

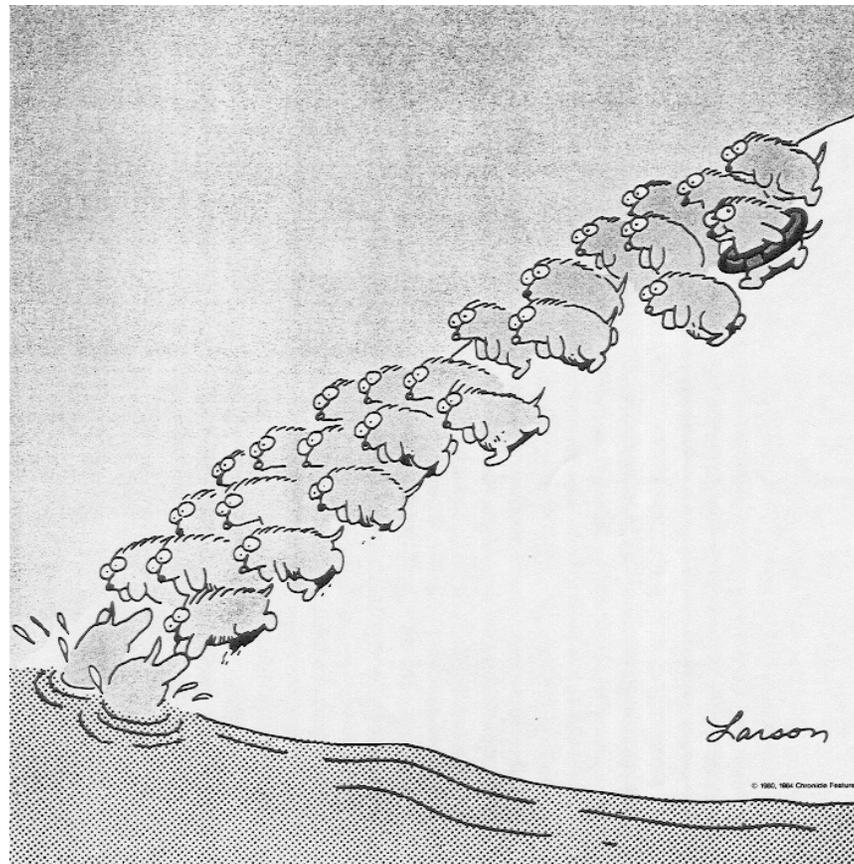
Valuation

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Some Initial Thoughts

" One hundred thousand lemmings cannot be wrong"

Graffiti



Misconceptions about Valuation

- Myth 1: A valuation is an objective search for “true” value
 - Truth 1.1: All valuations are biased. The only questions are how much and in which direction.
 - Truth 1.2: The direction and magnitude of the bias in your valuation is directly proportional to who pays you and how much you are paid.
- Myth 2.: A good valuation provides a precise estimate of value
 - Truth 2.1: There are no precise valuations
 - Truth 2.2: The payoff to valuation is greatest when valuation is least precise.
- Myth 3: . The more quantitative a model, the better the valuation
 - Truth 3.1: One’s understanding of a valuation model is inversely proportional to the number of inputs required for the model.
 - Truth 3.2: Simpler valuation models do much better than complex ones.

Approaches to Valuation

- **Discounted cashflow valuation**, relates the value of an asset to the present value of expected future cashflows on that asset.
- **Relative valuation**, estimates the value of an asset by looking at the pricing of 'comparable' assets relative to a common variable like earnings, cashflows, book value or sales.
- **Contingent claim valuation**, uses option pricing models to measure the value of assets that share option characteristics.

Discounted Cash Flow Valuation

- **What is it:** In discounted cash flow valuation, the value of an asset is the present value of the expected cash flows on the asset.
- **Philosophical Basis:** Every asset has an intrinsic value that can be estimated, based upon its characteristics in terms of cash flows, growth and risk.
- **Information Needed:** To use discounted cash flow valuation, you need
 - to estimate the life of the asset
 - to estimate the cash flows during the life of the asset
 - to estimate the discount rate to apply to these cash flows to get present value
- **Market Inefficiency:** Markets are assumed to make mistakes in pricing assets across time, and are assumed to correct themselves over time, as new information comes out about assets.

Valuing a Firm

- The value of the firm is obtained by discounting expected cashflows to the firm, i.e., the residual cashflows after meeting all operating expenses and taxes, but prior to debt payments, at the weighted average cost of capital, which is the cost of the different components of financing used by the firm, weighted by their market value proportions.

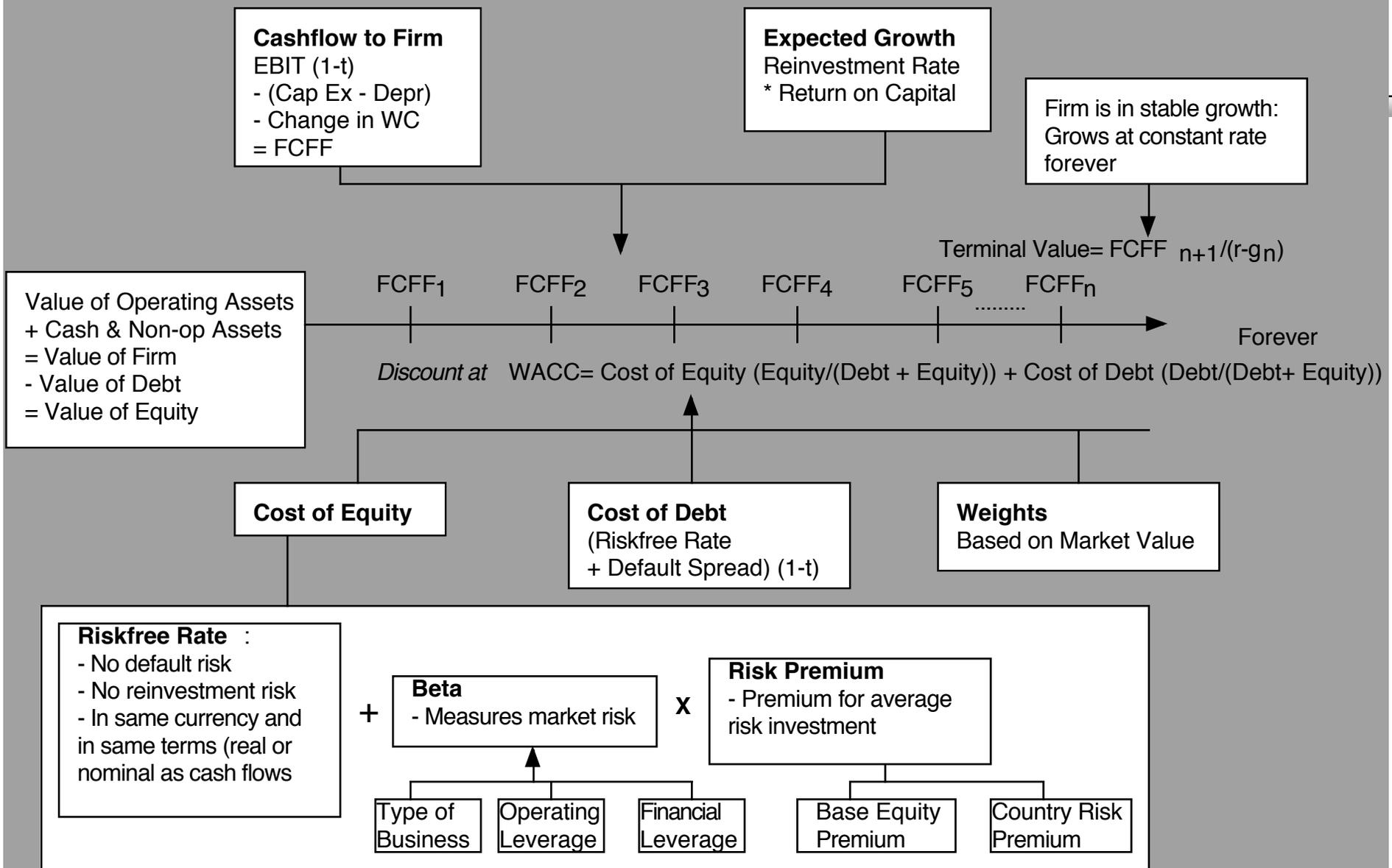
$$\text{Value of Firm} = \sum_{t=1}^{t=n} \frac{\text{CF to Firm}_t}{(1 + \text{WACC})^t}$$

where,

CF to Firm_t = Expected Cashflow to Firm in period t

WACC = Weighted Average Cost of Capital

DISCOUNTED CASHFLOW VALUATION



Titan Cements: Status Quo

Avg Reinvestment rate = 25.59%

Reinvestment Rate 25.59%

Return on Capital 18.42%

Current Cashflow to Firm

EBIT(1-t) : 154
 - Nt CpX 29
 - Chg WC 10
 = FCFF 115
 Reinvestment Rate = 39/115=25%

Expected Growth in EBIT (1-t)
 $.2559 * .1842 = .0471$
4.71%

Stable Growth
 $g = 4.31%$; Beta = 1.00;
 Country Premium = 0%
 Cost of capital = 7.20%
 ROC = 7.20%; Tax rate = 33%
 Reinvestment Rate = 59.85%

Terminal Value₅ = $78.4 / (.072 - .0431) = 2713$

Op. Assets 2,428
 + Cash: 81
 - Debt 396
 - Minor. Int. 101
 = Equity 2,011
 - Options 0
 Value/Share 52.68

EBIT	€ 232.47	€ 243.42	€ 254.90	€ 266.92	€ 279.50
EBIT(1-t)	€ 161.70	€ 169.33	€ 177.31	€ 185.67	€ 194.42
- Reinvestment	€ 41.39	€ 43.34	€ 45.38	€ 47.52	€ 49.76
= FCFF	€ 120.32	€ 125.99	€ 131.93	€ 138.15	€ 144.66

Term Yr
 291.5
 195.3
 116.9
 78.4

Discount at Cost of Capital (WACC) = $8.53% (.782) + 3.50% (0.218) = 7.43%$

Cost of Equity **8.53%**

Cost of Debt
 $(4.31% + .5% + .22%)(1 - .3044)$
 = 3.50%

Weights
 E = 78.2% D = 21.8%

Riskfree Rate:
 Euro riskfree rate = 4.31%

+

Beta
 0.96

x

Risk Premium
 4.44%

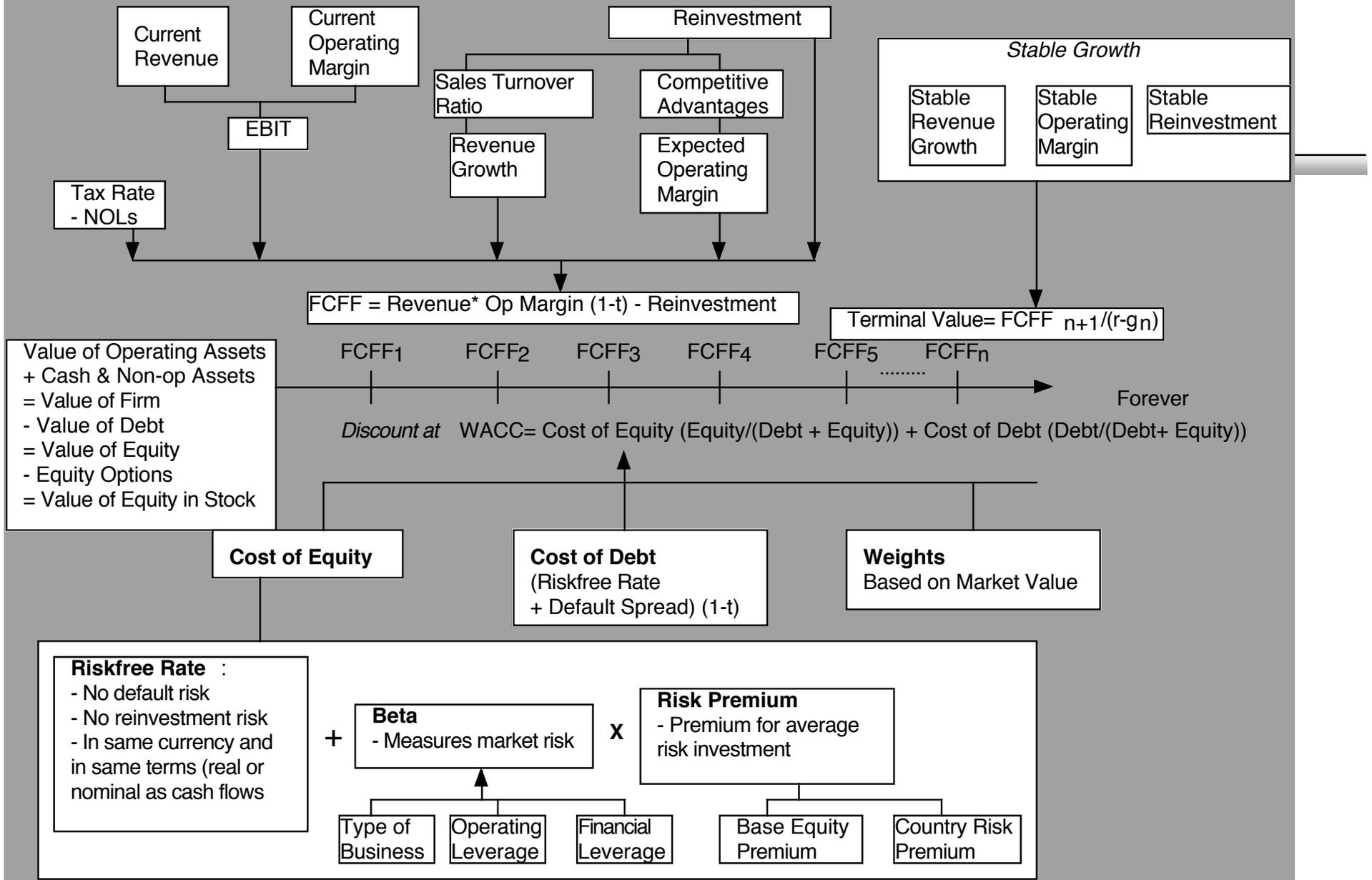
Unlevered Beta for Sectors: 0.80

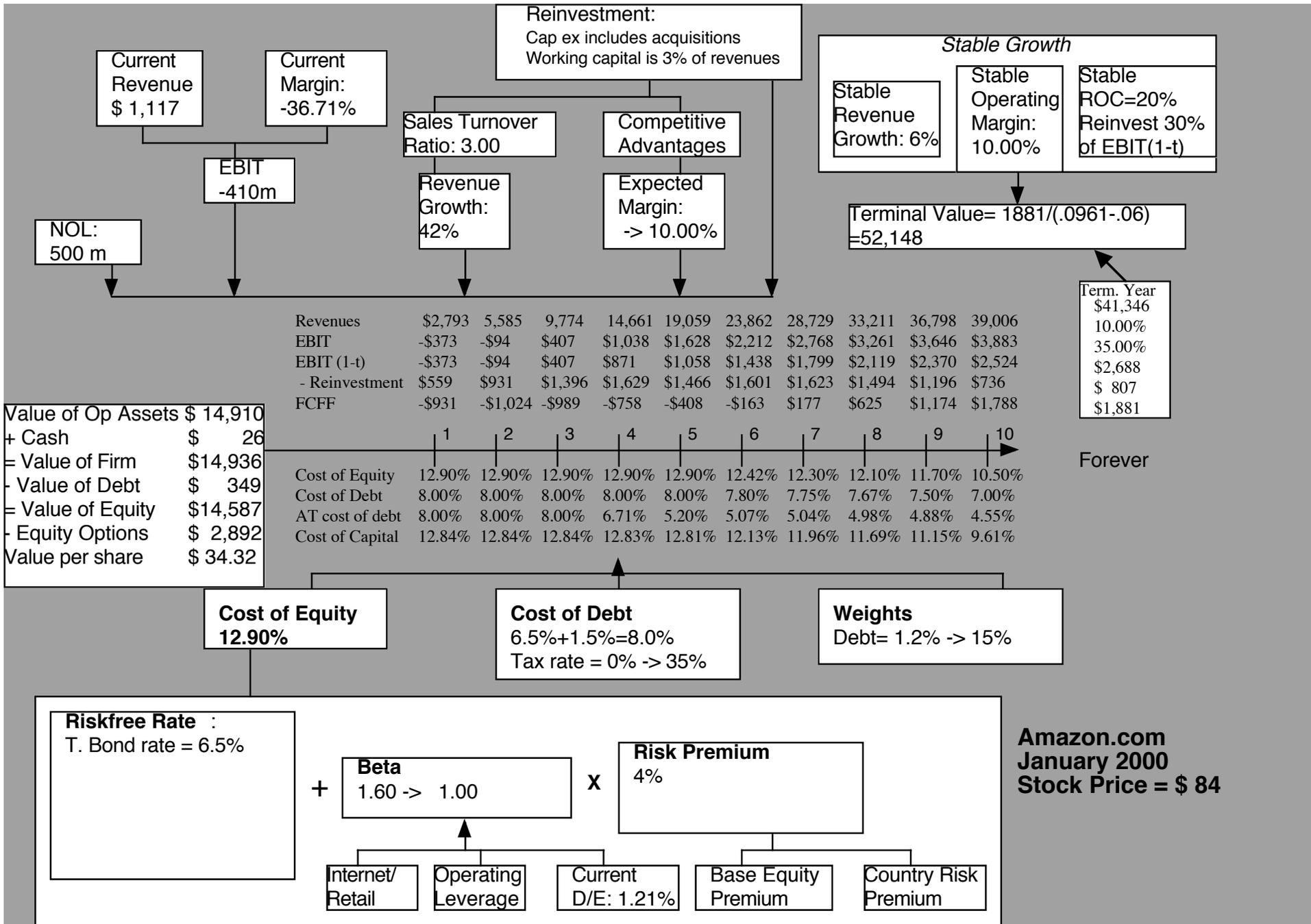
Firm's D/E Ratio: 27.9%

Mature risk premium 4%

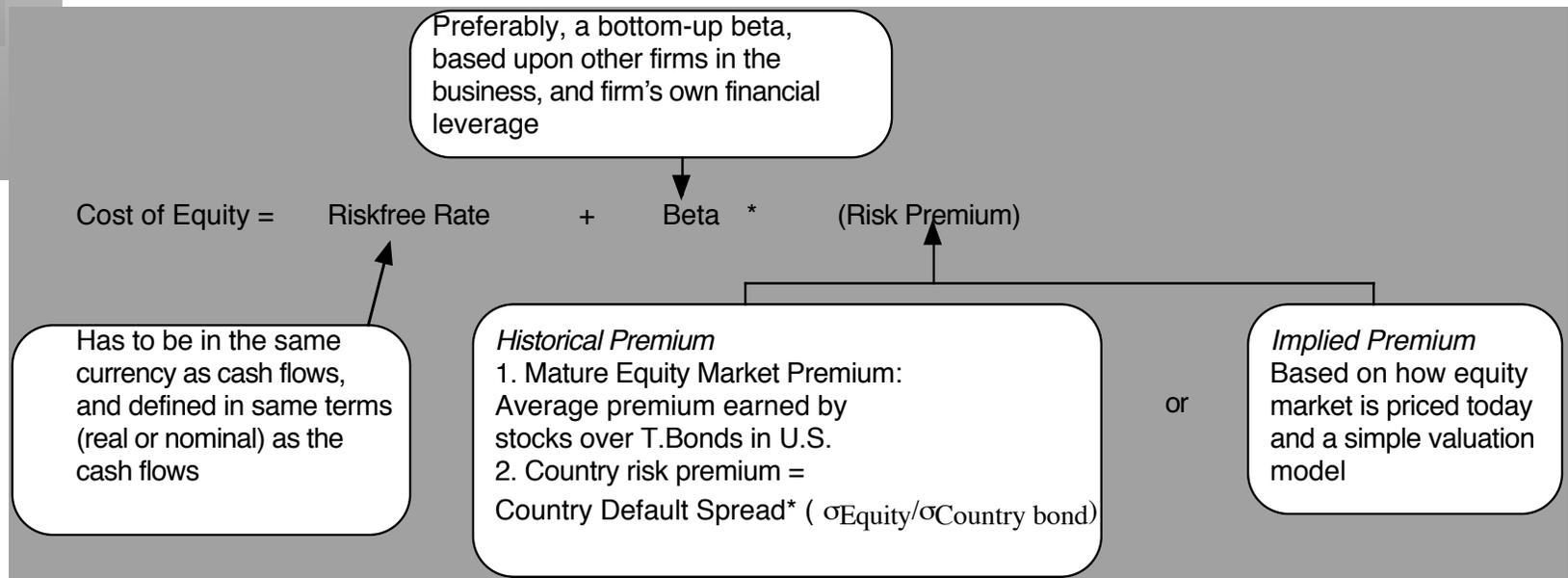
Country Equity Prem 0.44%

Discounted Cash Flow Valuation: High Growth with Negative Earnings





I. Discount Rates: Cost of Equity



A Simple Test

- You are valuing a Greek company in Euros for a US institutional investor and are attempting to estimate a risk free rate to use in the analysis. The risk free rate that you should use is
 - ❑ The interest rate on a US \$ denominated treasury bond (4.25%)
 - ❑ The interest rate on a Euro-denominated Greek government bond (4.52%)
 - ❑ The interest rate on a Euro-denominated bond issued by the German government (4.31%)

Everyone uses historical premiums, but..

- The historical premium is the premium that stocks have historically earned over riskless securities.
- Practitioners never seem to agree on the premium; it is sensitive to
 - How far back you go in history...
 - Whether you use T.bill rates or T.Bond rates
 - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

Historical Period	<i>Arithmetic average</i>		<i>Geometric Average</i>	
	Stocks - T.Bills	Stocks - T.Bonds	Stocks - T.Bills	Stocks - T.Bonds
1928-2003	7.92%	6.54%	5.99%	4.82%
1963-2003	6.09%	4.70%	4.85%	3.82%
1993-2003	8.43%	4.87%	6.68%	3.57%

Assessing Country Risk Using Currency Ratings: Western Europe

<i>Country</i>	<i>Rating</i>	<i>Default Spread over German Euro rate(in bp)</i>
Austria	Aaa	9
Belgium	Aaa	12
France	Aaa	6
Germany	Aaa	0
Greece	A1	22
Ireland	Aaa	3
Italy	Aa2	22
Netherlands	Aaa	9
Portugal	Aa2	14
Spain	Aaa	6

Assessing Country Risk using Ratings: The Rest of Europe

<i>Country</i>	<i>Rating</i>	<i>Default Spread</i>
Croatia	Baa3	145
Cyprus	A2	90
Czech Republic	Baa1	120
Hungary	A3	95
Latvia	Baa2	130
Lithuania	Ba1	250
Moldova	B3	650
Poland	Baa1	120
Romania	B3	650
Russia	B2	550
Slovakia	Ba1	250
Slovenia	A2	90
Turkey	B1	450

Using Country Ratings to Estimate Equity Spreads

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
 - One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
 - Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
 - Standard Deviation in Greek ASE(Equity) = 18%
 - Standard Deviation in Greek Euro Bond = 9%
 - Adjusted Equity Spread = 0.22% (18/9) = 0.44%

From Country Spreads to Corporate Risk premiums

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

$$E(\text{Return}) = \text{Riskfree Rate} + \text{Country Spread} + \text{Beta (US premium)}$$

Implicitly, this is what you are assuming when you use the local Government's dollar borrowing rate as your riskfree rate.

- Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.

$$E(\text{Return}) = \text{Riskfree Rate} + \text{Beta (US premium + Country Spread)}$$

- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

$$E(\text{Return}) = \text{Riskfree Rate} + \beta (\text{US premium}) + \lambda (\text{Country Spread})$$

Estimating Company Exposure to Country Risk

- Different companies should be exposed to different degrees to country risk. For instance, a Greek firm that generates the bulk of its revenues in the rest of Western Europe should be less exposed to country risk than one that generates all its business within Greece.

- The factor “ λ ” measures the relative exposure of a firm to country risk. One simplistic solution would be to do the following:

$$\lambda = \% \text{ of revenues domestically}_{\text{firm}} / \% \text{ of revenues domestically}_{\text{avg firm}}$$

For instance, if a firm gets 35% of its revenues domestically while the average firm in that market gets 70% of its revenues domestically

$$\lambda = 35\% / 70\% = 0.5$$

- There are two implications
 - A company’s risk exposure is determined by where it does business and not by where it is located
 - Firms might be able to actively manage their country risk exposures

Estimating E(Return) for Titan Cements

- Assume that the beta for Titan Cements is 0.95, and that the riskfree rate used is 4.31%. Also assume that the historical premium for the US (4.82%) is a reasonable estimate of a mature market risk premium.

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

$$E(\text{Return}) = 4.31\% + 0.44\% + 0.95 (4.82\%) = 9.33\%$$

- Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.

$$E(\text{Return}) = 4.31\% + 0.95 (4.82\% + 0.44\%) = 9.31\%$$

- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

$$E(\text{Return}) = 4.31\% + 0.95(4.82\%) + 0.56 (0.44\%) + 0.05(3\%) = 9.29\%$$

Titan is less exposed to Greek country risk than the typical Greek firm since it gets about 40% of its revenues in Greece; the average for Greek firms is 70%. In 2003, though, Titan got about 5% of its revenues from the Balkan states.

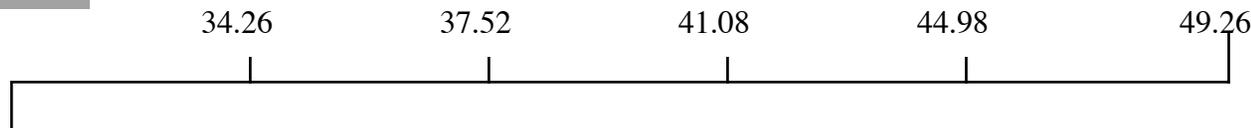
Implied Equity Premium for the S&P 500: January 1, 2004

- We can use the information in stock prices to back out how risk averse the market is and how much of a risk premium it is demanding.

In 2003, dividends & stock buybacks were 2.81% of the index, generating 31.29 in cashflows

Analysts expect earnings to grow 9.5% a year for the next 5 years as the economy comes out of a recession.

After year 5, we will assume that earnings on the index will grow at 4.25%, the same rate as the entire economy



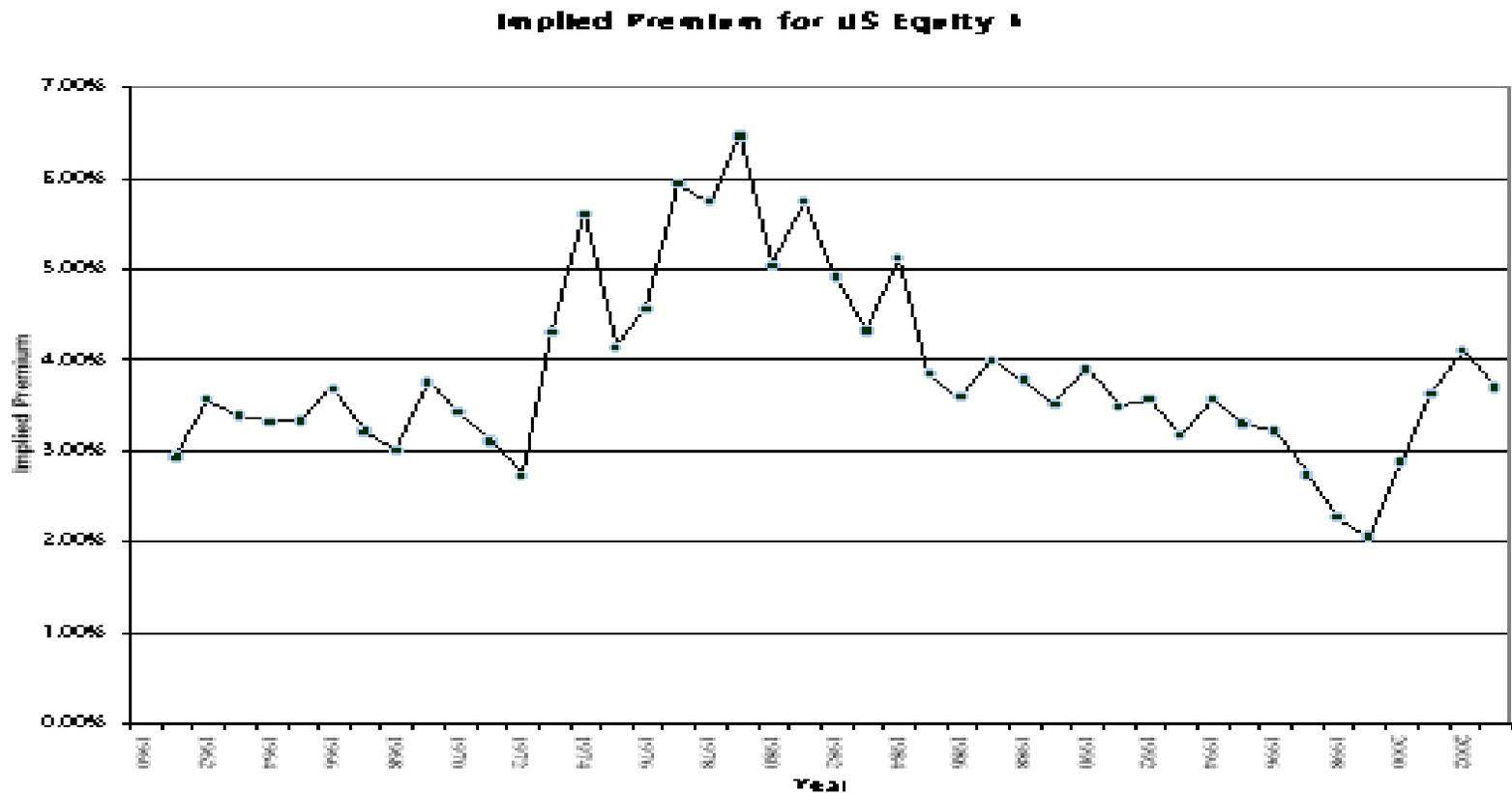
January 1, 2004
S&P 500 is at 1111.91

- If you pay the current level of the index, you can expect to make a return of 7.94% on stocks (which is obtained by solving for r in the following equation)

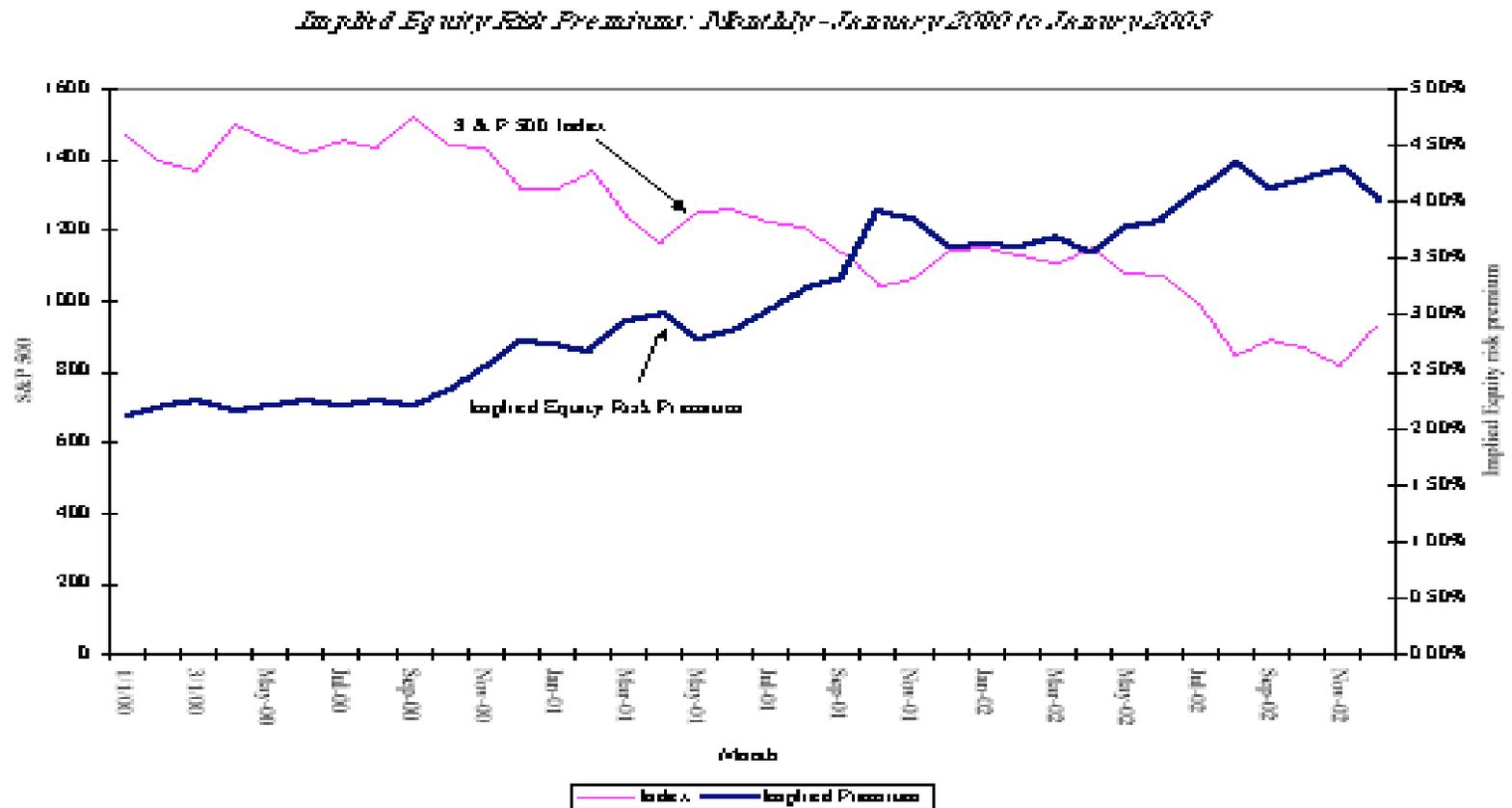
$$1111.91 = \frac{34.26}{(1+r)} + \frac{37.52}{(1+r)^2} + \frac{41.08}{(1+r)^3} + \frac{44.98}{(1+r)^4} + \frac{49.26}{(1+r)^5} + \frac{49.26(1.0425)}{(r-.0425)(1+r)^5}$$

- Implied Equity risk premium = Expected return on stocks - Treasury bond rate = 7.94% - 4.25% = 3.69%

Implied Premiums in the US



Implied Premiums: From Bubble to Bear Market... January 2000 to December 2002



Choosing an Equity Risk Premium

- The historical risk premium of 4.82% for the United States is too high a premium to use in valuation. It is much higher than the actual implied equity risk premium in the market
- The current implied equity risk premium requires us to assume that the market is correctly priced today. (If I were required to be market neutral, this is the premium I would use)
- The average implied equity risk premium between 1960-2003 in the United States is about 4%. We will use this as the premium for a mature equity market.

Implied Premium for Greek Market: May 14, 2004

- Level of the Index = 2467
- Dividends on the Index = 3.18% of 2467
- Other parameters
 - Riskfree Rate = 4.31% (Euros)
 - Expected Growth (in Euros)
 - Next 5 years = 9% (Used expected growth rate in Earnings)
 - After year 5 = 4.31%
- Solving for the expected return:
 - Expected return on Equity = 8.38%
 - Implied Equity premium = 8.38% - 4.31% = 4.07%
- Effect on valuation
 - Titan's value with historical premium (4%) + country (.44%) : 52.68 Euros/share
 - Tian's value with implied premium: 53.07 Euros per share

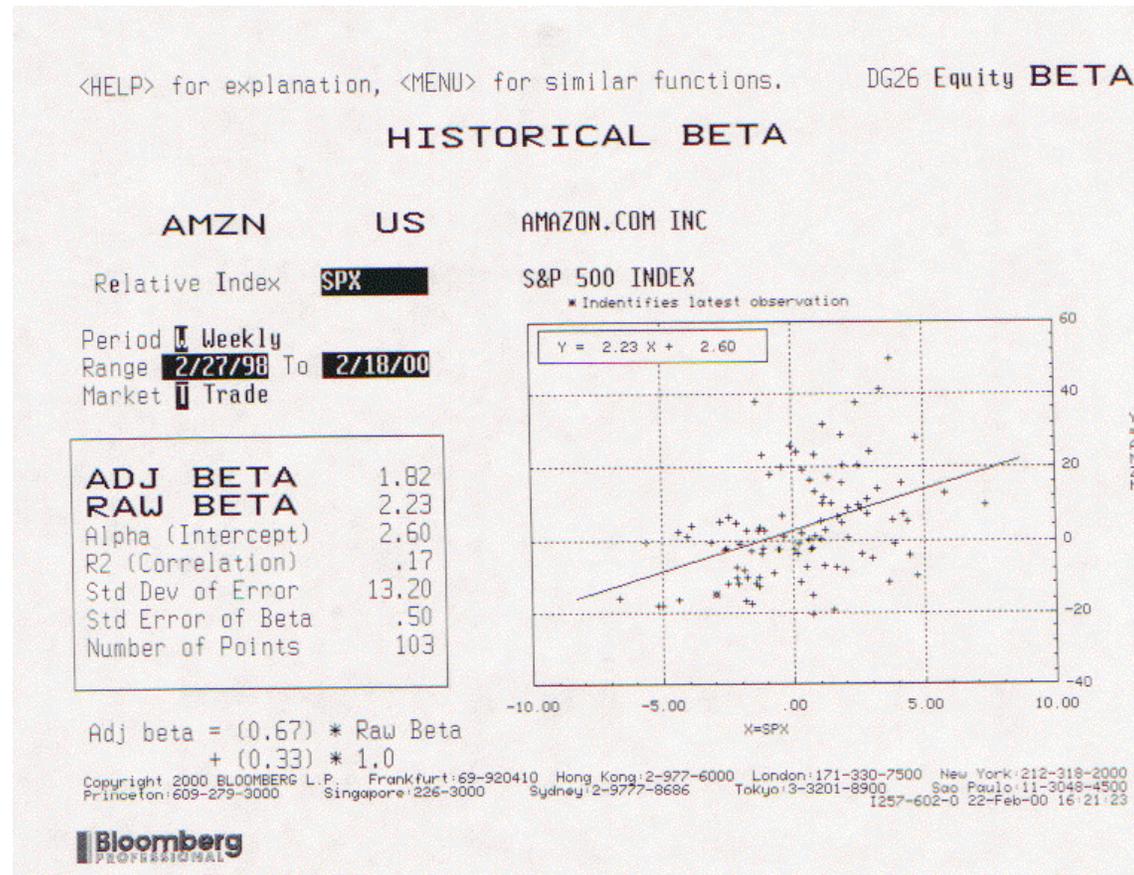
Estimating Beta

- The standard procedure for estimating betas is to regress stock returns (R_j) against market returns (R_m) -

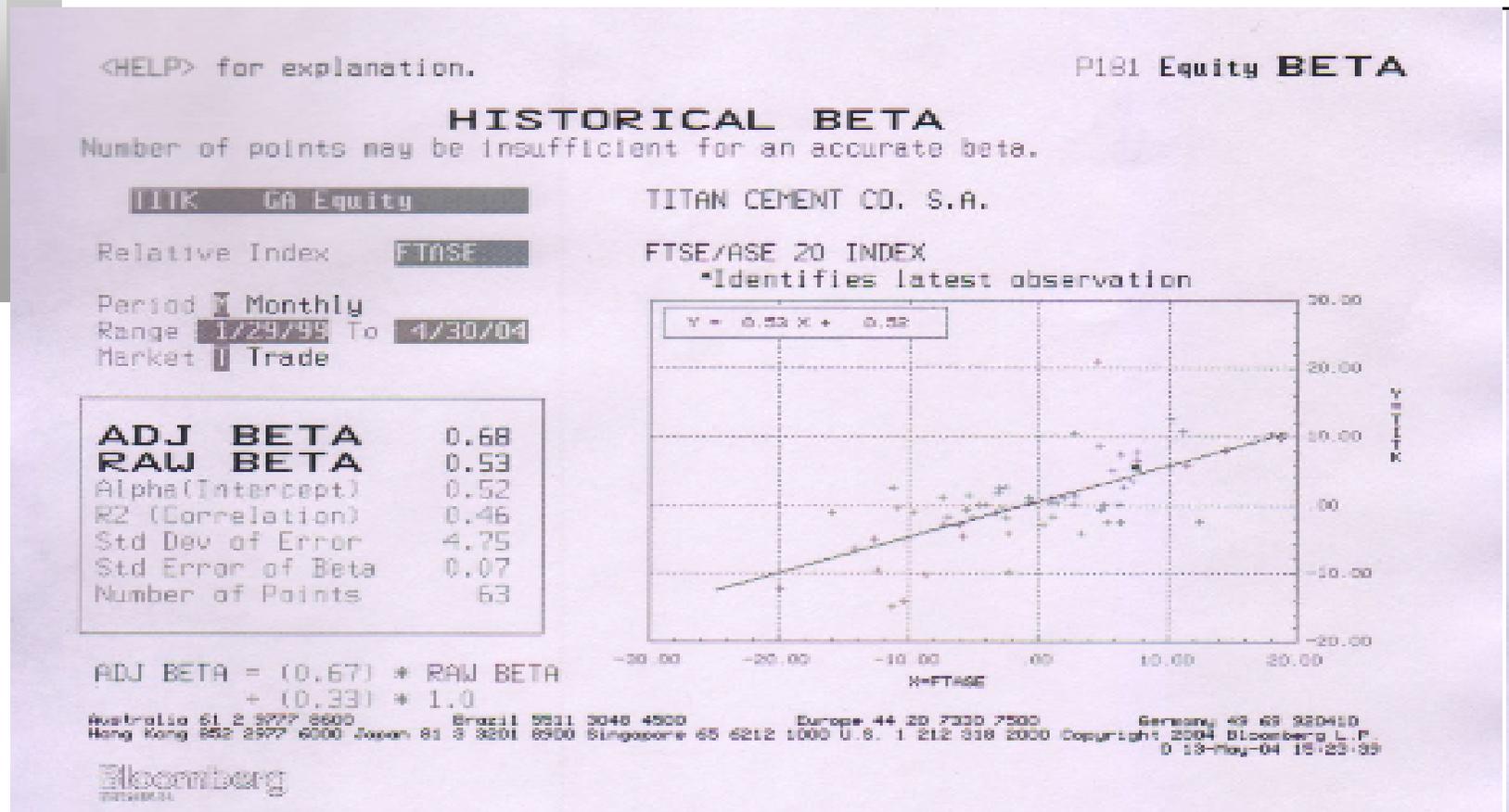
$$R_j = a + b R_m$$

- where a is the intercept and b is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.
- This beta has three problems:
 - It has high standard error
 - It reflects the firm's business mix over the period of the regression, not the current mix
 - It reflects the firm's average financial leverage over the period rather than the current leverage.

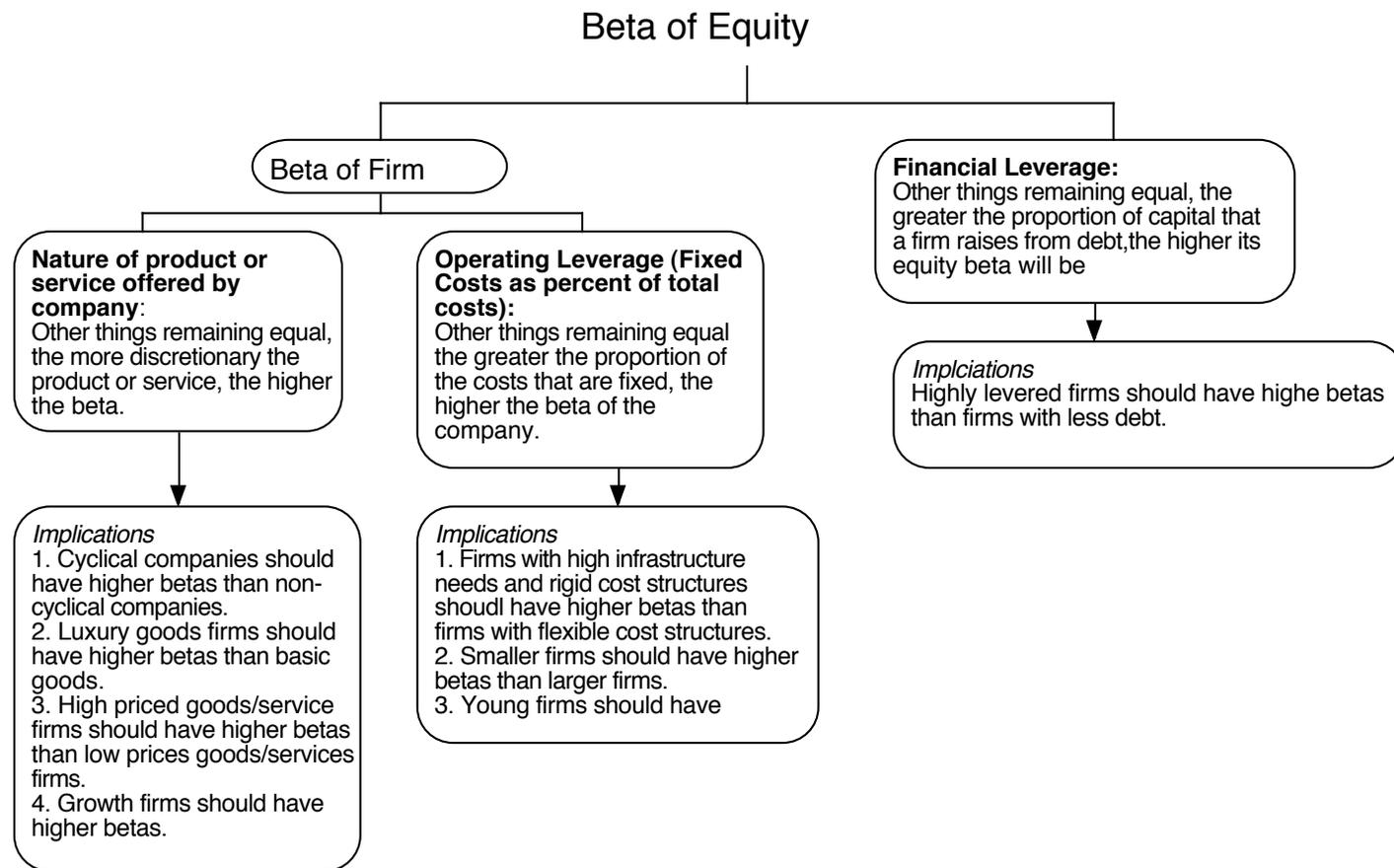
Beta Estimation: Amazon



Beta Estimation for Titan Cement: The Index Effect



Determinants of Betas



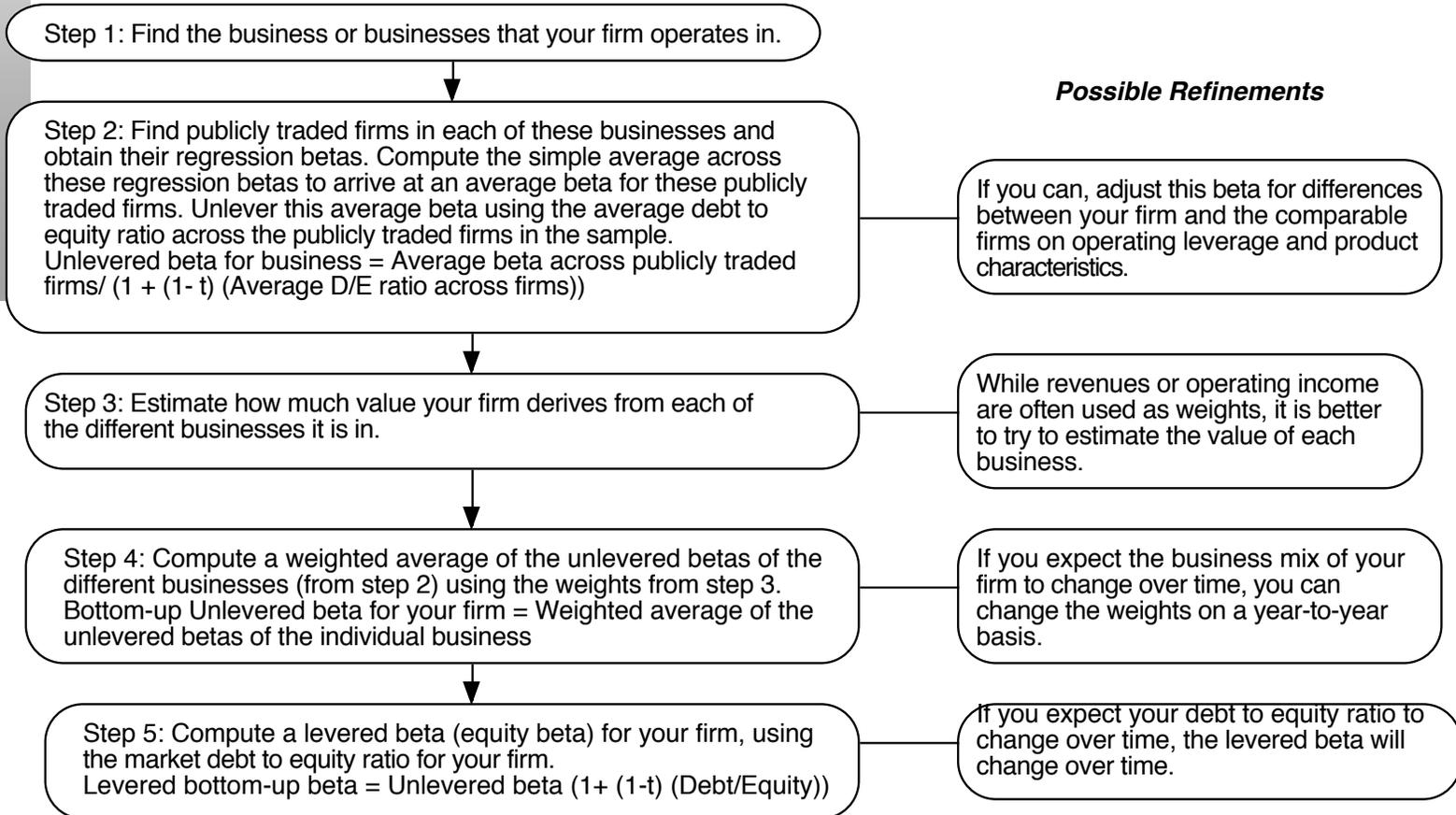
In a perfect world... we would estimate the beta of a firm by doing the following

Start with the beta of the business that the firm is in

Adjust the business beta for the operating leverage of the firm to arrive at the unlevered beta for the firm.

Use the financial leverage of the firm to estimate the equity beta for the firm
Levered Beta = Unlevered Beta (1 + (1- tax rate) (Debt/Equity))

Bottom-up Betas



Titan's Bottom-up Beta

<i>Business</i>	<i>Unlevered</i>	<i>D/E Ratio</i>	<i>Levered beta</i>	<i>Proportion of Value</i>
Cement	.80	27.9%	0.95	100%

Levered Beta = Unlevered Beta (1 + (1- tax rate) (D/E Ratio))
 = 0.80 (1 + (1-.3044) (.279)) = 0.95

A Hypothetical scenario: Assume that Titan had been in two businesses- cement and construction. You could estimate a beta for the combined firm as follows

<i>Comparable firms</i>						
<i>Business</i>	<i>Revenues</i>	<i>Value/Sales</i>	<i>Unlevered beta</i>	<i>Value</i>	<i>Weight</i>	<i>Weight*Beta</i>
Cement	750	1.33	0.80	1000	67%	.67*.80
Construct.	250	2.00	1.20	500	33%	.33*1.20
Firm						=.933

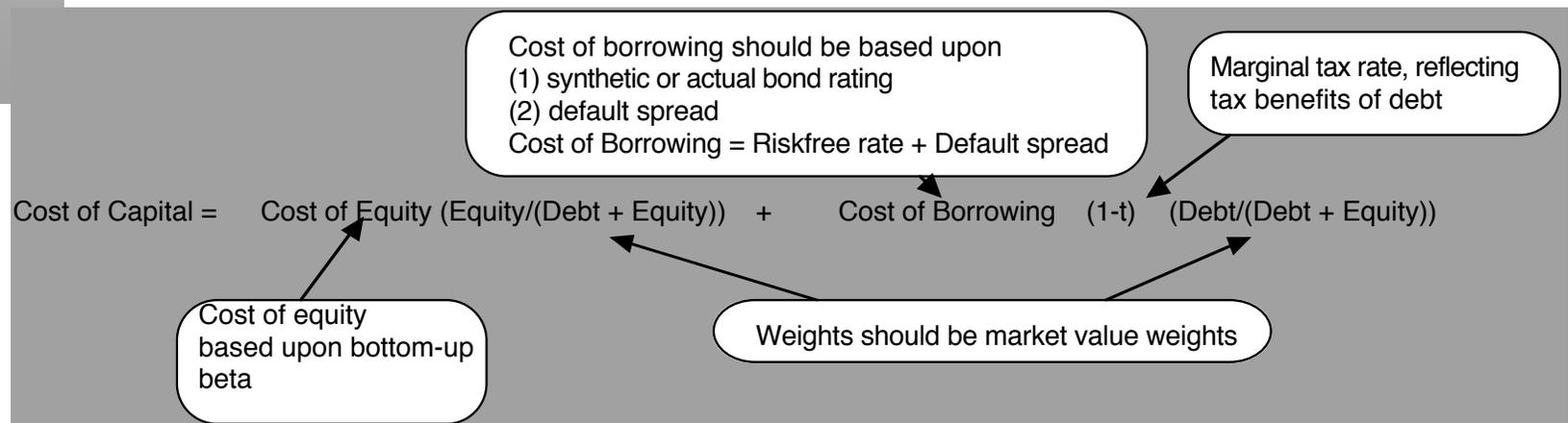
Amazon's Bottom-up Beta

Unlevered beta for firms in internet retailing = 1.60

Unlevered beta for firms in specialty retailing = 1.00

- Amazon is a specialty retailer, but its risk currently seems to be determined by the fact that it is an online retailer. Hence we will use the beta of internet companies to begin the valuation
- By the fifth year, we are estimating substantial revenues for Amazon and we move the beta towards to beta of the retailing business.

From Cost of Equity to Cost of Capital



Estimating Synthetic Ratings

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

$$\text{Interest Coverage Ratio} = \text{EBIT} / \text{Interest Expenses}$$

- For Titan's interest coverage ratio, we used the interest expenses and EBIT from 2003.

$$\text{Interest Coverage Ratio} = 222 / 19.4 = 11.44$$

- Amazon.com has negative operating income; this yields a negative interest coverage ratio, which should suggest a low rating. We computed an average interest coverage ratio of 2.82 over the next 5 years.

Interest Coverage Ratios, Ratings and Default Spreads

If Interest Coverage Ratio is		Estimated Bond Rating	Default Spread(1/00)	Default Spread(1/04)
> 8.50	(>12.50)	AAA	0.20%	0.35%
6.50 - 8.50	(9.5-12.5)	AA	0.50%	0.50%
5.50 - 6.50	(7.5-9.5)	A+	0.80%	0.70%
4.25 - 5.50	(6-7.5)	A	1.00%	0.85%
3.00 - 4.25	(4.5-6)	A-	1.25%	1.00%
2.50 - 3.00	(3.5-4.5)	BBB	1.50%	1.50%
2.25 - 2.50	(3.5 -4)	BB+	1.75%	2.00%
2.00 - 2.25	((3-3.5)	BB	2.00%	2.50%
1.75 - 2.00	(2.5-3)	B+	2.50%	3.25%
1.50 - 1.75	(2-2.5)	B	3.25%	4.00%
1.25 - 1.50	(1.5-2)	B -	4.25%	6.00%
0.80 - 1.25	(1.25-1.5)	CCC	5.00%	8.00%
0.65 - 0.80	(0.8-1.25)	CC	6.00%	10.00%
0.20 - 0.65	(0.5-0.8)	C	7.50%	12.00%
< 0.20	(<0.5)	D	10.00%	20.00%

For Titan, I used the interest coverage ratio table for smaller/riskier firms (the numbers in brackets) which yields a lower rating for the same interest coverage ratio.

Estimating the cost of debt for a firm

- The synthetic rating for Titan Cement is AA. Using the 2004 default spread of 0.50%, we estimate a cost of debt of 5.03% (using a riskfree rate of 4.31% and adding in the country default spread of 0.22%):

$$\begin{aligned} \text{Cost of debt} &= \text{Riskfree rate} + \text{Greek default spread} + \text{Company default spread} \\ &= 4.31\% + 0.22\% + 0.50\% = 5.03\% \end{aligned}$$

- The synthetic rating for Amazon.com in 2000 was BBB. The default spread for BBB rated bond was 1.50% in 2000 and the treasury bond rate was 6.5%.

$$\begin{aligned} \text{Pre-tax cost of debt} &= \text{Riskfree Rate} + \text{Default spread} \\ &= 6.50\% + 1.50\% = 8.00\% \end{aligned}$$

- The firm is paying no taxes currently. As the firm's tax rate changes and its cost of debt changes, the after tax cost of debt will change as well.

	1	2	3	4	5	6	7	8	9	10
Pre-tax	8.00%	8.00%	8.00%	8.00%	8.00%	7.80%	7.75%	7.67%	7.50%	7.00%
Tax rate	0%	0%	0%	16.13%	35%	35%	35%	35%	35%	35%
After-tax	8.00%	8.00%	8.00%	6.71%	5.20%	5.07%	5.04%	4.98%	4.88%	4.55%

Weights for the Cost of Capital Computation

- The weights used to compute the cost of capital should be the market value weights for debt and equity.
- There is an element of circularity that is introduced into every valuation by doing this, since the values that we attach to the firm and equity at the end of the analysis are different from the values we gave them at the beginning.
- As a general rule, the debt that you should subtract from firm value to arrive at the value of equity should be the same debt that you used to compute the cost of capital.

Estimating Cost of Capital: Amazon.com

■ Equity

- Cost of Equity = $6.50\% + 1.60 (4.00\%) = 12.90\%$
- Market Value of Equity = $\$ 84/\text{share} * 340.79 \text{ mil shs} = \$ 28,626 \text{ mil} (98.8\%)$

■ Debt

- Cost of debt = $6.50\% + 1.50\% (\text{default spread}) = 8.00\%$
- Market Value of Debt = $\$ 349 \text{ mil} (1.2\%)$

■ Cost of Capital

$$\text{Cost of Capital} = 12.9 \% (.988) + 8.00\% (1 - 0) (.012) = 12.84\%$$

Estimating Cost of Capital: Titan Cements

■ Equity

- Cost of Equity = $4.31\% + 0.95 (4\% + 0.44\%) = 8.53\%$
- Market Value of Equity = 1423 million Euros (78.2%)

■ Debt

- Cost of debt = $4.31\% + 0.22\% + 0.50\% = 5.03\%$
- Market Value of Debt = 396 million Euros (21.8%)

■ Cost of Capital

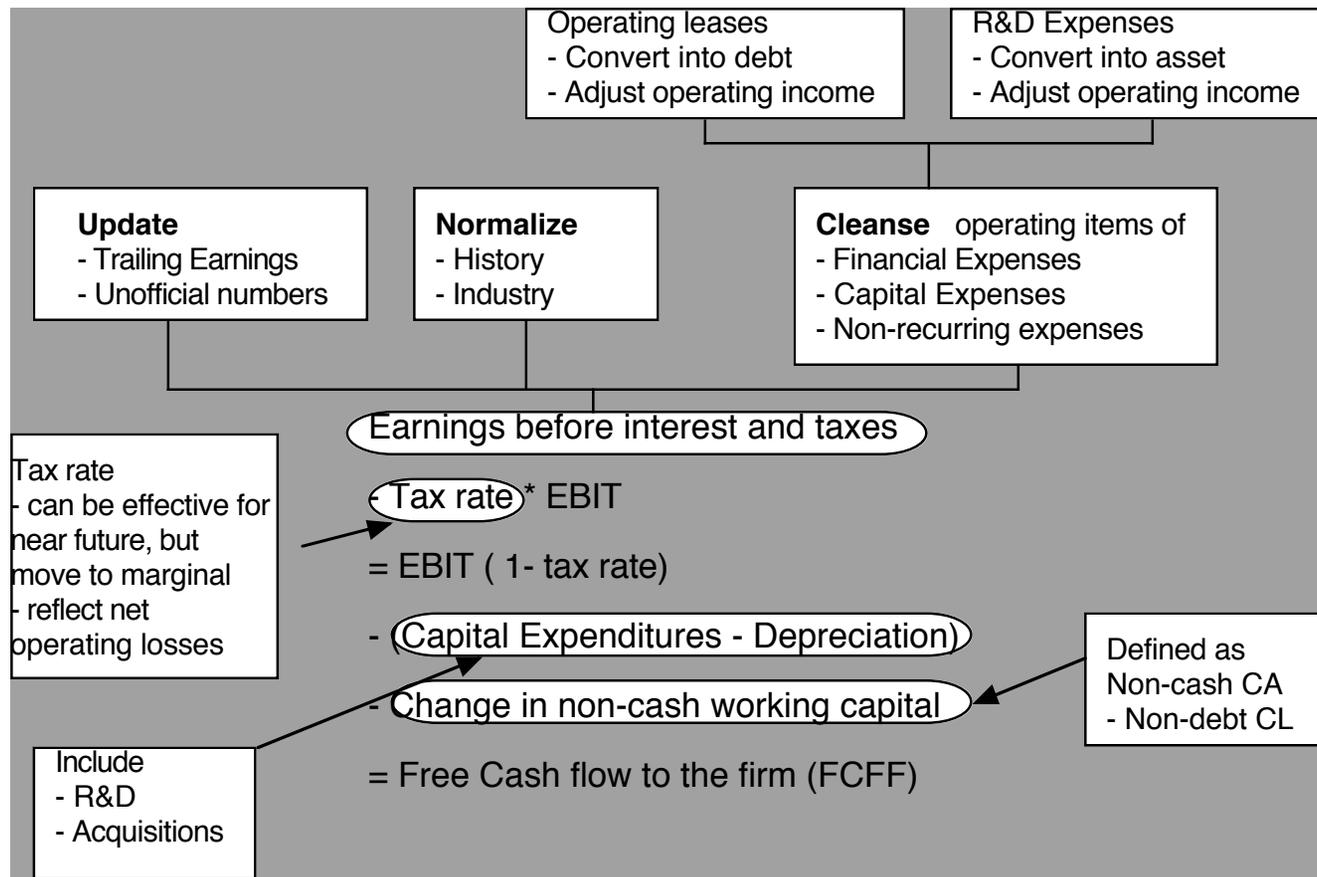
$$\text{Cost of Capital} = 8.53\% (.782) + 5.03\% (1 - .3044) (0.218) = 7.43\%$$

The book value of equity at Titan Cement is 498 million Euros

The book value of debt at Titan Cement is 399 million; Interest expense is 19 mil; Average maturity of debt = 4 years

Estimated market value of debt = $19 \text{ million (PV of annuity, 4 years, 5.03\%)} + \$ 399 \text{ million} / 1.0503^4 = 396 \text{ million Euros}$

II. Estimating Cash Flows to Firm



The Importance of Updating

- The operating income and revenue that we use in valuation should be updated numbers. One of the problems with using financial statements is that they are dated.
- As a general rule, it is better to use 12-month trailing estimates for earnings and revenues than numbers for the most recent financial year. This rule becomes even more critical when valuing companies that are evolving and growing rapidly.

	<i>Last 10-K</i>	<i>Trailing 12-month</i>
Revenues	\$ 610 million	\$1,117 million
EBIT	- \$125 million	- \$ 410 million

- The valuation of Titan is dated because there have been no financial statements released since the last 10K.

Normalizing Earnings: Amazon

Year	Revenues	Operating Margin	EBIT	
Tr12m	\$1,117	-36.71%	-\$410	
1	\$2,793	-13.35%	-\$373	
2	\$5,585	-1.68%	-\$94	
3	\$9,774	4.16%	\$407	
4	\$14,661	7.08%	\$1,038	
5	\$19,059	8.54%	\$1,628	
6	\$23,862	9.27%	\$2,212	
7	\$28,729	9.64%	\$2,768	
8	\$33,211	9.82%	\$3,261	
9	\$36,798	9.91%	\$3,646	
10	\$39,006	9.95%	\$3,883	
TY(11)	\$41,346	10.00%	\$4,135	Industry Average

Operating Leases at The Home Depot in 1998

- The pre-tax cost of debt at the Home Depot is 6.25%

Yr	Operating Lease Expense	Present Value
1	\$ 294	\$ 277
2	\$ 291	\$ 258
3	\$ 264	\$ 220
4	\$ 245	\$ 192
5	\$ 236	\$ 174
6-15	\$ 270	\$ 1,450 (PV of 10-yr annuity)

Present Value of Operating Leases = \$ 2,571

- Debt outstanding at the Home Depot = \$1,205 + \$2,571 = \$3,776 mil
(The Home Depot has other debt outstanding of \$1,205 million)
- Adjusted Operating Income = \$2,016 + 2,571 (.0625) = \$2,177 mil

Capitalizing R&D Expenses: Shire Pharmaceuticals

- To capitalize R&D,
 - Specify an amortizable life for R&D (2 - 10 years)
 - Collect past R&D expenses for as long as the amortizable life
 - Sum up the unamortized R&D over the period. (Thus, if the amortizable life is 5 years, the research asset can be obtained by adding up 1/5th of the R&D expense from five years ago, 2/5th of the R&D expense from four years ago...:
- R & D was assumed to have a 5-year life.

Year	R&D	Unamortized R&D	Amortization
Current	£48.12	1.00 £48.12	£0.00
-1	£37.42	0.80 £29.94	£7.48
-2	£28.99	0.60 £17.39	£5.80
-3	£17.88	0.40 £7.15	£3.58
-4	£8.18	0.20 £1.64	£1.64
-5	£4.56	0.00 £0.00	£0.91
		£104.24	£19.41

Value of research asset = £104.24

Amortization of research asset in 2000 = £19.41

Adjustment to Operating Income = + R&D - Amortization of R&D

Adjusted Operating Income = £41.03 + £48.12 - £19.41 = £69.74

The Effect of Net Operating Losses: Amazon.com's Tax Rate

Year	1	2	3	4	5
EBIT	-\$373	-\$94	\$407	\$1,038	\$1,628
Taxes	\$0	\$0	\$0	\$167	\$570
EBIT(1-t)	-\$373	-\$94	\$407	\$871	\$1,058
Tax rate	0%	0%	0%	16.13%	35%
NOL	\$500	\$873	\$967	\$560	\$0

After year 5, the tax rate becomes 35%.

Estimating Actual FCFF: Titan Cement

- EBIT = 222 million Euros
- Tax rate = 30.44%
- Net Capital expenditures = Cap Ex - Depreciation = 102.2 - 73.7 = 28.6 million
- Change in Working Capital = +10.2 million

Estimating FCFF

Current EBIT * (1 - tax rate) =	222 (1-.3044) =	154.4 Million
- (Capital Spending - Depreciation)		28.6
- Change in Working Capital		10.2
Current FCFF		115.6 Million Euros

Estimating FCFF: Amazon.com

- EBIT (Trailing 1999) = -\$ 410 million
- Tax rate used = 0% (Assumed Effective = Marginal)
- Capital spending (Trailing 1999) = \$ 243 million
- Depreciation (Trailing 1999) = \$ 31 million
- Non-cash Working capital Change (1999) = - 80 million
- Estimating FCFF (1999)
 - Current EBIT * (1 - tax rate) = - 410 (1-0) = - \$410 million
 - (Capital Spending - Depreciation) = \$212 million
 - Change in Working Capital = -\$ 80 million
 - Current FCFF = - \$542 million

IV. Expected Growth in EBIT and Fundamentals

- Reinvestment Rate and Return on Capital

$$g_{\text{EBIT}} = (\text{Net Capital Expenditures} + \text{Change in WC}) / \text{EBIT}(1-t) * \text{ROC}$$
$$= \text{Reinvestment Rate} * \text{ROC}$$

- **Proposition:** No firm can expect its operating income to grow over time without reinvesting some of the operating income in net capital expenditures and/or working capital.
- **Proposition:** The net capital expenditure needs of a firm, for a given growth rate, should be inversely proportional to the quality of its investments.

Normalizing Reinvestment: Titan Cements

	1999	2000	2001	2002	2003	Total
Cp Ex	\$136.65	\$50.54	\$81.00	\$113.30	\$102.30	\$483.79
Depreciation	\$89.53	\$39.26	\$40.87	\$80.94	\$73.70	\$324.30
EBIT	\$122.55	\$162.78	\$186.39	\$200.60	\$222.00	
EBIT(1-t)	\$85.25	\$113.23	\$129.65	\$139.54	\$154.42	\$622.09
Net Cap Ex as %	55.28%	9.96%	30.95%	23.19%	18.52%	25.64%
Revenues	562.6	622.7	982.9	1036.1	1035.7	4240
Non-cashh Curren	236.38	248.55	342.95	352.93	\$402.10	
Non-debt current	106.98	133.33	177.15	194.57	255	
Non-cash WC	129.4	115.22	165.8	158.36	147.1	715.88
as % of revenues	23.00%	18.50%	16.87%	15.28%	14.20%	16.88%

Expected Growth Estimate: Titan Cement

- Normalized Change in working capital = (Working capital as percent of revenues) * Change in revenues in 2003 = .1688 (1035.7-1036.1) = 0 mil Euros
- Normalized Net Cap Ex = Net Cap ex as % of EBIT(1-t) * EBIT (1-t) in 2001 = .2564*(222 (1-.3044)) = 30.59 million Euros
- Normalized reinvestment rate = 39.59/(222(1-.3044)) = 25.59%
- Return on capital = 222 (1-.3044)/ (477+361) = 18.42%
 - The book value of debt and equity from last year was used.
- Expected growth rate = .2559*.1842 = 4.71%

Revenue Growth and Operating Margins

- With negative operating income and a negative return on capital, the fundamental growth equation is of little use for Amazon.com
- For Amazon, the effect of reinvestment shows up in revenue growth rates and changes in expected operating margins:

Expected Revenue Growth in \$ = Reinvestment (in \$ terms) * (Sales/ Capital)

- The effect on expected margins is more subtle. Amazon's reinvestments (especially in acquisitions) may help create barriers to entry and other competitive advantages that will ultimately translate into high operating margins and high profits.

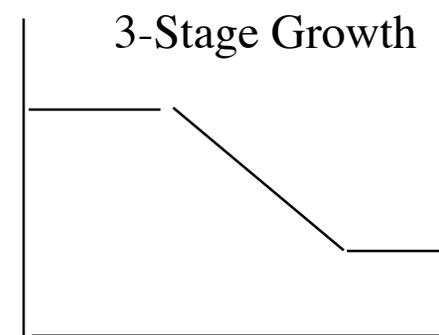
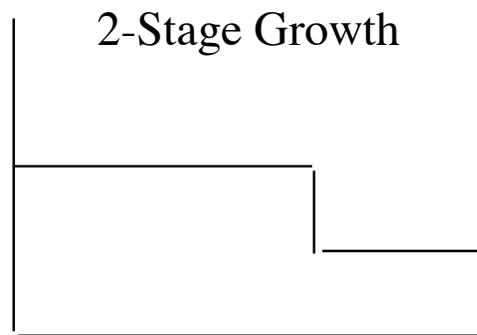
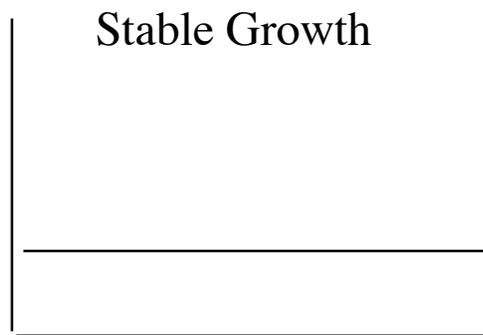
Growth in Revenues, Earnings and Reinvestment: Amazon

Year	Revenue Growth	Chg in Revenue	Reinvestment	Chg Rev/ Chg Reinvestment	ROC
1	150.00%	\$1,676	\$559	3.00	-76.62%
2	100.00%	\$2,793	\$931	3.00	-8.96%
3	75.00%	\$4,189	\$1,396	3.00	20.59%
4	50.00%	\$4,887	\$1,629	3.00	25.82%
5	30.00%	\$4,398	\$1,466	3.00	21.16%
6	25.20%	\$4,803	\$1,601	3.00	22.23%
7	20.40%	\$4,868	\$1,623	3.00	22.30%
8	15.60%	\$4,482	\$1,494	3.00	21.87%
9	10.80%	\$3,587	\$1,196	3.00	21.19%
10	6.00%	\$2,208	\$736	3.00	20.39%

Assume that firm can earn high returns because of established economies of scale.

V. Growth Patterns

- A key assumption in all discounted cash flow models is the period of high growth, and the pattern of growth during that period. In general, we can make one of three assumptions:
 - there is no high growth, in which case the firm is already in stable growth
 - there will be high growth for a period, at the end of which the growth rate will drop to the stable growth rate (2-stage)
 - there will be high growth for a period, at the end of which the growth rate will decline gradually to a stable growth rate(3-stage)



Determinants of Growth Patterns

- Size of the firm
 - Success usually makes a firm larger. As firms become larger, it becomes much more difficult for them to maintain high growth rates
- Current growth rate
 - While past growth is not always a reliable indicator of future growth, there is a correlation between current growth and future growth. Thus, a firm growing at 30% currently probably has higher growth and a longer expected growth period than one growing 10% a year now.
- Barriers to entry and differential advantages
 - Ultimately, high growth comes from high project returns, which, in turn, comes from barriers to entry and differential advantages.
 - The question of how long growth will last and how high it will be can therefore be framed as a question about what the barriers to entry are, how long they will stay up and how strong they will remain.

Stable Growth Characteristics

- In stable growth, firms should have the characteristics of other stable growth firms. In particular,
 - The risk of the firm, as measured by beta and ratings, should reflect that of a stable growth firm.
 - Beta should move towards one
 - The cost of debt should reflect the safety of stable firms (BBB or higher)
 - The debt ratio of the firm might increase to reflect the larger and more stable earnings of these firms.
 - The debt ratio of the firm might moved to the optimal or an industry average
 - If the managers of the firm are deeply averse to debt, this may never happen
 - The reinvestment rate of the firm should reflect the expected growth rate and the firm's return on capital
 - $\text{Reinvestment Rate} = \text{Expected Growth Rate} / \text{Return on Capital}$

Titan and Amazon.com: Stable Growth Inputs

	High Growth	Stable Growth
■ Titan Cement		
• Beta	0.95	1.00
• Debt Ratio	21.8%	21.8%
• Return on Capital	18.42%	7.20%
• Cost of Capital	7.43%	7.20%
• Expected Growth Rate	4.71%	4.31%
• Reinvestment Rate	25.59%	$4.31\%/7.20\% = 59.85\%$
■ Amazon.com		
• Beta	1.60	1.00
• Debt Ratio	1.20%	15%
• Return on Capital	Negative	20%
• Expected Growth Rate	NMF	6%
• Reinvestment Rate	>100%	$6\%/20\% = 30\%$

Dealing with Cash and Marketable Securities

- The simplest and most direct way of dealing with cash and marketable securities is to keep them out of the valuation - the cash flows should be before interest income from cash and securities, and the discount rate should not be contaminated by the inclusion of cash. (Use betas of the operating assets alone to estimate the cost of equity).
- Once the firm has been valued, add back the value of cash and marketable securities.
 - If you have a particularly incompetent management, with a history of overpaying on acquisitions, markets may discount the value of this cash.

Dealing with Cross Holdings

- When the holding is a majority, active stake, the value that we obtain from the cash flows includes the share held by outsiders. While their holding is measured in the balance sheet as a minority interest, it is at book value. To get the correct value, we need to subtract out the estimated market value of the minority interests from the firm value.
- When the holding is a minority, passive interest, the problem is a different one. The firm shows on its income statement only the share of dividends it receives on the holding. Using only this income will understate the value of the holdings. In fact, we have to value the subsidiary as a separate entity to get a measure of the market value of this holding.
- **Proposition 1:** It is almost impossible to correctly value firms with minority, passive interests in a large number of private subsidiaries.

Titan's Cash and Cross Holdings

- Titan has a majority interest in another company and the financial statements of that company are consolidated with those of Titan. The minority interests (representing the equity in the subsidiary that does not belong to Titan) are shown on the balance sheet at 52.40 million Euros.
- Estimated market value of minority interests = Book value of minority interest * P/BV of sector that subsidiary belongs to = $52.40 * 1.93 = 101.13$ million

Present Value of FCFF in high growth phase =	€532.34
+ Present Value of Terminal Value of Firm =	€ 1,895.41
= Value of operating assets of the firm =	€ 2,427.76
+ Value of Cash, Marketable Securities & Non-operating assets =	€ 81.20
= Value of Firm =	€ 2,508.96
- Market Value of outstanding debt =	€ 396.46
- Value of Minority Interests in Consolidated Company =	€ 101.13
= Market Value of Equity =	€ 2,011.36
/ Number of shares outstanding	/38.18
= Market Value of Equity/share =	€ 52.68

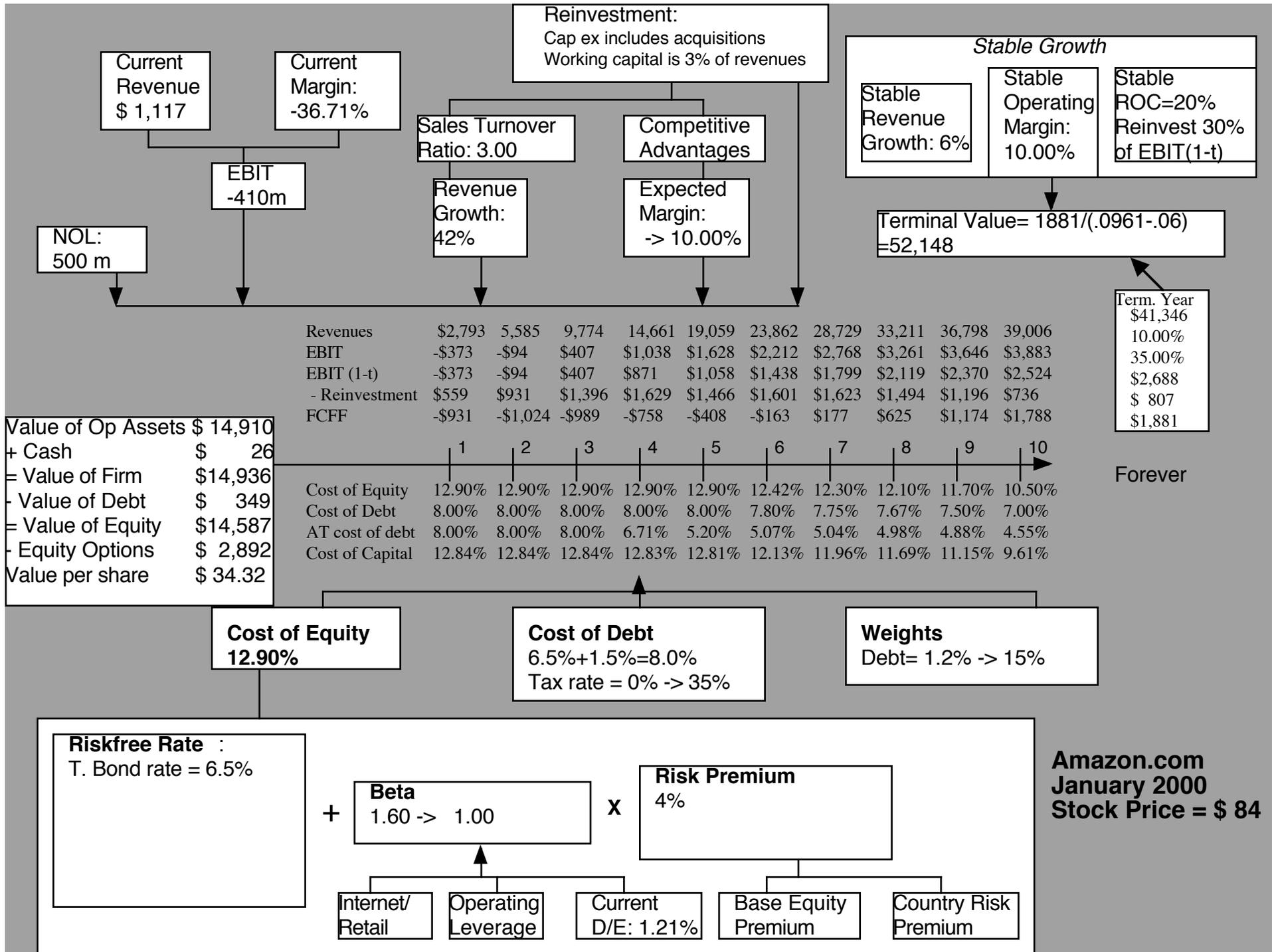
Amazon: Estimating the Value of Equity Options

■ Details of options outstanding

- Average strike price of options outstanding = \$ 13.375
- Average maturity of options outstanding = 8.4 years
- Standard deviation in $\ln(\text{stock price})$ = 50.00%
- Annualized dividend yield on stock = 0.00%
- Treasury bond rate = 6.50%
- Number of options outstanding = 38 million
- Number of shares outstanding = 340.79 million

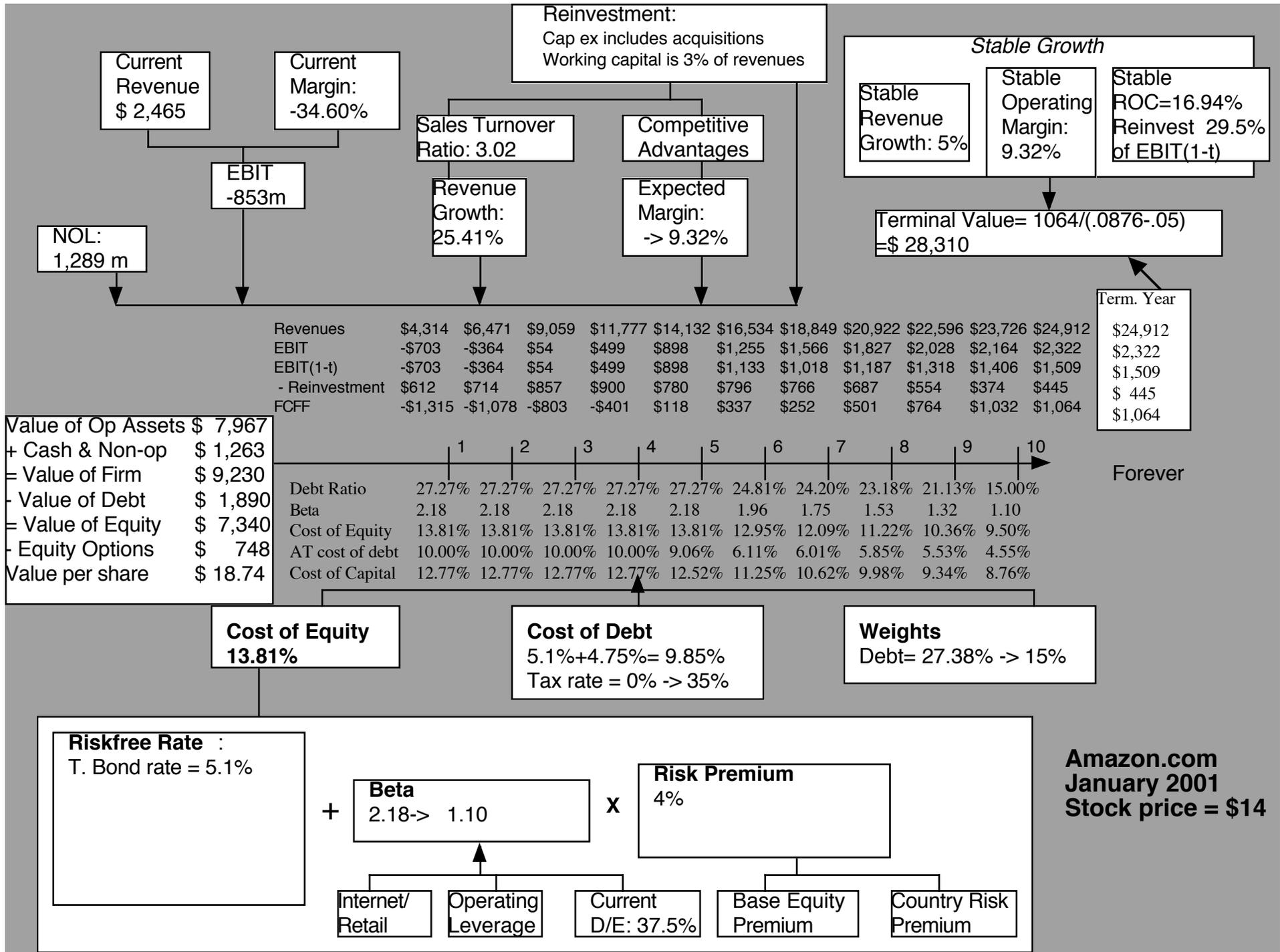
■ Value of options outstanding (using dilution-adjusted Black-Scholes model)

- Value of equity options = \$ 2,892 million



Amazon.com: Break Even at \$84?

	6%	8%	10%	12%	14%
30%	\$ (1.94)	\$ 2.95	\$ 7.84	\$ 12.71	\$ 17.57
35%	\$ 1.41	\$ 8.37	\$ 15.33	\$ 22.27	\$ 29.21
40%	\$ 6.10	\$ 15.93	\$ 25.74	\$ 35.54	\$ 45.34
45%	\$ 12.59	\$ 26.34	\$ 40.05	\$ 53.77	\$ 67.48
50%	\$ 21.47	\$ 40.50	\$ 59.52	\$ 78.53	\$ 97.54
55%	\$ 33.47	\$ 59.60	\$ 85.72	\$ 111.84	\$ 137.95
60%	\$ 49.53	\$ 85.10	\$ 120.66	\$ 156.22	\$ 191.77



Titan Cements: Status Quo

Avg Reinvestment rate = 25.59%

Reinvestment Rate 25.59%

Return on Capital 18.42%

Current Cashflow to Firm

EBIT(1-t) : 154
 - Nt CpX 29
 - Chg WC 10
 = FCFF 115
 Reinvestment Rate = 39/115=25%

Expected Growth in EBIT (1-t)
 $.2559 * .1842 = .0471$
4.71%

Stable Growth
 $g = 4.31%$; Beta = 1.00;
 Country Premium = 0%
 Cost of capital = 7.20%
 ROC = 7.20%; Tax rate = 33%
 Reinvestment Rate = 59.85%

Terminal Value₅ = $78.4 / (.072 - .0431) = 2713$

Op. Assets 2,428
 + Cash: 81
 - Debt 396
 - Minor. Int. 101
 = Equity 2,011
 - Options 0
 Value/Share 52.68

EBIT	€ 232.47	€ 243.42	€ 254.90	€ 266.92	€ 279.50
EBIT(1-t)	€ 161.70	€ 169.33	€ 177.31	€ 185.67	€ 194.42
- Reinvestment	€ 41.39	€ 43.34	€ 45.38	€ 47.52	€ 49.76
= FCFF	€ 120.32	€ 125.99	€ 131.93	€ 138.15	€ 144.66

Term Yr
 291.5
 195.3
 116.9
 78.4

Discount at Cost of Capital (WACC) = $8.53% (.782) + 3.50% (0.218) = 7.43%$

Cost of Equity **8.53%**

Cost of Debt
 $(4.31% + .5% + .22%)(1 - .3044)$
 = 3.50%

Weights
 E = 78.2% D = 21.8%

Riskfree Rate:
 Euro riskfree rate = 4.31%

+

Beta
 0.96

x

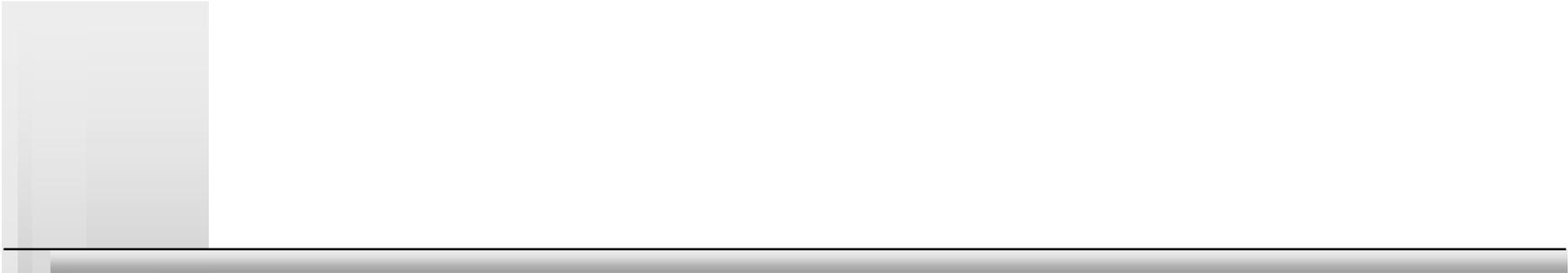
Risk Premium
 4.44%

Unlevered Beta for Sectors: 0.80

Firm's D/E Ratio: 27.9%

Mature risk premium 4%

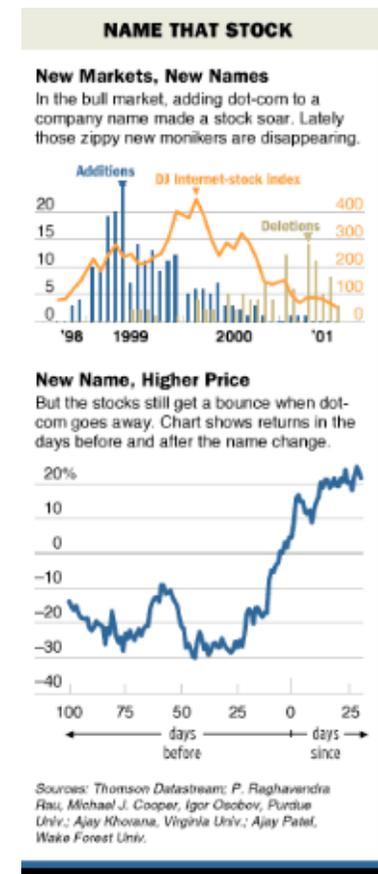
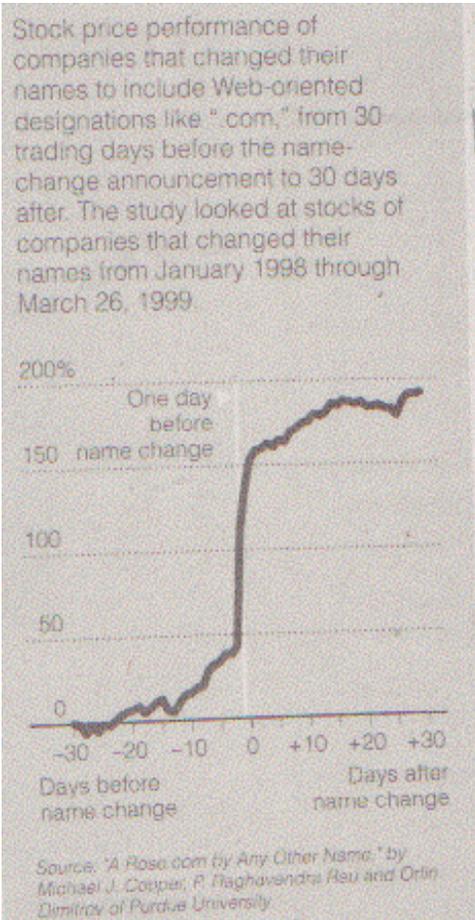
Country Equity Prem 0.44%



Value Enhancement: Back to Basics

Aswath Damodaran
<http://www.damodaran.com>

Price Enhancement versus Value Enhancement



The Paths to Value Creation

- Using the DCF framework, there are four basic ways in which the value of a firm can be enhanced:
 - The cash flows from existing assets to the firm can be increased, by either
 - increasing after-tax earnings from assets in place or
 - reducing reinvestment needs (net capital expenditures or working capital)
 - The expected growth rate in these cash flows can be increased by either
 - Increasing the rate of reinvestment in the firm
 - Improving the return on capital on those reinvestments
 - The length of the high growth period can be extended to allow for more years of high growth.
 - The cost of capital can be reduced by
 - Reducing the operating risk in investments/assets
 - Changing the financial mix
 - Changing the financing composition

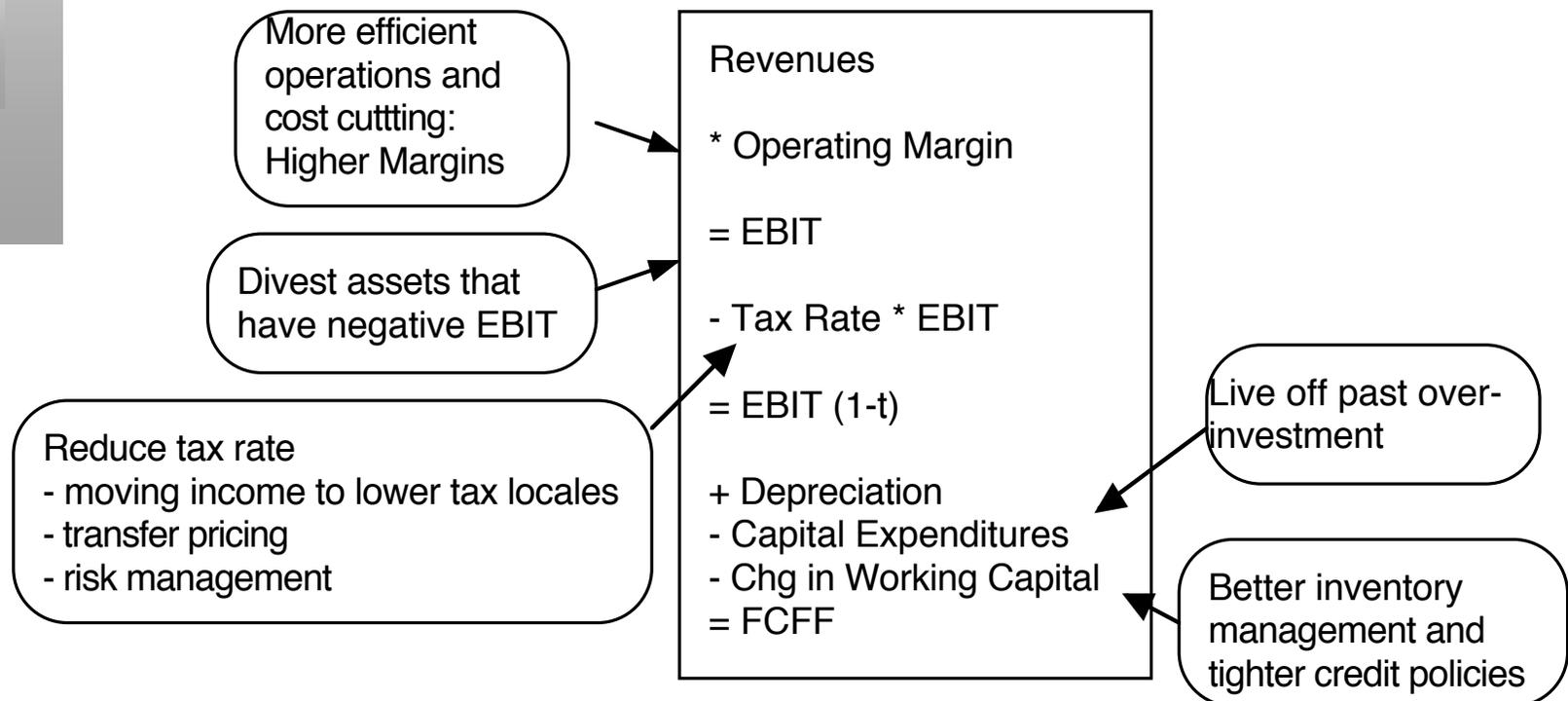
A Basic Proposition

- For an action to affect the value of the firm, it has to
 - Affect current cash flows (or)
 - Affect future growth (or)
 - Affect the length of the high growth period (or)
 - Affect the discount rate (cost of capital)
- **Proposition 1: Actions that do not affect current cash flows, future growth, the length of the high growth period or the discount rate cannot affect value.**

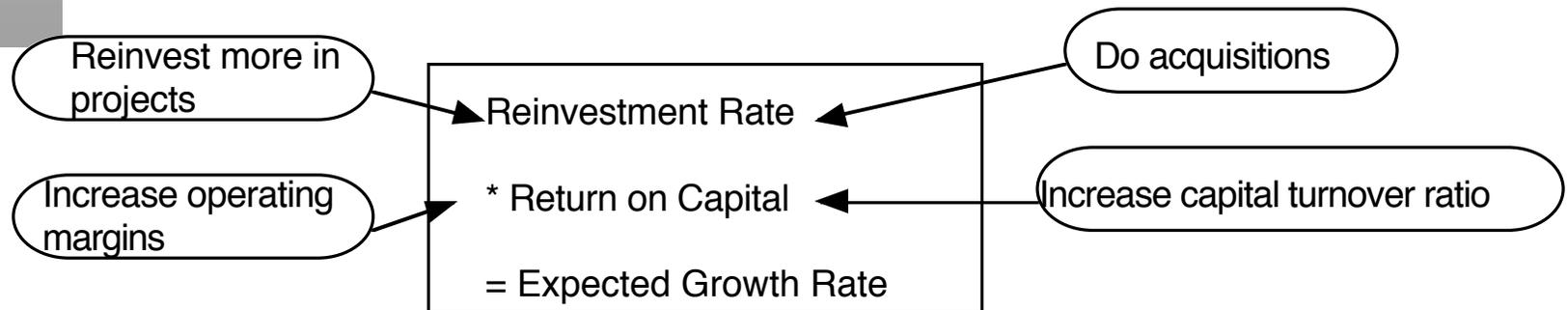
Value-Neutral Actions

- Stock splits and stock dividends change the number of units of equity in a firm, but cannot affect firm value since they do not affect cash flows, growth or risk.
- Accounting decisions that affect reported earnings but not cash flows should have no effect on value.
 - Changing inventory valuation methods from FIFO to LIFO or vice versa in financial reports but not for tax purposes
 - Changing the depreciation method used in financial reports (but not the tax books) from accelerated to straight line depreciation
 - Major non-cash restructuring charges that reduce reported earnings but are not tax deductible
 - Using pooling instead of purchase in acquisitions cannot change the value of a target firm.
- Decisions that create new securities on the existing assets of the firm (without altering the financial mix) such as tracking stock cannot create value, though they might affect perceptions and hence the price.

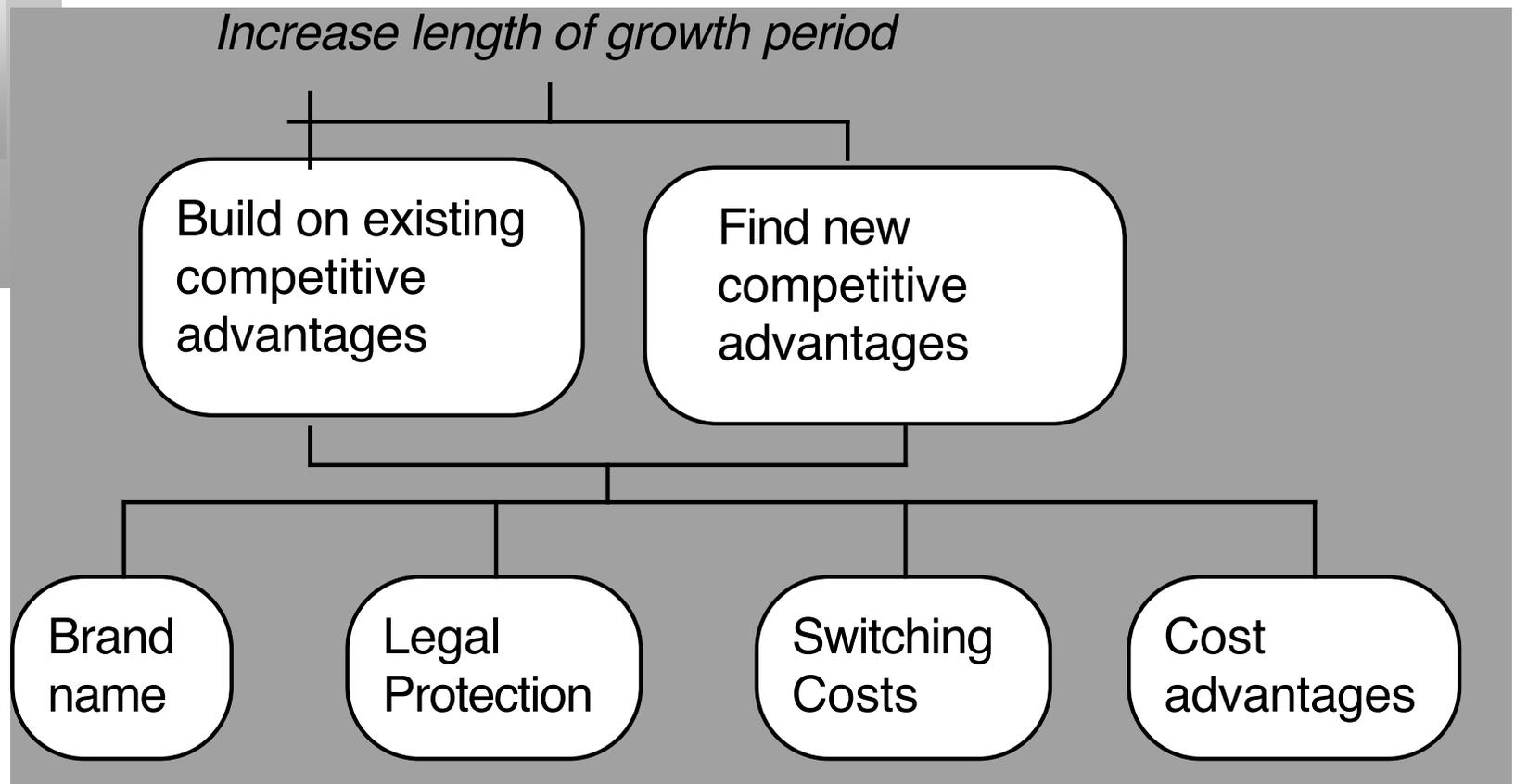
I. Ways of Increasing Cash Flows from Assets in Place



II. Value Enhancement through Growth



III. Building Competitive Advantages: Increase length of the growth period



3.1: The Brand Name Advantage

- Some firms are able to sustain above-normal returns and growth because they have well-recognized brand names that allow them to charge higher prices than their competitors and/or sell more than their competitors.
- Firms that are able to improve their brand name value over time can increase both their growth rate and the period over which they can expect to grow at rates above the stable growth rate, thus increasing value.

Illustration: Valuing a brand name: Coca Cola

	Coca Cola	Generic Cola Company
AT Operating Margin	18.56%	7.50%
Sales/BV of Capital	1.67	1.67
ROC	31.02%	12.53%
Reinvestment Rate	65.00% (19.35%)	65.00% (47.90%)
Expected Growth	20.16%	8.15%
Length	10 years	10 yea
Cost of Equity	12.33%	12.33%
E/(D+E)	97.65%	97.65%
AT Cost of Debt	4.16%	4.16%
D/(D+E)	2.35%	2.35%
Cost of Capital	12.13%	12.13%
Value	\$115	\$13

3.2: Patents and Legal Protection

- The most complete protection that a firm can have from competitive pressure is to own a patent, copyright or some other kind of legal protection allowing it to be the sole producer for an extended period.
- Note that patents only provide partial protection, since they cannot protect a firm against a competitive product that meets the same need but is not covered by the patent protection.
- Licenses and government-sanctioned monopolies also provide protection against competition. They may, however, come with restrictions on excess returns; utilities in the United States, for instance, are monopolies but are regulated when it comes to price increases and returns.

3.3: Switching Costs

- Another potential barrier to entry is the cost associated with switching from one firm's products to another.
- The greater the switching costs, the more difficult it is for competitors to come in and compete away excess returns.
- Firms that devise ways to increase the cost of switching from their products to competitors' products, while reducing the costs of switching from competitor products to their own will be able to increase their expected length of growth.

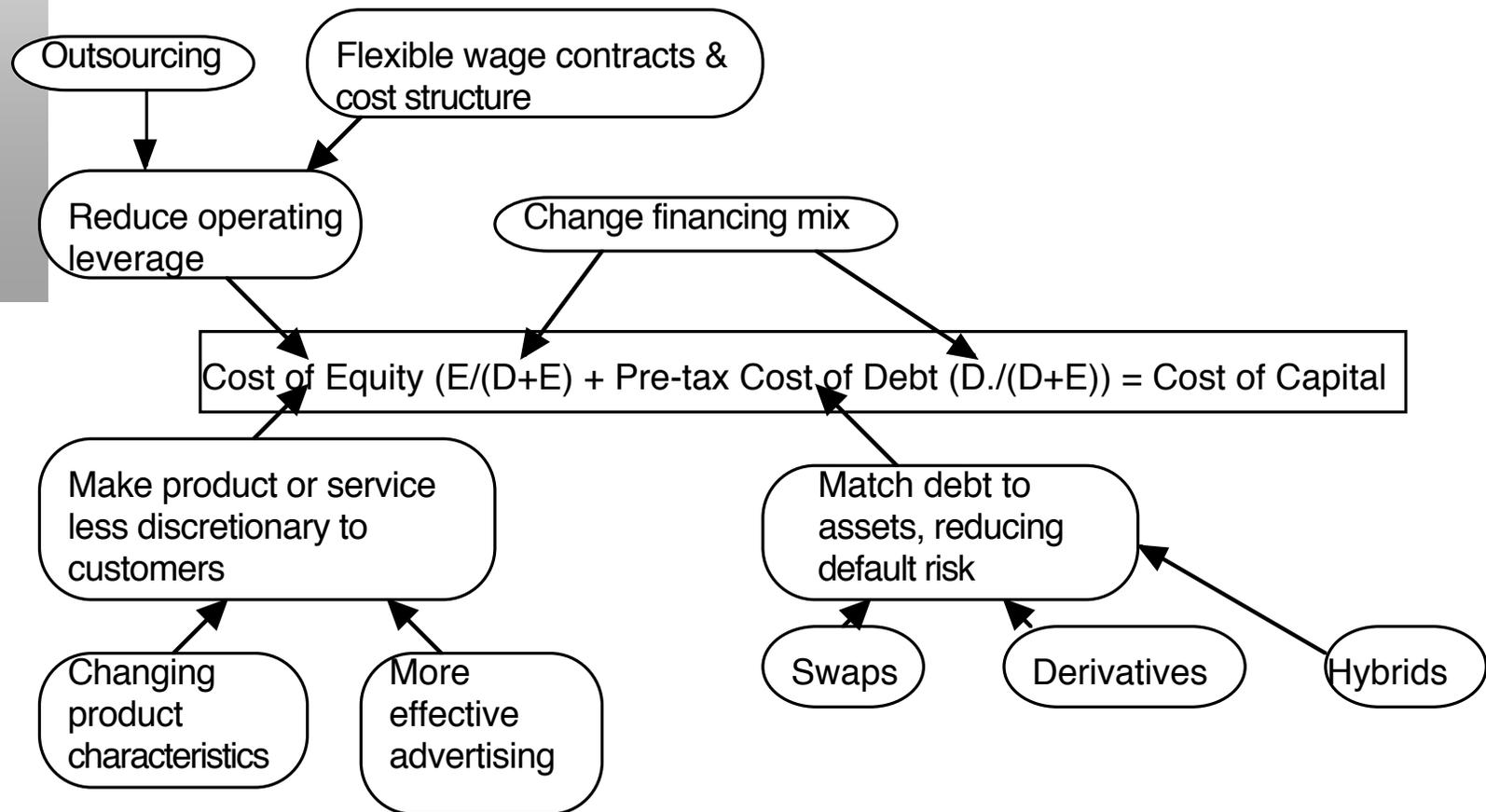
3.4: Cost Advantages

- There are a number of ways in which firms can establish a cost advantage over their competitors, and use this cost advantage as a barrier to entry:
 - In businesses, where scale can be used to reduce costs, economies of scale can give bigger firms advantages over smaller firms
 - Owning or having exclusive rights to a distribution system can provide firms with a cost advantage over its competitors.
 - Owning or having the rights to extract a natural resource which is in restricted supply (The undeveloped reserves of an oil or mining company, for instance)
- These cost advantages will show up in valuation in one of two ways:
 - The firm may charge the same price as its competitors, but have a much higher operating margin.
 - The firm may charge lower prices than its competitors and have a much higher capital turnover ratio.

Gauging Barriers to Entry

- Which of the following barriers to entry are most likely to work for Titan Cement?
 - Brand Name
 - Patents and Legal Protection
 - Switching Costs
 - Cost Advantages
- What about for Amazon.com?
 - Brand Name
 - Patents and Legal Protection
 - Switching Costs
 - Cost Advantages

Reducing Cost of Capital



Amazon.com: Optimal Debt Ratio

Debt Ratio	Beta	Cost of Equity	Bond Rating	Interest rate on debt	Tax Rate	Cost of Debt (after-tax)	WACC	Firm Value (G)
0%	1.58	12.82%	AAA	6.80%	0.00%	6.80%	12.82%	\$29,192
10%	1.76	13.53%	D	18.50%	0.00%	18.50%	14.02%	\$24,566
20%	1.98	14.40%	D	18.50%	0.00%	18.50%	15.22%	\$21,143
30%	2.26	15.53%	D	18.50%	0.00%	18.50%	16.42%	\$18,509
40%	2.63	17.04%	D	18.50%	0.00%	18.50%	17.62%	\$16,419
50%	3.16	19.15%	D	18.50%	0.00%	18.50%	18.82%	\$14,719
60%	3.95	22.31%	D	18.50%	0.00%	18.50%	20.02%	\$13,311
70%	5.27	27.58%	D	18.50%	0.00%	18.50%	21.22%	\$12,125
80%	7.90	38.11%	D	18.50%	0.00%	18.50%	22.42%	\$11,112
90%	15.81	69.73%	D	18.50%	0.00%	18.50%	23.62%	\$10,237

Titan : Optimal Capital Structure

Debt Ratio	Beta	Cost of Equity	Bond Rating	Interest rate on debt	Tax Rate	Cost of Debt (after-tax)	WACC	Firm Value (G)
0%	0.80	7.84%	AAA	4.88%	30.44%	3.39%	7.84%	\$1,658
10%	0.86	8.12%	AAA	4.88%	30.44%	3.39%	7.64%	\$1,732
20%	0.93	8.46%	A+	5.23%	30.44%	3.64%	7.49%	\$1,793
30%	1.03	8.90%	A-	5.53%	30.44%	3.85%	7.38%	\$1,841
40%	1.16	9.48%	B+	7.78%	30.44%	5.41%	7.85%	\$1,654
50%	1.35	10.30%	B-	10.53%	30.44%	7.32%	8.81%	\$1,371
60%	1.65	11.63%	CC	14.53%	28.47%	10.39%	10.89%	\$1,001
70%	2.20	14.08%	CC	14.53%	24.40%	10.98%	11.91%	\$883
80%	3.38	19.32%	C	16.53%	18.77%	13.43%	14.61%	\$675
90%	6.76	34.34%	C	16.53%	16.68%	13.77%	15.83%	\$609

Titan Cements: Restructured

Reinvest more in lower return projects

Return on Capital 15.00%

Current Cashflow to Firm

EBIT(1-t) :	154
- Nt CpX	29
- Chg WC	10
= FCFF	115
Reinvestment Rate = 39/115=25%	

Reinvestment Rate 50.00%

Expected Growth in EBIT (1-t)
 $.50 * .15 = .075$
7.50%

Stable Growth
 $g = 4.31\%$; Beta = 1.00;
 Country Premium = 0%
 Cost of capital = 6.78%
 ROC = 6.78%; Tax rate = 33%
 Reinvestment Rate = 63.5%

Terminal Value₅ = $81.2 / (.0678 - .0431) = 3283$

Op. Assets	2,688
+ Cash:	81
- Debt	396
- Minor. Int.	101
= Equity	2,279
- Options	0
Value/Share	59.68

EBIT	€ 238.65	€ 256.55	€ 275.79	€ 296.47	€ 318.71
EBIT(1-t)	€ 166.00	€ 178.46	€ 191.84	€ 206.23	€ 221.69
- Reinvestment	€ 83.00	€ 89.23	€ 95.92	€ 103.11	€ 110.85
= FCFF	€ 83.00	€ 89.23	€ 95.92	€ 103.11	€ 110.85

Term Yr

332.4
222.7
141.5
81.2

Discount at Cost of Capital (WACC) = $8.88\% (.70) + 3.85\% (0.30) = 7.27\%$

Optimize capital structure

Cost of Equity 8.88%

Cost of Debt
 $(4.31\% + \% + .22\%)(1 - .3044)$
 = 3.85%

Weights
 E = 70% D = 30%

Riskfree Rate:
Euro riskfree rate = 4.31%

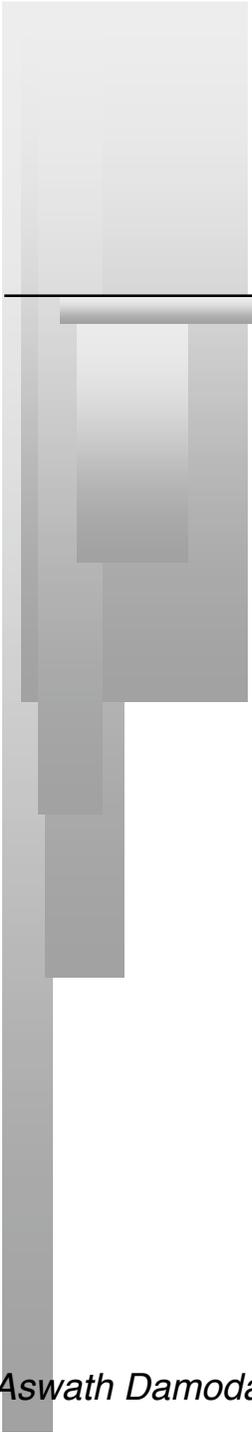
+ **Beta 1.03** X **Risk Premium 4.44%**

Unlevered Beta for Sectors: 0.80 Firm's D/E Ratio: 42.9%

Mature risk premium 4% Country Equity Prem 0.44%

The Value of Control?

- If the value of a firm run optimally is significantly higher than the value of the firm with the status quo (or incumbent management), you can write the value that you should be willing to pay as:
- $\text{Value of control} = \text{Value of firm optimally run} - \text{Value of firm with status quo}$
- Implications:
 - The value of control is greatest at poorly run firms.
 - Voting shares in poorly run firms should trade at a premium on non-voting shares if the votes associated with the shares will give you a chance to have a say in a hostile acquisition.
 - When valuing private firms, your estimate of value will vary depending upon whether you gain control of the firm. For example, 49% of a private firm may be worth less than 51% of the same firm.
 - 49% stake = 49% of status quo value
 - 51% stake = 51% of optimal value



Relative Valuation

Aswath Damodaran

What is relative valuation?

- In relative valuation, the value of an asset is compared to the values assessed by the market for similar or comparable assets.
- To do relative valuation then,
 - we need to identify comparable assets and obtain market values for these assets
 - convert these market values into standardized values, since the absolute prices cannot be compared This process of standardizing creates price multiples.
 - compare the standardized value or multiple for the asset being analyzed to the standardized values for comparable asset, controlling for any differences between the firms that might affect the multiple, to judge whether the asset is under or over valued

Relative valuation is pervasive...

- Most valuations on Wall Street are relative valuations.
 - Almost 85% of equity research reports are based upon a multiple and comparables.
 - More than 50% of all acquisition valuations are based upon multiples
 - Rules of thumb based on multiples are not only common but are often the basis for final valuation judgments.
- While there are more discounted cashflow valuations in consulting and corporate finance, they are often relative valuations masquerading as discounted cash flow valuations.
 - The objective in many discounted cashflow valuations is to back into a number that has been obtained by using a multiple.
 - The terminal value in a significant number of discounted cashflow valuations is estimated using a multiple.

Why relative valuation?

“If you think I’m crazy, you should see the guy who lives across the hall”

Jerry Seinfeld talking about Kramer in a Seinfeld episode

“ A little inaccuracy sometimes saves tons of explanation”



Michael Richards
Photo: George Langa

H.H. Munro

“ If you are going to screw up, make sure that you have lots of company”

Ex-portfolio manager

©NBC, Inc.

So, you believe only in intrinsic value? Here's why you should still care about relative value

- Even if you are a true believer in discounted cashflow valuation, presenting your findings on a relative valuation basis will make it more likely that your findings/recommendations will reach a receptive audience.
- In some cases, relative valuation can help find weak spots in discounted cash flow valuations and fix them.
- The problem with multiples is not in their use but in their abuse. If we can find ways to frame multiples right, we should be able to use them better.

Standardizing Value

- You can standardize either the equity value of an asset or the value of the asset itself, which goes in the numerator.
- You can standardize by dividing by the
 - Earnings of the asset
 - Price/Earnings Ratio (PE) and variants (PEG and Relative PE)
 - Value/EBIT
 - Value/EBITDA
 - Value/Cash Flow
 - Book value of the asset
 - Price/Book Value(of Equity) (PBV)
 - Value/ Book Value of Assets
 - Value/Replacement Cost (Tobin's Q)
 - Revenues generated by the asset
 - Price/Sales per Share (PS)
 - Value/Sales
 - Asset or Industry Specific Variable (Price/kwh, Price per ton of steel)

The Four Steps to Understanding Multiples

- Define the multiple
 - In use, the same multiple can be defined in different ways by different users. When comparing and using multiples, estimated by someone else, it is critical that we understand how the multiples have been estimated
- Describe the multiple
 - Too many people who use a multiple have no idea what its cross sectional distribution is. If you do not know what the cross sectional distribution of a multiple is, it is difficult to look at a number and pass judgment on whether it is too high or low.
- Analyze the multiple
 - It is critical that we understand the fundamentals that drive each multiple, and the nature of the relationship between the multiple and each variable.
- Apply the multiple
 - Defining the comparable universe and controlling for differences is far more difficult in practice than it is in theory.

Definitional Tests

- Is the multiple consistently defined?
 - **Proposition 1: Both the value (the numerator) and the standardizing variable (the denominator) should be to the same claimholders in the firm. In other words, the value of equity should be divided by equity earnings or equity book value, and firm value should be divided by firm earnings or book value.**
- Is the multiple uniformly estimated?
 - The variables used in defining the multiple should be estimated uniformly across assets in the “comparable firm” list.
 - If earnings-based multiples are used, the accounting rules to measure earnings should be applied consistently across assets. The same rule applies with book-value based multiples.

Descriptive Tests

- What is the average and standard deviation for this multiple, across the universe (market)?
- What is the median for this multiple?
 - The median for this multiple is often a more reliable comparison point.
- How large are the outliers to the distribution, and how do we deal with the outliers?
 - Throwing out the outliers may seem like an obvious solution, but if the outliers all lie on one side of the distribution (they usually are large positive numbers), this can lead to a biased estimate.
- Are there cases where the multiple cannot be estimated? Will ignoring these cases lead to a biased estimate of the multiple?
- How has this multiple changed over time?

Analytical Tests

- What are the fundamentals that determine and drive these multiples?
 - Proposition 2: Embedded in every multiple are all of the variables that drive every discounted cash flow valuation - growth, risk and cash flow patterns.
 - In fact, using a simple discounted cash flow model and basic algebra should yield the fundamentals that drive a multiple
- How do changes in these fundamentals change the multiple?
 - The relationship between a fundamental (like growth) and a multiple (such as PE) is seldom linear. For example, if firm A has twice the growth rate of firm B, it will generally not trade at twice its PE ratio
 - **Proposition 3: It is impossible to properly compare firms on a multiple, if we do not know the nature of the relationship between fundamentals and the multiple.**

Application Tests

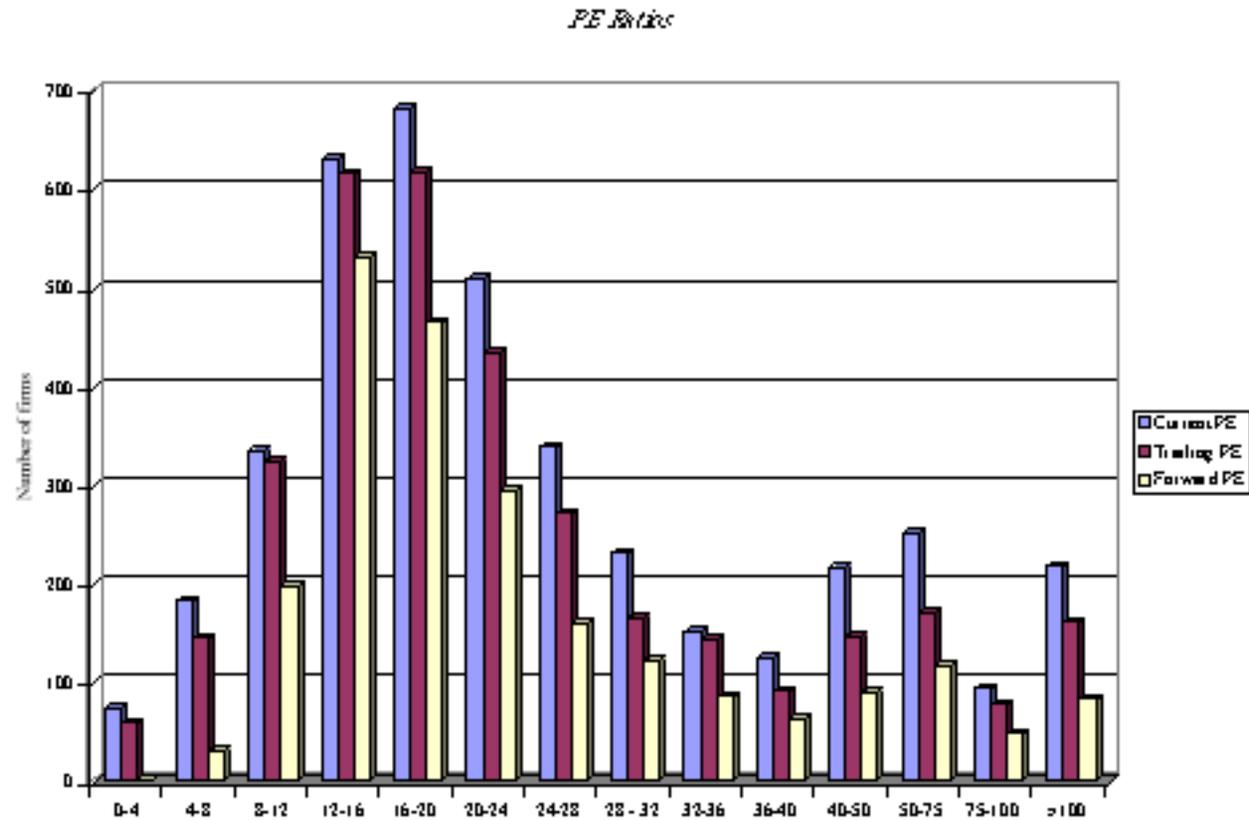
- Given the firm that we are valuing, what is a “comparable” firm?
 - While traditional analysis is built on the premise that firms in the same sector are comparable firms, valuation theory would suggest that a comparable firm is one which is similar to the one being analyzed in terms of fundamentals.
 - **Proposition 4: There is no reason why a firm cannot be compared with another firm in a very different business, if the two firms have the same risk, growth and cash flow characteristics.**
- Given the comparable firms, how do we adjust for differences across firms on the fundamentals?
 - **Proposition 5: It is impossible to find an exactly identical firm to the one you are valuing.**

Price Earnings Ratio: Definition

$$\text{PE} = \text{Market Price per Share} / \text{Earnings per Share}$$

- There are a number of variants on the basic PE ratio in use. They are based upon how the price and the earnings are defined.
- Price: is usually the current price
 - is sometimes the average price for the year
- EPS:
 - earnings per share in most recent financial year
 - earnings per share in trailing 12 months (Trailing PE)
 - forecasted earnings per share next year (Forward PE)
 - forecasted earnings per share in future year

PE Ratio: Distribution for the US: January 2004



PE: Deciphering the Distribution

	<i>Current PE</i>	<i>Trailing PE</i>	<i>Forward PE</i>
Mean	41.41	41.53	30.90
Standard Error	2.42	3.64	1.10
Median	20.76	19.39	19.21
Kurtosis	1062.81	700.63	252.62
Skewness	27.78	24.21	12.48
Minimum	0.40	1.22	2.57
Maximum	6841.25	7184.00	1430.00
Count	4032	3492	2281
500th largest	54.50	43.98	31.13
500th smallest	11.31	11.13	14.29

Comparing PE Ratios: Europe, Japan and Emerging Markets

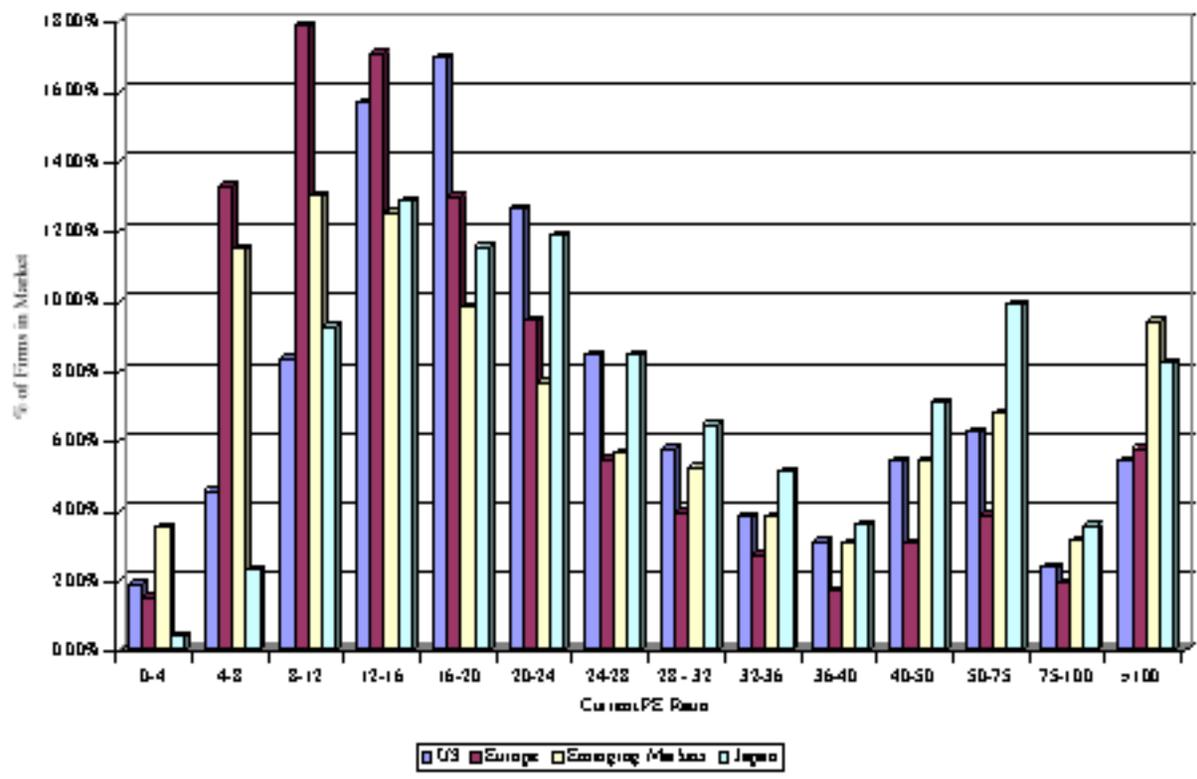
Median PE
Japan = 24.74

US = 20.76

Em. Mkts = 18.87

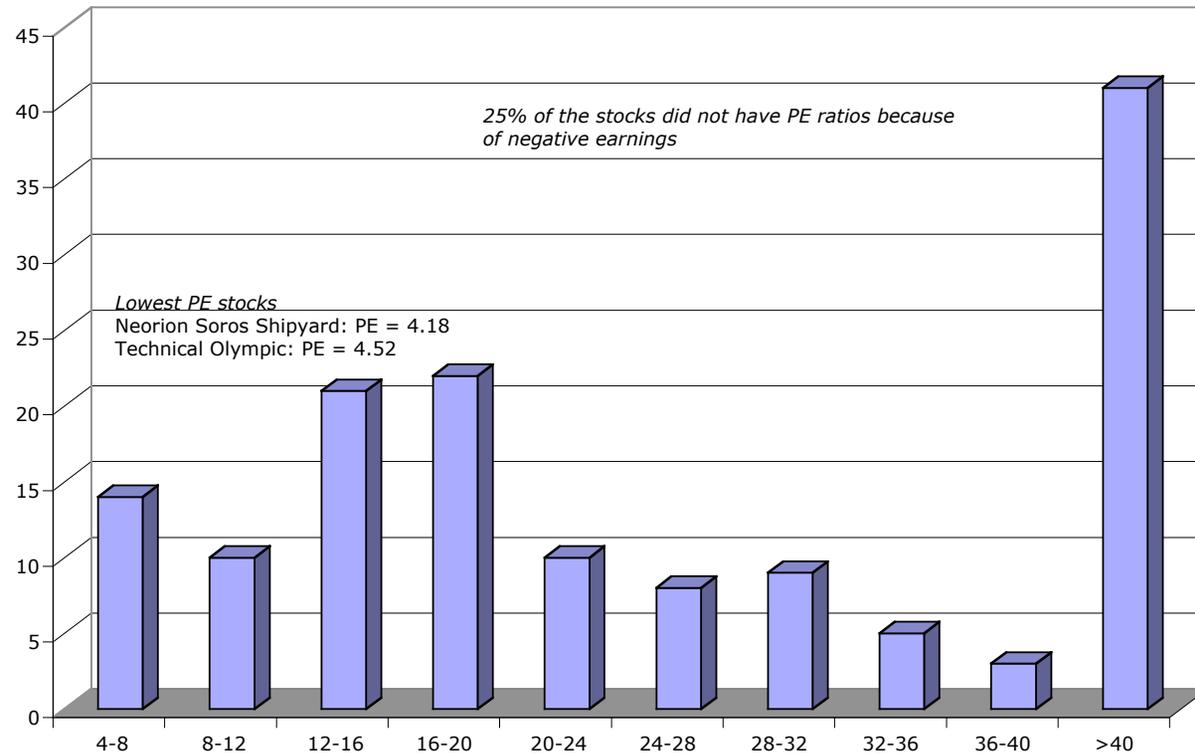
: 15.99

Comparing PE Ratios across Markets



What about Greece?

PE Ratio: Greece



PE Ratio: Understanding the Fundamentals

- To understand the fundamentals, start with a basic equity discounted cash flow model.

- With the dividend discount model,

$$P_0 = \frac{DPS_1}{r - g_n}$$

- Dividing both sides by the earnings per share,

$$\frac{P_0}{EPS_0} = PE = \frac{\text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

- If this had been a FCFE Model,

$$P_0 = \frac{FCFE_1}{r - g_n}$$

$$\frac{P_0}{EPS_0} = PE = \frac{(\text{FCFE/Earnings}) * (1 + g_n)}{r - g_n}$$

PE Ratio and Fundamentals

- **Proposition: Other things held equal, higher growth firms will have higher PE ratios than lower growth firms.**
- **Proposition: Other things held equal, higher risk firms will have lower PE ratios than lower risk firms**
- **Proposition: Other things held equal, firms with lower reinvestment needs will have higher PE ratios than firms with higher reinvestment rates.**
- **Of course, other things are difficult to hold equal since high growth firms, tend to have risk and high reinvestment rates.**

Using the Fundamental Model to Estimate PE For a High Growth Firm

- The price-earnings ratio for a high growth firm can also be related to fundamentals. In the special case of the two-stage dividend discount model, this relationship can be made explicit fairly simply:

$$P_0 = \frac{\text{EPS}_0 * \text{Payout Ratio} * (1+g) * \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)}{r-g} + \frac{\text{EPS}_0 * \text{Payout Ratio}_n * (1+g)^n * (1+g_n)}{(r-g_n)(1+r)^n}$$

- For a firm that does not pay what it can afford to in dividends, substitute FCFE/Earnings for the payout ratio.
- Dividing both sides by the earnings per share:

$$\frac{P_0}{\text{EPS}_0} = \frac{\text{Payout Ratio} * (1+g) * \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)}{r-g} + \frac{\text{Payout Ratio}_n * (1+g)^n * (1+g_n)}{(r-g_n)(1+r)^n}$$

Expanding the Model

- In this model, the PE ratio for a high growth firm is a function of growth, risk and payout, exactly the same variables that it was a function of for the stable growth firm.
- The only difference is that these inputs have to be estimated for two phases - the high growth phase and the stable growth phase.
- Expanding to more than two phases, say the three stage model, will mean that risk, growth and cash flow patterns in each stage.

A Simple Example

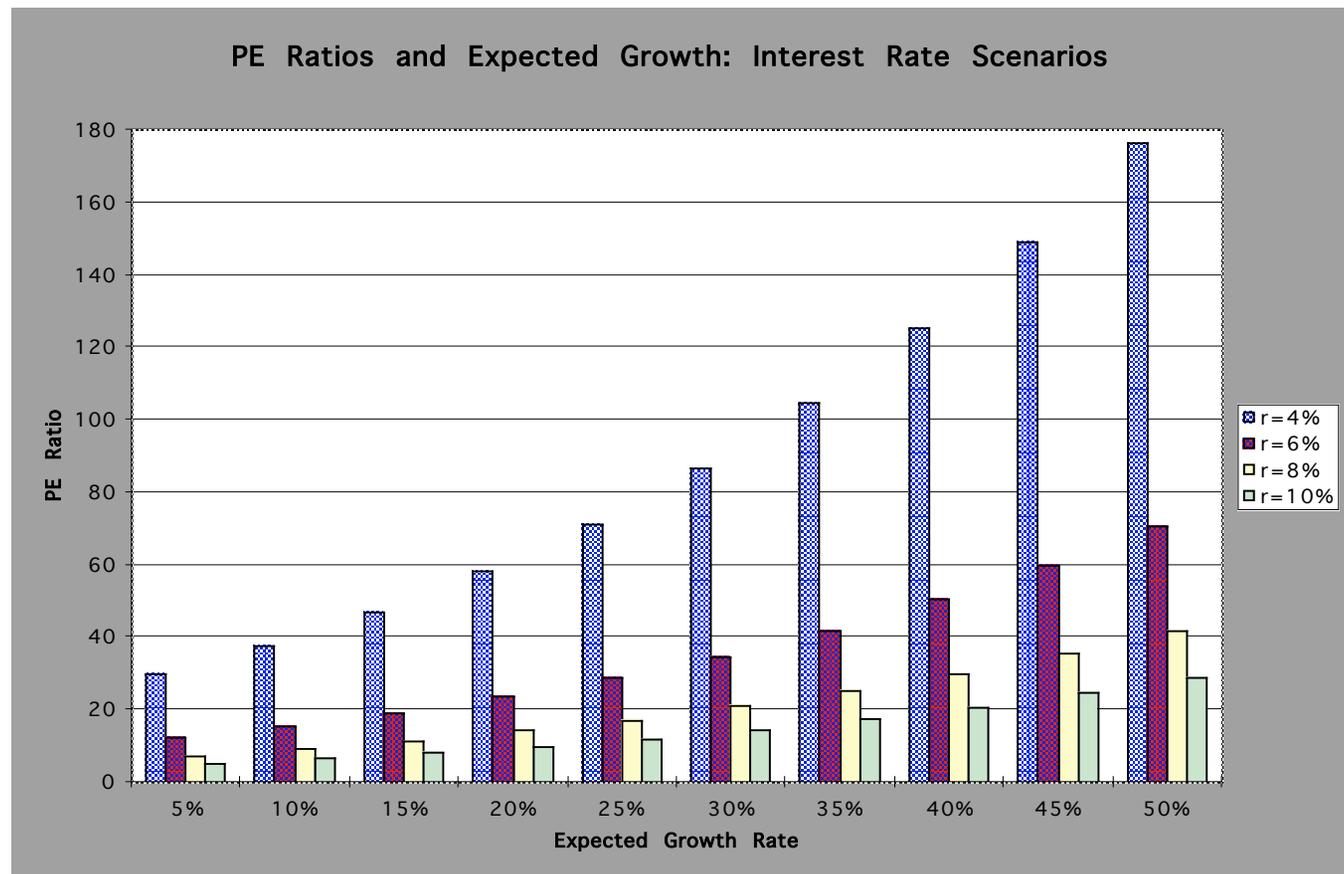
- Assume that you have been asked to estimate the PE ratio for a firm which has the following characteristics:

Variable	High Growth Phase	Stable Growth Phase
Expected Growth Rate	25%	8%
Payout Ratio	20%	50%
Beta	1.00	1.00
Number of years	5 years	Forever after year 5

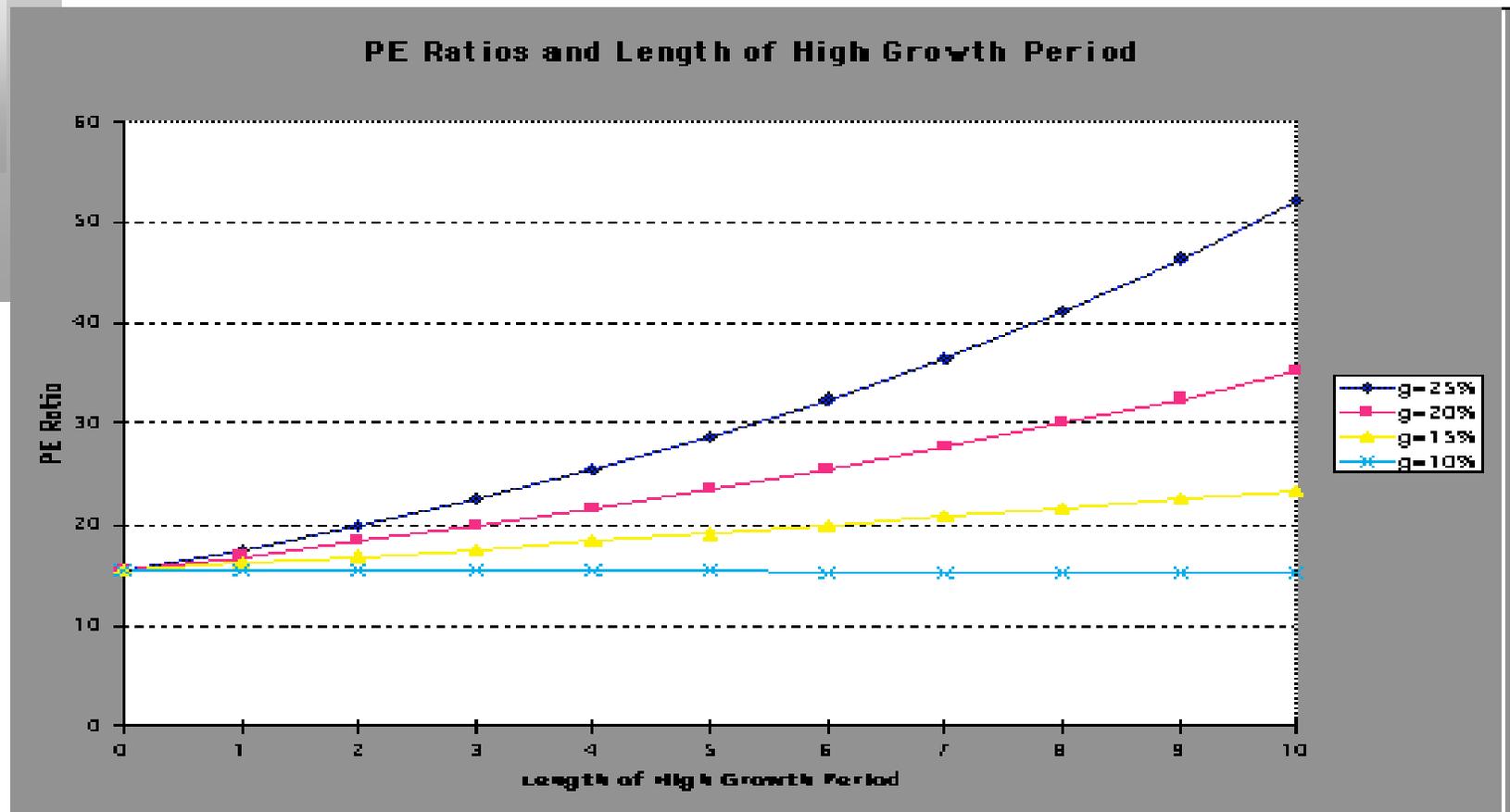
- Riskfree rate = T.Bond Rate = 6%
- Required rate of return = 6% + 1(5.5%)= 11.5%

$$PE = \frac{0.2 * (1.25) * \left(1 - \frac{(1.25)^5}{(1.115)^5}\right)}{(.115 - .25)} + \frac{0.5 * (1.25)^5 * (1.08)}{(.115 - .08) (1.115)^5} = 28.75$$

PE and Growth: Firm grows at $x\%$ for 5 years, 8% thereafter

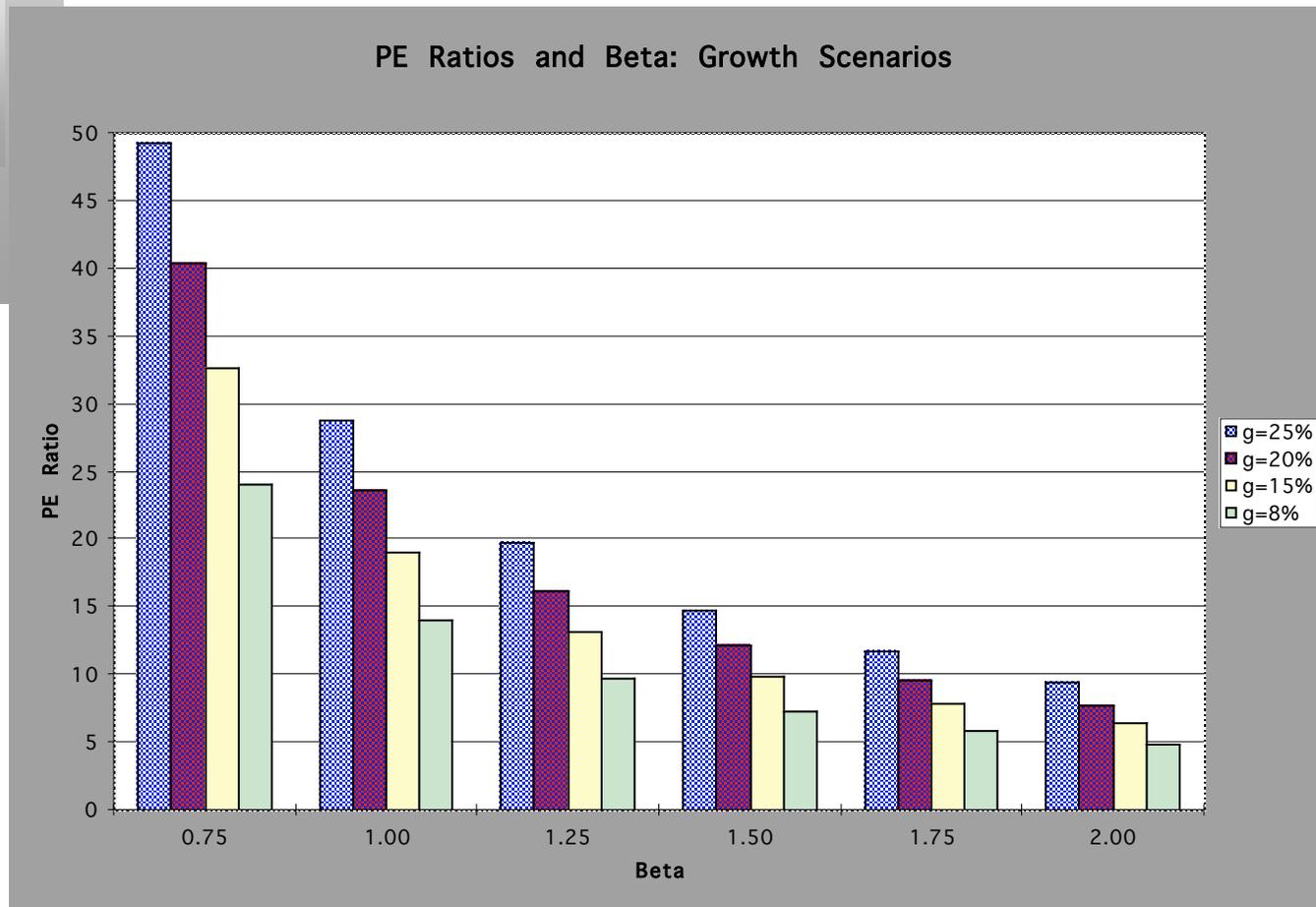


PE Ratios and Length of High Growth: 25% growth for n years; 8% thereafter

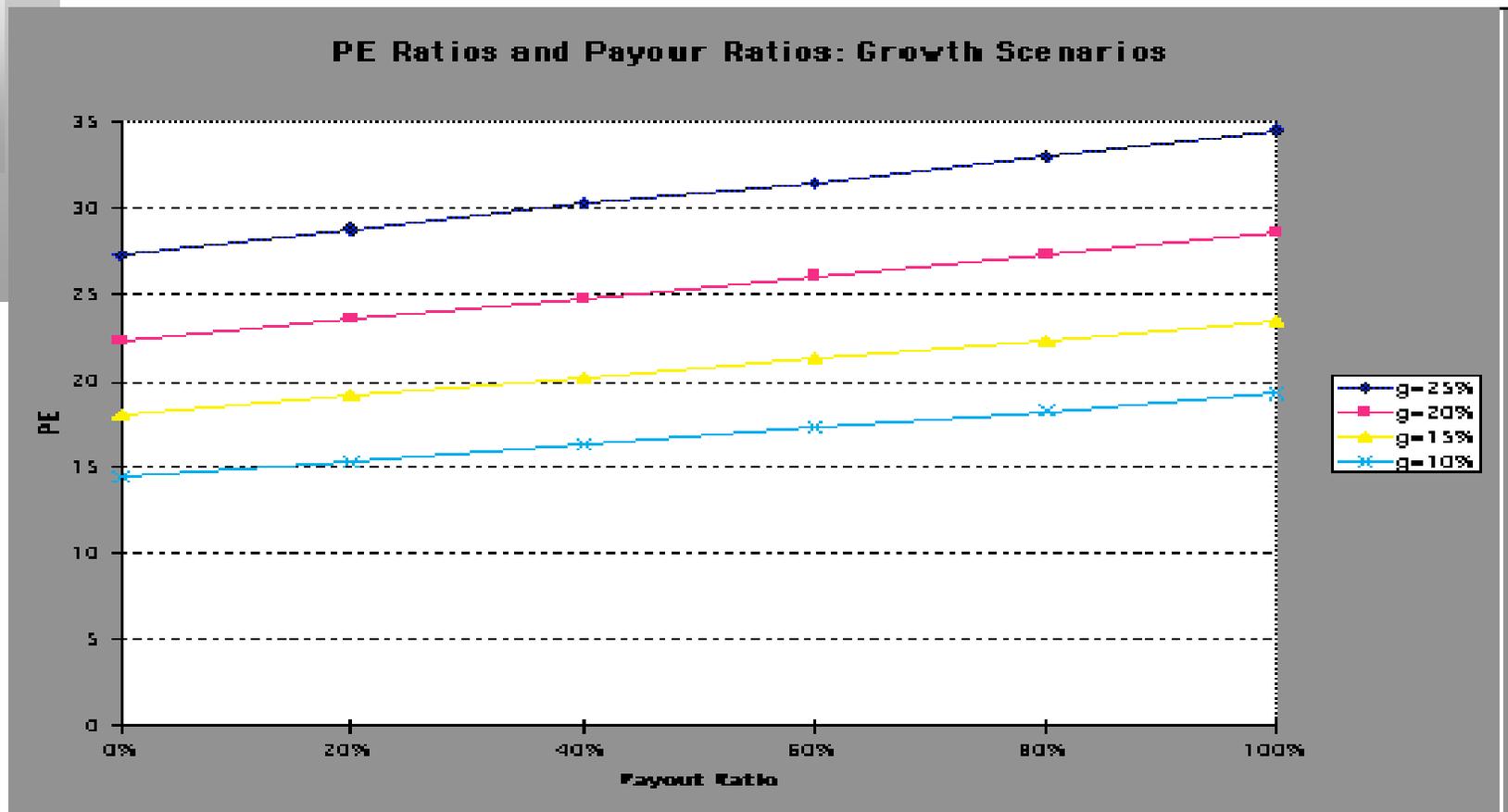


PE and Risk: Effects of Changing Betas on PE Ratio:

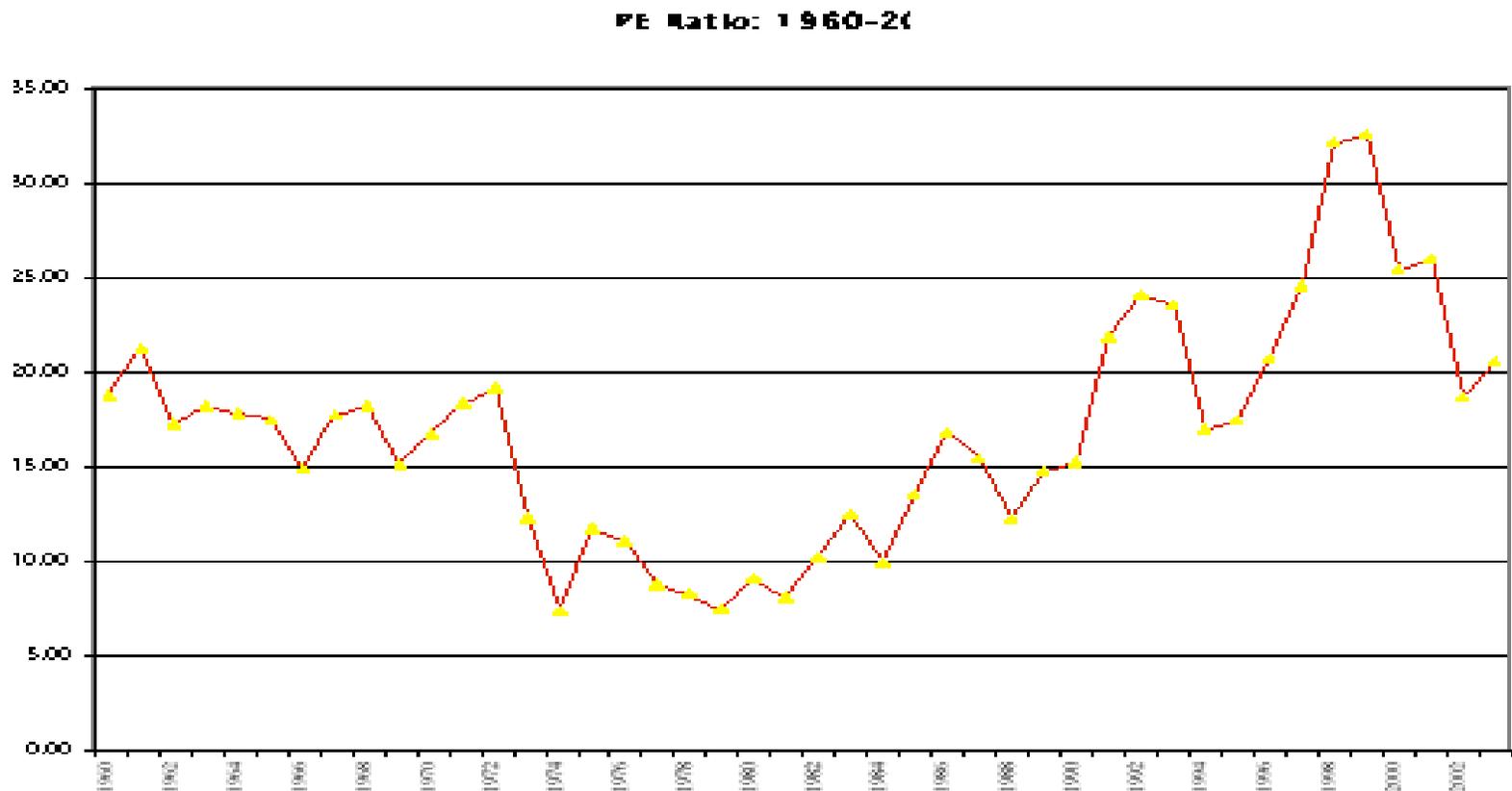
Firm with $x\%$ growth for 5 years; 8% thereafter



PE and Payout



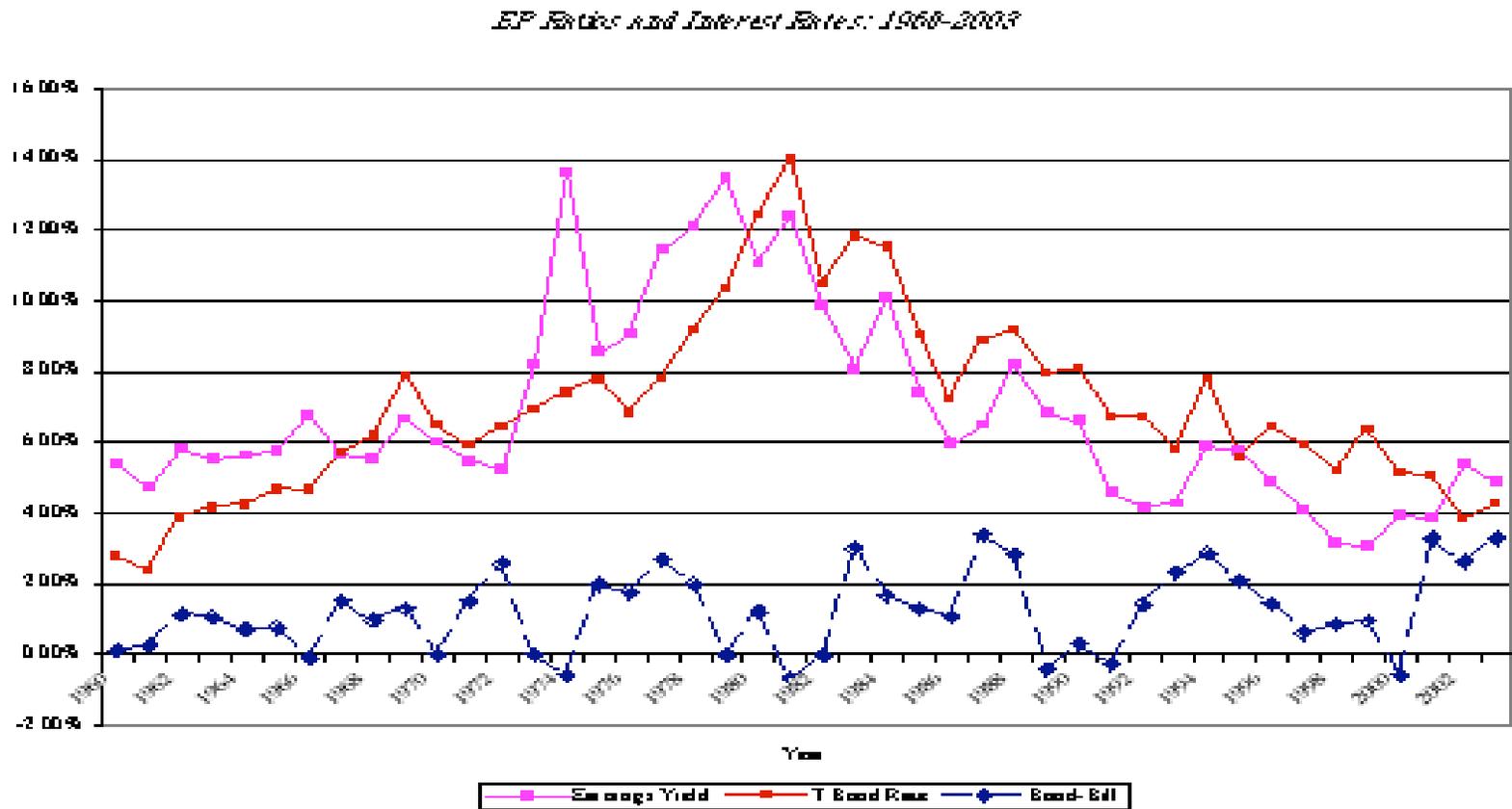
I. Comparisons of PE across time: PE Ratio for the S&P 500



Is low (high) PE cheap (expensive)?

- A market strategist argues that stocks are over priced because the PE ratio today is too high relative to the average PE ratio across time. Do you agree?
 - Yes
 - No
- If you do not agree, what factors might explain the higher PE ratio today?

E/P Ratios , T.Bond Rates and Term Structure



Regression Results

- There is a strong positive relationship between E/P ratios and T.Bond rates, as evidenced by the correlation of 0.69 between the two variables.,
- In addition, there is evidence that the term structure also affects the PE ratio.
- In the following regression, using 1960-2003 data, we regress E/P ratios against the level of T.Bond rates and a term structure variable (T.Bond - T.Bill rate)

$$\text{E/P} = 2.03\% + 0.753 \text{ T.Bond Rate} - 0.355 (\text{T.Bond Rate} - \text{T.Bill Rate})$$

(2.19) (6.38) (-1.38)

$$\text{R squared} = 50.85\%$$

II. Comparing PE Ratios across a Sector

<i>Company Name</i>	<i>PE</i>	<i>Growth</i>
<i>PT Indosat ADR</i>	<i>7.8</i>	<i>0.06</i>
<i>Telebras ADR</i>	<i>8.9</i>	<i>0.075</i>
<i>Telecom Corporation of New Zealand ADR</i>	<i>11.2</i>	<i>0.11</i>
<i>Telecom Argentina Stet - France Telecom SA ADR B</i>	<i>12.5</i>	<i>0.08</i>
<i>Hellenic Telecommunication Organization SA ADR</i>	<i>12.8</i>	<i>0.12</i>
<i>Telecomunicaciones de Chile ADR</i>	<i>16.6</i>	<i>0.08</i>
<i>Swisscom AG ADR</i>	<i>18.3</i>	<i>0.11</i>
<i>Asia Satellite Telecom Holdings ADR</i>	<i>19.6</i>	<i>0.16</i>
<i>Portugal Telecom SA ADR</i>	<i>20.8</i>	<i>0.13</i>
<i>Telefonos de Mexico ADR L</i>	<i>21.1</i>	<i>0.14</i>
<i>Matav RT ADR</i>	<i>21.5</i>	<i>0.22</i>
<i>Telstra ADR</i>	<i>21.7</i>	<i>0.12</i>
<i>Gilat Communications</i>	<i>22.7</i>	<i>0.31</i>
<i>Deutsche Telekom AG ADR</i>	<i>24.6</i>	<i>0.11</i>
<i>British Telecommunications PLC ADR</i>	<i>25.7</i>	<i>0.07</i>
<i>Tele Danmark AS ADR</i>	<i>27</i>	<i>0.09</i>
<i>Telekomunikasi Indonesia ADR</i>	<i>28.4</i>	<i>0.32</i>
<i>Cable & Wireless PLC ADR</i>	<i>29.8</i>	<i>0.14</i>
<i>APT Satellite Holdings ADR</i>	<i>31</i>	<i>0.33</i>
<i>Telefonica SA ADR</i>	<i>32.5</i>	<i>0.18</i>
<i>Royal KPN NV ADR</i>	<i>35.7</i>	<i>0.13</i>
<i>Telecom Italia SPA ADR</i>	<i>42.2</i>	<i>0.14</i>
<i>Nippon Telegraph & Telephone ADR</i>	<i>44.3</i>	<i>0.2</i>
<i>France Telecom SA ADR</i>	<i>45.2</i>	<i>0.19</i>
<i>Korea Telecom ADR</i>	<i>71.3</i>	<i>0.44</i>

PE, Growth and Risk

Dependent variable is: PE

R squared = 66.2% R squared (adjusted) = 63.1%

Variable	Coefficient	SE	t-ratio	prob
Constant	13.1151	3.471	3.78	0.0010
Growth rate	121.223	19.27	6.29	≤ 0.0001
Emerging Market	-13.8531	3.606	-3.84	0.0009

Emerging Market is a dummy: 1 if emerging market
0 if not

Is Telebras under valued?

- Predicted PE = $13.12 + 121.22 (.075) - 13.85 (1) = 8.35$
- At an actual price to earnings ratio of 8.9, Telebras is slightly overvalued.
- What about Hellenic Telecom?
 - If viewed as a developed market telecom
 $13.12 + 121.22 (0.12) - 13.85 (0) = 27.66$
It is dramatically undervalued at 12.8 times earnings
 - If viewed as an emerging market telecom
 $13.12 + 121.22 (0.12) - 13.85 (1) = 13.81$
It is close to fairly valued

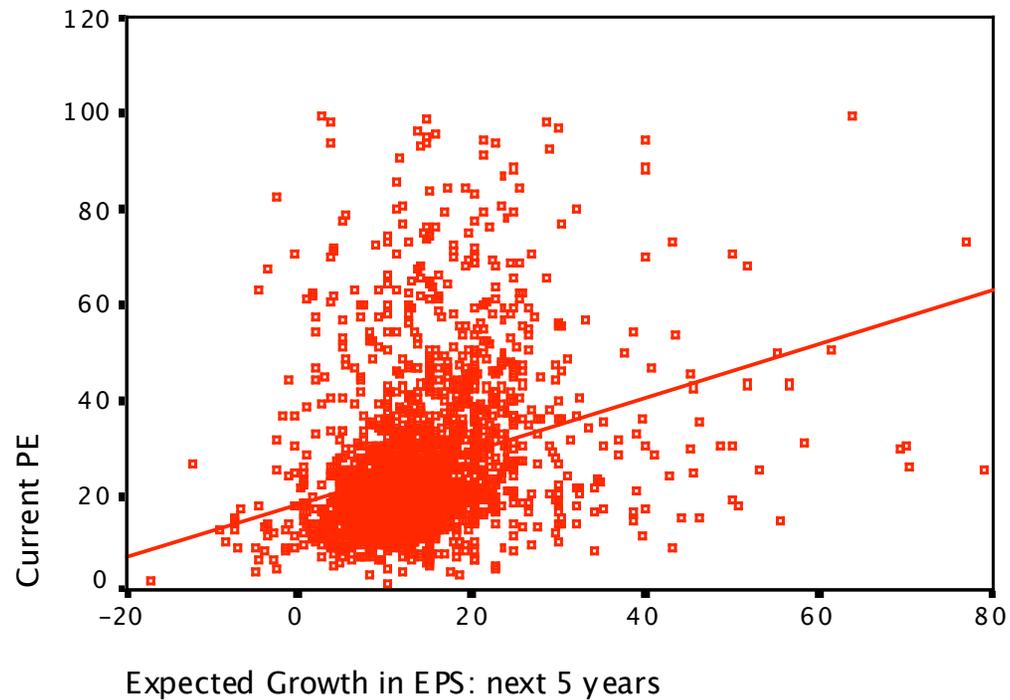
Using the entire crosssection: A regression approach

- In contrast to the 'comparable firm' approach, the information in the entire cross-section of firms can be used to predict PE ratios.
- The simplest way of summarizing this information is with a multiple regression, with the PE ratio as the dependent variable, and proxies for risk, growth and payout forming the independent variables.

PE versus Growth

Current PE vs Expected Growth in EPS

January 2004: US Companies



PE Ratio: Standard Regression for US stocks - January 2004

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.467 ^a	.218	.217	1049.7506205340

a. Predictor(s): (Constant), PAYOUT, Regression Beta, Expected Growth in EPS: next 5 years

Coefficients^{a,b}

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	9.475	.961		9.862	.000
	Expected Growth in EPS: next 5 years	.814	.046	.375	17.558	.000
	Regression Beta	6.283	.437	.298	14.375	.000
	PAYOUT	6.E-02	.014	.092	4.161	.000

a. Dependent Variable: Current PE

b. Weighted Least Squares Regression - Weighted by Market Cap

Problems with the regression methodology

- The basic regression assumes a linear relationship between PE ratios and the financial proxies, and that might not be appropriate.
- The basic relationship between PE ratios and financial variables itself might not be stable, and if it shifts from year to year, the predictions from the model may not be reliable.
- The independent variables are correlated with each other. For example, high growth firms tend to have high risk. This multi-collinearity makes the coefficients of the regressions unreliable and may explain the large changes in these coefficients from period to period.

The Multicollinearity Problem

Correlations

		Expected Growth in Revenues: next 5 years	Regression Beta	PAYOUT
Expected Growth in Revenues: next 5 years	Pearson Correlation	1	.031	-.325**
	Sig. (2-tailed)	.	.228	.000
	N	1472	1472	1185
Regression Beta	Pearson Correlation	.031	1	-.183**
	Sig. (2-tailed)	.228	.	.000
	N	1472	6933	4187
PAYOUT	Pearson Correlation	-.325**	-.183**	1
	Sig. (2-tailed)	.000	.000	.
	N	1185	4187	4187

** . Correlation is significant at the 0.01 level (2-tailed).

Using the PE ratio regression

- Assume that you were given the following information for Dell. The firm has an expected growth rate of 10%, a beta of 1.20 and pays no dividends. Based upon the regression, estimate the predicted PE ratio for Dell.
Predicted PE =
- Dell is actually trading at 22 times earnings. What does the predicted PE tell you?

The value of growth

Time Period	Value of extra 1% of growth	Equity Risk Premium
January 2004	0.812	3.69%
July 2003	1.228	3.88%
January 2003	2.621	4.10%
July 2002	0.859	4.35%
January 2002	1.003	3.62%
July 2001	1.251	3.05%
January 2001	1.457	2.75%
July 2000	1.761	2.20%
January 2000	2.105	2.05%

The value of growth is in terms of additional PE...

Value/Earnings and Value/Cashflow Ratios

- While Price earnings ratios look at the market value of equity relative to earnings to equity investors, Value earnings ratios look at the market value of the firm relative to operating earnings. Value to cash flow ratios modify the earnings number to make it a cash flow number.
- The form of value to cash flow ratios that has the closest parallels in DCF valuation is the value to Free Cash Flow to the Firm, which is defined as:

$$\text{Value/FCFF} = \frac{(\text{Market Value of Equity} + \text{Market Value of Debt} - \text{Cash})}{\text{EBIT} (1-t) - (\text{Cap Ex} - \text{Deprecn}) - \text{Chg in WC}}$$

- Consistency Tests:
 - If the numerator is net of cash (or if net debt is used, then the interest income from the cash should not be in denominator
 - The interest expenses added back to get to EBIT should correspond to the debt in the numerator. If only long term debt is considered, only long term interest should be added back.

Value of Firm/FCFF: Determinants

- Reverting back to a two-stage FCFF DCF model, we get:

$$V_0 = \frac{\text{FCFF}_0 (1+g) \left(1 - \frac{(1+g)^n}{(1+WACC)^n} \right)}{WACC - g} + \frac{\text{FCFF}_0 (1+g)^n (1+g_n)}{(WACC - g_n)(1+WACC)^n}$$

- V_0 = Value of the firm (today)
- FCFF_0 = Free Cashflow to the firm in current year
- g = Expected growth rate in FCFF in extraordinary growth period (first n years)
- $WACC$ = Weighted average cost of capital
- g_n = Expected growth rate in FCFF in stable growth period (after n years)

Value Multiples

- Dividing both sides by the FCFF yields,

$$\frac{V_0}{\text{FCFF}_0} = \frac{(1+g) \left(1 - \frac{(1+g)^n}{(1+WACC)^n} \right)}{WACC - g} + \frac{(1+g)^n (1+g_n)}{(WACC - g_n)(1+WACC)^n}$$

- The value/FCFF multiples is a function of
 - the cost of capital
 - the expected growth

Value/FCFF Multiples and the Alternatives

- Assume that you have computed the value of a firm, using discounted cash flow models. Rank the following multiples in the order of magnitude from lowest to highest?
 - Value/EBIT
 - Value/EBIT(1-t)
 - Value/FCFF
 - Value/EBITDA
- What assumption(s) would you need to make for the Value/EBIT(1-t) ratio to be equal to the Value/FCFF multiple?

Illustration: Using Value/FCFF Approaches to value a firm: MCI Communications

- MCI Communications had earnings before interest and taxes of \$3356 million in 1994 (Its net income after taxes was \$855 million).
- It had capital expenditures of \$2500 million in 1994 and depreciation of \$1100 million; Working capital increased by \$250 million.
- It expects free cashflows to the firm to grow 15% a year for the next five years and 5% a year after that.
- The cost of capital is 10.50% for the next five years and 10% after that.
- The company faces a tax rate of 36%.

$$\frac{V_0}{\text{FCFF}_0} = \frac{(1.15) \left(1 - \frac{(1.15)^5}{(1.105)^5} \right)}{.105 - .15} + \frac{(1.15)^5 (1.05)}{(.10 - .05)(1.105)^5} = 31.28$$

Multiple Magic

- In this case of MCI there is a big difference between the FCFF and short cut measures. For instance the following table illustrates the appropriate multiple using short cut measures, and the amount you would overpay by if you used the FCFF multiple.

Free Cash Flow to the Firm

= EBIT (1-t) - Net Cap Ex - Change in Working Capital

= 3356 (1 - 0.36) + 1100 - 2500 - 250 = \$ 498 million

	<i>\$ Value</i>	<i>Correct Multiple</i>
FCFF	\$498	31.28382355
EBIT (1-t)	\$2,148	7.251163362
EBIT \$ 3,356		4.640744552
EBITDA	\$4,456	3.49513885

Reasons for Increased Use of Value/EBITDA

1. The multiple can be computed even for firms that are reporting net losses, since earnings before interest, taxes and depreciation are usually positive.
2. For firms in certain industries, such as cellular, which require a substantial investment in infrastructure and long gestation periods, this multiple seems to be more appropriate than the price/earnings ratio.
3. In leveraged buyouts, where the key factor is cash generated by the firm prior to all discretionary expenditures, the EBITDA is the measure of cash flows from operations that can be used to support debt payment at least in the short term.
4. By looking at cashflows prior to capital expenditures, it may provide a better estimate of “optimal value”, especially if the capital expenditures are unwise or earn substandard returns.
5. By looking at the value of the firm and cashflows to the firm it allows for comparisons across firms with different financial leverage.

Value/EBITDA Multiple

- The Classic Definition

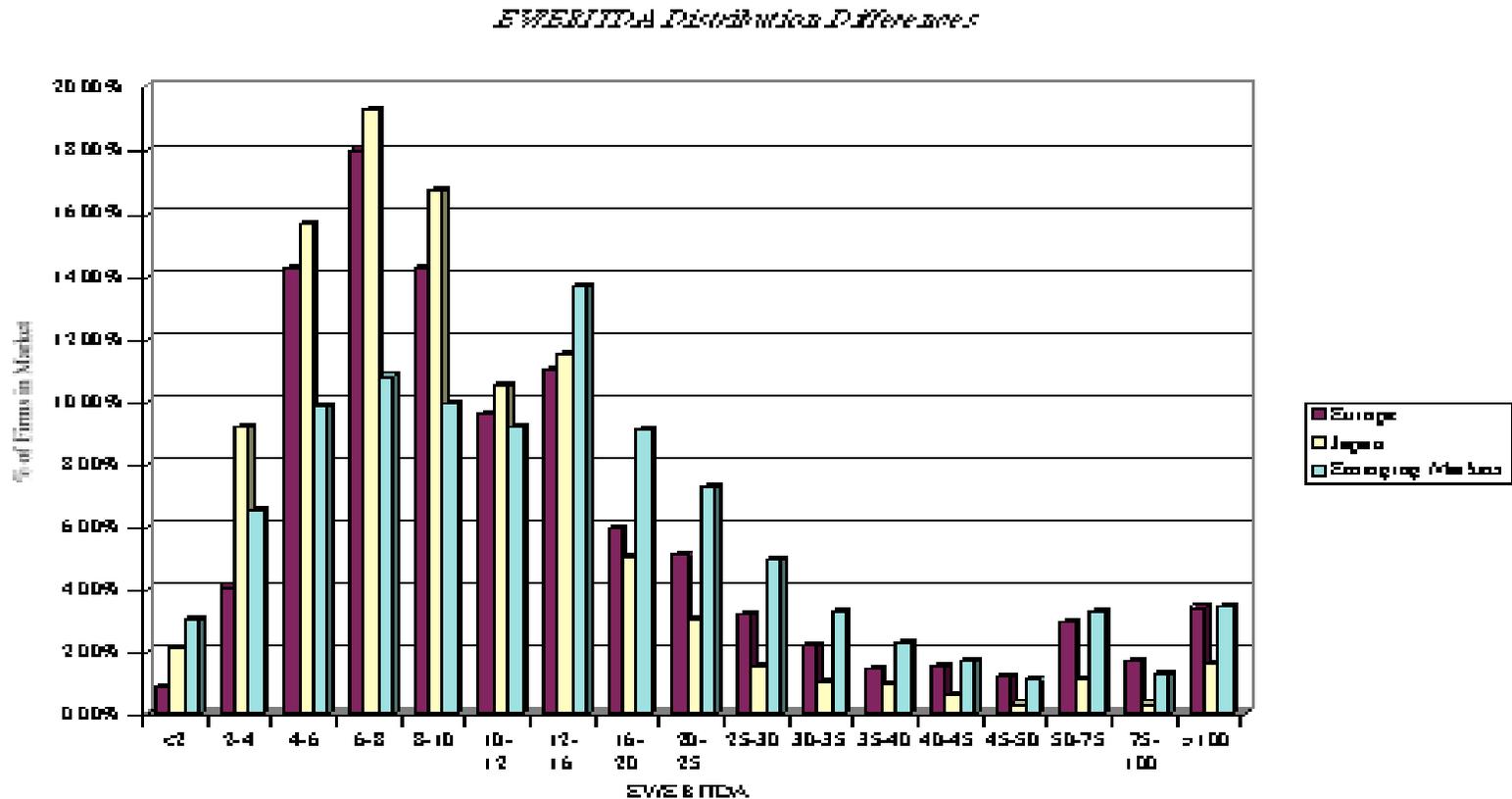
$$\frac{\text{Value}}{\text{EBITDA}} = \frac{\text{Market Value of Equity} + \text{Market Value of Debt}}{\text{Earnings before Interest, Taxes and Depreciation}}$$

- The No-Cash Version

$$\frac{\text{Enterprise Value}}{\text{EBITDA}} = \frac{\text{Market Value of Equity} + \text{Market Value of Debt} - \text{Cash}}{\text{Earnings before Interest, Taxes and Depreciation}}$$

- When cash and marketable securities are netted out of value, none of the income from the cash and securities should be reflected in the denominator.

Value/EBITDA Distribution: Europe, Japan and Emerging Markets



The Determinants of Value/EBITDA Multiples: Linkage to DCF Valuation

- Firm value can be written as:

$$V_0 = \frac{FCFF_1}{WACC - g}$$

- The numerator can be written as follows:

$$\begin{aligned} FCFF &= EBIT (1-t) - (Cex - Depr) - \Delta \text{ Working Capital} \\ &= (EBITDA - Depr) (1-t) - (Cex - Depr) - \Delta \text{ Working Capital} \\ &= EBITDA (1-t) + Depr (t) - Cex - \Delta \text{ Working Capital} \end{aligned}$$

From Firm Value to EBITDA Multiples

- Now the Value of the firm can be rewritten as,

$$\text{Value} = \frac{\text{EBITDA} (1 - t) + \text{Depr} (t) - \text{Cex} - \Delta \text{ Working Capital}}{\text{WACC} - g}$$

- Dividing both sides of the equation by EBITDA,

$$\frac{\text{Value}}{\text{EBITDA}} = \frac{(1 - t)}{\text{WACC} - g} + \frac{\text{Depr} (t)/\text{EBITDA}}{\text{WACC} - g} - \frac{\text{CEx}/\text{EBITDA}}{\text{WACC} - g} - \frac{\Delta \text{ Working Capital}/\text{EBITDA}}{\text{WACC} - g}$$

A Simple Example

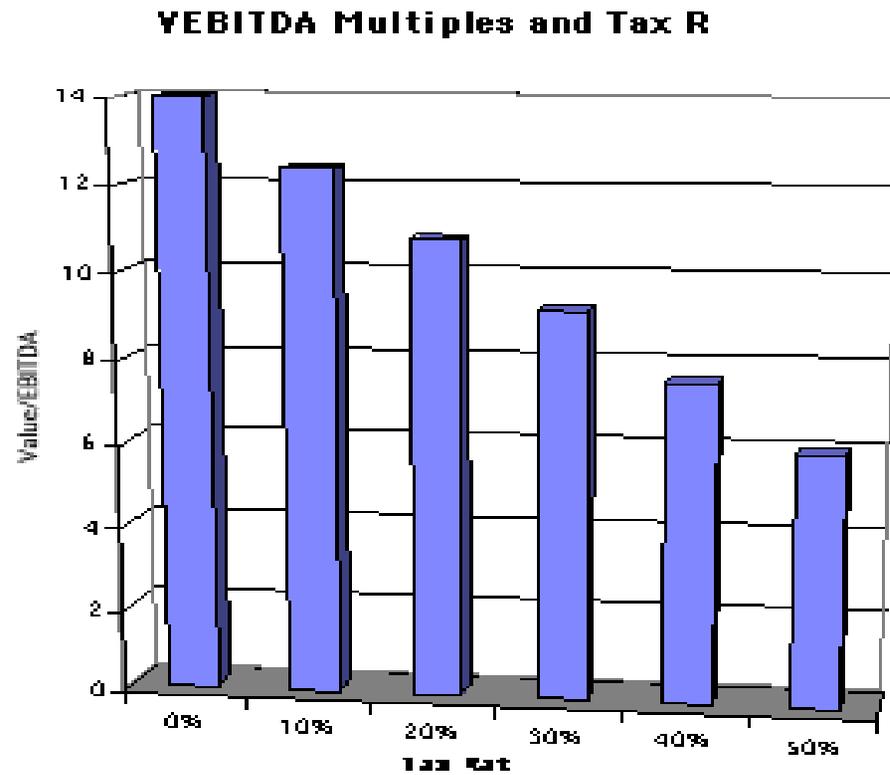
- Consider a firm with the following characteristics:
 - Tax Rate = 36%
 - Capital Expenditures/EBITDA = 30%
 - Depreciation/EBITDA = 20%
 - Cost of Capital = 10%
 - The firm has no working capital requirements
 - The firm is in stable growth and is expected to grow 5% a year forever.

Calculating Value/EBITDA Multiple

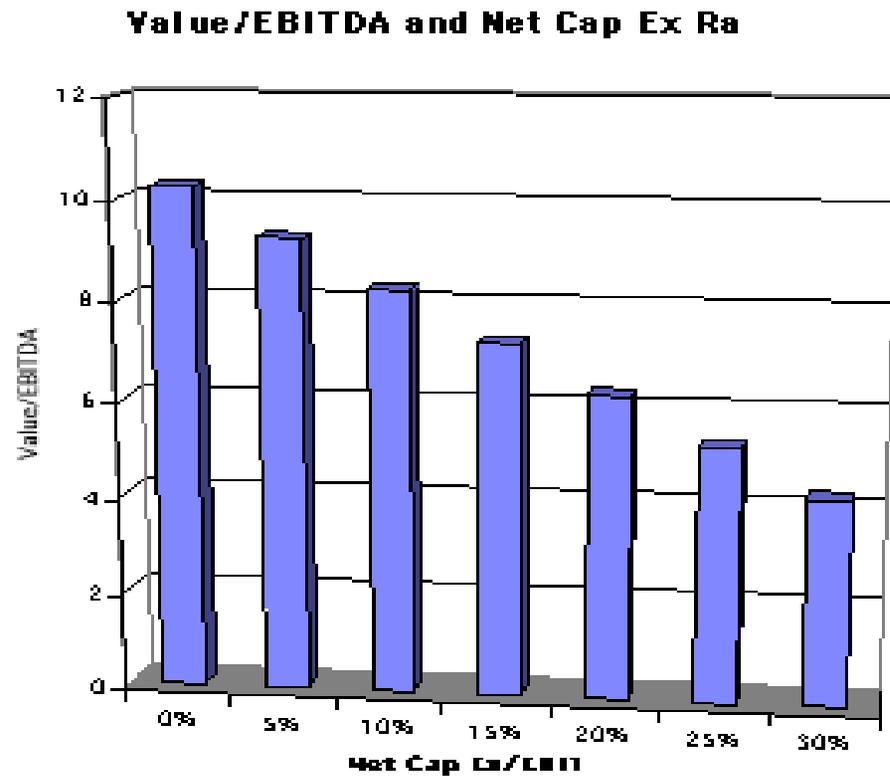
- In this case, the Value/EBITDA multiple for this firm can be estimated as follows:

$$\frac{\text{Value}}{\text{EBITDA}} = \frac{(1 - .36)}{.10 - .05} + \frac{(0.2)(.36)}{.10 - .05} - \frac{0.3}{.10 - .05} - \frac{0}{.10 - .05} = 8.24$$

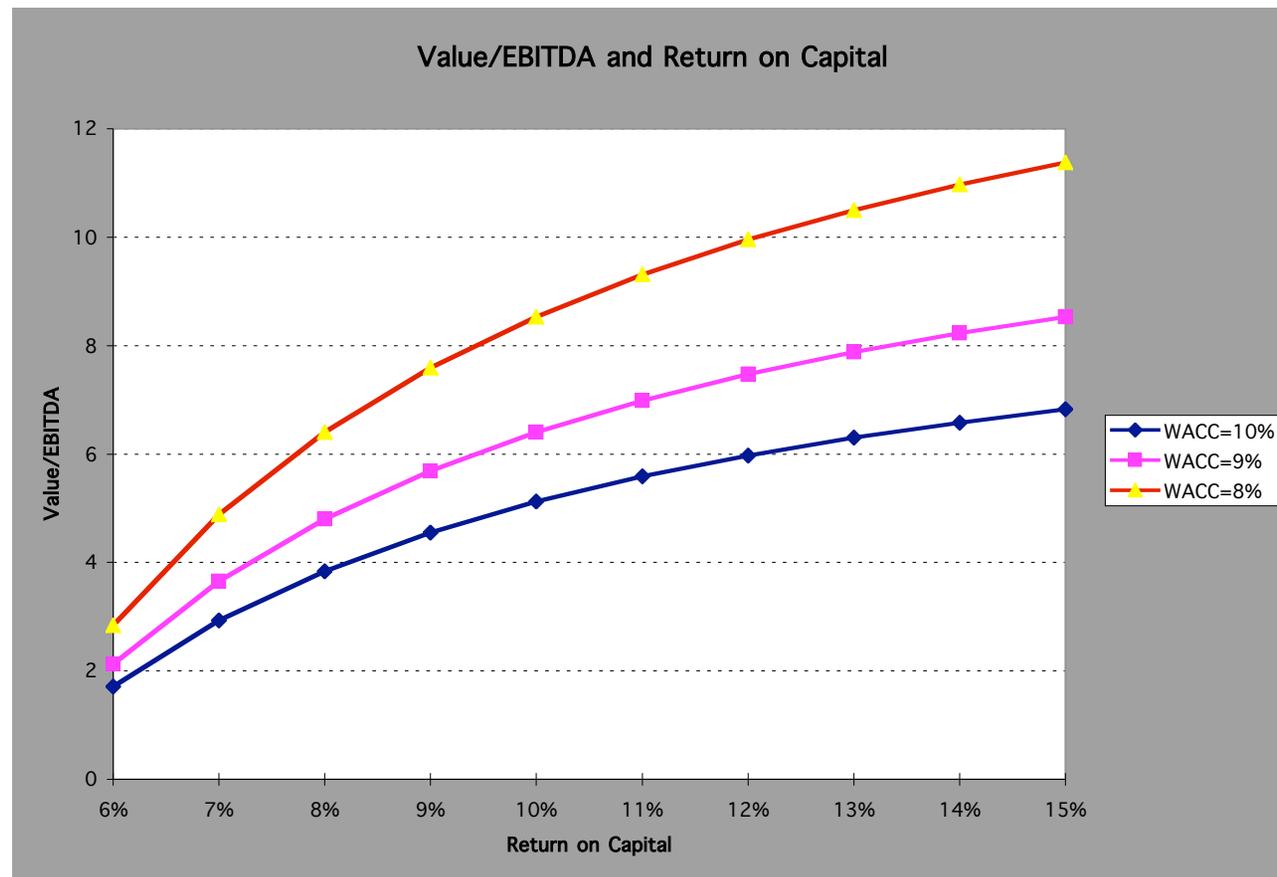
Value/EBITDA Multiples and Taxes



Value/EBITDA and Net Cap Ex



Value/EBITDA Multiples and Return on Capital



Value/EBITDA Multiple: Trucking Companies

<i>Company Name</i>	<i>Value</i>	<i>EBITDA</i>	<i>Value/EBITDA</i>
KLLM Trans. Svcs.	\$ 114.32	\$ 48.81	2.34
Ryder System	\$ 5,158.04	\$ 1,838.26	2.81
Rollins Truck Leasing	\$ 1,368.35	\$ 447.67	3.06
Cannon Express Inc.	\$ 83.57	\$ 27.05	3.09
Hunt (J.B.)	\$ 982.67	\$ 310.22	3.17
Yellow Corp.	\$ 931.47	\$ 292.82	3.18
Roadway Express	\$ 554.96	\$ 169.38	3.28
Marten Transport Ltd.	\$ 116.93	\$ 35.62	3.28
Kenan Transport Co.	\$ 67.66	\$ 19.44	3.48
M.S. Carriers	\$ 344.93	\$ 97.85	3.53
Old Dominion Freight	\$ 170.42	\$ 45.13	3.78
Trimac Ltd	\$ 661.18	\$ 174.28	3.79
Matlack Systems	\$ 112.42	\$ 28.94	3.88
XTRA Corp.	\$ 1,708.57	\$ 427.30	4.00
Covenant Transport Inc	\$ 259.16	\$ 64.35	4.03
Builders Transport	\$ 221.09	\$ 51.44	4.30
Werner Enterprises	\$ 844.39	\$ 196.15	4.30
Landstar Sys.	\$ 422.79	\$ 95.20	4.44
AMERCO	\$ 1,632.30	\$ 345.78	4.72
USA Truck	\$ 141.77	\$ 29.93	4.74
Frozen Food Express	\$ 164.17	\$ 34.10	4.81
Arnold Inds.	\$ 472.27	\$ 96.88	4.87
Greyhound Lines Inc.	\$ 437.71	\$ 89.61	4.88
USFreightways	\$ 983.86	\$ 198.91	4.95
Golden Eagle Group Inc.	\$ 12.50	\$ 2.33	5.37
Arkansas Best	\$ 578.78	\$ 107.15	5.40
Airlease Ltd.	\$ 73.64	\$ 13.48	5.46
Celadon Group	\$ 182.30	\$ 32.72	5.57
Amer. Freightways	\$ 716.15	\$ 120.94	5.92
Transfinancial Holdings	\$ 56.92	\$ 8.79	6.47
Vitran Corp. 'A'	\$ 140.68	\$ 21.51	6.54
Interpool Inc.	\$ 1,002.20	\$ 151.18	6.63
Intrenet Inc.	\$ 70.23	\$ 10.38	6.77
Swift Transportation	\$ 835.58	\$ 121.34	6.89
Landair Services	\$ 212.95	\$ 30.38	7.01
CNF Transportation	\$ 2,700.69	\$ 366.99	7.36
Budget Group Inc	\$ 1,247.30	\$ 166.71	7.48
Caliber System	\$ 2,514.99	\$ 333.13	7.55
Knight Transportation Inc	\$ 269.01	\$ 28.20	9.54
Heartland Express	\$ 727.50	\$ 64.62	11.26
Greyhound CDA Transn Corp	\$ 83.25	\$ 6.99	11.91
Mark VII	\$ 160.45	\$ 12.96	12.38
Coach USA Inc	\$ 678.38	\$ 51.76	13.11
US 1 Inds Inc.	\$ 5.60	\$ (0.17)	NA
Average			5.61

A Test on EBITDA

- Ryder System looks very cheap on a Value/EBITDA multiple basis, relative to the rest of the sector. What explanation (other than misvaluation) might there be for this difference?

Europe: Cross Sectional Regression

January 2004

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.542 ^a	.293	.292	1581.333005721082000

a. Predictors: (Constant), Tax Rate, Reinv Rate, Market Debt to Capital

Coefficients^{a,b}

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	8.419	1.279		6.580	.000
	Market Debt to Capital	.589	.021	.511	28.035	.000
	Reinv Rate	-.051	.009	-.099	-5.472	.000
	Tax Rate	-.152	.029	-.095	-5.236	.000

a. Dependent Variable: EV/EBITDA

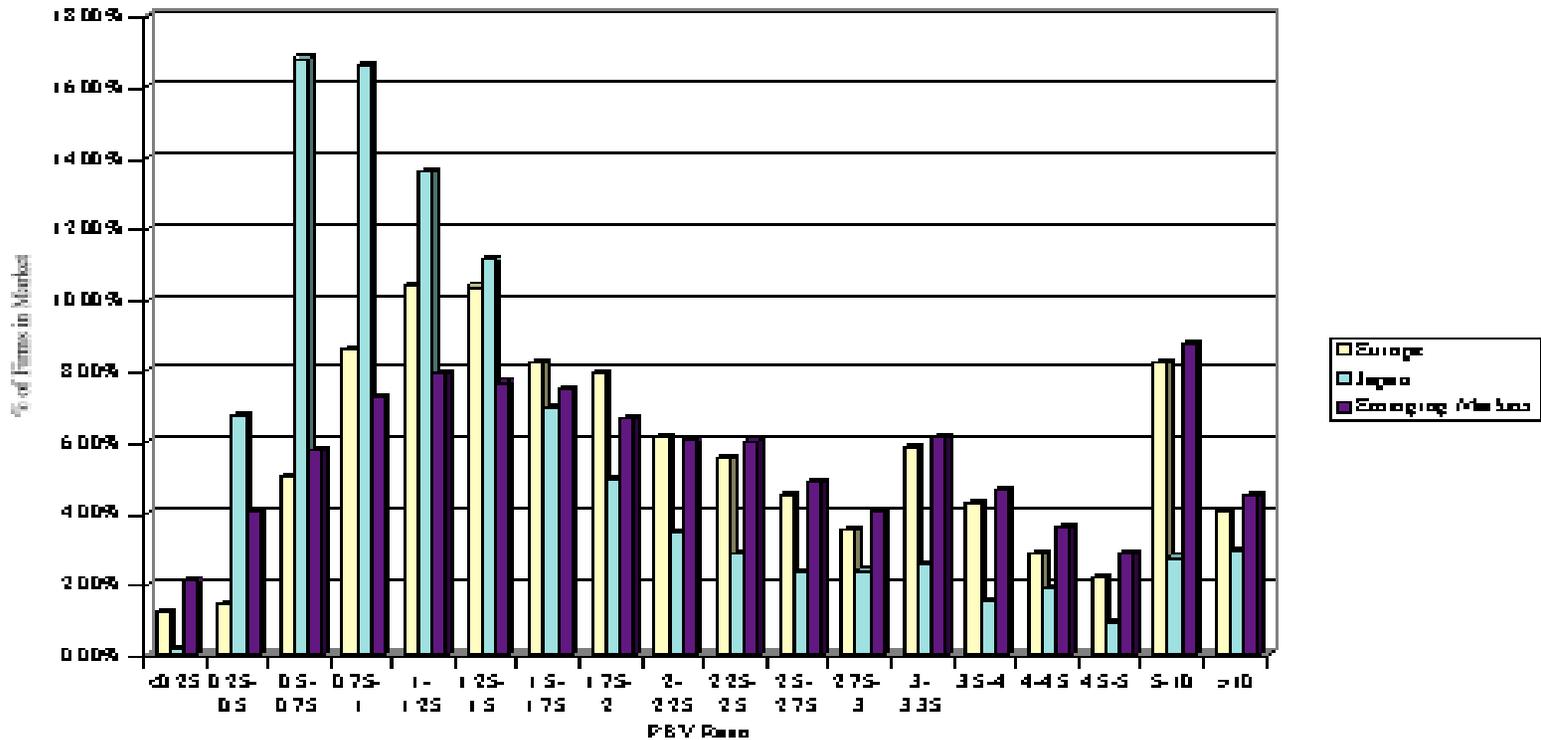
b. Weighted Least Squares Regression - Weighted by Market Capitalization

Price-Book Value Ratio: Definition

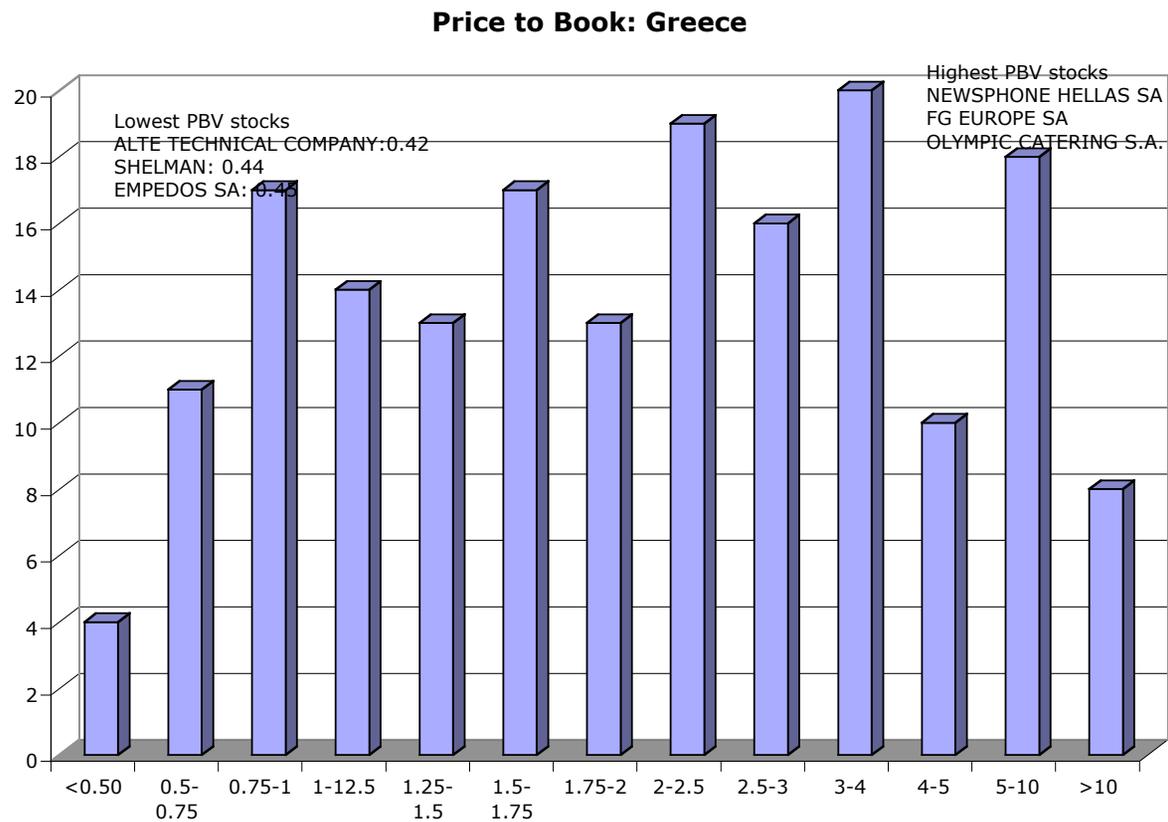
- The price/book value ratio is the ratio of the market value of equity to the book value of equity, i.e., the measure of shareholders' equity in the balance sheet.
- Price/Book Value =
$$\frac{\text{Market Value of Equity}}{\text{Book Value of Equity}}$$
- Consistency Tests:
 - If the market value of equity refers to the market value of equity of common stock outstanding, the book value of common equity should be used in the denominator.
 - If there is more than one class of common stock outstanding, the market values of all classes (even the non-traded classes) need to be factored in.

Price to Book Value: Europe, Japan and Emerging Markets

PBV Ratios: Different Markets



Price to Book: Greece in January 2004



Price Book Value Ratio: Stable Growth Firm

- Going back to a simple dividend discount model,

$$P_0 = \frac{DPS_1}{r - g_n}$$

- Defining the return on equity (ROE) = EPS_0 / Book Value of Equity, the value of equity can be written as:

$$P_0 = \frac{BV_0 * ROE * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

$$\frac{P_0}{BV_0} = \text{PBV} = \frac{ROE * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

- If the return on equity is based upon expected earnings in the next time period, this can be simplified to,

$$\frac{P_0}{BV_0} = \text{PBV} = \frac{ROE * \text{Payout Ratio}}{r - g_n}$$

PBV/ROE: European Banks

<i>Bank</i>	<i>Symbol</i>	<i>PBV</i>	<i>ROE</i>
Banca di Roma SpA	BAHQE	0.60	4.15%
Commerzbank AG	COHSO	0.74	5.49%
Bayerische Hypo und Vereinsbank AG	BAXWW	0.82	5.39%
Intesa Bci SpA	BAEWF	1.12	7.81%
Natexis Banques Populaires	NABQE	1.12	7.38%
Almanij NV Algemene Mij voor Nijver	ALPK	1.17	8.78%
Credit Industriel et Commercial	CIECM	1.20	9.46%
Credit Lyonnais SA	CREV	1.20	6.86%
BNL Banca Nazionale del Lavoro SpA	BAEXC	1.22	12.43%
Banca Monte dei Paschi di Siena SpA	MOGG	1.34	10.86%
Deutsche Bank AG	DEMXX	1.36	17.33%
Skandinaviska Enskilda Banken	SKHS	1.39	16.33%
Nordea Bank AB	NORDEA	1.40	13.69%
DNB Holding ASA	DNHLD	1.42	16.78%
ForeningsSparbanken AB	FOLG	1.61	18.69%
Danske Bank AS	DANKAS	1.66	19.09%
Credit Suisse Group	CRGAL	1.68	14.34%
KBC Bankverzekeringsholding	KBCBA	1.69	30.85%
Societe Generale	SODI	1.73	17.55%
Santander Central Hispano SA	BAZAB	1.83	11.01%
National Bank of Greece SA	NAGT	1.87	26.19%
San Paolo IMI SpA	SAOEL	1.88	16.57%
BNP Paribas	BNPRB	2.00	18.68%
Svenska Handelsbanken AB	SVKE	2.12	21.82%
UBS AG	UBQH	2.15	16.64%
Banco Bilbao Vizcaya Argentaria SA	BBFUG	2.18	22.94%
ABN Amro Holding NV	ABTS	2.21	24.21%
UniCredito Italiano SpA	UNCZA	2.25	15.90%
Rolo Banca 1473 SpA	ROGMBA	2.37	16.67%
Dexia	DECCT	2.76	14.99%
Average		1.60	14.96%

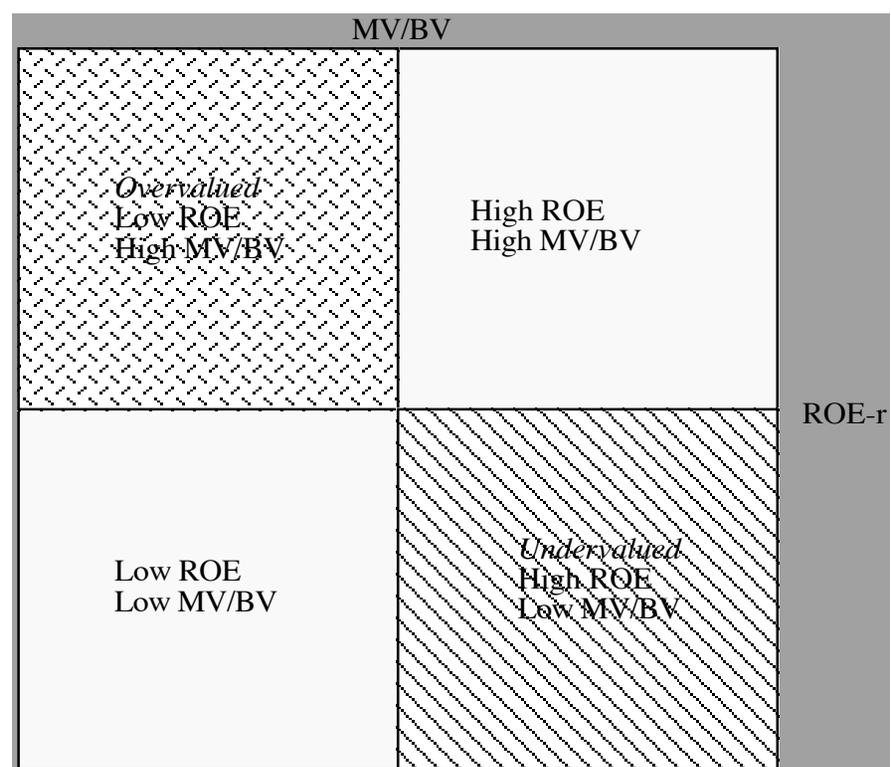
PBV versus ROE regression

- Regressing PBV ratios against ROE for banks yields the following regression:
$$\text{PBV} = 0.81 + 5.32 (\text{ROE}) \quad R^2 = 46\%$$
- For every 1% increase in ROE, the PBV ratio should increase by 0.0532.

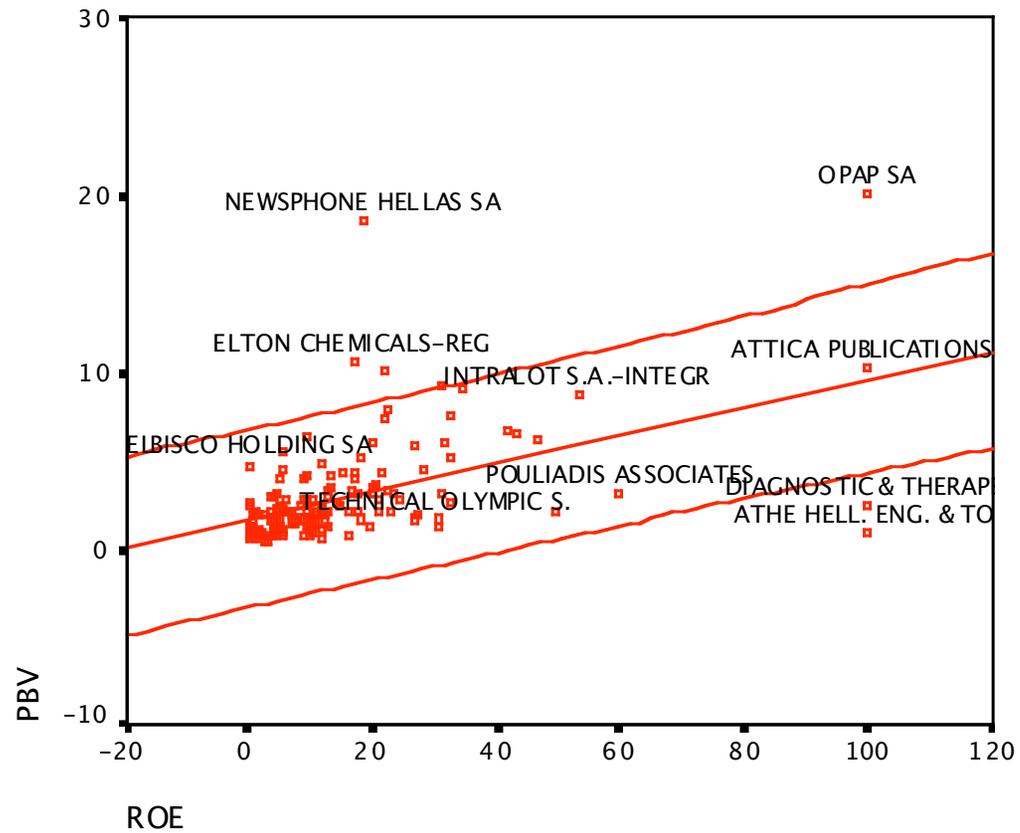
Under and Over Valued Banks?

<i>Bank</i>	<i>Actual</i>	<i>Predicted</i>	<i>Under or Over</i>
Banca di Roma SpA	0.60	1.03	-41.33%
Commerzbank AG	0.74	1.10	-32.86%
Bayerische Hypo und Vereinsbank AG	0.82	1.09	-24.92%
Intesa Bci SpA	1.12	1.22	-8.51%
Natexis Banques Populaires	1.12	1.20	-6.30%
Almanij NV Algemene Mij voor Nijver	1.17	1.27	-7.82%
Credit Industriel et Commercial	1.20	1.31	-8.30%
Credit Lyonnais SA	1.20	1.17	2.61%
BNL Banca Nazionale del Lavoro SpA	1.22	1.47	-16.71%
Banca Monte dei Paschi di Siena SpA	1.34	1.39	-3.38%
Deutsche Bank AG	1.36	1.73	-21.40%
Skandinaviska Enskilda Banken	1.39	1.68	-17.32%
Nordea Bank AB	1.40	1.54	-9.02%
DNB Holding ASA	1.42	1.70	-16.72%
ForeningsSparbanken AB	1.61	1.80	-10.66%
Danske Bank AS	1.66	1.82	-9.01%
Credit Suisse Group	1.68	1.57	7.20%
KBC Bankverzekeringsholding	1.69	2.45	-30.89%
Societe Generale	1.73	1.74	-0.42%
Santander Central Hispano SA	1.83	1.39	31.37%
National Bank of Greece SA	1.87	2.20	-15.06%
San Paolo IMI SpA	1.88	1.69	11.15%
BNP Paribas	2.00	1.80	11.07%
Svenska Handelsbanken AB	2.12	1.97	7.70%
UBS AG	2.15	1.69	27.17%
Banco Bilbao Vizcaya Argentaria SA	2.18	2.03	7.66%
ABN Amro Holding NV	2.21	2.10	5.23%
UniCredito Italiano SpA	2.25	1.65	36.23%
Rolo Banca 1473 SpA	2.37	1.69	39.74%
Dexia	2.76	1.61	72.04%

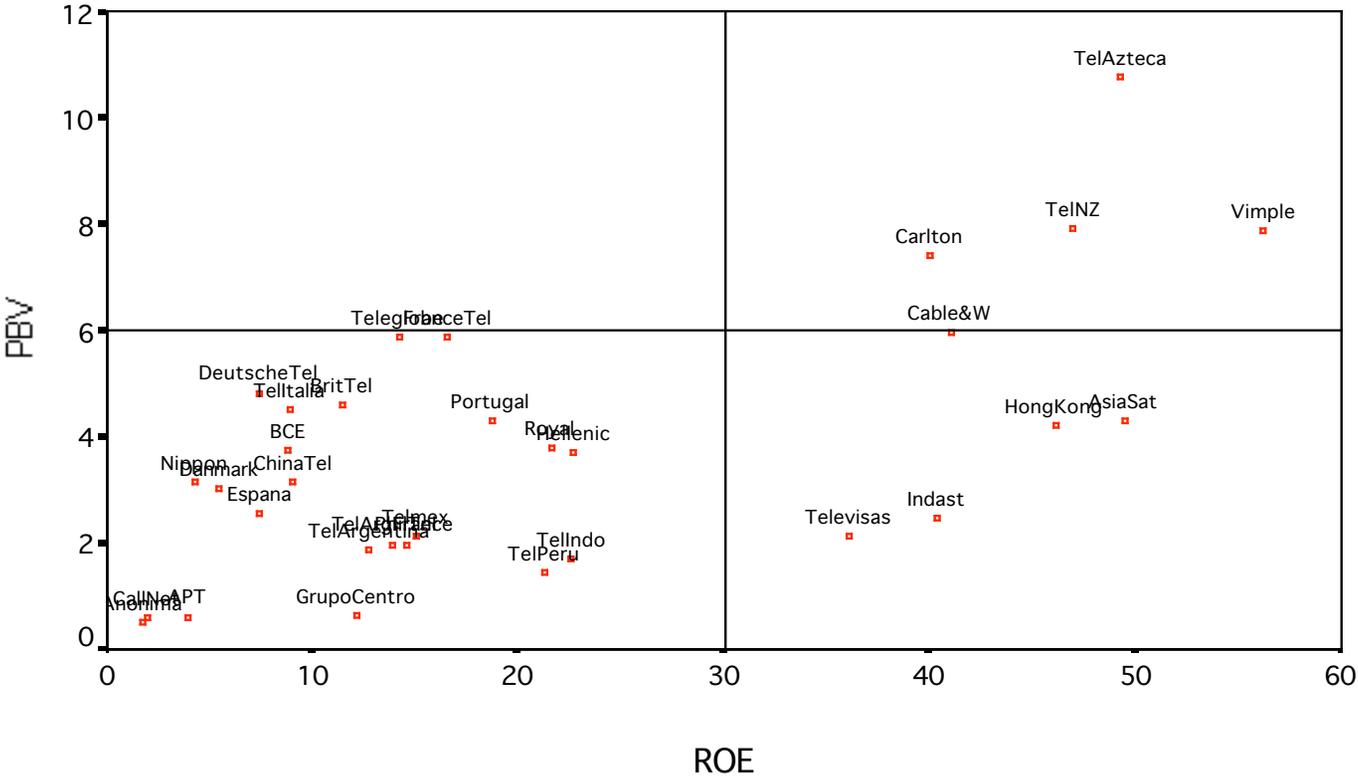
Looking for undervalued securities - PBV Ratios and ROE : The Valuation Matrix



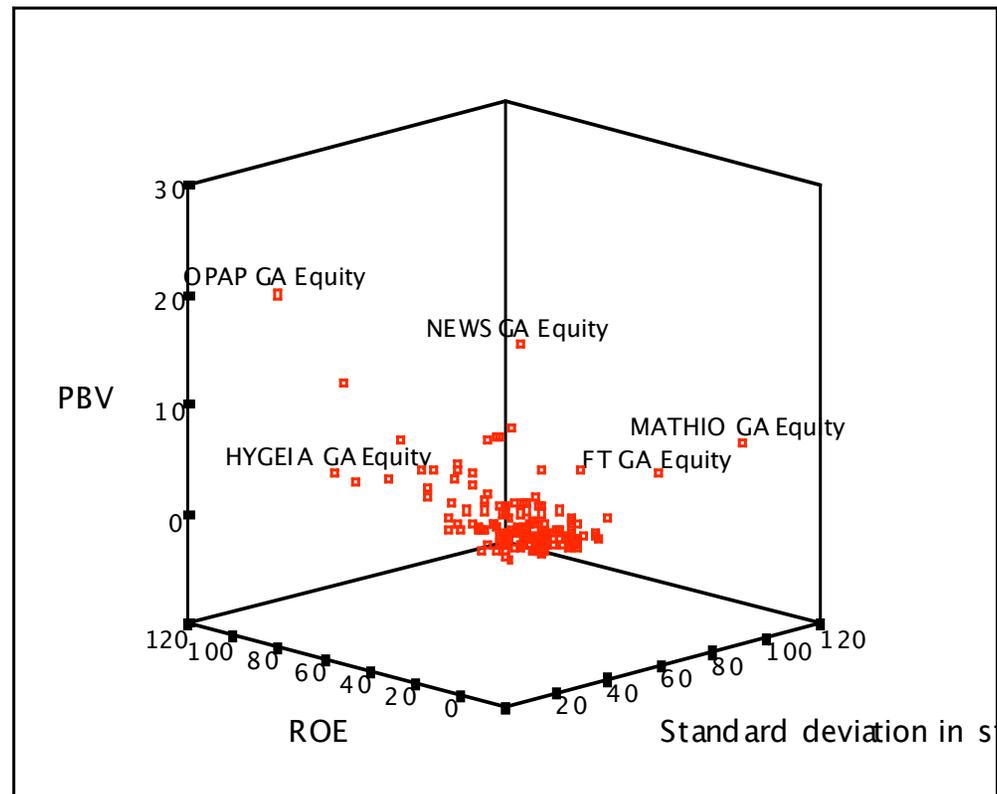
Price to Book vs ROE: Greek Stocks in January 2004



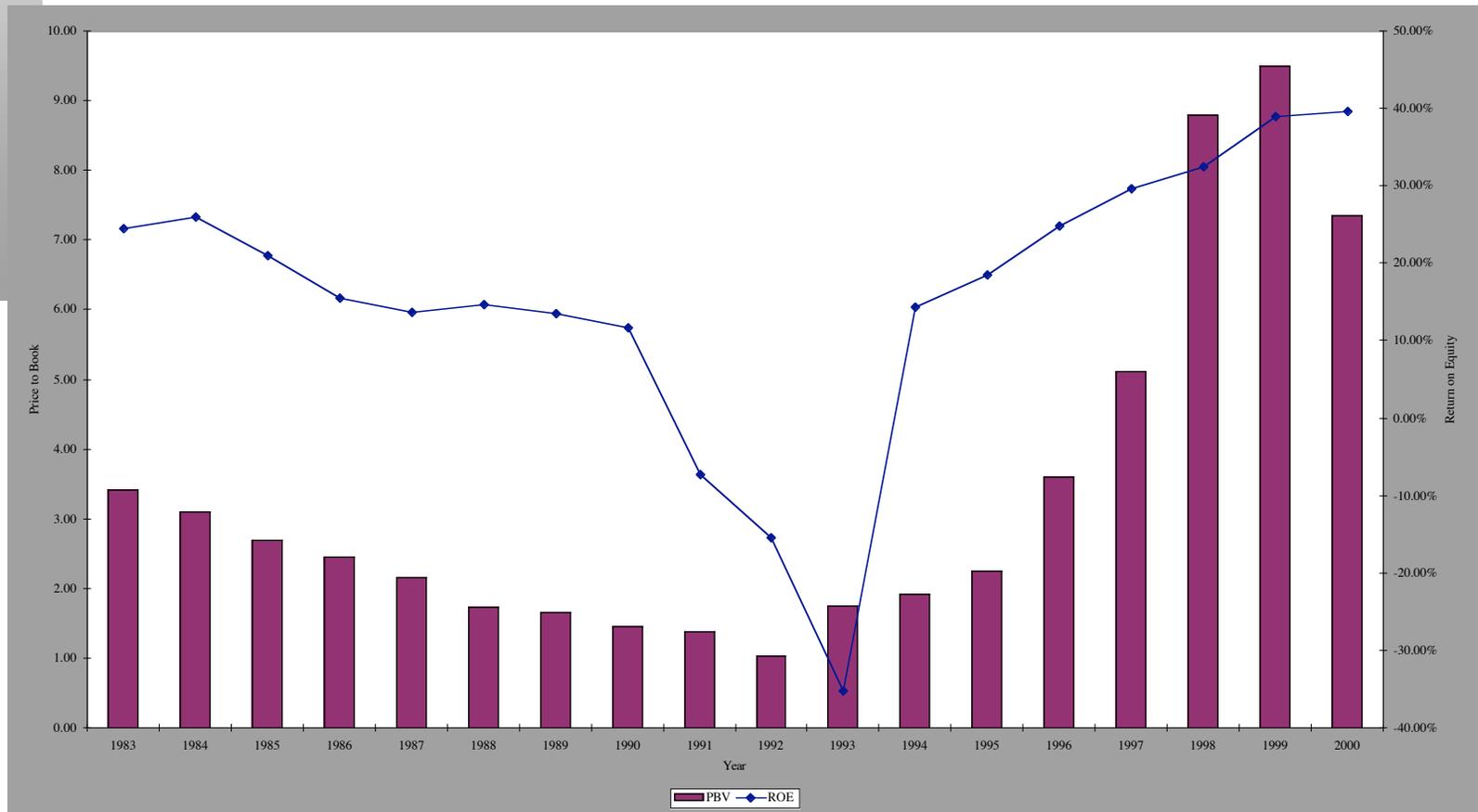
PBV Matrix: Telecom Companies



PBV, ROE and Risk: Greek Stocks



IBM: The Rise and Fall and Rise Again



PBV Ratio Regression- Europe

January 2004

Model Summary

Model	R	R Square ^a	Adjusted R Square	Std. Error of the Estimate
1	.830 ^b	.689	.689	154.44047748882220

a. For regression through the origin (the no-intercept model), R Square measures the proportion of the variability in the dependent variable about the origin explained by regression. This CANNOT be compared to R Square for models which include an intercept.

b. Predictors: ROE, Payout Ratio, BETA

Coefficients^{a,b,c}

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Payout Ratio	8.E-03	.002	.074	3.667	.000
	BETA	1.399	.114	.291	12.279	.000
	ROE	.104	.004	.537	28.148	.000

a. Dependent Variable: PBV

b. Linear Regression through the Origin

c. Weighted Least Squares Regression – Weighted by Market Capitalization

PBV Regression: Greece

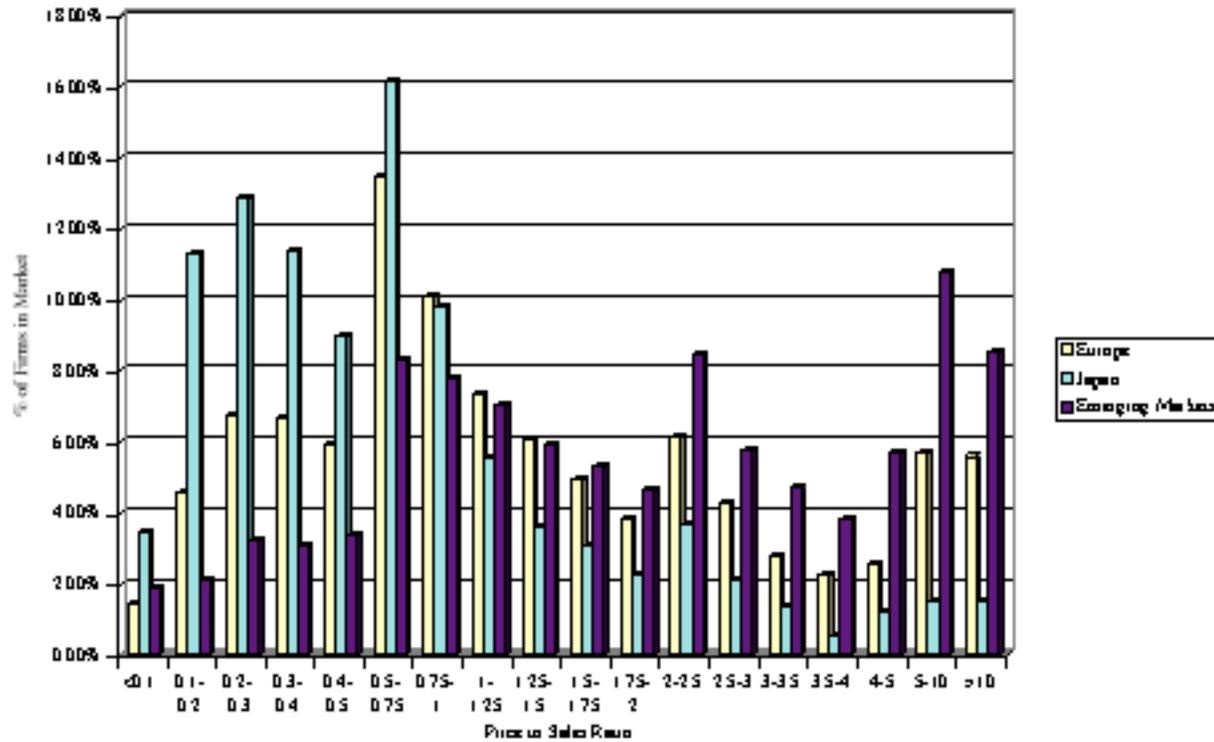
- Looking at 141 Greek stocks with PBV ratios and returns on equity available on them:
 - $PBV = 0.721 + 0.174 \text{ Return on Equity}$
(24.49)
- R squared for regression = 85.2%

Price Sales Ratio: Definition

- The price/sales ratio is the ratio of the market value of equity to the sales.
- Price/ Sales=
$$\frac{\text{Market Value of Equity}}{\text{Total Revenues}}$$
- Consistency Tests
 - The price/sales ratio is internally inconsistent, since the market value of equity is divided by the total revenues of the firm.

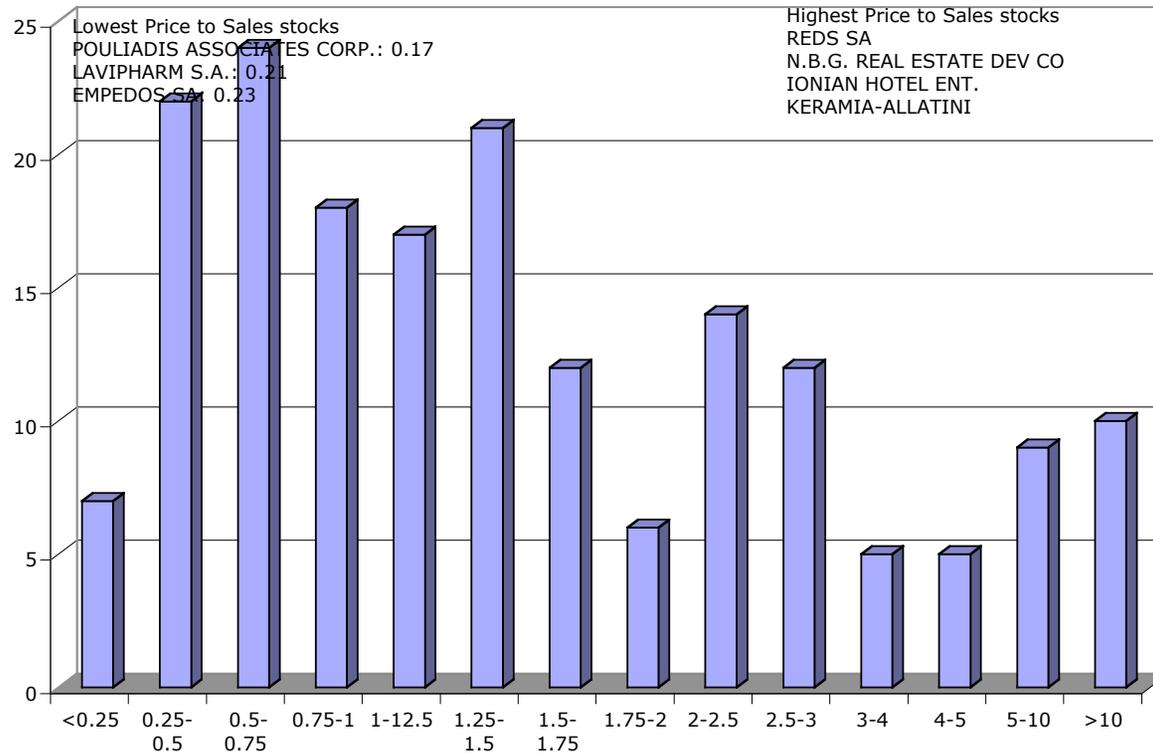
Price to Sales: Europe, Japan and Emerging Markets

Price to Sales Ratios: Market Comparisons



Price to Sales: Greece

Price to Sales: Greece



Price/Sales Ratio: Determinants

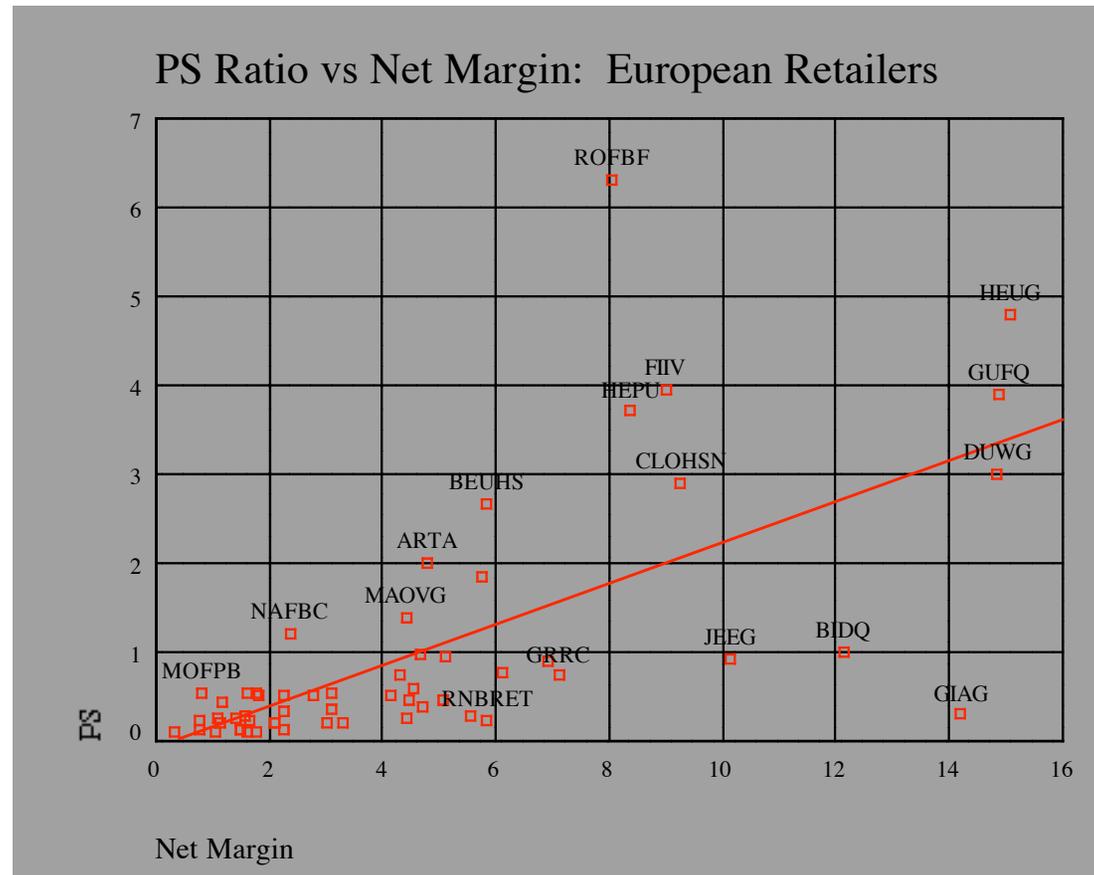
- The price/sales ratio of a stable growth firm can be estimated beginning with a 2-stage equity valuation model:

$$P_0 = \frac{DPS_1}{r - g_n}$$

- Dividing both sides by the sales per share:

$$\frac{P_0}{Sales_0} = PS = \frac{\text{Net Profit Margin} * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

PS/Margins: European Retailers - March 2002



Regression Results: PS Ratios and Margins

- Regressing PS ratios against net margins,

$$PS = -.06 + 22.90 (\text{Net Margin}) \quad R^2 = 45\%$$

- Thus, a 1% increase in the margin results in an increase of 0.23 in the price sales ratios.
- The regression also allows us to get predicted PS ratios for these firms

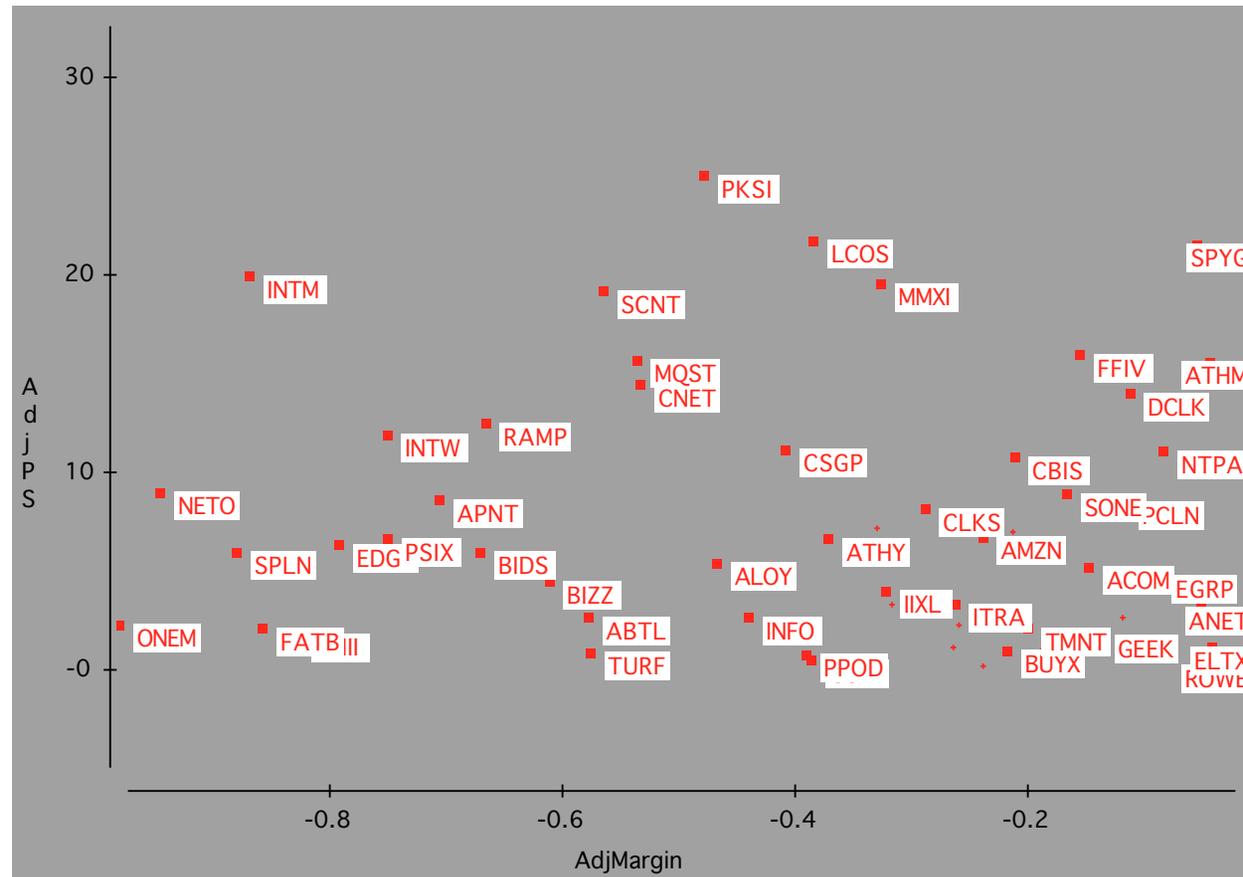
Current versus Predicted Margins

- One of the limitations of the analysis we did in these last few pages is the focus on current margins. Stocks are priced based upon expected margins rather than current margins.
- For most firms, current margins and predicted margins are highly correlated, making the analysis still relevant.
- For firms where current margins have little or no correlation with expected margins, regressions of price to sales ratios against current margins (or price to book against current return on equity) will not provide much explanatory power.
- In these cases, it makes more sense to run the regression using either predicted margins or some proxy for predicted margins.

Current versus Predicted Margins

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- In these cases, it makes more sense to run the regression using either predicted margins or some proxy for predicted margins.

A Case Study: The Internet Stocks



PS Ratios and Margins are not highly correlated

- Regressing PS ratios against current margins yields the following
$$PS = 81.36 - 7.54(\text{Net Margin}) \quad R^2 = 0.04$$

(0.49)
- This is not surprising. These firms are priced based upon expected margins, rather than current margins.

Solution 1: Use proxies for survival and growth: Amazon in early 2000

- Hypothesizing that firms with higher revenue growth and higher cash balances should have a greater chance of surviving and becoming profitable, we ran the following regression: (The level of revenues was used to control for size)

$$PS = 30.61 - 2.77 \ln(\text{Rev}) + 6.42 (\text{Rev Growth}) + 5.11 (\text{Cash/Rev})$$

(0.66) (2.63) (3.49)

R squared = 31.8%

$$\text{Predicted PS} = 30.61 - 2.77(7.1039) + 6.42(1.9946) + 5.11 (.3069) = 30.42$$

Actual PS = 25.63

Stock is undervalued, relative to other internet stocks.

Solution 2: Use forward multiples

- You can always estimate price (or value) as a multiple of revenues, earnings or book value in a future year. These multiples are called forward multiples.
- For young and evolving firms, the values of fundamentals in future years may provide a much better picture of the true value potential of the firm. There are two ways in which you can use forward multiples:
 - Look at value today as a multiple of revenues or earnings in the future (say 5 years from now) for all firms in the comparable firm list. Use the average of this multiple in conjunction with your firm's earnings or revenues to estimate the value of your firm today.
 - Estimate value as a multiple of current revenues or earnings for more mature firms in the group and apply this multiple to the forward earnings or revenues to the forward earnings for your firm. This will yield the expected value for your firm in the forward year and will have to be discounted back to the present to get current value.

Solution 2: Use forward multiples

- Global Crossing lost \$1.9 billion in 2001 and is expected to continue to lose money for the next 3 years. In a discounted cashflow valuation (see notes on DCF valuation) of Global Crossing, we estimated an expected EBITDA for Global Crossing in five years of \$ 1,371 million.
- The average enterprise value/ EBITDA multiple for healthy telecomm firms is 7.2 currently.
- Applying this multiple to Global Crossing's EBITDA in year 5, yields a value in year 5 of
 - Enterprise Value in year 5 = $1371 * 7.2 = \$9,871$ million
 - Enterprise Value today = $\$ 9,871 \text{ million} / 1.138^5 = \$5,172$ million
(The cost of capital for Global Crossing is 13.80%)
 - The probability that Global Crossing will not make it as a going concern is 77%.
 - Expected Enterprise value today = $0.23 (5172) = \$1,190$ million

PS Regression: Europe in January 2004

Model Summary

Model	R	R Square ^a	Adjusted R Square	Std. Error of the Estimate
1	.757 ^b	.574	.573	134.938678072015

a. For regression through the origin (the no-intercept model), R Square measures the proportion of the variability in the dependent variable about the origin explained by regression. This CANNOT be compared to R Square for models which include an intercept.

b. Predictors: Net Margin, Payout Ratio, BETA

Coefficients^{a,b,c}

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Payout Ratio	5.E-03	.002	.065	2.777	.006
	BETA	.937	.095	.261	9.909	.000
	Net Margin	.110	.004	.516	26.153	.000

a. Dependent Variable: PS

b. Linear Regression through the Origin

c. Weighted Least Squares Regression – Weighted by Market Capitalization

Choosing Between the Multiples

- As presented in this section, there are dozens of multiples that can be potentially used to value an individual firm.
- In addition, relative valuation can be relative to a sector (or comparable firms) or to the entire market (using the regressions, for instance)
- Since there can be only one final estimate of value, there are three choices at this stage:
 - Use a simple average of the valuations obtained using a number of different multiples
 - Use a weighted average of the valuations obtained using a number of different multiples
 - Choose one of the multiples and base your valuation on that multiple

Picking one Multiple

- This is usually the best way to approach this issue. While a range of values can be obtained from a number of multiples, the “best estimate” value is obtained using one multiple.
- The multiple that is used can be chosen in one of two ways:
 - Use the multiple that best fits your objective. Thus, if you want the company to be undervalued, you pick the multiple that yields the highest value.
 - Use the multiple that has the highest R-squared in the sector when regressed against fundamentals. Thus, if you have tried PE, PBV, PS, etc. and run regressions of these multiples against fundamentals, use the multiple that works best at explaining differences across firms in that sector.
 - Use the multiple that seems to make the most sense for that sector, given how value is measured and created.

A More Intuitive Approach

- Managers in every sector tend to focus on specific variables when analyzing strategy and performance. The multiple used will generally reflect this focus. Consider three examples.
 - In retailing: The focus is usually on same store sales (turnover) and profit margins. Not surprisingly, the revenue multiple is most common in this sector.
 - In financial services: The emphasis is usually on return on equity. Book Equity is often viewed as a scarce resource, since capital ratios are based upon it. Price to book ratios dominate.
 - In technology: Growth is usually the dominant theme. PEG ratios were invented in this sector.

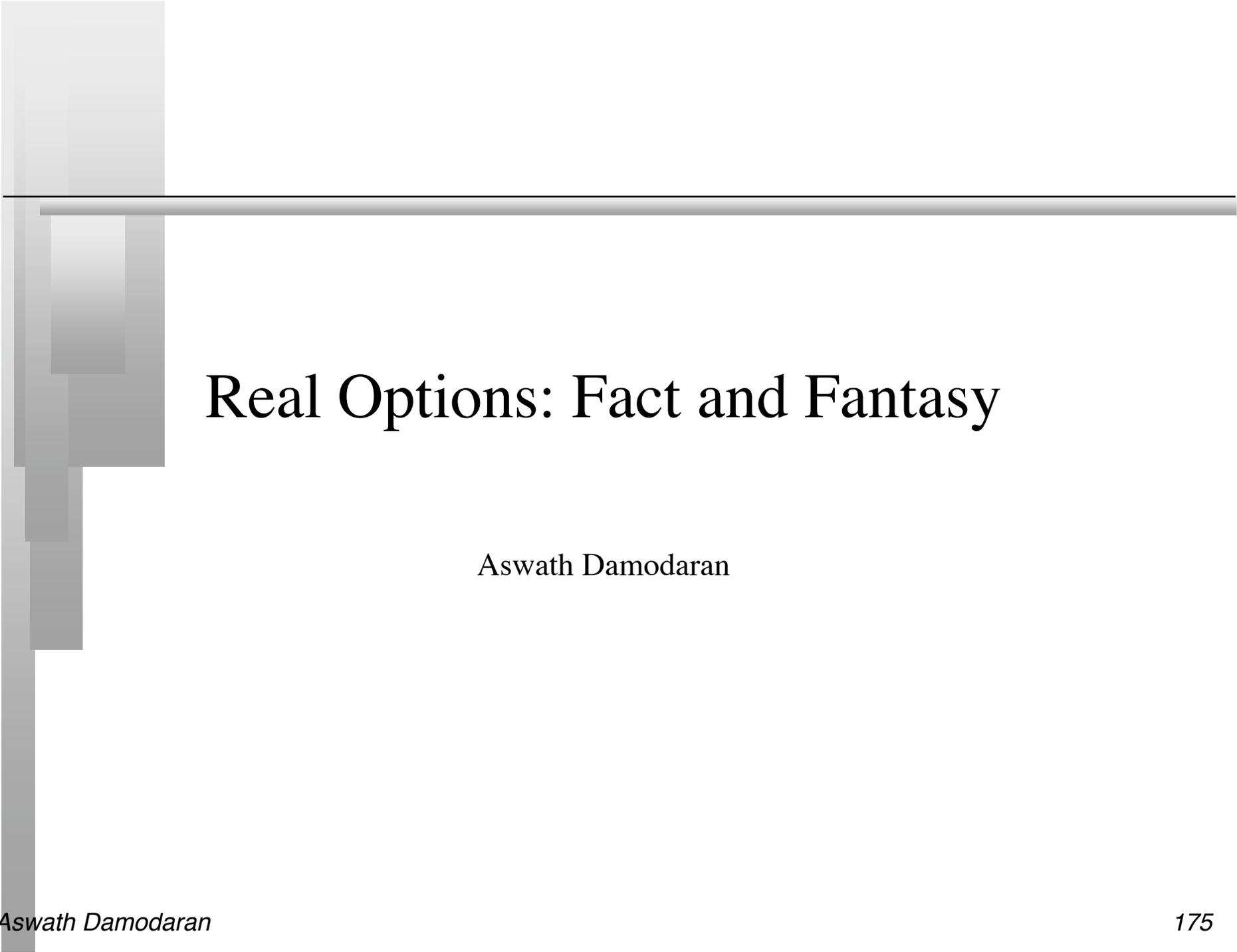
In Practice...

- As a general rule of thumb, the following table provides a way of picking a multiple for a sector

<i>Sector</i>	<i>Multiple Used</i>	<i>Rationale</i>
Cyclical Manufacturing	PE, Relative PE	Often with normalized earnings
High Tech, High Growth	PEG	Big differences in growth across firms
High Growth/No Earnings	PS, VS	Assume future margins will be good
Heavy Infrastructure	VEBITDA	Firms in sector have losses in early years and reported earnings can vary depending on depreciation method
REITa	P/CF	Generally no cap ex investments from equity earnings
Financial Services	PBV	Book value often marked to market
Retailing	PS	If leverage is similar across firms
	VS	If leverage is different

Reviewing: The Four Steps to Understanding Multiples

- Define the multiple
 - Check for consistency
 - Make sure that they are estimated uniformly
- Describe the multiple
 - Multiples have skewed distributions: The averages are seldom good indicators of typical multiples
 - Check for bias, if the multiple cannot be estimated
- Analyze the multiple
 - Identify the companion variable that drives the multiple
 - Examine the nature of the relationship
- Apply the multiple



Real Options: Fact and Fantasy

Aswath Damodaran

Underlying Theme: Searching for an Elusive Premium

- Traditional discounted cashflow models under estimate the value of investments, where there are options embedded in the investments to
 - Delay or defer making the investment (delay)
 - Adjust or alter production schedules as price changes (flexibility)
 - Expand into new markets or products at later stages in the process, based upon observing favorable outcomes at the early stages (expansion)
 - Stop production or abandon investments if the outcomes are unfavorable at early stages (abandonment)
- Put another way, real option advocates believe that you should be paying a premium on discounted cashflow value estimates.

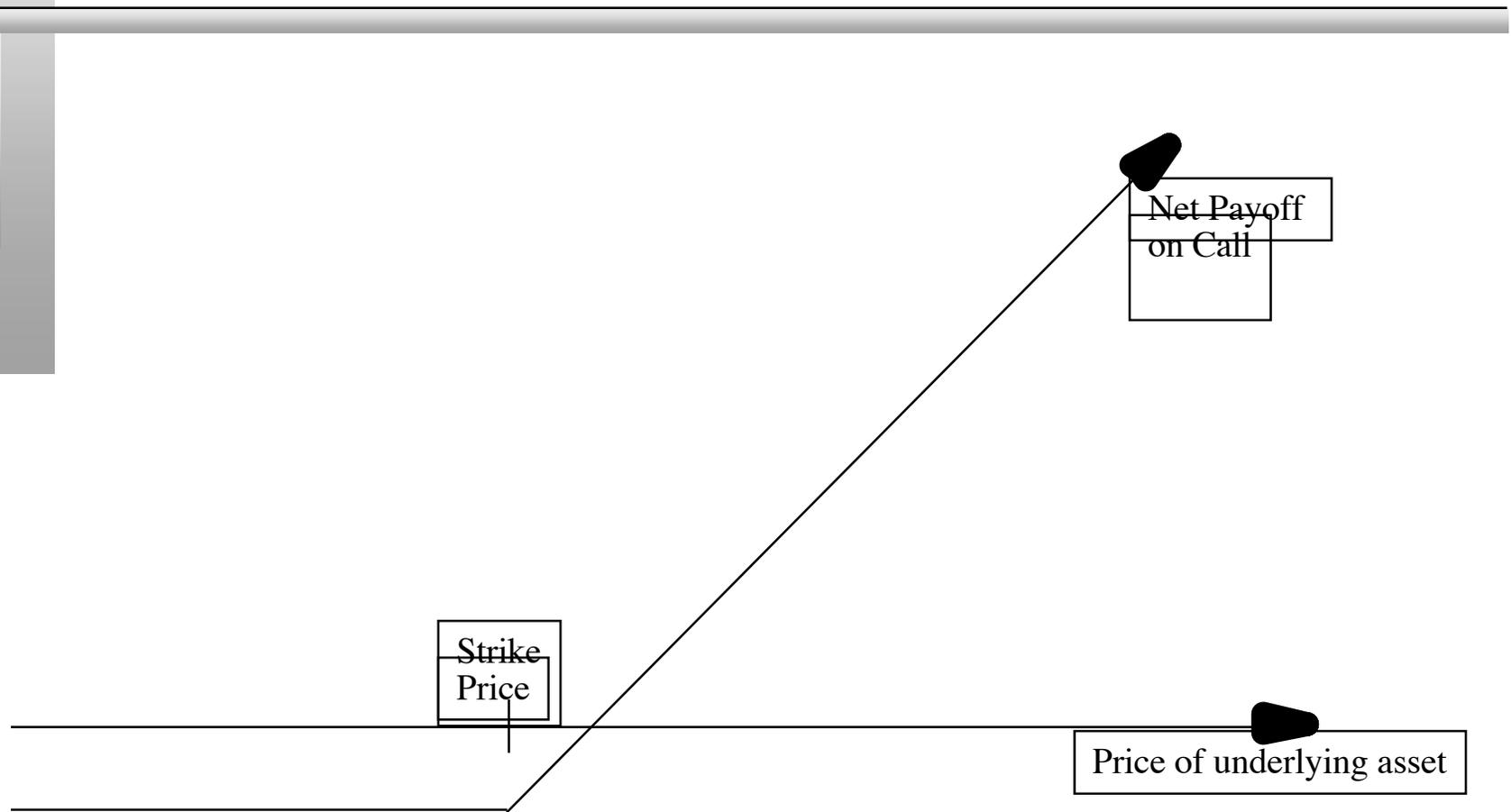
Three Basic Questions

- When is there a real option embedded in a decision or an asset?
- When does that real option have significant economic value?
- Can that value be estimated using an option pricing model?

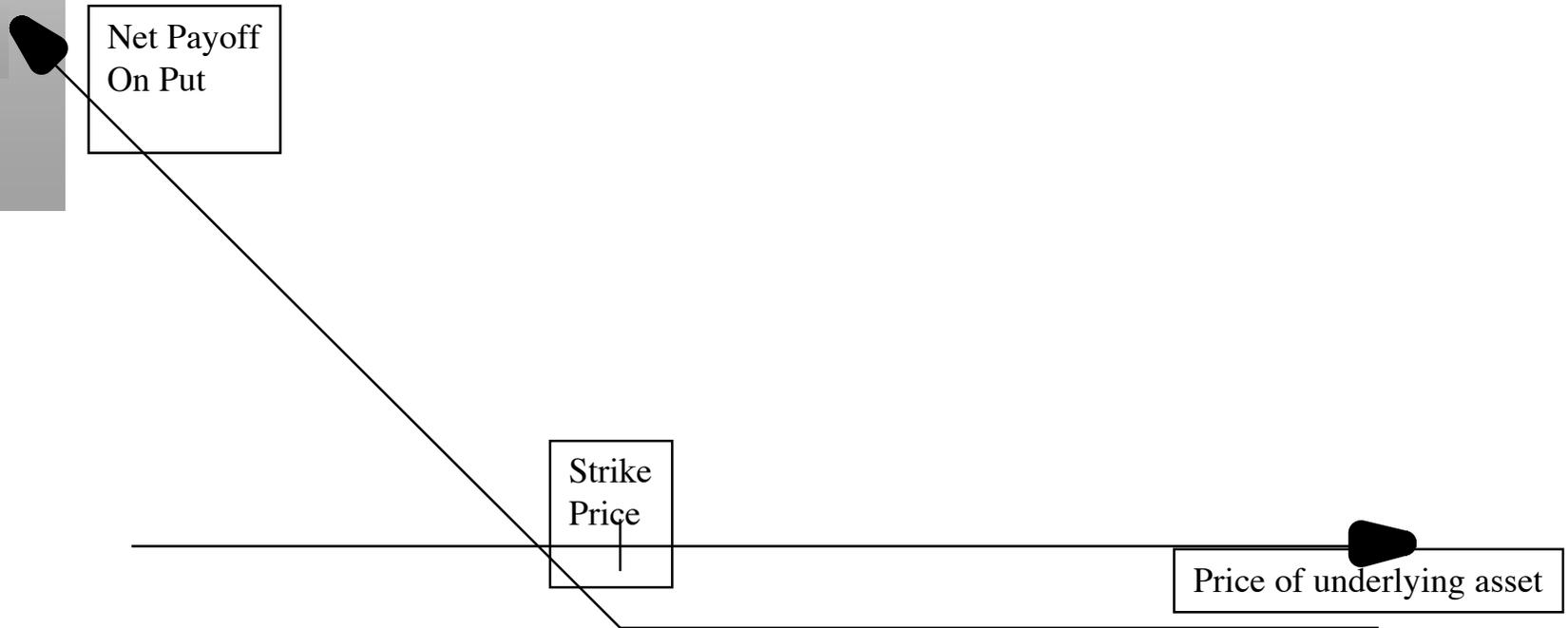
When is there an option embedded in an action?

- An option provides the holder with the **right** to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- There has to be a clearly defined underlying asset whose value changes over time in unpredictable ways.
- The payoffs on this asset (real option) have to be contingent on an specified event occurring within a finite period.

Payoff Diagram on a Call



Payoff Diagram on Put Option



When does the option have significant economic value?

- For an option to have significant economic value, there has to be a restriction on competition in the event of the contingency. In a perfectly competitive product market, no contingency, no matter how positive, will generate positive net present value.
- At the limit, real options are most valuable when you have exclusivity - you and only you can take advantage of the contingency. They become less valuable as the barriers to competition become less steep.

Determinants of option value

- Variables Relating to Underlying Asset
 - Value of Underlying Asset; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
 - Variance in that value; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
 - Expected dividends on the asset, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- Variables Relating to Option
 - Strike Price of Options; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
 - Life of the Option; both calls and puts benefit from a longer life.
- Level of Interest Rates; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

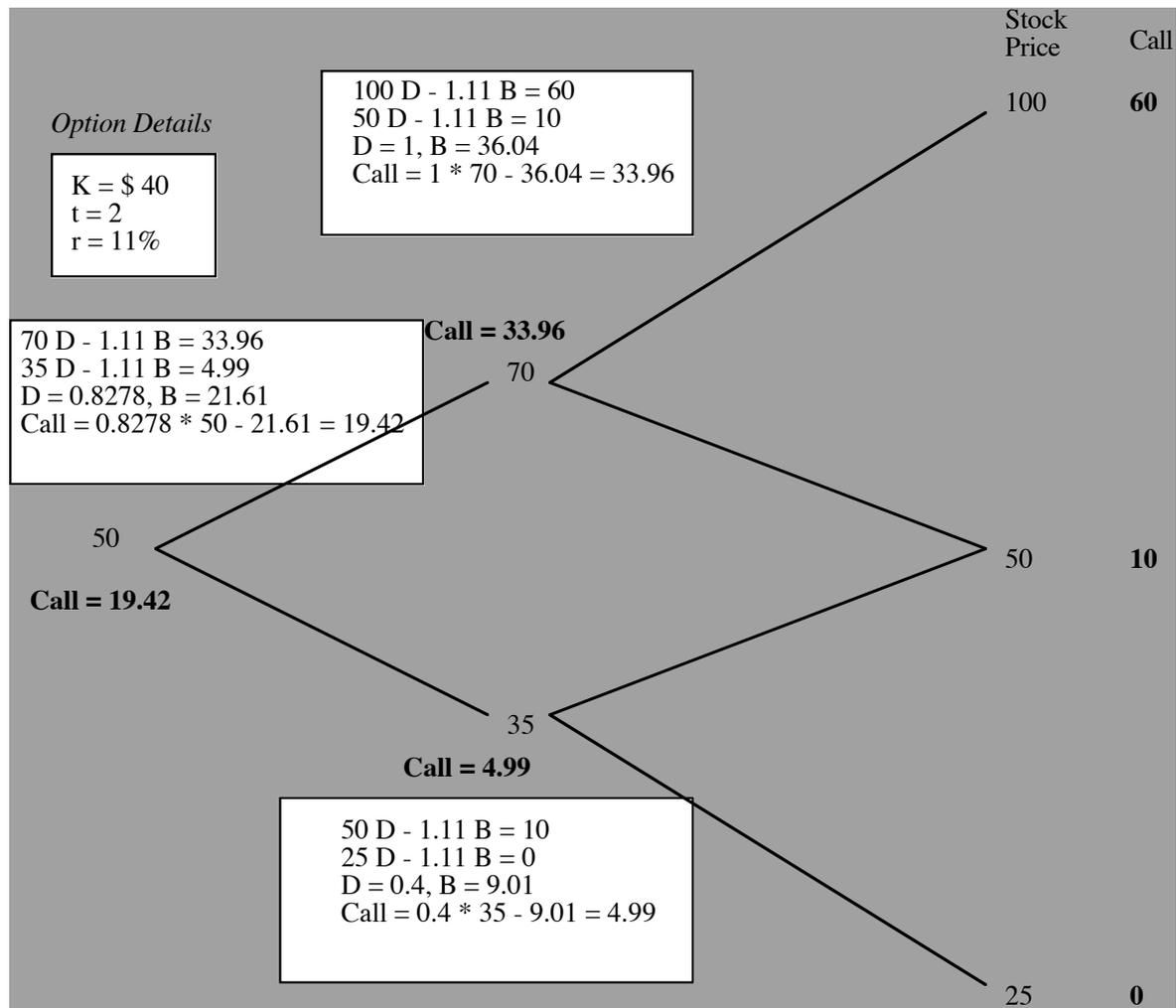
When can you use option pricing models to value real options?

- The notion of a replicating portfolio that drives option pricing models makes them most suited for valuing real options where
 - The underlying asset is traded - this yields not only observable prices and volatility as inputs to option pricing models but allows for the possibility of creating replicating portfolios
 - An active marketplace exists for the option itself.
 - The cost of exercising the option is known with some degree of certainty.
- When option pricing models are used to value real assets, we have to accept the fact that
 - The value estimates that emerge will be far more imprecise.
 - The value can deviate much more dramatically from market price because of the difficulty of arbitrage.

Creating a replicating portfolio

- The objective in creating a replicating portfolio is to use a combination of riskfree borrowing/lending and the underlying asset to create the same cashflows as the option being valued.
 - Call = Borrowing + Buying Δ of the Underlying Stock
 - Put = Selling Short Δ on Underlying Asset + Lending
 - The number of shares bought or sold is called the **option delta**.
- The principles of arbitrage then apply, and the value of the option has to be equal to the value of the replicating portfolio.

The Binomial Option Pricing Model



The Limiting Distributions....

- As the time interval is shortened, the limiting distribution, as $t \rightarrow 0$, can take one of two forms.
 - If as $t \rightarrow 0$, **price changes become smaller**, the limiting distribution is the normal distribution and the **price process is a continuous one**.
 - If as $t \rightarrow 0$, **price changes remain large**, the limiting distribution is the poisson distribution, i.e., a **distribution that allows for price jumps**.
- **The Black-Scholes model** applies when the **limiting distribution is the normal distribution**, and explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

The Black-Scholes Model

- The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected.
- The value of a call option in the Black-Scholes model can be written as a function of the following variables:

S = Current value of the underlying asset

K = Strike price of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the $\ln(\text{value})$ of the underlying asset

The Black Scholes Model

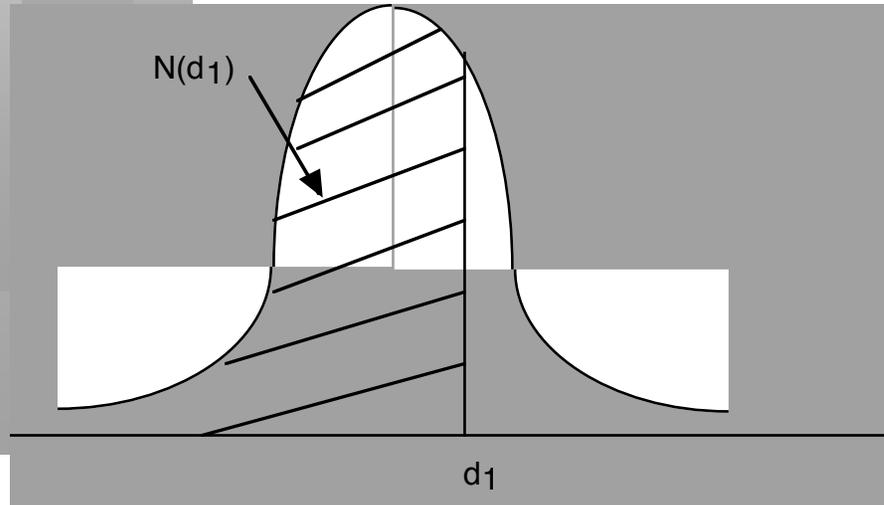
Value of call = $S N(d_1) - K e^{-rt} N(d_2)$

where,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

- $d_2 = d_1 - \sigma \sqrt{t}$
- The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to
 - Buy $N(d_1)$ shares of stock; $N(d_1)$ is called the option delta
 - Borrow $K e^{-rt} N(d_2)$

The Normal Distribution



d	$N(d)$	d	$N(d)$	d	$N(d)$
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

Adjusting for Dividends

- If the dividend yield ($y = \text{dividends} / \text{Current value of the asset}$) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S e^{-yt} N(d_1) - K e^{-rt} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r - y + \frac{\sigma^2}{2}) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

- The value of a put can also be derived:

$$P = K e^{-rt} (1 - N(d_2)) - S e^{-yt} (1 - N(d_1))$$

Choice of Option Pricing Models

- Most practitioners who use option pricing models to value real options argue for the binomial model over the Black-Scholes and justify this choice by noting that
 - Early exercise is the rule rather than the exception with real options
 - Underlying asset values are generally discontinuous.
- If you can develop a binomial tree with outcomes at each node, it looks a great deal like a decision tree from capital budgeting. The question then becomes when and why the two approaches yield different estimates of value.

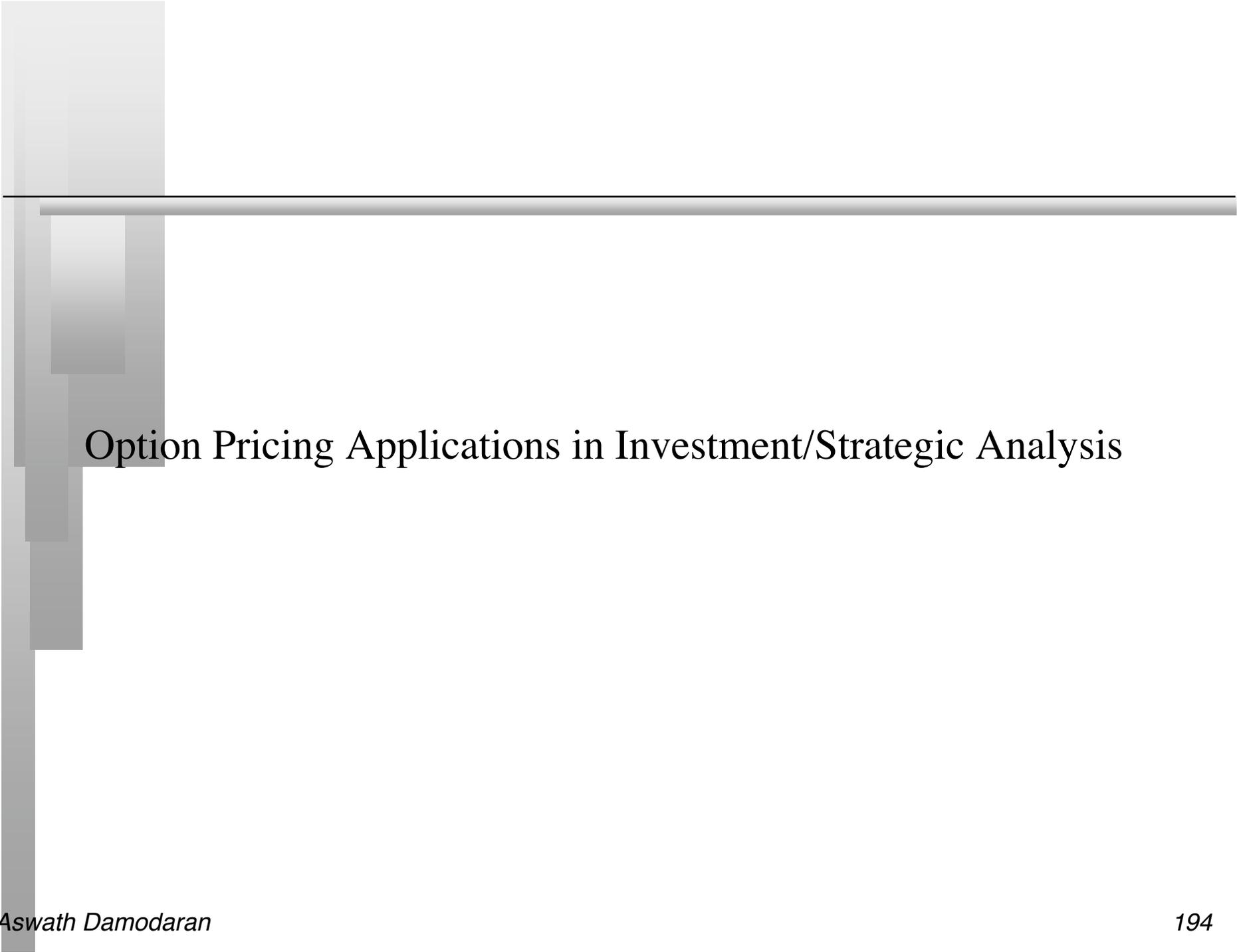
The Decision Tree Alternative

- Traditional decision tree analysis tends to use
 - One cost of capital to discount cashflows in each branch to the present
 - Probabilities to compute an expected value
 - **These values will generally be different from option pricing model values**
- If you modified decision tree analysis to
 - Use different discount rates at each node to reflect where you are in the decision tree (This is the Copeland solution) (or)
 - Use the riskfree rate to discount cashflows in each branch, estimate the probabilities to estimate an expected value and adjust the expected value for the market risk in the investment

Decision Trees could yield the same values as option pricing models

Key Tests for Real Options

- Is there an option embedded in this asset/ decision?
 - Can you identify the underlying asset?
 - Can you specify the contingency under which you will get payoff?
- Is there exclusivity?
 - If yes, there is option value.
 - If no, there is none.
 - If in between, you have to scale value.
- Can you use an option pricing model to value the real option?
 - Is the underlying asset traded?
 - Can the option be bought and sold?
 - Is the cost of exercising the option known and clear?



Option Pricing Applications in Investment/Strategic Analysis

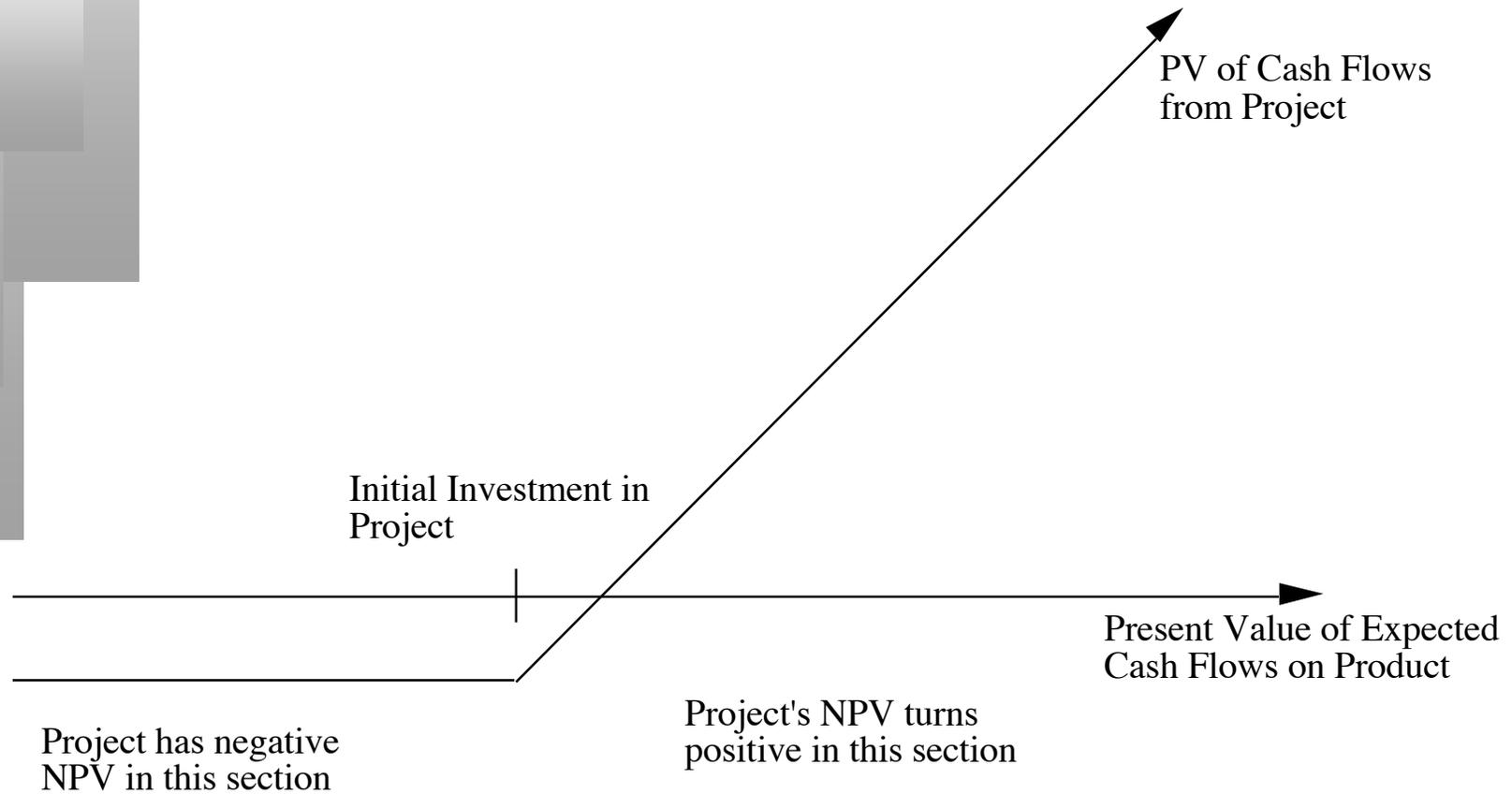
Options in Projects/Investments/Acquisitions

- One of the limitations of traditional investment analysis is that it is static and does not do a good job of capturing the options embedded in investment.
 - The first of these options is the option to delay taking a investment, when a firm has exclusive rights to it, until a later date.
 - The second of these options is taking one investment may allow us to take advantage of other opportunities (investments) in the future
 - The last option that is embedded in projects is the option to abandon a investment, if the cash flows do not measure up.
- These options all add value to projects and may make a “bad” investment (from traditional analysis) into a good one.

The Option to Delay

- When a firm has exclusive rights to a project or product for a specific period, it can delay taking this project or product until a later date.
- A traditional investment analysis just answers the question of whether the project is a “good” one if taken today.
- Thus, the fact that a project does not pass muster today (because its NPV is negative, or its IRR is less than its hurdle rate) does not mean that the rights to this project are not valuable.

Valuing the Option to Delay a Project



Insights for Investment Analyses

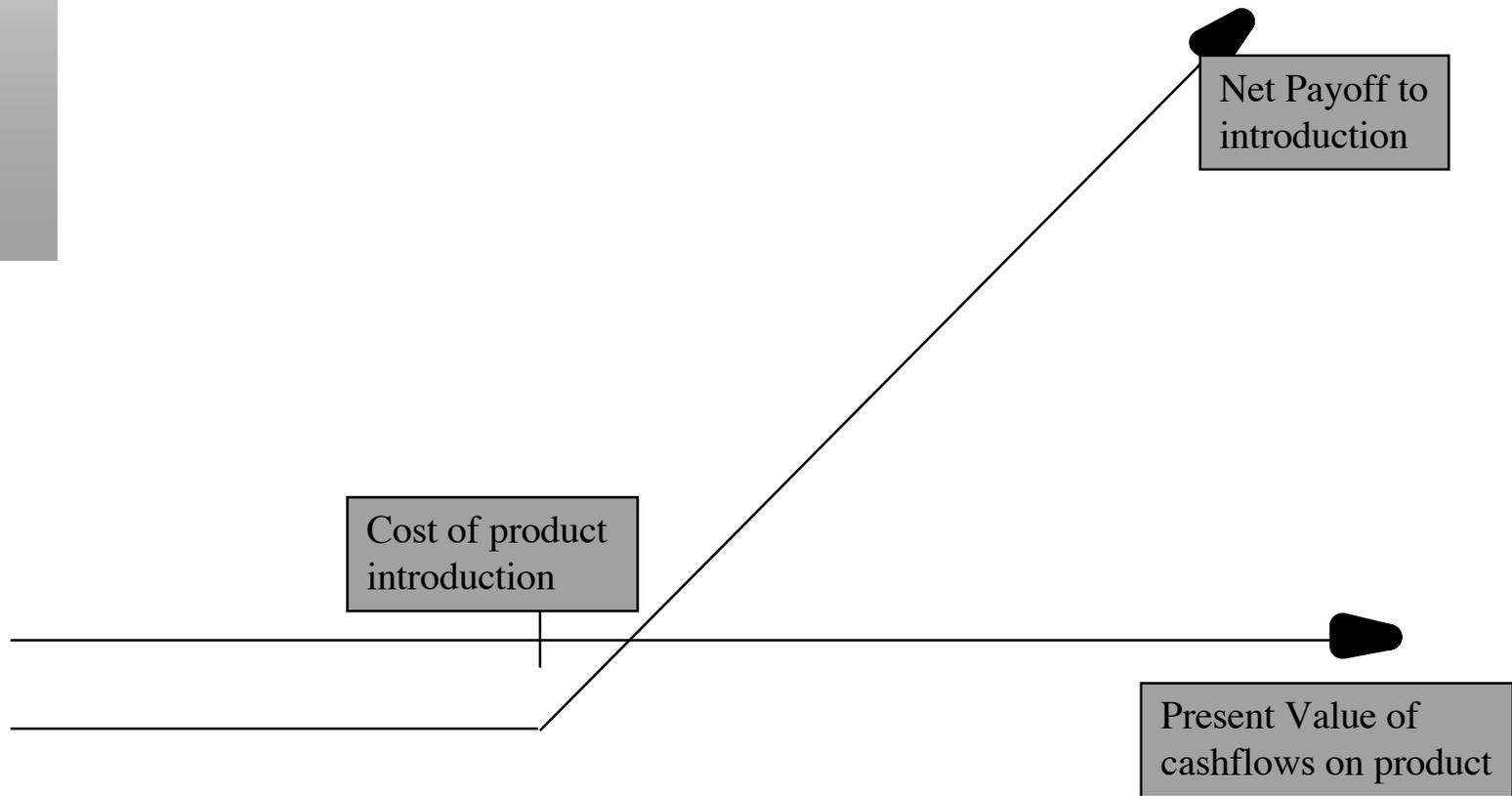
- Having the exclusive rights to a product or project is valuable, even if the product or project is not viable today.
- The value of these rights increases with the volatility of the underlying business.
- The cost of acquiring these rights (by buying them or spending money on development, for instance) has to be weighed off against these benefits.

Example 1: Valuing product patents as options

- A product patent provides the firm with the right to develop the product and market it.
- It will do so only if the present value of the expected cash flows from the product sales exceed the cost of development.
- If this does not occur, the firm can shelve the patent and not incur any further costs.
- If I is the present value of the costs of developing the product, and V is the present value of the expected cashflows from development, the payoffs from owning a product patent can be written as:

$$\begin{aligned} \text{Payoff from owning a product patent} &= V - I && \text{if } V > I \\ &= 0 && \text{if } V \leq I \end{aligned}$$

Payoff on Product Option



Obtaining Inputs for Patent Valuation

Input	Estimation Process
1. Value of the Underlying Asset	<ul style="list-style-type: none"> • Present Value of Cash Inflows from taking project now • This will be noisy, but that adds value.
2. Variance in value of underlying asset	<ul style="list-style-type: none"> • Variance in cash flows of similar assets or firms • Variance in present value from capital budgeting simulation.
3. Exercise Price on Option	<ul style="list-style-type: none"> • Option is exercised when investment is made. • Cost of making investment on the project ; assumed to be constant in present value dollars.
4. Expiration of the Option	<ul style="list-style-type: none"> • Life of the patent
5. Dividend Yield	<ul style="list-style-type: none"> • Cost of delay • Each year of delay translates into one less year of value-creating cashflows <p style="text-align: center;">Annual cost of delay = $\frac{1}{n}$</p>

Valuing a Product Patent: Avonex

- Biogen, a bio-technology firm, has a patent on Avonex, a drug to treat multiple sclerosis, for the next 17 years, and it plans to produce and sell the drug by itself. The key inputs on the drug are as follows:
 - PV of Cash Flows from Introducing the Drug Now = $S = \$ 3.422$ billion
 - PV of Cost of Developing Drug for Commercial Use = $K = \$ 2.875$ billion
 - Patent Life = $t = 17$ years Riskless Rate = $r = 6.7\%$ (17-year T.Bond rate)
 - Variance in Expected Present Values = $\sigma^2 = 0.224$ (Industry average firm variance for bio-tech firms)
 - Expected Cost of Delay = $y = 1/17 = 5.89\%$
 - $d1 = 1.1362$ $N(d1) = 0.8720$
 - $d2 = -0.8512$ $N(d2) = 0.2076$
- Call Value = $3,422 \exp^{(-0.0589)(17)} (0.8720) - 2,875 (\exp^{(-0.067)(17)} (0.2076)) = \$ 907$ million

Valuing a firm with patents

- The value of a firm with a substantial number of patents can be derived using the option pricing model.

Value of Firm = Value of commercial products (using DCF value
+ Value of existing patents (using option pricing)
+ (Value of New patents that will be obtained in the
future – Cost of obtaining these patents)

- The last input measures the efficiency of the firm in converting its R&D into commercial products. If we assume that a firm earns its cost of capital from research, this term will become zero.
- If we use this approach, we should be careful not to double count and allow for a high growth rate in cash flows (in the DCF valuation).

Value of Biogen's existing products

- Biogen had two commercial products (a drug to treat Hepatitis B and Intron) at the time of this valuation that it had licensed to other pharmaceutical firms.
- The license fees on these products were expected to generate \$ 50 million in after-tax cash flows each year for the next 12 years. To value these cash flows, which were guaranteed contractually, the pre-tax cost of debt of the guarantors (6.7%) was used:

$$\begin{aligned}\text{Present Value of License Fees} &= \$ 50 \text{ million } (1 - (1.067)^{-12}) / .067 \\ &= \$ 403.56 \text{ million}\end{aligned}$$

Value of Biogen's Future R&D

- Biogen continued to fund research into new products, spending about \$ 100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years, and 5% thereafter.
- It was assumed that every dollar invested in research would create \$ 1.25 in value in patents (valued using the option pricing model described above) for the next 10 years, and break even after that (i.e., generate \$ 1 in patent value for every \$ 1 invested in R&D).
- There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.

Value of Future R&D

<i>Yr</i>	<i>Value of Patents</i>	<i>R&D Cost</i>	<i>Excess Value</i>	<i>Present Value (at 15%)</i>
1	\$ 150.00	\$ 120.00	\$ 30.00	\$ 26.09
2	\$ 180.00	\$ 144.00	\$ 36.00	\$ 27.22
3	\$ 216.00	\$ 172.80	\$ 43.20	\$ 28.40
4	\$ 259.20	\$ 207.36	\$ 51.84	\$ 29.64
5	\$ 311.04	\$ 248.83	\$ 62.21	\$ 30.93
6	\$ 373.25	\$ 298.60	\$ 74.65	\$ 32.27
7	\$ 447.90	\$ 358.32	\$ 89.58	\$ 33.68
8	\$ 537.48	\$ 429.98	\$ 107.50	\$ 35.14
9	\$ 644.97	\$ 515.98	\$ 128.99	\$ 36.67
10	\$ 773.97	\$ 619.17	\$ 154.79	\$ 38.26
				\$ 318.30

Value of Biogen

- The value of Biogen as a firm is the sum of all three components – the present value of cash flows from existing products, the value of Avonex (as an option) and the value created by new research:

$$\begin{aligned}\text{Value} &= \text{Existing products} + \text{Existing Patents} + \text{Value: Future R\&D} \\ &= \$ 403.56 \text{ million} + \$ 907 \text{ million} + \$ 318.30 \text{ million} \\ &= \$1628.86 \text{ million}\end{aligned}$$

- Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.50 million) to arrive at a value per share:

$$\text{Value per share} = \$ 1,628.86 \text{ million} / 35.5 = \$ 45.88$$

The Real Options Test: Patents and Technology

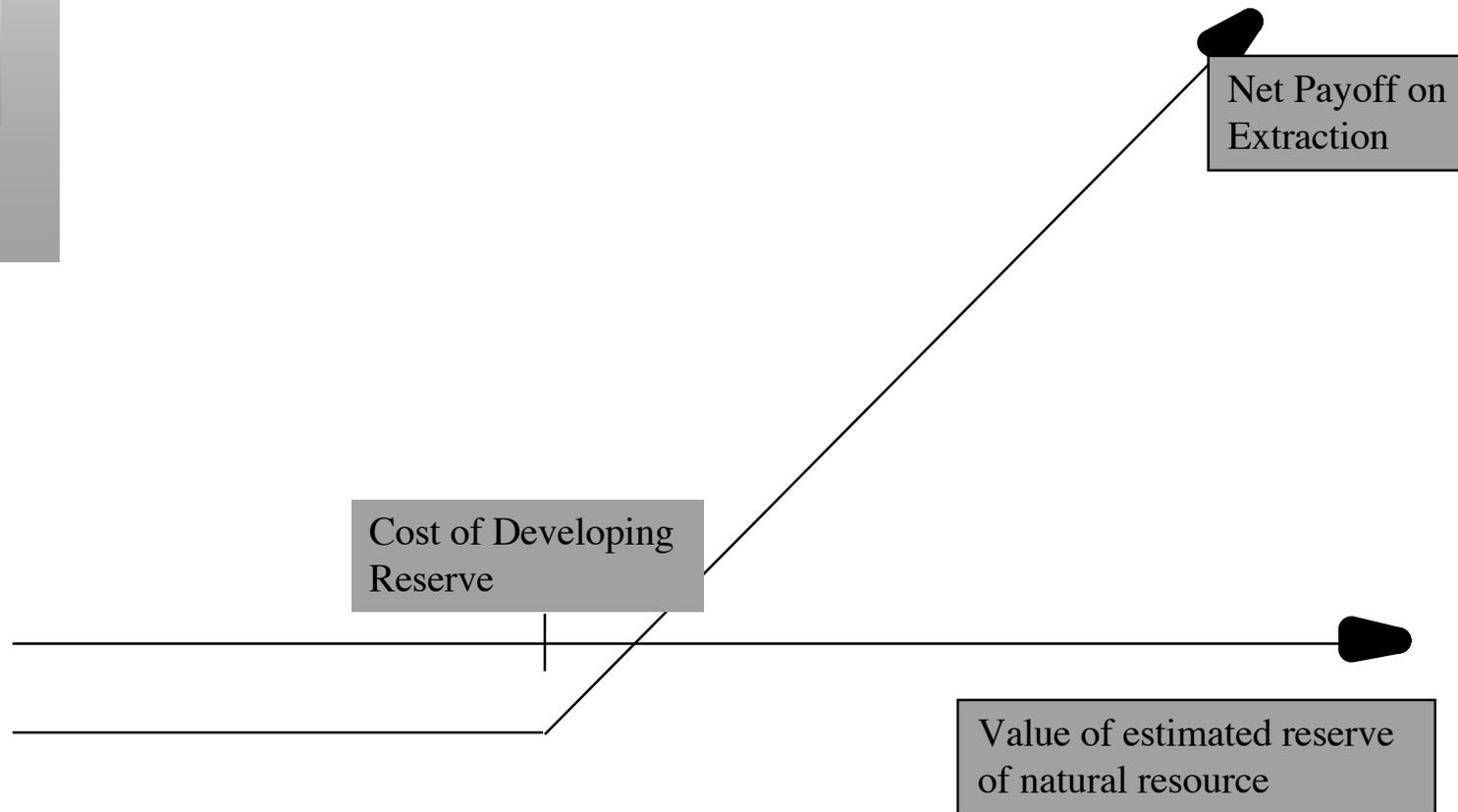
- The Option Test:
 - Underlying Asset: Product that would be generated by the patent
 - Contingency:
 - If PV of CFs from development > Cost of development: PV - Cost
 - If PV of CFs from development < Cost of development: 0
- The Exclusivity Test:
 - Patents restrict competitors from developing similar products
 - Patents do not restrict competitors from developing other products to treat the same disease.
- The Pricing Test
 - Underlying Asset: Patents are not traded. Not only do you therefore have to estimate the present values and volatilities yourself, you cannot construct replicating positions or do arbitrage.
 - Option: Patents are bought and sold, though not as frequently as oil reserves or mines.
 - Cost of Exercising the Option: This is the cost of converting the patent for commercial production. Here, experience does help and drug firms can make fairly precise estimates of the cost.
- Conclusion: You can estimate the value of the real option but the quality of your estimate will be a direct function of the quality of your capital budgeting. It works best if you are valuing a publicly traded firm that generates most of its value from one or a few patents - you can use the market value of the firm and the variance in that value then in your option pricing model.

Example 2: Valuing Natural Resource Options

- In a natural resource investment, the underlying asset is the resource and the value of the asset is based upon two variables - the quantity of the resource that is available in the investment and the price of the resource.
- In most such investments, there is a cost associated with developing the resource, and the difference between the value of the asset extracted and the cost of the development is the profit to the owner of the resource.
- Defining the cost of development as X , and the estimated value of the resource as V , the potential payoffs on a natural resource option can be written as follows:

$$\begin{aligned} \text{Payoff on natural resource investment} &= V - X && \text{if } V > X \\ &= 0 && \text{if } V \leq X \end{aligned}$$

Payoff Diagram on Natural Resource Firms



Estimating Inputs for Natural Resource Options

Input	Estimation Process
1. Value of Available Reserves of the Resource	<ul style="list-style-type: none"> Expert estimates (Geologists for oil.); The present value of the after-tax cash flows from the resource are then estimated.
2. Cost of Developing Reserve (Strike Price)	<ul style="list-style-type: none"> Past costs and the specifics of the investment
3. Time to Expiration	<ul style="list-style-type: none"> Relinquishment Period: if asset has to be relinquished at a point in time. Time to exhaust inventory - based upon inventory and capacity output.
4. Variance in value of underlying asset	<ul style="list-style-type: none"> based upon variability of the price of the resources and variability of available reserves.
5. Net Production Revenue (Dividend Yield)	<ul style="list-style-type: none"> Net production revenue every year as percent of market value.
6. Development Lag	<ul style="list-style-type: none"> Calculate present value of reserve based upon the lag.

Valuing an Oil Reserve

- Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil, where the present value of the development cost is \$12 per barrel and the development lag is two years.
- The firm has the rights to exploit this reserve for the next twenty years and the marginal value per barrel of oil is \$12 per barrel currently (Price per barrel - marginal cost per barrel).
- Once developed, the net production revenue each year will be 5% of the value of the reserves.
- The riskless rate is 8% and the variance in $\ln(\text{oil prices})$ is 0.03.

Inputs to Option Pricing Model

- Current Value of the asset = S = Value of the developed reserve discounted back the length of the development lag at the dividend yield = $\$12 * 50 / (1.05)^2 = \$ 544.22$
- (If development is started today, the oil will not be available for sale until two years from now. The estimated opportunity cost of this delay is the lost production revenue over the delay period. Hence, the discounting of the reserve back at the dividend yield)
- Exercise Price = Present Value of development cost = $\$12 * 50 = \600 million
- Time to expiration on the option = 20 years
- Variance in the value of the underlying asset = 0.03
- Riskless rate = 8%
- Dividend Yield = Net production revenue / Value of reserve = 5%

Valuing the Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 $d1 = 1.0359$ $N(d1) = 0.8498$
 $d2 = 0.2613$ $N(d2) = 0.6030$
- Call Value = $544.22 \exp^{(-0.05)(20)} (0.8498) - 600 (\exp^{(-0.08)(20)} (0.6030)) = \$ 97.08$ million
- This oil reserve, though not viable at current prices, still is a valuable property because of its potential to create value if oil prices go up.

Extending the option pricing approach to value natural resource firms

- Since the assets owned by a natural resource firm can be viewed primarily as options, **the firm itself can be valued using option pricing** models.
- The preferred approach would be to **consider each option separately**, value it and cumulate the values of the options to get the firm value.
- Since this information is likely to be **difficult to obtain** for large natural resource firms, such as oil companies, which own hundreds of such assets, a variant is to value the entire firm as one option.
- A purist would probably disagree, arguing that **valuing an option on a portfolio of assets (as in this approach) will provide a lower value than valuing a portfolio of options** (which is what the natural resource firm really own). Nevertheless, the value obtained from the model still provides an interesting perspective on the determinants of the value of natural resource firms.

Inputs to the Model

Input to model

Value of underlying asset

Exercise Price

Time to expiration on option

Riskless rate

Variance in value of asset

Dividend yield

Corresponding input for valuing firm

Value of cumulated estimated reserves of the resource owned by the firm, discounted back at the dividend yield for the development lag.

Estimated cumulated cost of developing estimated reserves

Average relinquishment period across all reserves owned by firm (if known) or estimate of when reserves will be exhausted, given current production rates.

Riskless rate corresponding to life of the option

Variance in the price of the natural resource

Estimated annual net production revenue as percentage of value of the reserve.

Valuing Gulf Oil

- Gulf Oil was the target of a takeover in early 1984 at \$70 per share (It had 165.30 million shares outstanding, and total debt of \$9.9 billion).
 - It had estimated reserves of 3038 million barrels of oil and the average cost of developing these reserves was estimated to be \$10 a barrel in present value dollars (The development lag is approximately two years).
 - The average relinquishment life of the reserves is 12 years.
 - The price of oil was \$22.38 per barrel, and the production cost, taxes and royalties were estimated at \$7 per barrel.
 - The bond rate at the time of the analysis was 9.00%.
 - Gulf was expected to have net production revenues each year of approximately 5% of the value of the developed reserves. The variance in oil prices is 0.03.

Valuing Undeveloped Reserves

- Inputs for valuing undeveloped reserves
 - Value of underlying asset = Value of estimated reserves discounted back for period of development lag = $3038 * (\$ 22.38 - \$7) / 1.05^2 = \$42,380.44$
 - Exercise price = Estimated development cost of reserves = $3038 * \$10 = \$30,380$ million
 - Time to expiration = Average length of relinquishment option = **12 years**
 - Variance in value of asset = Variance in oil prices = **0.03**
 - Riskless interest rate = **9%**
 - Dividend yield = Net production revenue/ Value of developed reserves = **5%**
- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - d1 = 1.6548 N(d1) = 0.9510
 - d2 = 1.0548 N(d2) = 0.8542
- Call Value = $42,380.44 \exp^{(-0.05)(12)} (0.9510) - 30,380 (\exp^{(-0.09)(12)} (0.8542)) = \$$
13,306 million

Valuing Gulf Oil

- In addition, Gulf Oil had free cashflows to the firm from its oil and gas production of \$915 million from already developed reserves and these cashflows are likely to continue for ten years (the remaining lifetime of developed reserves).
- The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:
 - Value of already developed reserves = $915 (1 - 1.125^{-10}) / .125 = \5065.83
- Adding the value of the developed and undeveloped reserves

Value of undeveloped reserves	= \$ 13,306 million	
Value of production in place	= \$ 5,066 million	
Total value of firm	= \$ 18,372 million	
Less Outstanding Debt	= \$ 9,900 million	
Value of Equity	= \$ 8,472 million	
Value per share	= \$ 8,472/165.3	= \$51.25

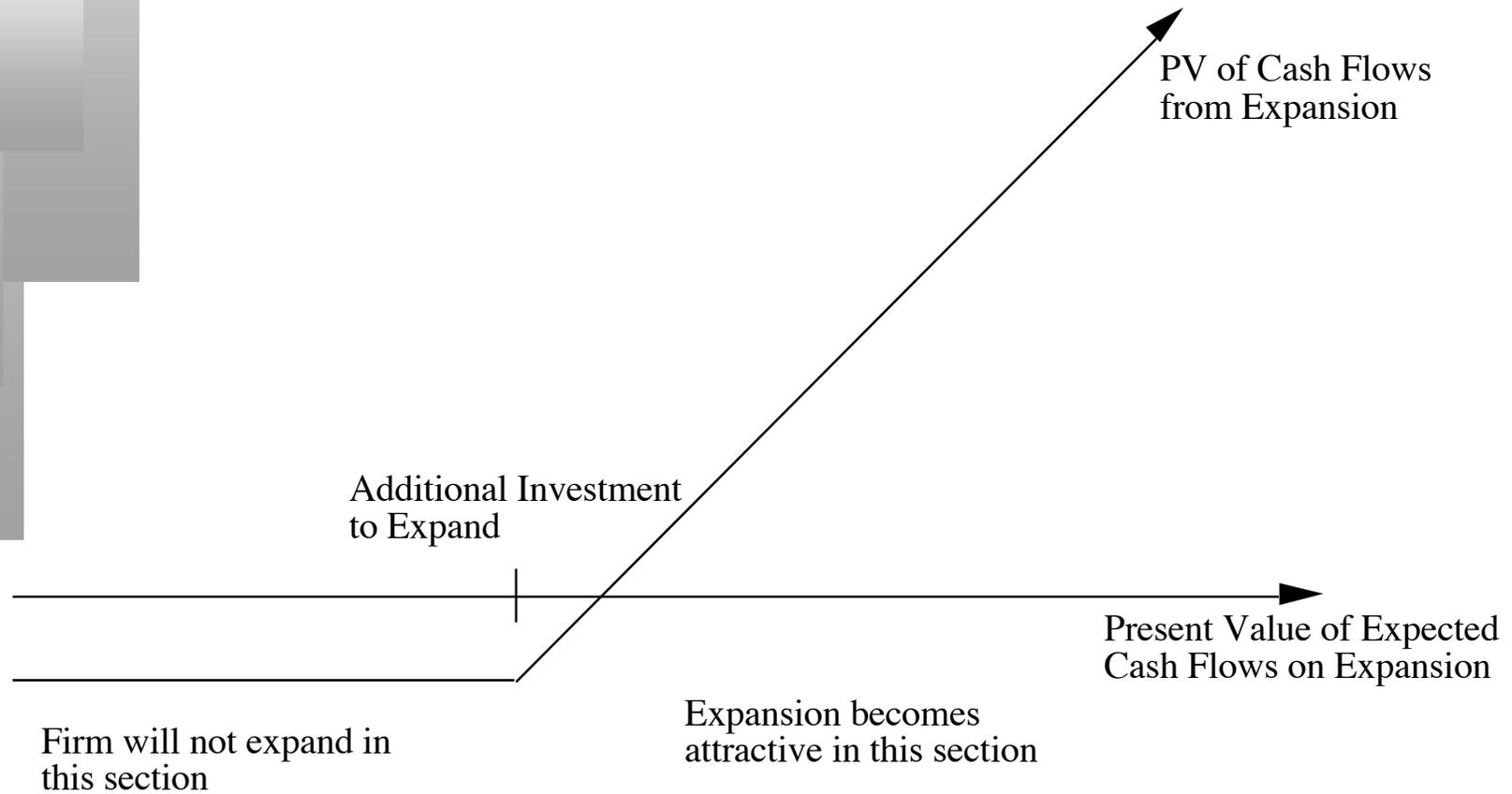
Putting Natural Resource Options to the Test

- The Option Test:
 - Underlying Asset: Oil or gold in reserve
 - Contingency: If value > Cost of development: Value - Dev Cost
If value < Cost of development: 0
- The Exclusivity Test:
 - Natural resource reserves are limited (at least for the short term)
 - It takes time and resources to develop new reserves
- The Option Pricing Test
 - Underlying Asset: While the reserve or mine may not be traded, the commodity is. If we assume that we know the quantity with a fair degree of certainty, you can trade the underlying asset
 - Option: Oil companies buy and sell reserves from each other regularly.
 - Cost of Exercising the Option: This is the cost of developing a reserve. Given the experience that commodity companies have with this, they can estimate this cost with a fair degree of precision.
- Real option pricing models work well with natural resource options.

The Option to Expand/Take Other Projects

- Taking a project today may allow a firm to consider and take other valuable projects in the future.
- Thus, even though a project may have a negative NPV, it may be a project worth taking if the option it provides the firm (to take other projects in the future) provides a more-than-compensating value.
- These are the options that firms often call “strategic options” and use as a rationale for taking on “negative NPV” or even “negative return” projects.

The Option to Expand



An Example of an Expansion Option

- Ambev is considering introducing a soft drink to the U.S. market. The drink will initially be introduced only in the metropolitan areas of the U.S. and the cost of this “limited introduction” is \$ 500 million.
- A financial analysis of the cash flows from this investment suggests that the present value of the cash flows from this investment to Ambev will be only \$ 400 million. Thus, by itself, the new investment has a **negative NPV of \$ 100 million**.
- If the initial introduction works out well, Ambev **could go ahead with a full-scale introduction to the entire market with an additional investment of \$ 1 billion** any time over the next 5 years. While the current expectation is that the cash flows from having this investment is only \$ 750 million, there is considerable uncertainty about both the potential for the drink, leading to significant variance in this estimate.

Valuing the Expansion Option

- Value of the Underlying Asset (S) = PV of Cash Flows from Expansion to entire U.S. market, if done now = \$ 750 Million
- Strike Price (K) = Cost of Expansion into entire U.S market = \$ 1000 Million
- We estimate the standard deviation in the estimate of the project value by using the annualized standard deviation in firm value of publicly traded firms in the beverage markets, which is approximately 34.25%.
 - Standard Deviation in Underlying Asset's Value = 34.25%
- Time to expiration = Period for which expansion option applies = 5 years

Call Value= \$ 234 Million

Considering the Project with Expansion Option

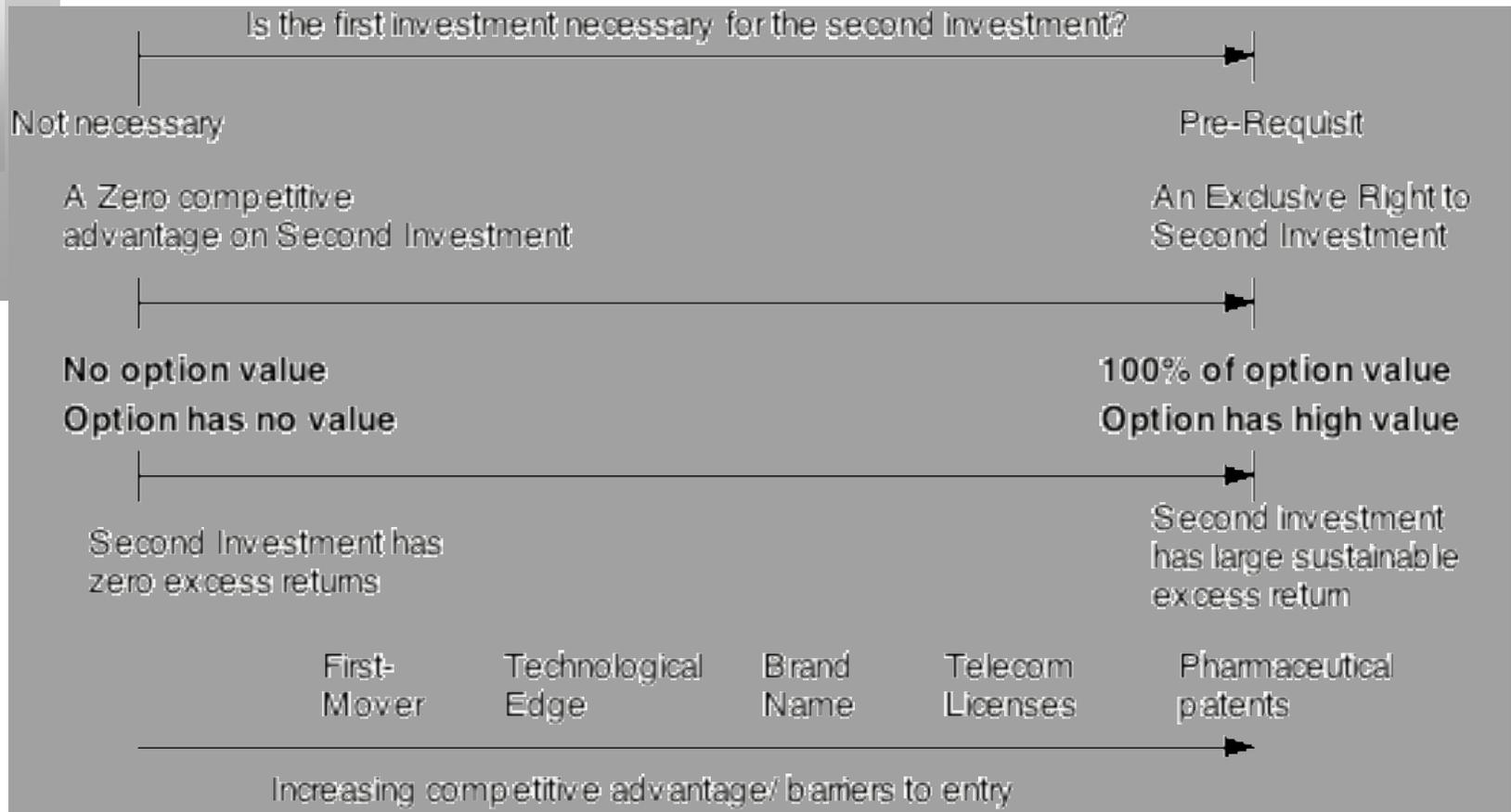
- NPV of Limited Introduction = \$ 400 Million - \$ 500 Million
= - \$ 100 Million
- Value of Option to Expand to full market= \$ 234 Million
- NPV of Project with option to expand
= - \$ 100 million + \$ 234 million
= \$ 134 million
- **Invest in the project**

The Real Options Test for Expansion Options

- The Options Test
 - Underlying Asset: Expansion Project
 - Contingency

If PV of CF from expansion > Expansion Cost: PV - Expansion Cost
If PV of CF from expansion < Expansion Cost: 0
- The Exclusivity Test
 - Barriers may range from strong (exclusive licenses granted by the government) to weaker (brand name, knowledge of the market) to weakest (first mover).
- The Pricing Test
 - Underlying Asset: As with patents, there is no trading in the underlying asset and you have to estimate value and volatility.
 - Option: Licenses are sometimes bought and sold, but more diffuse expansion options are not.
 - Cost of Exercising the Option: Not known with any precision and may itself evolve over time as the market evolves.
- Using option pricing models to value expansion options will not only yield extremely noisy estimates, but may attach inappropriate premiums to discounted cashflow estimates.

Opportunities and not Options...

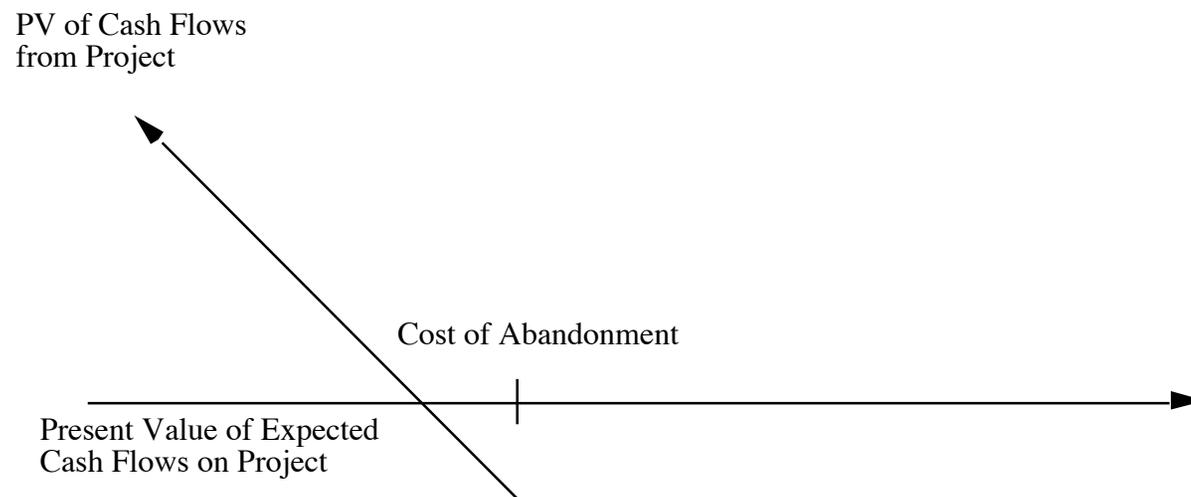


Internet Firms as Options

- Some analysts have justified the valuation of internet firms on the basis that you are buying the option to expand into a very large market. What do you think of this argument?
 - Is there an option to expand embedded in these firms?
 - Is it a valuable option?

The Option to Abandon

- A firm may sometimes have the option to abandon a project, if the cash flows do not measure up to expectations.
- If abandoning the project allows the firm to save itself from further losses, this option can make a project more valuable.



Valuing the Option to Abandon

- Embraer is considering a joint venture with Lear Aircraft to produce a small commercial airplane (capable of carrying 40-50 passengers on short haul flights)
 - Embraer will have to invest \$ 500 million for a 50% share of the venture
 - Its share of the present value of expected cash flows is 480 million.
- Lear Aircraft, which is eager to enter into the deal, offers to buy Embraer's 50% share of the investment anytime over the next five years for \$ 400 million, if Embraer decides to get out of the venture.
- A simulation of the cash flows on this time share investment yields a variance in the present value of the cash flows from being in the partnership is 0.16.
- The project has a life of 30 years.

Project with Option to Abandon

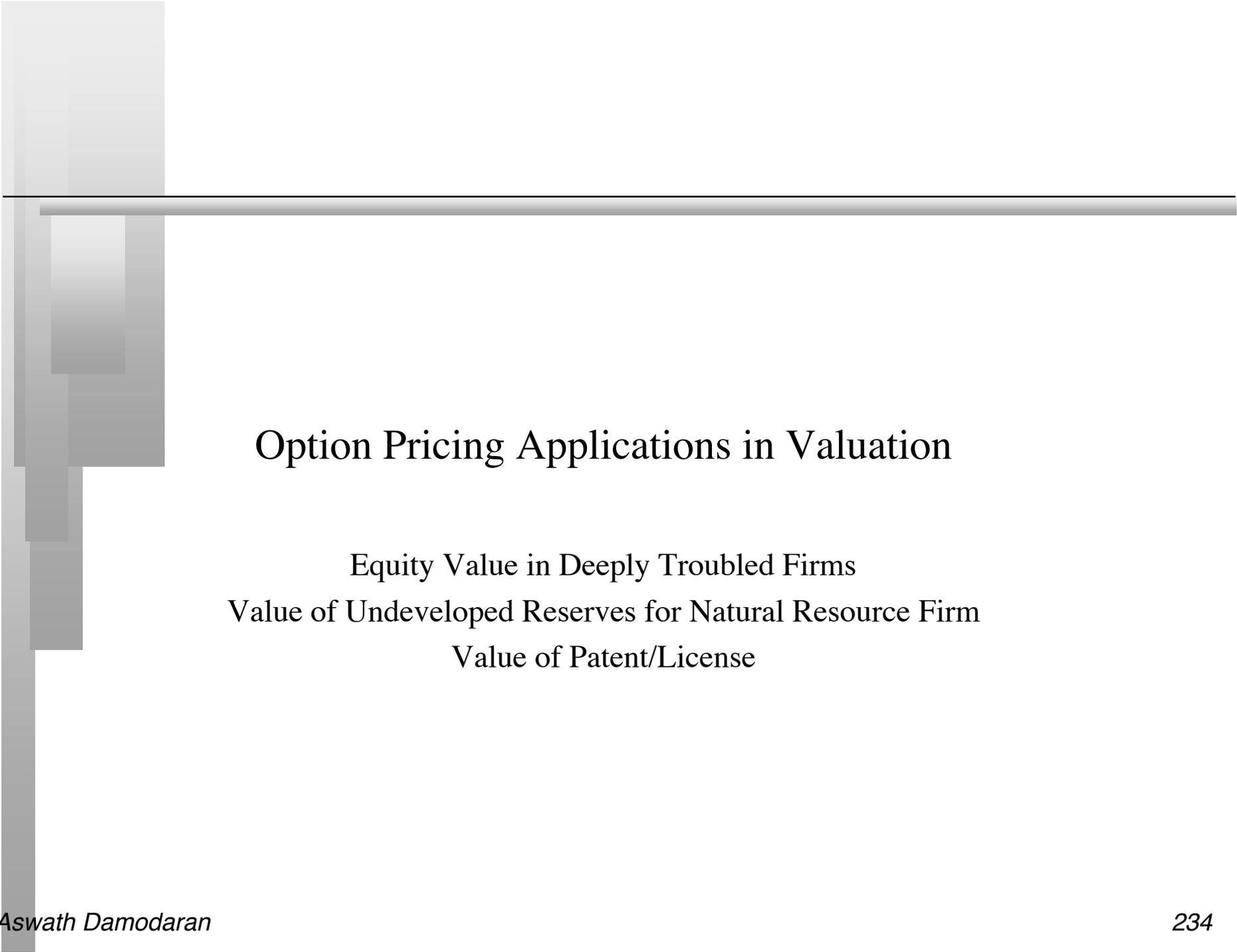
- Value of the Underlying Asset (S) = PV of Cash Flows from Project
= \$ 480 million
- Strike Price (K) = Salvage Value from Abandonment = \$ 400 million
- Variance in Underlying Asset's Value = 0.16
- Time to expiration = Life of the Project = 5 years
- Dividend Yield = $1/\text{Life of the Project} = 1/30 = 0.033$ (We are assuming that the project's present value will drop by roughly $1/n$ each year into the project)
- Assume that the five-year riskless rate is 6%. The value of the put option can be estimated as follows:

Should Embraer enter into the joint venture?

- Value of Put = $Ke^{-rt} (1-N(d2)) - Se^{-yt} (1-N(d1))$
= $400 (\exp^{(-0.06)(5)} (1-0.7496)) - 480 \exp^{(-0.033)(5)} (1-0.9105)$
= \$ 73.23 million
- The value of this abandonment option has to be added on to the net present value of the project of -\$ 20 million, yielding a total net present value with the abandonment option of \$ 53.23 million.

Implications for Investment Analysis

- Having a option to abandon a project can make otherwise unacceptable projects acceptable.
- Actions that increase the value of the abandonment option include
 - More cost flexibility, that is, making more of the costs of the projects into variable costs as opposed to fixed costs.
 - Fewer long-term contracts/obligations with employees and customers, since these add to the cost of abandoning a project
 - Finding partners in the investment, who are willing to acquire your investment in the future
- These actions will undoubtedly cost the firm some value, but this has to be weighed off against the increase in the value of the abandonment option.



Option Pricing Applications in Valuation

Equity Value in Deeply Troubled Firms
Value of Undeveloped Reserves for Natural Resource Firm
Value of Patent/License

Option Pricing Applications in Equity Valuation

- Equity in a troubled firm (i.e. a firm with high leverage, negative earnings and a significant chance of bankruptcy) can be viewed as a call option, which is the option to liquidate the firm.
- Natural resource companies, where the undeveloped reserves can be viewed as options on the natural resource.
- Start-up firms or high growth firms which derive the bulk of their value from the rights to a product or a service (eg. a patent)

Valuing Equity as an option

- The equity in a firm is a **residual claim**, i.e., equity holders lay claim to all cashflows left over after other financial claim-holders (debt, preferred stock etc.) have been satisfied.
- If a firm is liquidated, the same principle applies, with equity investors **receiving whatever is left over in the firm** after all outstanding debts and other financial claims are paid off.
- The **principle of limited liability**, however, protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm.

Equity as a call option

- The payoff to equity investors, on liquidation, can therefore be written as:

$$\begin{aligned} \text{Payoff to equity on liquidation} &= V - D && \text{if } V > D \\ &= 0 && \text{if } V \leq D \end{aligned}$$

where,

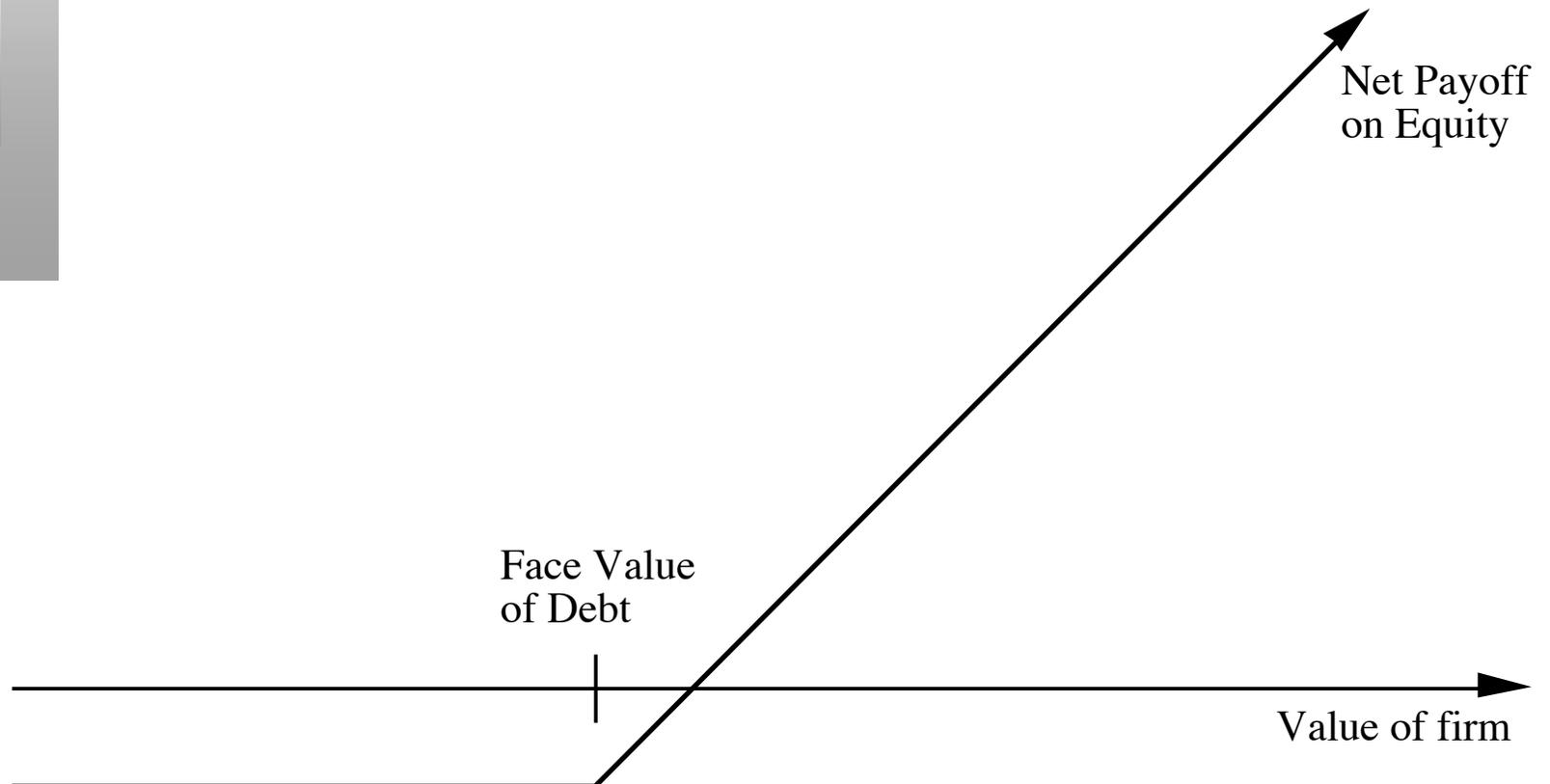
V = Value of the firm

D = Face Value of the outstanding debt and other external claims

- A call option, with a strike price of K, on an asset with a current value of S, has the following payoffs:

$$\begin{aligned} \text{Payoff on exercise} &= S - K && \text{if } S > K \\ &= 0 && \text{if } S \leq K \end{aligned}$$

Payoff Diagram for Liquidation Option



Application to valuation: A simple example

- Assume that you have a firm whose assets are currently valued at \$100 million and that the standard deviation in this asset value is 40%.
- Further, assume that the face value of debt is \$80 million (It is zero coupon debt with 10 years left to maturity).
- If the ten-year treasury bond rate is 10%,
 - how much is the equity worth?
 - What should the interest rate on debt be?

Model Parameters

- Value of the underlying asset = S = Value of the firm = \$ 100 million
- Exercise price = K = Face Value of outstanding debt = \$ 80 million
- Life of the option = t = Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16
- Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Valuing Equity as a Call Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - $d1 = 1.5994$ $N(d1) = 0.9451$
 - $d2 = 0.3345$ $N(d2) = 0.6310$
- Value of the call = $100 (0.9451) - 80 \exp^{(-0.10)(10)} (0.6310) = \75.94 million
- Value of the outstanding debt = $\$100 - \$75.94 = \$24.06$ million
- Interest rate on debt = $(\$ 80 / \$24.06)^{1/10} - 1 = 12.77\%$

The Effect of Catastrophic Drops in Value

- Assume now that a catastrophe wipes out half the value of this firm (the value drops to \$ 50 million), while the face value of the debt remains at \$ 80 million. What will happen to the equity value of this firm?
 - It will drop in value to \$ 25.94 million [\$ 50 million - market value of debt from previous page]
 - It will be worth nothing since debt outstanding > Firm Value
 - It will be worth more than \$ 25.94 million

Illustration : Value of a troubled firm

- Assume now that, in the previous example, the value of the firm were reduced to \$ 50 million while keeping the face value of the debt at \$80 million.
- This firm could be viewed as troubled, since it owes (at least in face value terms) more than it owns.
- The equity in the firm will still have value, however.

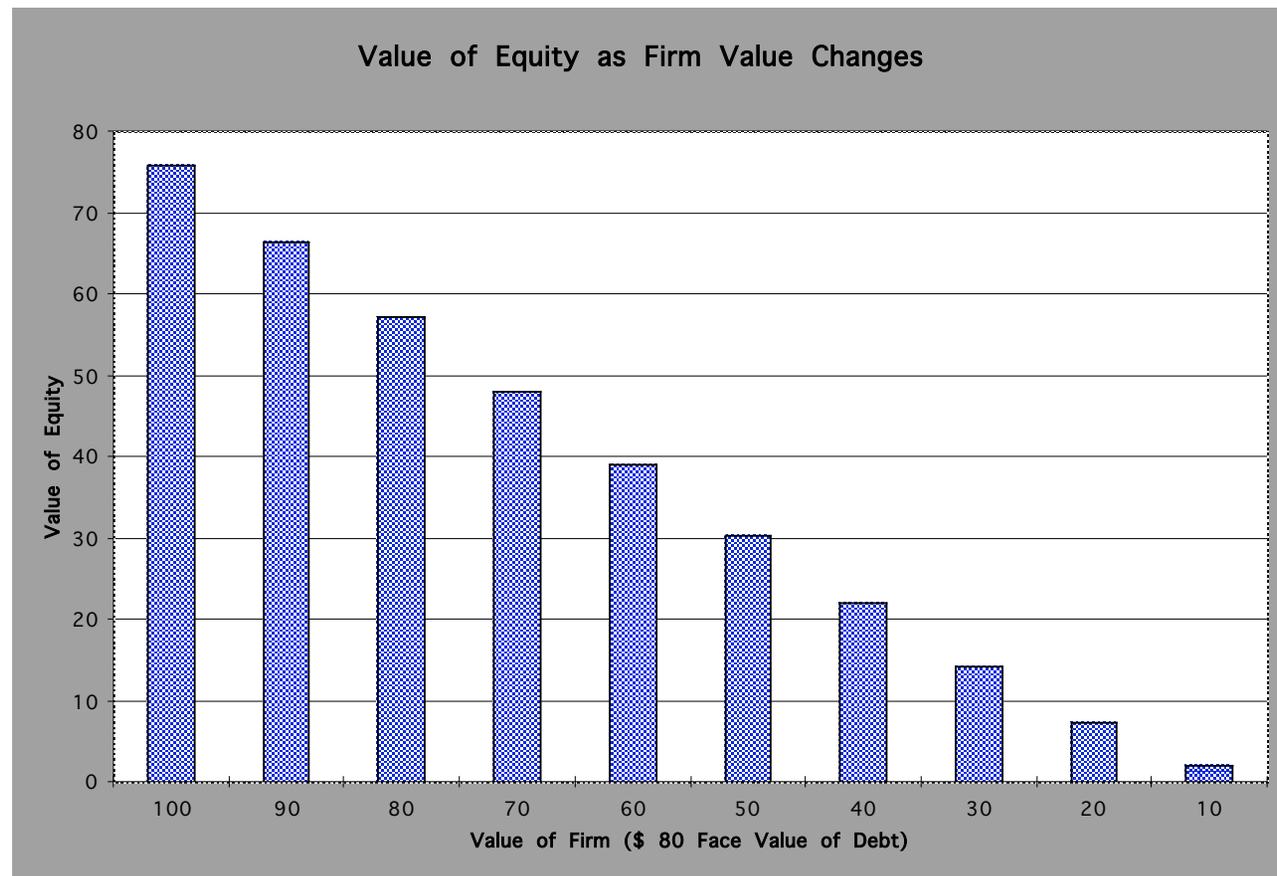
Valuing Equity in the Troubled Firm

- Value of the underlying asset = S = Value of the firm = \$ 50 million
- Exercise price = K = Face Value of outstanding debt = \$ 80 million
- Life of the option = t = Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16
- Riskless rate = r = Treasury bond rate corresponding to option life = 10%

The Value of Equity as an Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - $d1 = 1.0515$ $N(d1) = 0.8534$
 - $d2 = -0.2135$ $N(d2) = 0.4155$
- Value of the call = $50 (0.8534) - 80 \exp^{(-0.10)(10)} (0.4155) = \30.44 million
- Value of the bond = $\$50 - \$30.44 = \$19.56$ million
- The equity in this firm drops by, because of the option characteristics of equity.
- This might explain why stock in firms, which are in Chapter 11 and essentially bankrupt, still has value.

Equity value persists ..



Valuing equity in a troubled firm

- The first implication is that **equity will have value**, even if the **value of the firm** falls well **below the face value of the outstanding debt**.
- Such a firm will be viewed as **troubled** by investors, accountants and analysts, but that **does not mean that its equity is worthless**.
- Just as deep out-of-the-money traded options command value because of the possibility that the value of the underlying asset may increase above the strike price in the remaining lifetime of the option, **equity will command value because of the time premium on the option** (the time until the bonds mature and come due) and the possibility that the value of the assets may increase above the face value of the bonds before they come due.

Obtaining option pricing inputs - Some real world problems

- The examples that have been used to illustrate the use of option pricing theory to value equity have made some simplifying assumptions. Among them are the following:
 - (1) There were only two claim holders in the firm - debt and equity.
 - (2) There is only one issue of debt outstanding and it can be retired at face value.
 - (3) The debt has a zero coupon and no special features (convertibility, put clauses etc.)
 - (4) The value of the firm and the variance in that value can be estimated.

Real World Approaches to Getting inputs

Input	Estimation Process
Value of the Firm	<ul style="list-style-type: none"> • Cumulate market values of equity and debt (or) • Value the <u>assets in place</u> using FCFF and WACC (or) • Use cumulated market value of assets, if traded.
Variance in Firm Value	<ul style="list-style-type: none"> • If stocks and bonds are traded, $\sigma_{\text{firm}}^2 = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2 w_e w_d \rho_{ed} \sigma_e \sigma_d$ <p>where σ_e^2 = variance in the stock price w_e = MV weight of Equity σ_d^2 = the variance in the bond price w_d = MV weight of debt</p> <ul style="list-style-type: none"> • If not traded, use variances of similarly rated bonds. • Use average firm value variance from the industry in which company operates.
Value of the Debt	<ul style="list-style-type: none"> • If the debt is short term, you can use only the face or book value of the debt. • If the debt is long term and coupon bearing, add the cumulated nominal value of these coupons to the face value of the debt.
Maturity of the Debt	<ul style="list-style-type: none"> • Face value weighted duration of bonds outstanding (or) • If not available, use weighted maturity

Valuing Equity as an option - Eurotunnel in early 1998

- Eurotunnel has been a financial disaster since its opening
 - In 1997, Eurotunnel had earnings before interest and taxes of -£56 million and net income of -£685 million
 - At the end of 1997, its book value of equity was -£117 million
- It had £8,865 million in face value of debt outstanding
 - The weighted average duration of this debt was 10.93 years

Debt Type	Face Value	Duration
Short term	935	0.50
10 year	2435	6.7
20 year	3555	12.6
Longer	1940	18.2
<i>Total</i>	<i>£8,865 mil</i>	<i>10.93 years</i>

The Basic DCF Valuation

- The value of the firm estimated using projected cashflows to the firm, discounted at the weighted average cost of capital was £2,312 million.
- This was based upon the following assumptions –
 - Revenues will grow 5% a year in perpetuity.
 - The COGS which is currently 85% of revenues will drop to 65% of revenues in yr 5 and stay at that level.
 - Capital spending and depreciation will grow 5% a year in perpetuity.
 - There are no working capital requirements.
 - The debt ratio, which is currently 95.35%, will drop to 70% after year 5. The cost of debt is 10% in high growth period and 8% after that.
 - The beta for the stock will be 1.10 for the next five years, and drop to 0.8 after the next 5 years.
 - The long term bond rate is 6%.

Other Inputs

- The stock has been traded on the London Exchange, and the annualized std deviation based upon \ln (prices) is 41%.
- There are Eurotunnel bonds, that have been traded; the annualized std deviation in $\ln(\text{price})$ for the bonds is 17%.
 - The correlation between stock price and bond price changes has been 0.5. The proportion of debt in the capital structure during the period (1992-1996) was 85%.
 - Annualized variance in firm value
 $= (0.15)^2 (0.41)^2 + (0.85)^2 (0.17)^2 + 2 (0.15) (0.85)(0.5)(0.41)(0.17) = 0.0335$
- The 15-year bond rate is 6%. (I used a bond with a duration of roughly 11 years to match the life of my option)

Valuing Eurotunnel Equity and Debt

- Inputs to Model
 - Value of the underlying asset = S = Value of the firm = £2,312 million
 - Exercise price = K = Face Value of outstanding debt = £8,865 million
 - Life of the option = t = Weighted average duration of debt = 10.93 years
 - Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.0335
 - Riskless rate = r = Treasury bond rate corresponding to option life = 6%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - $d1 = -0.8337$ $N(d1) = 0.2023$
 - $d2 = -1.4392$ $N(d2) = 0.0751$
- Value of the call = $2312 (0.2023) - 8,865 \exp^{(-0.06)(10.93)} (0.0751) = \text{£}122$ million
- Appropriate interest rate on debt = $(8865/2190)^{(1/10.93)} - 1 = 13.65\%$

In Closing...

- There are real options everywhere.
- Most of them have no significant economic value because there is no exclusivity associated with using them.
- When options have significant economic value, the inputs needed to value them in a binomial model can be used in more traditional approaches (decision trees) to yield equivalent value.
- The real value from real options lies in
 - Recognizing that building in flexibility and escape hatches into large decisions has value
 - Insights we get on understanding how and why companies behave the way they do in investment analysis and capital structure choices.

Back to Lemmings...

