

Chapter 6

Notation: PVA = PV of an annuity, APV: Annuity given PV. PV = Present value, given future value, FP = Future value, given present value, AFV= Annuity, given future value)

6-1

Project	Investment	NPV	PI	
A	\$25	\$10	0.40	
B	\$30	\$25	0.83	Accept
C	\$40	\$20	0.50	Accept
D	\$10	\$10	1.00	Accept
E	\$15	\$10	0.67	Accept
F	\$60	\$20	0.33	
G	\$20	\$10	0.50	Accept
H	\$25	\$20	0.80	Accept
I	\$35	\$10	0.29	
J	\$15	\$5	0.33	

b. Cost of Capital Rationing Constraint = NPV of rejected projects = \$45 million

6-2: Linear Programming Problem

Maximize

$$20X_1 + 20X_2 + 15X_3 + 20X_4 + 30X_5 + 10X_6 + 20X_7 + 35X_8 + 25X_9 + 10X_{10}$$

subject to

$$20X_1 + 25X_2 + 30X_3 + 15X_4 + 40X_5 + 10X_6 + 20X_7 + 30X_8 + 35X_9 + 25X_{10} \leq 100$$

$$10X_1 + 15X_2 + 30X_3 + 15X_4 + 25X_5 + 10X_6 + 15X_7 + 25X_8 + 25X_9 + 15X_{10} \leq 75$$

$X_1 \dots X_{10} = 0$ or 1 (ensures that fractions of projects cannot be taken.)

6-3

$$NPV(I) = -12,000 - 500/0.1 = -17,000$$

$$\text{Annualized Cost (I)} = -17,000 \times 0.1 = -1,700$$

Remember that this is a perpetuity: $PV = A/i$; $A = PV \times i$;

$$NPV(II) = -5,000 - 1,000(1 - (1.1)^{-20})/0.1 = -13,514 \quad \text{Annualized cost (II)} = -1,587$$

(Annuity, given PV = -\$13,514, n = 20 years and r = 10%)

$$NPV(III) = -3,500 - 1,200(1 - (1.1)^{-15})/0.1 = -12,627 \quad \text{Annualized cost(III)} = -1,660$$

(Annuity, given PV = -\$12,627, n = 15 years and r = 10%)

CHOOSE OPTION II (GAS HEATING SYSTEM)

6-4

$$NPV \text{ of Wood Siding} = -5,000 - 1,000 (PVA, 10, 10\%) = \$(11,145)$$

$$\text{Annualized Cost of Wood Siding} = -11,145 * (APV, 10, 10\%) = \$(1,813.63)$$

$$\text{Annualized Cost of Aluminum Siding investment} = -15,000 * .1 = -1,500 \text{ (Perpetuity)}$$

$$\text{Annual Maintenance Cost for Aluminum Siding} = 1,813.63 - 1,500 = 313.63$$

6-5

Annualized Cost for 1-year subscription = \$20.00
 Annualized Cost for 2-year subscription = \$ 36 (APV,20%,2) = \$23.56
 Annualized Cost for 3-year subscription = \$ 45 (APV,20%,3) = \$21.36
 Hence you should choose the 1-year subscription.

6-6

a. Initial investment = 10 million (Distribution system) + 1 million (WC) = 11 million
 (I am assuming that working capital investments have to be made at the start of each year. If they have to be made at the end of each year, the investment of \$ 1 million will be at the end of year 1.)

b.

Incremental Revenues	10,000,000	
Variable Costs (40%)	4,000,000	(No additional fixed costs)
Advertising Costs	1,000,000	
Depreciation	1,000,000	
Taxable Income	4,000,000	
Taxes	1,600,000	
After-tax Income	2,400,000	
+ Depreciation	1,000,000	
After-tax Cashflow	3,400,000	

c. NPV = -11,000,000 + 3,400,000 (PVA,10 years,8%) + 1,000,000 (PF, 10 years, 8%)
 = \$ 12,277,470

(The working capital is assumed to be salvaged at the end of year 10)

d. Precise Breakeven :

Let z be the incremental revenue

$$(-10,000,000 - 0.1z) + (0.6z - 1,000,000 - 1,000,000 - 0.4(.6z - 1,000,000 - 1,000,000) + 1,000,000)(PVA, 10yrs, 8\%) + .1z/1.08^{10} = 0$$

,Introducing the present value factors, PVA (\$1, 10%,8) = 6.71 and $1/1.08^{10} = 0.4632$

$$(-10,000,000 - 0.1z) + (0.6z - 1,000,000 - 1,000,000 - 0.4(.6z - 1,000,000 - 1,000,000) + 1,000,000)(6.71) + .1z/(0.4632) = 0$$

$z = 4,802,025$ or an increase 4.80% from initial level of 10%.

(Plug it back above and you should get a NPV of zero)

6-7

Incremental investment in new machine = Cost of new machine – Salvage value of old machine = 2,000,000 – 300,000 = \$1,700,000 or \$1.7 million

Depreciation is based upon book value. The existing machine has an annual depreciation tax advantage = $(500000/5) * 0.40 = 40,000$. The present value of this annuity equals

$$\frac{40000}{.1} \left(1 - \frac{1}{1.1^5} \right) = 151631.47$$

The new machine has an annual depreciation tax advantage = $(2000000/10) \cdot .4 = 80,000$.

The present value of this annuity equals $\frac{80000}{.1} \left(1 - \frac{1}{1.1^{10}}\right) = 491565.37$. Net Cost of the

New Machine = $-1,700,000 + 491,565 - 151,531 = \$1,360,066$.

Solving, for the annual savings that we would need each year for the next 10 years, we get

Annual Savings = $\$1,360,066$ (Annuity given PV, 10 years, 10%) = $\$221,344$

(I am assuming no capital gains taxes. If there are capital gains taxes, the initial investment will be reduced by the tax savings from capital losses from the sale of the old machine).

6-8

	1	2	3	4	5	
Revenues	\$15,000	\$15,750	\$16,538	\$17,364	\$18,233	
- Op. Exp.	\$7,500	\$7,875	\$8,269	\$8,682	\$9,116	
- Depreciation	\$8,000	\$8,000	\$8,000	\$8,000	\$8,000	
EBIT	\$(500)	\$(125)	\$269	\$682	\$1,116	
- Taxes	\$(200)	\$(50)	\$108	\$273	\$447	
EBIT (1-t)	\$(300)	\$(75)	\$161	\$409	\$670	
+ Depreciation	\$8,000	\$8,000	\$8,000	\$8,000	\$8,000	
ATCF	\$7,700	\$7,925	\$8,161	\$8,409	\$8,670	
PV at 12%	\$6,875	\$6,318	\$5,809	\$5,344	\$4,919	\$29,266

a. NPV = $-50,000 + \$29,266 + \$10,000/1.12^5 = \$(15,060)$

b. Present Value from Additional Book Sales

Year	Sales	Pre-tax Operating margin	After-tax operating margin
0			
1	20000	8000	4800
2	22000	8800	5280
3	24200	9680	5808
4	26620	10648	6388.8
5	29282	11712.8	7027.68
		NPV (@12%)	\$20,677

The present value of the cashflows accruing from the additional book sales equals $\$20,677$

c. The net effect is equal to $\$20,677 - \$15,060 = \$5,617$. Since it is positive the coffee shop should be opened.

6-9

NPV of less expensive lining = $-2000 - 80$ (PVA, 20%, 3 years) = $\$(2,169)$

Annualized Cost of less expensive lining = -2168.52 (APV, 20%, 3 years) = $\$(1,029)$

Key question: how long does the more exp. lining have to last to have an Annualized cost < -1029.45?

NPV of more expensive lining = $-4000 - 160 (PVA, 20\%, n \text{ years})$

Annualized cost of more expensive lining = $NPV (APV, 20\%, n \text{ years})$

Try different lifetimes. You will find that the annualized cost declines as you increase the lifetime and that it becomes lower than 1,029.45 at 14 years.

6-10

NPV(A) = $-50,000 - 9,000 (PVA, 8\%, 20 \text{ years}) + 10,000/1.08^{20} = \$(136,218)$

Annualized Cost(A) = $NPV(PVA, 8\%, 20 \text{ years}) = \$13,874$

NPV(B) = $-120,000 - 6,000(PVA, 8\%, 40 \text{ years}) + 20,000/1.08^{40} = \$(190,627)$

Annualized Cost(B) = $NPV(APV, 8\%, 40 \text{ years}) = \$15,986.$

Hence it is optimal to go with the first option.

6-11

NPV of Project A = $-5,000,000 + 2,500,000 (PVA, 10\%, 5) = \$4,476,967$

Equivalent Annuity for Project A = $4,476,967 (APV, 10\%, 5) = \$1,181,013$

NPV of Project B = $1,000,000 (PVA, 10\%, 10) + 2,000,000/1.1^{10} = \$6,915,654$

Equivalent Annuity for Project B = $6,915,654 (APV, 10\%, 10) = \$1,125,491$

NPV of Project C = $2,500,000/.1 - 10,000,000 - 5,000,000/1.1^{10} = \$13,072,284$

Equivalent Annuity for Project C = $13,072,284 * 0.1 = \$1,307,228.$

In this case, we'd go with project C, which has the highest equivalent annuity.

6-12

Annualized Cost of inexpensive machines = $- 2,000 (APV, 12\%, 3) - 150 = \(983)

Annualized Cost of expensive machines = $- 4,000(APV, 12\%, 5) - 50 = \$(1,160)$

I would pick the less expensive machine. They are cheaper on an annual basis.

6-13

Annualized Cost of spending \$400,000 right now = $\$400,000 (.10) = \$40,000$

(I am assuming that repaving and repairing the roads is a one-time cost. To the extent that is not true, I have to compute the annualized cost over whatever period the roads will last)

Maximum Additional Cost that the Town can bear = $\$100,000 - \$40,000 = \$60,000$

Annual expenditures will have to drop more than \$40,000 for the second option to be cheaper.

6-14

Initial Cost of First Strategy = \$10 million

Initial Cost of Second Strategy = \$40 million

Additional Initial Cost associated with Second Strategy = \$30 million

Additional Annual Cash Flow needed for Second Strategy to be viable:

= $\$30 \text{ million } (APV, 12\%, 15 \text{ years}) = \4.40 million.

Size of Market under First Strategy = $0.05 * \$200 \text{ million} = \10 million

Size of Market under Second Strategy = $0.10 * \$200 \text{ million} = \20 million

Additional Sales Associated with Second Strategy = \$10 million

After-tax Operating Margin needed to break even with second strategy = 44%

6-15

Project	Initial Investment	NPV	PI	IRR
I	10	3	0.30	21%
II	5	2.5	0.50	28%
III	15	4	0.27	19%
IV	10	4	0.40	24%
V	5	2	0.40	20%

- The PI would suggest that the firm invest in projects II, IV and V.
- Based on the IRR, I would still take projects II, IV and V. While project I has a higher IRR than project V, it breaks my budget constraint.
- The differences arise because of the reinvestment rate assumptions ; with the IRR, intermediate cash flows are reinvested at the IRR; with the PI, cash flows are reinvested at the cost of capital.

6-16

	Years 1- 10
ATCF : Store	10,000
- CF from Lost Sales	-1,200
Net ATCF	8,800

$$\text{NPV} = -50,000 + 8,800 (\text{PVA}, 14\%, 10 \text{ years}) = \$(4,098)$$

I would not open the store.

6-17

Initial Investment = - \$150,000

Annual Cash Flows from Baby-sitting Service

Additional Revenues \$1,000,000

$$\text{ATCF} = \$1,000,000 (.10) - \$ 60,000 = \$40,000$$

$$\text{NPV} = -150,000 + \$40,000 (\text{PVA}, 12\%, 10 \text{ years}) = \$76,010$$

(I am ignoring the tax benefits from the expenses, since the expenses will be tax deductible. With a 40% tax rate, the after-tax expense would be only \$36,000 and the NPV would be \$211,614.)

Yes. I would open the service.

6-18

$$\text{Total Cost of Buying Computers} = \$2,500 * 5,000 = \$12,500,000$$

$$\text{- PV of Salvage} = \$2,500,000 / 1.1^3 = \$1,878,287$$

$$\text{- PV of Depreciation} = \$3,333,333 * .4 * (\text{PVA}, 10\%, 3) = \$3,315,802$$

$$\text{Net Cost of Buying Computers} = \$7,305,911$$

$$\text{Annualized Cost of Buying Computers} = \$7,305,911 (\text{APV}, 10\%, 3) = \$2,937,815$$

$$\text{Annualized Cost of Leasing} = \$5,000,000 (1-.4) = \$3,000,000$$

It is slightly cheaper to buy the computers rather than lease them.

6-19

a. There is no cost the first three years. The after-tax salary paid in last two years is an opportunity cost = $80,000 \cdot 0.6 / 1.1^4 + 80,000 \cdot 0.6 / 1.1^5 = \$62,589$

b. The opportunity cost is the difference in PV of investing in year 4 instead of year 8 = $250,000 / 1.1^4 - 250,000 / 1.1^8 = \$54,126$

I am assuming that these are capital expenditures and that they are not tax deductible at the time of the expense. I am ignoring depreciation.

c. The present value of after-tax rental payments over five years is the opportunity cost = $(3000 \cdot 0.6)(PVA, 10\%, 5 \text{ years}) = \$6,823$

d. After-tax cash flow = $(400,000 - 160,000) - (240,000 - 100,000) \cdot 0.4 = \$184,000$

e. NPV = $-500,000 - 62,589 - 54,126 - 6,823 + 184,000(1 - (1.1)^{-5}) / .1 = \$73,967$

6-20 a.

Year	Old Product	New Product	Excess/Shortfall	
1	50	30	20	
2	52.5	33	14.5	
3	55.13	36.3	8.58	
4	57.88	39.93	2.19	
5	60.78	43.92	-4.7	OUT OF CAPACITY
6	63.81	48.32	-12.13	
7	67	53.15	-20.15	
8	70.36	58.46	-28.82	
9	73.87	64.31	-38.18	
10	77.57	70.74	-48.3	

b. Contribution margin for 1% of capacity for OLD = $(100 - 50) / 50 = \$1.00$
for NEW = $(80 - 44) / 30 = \$1.20$

You will lose less cutting back on old product.

Year	Lost Capacity	\$BT loss (m)	\$AT loss (m)	PV (loss)
5	-4.7	(\$4.70)	(\$2.82)	(\$1.75)
6	-12.13	(\$12.13)	(\$7.28)	(\$4.11)
7	-20.15	(\$20.15)	(\$12.09)	(\$6.20)
8	-28.82	(\$28.82)	(\$17.29)	(\$8.07)
9	-38.18	(\$38.18)	(\$22.91)	(\$9.72)
10	-48.3	(\$48.30)	(\$28.98)	(\$11.17)

Total opportunity cost = \$(41.02)

c. PV of Building facility in year 5 = \$31.05

PV of depreciation benefits on this building

$$= 2 \text{ million} * 0.4 * (\text{PVA}, 10\%, 25) * (\text{PF}, 10\%, 5) = \$4.51$$

Opportunity cost of building facility early = $-31.05 + 4.51 = -\$26.54$ million

I would rather build the facility than cut back on sales, since it has a lower PV of costs

6-21

Year	Potential			
	sales	Lost sales	Lost profits	PV lost profits
1	27,500	0	\$0	\$0
2	30,250	250	\$9,000	\$7,438
3	33,275	3,275	\$117,900	\$88,580
4	36,603	6,603	\$237,690	\$162,345
5	40,263	10,263	\$369,459	\$229,405
6	44,289	14,289	\$514,405	\$290,368
7	48,718	18,718	\$673,845	\$345,789
8	50,000	20,000	\$720,000	\$335,885
9	50,000	20,000	\$720,000	\$305,350
10	50,000	20,000	\$720,000	\$277,591

OPPORTUNITY COST \$2,042,753

This is based on the assumption that the production of tennis racquets will be cut back when you run out of capacity and that you lose \$36 per racquet not sold ($100 - 40 * .4$).