

CHAPTER 27:

OPTION PRICING THEORY

27-1

- a. False. The reverse is true.
- b. True. Higher variance increases option value.
- c. True. Otherwise, arbitrage will be possible.
- d. False. Put-call parity can cut both ways.
- e. True. Dividends reduce the stock price.
- f. True.
- g. False. Some deep-in-the-money put options will be exercised early.
- h. True. The time premium decreases.
- i. True. The value of early exercise is more likely to overwhelm the time premium.
- j. False. It is the variance that matters, not the beta.

27-2

a.

At $t = 1$

if the stock price = \$70

Delta = 0.80

Borrowing = \$36.03

Option Value = \$19.96

if the stock price = \$35

Delta = 0.00

Borrowing = \$0.00

Option Value = \$0.00

At $t = 0$.

Delta = 0.5704

Borrowing = \$17.99

Option Value = \$10.53

Value of the Call ($K = 60, t = 2$) = \$10.53

b.

At $t = 1$,

if the stock price = \$70

Delta = -0.20

if the stock price = \$35

Delta = -1.00

Borrowing = -\$18.02
Option Value = \$4.02

Borrowing = -\$54.05
Option Value = \$19.05

At $t=0$,
Delta = -0.4296
Borrowing = \$30.72
Option Value = \$9.23
Value of the Put ($K = 60, t = 2$) = \$9.23

c. At $t = 0$, the call can be replicated by borrowing \$17.99 and buying 0.57 shares of stock.

At $t = 1$, the call can be replicated by borrowing \$36.04 and buying 0.8 shares of stock if the stock price goes to \$70, and by doing nothing if the stock price goes to \$35.

d. At $t = 0$, the put can be replicated by selling short 0.43 shares of stock and lending \$30.71.

At $t = 1$, the put can be replicated by selling short 0.2 shares of stock and lending \$18.02 if the stock price goes to \$70 and by selling short one share of stock and lending \$54.05 if the stock price goes to \$35.

27-3

a. The values of the option parameters are as follows:

$S = \$83$

$K = \$85$

$t = 0.25$

$r = 3.80\%$

Variance = 0.09

Value of call = \$4.42

b. To replicate this call, you would have to:

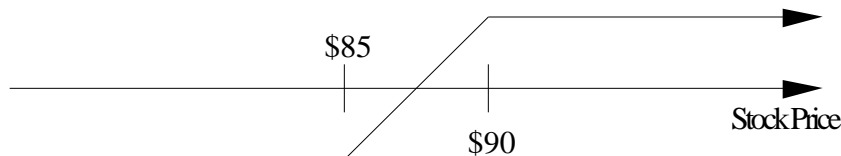
Buy 0.4919 Shares of Stock (this is $N(d1)$ from the model) and

Borrow $K e^{-rt} N(d2) = 85 \exp^{-(0.038)(0.25)} (0.4324) = \36.40

c. At an implied variance of 0.075, the call has a value of approximately \$4.00 (the market price).

Implied Standard Deviation = $\text{Sqrt}(0.075) = 0.27$

d.



e.

Value of Three-month Put = $C - S + Ke^{-rt} = \$4.42 - \$83 + 85 \exp^{-(0.038)(0.25)} = \5.62

27-4

- a.
- $S = \$28.75$
 - $K = \$30$
 - $t = 0.25$
 - $r = 3.60\%$
 - $s^2 = 0.04$
 - PV of Expected Dividends = $\$0.28/(1.036)^{2/12} = \0.28
 - Value of Call = $\$0.64$

b. The payment of a dividend reduces the expected stock price, and hence reduces the value of calls and increases the value of puts.

27-5

- a. First value the three-month call, as above:

Value of Call = $\$0.64$

Then, value a call to the first (and only) dividend payment,

- $S = \$28.75$
- $K = \$30$
- $t = 2/12$
- $r = 3.60\%$
- $s^2 = 0.04$
- $y = 0$ (since it assumes exercise before the dividend payment)
- Value of Call = $\$0.51$

Since the value of the three-month call is higher, there is no anticipated exercise.

b. If the dividend payment is large enough, it may pay to exercise the call just before the ex-dividend day (before the stock price drops) rather than wait until expiration. This early exercise is more likely for call options:

- (a) the larger the dividend on the stock, and
- (b) the closer the option is to expiration.

27-6

- a. You would need to borrow $Ke^{-rt} N(d2) = 90 \exp(-0.04)(0.25) (0.4500) = \40.10

b. You would need to buy 0.575 shares of stock.

27-7

- a.
- $S = \$4.00$
 - $K = \$4.25$
 - $r = 5\%$
 - $t = 1$
 - Variance = 0.36
 - Value of Warrant = $\$0.93$

b. Adjusted Stock Price = (Stock Price * Number of Shares Outstanding) +

$(\text{Warrant price} * \text{Number of Warrants Outstanding}) / (\text{Number of Shares} + \text{Number of Warrants})$

$$= (\$4.00 * 11,000,000 + \$0.93 * 550,000) / (11,550,000) = \$3.85$$

(To avoid the circular reasoning problem, the price from the no-dilution case is used.)

Adjusted Exercise Price = \$4.25

$r = 5\%$

$t = 1$

Variance = 0.36

Value of Warrant = \$0.80

(If you are using a spreadsheet with iterations turned on, and are feeding the option prices back to calculate the adjusted stock price, the value of the warrants is still \$0.80.)

c. Dilution increases the number of shares outstanding. For any given value of equity, each share is worth less.

27-8

a. $S = 250$
 $K = 275$
 $t = 5$
 $r = 5\%$
 $s^2 = (0.15)^2$
 $y = 0.03$
Value of call = \$29.09

b. Value of put with same parameters = \$28.09

c.

(1) The variance will be unchanged for the life of the option. This is likely to be violated because stock price variances do change substantially over time.

(2) There will be no early exercise. This is reasonable and is unlikely to be violated.

(3) Any deviations from the option value will be arbitrated away.

While there are plenty of arbitrageurs eager to exploit deviations from true value, arbitraging an index is clearly more difficult to do than arbitraging an individual stock.

27-9

New Security = AT & T stock - Call ($K = 60$) + Put ($K = 45$) = \$50 - \$2.35 + \$3.55 = \$51.20

The call with a strike price of \$60 is sold, eliminating upside potential above \$60.

The put with a strike price of \$45 is bought, providing downside protection.

27-10

The option values increase when the volatility increases because options have limited downside (the price of the option) and unlimited upside. Thus, volatility always works in the favor of an option buyer.

27-11

Buying a put would allow you to lock in the profits, at least for the life of the put. This will allow you to sell the stock back at the exercise price.

27-12

Calls have unlimited upside, since stock prices, at least in theory, could go to infinity. Puts, on the other hand, have a maximum profit, since stock prices cannot drop below zero.

27-13

These positions are referred to as spreads. These positions are designed to let people speculate on the direction of stock prices, while minimizing losses. Alternatively, these positions can be used to take advantage of option mispricing.

27-14

Less time left on the option translates into less time for the price of the underlying asset to move up (in the case of calls) and down (in the case of a put). This, in turn, will lower the value of the option.

27-15

If the tax benefit from exercising (i.e. the premium originally paid on the call can be claimed as a capital loss) exceeds the time premium on the option, it may pay to exercise.

27-16

Assume:

K = Strike prices of both the call and the put

S = Stock price

C = call premium

P = put premium

Then, the straddle is profitable if $S > K + C + P$ or $S < K - C - P$.

I would use this strategy if I believe that the market would have wide swings toward either direction.

27-17

$d_1 = [\ln(20/25) + (.0625 - .04 + .2 / 2) * 1] / \text{Sqrt}(.2*1) = -0.2251$, $N(d_1) = 0.4110$

$$d_2 = -0.2251 - \text{Sqrt}(.2*1) = -0.6723, N(d_2) = 0.2507$$

$$\text{The value of the call option} = 20 e^{-0.04} 0.4110 - 25 e^{-0.0625} 0.2507 = \$2.01$$

27-18

$$d_1 = [\ln(61/61) + (.0625 + .15 / 2) * 0.5] / \text{Sqrt}(.15*0.5) = 0.2510, N(d_1) = 0.5991$$

$$d_2 = 0.2510 - \text{Sqrt}(.15*0.5) = -0.0228, N(d_2) = 0.4909$$

$$\text{The value of the put option} = 61*(0.5991 - 1) - 61e^{-0.0625*0.5} (0.4909 - 1) = \$5.64$$

$$\text{The profit per share} = 61 - 24 - 5.64 = \$31.36$$

27-19

$$\text{The price of a call} = 61*0.5991 - 61e^{-0.0625*0.5} *0.4909 = \$7.52$$

$$\text{Now the profit per share} = 61 + 7.52 - 24 - 5.64 = \$38.88$$

Selling the call increases the risk of this strategy.