

CHAPTER 3

THE BASICS OF RISK

Risk, in traditional terms, is viewed as a negative and something to be avoided. Webster's dictionary, for instance, defines risk as "exposing to danger or hazard". The Chinese symbols for risk, reproduced below, give a much better description of risk –

危机

The first symbol is the symbol for "danger", while the second is the symbol for "opportunity", making risk a mix of danger and opportunity. It illustrates very clearly the tradeoff that every investor and business has to make – between the "higher rewards" that potentially come with the opportunity and the "higher risk" that has to be borne as a consequence of the danger. The key test in finance is to ensure that when an investor is exposed to risk that he or she is "appropriately" rewarded for taking this risk.

In this chapter, we will lay the foundations for analyzing risk in corporate finance and present alternative models for measuring risk and converting these risk measures into "acceptable" hurdle rates.

Motivation and Perspective in Analyzing Risk

Why do we need a model that measures risk and estimates expected return? A good model for risk and return provides us with the tools to measure the risk in any investment and uses that risk measure to come up with the appropriate expected return on that investment; this expected return provides us with the hurdle rate in project analysis.

What makes the measurement of risk and expected return so challenging is that it can vary depending upon whose perspective we adopt. When analyzing Disney's risk, for instance, we can measure it from the viewpoint of Disney's managers. Alternatively, we can argue that Disney's equity is owned by its stockholders, and that it is their perspective on risk that should matter. Disney's stockholders, many of whom hold the stock as one investment in a larger portfolio, might perceive the risk in Disney very differently from Disney's managers, who might have the bulk of their capital, human and

financial, invested in the firm. In this chapter, we will argue that risk in an equity investment has to be perceived through the eyes of investors in the firm. Since firms like Disney have thousands of investors, often with very different perspectives, we will go further. We will assert that risk has to be measured from the perspective of not just any investor in the stock, but of the **marginal investor**, defined to be the investor most likely to be trading on the stock at any given point in time. The objective in corporate finance is the maximization of firm value and stock price. If we want to stay true to this objective, we have to consider the viewpoint of those who set the stock prices, and they are the marginal investors.

Finally, the risk in a company can be viewed very differently by investors in its stock (equity investors) and by lenders to the firm (bondholders and bankers). Equity investors who benefit from upside as well as downside tend to take a much more sanguine view of risk than lenders who have limited upside but potentially high downside. We will consider how to measure equity risk in the first part of the chapter and risk from the perspective of lenders in the latter half of the chapter.

We will be presenting a number of different risk and return models in this chapter. In order to evaluate the relative strengths of these models, it is worth reviewing the characteristics of a good risk and return model.

1. It should come up with a measure of risk that applies to all assets and not be asset-specific.
2. It should clearly delineate what types of risk are rewarded and what are not, and provide a rationale for the delineation.
3. It should come up with standardized risk measures, i.e., an investor presented with a risk measure for an individual asset should be able to draw conclusions about whether the asset is above-average or below-average risk.
4. It should translate the measure of risk into a rate of return that the investor should demand as compensation for bearing the risk.
5. It should work well not only at explaining past returns, but also in predicting future expected returns.

Every risk and return model is flawed, and we should not let the perfect be the enemy of a good or even an adequate model.

Equity Risk and Expected Returns

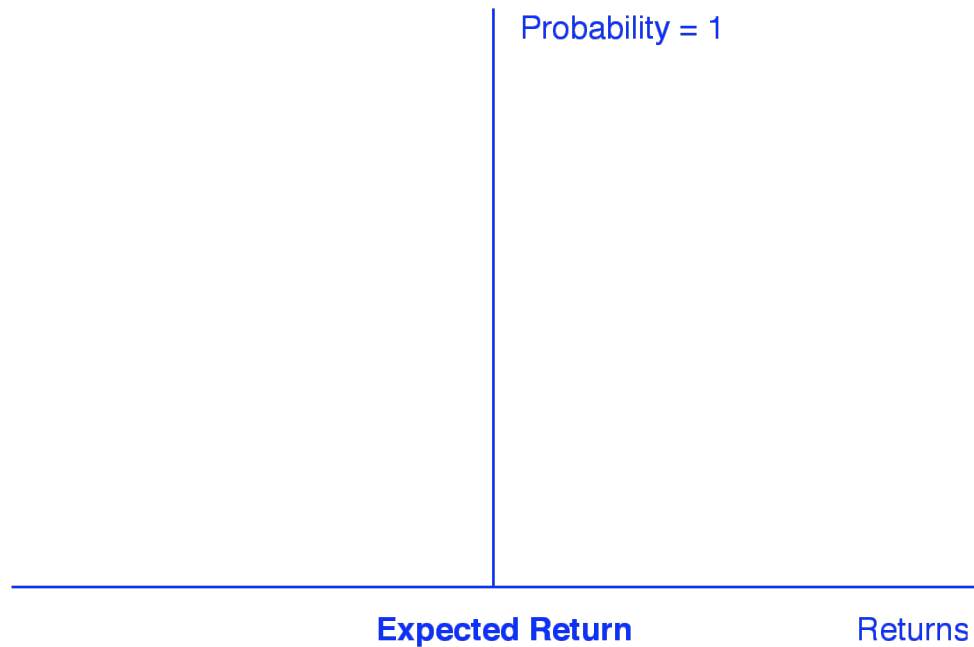
To understand how risk is viewed in corporate finance, we will present the analysis in three steps. First, we will define risk in terms of the distribution of actual returns around an expected return. Second, we will differentiate between risk that is specific to an investment or a few investments and risk that affects a much wider cross section of investments. We will argue that when the marginal investor is well diversified, it is only the latter risk, called market risk that will be rewarded. Third, we will look at alternative models for measuring this market risk and the expected returns that go with this risk.

I. Measuring Risk

Investors who buy an asset expect to make a return over the time horizon that they will hold the asset. The actual return that they make over this holding period may be very different from the expected return, and this is where the risk comes in. Consider an investor with a 1-year time horizon buying a 1-year Treasury bill (or any other default-free one-year bond) with a 5% expected return. At the end of the 1-year holding period, the actual return that this investor would have on this investment will always be 5%, which is equal to the expected return. The return distribution for this investment is shown in Figure 3.1.

Variance in Returns: This is a measure of the squared difference between the actual returns and the expected returns on an investment.

Figure 3.1: Returns on a Riskfree Investment



This is a riskless investment, at least in nominal terms.

To provide a contrast, consider an investor who invests in Disney. This investor, having done her research, may conclude that she can make an expected return of 30% on Disney over her 1-year holding period. The actual return over this period will almost certainly not be equal to 30%; it might be much greater or much lower. The distribution of returns on this investment is illustrated in Figure 3.2:

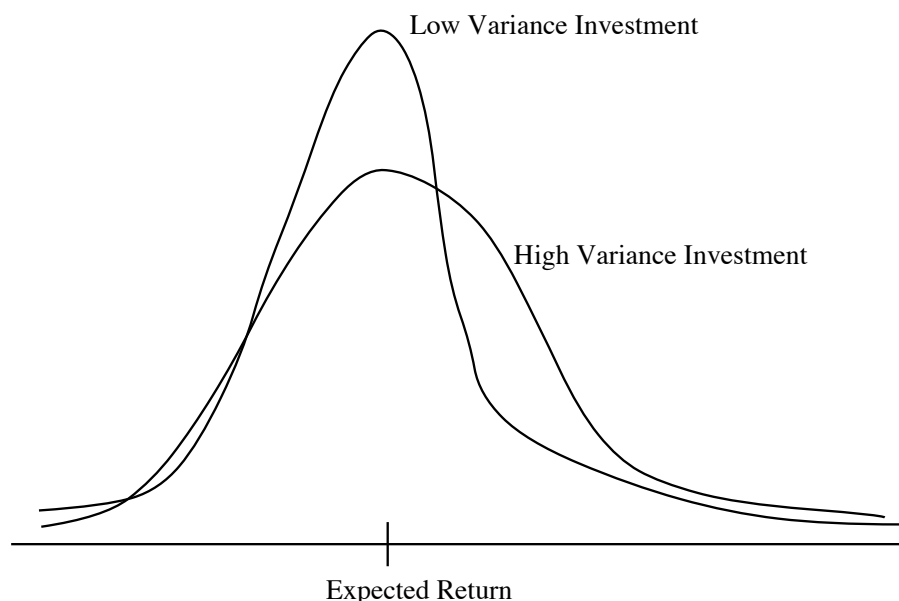
Figure 3.2: Probability Distribution for Risky Investment



In addition to the expected return, an investor now has to consider the following. First, the spread of the actual returns around the expected return is captured by the variance or standard deviation of the distribution; the greater the deviation of the actual returns from expected returns, the greater the variance. Second, the bias towards positive or negative returns is captured by the skewness of the distribution. The distribution above is positively skewed, since there is a greater likelihood of large positive returns than large negative returns. Third, the shape of the tails of the distribution is measured by the kurtosis of the distribution; fatter tails lead to higher kurtosis. In investment terms, this captures the tendency of the price of this investment to “jump” in either direction.

In the special case of the normal distribution, returns are symmetric and investors do not have to worry about skewness and kurtosis, since there is no skewness and a normal distribution is defined to have a kurtosis of zero. In that case, it can be argued that investments can be measured on only two dimensions - (1) the 'expected return' on the investment comprises the reward, and (2) the variance in anticipated returns comprises the risk on the investment. Figure 3.3 illustrates the return distributions on two investments with symmetric returns-

Figure 3.3: Return Distribution Comparisons



In this scenario, an investor faced with a choice between two investments with the same standard deviation but different expected returns, will always pick the one with the higher expected return.

In the more general case, where distributions are neither symmetric nor normal, it is still conceivable, though unlikely, that investors still choose between investments on the basis of only the expected return and the variance, if they possess utility functions¹ that allow them to do so. It is far more likely, however, that they prefer positive skewed distributions to negatively skewed ones, and distributions with a lower likelihood of jumps (lower kurtosis) over those with a higher likelihood of jumps (higher kurtosis). In this world, investors will trade off the good (higher expected returns and more positive skewness) against the bad (higher variance and kurtosis) in making investments. Among the risk and return models that we will be examining, one (the capital asset pricing model or the CAPM) explicitly requires that choices be made only in terms of expected returns

¹ A utility function is a way of summarizing investor preferences into a generic term called 'utility' on the basis of some choice variables. In this case, for instance, investor utility or satisfaction is stated as a function of wealth. By doing so, we effectively can answer questions such as – Will an investor be twice as happy if he has twice as much wealth? Does each marginal increase in wealth lead to less additional utility than the prior marginal increase? In one specific form of this function, the quadratic utility function, the entire utility of an investor can be compressed into the expected wealth measure and the standard deviation in that wealth, which provides a justification for the use of a framework where only the expected return (mean) and its standard deviation (variance) matter.

and variances. While it does ignore the skewness and kurtosis, it is not clear how much of a factor these additional moments of the distribution are in determining expected returns.

In closing, we should note that the return moments that we run into in practice are almost always estimated using past returns rather than future returns. The assumption we are making when we use historical variances is that past return distributions are good indicators of future return distributions. When this assumption is violated, as is the case when the asset's characteristics have changed significantly over time, the historical estimates may not be good measures of risk.



: 3.1: Do you live in a mean-variance world?

Assume that you had to pick between two investments. They have the same expected return of 15% and the same standard deviation of 25%; however, investment A offers a very small possibility that you could quadruple your money, while investment B's highest possible payoff is a 60% return. Would you

- be indifferent between the two investments, since they have the same expected return and standard deviation?
- prefer investment A, because of the possibility of a high payoff?
- prefer investment B, because it is safer?



Risk Assessment: A Behavioral Perspective

The mean-variance framework for assessing risk is focused on measuring risk quantitatively, often with one number – a standard deviation. While this focus is understandable, because it introduces discipline into the process and makes it easier for us to follow up and measure expected returns, it may not fully capture the complicated relationship that we, as human beings, have with risk. Behavioral finance scholars present three aspects of risk assessment that are at variance with the mean-variance school's view of risk:

- Loss aversion: In experiments with human subjects, there is evidence that individuals are affected far more negatively by a loss than they are helped by an equivalent gain, and that they generally measure losses in dollar terms rather than percentage terms. Put another way, investors are loss averse rather than risk averse. Consequently,

investments where there is even a small chance of a significant loss in wealth will be viewed as risky, even if they have only a small standard deviation.

- b. Familiarity bias: Individuals seem to perceive less risk with investments that they are familiar with than with unfamiliar investments. Thus, they see less risk in a domestic company with a long provenance than they do in an emerging market firm. This may explain why there is a “home bias” in portfolios, where investors over invest in investments in their domestic market and under invest in foreign investments. In an extension of this bias, the risk that individuals perceive in an activity or investment is inversely proportional to the difficulty they face in understanding it.
- c. Emotional factors: There is an emotional component to risk that quantitative risk measures cannot capture. This component can have both a positive affect, where gains accentuate positive affects (happiness and optimism) and losses feed into negative affects (worry and anxiety). More generally, investor moods can affect risk perceptions, with investments that are viewed as relatively safe in buoyant times becoming risky when investor moods shift.

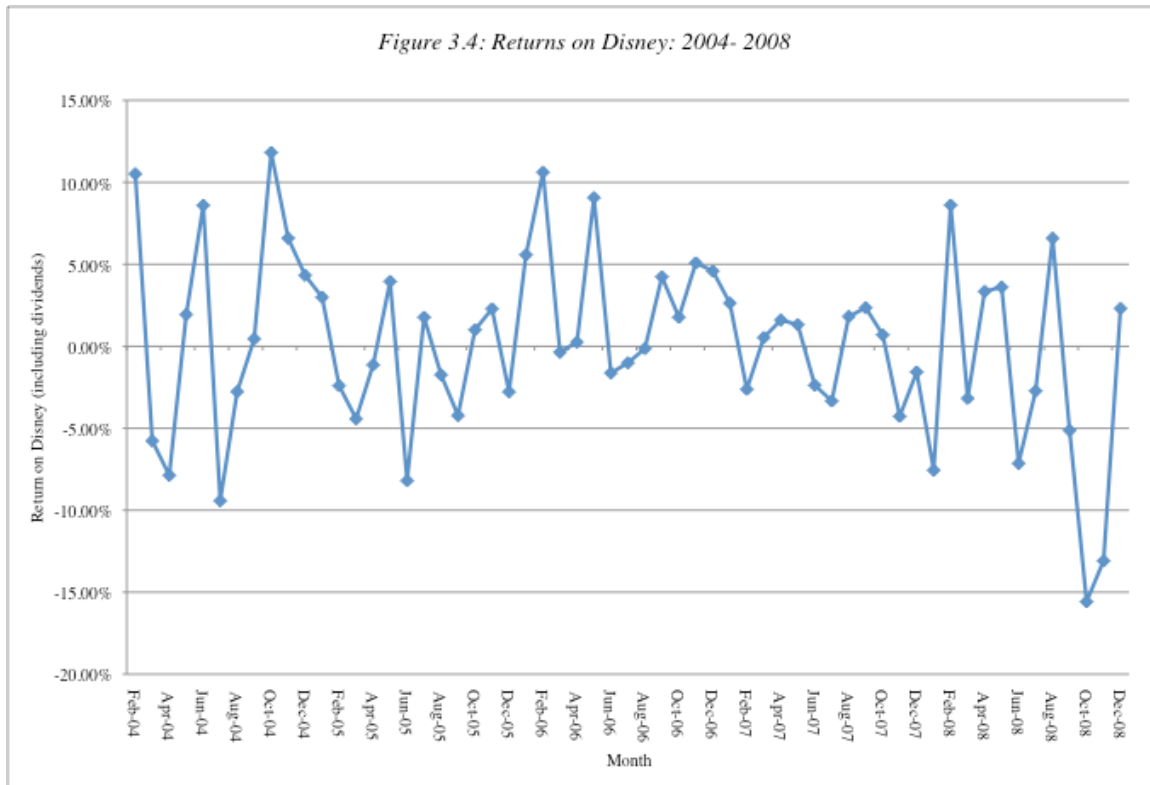
In recent years, there have been attempts to build composite risk measures that bring these behavioral components into the analysis. While no consensus has emerged, it may explain why quantitative measures of risk (such as standard deviation) for firm may deviate from the many qualitative risk measures that often exist for the same firm.

Illustration 3.1: Calculation of standard deviation using historical returns: Disney

We collected the data on the returns we would have made on a monthly basis for every month from January 2004 to December 2008 on an investment in Disney stock. To compute the returns, we looked at the price change in each month (with $Price_t$ being the price at the end of month t) and dividends if any during the month ($Dividends_t$):

$$Return_t = \frac{(Price_t - Price_{t-1} + Dividends_t)}{Price_{t-1}}$$

The monthly returns are graphed in figure 3.4:



Disney's returns reflect the risk that an investor in the stock would have faced over the period, with October 2004 being the best month (with a return of 11.82%) and October 2008 representing the worst month (with a return of -15.58%).

Looking at the summary statistics, the average monthly return on Disney over the 59 months was 0.18%. In fact, we started the period, in January 2004, with a stock price of \$23.68 and ended the period on December 31, 2008 with a stock price of \$22.69. However, the stock did pay an annual dividend that increased from \$0.24 in 2004 to \$0.35 in 2008. To measure the volatility or risk in the stock, we estimated the standard deviation in monthly returns over this period to be 5.59%; the variance in monthly returns was 31.25%.² To convert monthly values to annualized ones:

$$\text{Annualized Standard Deviation} = 5.59\% * \sqrt{12} = 19.36\%$$

$$\text{Annualized Variance} = 31.25\% * 12 = 374.98\%^2$$

² The variance is percent squared. In other words, if you stated the standard deviation of 9.96% in decimal terms, it would be 0.0996 but the variance of 99.15% would be 0.009915 in decimal terms.

Without making comparisons to the standard deviations in stock returns of other companies, we cannot really draw any conclusions about the relative risk of Disney by just looking at its standard deviation.



optvar.xls is a dataset on the web that summarizes average standard deviations of equity values by industry group in the United States.



3.2. Upside and Downside Risk

You are looking at the historical standard deviations over the last 5 years on two investments. Both have standard deviations of 35% in returns during the period, but one had a return of -10% during the period, whereas the other had a return of +40% during the period. Would you view them as equally risky?

- a. Yes
- b. No

Why do we not differentiate between “upside risk” and “downside risk” in finance?

In Practice: Estimating only downside risk

The variance of a return distribution measures the deviation of actual returns from the expected return. In estimating the variance, we consider not only actual returns that fall below the average return (downside risk) but also those that lie above it (upside risk). As investors, it is the downside that we generally consider as risk. There is an alternative measure called the semi-variance that considers only downside risk. To estimate the semi-variance, we calculate the deviations of actual returns from the average return only if the actual return is less than the expected return; we ignore actual returns that are higher than the average return.

$$\text{Semi-variance} = \sum_{t=1}^{t=n} \frac{(R_t - \text{Average Return})^2}{n}$$

n = number of periods where actual return < Average return

With a normal distribution, the semi-variance will generate a value identical to the variance, but for any non-symmetric distribution, the semi-variance will yield different values than the variance. In general, a stock that generates small positive returns in most

periods but very large negative returns in a few periods will have a semi-variance that is much higher than the variance.

II. Rewarded and Unrewarded Risk

Risk, as we have defined it in the previous section, arises from the deviation of actual returns from expected returns. This deviation, however, can occur for any number of reasons, and these reasons can be classified into two categories - those that are specific to the investment being considered (called firm specific risks) and those that apply across most or all investments (market risks).

The Components of Risk

When a firm makes an investment, in a new asset or a project, the return on that investment can be affected by several variables, most of which are not under the direct control of the firm. Some of the risk comes directly from the investment, a portion from competition, some from shifts in the industry, some from changes in exchange rates and some from macroeconomic factors. A portion of this risk, however, will be eliminated by the firm itself over the course of multiple investments and another portion by investors as they hold diversified portfolios.

The first source of risk is project-specific; an individual project may have higher or lower cashflows than expected, either because the firm misestimated the cashflows for that project or because of factors specific to that project. When firms take a large number of similar projects, it

Project Risk: This is risk that affects only the project under consideration, and may arise from factors specific to the project or estimation error.

can be argued that much of this risk should be diversified away in the normal course of business. For instance, Disney, while considering making a new movie, exposes itself to estimation error - it may under or over estimate the cost and time of making the movie, and may also err in its estimates of revenues from both theatrical release and the sale of merchandise. Since Disney releases several movies a year, it can be argued that some or

much of this risk should be diversifiable across movies produced during the course of the year.³

The second source of risk is competitive risk, whereby the earnings and cashflows on a project are affected (positively or negatively) by the actions

Competitive Risk: This is the unanticipated effect on the cashflows in a project of competitor actions - these effects can be positive or negative.

of competitors. While a good project analysis will build in the expected reactions of competitors into estimates of profit margins and growth, the actual actions taken by competitors may differ from these expectations. In most cases, this component of risk will affect more than one project, and is therefore more difficult to diversify away in the normal course of business by the firm. Disney, for instance, in its analysis of revenues from its theme parks division may err in its assessments of the strength and strategies of competitors like Universal Studios. While Disney cannot diversify away its competitive risk, stockholders in Disney can, if they are willing to hold stock in the competitors.⁴

The third source of risk is industry-specific risk — those factors that impact the earnings and cashflows of a specific industry. There are three sources of industry-specific risk. The first is *technology risk*, which reflects the effects of technologies that change or evolve in ways different from those expected when a project was originally analyzed. The second source is *legal risk*, which reflects the effect of changing laws and regulations.

The third is *commodity risk*, which reflects the effects of price changes in commodities and services that are used or produced disproportionately by a specific industry. Disney, for instance, in assessing the prospects of its broadcasting division

Industry-Specific Risk: These are unanticipated effects on project cashflows of industry-wide shifts in technology, changes in laws or in the price of a commodity.

(ABC) is likely to be exposed to all three risks; to technology risk, as the lines between television entertainment and the internet are increasingly blurred by companies like

³ To provide an illustration, Disney released *Treasure Planet*, an animated movie, in 2002, which cost \$140 million to make and resulted in a \$98 million write-off. A few months later, *Finding Nemo*, another animated Disney movie made hundreds of millions of dollars and became one of the biggest hits of 2003.

Microsoft, to legal risk, as the laws governing broadcasting change and to commodity risk, as the costs of making new television programs change over time. A firm cannot diversify away its industry-specific risk without diversifying across industries, either with new projects or through acquisitions. Stockholders in the firm should be able to diversify away industry-specific risk by holding portfolios of stocks from different industries.

The fourth source of risk is international risk. A firm faces this type of risk when it generates revenues or has costs outside its domestic market. In such cases, the earnings and cashflows will be affected by unexpected exchange rate movements or by political developments. Disney, for instance, is clearly exposed to this risk with its theme park in Hong Kong. Some of this risk may be diversified away by the firm in the normal course of business by investing in projects in different countries whose currencies may not all move in the same direction. McDonalds, for instance, operates in many different countries and should be able to diversify away some (though not all) of its exposure to international risk. Companies can also reduce their exposure to the exchange rate component of this risk by borrowing in the local currency to fund projects. Investors should be able to reduce their exposure to international risk by diversifying globally.

International Risk: This is the additional uncertainty created in cashflows of projects by unanticipated changes in exchange rates and by political risk in foreign markets.

The final source of risk is market risk: macroeconomic factors that affect essentially all companies and all projects, to varying degrees. For example, changes in interest rates will affect the value of projects already taken and those yet to be taken both directly, through the discount rates, and indirectly, through the cashflows. Other factors that affect all investments include the term structure (the difference between short and long term rates), the risk preferences of investors (as investors become more risk averse, more risky investments will lose value), inflation, and economic growth. While expected

Market Risk: Market risk refers to the unanticipated changes in project cashflows created by changes in interest rates, inflation rates and the economy that affect all firms, though to differing degrees.

⁴ Firms could conceivably diversify away competitive risk by acquiring their existing competitors. Doing so would expose them to attacks under the anti-trust law, however and would not eliminate the risk from as yet unannounced competitors.

values of all these variables enter into project analysis, unexpected changes in these variables will affect the values of these investments. Neither investors nor firms can diversify away this risk since all risky investments bear some exposure to this risk.



: 3.3. Risk is in the eyes of the beholder

A privately owned firm will generally end up with a higher discount rate for a project than would an otherwise similar publicly traded firm with diversified investors.

a. True

b. False

Does this provide a rationale for why a private firm may be acquired by a publicly traded firm?

Why Diversification Reduces or Eliminates Firm-Specific Risk

Why do we distinguish between the different types of risk? Risk that affect one of a few firms, i.e., firm specific risk, can be reduced or even eliminated by investors as they hold more diverse portfolios due to two reasons.

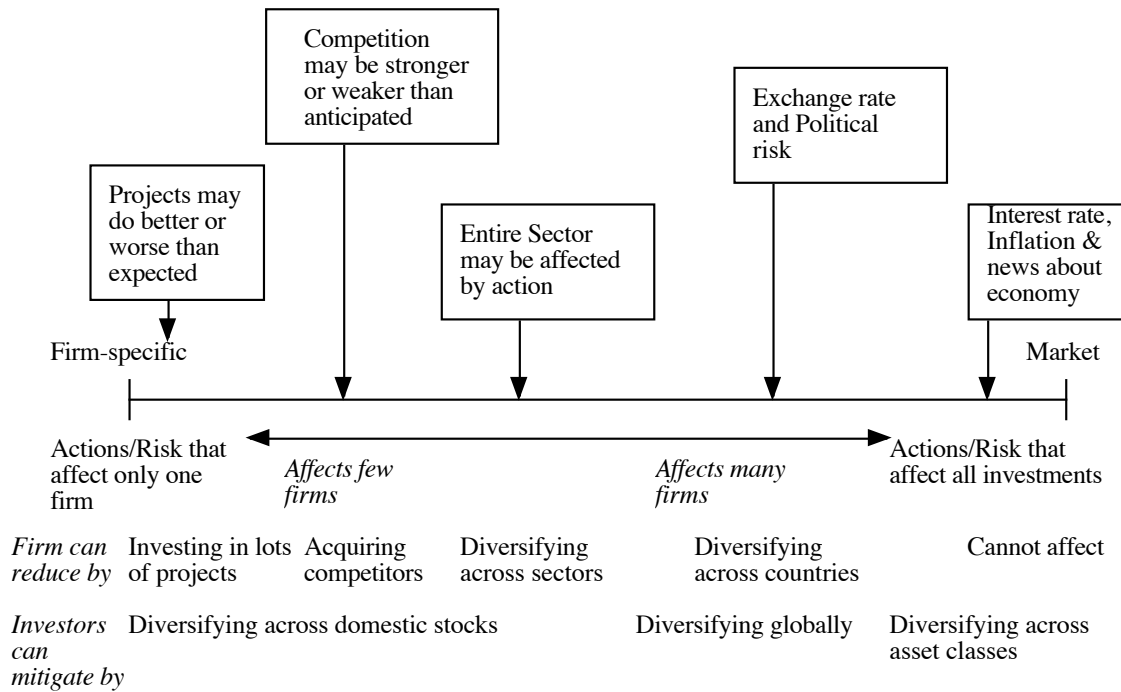
- The first is that each investment in a diversified portfolio is a much smaller percentage of that portfolio. Thus, any risk that increases or reduces the value of only that investment or a small group of investments will have only a small impact on the overall portfolio.

Diversification: This is the process of holding many investments in a portfolio, either across the same asset class (eg. stocks) or across asset classes (real estate, bonds etc.

- The second is that the effects of firm-specific actions on the prices of individual assets in a portfolio can be either positive or negative for each asset for any period. Thus, in large portfolios, it can be reasonably argued that this risk will average out to be zero and thus not impact the overall value of the portfolio.

In contrast, risk that affects most of all assets in the market will continue to persist even in large and diversified portfolios. For instance, other things being equal, an increase in interest rates will lower the values of most assets in a portfolio. Figure 3.5 summarizes the different components of risk and the actions that can be taken by the firm and its investors to reduce or eliminate this risk.

Figure 3.5: A Break Down of Risk



While the intuition for diversification reducing risk is simple, the benefits of diversification can also be shown statistically. In the last section, we introduced standard deviation as the measure of risk in an investment and calculated the standard deviation for an individual stock (Disney). When you combine two investments that do not move together in a portfolio, the standard deviation of that portfolio can be lower than the standard deviation of the individual stocks in the portfolio. To see how the magic of diversification works, consider a portfolio of two assets. Asset A has an expected return of μ_A and a variance in returns of σ_A^2 , while asset B has an expected return of μ_B and a variance in returns of σ_B^2 . The correlation in returns between the two assets, which measures how the assets move together, is ρ_{AB} .⁵ The expected returns and variance of a two-asset portfolio can be written as a function of these inputs and the proportion of the portfolio going to each asset.

$$\mu_{\text{portfolio}} = w_A \mu_A + (1 - w_A) \mu_B$$

$$\sigma_{\text{portfolio}}^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2 w_A w_B \rho_{AB} \sigma_A \sigma_B$$

where

w_A = Proportion of the portfolio in asset A

The last term in the variance formulation is sometimes written in terms of the covariance in returns between the two assets, which is

$$\sigma_{AB} = \rho_{AB} \sigma_A \sigma_B$$

The savings that accrue from diversification are a function of the correlation coefficient. Other things remaining equal, the higher the correlation in returns between the two assets, the smaller are the potential benefits from diversification. The following example illustrates the savings from diversification.



Under Diversification: A Behavioral Perspective

The argument that investors should diversify is impeccable, at least in a mean-variance world full of rational investors. The reality, though, is that most investors do not diversify. In one of the earliest studies of this phenomenon, Blume, Crockett and Friend (1974) examined the portfolios of individual investors and reported that 34% of the investors held only one dividend paying stock in their portfolios, 55% held between one and ten stocks and that only 11% held more than 10 stocks. While these investors could be granted the excuse that mutual funds were neither as prevalent nor as accessible as they are today, Goetzmann and Kumar looked at 60,000 investors at a discount brokerage house between 1991 and 1996 and conclude that there has been little improvement on the diversification front, and that the absence of diversification cannot be explained away easily with transactions costs.⁶ While some researchers have tried to find explanations within the conventional finance framework, behavioral economists offer three possible reasons:

- a. The Gambling instinct: One possible explanation is that investors construct their portfolios, as layered pyramids, with the bottom layer designed for downside protection and the top layer for risk seeking and upside potential. Investing in one or

⁵ The correlation is a number between -1 and $+1$. If the correlation is -1 , the two stocks move in lock step but in opposite directions. If the correlation is $+1$, the two stocks move together in synch.

⁶ Goetzmann, W.N. and A. Kumar, 2008, Equity Portfolio Diversification, Review of Finance. V12, 433-463. They find that 25% of investors hold only one stock and 50% of investors hold two or three stocks in their portfolios.

a few stocks in the top layer may not yield efficient risk taking portfolios, but they offer more upside. In a sense, these investments are closer to lottery tickets than to financial investments.⁷

- b. Over confidence: Goetzmann and Kumar note that investors who overweight specific industries or stock characteristics such as volatility tend to be less diversified than investors who hold wider portfolios. They argue that this is consistent with investors being over confident in their own abilities to find winners and thus not diversifying.
- c. Narrow framing and estimation biases: Investors who frame their investment decisions narrowly (looking at pieces of their portfolio rather than the whole) or misestimate correlations (by assuming that individual stocks are more highly correlated with each other than they really are) will hold less diversified portfolios.

In summary, many individual investors and some institutional investors seem to ignore the lessons of diversification and choose to hold narrow portfolios. Their perspective on risk may vary from more diversified investors in the same companies.

Illustration 3.2: Variance of a portfolio: Disney and Aracruz ADR

In illustration 3.1, we computed the average return and standard deviation of returns on Disney between January 2004 and December 2008. While Aracruz is a Brazilian stock, it has been listed and traded as an American Depositary Receipt (ADR) in the U.S. market over the same period.⁸ Using the same 60 months of data on Aracruz, we computed the average return and standard deviation on its returns over the same period:

	<i>Disney</i>	<i>Aracruz ADR</i>
Average Monthly Return	0.18%	-0.74%
Standard Deviation in Monthly Returns	5.59%	14.87%

Between 2004 and 2008, Disney generated higher returns than Aracruz, with lower volatility. With the benefit of hindsight, Disney would have been a much better

⁷ Shefrin, H. and M. Statman, 2000, Behavioral Portfolio Theory, Journal of Financial and Quantitative Analysis, v35, pp 127-151.

⁸ Like most foreign stocks, Aracruz has a listing for depository receipts or ADRs on the U.S. exchanges. Effectively, a bank buys shares of Aracruz in Brazil and issues dollar denominated shares in the United

investment than Aracruz, at least over this period. There are two points worth making. The first is that Aracruz generated an average monthly return of 2.47% from January 2004 to April 2008; the stock price dropped from an all time high of \$90.74 to \$8.39 between May and December of 2008. The second is that these returns were on the Aracruz ADR and thus in dollar terms. These returns are therefore affected both by the stock price performance of Aracruz (in Brazilian Reals(BR)) and the \$/BR exchange rate. A contributing factor to decline in the ADR price in the latter part of 2008 was the precipitous fall in the value of the BR, relative to the dollar.

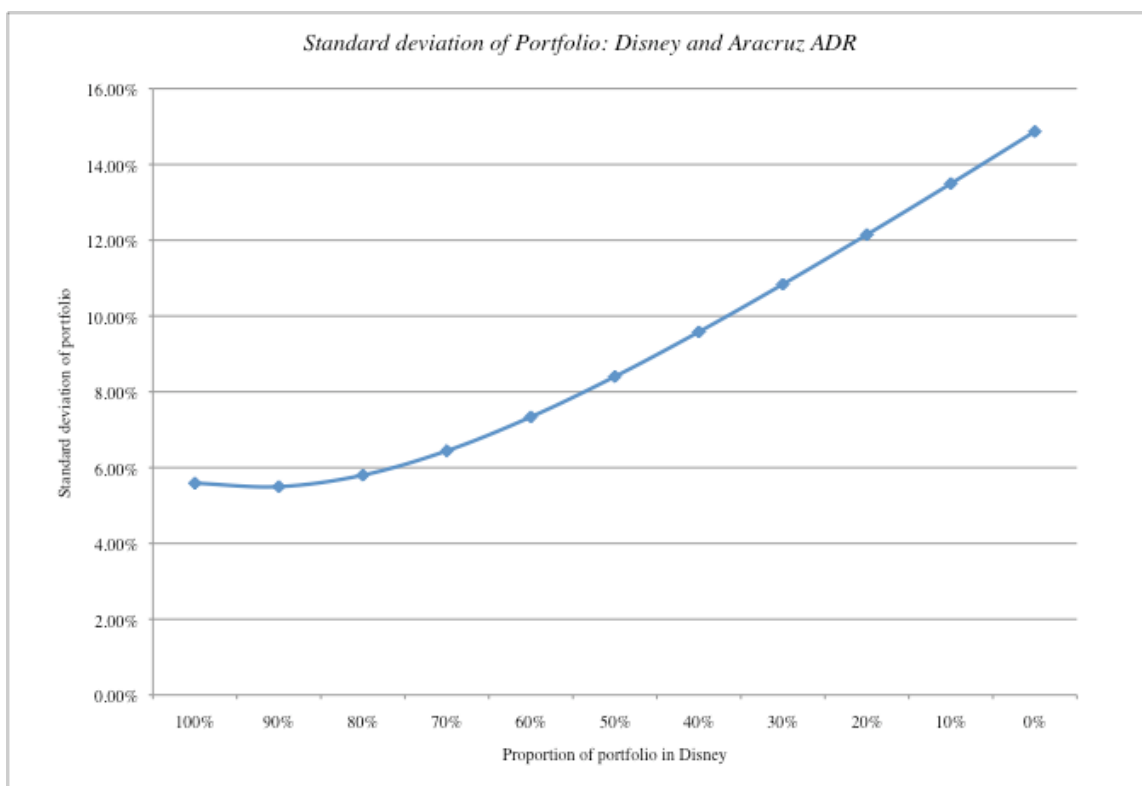
To examine how a combination of Disney and the Aracruz ADR would do as an investment, we computed the correlation between the two stocks over the 60-month period to be 0.1807. Consider now a portfolio that is invested 90% in Disney and 10% in the Aracruz ADR. The variance and the standard deviation of the portfolio can be computed as follows:

$$\begin{aligned}\text{Variance of portfolio} &= w_{\text{Dis}}^2 \sigma_{\text{Dis}}^2 + (1 - w_{\text{Dis}})^2 \sigma_{\text{Ara}}^2 + 2 w_{\text{Dis}} w_{\text{Ara}} \rho_{\text{Dis,Ara}} \sigma_{\text{Dis}} \sigma_{\text{Ara}} \\ &= (.9)^2(.0559)^2 + (.1)^2(.1487)^2 + 2 (.9)(.1)(.1807)(.0559)(.1487) \\ &= .003023\end{aligned}$$

$$\text{Standard Deviation of Portfolio} = \sqrt{.003023} = .0550 \text{ or } 5.50\%$$

The portfolio is less risky than either of the two stocks that go into it. In figure 3.6, we graph the standard deviation in the portfolio as a function of the proportion of the portfolio invested in Disney:

States to interested investors. Aracruz's ADR price tracks the price of the local listing while reflecting exchange rate changes.



As the proportion of the portfolio invested in Aracruz shifts towards 100%, the standard deviation of the portfolio converges on the standard deviation of Aracruz.

Aracruz's travails between May and December of 2008 also provide some insight into the essence of firm specific and market risk. The company had reported healthy profits from 2004 through 2007 but some of those profits came from managers speculating with derivatives (options and futures) that the Brazilian Real would continue to strengthen against the U.S. dollar. When the tide turned in 2008, and the Brazilian Real started weakening, the derivative bets made by the firm generated losses in excess of \$ 2 billion, which, in turn caused the drop in the stock price. The global market collapse in the last three months of the year accelerated the fall. The speculative losses from exchange rate bets are clearly firm specific risk but the losses accruing from the global crisis can be attributed to market risk.

Identifying the Marginal Investor

The marginal investor in a firm is the investor who is most likely to be trading at the margin and therefore has the most influence on the pricing of its equity. In some

cases, this may be a large institutional investor, but institutional investors themselves can differ in several ways. The institution may be a taxable mutual fund or a tax-exempt pension fund, may be domestically or internationally diversified, and vary on investment philosophy. In some cases, the marginal investors may be individuals, and here again there can be wide differences depending upon how diversified these individuals are, and what their investment objectives may be. In still other cases, the marginal investors may be insiders in the firm who own a significant portion of the equity of the firm and are involved in the management of the firm.

While it is difficult to identify the marginal investor in a firm, we would begin by breaking down the percent of the firm's stock held by individuals, institutions and insiders in the firm. This information, which is available widely for US stocks, can then be analyzed to yield the following conclusions:

- If the firm has relatively small institutional holdings but substantial holdings by wealthy individual investors, the marginal investor is *an individual investor with a significant equity holding* in the firm. In this case, we have to consider how diversified that individual investor's portfolio is to assess project risk. If the individual investor is not diversified, this firm may have to be treated like a private firm, and the cost of equity has to include a premium for all risk, rather than just non-diversifiable risk. If on the other hand, the individual investor is a wealthy individual with significant stakes in a large number of firms, a large portion of the risk may be diversifiable.
- If the firm has small institutional holdings and small insider holdings, its stock is held by large numbers of *individual investors with small equity holdings*. In this case, the marginal investor is an individual investor, with a portfolio that may be only partially diversified. For instance, phone and utility stocks in the United States, at least until recently, had holdings dispersed among thousands of individual investors, who held the stocks for their high dividends. This preference for dividends meant, however, that these investors diversified across only those sectors where firms paid high dividends.
- If the firm has significant institutional holdings and small insider holdings, the marginal investor is almost always a *diversified, institutional investor*. In fact, we can

learn more about what kind of institutional investor holds stock by examining the top 15 or 20 largest stockholders in the firms, and then categorizing them by tax status (mutual funds versus pension funds), investment objective (growth or value) and globalization (domestic versus international).

- If the firm has significant institutional holdings and large insider holdings, the choice for marginal investor becomes a little more complicated. Often, in these scenarios, the large insider is the founder or original owner for the firm, and often, this investor continues to be involved in the top management of firm. Microsoft and Dell are good examples, with Bill Gates and Michael Dell being the largest stockholders in the firms. In most of these cases, however, the insider owner seldom trades the stock, and his or her wealth is determined by the level of the stock price, which is determined by institutional investors trading the stock. We would argue that the institutional investor is the marginal investor in these firms as well, though *the leading stockholder will influence* the final decision.

Thus, by examining the percent of stock held by different groups, and the largest investors in a firm, we should have a sense of who the marginal investor in the firm is, and how best to assess and risk in corporate financial analysis.

Why do we care about the marginal investor? Since the marginal investors are assumed to set prices, their assessments of risk should govern how the rest of us think about risk. Thus, if the marginal investors are diversified institution, the only risk that they see in a company is the risk that they cannot diversify away and managers at the firm should be considering only that risk, when making investments. If the marginal investors are undiversified individuals, they will care about all risk in a company and the firm should therefore consider all risk, when making investments.

Illustration 3.3: Identifying the Marginal Investor

Who are the marginal investors in Disney, Aracruz, Tata Chemicals and Deutsche Bank? We begin to answer this question by examining whether insiders own a significant portion of the equity in the firm and are involved in the top management of the firm. Although no such investors exist at Deutsche Bank, there are significant insider holdings at the other three companies:

- While the shares held by the Disney family have dwindled to less than 1%, Disney's acquisition of Pixar has resulted in Steve Jobs becoming the largest single stockholder in the company, owning about 7% of the stock in the company.
- At Aracruz, the voting shares are held by the Votorantim Group (84%) and the Brazilian National Development Bank (BNDES), while the non-voting shares are held by a mix of institutional and individual investors.
- At Tata Chemicals, the Tata family control (even if they might not hold) a significant portion of the stock through other Tata companies in the group.

However, we do not believe that insiders represent the marginal investors at any of these companies because their holdings are static for two reasons. One is that their capacity to trade is restricted as insiders, especially in the case of Disney.⁹ The other is that trading may result in loss of the control they exercise over the firm, at least at Tata Chemicals and Aracruz. Consequently, we examine the proportion of stock held in each of the firms by individuals, insiders and institutions, in table 3.1.

Table 3.1: Investors in Disney, Aracruz, Deutsche Bank and Tata Chemicals

	<i>Disney</i>	<i>Deutsche Bank</i>	<i>Aracruz (non-voting)</i>	<i>Tata Chemicals</i>
Institutions	72%	76%	32%	47%
Individuals	21%	23%	60%	24%
Insiders	7%	1%	8%	29%*

Source: Value Line, Morningstar, Bloomberg.

All four companies are widely held by institutional investors, and foreign institutional investors hold significant portions of Aracruz and Tata Chemicals. In table 3.2, we examine the ten largest investors in each firm at the end of 2008 in Table 8.5, with the percent of the firm's stock held by each (in brackets).

Table 3.2: Largest Stockholders in Disney, Deutsche Bank and Tata Chemicals

<i>Disney</i>	<i>Deutsche Bank</i>	<i>Aracruz Preferred</i>	<i>-Tata Chemicals</i>
Steven Jobs (7.43%)	Deutsche Post (8.05%)	BB DTVM (0.89%)	Tata Sons (14.26%)
Fidelity (4.86%)	Allianz (6.81%)	Barclays(0.34%)	Life Insurance Co (11.71%)
State Street (3.97%)	AXA (4.64%)	Banco Itau (0.32%)	Tata Investment (6.8%)
Barclays (3.79%)	Credit Suisse (3.55%)	Banco Barclays (0.19%)	Tata Tea (6.54%)
Vanguard Group (3.07%)	Deutsche Bank	Vanguard Group (0.18%)	New India Assur.

⁹ Insider trading laws in the United States restrict insiders from trading on material information and also require filings of any trades that are made.

	(3.52%)		(2.58%)
Southeastern Asset (2.40%)	Barclays (3.02%)	UBS Strategy (0.17%)	Hindustan Lever (2.14%)
State Farm Mutual (2.27%)	Blackrock (2.35%)	Banco Itau (0.17%)	General Insurance (2.12%)
AXA (2.13%)	UBS (1.65%)	Dimensional Fund (0.10%)	United India Insur. (1.13%)
Wellington Mgmt (1.87%)	Deka (1.52%)	Banco Bradesco (0.09%)	National Insurance (1.01%)
Massachusetts Finl (1.57%)	Dekabank (1.44%)	Landesbank (0.08%)	Templeton Funds (1.01%)

Source: Bloomberg

Nine of the ten largest investors in Disney are institutional investors, suggesting that we are on safe grounds assuming that the marginal investor in Disney is likely to be both institutional and diversified. The two largest investors in Deutsche Bank are Allianz, the German insurance giant, and Deutsche Post, the privatized German postal company, reflecting again the cross-holding corporate governance structure favored by German corporations. However, the investors below Allianz are all institutional investors, and about half of them are non-German. Here again, we can safely assume that the marginal investor is likely to be institutional and broadly diversified across at least European equities rather than just German stocks. The common shares in Aracruz, where the voting rights reside, is held by a handful of controlling stockholders, but trading in this stock is light. The preferred shares are widely dispersed among a mix of domestic and international institutional investors. While there is a clear danger here that the company will be run for the benefit of the voting shareholders, the price of the voting stock is closely linked to the price of the preferred shares. Self-interest alone should induce the voting shareholders to consider the investors in the preferred shares as the marginal investors in the company. Finally, with Tata Chemicals, four of the ten largest investors are other Tata companies and those holdings are seldom traded. All of the remaining large investors are institutional investors, with about 12% of the stock held by foreign institutional investors.

In summary, then, we are on very safe ground with Disney and Deutsche Bank, when we assume that only the risk that cannot be diversified away should be considered when the company makes investments. We are on less secure ground with Aracruz and Tata Chemicals, because of the heavy influence of insiders, but we feel that institutional

investors exercise enough influence on how equity is priced at both firms for us to make the same assumption.

Why is the marginal investor assumed to be diversified?

The argument that investors can reduce their exposure to risk by diversifying can be easily made, but risk and return models in finance go further. They argue that the marginal investor, who sets prices for investments, is well diversified; thus, the only risk that will be priced in the risk as perceived by that investor. The justification that can be offered is a simple one. The risk in an investment will always be perceived to be higher for an undiversified investor than to a diversified one, since the latter does not consider any firm-specific risk while the former does. If both investors have the same perceptions about future earnings and cashflows on an asset, the diversified investor will be willing to pay a higher price for that asset because of his or her risk perceptions. Consequently, the asset, over time, will end up being held by diversified investors.

While this argument is a powerful one for stocks and other assets, which are traded in small units and are liquid, it is less so for investments that are large and illiquid. Real estate in most countries is still held by investors who are undiversified and have the bulk of their wealth tied up in these investments. The benefits of diversification are strong enough, however, that securities such as real estate investment trusts and mortgage-backed bonds were created to allow investors to invest in real estate and stay diversified at the same time.

Note that diversification does not require investors to give up their pursuit of higher returns. Investors can be diversified and try to beat the market at the same time. For instance, investors who believe that they can do better than the market by buying stocks trading at low PE ratios can still diversify by holding low PE stocks in a number of different sectors at the same time.



: 3.4. Management Quality and Risk

A well managed firm is less risky than a firm that is badly managed.

- a. True
- b. False

In Practice: Who should diversify? The Firm or Investors?

As we noted in the last section, the exposure to each type of risk can be mitigated by either the firm or by investors in the firm. The question of who should do it can be answered fairly easily by comparing the costs faced by each. As a general rule, a firm should embark on actions that reduce risk only if it is cheaper for it to do so than it is for its investors. With a publicly traded firm, it will usually be much cheaper for investors to diversify away risk than it is for the firm. Consider, for instance, risk that affects an entire sector. A firm can reduce its exposure to this risk by either acquiring other firms, paying large premiums over the market price, or by investing large amounts in businesses where it does not have any expertise. Investors in the firm, on the other hand, can accomplish the same by expanding their portfolios to include stocks in other sectors or even more simply by holding diversified mutual funds. Since the cost of diversifying for investors is very low, firms should try to diversify away risk only if the cost is minimal or if the risk reduction is a side benefit from an action with a different objective. One example would be project risk. Since Disney is in the business of making movies, the risk reduction that comes from making lots of movies is essentially costless.

The choice is more complicated for private businesses. The owners of these businesses often have the bulk of their wealth invested in these businesses and they can either try to take money out of the businesses and invest it elsewhere or they can diversify their businesses. In fact, many family businesses in Latin America and Asia became conglomerates as they expanded, partly because they wanted to spread their risks.

III. Measuring Market Risk

While most risk and return models in use in corporate finance agree on the first two steps of this process, i.e., that risk comes from the distribution of actual returns around the expected return and that risk should be measured from the perspective of a marginal investor who is well diversified, they part ways on how to measure the non-diversifiable or market risk. In this section, we will provide a sense of how each of the four basic models - the capital asset pricing model (CAPM), the arbitrage pricing model (APM) and the multi-factor model - approaches the issue of measuring market risk.

A. The Capital Asset Pricing Model

The risk and return model that has been in use the longest and is still the standard in most real world analyses is the capital asset pricing model (CAPM). While it has come in for its fair share of criticism over the years, it provides a useful starting point for our discussion of risk and return models.

1. Assumptions

While diversification has its attractions in terms of reducing the exposure of investors to firm specific risk, most investors limit their diversification to holding relatively few assets. Even large mutual funds are reluctant to hold more than a few hundred stocks, and many of them hold as few as 10 to 20 stocks. There are two reasons for this reluctance. The first is that the marginal benefits of diversification become smaller as the portfolio gets more diversified - the twenty-first asset added will generally provide a much smaller reduction in firm specific risk than the fifth asset added, and may not cover the marginal costs of diversification, which include transactions and monitoring costs. The second is that many investors (and funds) believe that they can find under valued assets and thus choose not to hold those assets that they believe to be correctly or over valued.

Riskless Asset: A riskless asset is one, where the actual return is always equal to the expected return.

The capital asset pricing model assumes that there are no transactions costs, all assets are traded and that investments are infinitely divisible (i.e., you can buy any fraction of a unit of the asset). It also assumes that there is no private information and that investors therefore cannot find under or over valued assets in the market place. By making these assumptions, it eliminates the factors that cause investors to stop diversifying. With these assumptions in place, the logical end limit of diversification is to hold every traded risky asset (stocks, bonds and real assets included) in your portfolio, in proportion to their market value¹⁰. This portfolio of every traded risky asset in the market place is called the market portfolio.

¹⁰ If investments are not held in proportion to their market value, investors are still losing some diversification benefits. Since there is no gain from over weighting some sectors and under weighting others in a market place where the odds are random of finding under valued and over valued assets, investors will not do so.

2. Implications for Investors

If every investor in the market holds the same market portfolio, how exactly do investors reflect their risk aversion in their investments? In the capital asset pricing model, investors adjust for their risk preferences in their allocation decisions, where they decide how much to invest in an asset with guaranteed returns – a riskless asset - and how much in risky assets (market portfolio). Investors who are risk averse might choose to put much or even all of their wealth in the riskless asset. Investors who want to take more risk will invest the bulk or even all of their wealth in the market portfolio. Those investors who invest all their wealth in the market portfolio and are still desirous of taking on more risk, would do so by borrowing at the riskless rate and investing in the same market portfolio as everyone else.

These results are predicated on two additional assumptions. First, there exists a riskless asset. Second, investors can lend and borrow at this riskless rate to arrive at their optimal allocations. There are variations of the CAPM that allow these assumptions to be relaxed and still arrive at conclusions that are consistent with the general model.



: 3.5. Efficient Risk Taking

In the capital asset pricing model, the most efficient way to take a lot of risk is to

- a. Buy a well-balanced portfolio of the riskiest stocks in the market
- b. Buy risky stocks that are also undervalued
- c. Borrow money and buy a well diversified portfolio

3. Measuring the Market Risk of an Individual Asset

The risk of any asset to an investor is the risk added on by that asset to the investor's overall portfolio. In the CAPM world, where all investors hold the market portfolio, the risk of an individual asset to an investor will be the risk that this asset adds on to the market portfolio. Intuitively, assets that move more with the market portfolio will tend to be riskier than assets that move less, since the movements that are unrelated to the market portfolio will not affect the overall value of the portfolio when an asset is added on to the portfolio. Statistically, this added risk is measured by the *covariance* of the asset with the market portfolio.

The covariance is a non-standardized measure of market risk; knowing that the covariance of Disney with the Market Portfolio is 55% does not provide a clue as to whether Disney is riskier or safer than the average asset. We therefore standardize the risk measure by dividing the covariance of each asset with the market portfolio by the variance of the market portfolio. This yields the beta of the asset:

$$\text{Beta of an asset } i = \frac{\text{Covariance of asset } i \text{ with Market Portfolio}}{\text{Variance of the Market Portfolio}}$$

Since the covariance of the market portfolio with itself is its variance, the beta of the market portfolio, and by extension, the average asset in it, is one. Assets that are riskier than average (using this measure of risk) will have betas that exceed one and assets that are safer than average will have betas that are lower than one. The riskless asset will have a beta of zero.

4. Getting Expected Returns

The fact that every investor holds some combination of the riskless asset and the market portfolio leads to the next conclusion, which is that the expected return on an asset is linearly related to the

Beta: The beta of any investment in the CAPM is a standardized measure of the risk that it adds to the market portfolio.

beta of the asset. In particular, the expected return on an asset can be written as a function of the risk-free rate and the beta of that asset;

Expected Return on asset i

$$= R_f + \beta_i [E(R_m) - R_f]$$

$$= \text{Risk-free rate} + \text{Beta of asset } i * (\text{Risk premium on market portfolio})$$

where,

$$E(R_i) = \text{Expected Return on asset } i$$

$$R_f = \text{Risk-free Rate}$$

$$E(R_m) = \text{Expected Return on market portfolio}$$

$$\beta_i = \text{Beta of asset } i$$

To use the capital asset pricing model, we need three inputs. While we will look at the estimation process in far more detail in the next chapter, each of these inputs is estimated as follows:

- The riskless asset is defined to be an asset where the investor knows the expected return with certainty for the time horizon of the analysis. Consequently, the riskless rate used will vary depending upon whether the time period for the expected return is one year, five years or ten years.
- The risk premium is the premium demanded by investors for investing in the market portfolio, which includes all risky assets in the market, instead of investing in a riskless asset. Thus, it does not relate to any individual risky asset but to risky assets as a class.
- The beta, which we defined to be the covariance of the asset divided by the market portfolio, is the only firm-specific input in this equation. In other words, the only reason two investments have different expected returns in the capital asset pricing model is because they have different betas.

In summary, in the capital asset pricing model all of the market risk is captured in one beta, measured relative to a market portfolio, which at least in theory should include all traded assets in the market place held in proportion to their market value.



: 3.6. What do negative betas mean?

In the capital asset pricing model, there are assets that can have betas that are less than zero. When this occurs, which of the following statements describes your investment?

- This investment will have an expected return less than the riskless rate
- This investment insures your “diversified portfolio” against some type of market risk
- Holding this asset makes sense only if you are well diversified
- All of the above

In Practice: Index Funds and Market Portfolios

Many critics of the capital asset pricing model seize on its conclusion that all investors in the market will hold the market portfolio, which includes all assets in proportion to their market value, as evidence that it is an unrealistic model. But is it? It is true that not all assets in the world are traded and that there are transactions costs. It is also true that investors sometimes trade on inside information and often hold undiversified portfolios. However, we can create portfolios that closely resemble the market portfolio using index funds. An index fund replicates an index by buying all of the

stocks in the index, in the same proportions that they form of the index. The earliest and still the largest one is the Vanguard 500 Index fund, which replicates the S&P 500 index. Today, we have access to index funds that replicate smaller companies in the United States, European stocks, Latin American markets and Asian equities as well as bond and commodity markets. An investor can create a portfolio composed of a mix of index funds – the weights on each fund should be based upon market values of the underlying market – which resembles the market portfolio; the only asset class that is usually difficult to replicate is real estate.

B. The Arbitrage Pricing Model

The restrictive assumptions in the capital asset pricing model and its dependence upon the market portfolio have for long been viewed with skepticism by both academics and practitioners. In the late seventies, an alternative and more general model for measuring risk called the arbitrage pricing model was developed.¹¹

1. Assumptions

The arbitrage pricing model is built on the simple premise that two investments with the same exposure to risk should be priced to earn the same

Arbitrage: An investment that requires no investment, involves no risk but still delivers a sure profit.

expected returns. An alternate way of saying this is that if two portfolios have the same exposure to risk but offer different expected returns, investors can buy the portfolio that has the higher expected returns and sell the one with lower expected returns, until the expected returns converge.

Like the capital asset pricing model, the arbitrage pricing model begins by breaking risk down into two components. The first is firm specific and covers information that affects primarily the firm. The second is the market risk that affects all investment; this would include unanticipated changes in a number of economic variables, including gross national product, inflation, and interest rates. Incorporating this into the return model above

¹¹ Ross, Stephen A., 1976, *The Arbitrage Theory Of Capital Asset Pricing*, Journal of Economic Theory, v13(3), 341-360.

$$R = E(R) + m + \varepsilon$$

where m is the market-wide component of unanticipated risk and ε is the firm-specific component.

2. The Sources of Market-Wide Risk

While both the capital asset pricing model and the arbitrage pricing model make a distinction between firm-specific and market-wide risk, they part ways when it comes to measuring the market risk. The CAPM assumes that all of the market risk is captured in the market portfolio, whereas the arbitrage pricing model allows for multiple sources of market-wide risk, and measures the sensitivity of investments to each source with what a factor betas. In general, the market component of unanticipated returns can be decomposed into economic factors:

$$\begin{aligned} R &= R + m + \varepsilon \\ &= R + (\beta_1 F_1 + \beta_2 F_2 + \dots + \beta_n F_n) + \varepsilon \end{aligned}$$

where

β_j = Sensitivity of investment to unanticipated changes in factor j

F_j = Unanticipated changes in factor j

3. The Effects of Diversification

The benefits of diversification have been discussed extensively in our treatment of the capital asset pricing model. The primary point of that discussion was that diversification of investments into portfolios eliminate firm-specific risk. The arbitrage pricing model makes the same point and concludes that the return on a portfolio will not have a firm-specific component of unanticipated returns. The return on a portfolio can then be written as the sum of two weighted averages -that of the anticipated returns in the portfolio and that of the factor betas:

$$\begin{aligned} R_p &= (w_1 R_1 + w_2 R_2 + \dots + w_n R_n) + (w_1 \beta_{1,1} + w_2 \beta_{1,2} + \dots + w_n \beta_{1,n}) F_1 + \\ &\quad (w_1 \beta_{2,1} + w_2 \beta_{2,2} + \dots + w_n \beta_{2,n}) F_2 \dots \end{aligned}$$

where,

w_j = Portfolio weight on asset j

R_j = Expected return on asset j

$\beta_{i,j}$ = Beta on factor i for asset j

Note that the firm specific component of returns (ϵ) in the individual firm equation disappears in the portfolio as a result of diversification.

4. Expected Returns and Betas

The fact that the beta of a portfolio is the weighted average of the betas of the assets in the portfolio, in conjunction with the absence of arbitrage, leads to the conclusion that expected returns should be linearly related to betas. To see why, assume that there is only one factor and that there are three portfolios. Portfolio A has a beta of 2.0, and an expected return on 20%; portfolio B has a beta of 1.0 and an expected return of 12%; and portfolio C has a beta of 1.5, and an expected return on 14%. Note that the investor can put half of his wealth in portfolio A and half in portfolio B and end up with a portfolio with a beta of 1.5 and an expected return of 16%. Consequently no investor will choose to hold portfolio C until the prices of assets in that portfolio drop and the expected return increases to 16%. Alternatively, an investor can buy the combination of portfolio A and B, with an expected return of 16%, and sell portfolio C with an expected return of 15%, and pure profit of 1% without taking any risk and investing any money. To prevent this “arbitrage” from occurring, the expected returns on every portfolio should be a linear function of the beta to prevent this. This argument can be extended to multiple factors, with the same results. Therefore, the expected return on an asset can be written as

$$E(R) = R_f + \beta_1 [E(R_1) - R_f] + \beta_2 [E(R_2) - R_f] + \dots + \beta_n [E(R_n) - R_f]$$

where

R_f = Expected return on a zero-beta portfolio

$E(R_j)$ = Expected return on a portfolio with a factor beta of 1 for factor j, and zero for all other factors.

The terms in the brackets can be considered to be risk premiums for each of the factors in the model.

Note that the capital asset pricing model can be considered to be a special case of the arbitrage pricing model, where there is only one economic factor driving market-wide returns and the market portfolio is the factor.

$$E(R) = R_f + \beta_m (E(R_m) - R_f)$$

5. The APM in Practice

The arbitrage pricing model requires estimates of each of the factor betas and factor risk premiums in addition to the riskless rate. In practice, these are usually estimated using historical data on stocks and a

Arbitrage:	An investment opportunity with no risk that earns a return higher than the riskless rate.
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statistical technique called factor analysis. Intuitively, a factor analysis examines the historical data looking for common patterns that affect broad groups of stocks (rather than just one sector or a few stocks). It provides two output measures:

1. It specifies the number of common factors that affected the historical data that it worked on.
2. It measures the beta of each investment relative to each of the common factors, and provides an estimate of the actual risk premium earned by each factor.

The factor analysis does not, however, identify the factors in economic terms.

In summary, in the arbitrage-pricing model the market or non-diversifiable risk in an investment is measured relative to multiple unspecified macro economic factors, with the sensitivity of the investment relative to each factor being measured by a factor beta. The number of factors, the factor betas and factor risk premiums can all be estimated using a factor analysis.

C. Multi-factor Models for risk and return

The arbitrage pricing model's failure to identify specifically the factors in the model may be a strength from a statistical standpoint, but it is a clear weakness from an intuitive standpoint. The solution seems simple: Replace the unidentified statistical factors with specified economic factors, and the resultant model should be intuitive while still retaining much of the strength of the arbitrage pricing model. That is precisely what multi-factor models do.

Deriving a Multi-Factor Model

Multi-factor models generally are not based on extensive economic rationale but are determined by the data. Once the number of factors has been identified in the arbitrage pricing model, the behavior of the factors over time can be extracted from the data. These factor time series can then be compared to the time series of macroeconomic

variables to see if any of the variables are correlated, over time, with the identified factors.

For instance, a study from the 1980s suggested that the following macroeconomic variables were highly correlated with the factors that come out of factor analysis: industrial production, changes in the premium

Unanticipated Inflation: This is the difference between actual inflation and expected inflation.

paid on corporate bonds over the riskless rate, shifts in the term structure, unanticipated inflation, and changes in the real rate of return.¹² These variables can then be correlated with returns to come up with a model of expected returns, with firm-specific betas calculated relative to each variable. The equation for expected returns will take the following form:

$$E(R) = R_f + \beta_{\text{GNP}} (E(R_{\text{GNP}}) - R_f) + \beta_i (E(R_i) - R_f) \dots + \beta_\delta (E(R_\delta) - R_f)$$

where

β_{GNP} = Beta relative to changes in industrial production

$E(R_{\text{GNP}})$ = Expected return on a portfolio with a beta of one on the industrial production factor, and zero on all other factors

β_i = Beta relative to changes in inflation

$E(R_i)$ = Expected return on a portfolio with a beta of one on the inflation factor, and zero on all other factors

The costs of going from the arbitrage pricing model to a macroeconomic multi-factor model can be traced directly to the errors that can be made in identifying the factors. The economic factors in the model can change over time, as will the risk premium associated with each one. For instance, oil price changes were a significant economic factor driving expected returns in the 1970s but are not as significant in other time periods. Using the wrong factor(s) or missing a significant factor in a multi-factor model can lead to inferior estimates of cost of equity.

In summary, multi factor models, like the arbitrage pricing model, assume that market risk can be captured best using multiple macro economic factors and estimating betas

¹² Chen, N., R. Roll and S.A. Ross, 1986, *Economic Forces and the Stock Market*, Journal of Business, 1986, v59, 383-404.

relative to each. Unlike the arbitrage pricing model, multi factor models do attempt to identify the macro economic factors that drive market risk.

D. Proxy Models

All of the models described so far begin by thinking about market risk in economic terms and then developing models that might best explain this market risk. All of them, however, extract their risk parameters by looking at historical data. There is a final class of risk and return models that start with past returns on individual stocks, and then work backwards by trying to explain differences in returns across long time periods using firm characteristics. In other words, these models try to find common characteristics shared by firms that have historically earned higher returns and identify these characteristics as proxies for market risk.

Book-to-Market Ratio: This is the ratio of the book value of equity to the market value of equity.

Fama and French, in a highly influential study of the capital asset pricing model in the early 1990s, note that actual returns over long time periods have been highly correlated with price/book value ratios and market capitalization.¹³ In particular, they note that firms with small market capitalization and low price to book ratios earned higher returns between 1963 and 1990. They suggest that these measures and similar ones developed from the data be used as proxies for risk and that the regression coefficients be used to estimate expected returns for investments. They report the following regression for monthly returns on stocks on the NYSE, using data from 1963 to 1990:

$$R_t = 1.77\% - 0.11 \ln (MV) + 0.35 \ln (BV/MV)$$

where

MV = Market Value of Equity

BV/MV = Book Value of Equity / Market Value of Equity

The values for market value of equity and book-price ratios for individual firms, when plugged into this regression, should yield expected monthly returns. For instance, a firm

¹³ Fama, E.F. and K.R. French, 1992, *The Cross-Section of Expected Returns*, Journal of Finance, v47, 427-466.

with a market value of \$ 100 million and a book to market ratio of 0.5 would have an expected monthly return of 1.02%.

$$R_t = 1.77\% - 0.11 \ln(100) + 0.35 \ln(0.5) = 1.02\%$$

As data on individual firms has become richer and more easily accessible in recent years, these proxy models have expanded to include additional variables. In particular, researchers have found that price momentum (the rate of increase in the stock price over recent months) also seems to help explain returns; stocks with high price momentum tend to have higher returns in following periods.

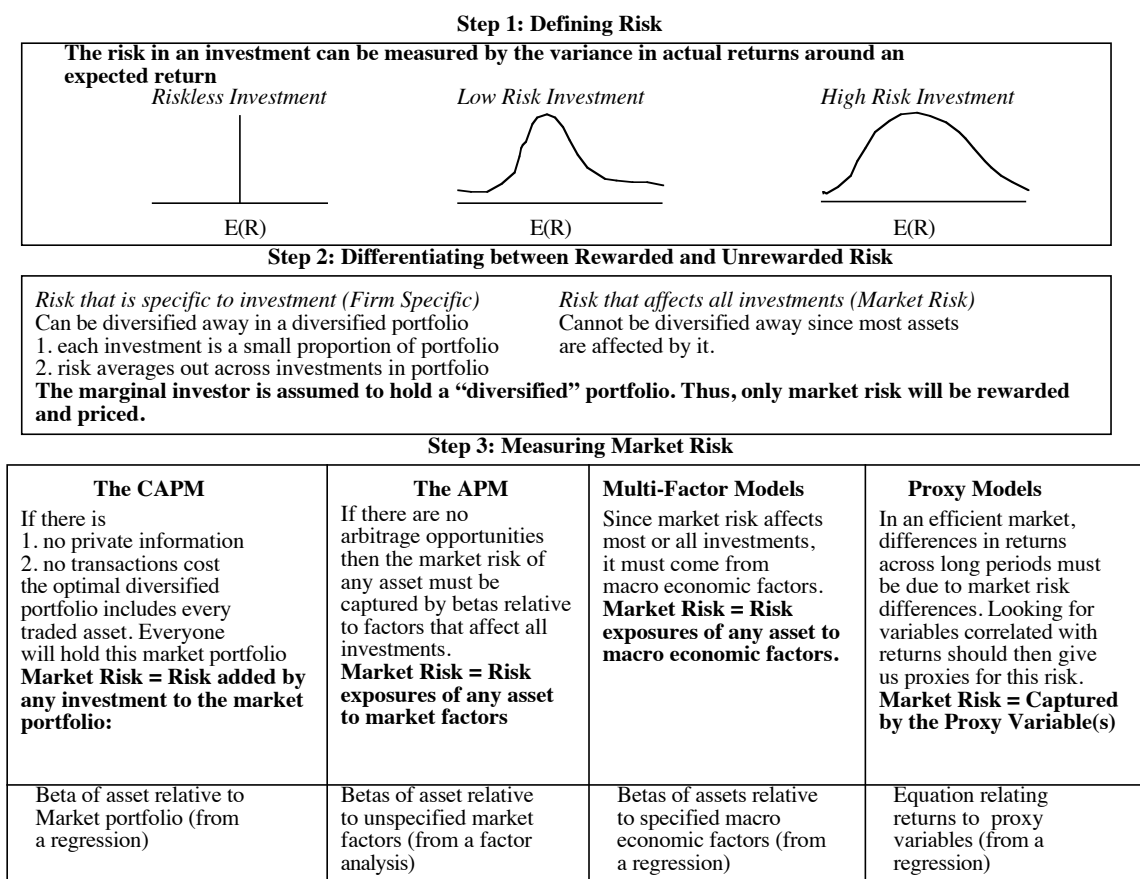
In summary, proxy models measure market risk using firm characteristics as proxies for market risk, rather than the macro economic variables used by conventional multi-factor models¹⁴. The firm characteristics are identified by looking at differences in returns across investments over very long time periods and correlating with identifiable characteristics of these investments.

A Comparative Analysis of Risk and Return Models

All the risk and return models developed in this chapter have common ingredients. They all assume that only market-wide risk is rewarded, and they derive the expected return as a function of measures of this risk. Figure 3.7 presents a comparison of the different models:

¹⁴ Adding to the confusion, researchers in recent years have taken to describing proxy models also as multi factor models.

Figure 3.7: Competing Models for Risk and Return in Finance



The capital asset pricing model makes the most assumptions but arrives at the simplest model, with only one risk factor requiring estimation. The arbitrage pricing model makes fewer assumptions but arrives at a more complicated model, at least in terms of the parameters that require estimation. In general, the CAPM has the advantage of being a simpler model to estimate and to use, but it will under perform the richer multi factor models when the company is sensitive to economic factors not well represented in the market index. For instance, oil companies, which derive most of their risk from oil price movements, tend to have low CAPM betas. Using a multi factor model, where one of the factors may be capturing oil and other commodity price movements, will yield a better estimate of risk and higher cost of equity for these firms¹⁵.

¹⁵ Weston, J.F. and T.E. Copeland, 1992, *Managerial Finance*, Dryden Press. They used both approaches to estimate the cost of equity for oil companies in 1989 and came up with 14.4% with the CAPM and 19.1% using the arbitrage pricing model.

The biggest intuitive block in using the arbitrage pricing model is its failure to identify specifically the factors driving expected returns. While this may preserve the flexibility of the model and reduce statistical problems in testing, it does make it difficult to understand what the beta coefficients for a firm mean and how they will change as the firm changes (or restructures).

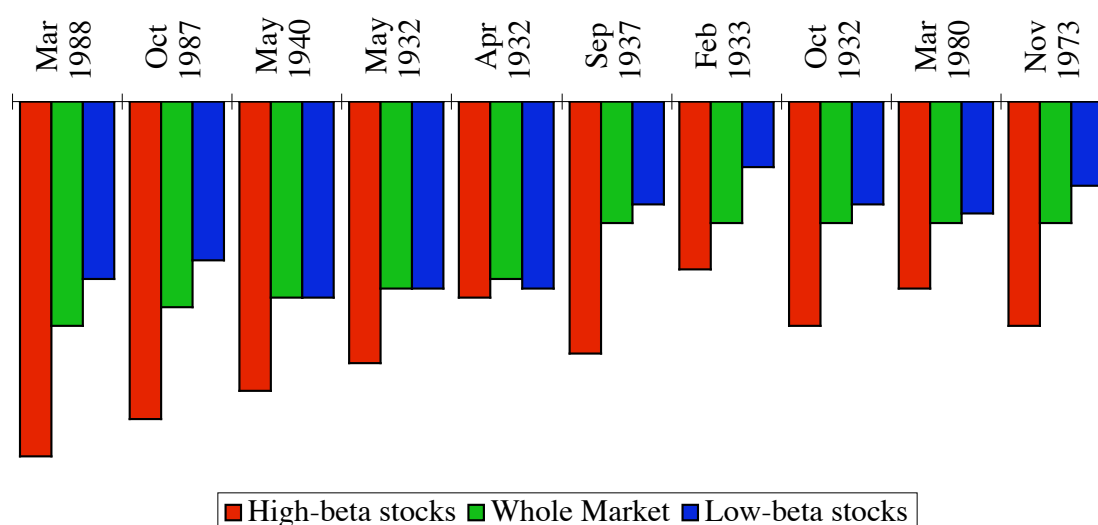
Does the CAPM work? Is beta a good proxy for risk, and is it correlated with expected returns? The answers to these questions have been debated widely in the last two decades. The first tests of the model suggested that betas and returns were positively related, though other measures of risk (such as variance) continued to explain differences in actual returns. This discrepancy was attributed to limitations in the testing techniques. In 1977, Roll, in a seminal critique of the model's tests, suggested that since the market portfolio (which should include every traded asset of the market) could never be observed, the CAPM could never be tested, and that all tests of the CAPM were therefore joint tests of both the model and the market portfolio used in the tests, i.e., all any test of the CAPM could show was that the model worked (or did not) given the proxy used for the market portfolio.¹⁶ He argued that in any empirical test that claimed to reject the CAPM, the rejection could be of the proxy used for the market portfolio rather than of the model itself. Roll noted that there was no way to ever prove that the CAPM worked, and thus, no empirical basis for using the model.

The study by Fama and French quoted in the last section examined the relationship between the betas of stocks and annual returns between 1963 and 1990 and concluded that there was little relationship between the two. They noted that market capitalization and book-to-market value explained differences in returns across firms much better than did beta and were better proxies for risk. These results have been contested on two fronts. First, Amihud, Christensen, and Mendelson, used the same data, performed different statistical tests, and showed that betas did, in fact, explain returns during the time period.¹⁷ Second, Chan and Lakonishok look at a much longer time series of returns from 1926 to 1991 and found that the positive relationship between betas and

¹⁶ Roll, R., 1977, A Critique of the Asset Pricing Theory's Tests: Part I: On Past and Potential Testability of Theory, *Journal of Financial Economics*, v4, 129-176.

returns broke down only in the period after 1982.¹⁸ They attribute this breakdown to indexing, which they argue has led the larger, lower-beta stocks in the S & P 500 to outperform smaller, higher-beta stocks. They also find that betas are a useful guide to risk in extreme market conditions, with the riskiest firms (the 10% with highest betas) performing far worse than the market as a whole, in the ten worst months for the market between 1926 and 1991 (See Figure 3.8).

Figure 3.8: Returns and Betas: Ten Worst Months between 1926 and 1991



While the initial tests of the APM and the multi-factor models suggested that they might provide more promise in terms of explaining differences in returns, a distinction has to be drawn between the use of these models to explain differences in past returns and their use to get expected returns for the future. The competitors to the CAPM clearly do a much better job at explaining past returns since they do not constrain themselves to one factor, as the CAPM does. This extension to multiple factors does become more of a problem when we try to project expected returns into the future, since the betas and premiums of each of these factors now have to be estimated. As the factor premiums and

¹⁷ Amihud, Y., B. Christensen and H. Mendelson, 1992, *Further Evidence on the Risk-Return Relationship*, Working Paper, New York University.

¹⁸ Chan, L.K. and J. Lakonishok, 1993, *Are the reports of Beta's death premature?*, Journal of Portfolio Management, v19, 51-62.

betas are themselves volatile, the estimation error may wipe out the benefits that could be gained by moving from the CAPM to more complex models. The regression models that were offered as an alternative are even more exposed to this problem, since the variables that work best as proxies for market risk in one period (such as size) may not be the ones that work in the next period. This may explain why multi-factor models have been accepted more widely in evaluating portfolio performance evaluation than in corporate finance; the former is focused on past returns whereas the latter is concerned with future expected returns.

Ultimately, the survival of the capital asset pricing model as the default model for risk in real world application is a testament both to its intuitive appeal and the failure of more complex models to deliver significant improvement in terms of expected returns. We would argue that a judicious use of the capital asset pricing model, without an over reliance on historical data, in conjunction with the accumulated evidence¹⁹ presented by those who have developed the alternatives to the CAPM, is still the most effective way of dealing with risk in modern corporate finance.

In Practice: Implied Costs of Equity and Capital

The controversy surrounding the assumptions made by each of the risk and return models outlined above and the errors that are associated with the estimates from each has led some analysts to use an alternate approach for companies that are publicly traded. With these companies, the market price represents the market's best estimate of the value of the company today. If you assume that the market is right and you are willing to make assumptions about expected growth in the future, you can back out a cost of equity from the current market price. For example, assume that a stock is trading at \$ 50 and that dividends next year are expected to be \$2.50. Furthermore, assume that dividends will grow 4% a year in perpetuity. The cost of equity implied in the stock price can be estimated as follows:

Stock price = \$ 50 = Expected dividends next year/ (Cost of equity – Expected growth rate)

$$\$ 50 = 2.50/(r - .04)$$

Solving for r , $r = 9\%$. This approach can be extended to the entire firm and to compute the cost of capital.

While this approach has the obvious benefit of being model free, it has its limitations. In particular, our cost of equity will be a function of our estimates of growth and cashflows. If we use overly optimistic estimates of expected growth and cashflows, we will under estimate the cost of equity. It is also built on the presumption that the market price is right.

The Risk in Borrowing: Default Risk and the Cost of Debt

When an investor lends to an individual or a firm, there is the possibility that the borrower may default on interest and principal payments on the borrowing. This possibility of default is called the default risk. Generally speaking, borrowers with higher default risk should pay higher interest rates on their borrowing than those with lower default risk. This section examines the measurement of default risk, and the relationship of default risk to interest rates on borrowing.

In contrast to the general risk and return models for equity, which evaluate the effects of market risk on expected returns, models of default risk measure the consequences of firm-specific default risk on promised returns. While diversification can be used to explain why firm-specific risk will not be priced into expected returns for equities, the same rationale cannot be applied to securities that have limited upside potential and much greater downside potential from firm-specific events. To see what we mean by limited upside potential, consider investing in the bond issued by a company. The coupons are fixed at the time of the issue, and these coupons represent the promised cash flow on the bond. The best-case scenario for you as an investor is that you receive the promised cash flows; you are not entitled to more than these cash flows even if the company is wildly successful. All other scenarios contain only bad news, though in varying degrees, with the delivered cash flows being less than the promised cash flows.

¹⁹ Barra, a leading beta estimation service, adjusts betas to reflect differences in fundamentals across firms (such as size and dividend yields). It is drawing on the regression studies that have found these to be good proxies for market risk.

Consequently, the expected return on a corporate bond is likely to reflect the firm-specific default risk of the firm issuing the bond.

The Determinants of Default Risk

The default risk of a firm is a function of its capacity to generate cashflows from operations and its financial obligations - including interest and principal payments.²⁰ It is also a function of the how liquid a firm's assets are since firms with more liquid assets should have an easier time liquidating them, in a crisis, to meet debt obligations. Consequently, the following propositions relate to default risk:

- Firms that generate high cashflows relative to their financial obligations have lower default risk than do firms that generate low cashflows relative to obligations. Thus, firms with significant current investments that generate high cashflows, will have lower default risk than will firms that do not.
- The more stable the cashflows, the lower is the default risk in the firm. Firms that operate in predictable and stable businesses will have lower default risk than will otherwise similar firms that operate in cyclical and/or volatile businesses, for the same level of indebtedness.
- The more liquid a firm's assets, for any given level of operating cashflows and financial obligations, the less default risk in the firm.

For as long as there have been borrowers, lenders have had to assess default risk. Historically, assessments of default risk have been based on financial ratios to measure the cashflow coverage (i.e., the magnitude of cashflows relative to obligations) and control for industry effects, to capture the variability in cashflows and the liquidity of assets.

Default Risk and Interest rates

When banks did much of the lending to firms, it made sense for banks to expend the resources to make their own assessments of default risk, and they still do for most

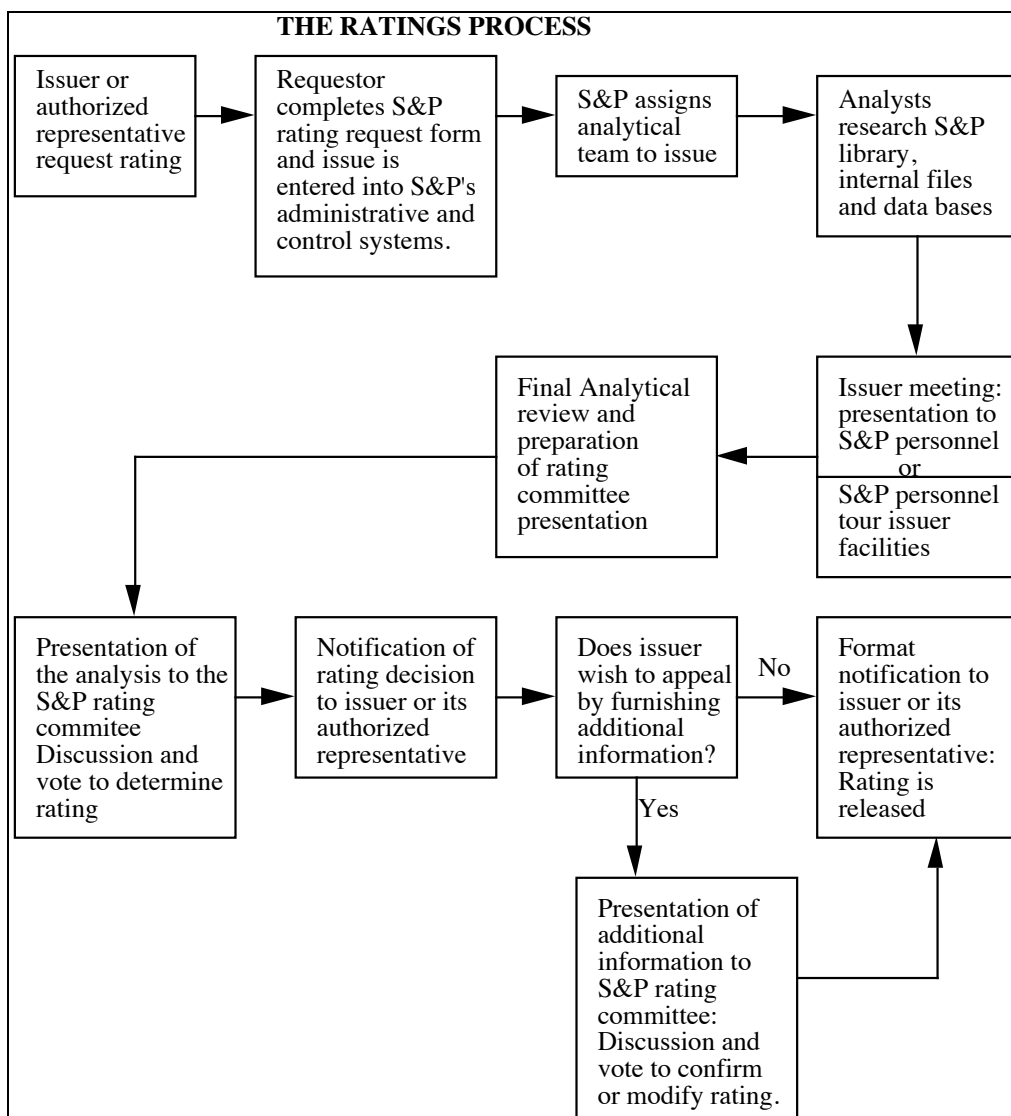
²⁰ Financial obligation refers to any payment that the firm has legally obligated itself to make, such as interest and principal payments. It does not include discretionary cashflows, such as dividend payments or new capital expenditures, which can be deferred or delayed, without legal consequences, though there may be economic consequences.

lenders. The advent of the corporate bond market created a demand for third party assessments of default risk on the part of bondholders. This demand came from the need for economies of scale, since few individual bondholders had the resources to make the assessment themselves. In the United States, this led to the growth of ratings agencies like Standard and Poor's and Moody's which made judgments of the default risk of corporations, using a mix of private and public information, converted these judgments into measures of default risk (bond rating) and made these ratings public. Investors buying corporate bonds could therefore use the bond ratings as a shorthand measure of default risk.

The Ratings Process

The process of rating a bond starts when a company requests a rating from the ratings agency. This request is usually precipitated by a desire on the part of the company to issue bonds. While ratings are not a legal pre-requisite for bond issues, it is unlikely that investors in the bond market will be willing to buy bonds issued by a company that is not well known and has shown itself to be unwilling to put itself through the rigor of a bond rating process. It is not surprising, therefore, that the largest number of rated companies are in the United States, which has the most active corporate bond markets, and that there are relatively few rated companies in Europe, where bank lending remains the norm for all but the largest companies.

The ratings agency then collects information from both publicly available data, such as financial statements, and the company itself, and makes a decision on the rating. If it disagrees with the rating, the company is given the opportunity to present additional information. This process is presented schematically for one ratings agency, Standard and Poor's (S&P), in Figure 3.9:



The ratings assigned by these agencies are letter ratings. A rating of AAA from Standard and Poor's and Aaa from Moody's represents the highest rating granted to firms that are viewed as having the lowest default risk. As the default risk increases, the ratings decrease toward D for firms in default (Standard and Poor's). Table 3.1 provides a description of the bond ratings assigned by the two agencies.

Table 3.1: Index of Bond Ratings

Standard and Poor's		Moody's	
AAA	The highest debt rating assigned. The borrower's capacity to repay debt is extremely strong.	Aaa	Judged to be of the best quality with a small degree of risk.
AA	Capacity to repay is strong and differs from the highest quality	Aa	High quality but rated lower than Aaa because margin of protection

	only by a small amount.		may not be as large or because there may be other elements of long-term risk.
A	Has strong capacity to repay; Borrower is susceptible to adverse effects of changes in circumstances and economic conditions.	A	Bonds possess favorable investment attributes but may be susceptible to risk in the future.
BBB	Has adequate capacity to repay, but adverse economic conditions or circumstances are more likely to lead to risk.	Baa	Neither highly protected nor poorly secured; adequate payment capacity.
BB,B, CCC, CC	Regarded as predominantly speculative, BB being the least speculative and CC the most.	Ba B	Judged to have some speculative risk. Generally lacking characteristics of a desirable investment; probability of payment small.
D	In default or with payments in arrears.	Caa Ca C	Poor standing and perhaps in default. Very speculative; often in default. Highly speculative; in default.

In Practice: Investment Grade and Junk Bonds

While ratings can range from AAA (safest) to D (in default), a rating at or above BBB by Standard and Poor's (Baa for Moody's) is categorized as investment grade, reflecting the view of the ratings agency that there is relatively little default risk in investing in bonds issued by these firms. Bonds rated below BBB are generally categorized as junk bonds or as high-yield bonds. While it is an arbitrary dividing line, it is an important one for two reasons. First, many investment portfolios are restricted from investing in bonds below investment grade. Thus, the market for investment grade bonds tends to be wider and deeper than that for bonds below that grade. Second, firms that are not rated investment grade have a tougher time when they try to raise new funding and they also pay much higher issuance costs when they do. In fact, until the early 1980s, firms below investment grade often could not issue new bonds.²¹ The perception that they are exposed to default risk also creates a host of other costs including tighter supplier credit and debt covenants.

²¹ In the early 1980s, Michael Milken and Drexel Burnham that created the junk bond market, allowing for original issuance of junk bonds. They did this primarily to facilitate hostile takeovers by the raiders of the era.

Determinants of Bond Ratings

The bond ratings assigned by ratings agencies are primarily based upon publicly available information, though private information conveyed by the firm to the rating agency does play a role. The rating that is assigned to a company's bonds will depend in large part on financial ratios that measure the capacity of the company to meet debt payments and generate stable and predictable cashflows. While a multitude of financial ratios exist, table 3.2 summarizes some of the key ratios that are used to measure default risk:

Table 3.2: Financial Ratios used to measure Default Risk

Ratio	Description
Pretax Interest Coverage	$= (\text{Pretax Income from Continuing Operations} + \text{Interest Expense}) / \text{Gross Interest}$
EBITDA Interest Coverage	$= \text{EBITDA} / \text{Gross Interest}$
Funds from Operations / Total Debt	$= (\text{Net Income from Continuing Operations} + \text{Depreciation}) / \text{Total Debt}$
Free Operating Cashflow/ Total Debt	$= (\text{Funds from Operations} - \text{Capital Expenditures} - \text{Change in Working Capital}) / \text{Total Debt}$
Pretax Return on Permanent Capital	$= (\text{Pretax Income from Continuing Operations} + \text{Interest Expense}) / (\text{Average of Beginning of the year and End of the year of long and short term debt, minority interest and Shareholders Equity})$
Operating Income/Sales (%)	$= (\text{Sales} - \text{COGS (before depreciation)} - \text{Selling Expenses} - \text{Administrative Expenses} - \text{R\&D Expenses}) / \text{Sales}$
Long Term Debt/ Capital	$= \text{Long Term Debt} / (\text{Long Term Debt} + \text{Equity})$
Total Debt/Capitalization	$= \text{Total Debt} / (\text{Total Debt} + \text{Equity})$

There is a strong relationship between the bond rating a company receives and its performance on these financial ratios. Table 3.3 provides a summary of the median ratios from 2006 to 2008 for different S&P ratings classes for manufacturing firms.

Table 3.3: Financial Ratios by Bond Rating: 2006-2008

	<i>AAA</i>	<i>AA</i>	<i>A</i>	<i>BBB</i>	<i>BB</i>	<i>B</i>	<i>CCC</i>
EBIT interest cov. (x)	17.5	10.8	6.8	3.9	2.3	1.0	0.2
EBITDA interest cov.	21.8	14.6	9.6	6.1	3.8	2.0	1.4
Funds flow/total debt	105.8	55.8	46.1	30.5	19.2	9.4	5.8

Free oper. cash flow/total debt (%)	55.4	24.6	15.6	6.6	1.9	-4.5	-14.0
Return on capital (%)	28.2	22.9	19.9	14.0	11.7	7.2	0.5
Oper.income/sales (%)	29.2	21.3	18.3	15.3	15.4	11.2	13.6
Long-term debt/capital (%)	15.2	26.4	32.5	41.0	55.8	70.7	80.3
Total Debt/ Capital (%)	26.9	35.6	40.1	47.4	61.3	74.6	89.4
Number of firms	10	34	150	234	276	240	23

Note that the pre-tax interest coverage ratio and the EBITDA interest coverage ratio are stated in terms of times interest earned, whereas the rest of the ratios are stated in percentage terms.

Not surprisingly, firms that generate income and cashflows that are significantly higher than debt payments that are profitable and that have low debt ratios are more likely to be highly rated than are firms that do not have these characteristics. There will be individual firms whose ratings are not consistent with their financial ratios, however, because the ratings agency does bring subjective judgments into the final mix. Thus, a firm that performs poorly on financial ratios but is expected to improve its performance dramatically over the next period may receive a higher rating than that justified by its current financials. For most firms, however, the financial ratios should provide a reasonable basis for guessing at the bond rating.



There is a dataset on the web that summarizes key financial ratios by bond rating class for the United States in the most recent period for which the data is available.

Bond Ratings and Interest Rates

The interest rate on a corporate bond should be a function of its default risk. If the rating is a good measure of the default risk, higher rated bonds should be priced to yield lower interest rates than would lower rated bonds. The difference between the interest rate on a bond with default risk and a default-free government bond is called the default spread. This default spread will vary by maturity of the bond and can also change from period to period, depending on economic conditions. Table 3.4 summarizes default

spreads in early 2009 for ten-year bonds in each ratings class (using S&P ratings) and the market interest rates on these bonds, based upon a treasury bond rate of 3.5%.

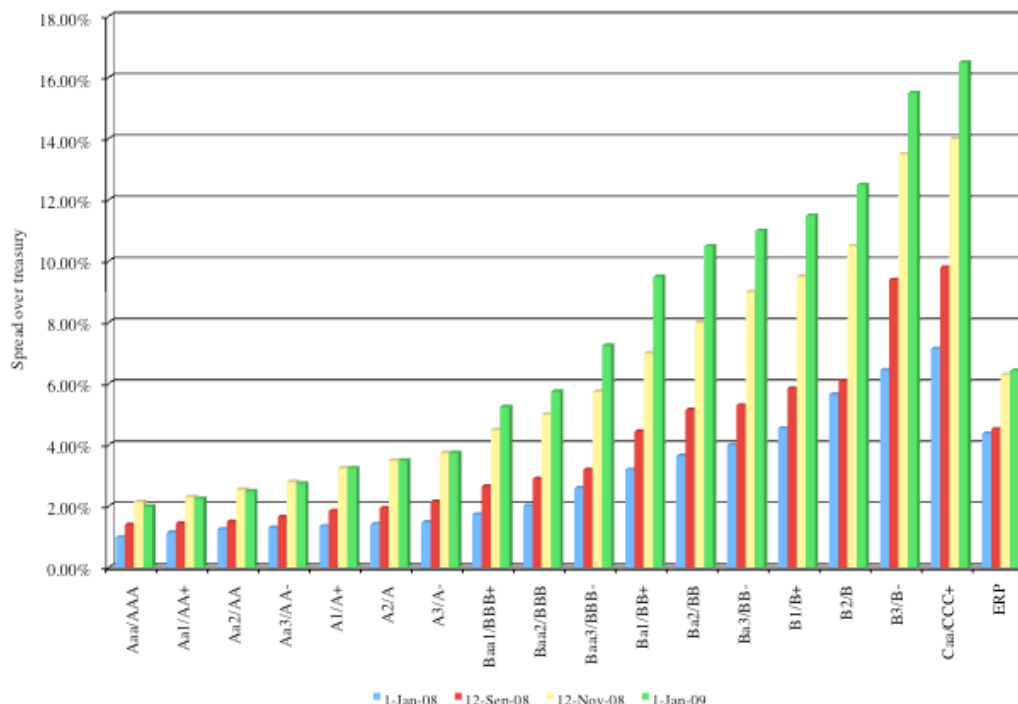
Table 3.4: Default Spreads for Ratings Classes: January 2009

<i>Rating</i>	<i>Default Spread</i>	<i>Interest rate on bond</i>
AAA	1.25%	4.75%
AA	1.75%	5.25%
A+	2.25%	5.75%
A	2.50%	6.00%
A-	3.00%	6.50%
BBB	3.50%	7.00%
BB	4.25%	7.75%
B+	5.00%	8.50%
B	6.00%	9.50%
B-	7.25%	10.75%
CCC	8.50%	12.00%
CC	10.00%	13.50%
C	12.00%	15.50%
D	15.00%	18.50%

Source: bondsonline.com

Table 3.4 provides default spreads at a point in time, but default spreads not only vary across time but they can vary for bonds with the same rating but different maturities. For the bonds with higher ratings, the default spread generally widens for the longer maturities. For bonds with lower ratings, the spreads may decrease as we go to longer maturities, reflecting the fact that near term default risk is greater than long term default risk. Historically, default spreads for every ratings class have increased during recessions and decreased during economic booms. In figure 3.10, we take a look at the evolution of default spreads for different bond rating classes through 2008:

Figure 3.10: Default Spreads on Ratings Classes



Note how much default spreads widened through 2008. The practical implication of this phenomenon is that default spreads for bonds have to be re-estimated at regular intervals, especially if the economy shifts from low to high growth or vice versa.

A final point worth making here is that everything that has been said about the relationship between interest rates and bond ratings could be said more generally about interest rates and default risk. The existence of ratings is a convenience that makes the assessment of default risk a little easier for us when analyzing companies. In its absence, we would still have to assess default risk on our own and come up with estimates of the default spread we would charge if we were lending to a firm.



ratings.xls: There is a dataset on the web that summarizes default spreads by bond rating class for the most recent period.

In Practice: Ratings Changes and Interest Rates

The rating assigned to a company can change at the discretion of the ratings agency. The change is usually triggered by a change in a firm's operating health, a new

security issue by the firm or by new borrowing. Other things remaining equal, ratings will drop if the operating performance deteriorates or if the firm borrows substantially more and improve if it reports better earnings or if it raises new equity. In either case, though, the ratings agency is reacting to news that the rest of the market also receives. In fact, ratings agencies deliberate before making ratings changes, often putting a firm on a credit watch list before changing its ratings. Since markets can react instantaneously, it should come as no surprise that bond prices often decline before a ratings drop and increase before a ratings increase. In fact, studies indicate that much of the bond price reaction to deteriorating credit quality precedes a ratings drop.

This does not mean that there is no information in a ratings change. When ratings are changed, the market still reacts but the reactions tend to be small. The biggest service provided by ratings agencies may be in providing a measure of default risk that is comparable across hundreds of rated firms, thus allowing bond investors a simple way of categorizing their potential investments.

Conclusion

Risk, as we define it in finance, is measured based upon deviations of actual returns on an investment from its' expected returns. There are two types of risk. The first, which we call equity risk, arises in investments where there are no promised cash flows, but there are expected cash flows. The second,, default risk, arises on investments with promised cash flows.

On investments with equity risk, the risk is best measured by looking at the variance of actual returns around the expected returns, with greater variance indicating greater risk. This risk can be broken down into risk that affects one or a few investments, which we call firm specific risk, and risk that affects many investments, which we refer to as market risk. When investors diversify, they can reduce their exposure to firm specific risk. By assuming that the investors who trade at the margin are well diversified, we conclude that the risk we should be looking at with equity investments is the market risk. The different models of equity risk introduced in this chapter share this objective of measuring market risk, but they differ in the way they do it. In the capital asset pricing model, exposure to market risk is measured by a market beta, which estimates how much

risk an individual investment will add to a portfolio that includes all traded assets. The arbitrage pricing model and the multi-factor model allow for multiple sources of market risk and estimate betas for an investment relative to each source. Regression or proxy models for risk look for firm characteristics, such as size, that have been correlated with high returns in the past and use these to measure market risk. In all these models, the risk measures are used to estimate the expected return on an equity investment. This expected return can be considered the cost of equity for a company.

On investments with default risk, risk is measured by the likelihood that the promised cash flows might not be delivered. Investments with higher default risk should have higher interest rates, and the premium that we demand over a riskless rate is the default premium. For most US companies, default risk is measured by rating agencies in the form of a company rating; these ratings determine, in large part, the interest rates at which these firms can borrow. Even in the absence of ratings, interest rates will include a default premium that reflects the lenders' assessments of default risk. These default-risk adjusted interest rates represent the cost of borrowing or debt for a business.

Problems and Questions

1. The following table lists the stock prices for Microsoft from 1989 to 1998. The company did not pay any dividends during the period

Year	Price
1989	\$ 1.20
1990	\$ 2.09
1991	\$ 4.64
1992	\$ 5.34
1993	\$ 5.05
1994	\$ 7.64
1995	\$ 10.97
1996	\$ 20.66
1997	\$ 32.31
1998	\$ 69.34

- a. Estimate the average annual return you would have made on your investment
- b. Estimate the standard deviation and variance in annual returns
- c. If you were investing in Microsoft today, would you expect the historical standard deviations and variances to continue to hold? Why or why not?

2. Unicom is a regulated utility serving Northern Illinois. The following table lists the stock prices and dividends on Unicom from 1989 to 1998.

Year	Price	Dividends
1989	\$ 36.10	\$ 3.00
1990	\$ 33.60	\$ 3.00
1991	\$ 37.80	\$ 3.00
1992	\$ 30.90	\$ 2.30
1993	\$ 26.80	\$ 1.60
1994	\$ 24.80	\$ 1.60
1995	\$ 31.60	\$ 1.60
1996	\$ 28.50	\$ 1.60
1997	\$ 24.25	\$ 1.60
1998	\$ 35.60	\$ 1.60

- a. Estimate the average annual return you would have made on your investment
- b. Estimate the standard deviation and variance in annual returns
- c. If you were investing in Unicom today, would you expect the historical standard deviations and variances to continue to hold? Why or why not?

3. The following table summarizes the annual returns you would have made on two companies – Scientific Atlanta, a satellite and data equipment manufacturer, and AT&T, the telecomm giant, from 1988 to 1998.

Year	Scientific Atlanta	AT&T
1989	80.95%	58.26%
1990	-47.37%	-33.79%
1991	31%	29.88%
1992	132.44%	30.35%
1993	32.02%	2.94%
1994	25.37%	-4.29%
1995	-28.57%	28.86%
1996	0.00%	-6.36%
1997	11.67%	48.64%
1998	36.19%	23.55%

- Estimate the average and standard deviation in annual returns in each company
 - Estimate the covariance and correlation in returns between the two companies
 - Estimate the variance of a portfolio composed, in equal parts, of the two investments
4. You are in a world where there are only two assets, gold and stocks. You are interested in investing your money in one, the other or both assets. Consequently you collect the following data on the returns on the two assets over the last six years.

	Gold	Stock Market
Average return	8%	20%
Standard deviation	25%	22%
Correlation	-.4	

- If you were constrained to pick just one, which one would you choose?
- A friend argues that this is wrong. He says that you are ignoring the big payoffs that you can get on gold. How would you go about alleviating his concern?
- How would a portfolio composed of equal proportions in gold and stocks do in terms of mean and variance?
- You now learn that GPEC (a cartel of gold-producing countries) is going to vary the amount of gold it produces with stock prices in the US. (GPEC will produce less gold when stock markets are up and more when it is down.) What effect will this have on your portfolios? Explain.

5. You are interested in creating a portfolio of two stocks – Coca Cola and Texas Utilities. Over the last decade, an investment in Coca Cola stock would have earned an average annual return of 25%, with a standard deviation in returns of 36%. An investment in Texas Utilities stock would have earned an average annual return of 12%, with a standard deviation of 22%. The correlation in returns across the two stocks is 0.28.

- a. Assuming that the average and standard deviation, estimated using past returns, will continue to hold in the future, estimate the average returns and standard deviation of a portfolio composed 60% of Coca Cola and 40% of Texas Utilities stock.
- b. Estimate the minimum variance portfolio.
- c. Now assume that Coca Cola's international diversification will reduce the correlation to 0.20, while increasing Coca Cola's standard deviation in returns to 45%. Assuming all of the other numbers remain unchanged, answer (a) and (b).

6. Assume that you have half your money invested in Times Mirror, the media company, and the other half invested in Unilever, the consumer product giant. The expected returns and standard deviations on the two investments are summarized below:

	Times Mirror	Unilever
Expected Return	14%	18%
Standard Deviation	25%	40%

Estimate the variance of the portfolio as a function of the correlation coefficient (Start with -1 and increase the correlation to $+1$ in 0.2 increments).

7. You have been asked to analyze the standard deviation of a portfolio composed of the following three assets:

Investment	Expected Return	Standard Deviation
Sony Corporation	11%	23%
Tesoro Petroleum	9%	27%
Storage Technology	16%	50%

You have also been provided with the correlations across these three investments:

	Sony	Tesoro	Storage Tech
Sony	1.00	-0.15	0.20
Tesoro	-0.15	1.00	-0.25

Storage Tech 0.20 -0.25 1.00

Estimate the variance of a portfolio, equally weighted across all three assets.

8. You have been asked to estimate a Markowitz portfolio across a universe of 1250 assets.

- a. How many expected returns and variances would you need to compute?
- b. How many covariances would you need to compute to obtain Markowitz portfolios?

9. Assume that the average variance of return for an individual security is 50 and that the average covariance is 10. What is the expected variance of a portfolio of 5, 10, 20, 50 and 100 securities. How many securities need to be held before the risk of a portfolio is only 10% more than the minimum?

10. Assume you have all your wealth (a million dollars) invested in the Vanguard 500 index fund, and that you expect to earn an annual return of 12%, with a standard deviation in returns of 25%. Since you have become more risk averse, you decide to shift \$ 200,000 from the Vanguard 500 index fund to treasury bills. The T.bill rate is 5%. Estimate the expected return and standard deviation of your new portfolio.

11. Every investor in the capital asset pricing model owns a combination of the market portfolio and a riskless asset. Assume that the standard deviation of the market portfolio is 30%, and that the expected return on the portfolio is 15%. What proportion of the following investor's wealth would you suggest investing in the market portfolio and what proportion in the riskless asset? (The riskless asset has an expected return of 5%)

- a. an investor who desires a portfolio with no standard deviation
- b. an investor who desires a portfolio with a standard deviation of 15%
- c. an investor who desires a portfolio with a standard deviation of 30%
- d. an investor who desires a portfolio with a standard deviation of 45%
- e. an investor who desires a portfolio with an expected return of 12%

12. The following table lists returns on the market portfolio and on Scientific Atlanta, each year from 1989 to 1998.

<i>Year</i>	<i>Scientific Atlanta</i>	<i>Market Portfolio</i>
1989	80.95%	31.49%
1990	-47.37%	-3.17%

1991	31%	30.57%
1992	132.44%	7.58%
1993	32.02%	10.36%
1994	25.37%	2.55%
1995	-28.57%	37.57%
1996	0.00%	22.68%
1997	11.67%	33.10%
1998	36.19%	28.32%

- Estimate the covariance in returns between Microsoft and the market portfolio
- Estimate the variances in returns on both investments
- Estimate the beta for Microsoft

13. United Airlines has a beta of 1.50. The standard deviation in the market portfolio is 22% and United Airlines has a standard deviation of 66%

- Estimate the correlation between United Airlines and the market portfolio.
- What proportion of United Airlines' risk is market risk?

14. You are using the arbitrage pricing model to estimate the expected return on Bethlehem Steel, and have derived the following estimates for the factor betas and risk premia:

Factor	Beta	Risk Premia
1	1.2	2.5%
2	0.6	1.5%
3	1.5	1.0%
4	2.2	0.8%
5	0.5	1.2%

- Which risk factor is Bethlehem Steel most exposed to? Is there any way, within the arbitrage pricing model, to identify the risk factor?
- If the riskfree rate is 5%, estimate the expected return on Bethlehem Steel
- Now assume that the beta in the capital asset pricing model for Bethlehem Steel is 1.1, and that the risk premium for the market portfolio is 5%. Estimate the expected return, using the CAPM.
- Why are the expected returns different using the two models?

15. You are using the multi-factor model to estimate the expected return on Emerson Electric, and have derived the following estimates for the factor betas and risk premia:

<i>Macro-economic Factor</i>	<i>Measure</i>	<i>Beta</i>	<i>Risk Premia ($R_{factor}-R_f$)</i>
Level of Interest rates	T.bond rate	0.5	1.8%
Term Structure	T.bond rate – T.bill rate	1.4	0.6%
Inflation rate	CPI	1.2	1.5%
Economic Growth	GNP Growth rate	1.8	4.2%

With a riskless rate of 6%, estimate the expected return on Emerson Electric.

16. The following equation is reproduced from the study by Fama and French of returns between 1963 and 1990.

$$R_t = .0177 - 0.11 \ln (MV) + 0.35 \ln (BV/MV)$$

where MV is the market value of equity in hundreds of millions of dollar and BV is the book value of equity in hundreds of millions of dollars. The return is a monthly return.

- Estimate the expected annual return on Lucent Technologies. The market value of equity is \$ 240 billion, and the book value of equity is \$ 13.5 billion.
- Lucent Technologies has a beta of 1.55. If the riskless rate is 6%, and the risk premium for the market portfolio is 5.5%, estimate the expected return.
- Why are the expected returns different under the two approaches?

Live Case Study

Stockholder Analysis

Objective: To find out who the average and marginal investors in the company are. This is relevant because risk and return models in finance assume that the marginal investor is well diversified.

Key Questions:

- Who is the average investor in this stock? (Individual or pension fund, taxable or tax-exempt, small or large, domestic or foreign)
- Who is the marginal investor in this stock?

Framework for Analysis

1. Who holds stock in this company?

- How many stockholders does the company have?
- What percent of the stock is held by institutional investors?
- Does the company have listings in foreign markets? (If you can, estimate the percent of the stock held by non-domestic investors)

2. Insider Holdings

- Who are the insiders in this company? (Besides the managers and directors, anyone with more than 5% is treated as an insider)
- What role do the insiders play in running the company?
- What percent of the stock is held by insiders in the company?
- What percent of the stock is held by employees overall? (Include the holdings by employee pension plans)
- Have insiders been buying or selling stock in this company in the most recent year?

Getting Information on Stockholder Composition

Information about insider and institutional ownership of firms is widely available since both groups have to file with the SEC. These SIC filings are used to develop rankings of the largest holders of stock in firms. Insider activity (buying and selling) is

also recorded by the SEC, though the information is not available until a few weeks after the filing.

Online sources of information:

<http://www.stern.nyu.edu/~adamodar/cfin2E/project/data.htm>

