Chapter 3

3-1

a. Current Savings Needed = $\$500,000/1.1^{10}$ = $192,772

b. Solve the equation: $(x/0.1)[1-(1/1.1)^{10}]/(1.1)^{10} = \$500,000$ to get x = $31,372.76.

3-2

Present Value of $1,500 growing at 5% a year for next 15 years = $18,093

Future Value = $\$18,093\ (1.08^{15})$ = $57,394

3-3

Annual Percentage Rate = 8%

Monthly Rate = 8%/12 = 0.67%

Monthly Payment needed for 30 years is obtained as the solution to

$(x/0.0067)[1-(1/1.0067)^{360}] = \$200,000$. $x =$ $1,473

3-4

a. Discounted Price Deal

Monthly Cost of borrowing $18,000 at 9% APR = $373.65

[A monthly rate of 0.75% is used]

b. Special Financing Deal

Monthly Cost of borrowing $ 20,000 at 3% APR = $359.37

[A monthly rate of 0.25% is used]

The second deal is the better one.

3-5

a. Year-end Annuity Needed to have $100 million available in 10 years = $6.58 million

[FV = $ 100, r = 9%, n = 10 years]

b. Year-beginning Annuity Needed to have $100 million in 10 years = $6.04 million

3-6

Effective Annualized Interest Rate = $(1+.09/52)^{52} - 1$ = 9.41%

3-7

Annuity given current savings of $250,000 and n=25 = $17,738.11

3-8

PV of first annuity - $20,000 a year for next 10 years = $128,353.15

PV of second annuity discounted back 10 years = $81,326.64

Sum of the present values of the annuities = $209,679.79

If annuities are paid at the start of each period,

PV of first annuity - $20,000 at beginning of each year = $139,904.94

PV of second annuity discounted back 10 years = $88,646.04

Sum of the present values of the annuities = $228,550.97

3-9

PV of deficit reduction can be computed as follows –

Year	Deficit Reduction	PV
1	$25.00	$23.15
2	$30.00	$25.72
3	$35.00	$27.78
4	$40.00	$ 29.40
5	$45.00	$30.63
6	$55.00	$34.66
7	$60.00	$35.01
8	$65.00	$35.12
9	$70.00	$35.02
10	$75.00	$34.74
Sum	$500.00	$311.22

The true deficit reduction is $ 311.22 million.

3-10

a. Annuity needed at 6% = 1.8967 billions

b. Annuity needed at 8% = 1.7257 billions

Savings = 0.1710 billions

This cannot be viewed as real savings, since there will be greater risk associated with the higher-return investments.

3-11

a.

Year	**Nominal**	**PV**
0	$5.50	$5.50
1	$4.00	$3.74
2	$4.00	$3.49
3	$4.00	$3.27
4	$4.00	$3.05
5	$7.00	$4.99
Sum	**$28.50**	**$24.04**

b. Let the sign up bonus be reduced by X.

Then the cash flow in year 5 will have to be raised by X + 1.5 million to get the nominal value of the contract to be equal to $30 million. Since the present value cannot change,

$X - (X+1.5)/1.07^5 = 0$

$X (1.075^5 - 1) = 1.5$

$X = 1.5/ (1.075^5 -1) = \$3.73$ million

The sign up bonus has to be reduced by $3.73 million and the final year's cash flow has to be increased by $5.23 million, to arrive at a contract with a nominal value of $30 million and a present value of $24.04 million.

3-12

a.

	Chatham	South Orange
Mortgage:	$300,000	$200,000
Monthly Payment.	$2,201	$1,468
Annual Payments	$26,416	$17,610
Property Tax	$6,000	$12,000
Total Payment	$32,416	$29,610

b. Mortgage payments will end after 30 years. Property taxes are not only a perpetuity; they are a growing perpetuity. Therefore, they are likely to be more onerous.

c. If property taxes are expected to grow at 3% annually forever,
PV of property taxes = Property tax * (1 +g) / (r -g)
For Chatham, PV of property tax = $6,000 *1.03/(.08-.03) = $123,600
For South Orange, PV of property tax = $12,000 *1.03/(.08-.03) = $247,200
To make the comparison, add these to the house prices,
Cost of the Chatham house = $400,000 + $123,600 = $523,600
Cost of the South Orange house = $300,000 + $247,200 = $547,200
The Chatham house is cheaper.

3-13

a. Monthly Payments at 10% on current loan = $1,755.14 (Monthly rate used=(10/12)%)

b. Monthly Payments at 9% on refinanced mortgage = $1,609.25 (Monthly rate=0.75%)
Monthly Savings from refinancing = $145.90

c. Present Value of Savings at 8% for 60 months = $7,195.56 (Monthly rate = (8/12)%)
Refinancing Cost = 3% of $200,000 =$6,000
You would refinance.

d. Annual Savings needed to cover $ 6,000 in refinancing cost = $121.66
Monthly Payment with Savings = $1,755.14 - $121.66 = $1,633.48
Interest Rate at which Monthly Payment is $ 1,633.48 = 9.17%

3-14

a. Present Value of Cash Outflows after age 65 = $300,000 + PV of $30,000 each year for 35 years = $649,637.05

b. FV of Current Savings of $ 50,000 = $503,132.84
Shortfall at the end of the 30th year = $146,504.21
Annuity needed each year for next 30 years for FV of $ 146,504 = $1,293.26

c. Without the current savings,
Annuity needed each year for 25 years for FV of $ 649,637.05 = $8,886.24

3-15
Assuming a discount rate of 10 percent, the present value of what you can pay the player is the present value of an annuity of $1.5m. over 3 years, or $\frac{1.5}{0.1}\ 1-\frac{1}{(1.1)^3}$ = $3.7302 m.

If the number of years can be extended, then the nominal value of a contract with a present value of no more than $3.7302 m. can still be $5m. For example, if the number of years can be 6, an annual payment of *x* will result in a present value of $3.7302m., where *x* solves 3.7302 = $\frac{x}{0.1}\ 1-\frac{1}{(1.1)^6}$. Solving this, we find *x* = 0.8565m. The nominal value of the contract is, therefore, 6(0.8565m.) = $5.1389m. > $5m.

3-16
a. Estimated Funds at end of 10 years:
FV of $5 million at end of 10th year = $10.79 (in millions)
FV of inflows of $2 million each year for next 5 years = $17.24
FV of outflows of $3 million each year for years 6-10 = $17.60
Expected Balance at the end of the tenth year = $ 10.43 million

b. Perpetuity that can be paid out of these funds = $10.43 (.08) = $0.83 million

3-17
a. Amount needed in the bank to withdraw $ 80,000 each year for 25 years = $1,127,516

b. Future Value of Existing Savings in the Bank = $407,224
Shortfall in Savings = $1,127,516 - $407,224 = $720,292
Annual Savings needed to get FV of $720,292 = $57,267

c. If interest rates drop to 4% after the 10th year,
Annuity based upon interest rate of 4% and PV of $1,127,516 = $72,175
The decline in the amount of withdrawal = $80,000 - $72,175 = $ 7825

3-18
a. Value of Store = $100,000 (1.05)/(.10-.05) = $2,100,000

b. Growth rate needed to justify a value of $ 2.5 million,
Solving for g, 100,000(1+g)/(.10-g) = 2,500,000. We find g = 5.77%