a call option, we have to change the way we think about its value and what determines its value. In this section, we will consider a number of potential implications for equity investors and bondholders in the firm.

When will equity be worthless?

In discounted cash flow valuation, we argue that equity is worthless if what we own (the value of the firm) is less than what we owe. The first implication of viewing equity as a call option is that equity will have value, even if the value of the firm falls well below the face value of the outstanding debt. While the firm will be viewed as troubled by investors, accountants and analysts, its equity is not worthless. In fact, just as deep out-of-the-money traded call options command value because of the possibility that the value of the underlying asset may increase above the strike price in the remaining lifetime of the option, equity commands value because of the time premium on the option (the time until the bonds mature and come due) and the possibility that the value of the assets may increase above the face value of the bonds before they come due.

Illustration 12.19: Firm Value and Equity Value

Revisiting the preceding example, assume that the value of the firm drops to \$50 million, below the face value of the outstanding debt (\$80 million). Assume that all the other inputs remain unchanged. The parameters of equity as a call option are as follows:

Value of the underlying asset = S = Value of the firm = \$ 50 million

Exercise price = K = Face Value of outstanding debt = \$ 80 million

Life of the option = t = Life of zero-coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based upon these inputs, the Black-Scholes model provides the following value for the call.

$$d_1 = 1.0515 \qquad N(d_1) = 0.8534$$

$$d_2 = -0.2135 \qquad N(d_2) = 0.4155$$

$$\text{Value of the call (equity)} = 50 (0.8534) - 80 \exp^{(-0.10)(10)} (0.4155) = \$30.44 \text{ million}$$

$$\text{Value of the bond} = \$50 - \$30.44 = \$19.56 \text{ million}$$

As we can see, the equity in this firm retains value, because of the option characteristics of equity. In fact, equity continues to have value in this example even if the firm value drops to \$10 million or below.

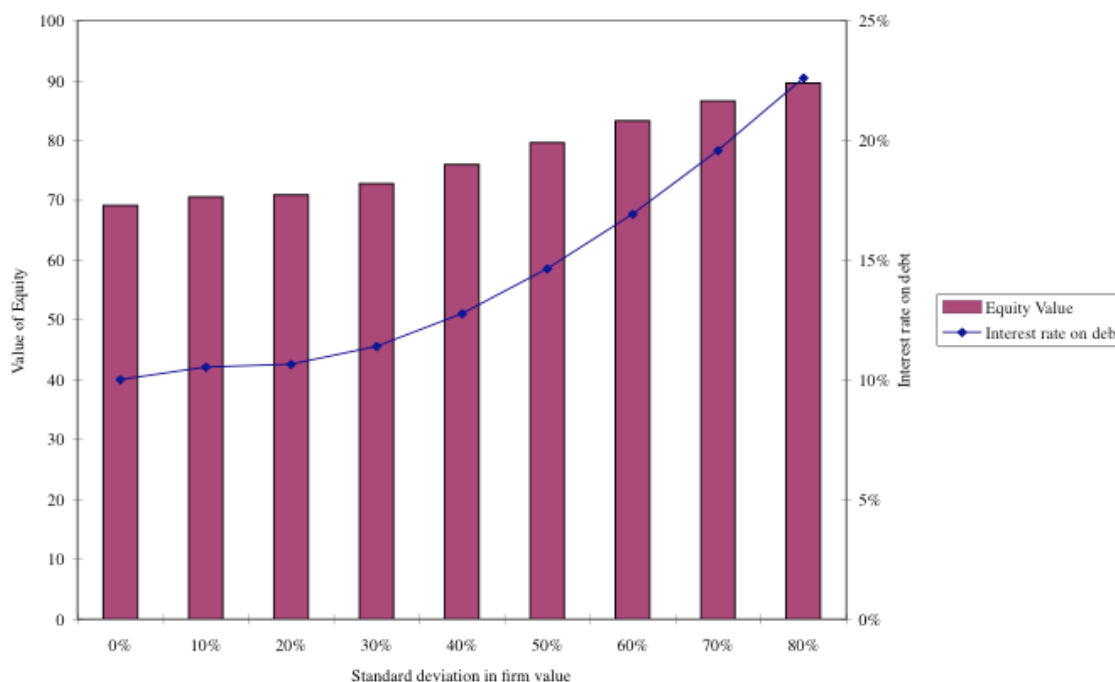
Increasing Risk can increase Equity Value

In traditional discounted cash flow valuation, higher risk almost always translates into lower value for equity investors. When equity takes on the characteristics of a call option, we should not expect this relationship to continue to hold. Risk can become our ally, when we are equity investors in a troubled firm. In essence, we have little to lose and much to gain from swings in firm value.

Illustration 12.20: Equity Value and Volatility

Let us revisit the valuation in Illustration 12.8. The value of the equity is a function of the variance in firm value, which we assumed to be 40%. If we change this variance, holding all else constant, the value of the equity will change as evidenced in Figure 12.8.

Figure 17.3: Equity Value and Standard Deviation in Firm Value



Note that the value of equity increases, if we hold firm value constant, as the standard deviation increases. The interest rate on debt also increases as the standard deviation increases.

Probability of Default and Default Spreads

One of the more interesting pieces of output from the option pricing model is the risk-neutral probability of default that we can obtain for the firm. In the Black-Scholes model, we can estimate this value from $N(d_2)$, which is the risk-neutral probability that $S > K$, which in this model is the probability that the value of the firm's asset will exceed the face value of the debt.

$$\text{Risk-neutral probability of default} = 1 - N(d_2)$$

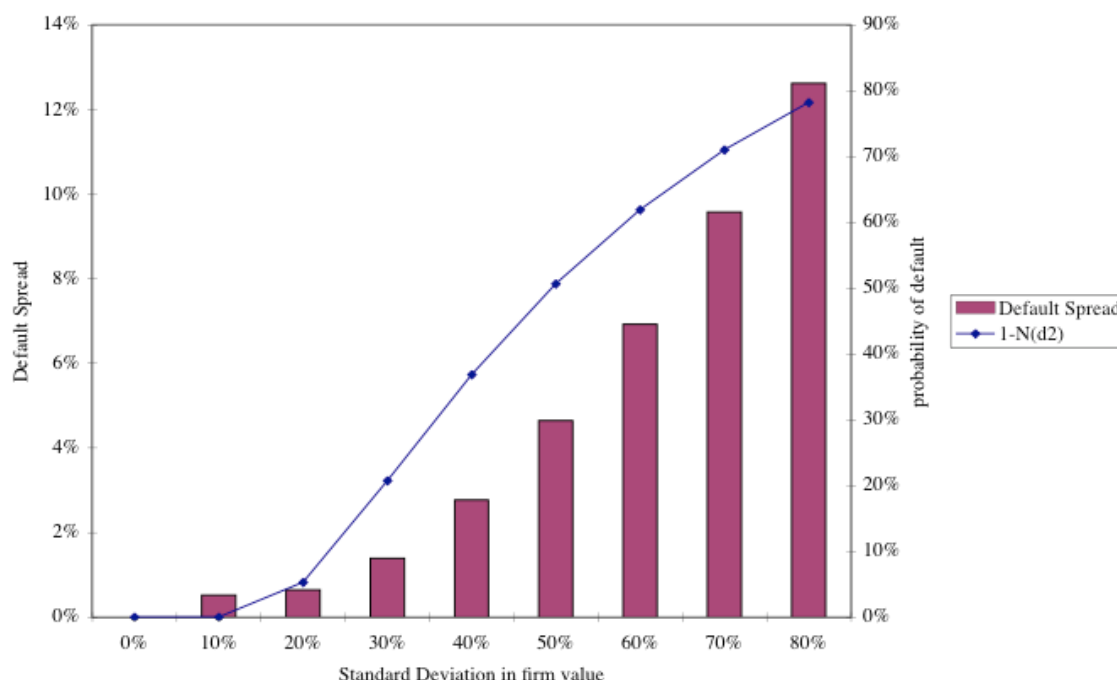
In addition, the interest rate from the debt allows us to estimate the appropriate default spread to charge on bonds.

You can see the potential in applying this model to bank loan portfolios to extract both the probability of default and to measure whether you are charging an interest rate that is high enough on the debt. In fact, there are commercial services that use fairly sophisticated option-pricing models to estimate both values for firms.

Illustration 12.21: Probabilities of default and Default Spreads

We return to Illustration 12.8 and estimate the probability of default as $N(d_2)$ and the default spread, measured as the difference between the interest rate on a firm's debt and the riskfree rate, as a function of the variance. These values are graphed in Figure 12.9.

Figure 17.4: Risk Neutral Probability of default and Default spreads



Note that the probability of default climbs very quickly as the standard deviation in firm value increases and the default spread follows it along.

Estimating the Value of Equity as an Option

The examples we have used thus far to illustrate the application of option pricing to value equity have included some simplifying assumptions. Among them are the following.

1. There are only two claimholders in the firm - debt and equity.
2. There is only one issue of debt outstanding and it can be retired at face value.
3. The debt has a zero coupon and no special features (convertibility, put clauses, etc.)
4. The value of the firm and the variance in that value can be estimated.

Each of these assumptions is made for a reason. First, by restricting the claimholders to just debt and equity, we make the problem more tractable; introducing other claimholders

such as preferred stock makes it more difficult to arrive at a result, albeit not impossible. Second, by assuming only one zero-coupon debt issue that can be retired at face value any time prior to maturity, we align the features of the debt more closely to the features of the strike price on a standard option. Third, if the debt is coupon debt, or more than one debt issue is outstanding, the equity investors can be forced to exercise (liquidate the firm) at these earlier coupon dates if they do not have the cash flows to meet their coupon obligations.

Finally, knowing the value of the firm and the variance in that value makes the option pricing possible, but it also raises an interesting question about the usefulness of option pricing in equity valuation. If the bonds of the firm are publicly traded, the market value of the debt can be subtracted from the value of the firm to obtain the value of equity much more directly. The option pricing approach does have its advantages, however. Specifically, when the debt of a firm is not publicly traded, option pricing theory can provide an estimate of value for the equity in the firm. Even when the debt is publicly traded, the bonds may not be correctly valued and the option pricing framework can be useful in evaluating the values of debt and equity. Finally, relating the values of debt and equity to the variance in firm value provides some insight into the redistributive effects of actions taken by the firm.

Inputs for Valuing Equity as an Option

Since most firms do not fall into the neat framework developed above (such as having only one zero-coupon bond outstanding), we have to make some compromises to use this model in valuation.

Value of the Firm

We can obtain the value of the firm in one of four ways. In the first, we cumulate the market values of outstanding debt and equity, assuming that all debt and equity are traded, to obtain firm value. The option pricing model then reallocates the firm value between debt and equity. This approach, while simple, is internally inconsistent. We start with one set of market values for debt and equity and, using the option pricing model, end up with entirely different values for each.

In the second, we estimate the market values of the assets of the firm by discounting expected cash flows at the cost of capital. The one consideration that we need

to keep in mind is that the value of the firm in an option pricing model should be the value obtained on liquidation. This may be less than the total firm value, which includes expected future investments and it may also be reduced to reflect the cost of liquidation. If we estimate the firm value using a discounted cash flow model, then this would suggest that only existing investments⁴⁰ should be considered while estimating firm value. The biggest problem with this approach is that financial distress can affect operating income and thus the value that we obtain by using current operating income may be too low.

In the third approach, we estimate a multiple of revenues by looking at healthy firms in the same business and apply this multiple to the revenues of the firm we are valuing. Implicitly, we are assuming that a potential buyer, in the event of liquidation, will pay this value.

We can use the fourth approach for firms that have separable assets that are individually traded. Here, we cumulate the value of the market values of the assets to arrive at firm value. For example, we can value a troubled real estate firm that owns five properties by valuing each property separately and then aggregating the values.

Variance in Firm value

We can obtain the variance in firm value directly if both stocks and bonds in the firm are traded. Defining σ_e^2 as the variance in the stock price and σ_d^2 as the variance in the bond price, w_e as the market-value weight of equity and w_d as the market-value weight of debt, we can write the variance in firm value as:⁴¹

$$\sigma_{firm}^2 = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2w_e w_d \rho_{ed} \sigma_e \sigma_d$$

where ρ_{ed} is the correlation between the stock and the bond prices. When the bonds of the firm are not traded, we can use the variance of similarly rated bonds as the estimate of σ_d^2 and the correlation between similarly rated bonds and the firm's stock as the estimate of ρ_{ed} .

When companies get into financial trouble, this approach can yield misleading results as both its stock prices and its bond prices become more volatile. An alternative that often yields more reliable estimates is to use the average variance in firm value for

⁴⁰ Technically, this can be done by putting the firm into stable growth and valuing it as a stable growth firm, where reinvestments are used to either preserve or augment existing assets.

other firms in the sector. Thus, the value of equity in a deeply troubled steel company can be estimated using the average variance in firm value of all traded steel companies.

Maturity of the Debt

Most firms have more than one debt issue on their books and much of the debt comes with coupons. Since the option pricing model allows for only one input for the time to expiration, we have to convert these multiple bonds issues and coupon payments into one equivalent zero-coupon bond.

- One solution, which takes into account both the coupon payments and the maturity of the bonds, is to estimate the duration of each debt issue and calculate a face-value-weighted average of the durations of the different issues. This value-weighted duration is then used as a measure of the time to expiration of the option.
- An approximation is to use the face-value weighted maturity of the debt converted to the maturity of the zero-coupon bond in the option pricing model.

Face Value of Debt

When a distressed firm has multiple debt issues outstanding, we have three choices when it comes to what we use as the face value of debt:

- We could add up the principal due on all of the debt of the firm and consider it to be the face value of the hypothetical zero coupon bond that we assume that the firm has issued. The limitation of this approach is that it will understate what the firm will truly have to pay out over the life of the debt, since there will be coupon payments and interest payments during the period.
- At the other extreme, we could add the expected interest and coupon payments that will come due on the debt to the principal payments to come up with a cumulated face value of debt. Since the interest payments occur in the near years and the principal payments are due only when the debt comes due, we are mixing cash flows up at different points in time when we do this. This is, however, the simplest approach of dealing with intermediate interest payments coming due.
- We can consider only the principal due on the debt as the face value of the debt and the interest payments each year, specified as a percent of firm value, can take

⁴¹ This is an extension of the variance formula for a two-asset portfolio.

the place of the dividend yield in the option pricing model. In effect, each year that the firm remains in existence, we would expect to see the value of the firm decline by the expected payments on the debt.

Illustration 12.22: Valuing Equity as an option – Aracruz in 2009

For Aracruz, 2008 was a very bad year, with losses from derivatives in the billions leading the firm to the brink of disaster. In June 2009, the firm owed in excess of 9.8 billion \$R in debt and had operating income of only 574 million \$R. Even if we assume that the firm reverts back to its average profitability between 2003 and 2007, the firm will generate pre-tax operating income of only 1.007 billion \$R (see chapter 8). Assuming a perpetual growth rate of 7% growth rate (in nominal \$R), a return on capital of 15% and using the cost of capital of 18.37% (also in nominal \$R, estimated in chapter 4) allows us to estimate the value of the operating assets:

$$\begin{aligned}\text{Value of Operating Assets} &= \frac{\text{EBIT}(1-t)(1-\frac{g}{\text{ROC}})(1+g)}{(\text{Cost of capital} - g)} \\ &= \frac{1007(1-.34)(1-\frac{.07}{.15})(1.07)}{(.1837-.07)} = \$R\ 5,807 \text{ million}\end{aligned}$$

Since this is well below the face value of the debt, it would be difficult to justify the value of equity, using an intrinsic valuation model.

We can try to estimate the value of Aracruz, as an equity option, assuming that the debt has a weighted average duration of 3 years and the industry average standard deviation of 34% as the standard deviation in firm value.⁴² In summary, the inputs to the option pricing model are as follows:

Value of the underlying asset = S = Value of the firm = R\$ 5,807 million

Exercise price = K = Face Value of outstanding debt = R\$ 9,835 million

Life of the option = t = Weighted average duration of debt = 3 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = $(0.34)^2 = 0.115$

Riskless rate = r = Riskfree corresponding to option life = 6.5%

Based upon these inputs, we estimate the following value for the call:

$$d_1 = 0.2691$$

$$N(d_1) = 0.3939$$

$$d_2 = -0.8580$$

$$N(d_2) = 0.1954$$

$$\text{Value of the call} = 5,807(0.3939) - 9,835e^{(-0.065)(3)}(0.1954) = \$706 \text{ million}$$

If we treat this as the value of equity, it yields a value per share of R\$1.20 a share, which is much lower than the stock price of R\$ 15.14 per share.

The option pricing framework, in addition to yielding a value for Aracruz equity, yields some valuable insight into the drivers of value for this equity. While it is certainly important that the firm try to bring costs under control and increase operating margins, the two most critical variables determining equity value are the duration of the debt and the variance in firm value. Any action that increases (decreases) the debt duration will have a positive (negative) effect on equity value. Thus, the results of debt renegotiation talks that were ongoing at the time of this analysis could have a significant effect on value.

Reconciling Different Valuations

The standard approaches to valuation—discounted cash flow valuation and relative valuation—yield different values for Disney.⁴³ In fact, Disney is under valued using a discounted cash flow model but is closer to being fairly valued using relative valuation models. Even within relative valuation, we arrive at different estimates of value, depending on which multiple we use and the firms on which we based the relative valuation.

The differences in value between discounted cash flow valuation and relative valuation come from different views of market efficiency or, put more precisely, market inefficiency. In discounted cash flow valuation, we assume that markets make mistakes, they correct these mistakes over time, and these mistakes can often occur across entire sectors or even the entire market. In relative valuation, we assume that although markets

⁴² This is the industry average for firm value variances of paper and pulp companies.

⁴³ See Kaplan, S. and R. Ruback, 1995, *The Valuation of Cash Flow Forecasts: An Empirical Analysis*, Journal of Finance, v50, 1059-1093. They examine valuations in acquisitions and find that discounted cash flow models better explain prices paid than relative valuation models.

make mistakes on individual stocks, they are correct on average. In other words, when we value Disney relative to other entertainment companies, we are assuming that the market has priced these companies correctly, on average, even though it might have made mistakes in the pricing of each of them individually. Thus, a stock may be overvalued on a discounted cash flow basis but undervalued on a relative basis, if the firms used in the relative valuation are all overpriced by the market. The reverse would occur, if an entire sector or market were underpriced.

To conclude, we suggest the following broad guidelines on gauging value using different approaches:

- The discounted cash flow models are built on the implicit assumption of long time horizons, giving markets time to correct their errors.
- When using relative valuation, it is dangerous to base valuations on multiples where the differences across firms cannot be explained well using financial fundamentals—growth, risk, and cash flow patterns. One of the advantages of using the regression approach described in the later part of this chapter is that the R^2 and t -statistics from the regressions yield a tangible estimate of the strength (or weakness) of this relationship.



12.12. Valuing an IPO

If you were an investment banker pricing an IPO, would you primarily use discounted cash flow valuation, relative valuation, or a combination of the two?

- a. Relative valuation, because the buyers of the IPO will look at comparables
- b. Discounted cash flow valuation, because it reflects intrinsic value
- c. The higher of the two values, because it is my job to get the highest price I can for my client
- d. None of the above

Explain.

Conclusion

There are three basic approaches to valuation. The first is discounted cash flow valuation, in which the value of any asset is estimated by computing the present value of

the expected cash flows on it. The actual process of estimation, in either case, generally requires four inputs:

- the length of the period for which a firm or asset can be expected to generate growth greater than the stable growth rate (which is constrained to be close to the growth rate of the economy in which the firm operates),
- the cash flows during the high-growth period,
- the terminal value at the end of the high growth period, and
- a discount rate.

The expected growth potential will vary across firms, with some firms already growing at a stable growth rate and others for which the expectation, at least, is that high growth will last for some period into the future. We can value the operating assets of a firm by discounting cash flows before debt payments but after reinvestment at the cost of capital. Adding the value of cash and nonoperating assets give us firm value, and subtracting out debt yields the value of equity. We can also value equity directly by discounting cash flows after debt payments and reinvestment needs at the cost of equity.

The second approach to valuation is relative valuation, where the value of any asset is estimated by looking at how similar assets are priced in the market. The key steps in this approach are defining comparable firms or assets and choosing a standardized measure of value (usually value as a multiple of earnings, cash flows, or book value) to compare the firms. To compare multiples across companies, we have to control for differences in growth, risk, and cash flows, just as we would have in discounted cash flow valuation.

In the final approach to valuation, we assume that equity investors own the option to liquidate the firm's assets and claim the difference between asset value and debt outstanding for themselves. This approach works for highly levered and distressed firms and is the only one where equity value increases as risk increases.

Live Case Study

Valuation

Objective To value your firm, based on its existing management, and your expectations for the future.

Key Questions

- What type of cash flow (dividends, FCFE, or FCFF) would you choose to discount for this firm?
- What growth pattern would you pick for this firm? How long will high growth and excess returns last?
- When will your firm be in stable growth and what will your firm look like when it reaches stable growth?
- What is your estimate of value of equity in this firm? How does this compare to the market value?

Framework for Analysis

1. Cash Flow Estimation

- What is this firm's accounting operating income? Would you adjust it for your valuation?
- What is your firm's effective tax rate? What is its marginal tax rate? Which would you use in your valuation?
- How much did your firm reinvest last year in internal investments, acquisitions, R&D, and working capital?

2. Growth Pattern Choice

- How fast have this company's earnings grown historically?
- How quickly do analysts expect this company's earnings to grow in the future?
- What do the fundamentals suggest about earnings growth at this company? (How much is being reinvested and at what rate of return?)
- If there is anticipated high growth with excess returns, what are the barriers to entry that will allow these excess returns to continue? For how long?

3. *Valuation*

- What is the value of the operating assets of the firm, based on a discounted cash flow model?
- Does the firm have cash and nonoperating assets, and what are their values?
- Are there equity options outstanding (management options, convertible bonds) and how much are they worth?
- What is the value of equity per share?

4. *Relative Valuation*

- What multiple would you use to value the firm or its equity?
- What industry does the firm belong to, and what are the comparable firms?
- How does your firm's valuation (in multiple terms) compare to those of the other firms in the industry?
- What value would you assign your firm (or its equity), given how comparable firms are valued?

Getting Information for valuation

Most of the information that you need for valuation come from your current or past financial statements. You will also need a beta (see risk and return section) and a debt ratio (see risk and return section) to estimate the FCFE. You can get analyst estimates of growth in several sources including Zacks and I/B/E/S.

Online sources of

informationwww.stern.nyu.edu/~adamodar/cfin2E/project/data.htm.

Problems and Questions

In the problems below, you can use a risk premium of 5.5 percent and a tax rate of 40 percent if none is specified.

1. Vernon Enterprises has current after-tax operating income of \$100 million and a cost of capital of 10 percent. The firm earns a return on capital equal to its cost of capital.
 - a. Assume that the firm is in stable growth, growing 5 percent a year forever; estimate the firm's reinvestment rate.
 - b. Given this reinvestment rate, estimate the value of the firm.
 - c. What is the value of the firm, if you assume a zero reinvestment rate and no growth?

2. Assume in the previous question with Vernon Enterprises that the firm will earn a return on capital of 15 percent in perpetuity.
 - a. Assume that the firm is in stable growth, growing 5 percent a year forever; estimate the firm's reinvestment rate.
 - b. Given this reinvestment rate, estimate the value of the firm.

3. Cello is a manufacturer of pianos. It earned an after-tax return on capital of 10 percent last year and expects to maintain this next year. If the current year's after-tax operating income is \$100 million and the firm reinvests 50 percent of this income back, estimate the FCFF next year. (After-Tax Operating Income = $EBIT(1 - t)$).

4. Cell Phone is a cellular firm that reported net income of \$50 million in the most recent financial year. The firm had \$1 billion in debt, on which it reported interest expenses of \$100 million in the most recent financial year. The firm had depreciation of \$100 million for the year, and capital expenditures were 200 percent of depreciation. The firm has a cost of capital of 11 percent. Assuming that there is no working capital requirement, and a constant growth rate of 4 percent in perpetuity, estimate the value of the firm.

5. Netsoft is a company that manufactures networking software. In the current year, the firm reported operating earnings before interest and taxes of \$200 million (operating earnings does not include interest income), and these earnings are expected to grow 4

percent a year in perpetuity. In addition, the firm has a cash balance of \$250 million on which it earned interest income of \$20 million. The unlevered beta for other networking software firm is 1.20, and these firms have on average cash balances of 10 percent of firm value. If Netsoft has a debt ratio of 15 percent, a tax rate of 40 percent, a return on capital of 10 percent on operating assets, and a cost of debt of 10 percent, estimate the value of the firm. (The risk-free rate is 6 percent, and you can assume a market risk premium of 5.5 percent.)

6. Gemco Jewelers earned \$5 million in after-tax operating income in the most recent year. The firm also had capital expenditures of \$4 million and depreciation of \$2 million during the year, and the noncash working capital at the end of the year was \$10 million.

- a. Assuming that the firm's operating income will grow 20 percent next year, and that all other items (capital expenditures, depreciation, and noncash working capital) will grow at the same rate, estimate the FCFF next year.
- b. If the firm can grow at 20 percent for the next five years, estimate the present value of the FCFF over that period. You can assume a cost of capital of 12 percent.
- c. After year five, the firm's capital expenditures will decline to 125 percent of revenues, and the growth rate will drop to 5 percent (in both operating income and noncash working capital). In addition, the cost of capital will decline to 10 percent. Estimate the terminal value of the firm at the end of year five.
- d. Estimate the total value of the operating assets of the firm.

7. Now assume that Gemco Jewelers has \$10 million in cash and nonoperating assets and that the firm has \$15 million in outstanding debt.

- a. Estimate the value of equity in the firm.
- b. If the firm has 5 million shares outstanding, estimate the value of equity per share.
- c. How would your answer to **b** change if you learn that the firm has 1 million options outstanding, with an exercise price of \$5 and five years to maturity? (The estimated value per option is \$7.)

8. Union Pacific Railroad reported net income of \$770 million after interest expenses of \$320 million in a recent financial year. (The corporate tax rate was 36 percent.) It

reported depreciation of \$960 million in that year, and capital spending was \$1.2 billion. The firm also had \$4 billion in debt outstanding on the books, was rated AA (carrying a yield to maturity of 8 percent), and was trading at par (up from \$3.8 billion at the end of the previous year). The beta of the stock is 1.05, and there were 200 million shares outstanding (trading at \$60 per share), with a book value of \$5 billion. Union Pacific paid 40 percent of its earnings as dividends and working capital requirements are negligible. (The Treasury bond rate is 7 percent.)

- a. Estimate the FCFF for the most recent financial year.
- b. Estimate the value of the firm now.
- c. Estimate the value of equity and the value per share now.

9. Lockheed, one of the largest defense contractors in the United States, reported EBITDA of \$1,290 million in a recent financial year, prior to interest expenses of \$215 million and depreciation charges of \$400 million. Capital expenditures amounted to \$450 million during the year, and working capital was 7 percent of revenues (which were \$13,500 million). The firm had debt outstanding of \$3.068 billion (in book value terms), trading at a market value of \$3.2 billion, and yielding a pretax interest rate of 8 percent. There were 62 million shares outstanding, trading at \$64 per share, and the most recent beta is 1.10. The tax rate for the firm is 40 percent. (The Treasury bond rate is 7 percent.) The firm expects revenues, earnings, capital expenditures, and depreciation to grow at 9.5 percent a year for the next 5 years, after which the growth rate is expected to drop to 4 percent. (Even though this is unrealistic, you can assume that capital spending will offset depreciation in the stable-growth period.) The company also plans to lower its debt/equity ratio to 50 percent for the steady state (which will result in the pretax interest rate dropping to 7.5 percent).

- a. Estimate the value of the firm.
- b. Estimate the value of the equity in the firm and the value per share.

10. In the face of disappointing earnings results and increasingly assertive institutional stockholders, Eastman Kodak was considering the sale of its health division, which earned \$560 million in EBIT in the most recent year on revenues of \$5.285 billion. The expected growth in earnings was expected to moderate to 6 percent for the next 5 years,

and to 4 percent after that. Capital expenditures in the health division amounted to \$420 million in the most recent year, whereas depreciation was \$350 million. Both are expected to grow 4 percent a year in the long run. Working capital requirements are negligible.

The average beta of firms competing with Eastman Kodak's health division is 1.15. Although Eastman Kodak has a debt ratio ($D/[D + E]$) of 50 percent, the health division can sustain a debt ratio ($D/[D + E]$) of only 20 percent, which is similar to the average debt ratio of firms competing in the health sector. At this level of debt, the health division can expect to pay 7.5 percent on its debt, before taxes. (The tax rate is 40 percent, and the Treasury bond rate is 7 percent.)

- a. Estimate the cost of capital for the division.
- b. Estimate the value of the division.

11. You have been asked to value Alcoa and have come up with the following inputs.

- The stock has a beta of 0.90, estimated over the last five years. During this period, the firm had an average debt/equity ratio of 20 percent and an average cash balance of 15 percent.
- The firm's current market value of equity is 1.6 billion and its current market value of debt is \$800 million. The current cash balance is \$500 million.
- The firm earned earnings before interest and taxes of \$450 million, which includes the interest income on the current cash balance of \$50 million. The firm's tax rate is 40 percent.
- The firm is in stable growth, and its earnings from operations are expected to grow 5 percent a year. The net capital expenditures next year are expected to be \$90 million. Estimate the value of the noncash assets of the firm, its total value, and the value of its equity.

12. You are analyzing a valuation done on a stable firm by a well-known analyst. Based on the expected FCFF next year of \$30 million, and an expected growth rate of 5 percent, the analyst has estimated a value of \$750 million. However, he has made the mistake of using the book values of debt and equity in his calculation. Although you do not know the book value weights he used, you know that the firm has a cost of equity of 12 percent

and an after-tax cost of debt of 6 percent. You also know that the market value of equity is three times the book value of equity, and the market value of debt is equal to the book value of debt. Estimate the correct value for the firm.

13. You have been asked to value Office Help, a private firm providing office support services in the New York area.

- The firm reported pretax operating income of \$10 million in its most recent financial year on revenues of \$100 million. In the most recent financial year, you note that the owners of the business did not pay themselves a salary. You believe that a fair salary for their services would be \$1.5 million a year.
- The cost of capital for comparable firms that are publicly traded is 9 percent. (You can assume that this firm will have similar leverage and cost of capital.)
- The firm is in stable growth and expects to grow 5 percent a year in perpetuity. The tax rate is 40 percent.

The average illiquidity discount applied to private firms is 30 percent, but you have run a regression and arrived at the following estimate for the discount:

$$\text{Illiquidity Discount} = 0.30 - 0.04 (\ln [\text{Revenues in millions}])$$

Estimate the value of Office Help for sale in a private transaction (to an individual).

14. National City, a bank holding company, reported earnings per share of \$2.40 and paid dividends per share of \$1.06. The earnings had grown 7.5 percent a year over the prior five years, and were expected to grow 6 percent a year in the long run. The stock had a beta of 1.05 and traded for ten times earnings. The Treasury bond rate was 7 percent.

- Estimate the P/E ratio for National City.
- What long-term growth rate is implied in the firm's current PE ratio?

15. The following were the P/E ratios of firms in the aerospace/defense industry at the with additional data on expected growth and risk:

<i>Company</i>	<i>P/E Ratio</i>	<i>Expected Growth</i>	<i>Beta</i>	<i>Payout</i>
Boeing	17.3	3.5%	1.10	28%
General Dynamics	15.5	11.5%	1.25	40%
General Motors—Hughes	16.5	13.0%	0.85	41%

Grumman	11.4	10.5%	0.80	37%
Lockheed	10.2	9.5%	0.85	37%
Logicon	12.4	14.0%	0.85	11%
Loral	13.3	16.5%	0.75	23%
Martin Marietta	11.0	8.0%	0.85	22%
McDonnell Douglas	22.6	13.0%	1.15	37%
Northrop	9.5	9.0%	1.05	47%
Raytheon	12.1	9.5%	0.75	28%
Rockwell	13.9	11.5%	1.00	38%
Thiokol	8.7	5.5%	0.95	15%
United Industrial	10.4	4.5%	0.70	50%

- Estimate the average and median P/E ratios. What, if anything, would these averages tell you?
- An analyst concludes that Thiokol is undervalued because its P/E ratio is lower than the industry average. Under what conditions is this statement true? Would you agree with it here?
- Using the PEG ratio, assess whether Thiokol is undervalued. What are you assuming about the relationship between value and growth when you use PEG ratios?
- Using a regression, control for differences across firms on risk, growth, and payout. Specify how you would use this regression to spot under- and overvalued stocks. What are the limitations of this approach?

16. NCH, which markets cleaning chemicals, insecticides, and other products, paid dividends of \$2.00 per share on earnings of \$4.00 per share. The book value of equity per share was \$40.00, and earnings are expected to grow 6 percent a year in the long term. The stock has a beta of 0.85, and sells for \$60 per share. The Treasury bond rate is 7 percent.

- Based on these inputs, estimate the price/book value ratio for NCH.
- How much would the return on equity have to increase to justify the price/book value ratio at which NCH sells for currently?

17. You are trying to estimate a price per share on an IPO of a company involved in environmental waste disposal. The company has a book value per share of \$20 and earned \$3.50 per share in the most recent time period. Although it does not pay dividends, the capital expenditures per share were \$2.50 higher than depreciation per share in the most recent period, and the firm uses no debt financing. Analysts project that earnings for the company will grow 25 percent a year for the next five years. You have data on other companies in the environment waste disposal business:

<i>Company</i>	<i>Price</i>	<i>BV/Share</i>	<i>EPS</i>	<i>DPS</i>	<i>Beta</i>	<i>Exp. Growth</i>
Air & Water	\$9.60	\$8.48	\$0.40	\$0.00	1.65	10.5%
Allwaste	\$5.40	\$3.10	\$0.25	\$0.00	1.10	18.5%
Browning Ferris	\$29.00	\$11.50	\$1.45	\$0.68	1.25	11.0%
Chemical Waste	\$9.40	\$3.75	\$0.45	\$0.15	1.15	2.5%
Groundwater	\$15.00	\$14.45	\$0.65	\$0.00	1.00	3.0%
Intn'l Tech.	\$3.30	\$3.35	\$0.16	\$0.00	1.10	11.0%
Ionics	\$48.00	\$31.00	\$2.20	\$0.00	1.00	14.5%
Laidlaw	\$6.30	\$5.85	\$0.40	\$0.12	1.15	8.5%
OHM	\$16.00	\$5.65	\$0.60	\$0.00	1.15	9.50%
Rollins	\$5.10	\$3.65	\$0.05	\$0.00	1.30	1.0%
Safety-Kleen	\$14.00	\$9.25	\$0.80	\$0.36	1.15	6.50%

The average debt/equity ratio of these firms is 20 percent, and the tax rate is 40 percent.

- Estimate the average price/book value ratio for these comparable firms. Would you use this average P/BV ratio to price the IPO?
- What subjective adjustments would you make to the price/book value ratio for this firm and why?

18. Longs Drug, a large U.S. drugstore chain operating primarily in northern California, had sales per share of \$122 on which it reported earnings per share of \$2.45 and paid a dividend per share of \$1.12. The company is expected to grow 6 percent in the long run, and has a beta of 0.90. The current Treasury bond rate is 7 percent.

- Estimate the appropriate price/sales multiple for Longs Drug.

b. The stock is currently trading for \$34 per share. Assuming the growth rate is estimated correctly, what would the profit margin need to be to justify this price per share?

19. You have been asked to assess whether Walgreen, a drugstore chain, is correctly priced relative to its competitors in the drugstore industry. The following are the price/sales ratios, profit margins, and other relative details of the firms in the drugstore industry.

<i>Company</i>	<i>P/S Ratio</i>	<i>Profit Margin</i>	<i>Payout</i>	<i>Expected Growth</i>	<i>Beta</i>
Arbor Drugs	0.42	3.40%	18%	14.0%	1.05
Big B	0.30	1.90%	14%	23.5%	0.70
Drug Emporium	0.10	0.60%	0%	27.5%	0.90
Fay's	0.15	1.30%	37%	11.5%	0.90
Genovese	0.18	1.70%	26%	10.5%	0.80
Longs Drug	0.30	2.00%	46%	6.0%	0.90
Perry Drugs	0.12	1.30%	0%	12.5%	1.10
Rite-Aid	0.33	3.20%	37%	10.5%	0.90
<i>Walgreen</i>	<i>0.60</i>	<i>2.70%</i>	<i>31%</i>	<i>13.5%</i>	<i>1.15</i>

Based entirely on a subjective analysis, do you think that Walgreen is overpriced because its price/sales ratio is the highest in the industry? If it is not, how would you rationalize its value?

20. Time Warner is considering a sale of its publishing division. The division had earnings EBITDA of \$550 million in the most recent year (depreciation was \$150 million), growing at an estimated 5 percent a year (you can assume that depreciation grows at the same rate). The return on capital in the division is 15 percent, and the corporate tax rate is 40 percent. If the cost of capital for the division is 9 percent, estimate the following:

- Value/FCFF multiple.
- Value/EBIT multiple.
- Value/EBITDA multiple.