Valuation

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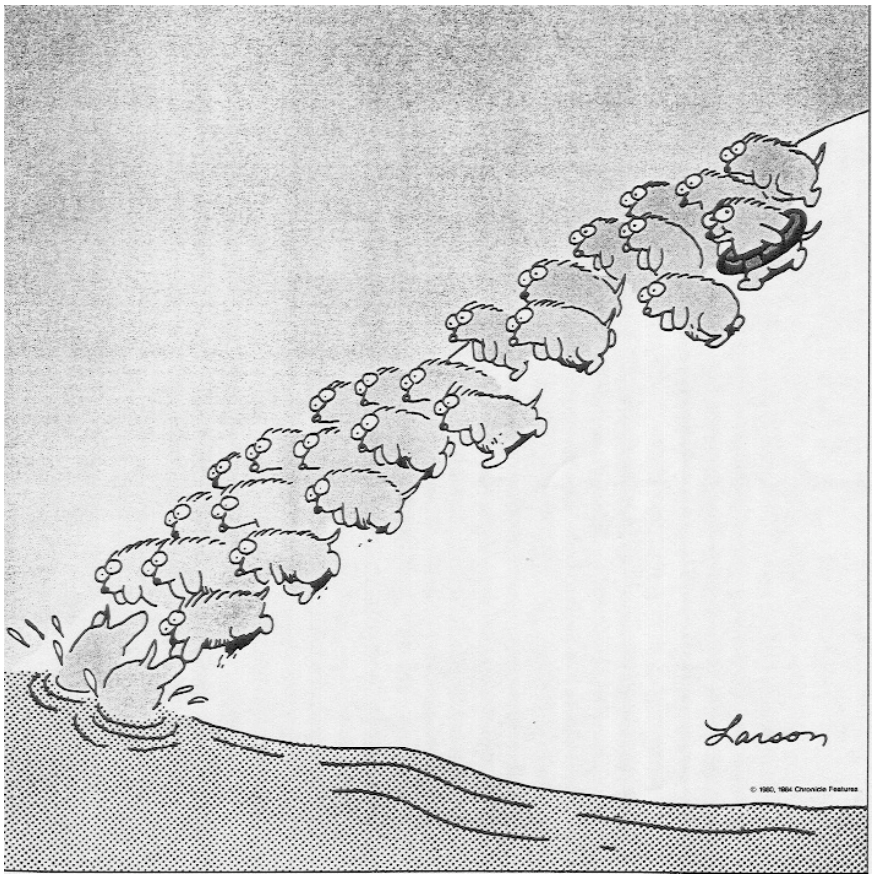
Some Initial Thoughts

" One hundred thousand lemmings cannot be wrong"

Graffiti



Misconceptions about Valuation

- Myth 1: A valuation is an objective search for "true" value
  - Truth 1.1: All valuations are biased. The only questions are how much and in which direction.
  - Truth 1.2: The direction and magnitude of the bias in your valuation is directly proportional to who pays you and how much you are paid.
- Myth 2.: A good valuation provides a precise estimate of value
  - Truth 2.1: There are no precise valuations
  - Truth 2.2: The payoff to valuation is greatest when valuation is least precise.
- Myth 3: . The more quantitative a model, the better the valuation
  - Truth 3.1: One's understanding of a valuation model is inversely proportional to the number of inputs required for the model.
  - Truth 3.2: Simpler valuation models do much better than complex ones.

Approaches to Valuation

- **Discounted cashflow valuation**, relates the value of an asset to the present value of expected future cashflows on that asset.
- **Relative valuation**, estimates the value of an asset by looking at the pricing of 'comparable' assets relative to a common variable like earnings, cashflows, book value or sales.
- **Contingent claim valuation**, uses option pricing models to measure the value of assets that share option characteristics.

Discounted Cash Flow Valuation

- **What is it**: In discounted cash flow valuation, the value of an asset is the present value of the expected cash flows on the asset.
- **Philosophical Basis**: Every asset has an intrinsic value that can be estimated, based upon its characteristics in terms of cash flows, growth and risk.
- **Information Needed**: To use discounted cash flow valuation, you need
  - to estimate the life of the asset
  - to estimate the cash flows during the life of the asset
  - to estimate the discount rate to apply to these cash flows to get present value
- **Market Inefficiency**: Markets are assumed to make mistakes in pricing assets across time, and are assumed to correct themselves over time, as new information comes out about assets.

Discounted Cashflow Valuation: Basis for Approach

$$\text{Value of asset} = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} \ldots\ldots + \frac{CF_n}{(1+r)^n}$$

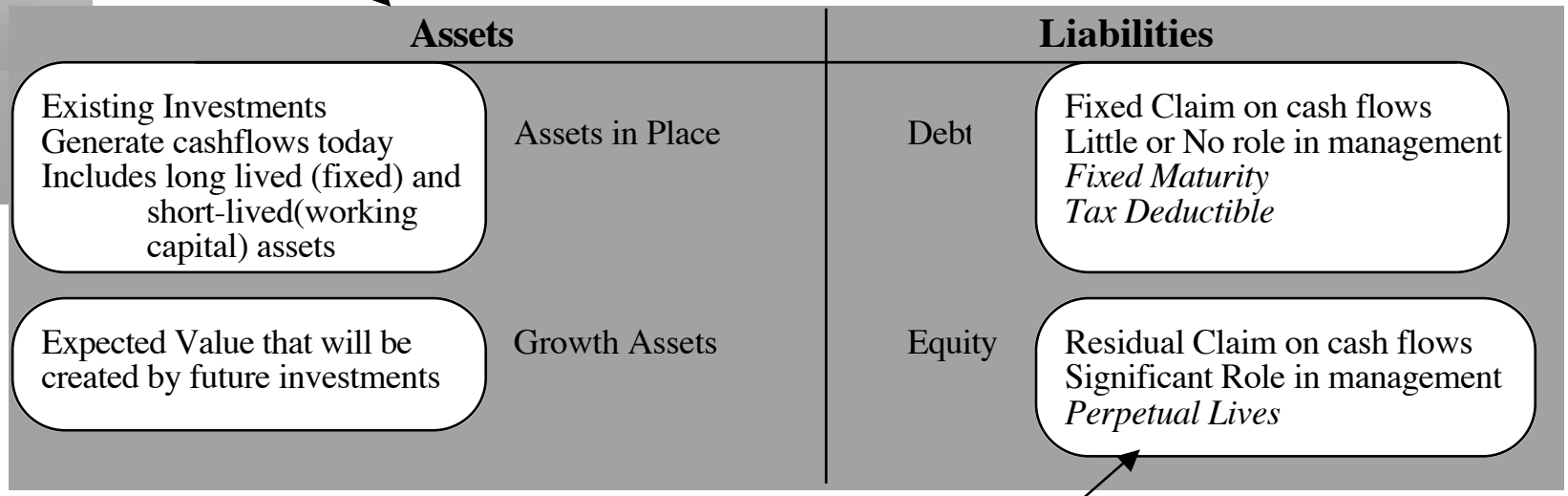
where CF_t is the expected cash flow in period t, r is the discount rate appropriate given the riskiness of the cash flow and n is the life of the asset.

Proposition 1: For an asset to have value, the expected cash flows have to be positive some time over the life of the asset.

Proposition 2: Assets that generate cash flows early in their life will be worth more than assets that generate cash flows later; the latter may however have greater growth and higher cash flows to compensate.

DCF Choices: Equity Valuation versus Firm Valuation

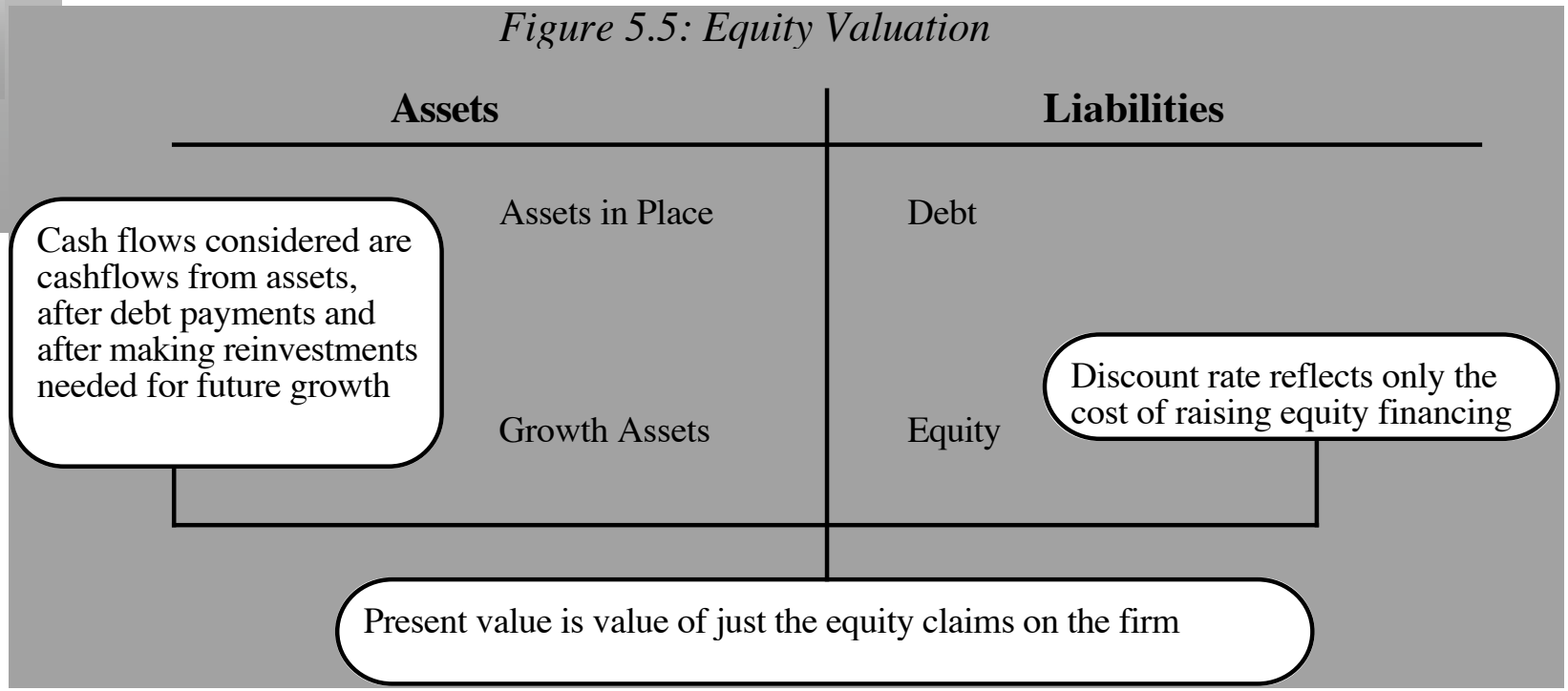
Firm Valuation: Value the entire business



Equity valuation: Value just the equity claim in the business

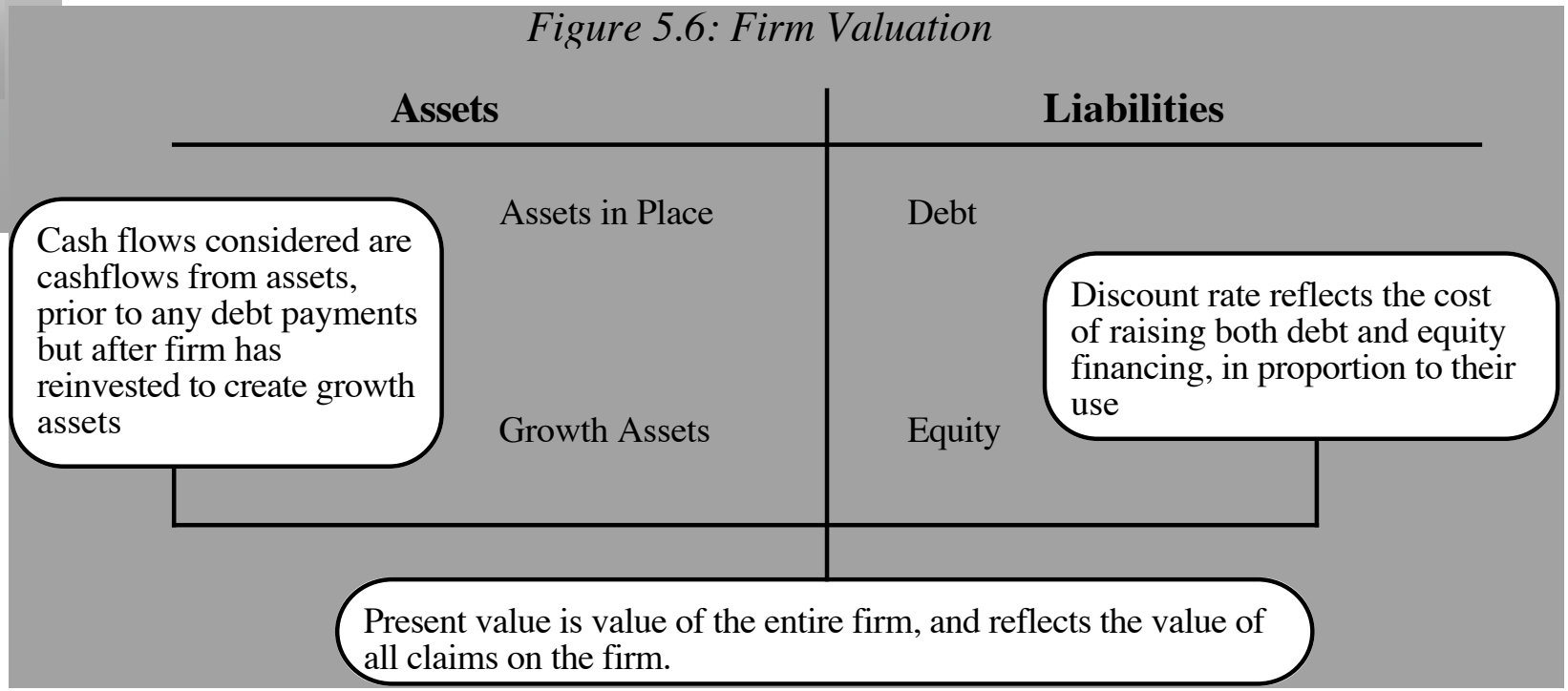
Equity Valuation

Figure 5.5: Equity Valuation

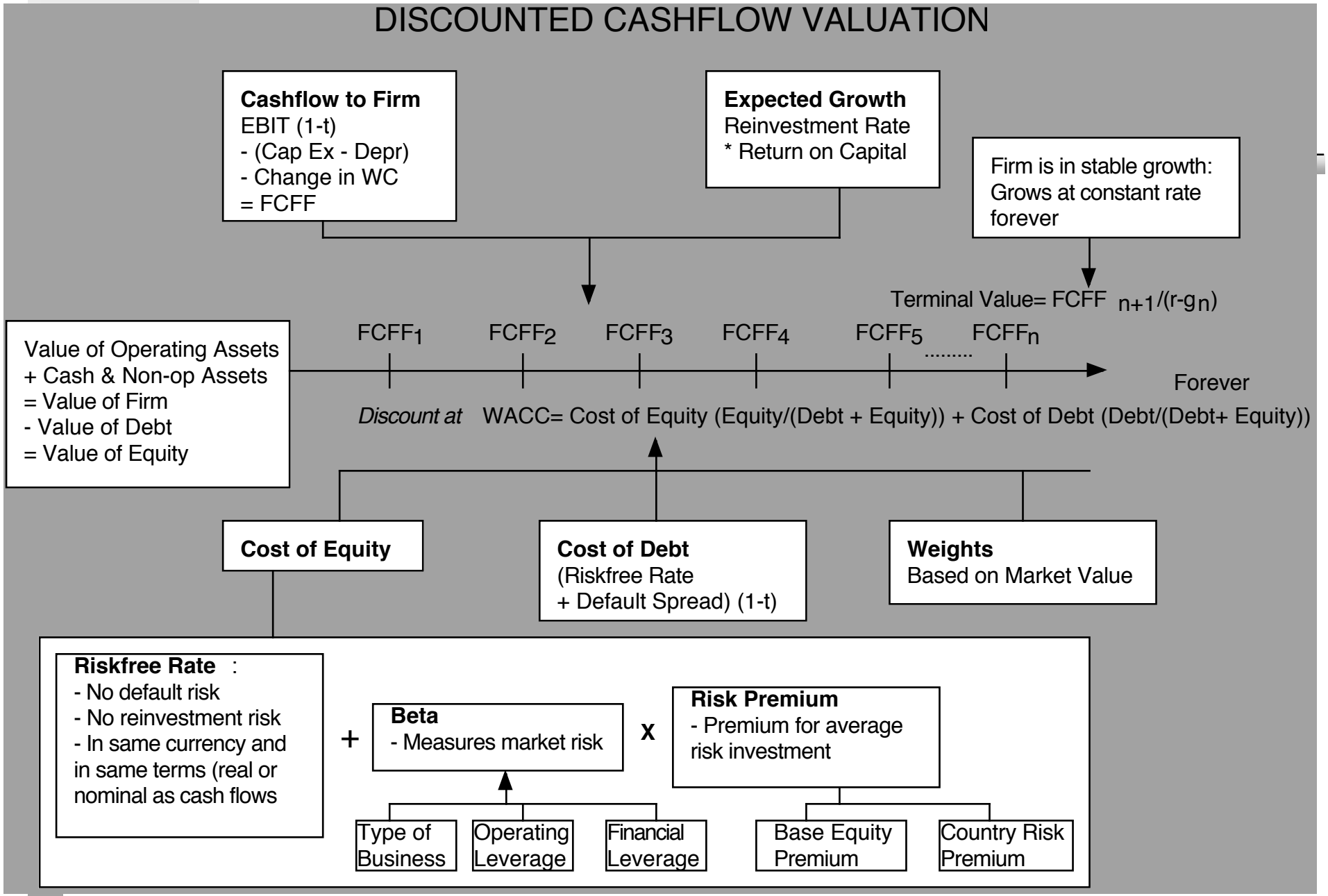


Firm Valuation

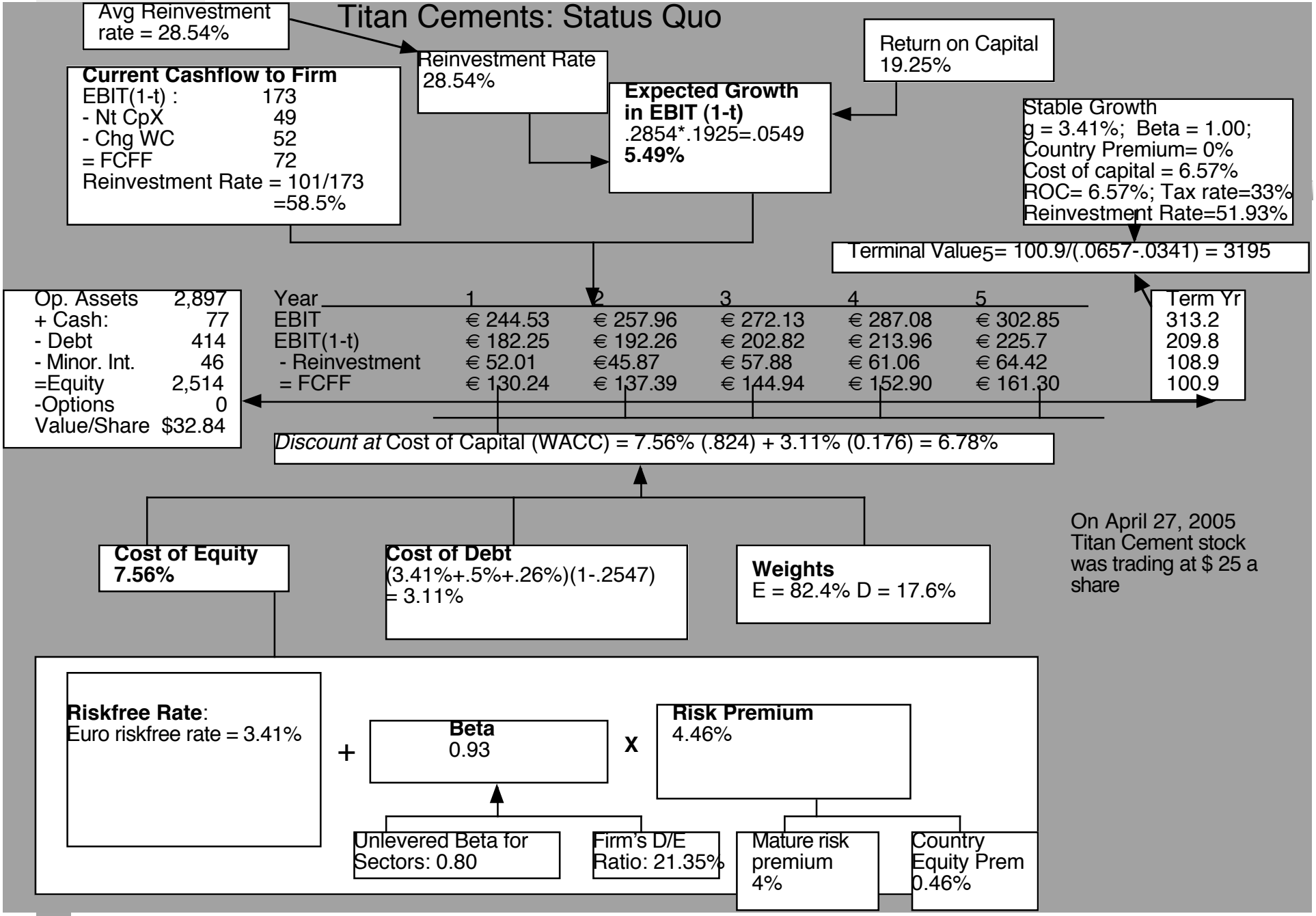
Figure 5.6: Firm Valuation



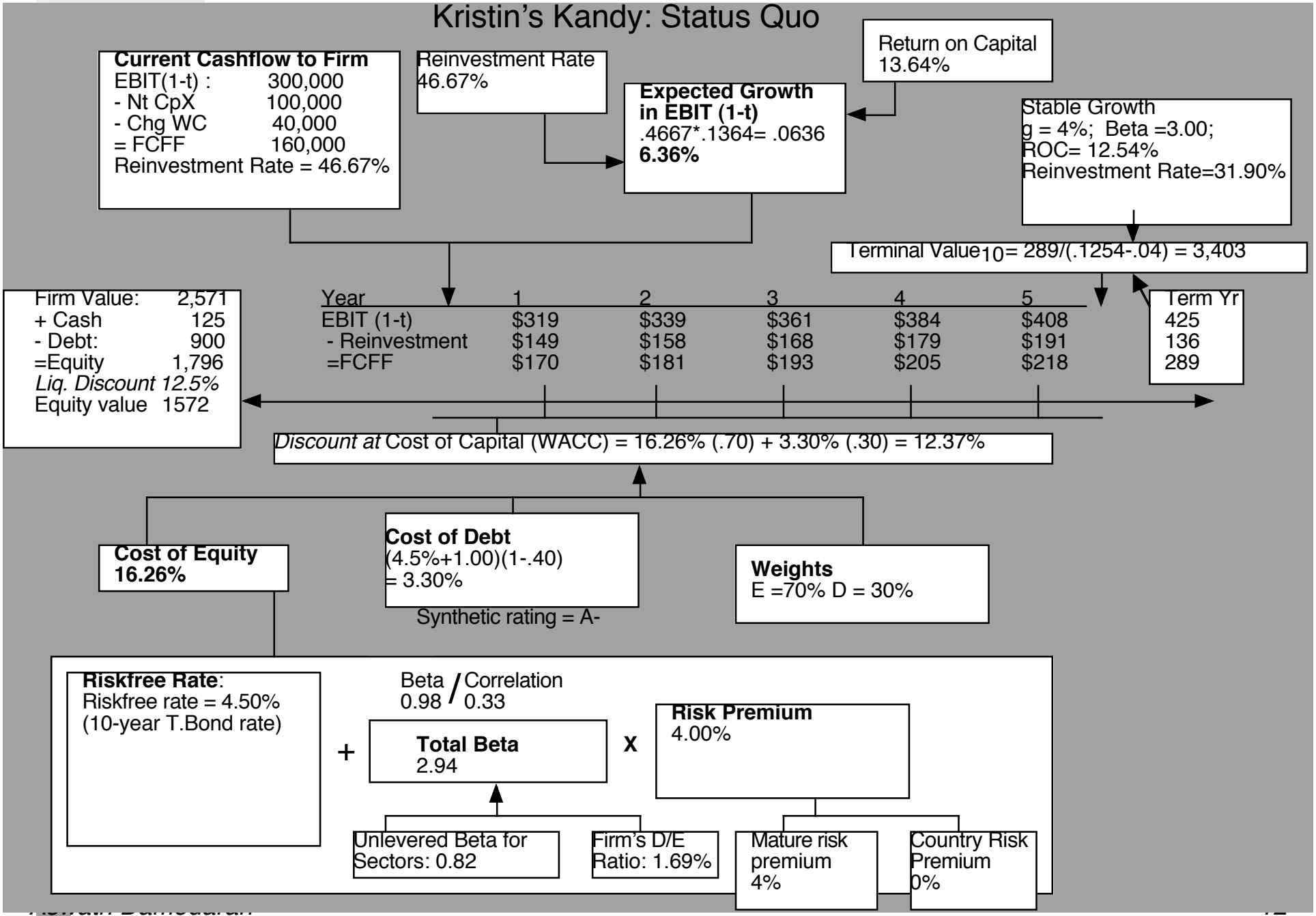
DISCOUNTED CASHFLOW VALUATION

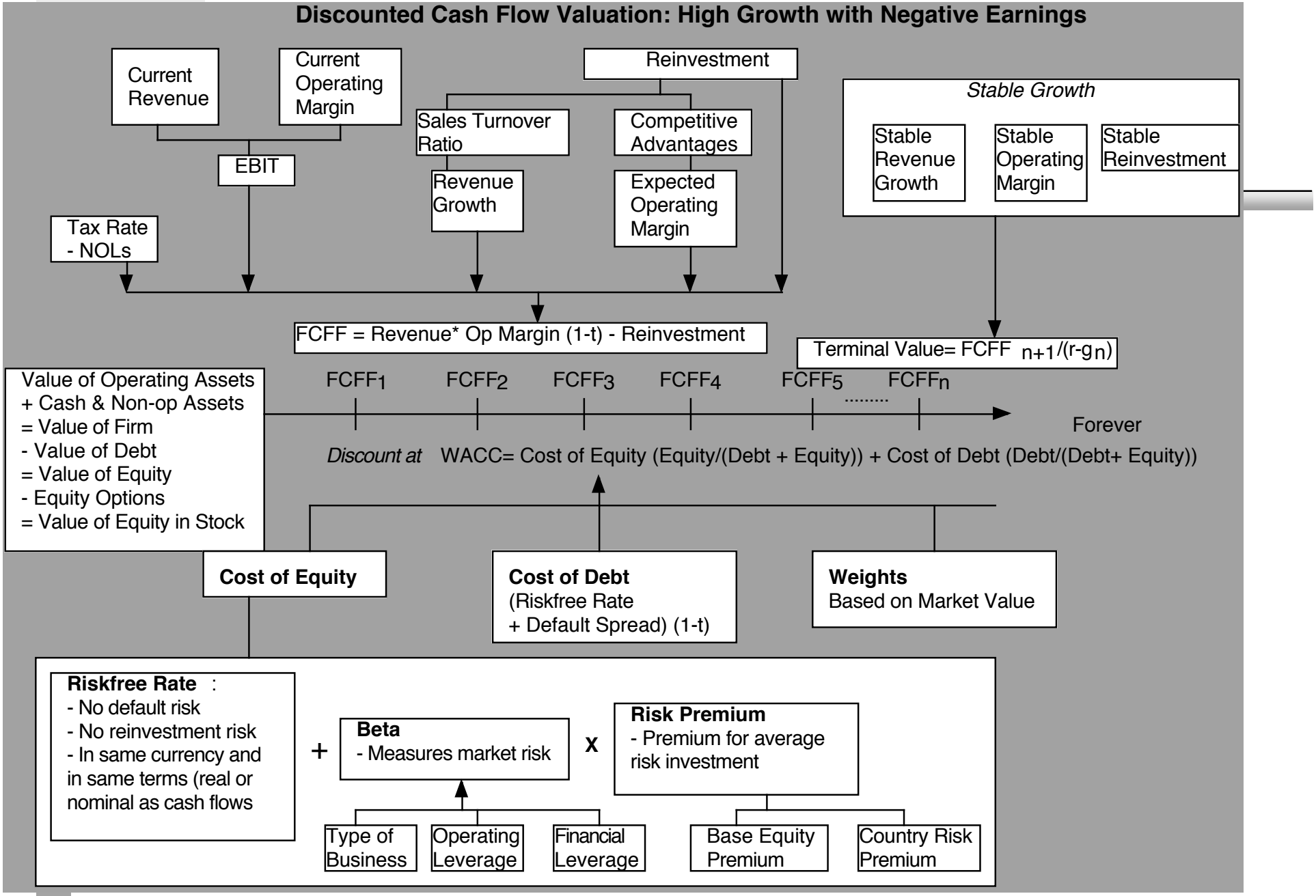


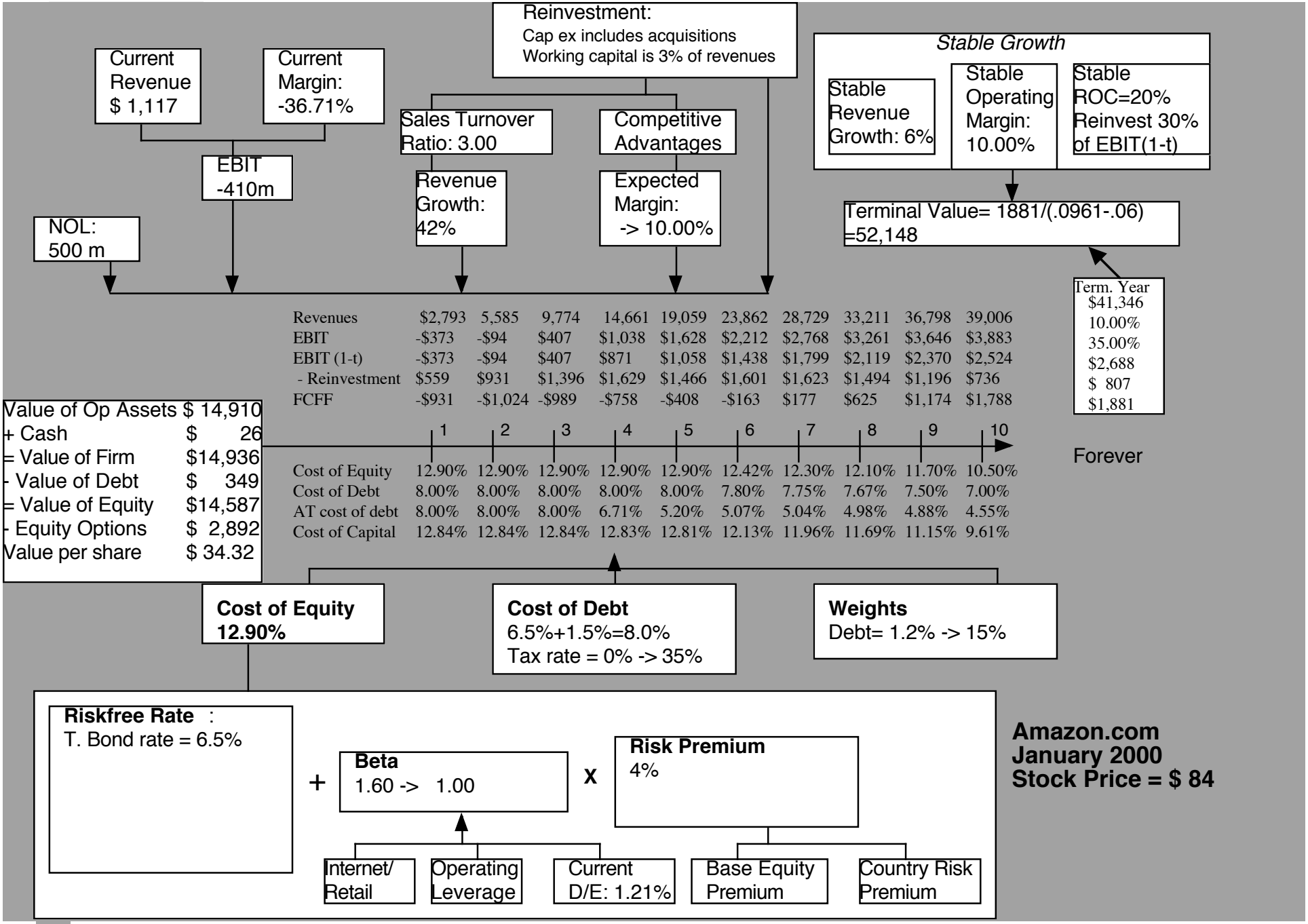
Titan Cements: Status Quo



Kristin's Kandy: Status Quo

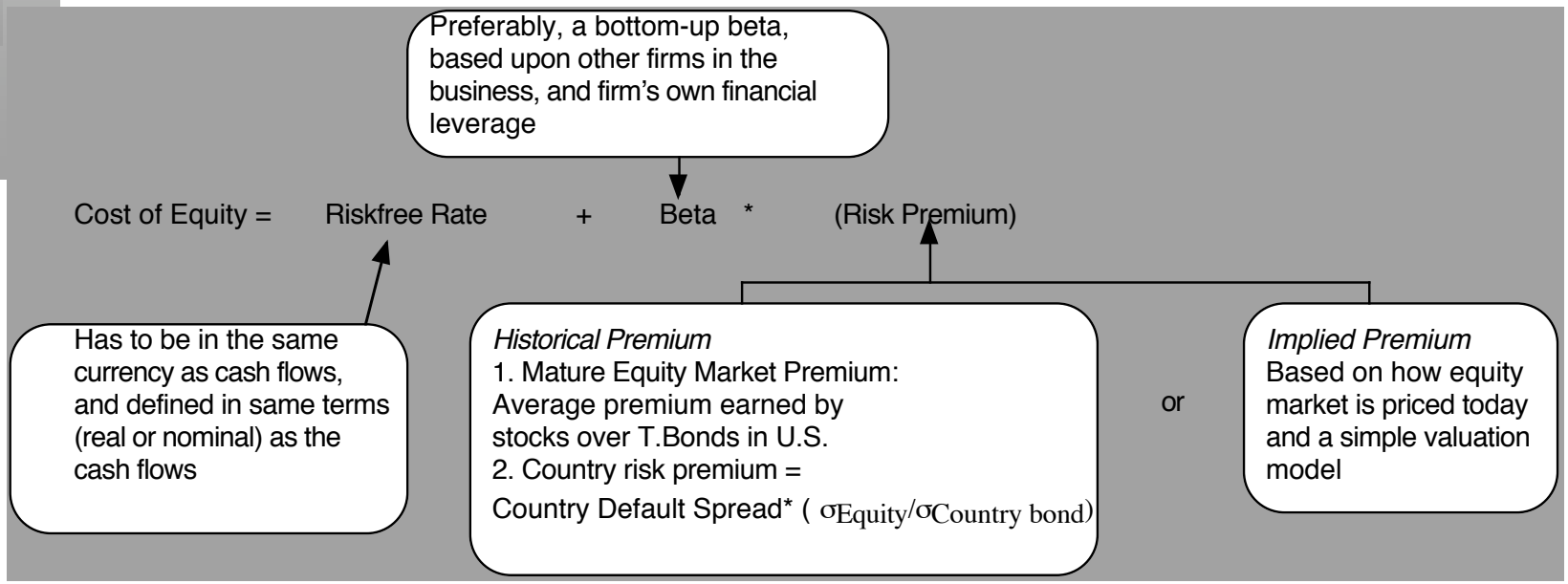



Discounted Cash Flow Valuation: High Growth with Negative Earnings
Current Revenue
Current Operating Margin
EBIT
Tax Rate - NOLs
Reinvestment
Sales Turnover Ratio
Revenue Growth
Competitive Advantages
Expected Operating Margin
Stable Growth
Stable Revenue Growth
Stable Operating Margin
Stable Reinvestment
FCFF = Revenue* Op Margin (1-t) - Reinvestment
Terminal Value= FCFF n+1/(r-gn)
Value of Operating Assets
+ Cash & Non-op Assets
= Value of Firm
- Value of Debt
= Value of Equity
- Equity Options
= Value of Equity in Stock
FCFF1 FCFF2 FCFF3 FCFF4 FCFF5 ..... FCFFn
Forever
Discount at WACC= Cost of Equity (Equity/(Debt + Equity)) + Cost of Debt (Debt/(Debt+ Equity))
Cost of Equity
Cost of Debt
(Riskfree Rate
+ Default Spread) (1-t)
Weights
Based on Market Value
Riskfree Rate :
- No default risk
- No reinvestment risk
- In same currency and in same terms (real or nominal as cash flows
+
Beta
- Measures market risk
x
Risk Premium
- Premium for average risk investment
Type of Business
Operating Leverage
Financial Leverage
Base Equity Premium
Country Risk Premium


Reinvestment:
Cap ex includes acquisitions
Working capital is 3% of revenues
Current Revenue $ 1,117
Current Margin: -36.71%
EBIT -410m
NOL: 500 m
Sales Turnover Ratio: 3.00
Revenue Growth: 42%
Competitive Advantages
Expected Margin: -> 10.00%
Stable Growth
Stable Revenue Growth: 6%
Stable Operating Margin: 10.00%
Stable ROC=20% Reinvest 30% of EBIT(1-t)
Terminal Value= 1881/(.0961-.06) =52,148
Term. Year
$41,346
10.00%
35.00%
$2,688
$ 807
$1,881
Revenues $2,793 5,585 9,774 14,661 19,059 23,862 28,729 33,211 36,798 39,006
EBIT -$373 -$94 $407 $1,038 $1,628 $2,212 $2,768 $3,261 $3,646 $3,883
EBIT (1-t) -$373 -$94 $407 $871 $1,058 $1,438 $1,799 $2,119 $2,370 $2,524
- Reinvestment $559 $931 $1,396 $1,629 $1,466 $1,601 $1,623 $1,494 $1,196 $736
FCFF -$931 -$1,024 -$989 -$758 -$408 -$163 $177 $625 $1,174 $1,788
1 2 3 4 5 6 7 8 9 10
Forever
Cost of Equity 12.90% 12.90% 12.90% 12.90% 12.90% 12.42% 12.30% 12.10% 11.70% 10.50%
Cost of Debt 8.00% 8.00% 8.00% 8.00% 8.00% 7.80% 7.75% 7.67% 7.50% 7.00%
AT cost of debt 8.00% 8.00% 8.00% 6.71% 5.20% 5.07% 5.04% 4.98% 4.88% 4.55%
Cost of Capital 12.84% 12.84% 12.84% 12.83% 12.81% 12.13% 11.96% 11.69% 11.15% 9.61%
Value of Op Assets $ 14,910
+ Cash $ 26
= Value of Firm $14,936
- Value of Debt $ 349
= Value of Equity $14,587
- Equity Options $ 2,892
Value per share $ 34.32
Cost of Equity 12.90%
Cost of Debt 6.5%+1.5%=8.0% Tax rate = 0% -> 35%
Weights Debt= 1.2% -> 15%
Riskfree Rate : T. Bond rate = 6.5%
+
Beta 1.60 -> 1.00
x
Risk Premium 4%
Internet/ Retail
Operating Leverage
Current D/E: 1.21%
Base Equity Premium
Country Risk Premium
Amazon.com
January 2000
Stock Price = $ 84

I. Estimating Discount Rates

Cost of Equity



A Simple Test

- You are valuing a Greek company in Euros for a US institutional investor and are attempting to estimate a risk free rate to use in the analysis. The risk free rate that you should use is
  - The interest rate on a US $ denominated treasury bond (4.25%)
  - The interest rate on a Euro-denominated Greek government bond (3.67%)
  - The interest rate on a Euro-denominated bond issued by the German government (3.41%)

Everyone uses historical premiums, but..

- The historical premium is the premium that stocks have historically earned over riskless securities.
- Practitioners never seem to agree on the premium; it is sensitive to
  - How far back you go in history…
  - Whether you use T.bill rates or T.Bond rates
  - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

	Arithmetic average		*Geometric Average*	
Historical Period	Stocks - T.Bills	Stocks - T.Bonds	Stocks - T.Bills	Stocks - T.Bonds
1928-2004	7.92%	6.53%	6.02%	4.84%
1964-2004	5.82%	4.34%	4.59%	3.47%
1994-2004	8.60%	5.82%	6.85%	4.51%

Assessing Country Risk Using Currency Ratings: Western Europe

Country	*Rating*	*Default Spread over German Euro (in bp)*
Austria	Aaa	0
Belgium	Aaa	10
France	Aaa	4
Germany	Aaa	0
Greece	A1	26
Ireland	Aaa	6
Italy	Aa2	16
Portugal	Aa2	10
Spain	Aaa	3

Assessing Country Risk using Ratings: Beyond the EU

Country	*Rating*	*Default Spread*
Croatia	Baa3	145
Cyprus	A2	90
Czech Republic	Baa1	120
Hungary	A3	95
Latvia	Baa2	130
Lithuania	Ba1	250
Moldova	B3	650
Poland	Baa1	120
Romania	B3	650
Russia	B2	550
Slovakia	Ba1	250
Slovenia	A2	90
Turkey	B1	450

Using Country Ratings to Estimate Equity Spreads

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
  - One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
  - Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
    - Standard Deviation in Greek ASE(Equity) = 16%
    - Standard Deviation in Greek Euro Bond = 9%
    - Adjusted Equity Spread = 0.26% (16/9) = 0.46%

From Country Risk Premiums to Corporate Risk premiums

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

  E(Return) = Riskfree Rate + Country ERP + Beta (US premium)

- Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.

  E(Return) = Riskfree Rate + Beta (US premium + Country ERP)

- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

  E(Return)=Riskfree Rate+ β (US premium) + λ (Country ERP)

  Country ERP: Additional country equity risk premium

Estimating Company Exposure to Country Risk

- Different companies should be exposed to different degrees to country risk. For instance, a Greek firm that generates the bulk of its revenues in the rest of Western Europe should be less exposed to country risk than one that generates all its business within Greece.
- The factor "λ" measures the relative exposure of a firm to country risk. One simplistic solution would be to do the following:

  λ = % of revenues domestically$_{firm}$/ % of revenues domestically$_{avg\ firm}$

  For instance, if a firm gets 35% of its revenues domestically while the average firm in that market gets 70% of its revenues domestically

  $$\lambda = 35\%/\ 70\ \% = 0.5$$

- There are two implications
  - A company's risk exposure is determined by where it does business and not by where it is located
  - Firms might be able to actively manage their country risk exposures

Estimating E(Return) for Titan Cements

- Assume that the beta for Titan Cements is 0.95, and that the riskfree rate used is 3.41%. Also assume that the historical premium for the US (4.84%) is a reasonable estimate of a mature market risk premium.
- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

E(Return) = 3.41% + 0.46% + 0.93 (4.84%) = 8.37%

- Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.

E(Return) = 3.41% + 0.93 (4.84%+ 0.46%) = 8.34%

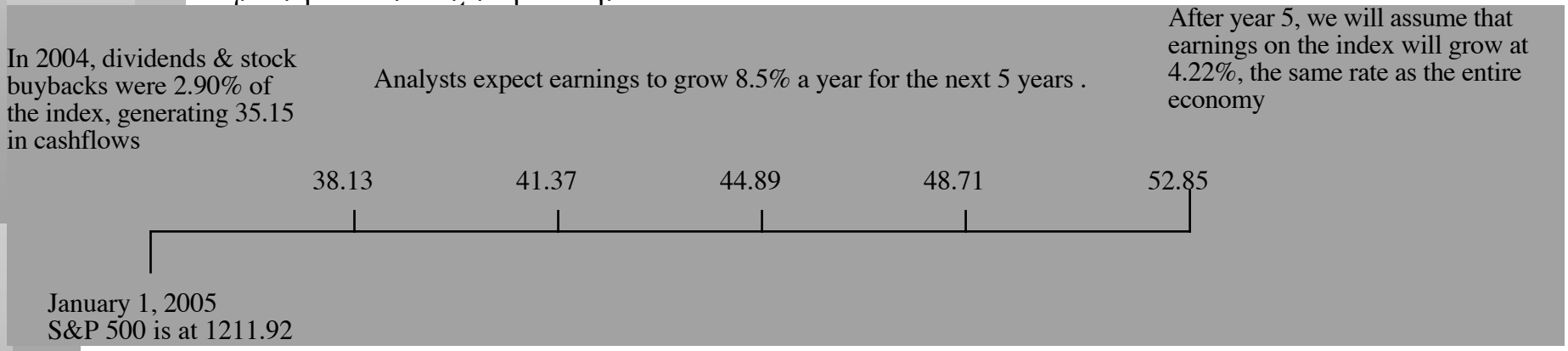
- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

E(Return)= 3.41% + 0.93(4.84%) + 0.56 (0.46%) + 0.14(3%) = 8.59%

Titan is less exposed to Greek country risk than the typical Greek firm since it gets about 40% of its revenues in Greece; the average for Greek firms is 70%. In 2004, though, Titan got about 14% of it's revenues from the Balkan states.

Implied Equity Premiums

- We can use the information in stock prices to back out how risk averse the market is and how much

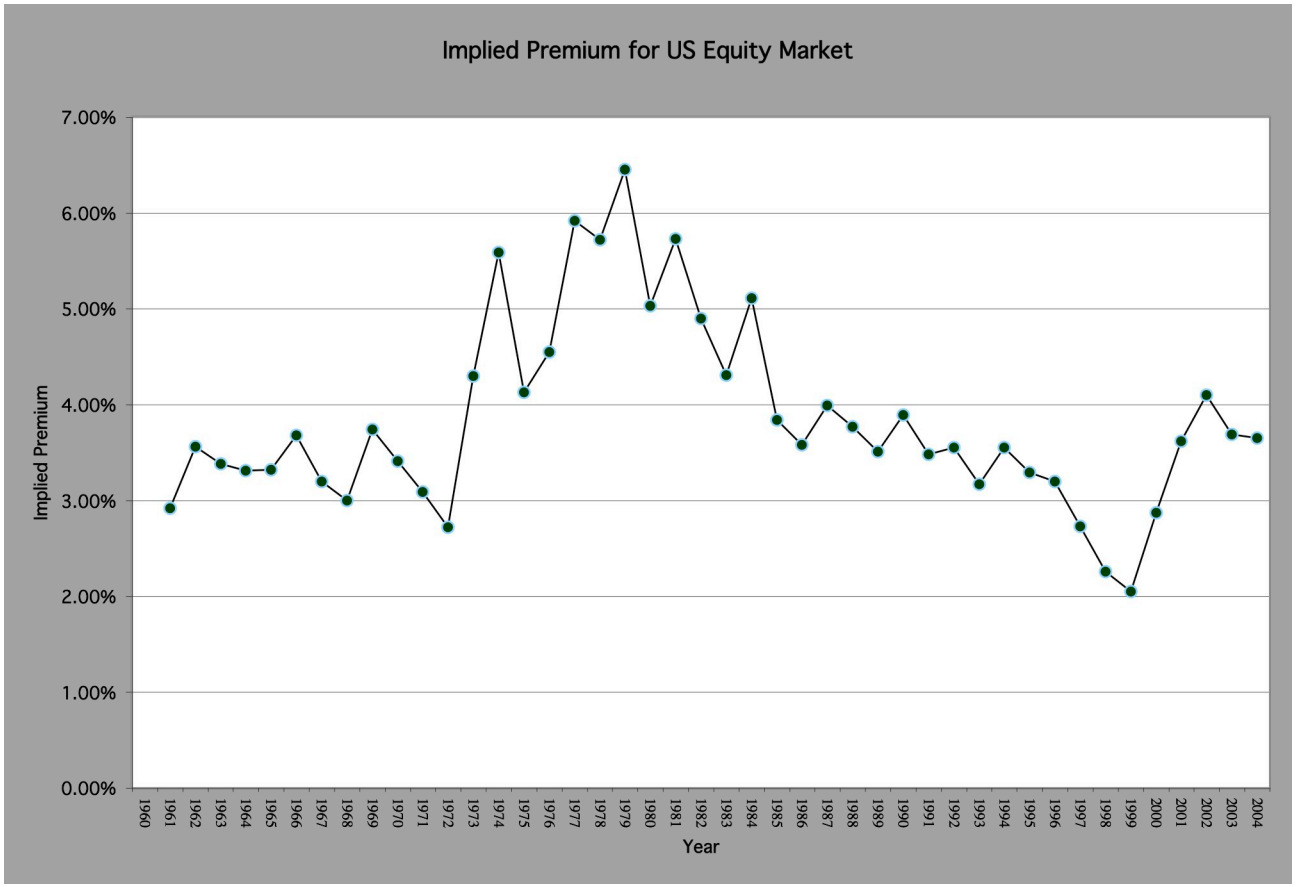


- If you pay the current level of the index, you can expect to make a return of 7.87% on stocks (which is obtained by solving for r in the following equation)

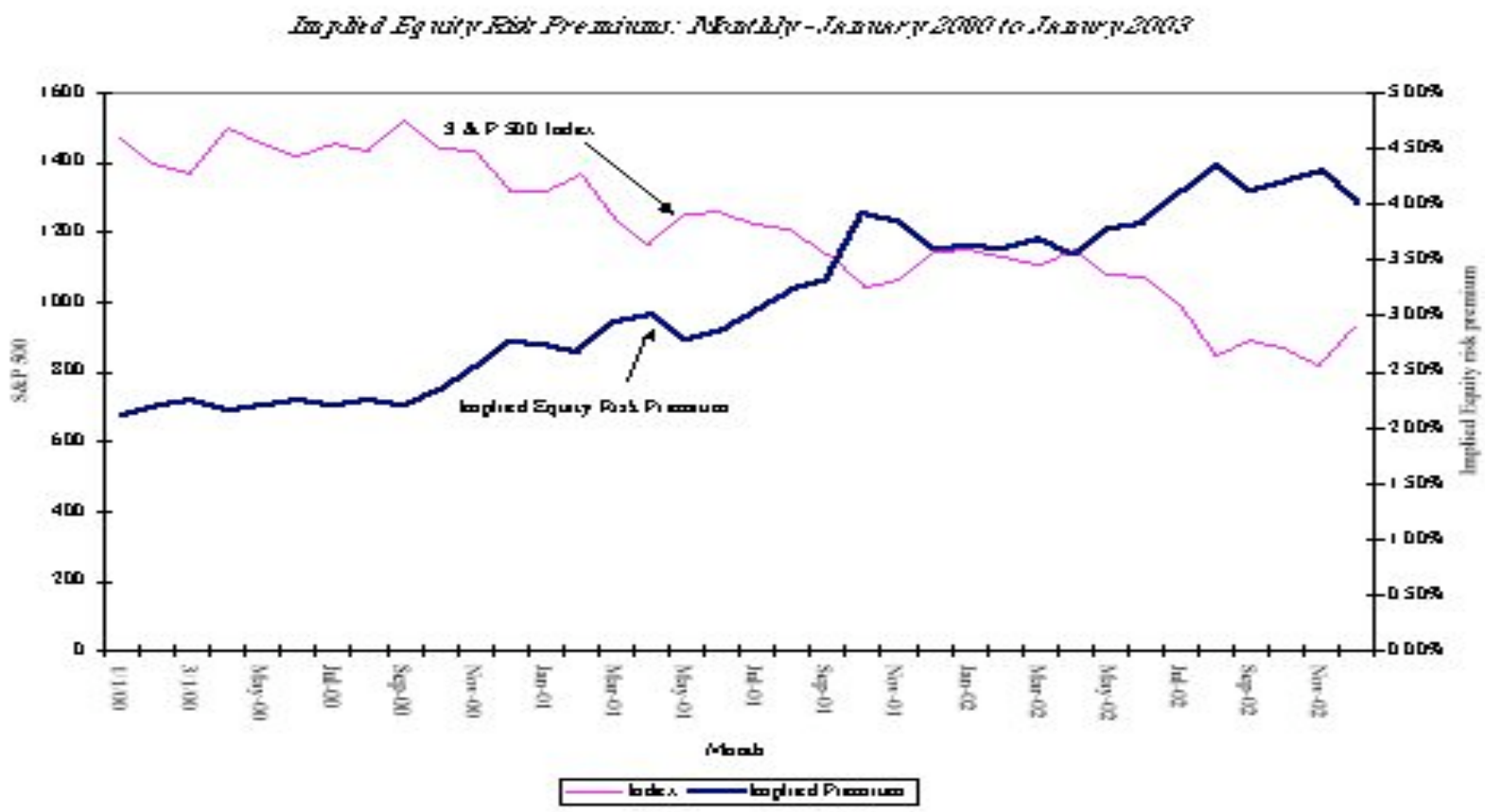
$$1211.92 = \frac{38.13}{(1+r)} + \frac{41.37}{(1+r)^2} + \frac{44.89}{(1+r)^3} + \frac{48.71}{(1+r)^4} + \frac{52.85}{(1+r)^5} + \frac{52.85(1.0422)}{(r-.0422)(1+r)^5}$$

- In % - 4.22% = 3.65%

Implied Premiums in the US



Implied Premiums: From Bubble to Bear Market… January 2000 to December 2002



Choosing an Equity Risk Premium

- The historical risk premium of 4.84% for the United States is too high a premium to use in valuation. It is much higher than the actual implied equity risk premium in the market
- The current implied equity risk premium requires us to assume that the market is correctly priced today. (If I were required to be market neutral, this is the premium I would use)
- The average implied equity risk premium between 1960-2004 in the United States is about 4%. We will use this as the premium for a mature equity market.

Implied Premium for Greek Market: April 27, 2005

- Level of the Index = 2786
- Dividends on the Index = 3.28% of 2786
- Other parameters
  - Riskfree Rate = 3.41% (Euros)
  - Expected Growth (in Euros)
    - Next 5 years = 8% (Used expected growth rate in Earnings)
    - After year 5 = 3.41%
- Solving for the expected return:
  - Expected return on Equity = 7.56%
  - Implied Equity premium = 7.56% - 3.41% = 4.15%
- Effect on valuation
  - Titan's value with historical premium (4%) + country (.46%) : 32.84 Euros/share
  - Titan's value with implied premium: 32.67 Euros per share

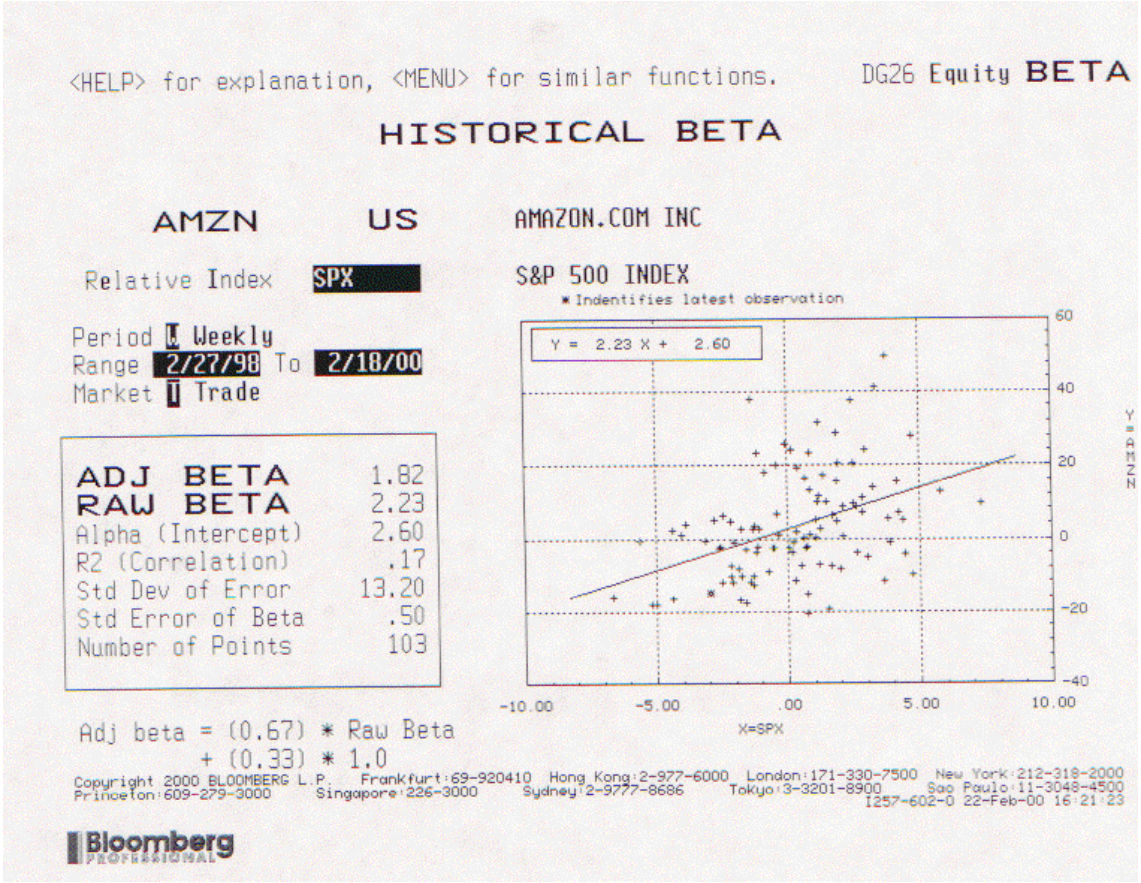
Estimating Beta

- The standard procedure for estimating betas is to regress stock returns (R_j) against market returns (R_m) -

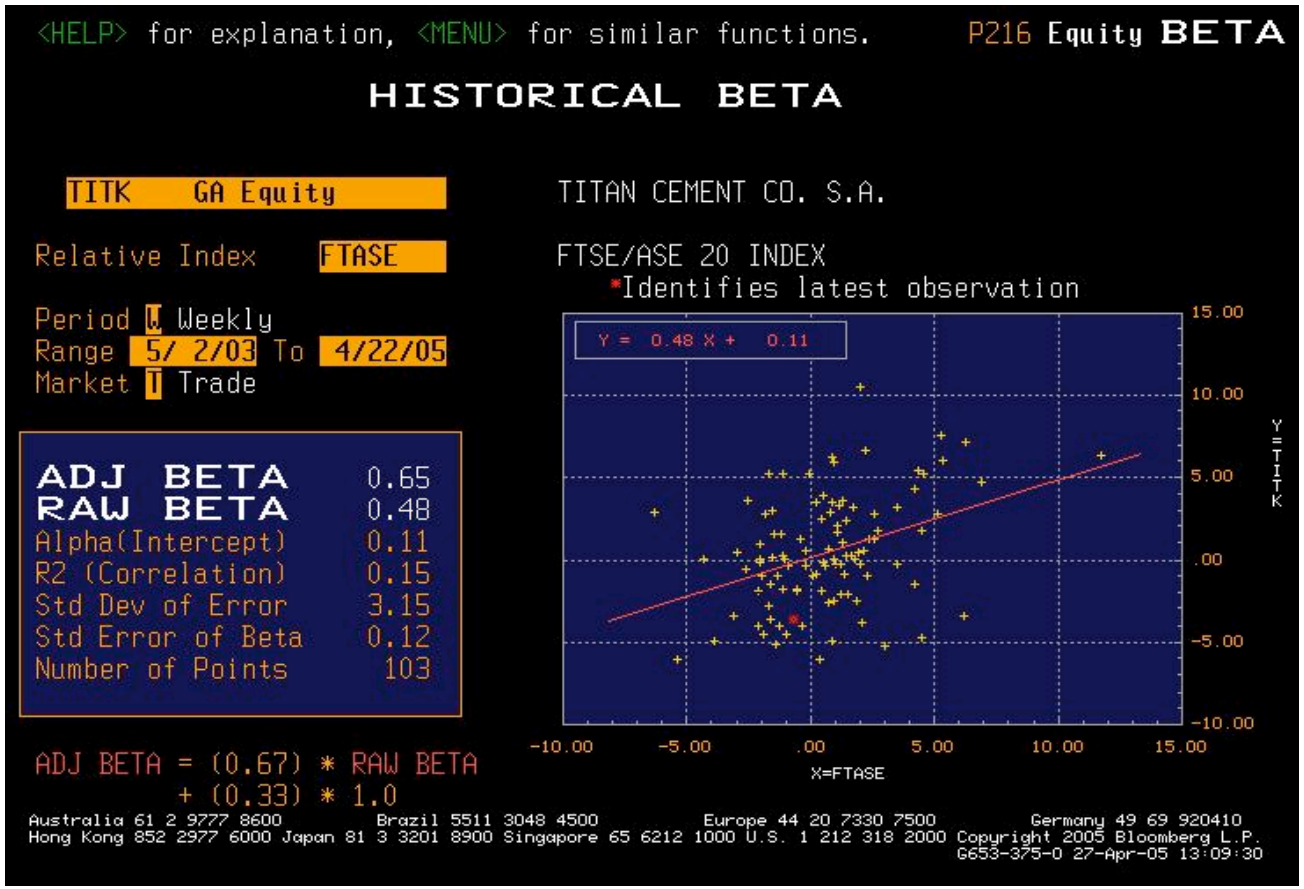
$$R_j = a + b\ R_m$$

  - where a is the intercept and b is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.
- This beta has three problems:
  - It has high standard error
  - It reflects the firm's business mix over the period of the regression, not the current mix
  - It reflects the firm's average financial leverage over the period rather than the current leverage.

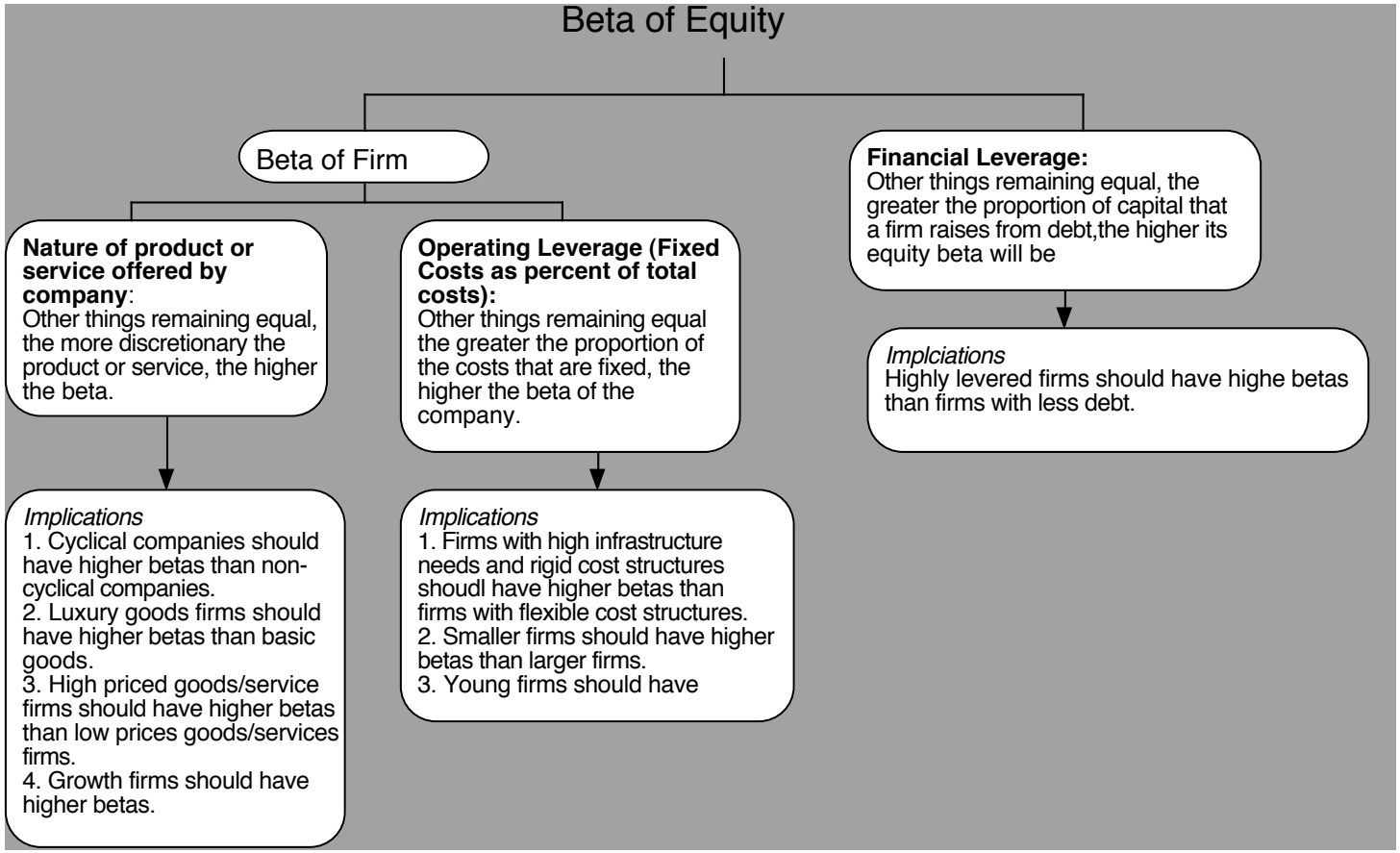
Beta Estimation: Amazon



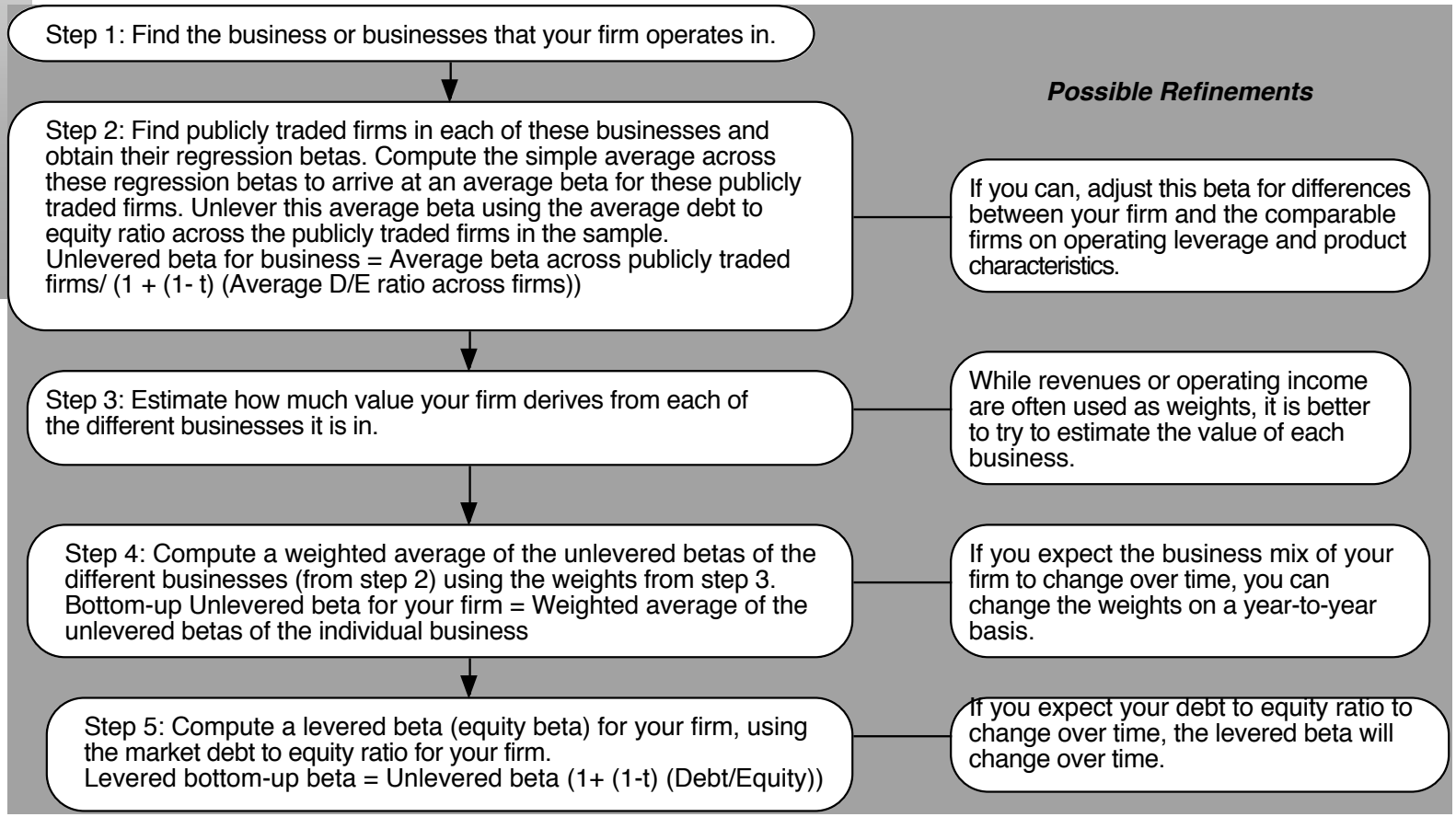
Beta Estimation for Titan Cement: The Index Effect



Determinants of Betas



Bottom-up Betas



Bottom up Beta Estimates

Company	*Comparable Companies*	*Unlevered Beta*	*Levered Beta*
Titan Cement	Global Cement companies	0.80	0.80 (1 + (1-.2547) (.2135) = 0.93
Amazon (First 5 years)	Internet Retailers	1.58	1.58 (1- (1-0) (.0121) = 1.60
Amazon (After year 5)	Specialty Retailers		1.00
Kristin Kandy	Food Processing companies with market cap < $ 250 million	0.78	0.78 (1+(1-.4) (30/70)) = 0.98

Small Firm and Other Premiums

- It is common practice to add premiums on to the cost of equity for firm-specific characteristics. For instance, many analysts add a small stock premium of 3-3.5% (historical premium for small stocks over the market) to the cost of equity for smaller companies.
- Adding arbitrary premiums to the cost of equity is always a dangerous exercise. If small stocks are riskier than larger stocks, we need to specify the reasons and try to quantify them rather than trust historical averages. (You could argue that smaller companies are more likely to serve niche (discretionary) markets or have higher operating leverage and adjust the beta to reflect this tendency).

Is Beta an Adequate Measure of Risk for a Private Firm?

The owners of most private firms are not diversified. Beta measures the risk added on to a diversified portfolio. Therefore, using beta to arrive at a cost of equity for a private firm will

a) Under estimate the cost of equity for the private firm
b) Over estimate the cost of equity for the private firm
c) Could under or over estimate the cost of equity for the private firm

Total Risk versus Market Risk

- Adjust the beta to reflect total risk rather than market risk. This adjustment is a relatively simple one, since the R squared of the regression measures the proportion of the risk that is market risk.

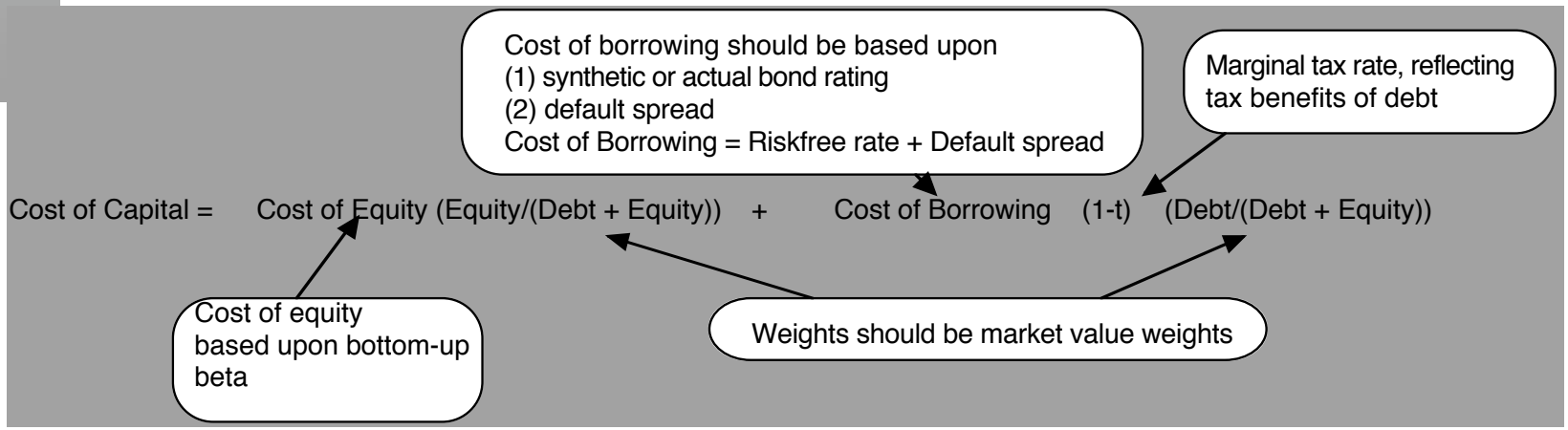
  Total Beta = Market Beta / Correlation of the sector with the market

- To estimate the beta for Kristin Kandy, we begin with the bottom-up unlevered beta of food processing companies:
  - Unlevered beta for publicly traded food processing companies = 0.78
  - Average correlation of food processing companies with market = 0.333
  - Unlevered total beta for Kristin Kandy = 0.78/0.333 = 2.34
  - Debt to equity ratio for Kristin Kandy = 0.3/0.7 (assumed industry average)
  - Total Beta = 2.34 (1- (1-.40)(30/70)) = 2.94
  - Total Cost of Equity = 4.50% + 2.94 (4%) = 16.26%

When would you use this total risk measure?

- Under which of the following scenarios are you most likely to use the total risk measure:
  - when valuing a private firm for an initial public offering
  - when valuing a private firm for sale to a publicly traded firm
  - when valuing a private firm for sale to another private investor
- Assume that you own a private business. What does this tell you about the best potential buyer for your business?

From Cost of Equity to Cost of Capital



Estimating Synthetic Ratings

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

  Interest Coverage Ratio = EBIT / Interest Expenses

- For Titan's interest coverage ratio, we used the interest expenses and EBIT from 2004.

  Interest Coverage Ratio = 232/ 19.4 = 11.95

- For Kristin Kandy, we used the interest expenses and EBIT from the most recent financial year:

  Interest Coverage Ratio = 500,000/ 85,000 = 5.88

- Amazon.com has negative operating income; this yields a negative interest coverage ratio, which should suggest a D rating. We computed an average interest coverage ratio of 2.82 over the next 5 years.

Interest Coverage Ratios, Ratings and Default Spreads

If Interest Coverage Ratio is		Estimated Bond Rating	Default Spread(1/00)	Default Spread(1/04)
> 8.50	(>12.50)	AAA	0.20%	0.35%
6.50 - 8.50	(9.5-12.5)	AA	0.50%	0.50%
5.50 - 6.50	(7.5-9.5)	A+	0.80%	0.70%
4.25 - 5.50	(6-7.5)	A	1.00%	0.85%
3.00 - 4.25	(4.5-6)	A–	1.25%	1.00%
2.50 - 3.00	(3.5-4.5)	BBB	1.50%	1.50%
2.25 - 2.50	(3.5 -4)	BB+	1.75%	2.00%
2.00 - 2.25	((3-3.5)	BB	2.00%	2.50%
1.75 - 2.00	(2.5-3)	B+	2.50%	3.25%
1.50 - 1.75	(2-2.5)	B	3.25%	4.00%
1.25 - 1.50	(1.5-2)	B –	4.25%	6.00%
0.80 - 1.25	(1.25-1.5)	CCC	5.00%	8.00%
0.65 - 0.80	(0.8-1.25)	CC	6.00%	10.00%
0.20 - 0.65	(0.5-0.8)	C	7.50%	12.00%
< 0.20	(<0.5)	D	10.00%	20.00%

For Titan and Kristing Kandy, I used the interest coverage ratio table for smaller/riskier firms (the numbers in brackets) which yields a lower rating for the same interest coverage ratio.

Estimating the cost of debt for a firm

- The synthetic rating for Titan Cement is AA. Using the 2004 default spread of 0.50%, we estimate a cost of debt of 4.17% (using a riskfree rate of 3.41% and adding in the country default spread of 0.26%):

  Cost of debt = Riskfree rate + Greek default spread + Company default spread

  =3.41% + 0..26%+ 0.50% = 4.17%

- The synthetic rating for Kristin Kandy is A-. Using the 2004 default spread of 1.00% and a riskfree rate of 4.50%, we estimate a cost of debt of 5.50%.

  Cost of debt = Riskfree rate + Default spread =4.50% + 1.00% = 5.50%

- The synthetic rating for Amazon.com in 2000 was BBB. The default spread for BBB rated bond was 1.50% in 2000 and the treasury bond rate was 6.5%.

  Cost of debt = Riskfree Rate + Default spread = 6.50% + 1.50% = 8.00%

Weights for the Cost of Capital Computation

- The weights used to compute the cost of capital should be the market value weights for debt and equity.
- There is an element of circularity that is introduced into every valuation by doing this, since the values that we attach to the firm and equity at the end of the analysis are different from the values we gave them at the beginning.
- For private companies, neither the market value of equity nor the market value of debt is observable. Rather than use book value weights, you should try
  - Industry average debt ratios for publicly traded firms in the business
  - Target debt ratio (if management has such a target)
  - Estimated value of equity and debt from valuation (through an iterative process)

Estimating Cost of Capital: Amazon.com

- Equity
  - Cost of Equity = 6.50% + 1.60 (4.00%) = 12.90%
  - Market Value of Equity = $ 84/share* 340.79 mil shs = $ 28,626 mil (98.8%)
- Debt
  - Cost of debt = 6.50% + 1.50% (default spread) = 8.00%
  - Market Value of Debt = $ 349 mil (1.2%)
- Cost of Capital

  Cost of Capital = 12.9 % (.988) + 8.00% (1- 0) (.012)) = 12.84%

Estimating Cost of Capital: Titan Cements

- Equity
  - Cost of Equity = 3.41% + 0.93 (4%+ 0.46%) = 7.56%
  - Market Value of Equity =1940 million Euros (82.4%)
- Debt
  - Cost of debt = 3.41% + 0.26% + 0.50%= 4.17%
  - Market Value of Debt = 414 million Euros (17.6%)
- Cost of Capital

Cost of Capital = 7.56 % (.824) + 4.17% (1- .2547) (0.176)) = 6.78%

The book value of equity at Titan Cement is 542 million Euros

The book value of debt at Titan Cement is 405 million; Interest expense is 19 mil; Average maturity of debt = 4 years

Estimated market value of debt = 19 million (PV of annuity, 4 years, 4.17%) + $ 405 million/1.0417^4 = 414 million Euros

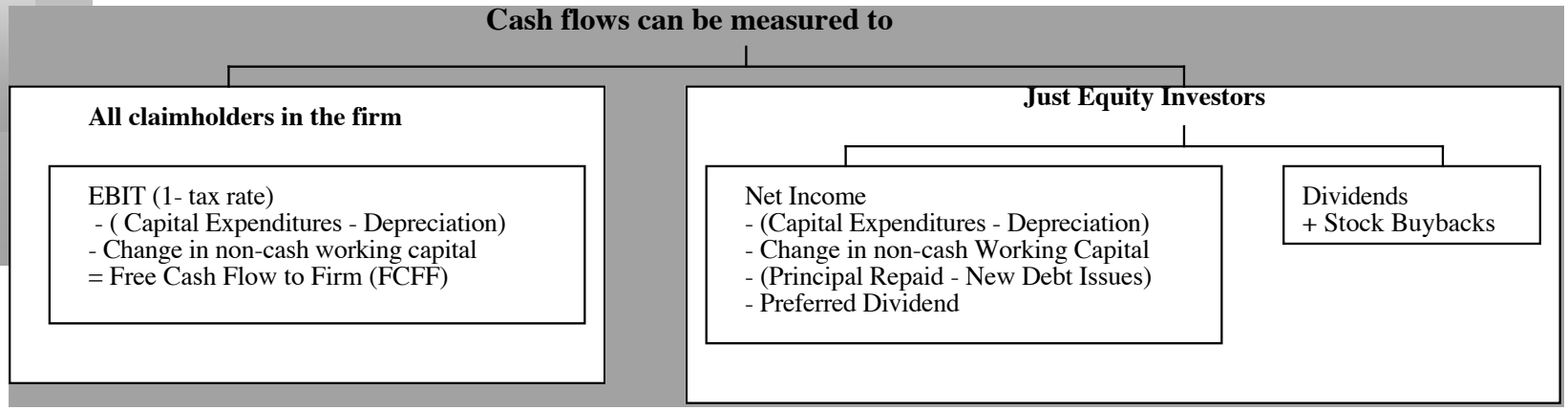
Estimating Cost of Capital: Kristin Kandy

- Equity
  - Cost of Equity = 4.50% + 2.94 (4%) = 16.26%
  - Equity as percent of capital = 70%
- Debt
  - Pre-tax Cost of debt = 4.50% + 1.00% = 5.50%
  - Marginal tax rate = 40%
  - Debt as percent of capital = 30% (Industry average)
- Cost of Capital

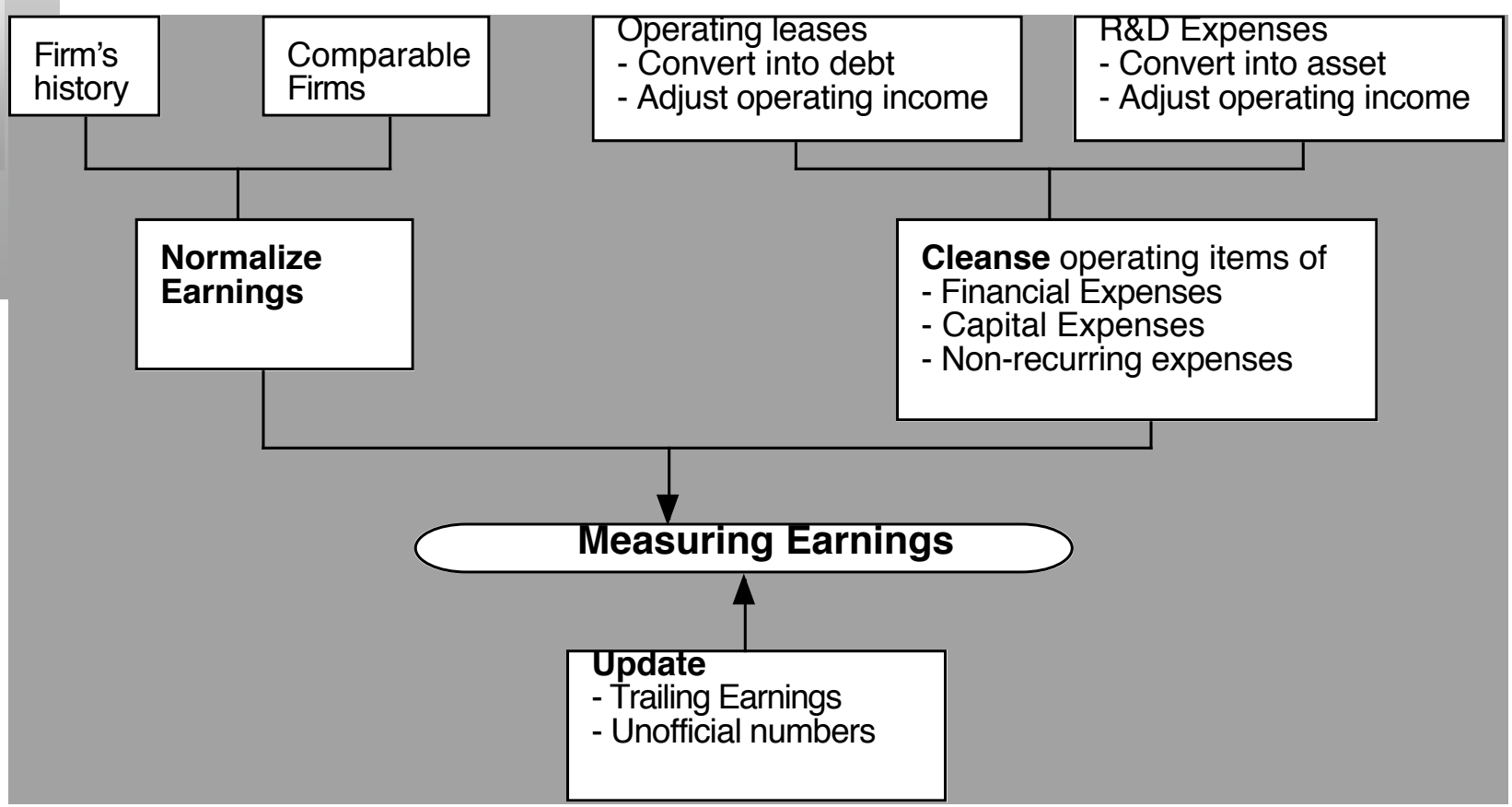
  Cost of Capital = 16.26% (.70) + 5.50% (1-.40) (.30) = 12.37%

II. Estimating Cashflows and Growth

Defining Cashflow



From Reported to Actual Earnings



Dealing with Operating Lease Expenses

- Operating Lease Expenses are treated as operating expenses in computing operating income. In reality, operating lease expenses should be treated as financing expenses, with the following adjustments to earnings and capital:
- Debt Value of Operating Leases = Present value of Operating Lease Commitments at the pre-tax cost of debt
- When you convert operating leases into debt, you also create an asset to counter it of exactly the same value.
- Adjusted Operating Earnings

  Adjusted Operating Earnings = Operating Earnings + Operating Lease Expenses - Depreciation on Leased Asset

  - As an approximation, this works:

  Adjusted Operating Earnings = Operating Earnings + Pre-tax cost of Debt * PV of Operating Leases.

Operating Leases at The Gap in 2003

- The Gap has conventional debt of about $ 1.97 billion on its balance sheet and its pre-tax cost of debt is about 6%. Its operating lease payments in the 2003 were $978 million and its commitments for the future are below:

Year	Commitment (millions)	Present Value (at 6%)
1	$899.00	$848.11
2	$846.00	$752.94
3	$738.00	$619.64
4	$598.00	$473.67
5	$477.00	$356.44
6&7	$982.50 each year	$1,346.04
Debt Value of leases =		$4,396.85 (Also value of leased asset)

- Debt outstanding at The Gap = $1,970 m + $4,397 m = $6,367 m
- Adjusted Operating Income = Stated OI + OL exp this year - Deprec'n

= $1,012 m + 978 m - 4397 m /7 = $1,362 million (7 year life for assets)

- Approximate OI = $1,012 m + $ 4397 m (.06) = $1,276 m

The Collateral Effects of Treating Operating Leases as Debt

Conventional Accounting	*Operating Leases Treated as Debt*
Income Statement EBIT& Leases = 1,990 - Op Leases = 978 EBIT = 1,012	*Income Statement* EBIT& Leases = 1,990 - Deprecn: OL= 628 EBIT = 1,362 Interest expense will rise to reflect the conversion of operating leases as debt. Net income should not change.
Balance Sheet Off balance sheet (Not shown as debt or as an asset). Only the conventional debt of $1,970 million shows up on balance sheet	*Balance Sheet* Asset / Liability OL Asset 4397 / OL Debt 4397 Total debt = 4397 + 1970 = $6,367 million
Cost of capital = 8.20%(7350/9320) + 4% (1970/9320) = 7.31% Cost of equity for The Gap = 8.20% After-tax cost of debt = 4% Market value of equity = 7350	Cost of capital = 8.20%(7350/13717) + 4% (6367/13717) = 6.25%
Return on capital = 1012 (1-.35)/(3130+1970) = 12.90%	Return on capital = 1362 (1-.35)/(3130+6367) = 9.30%

R&D Expenses: Operating or Capital Expenses

- Accounting standards require us to consider R&D as an operating expense even though it is designed to generate future growth. It is more logical to treat it as capital expenditures.
- To capitalize R&D,
  - Specify an amortizable life for R&D (2 - 10 years)
  - Collect past R&D expenses for as long as the amortizable life
  - Sum up the unamortized R&D over the period. (Thus, if the amortizable life is 5 years, the research asset can be obtained by adding up 1/5th of the R&D expense from five years ago, 2/5th of the R&D expense from four years ago...:

Capitalizing R&D Expenses: Cisco in 1999

- R & D was assumed to have a 5-year life.

Year	*R&D Expense*	*Unamortized portion*	*Amortization this year*	
1999 (current)	1594.00	1.00	1594.00	
1998	1026.00	0.80	820.80	$205.20
1997	698.00	0.60	418.80	$139.60
1996	399.00	0.40	159.60	$79.80
1995	211.00	0.20	42.20	$42.20
1994	89.00	0.00	0.00	$17.80
Total			$ 3,035.40	$ 484.60

Value of research asset = $ 3,035.4 million

Amortization of research asset in 1998 = $ 484.6 million

Increase in Operating Income = $ 1,594 million - 484.6 million = 1,109.4 million

The Effect of Capitalizing R&D

Conventional Accounting	*R&D treated as capital expenditure*
Income Statement EBIT& R&D = 5,049 - R&D = 1,594 EBIT = 3,455 EBIT (1-t) = 2,246	*Income Statement* EBIT& R&D = 5,049 - Amort: R&D = 485 EBIT = 4,564 (Increase of 1,109) EBIT (1-t) = 2,967 Ignored tax benefit = (1594-485)(.35) = 388 Adjusted EBIT (1-t) = 2967 + 388 = 3354 (Increase of $1,109 million) Net Income will also increase by $1,109 million
Balance Sheet Off balance sheet asset. Book value of equity at $11,722 million is understated because biggest asset is off the books.	*Balance Sheet* Asset Liability R&D Asset 3035 Book Equity +3035 Total Book Equity = 11722+3035 = 14757
Capital Expenditures Conventional net cap ex of $98 million	*Capital Expenditures* Net Cap ex = 98 + 1594 – 485 = 1206
Cash Flows EBIT (1-t) = 2246 - Net Cap Ex = 98 FCFF = 2148	*Cash Flows* EBIT (1-t) = 3354 - Net Cap Ex = 1206 FCFF = 2148
Return on capital = 2246/11722 (no debt) = 19.16%	Return on capital = 3354/14757 = 22.78%

What tax rate?

- The tax rate that you should use in computing the after-tax operating income should be
  - The effective tax rate in the financial statements (taxes paid/Taxable income)
  - The tax rate based upon taxes paid and EBIT (taxes paid/EBIT)
  - The marginal tax rate for the country in which the company operates
  - The weighted average marginal tax rate across the countries in which the company operates
  - None of the above
  - Any of the above, as long as you compute your after-tax cost of debt using the same tax rate

Capital expenditures should include

- Research and development expenses, once they have been re-categorized as capital expenses. The adjusted net cap ex will be

  Adjusted Net Capital Expenditures = Net Capital Expenditures + Current year's R&D expenses - Amortization of Research Asset

- Acquisitions of other firms, since these are like capital expenditures. The adjusted net cap ex will be

  Adjusted Net Cap Ex = Net Capital Expenditures + Acquisitions of other firms - Amortization of such acquisitions

  Two caveats:

  1. Most firms do not do acquisitions every year. Hence, a normalized measure of acquisitions (looking at an average over time) should be used
  2. The best place to find acquisitions is in the statement of cash flows, usually categorized under other investment activities

Cisco's Net Capital Expenditures in 1999

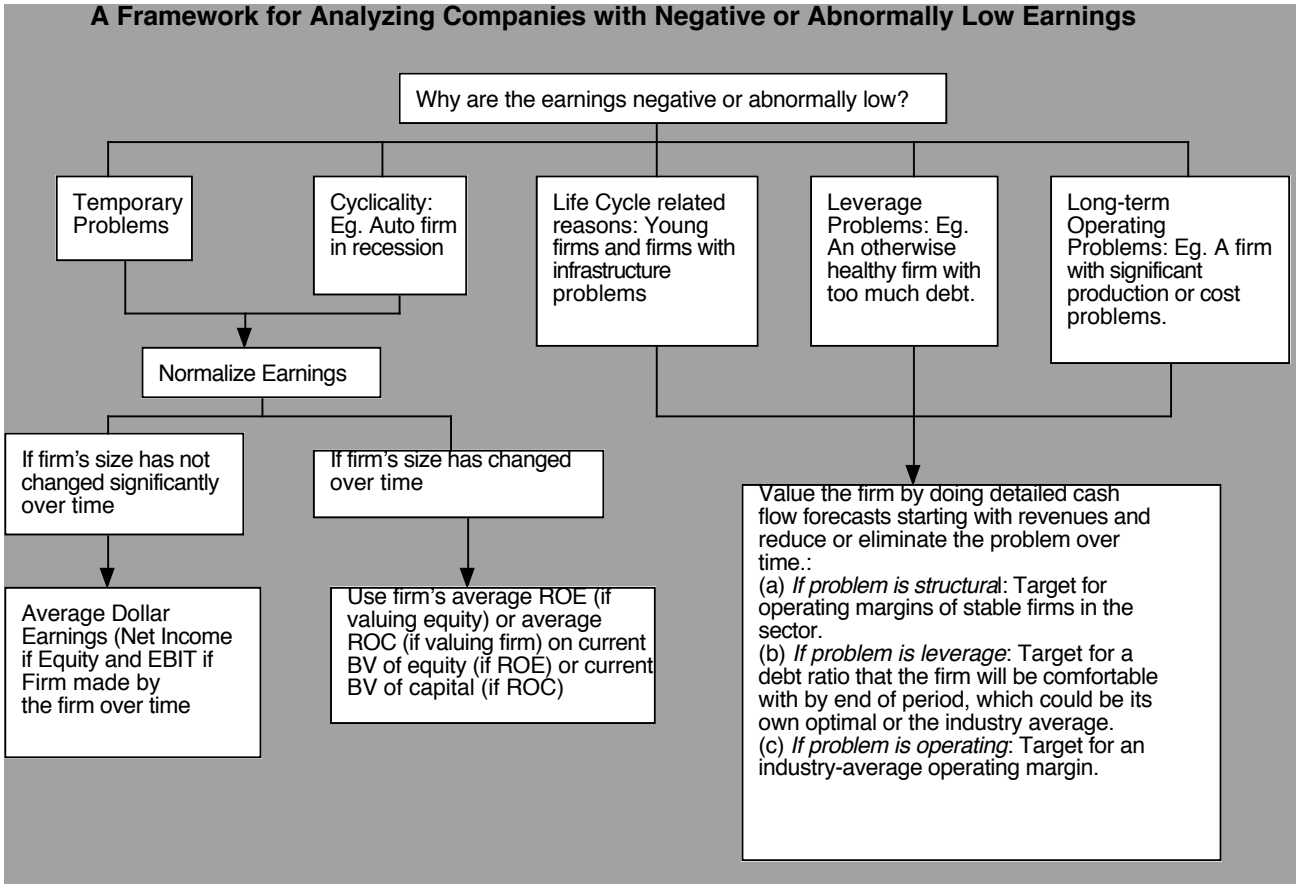
Cap Expenditures (from statement of CF)	= $ 584 mil
- Depreciation (from statement of CF)	= $ 486 mil
Net Cap Ex (from statement of CF)	= $ 98 mil
+ R & D expense	= $ 1,594 mil
- Amortization of R&D	= $ 485 mil
+ Acquisitions	= $ 2,516 mil
Adjusted Net Capital Expenditures	= $3,723 mil

(Amortization was included in the depreciation number)

Working Capital Investments

- In accounting terms, the working capital is the difference between current assets (inventory, cash and accounts receivable) and current liabilities (accounts payables, short term debt and debt due within the next year)
- A cleaner definition of working capital from a cash flow perspective is the difference between non-cash current assets (inventory and accounts receivable) and non-debt current liabilities (accounts payable)
- Any investment in this measure of working capital ties up cash. Therefore, any increases (decreases) in working capital will reduce (increase) cash flows in that period.
- When forecasting future growth, it is important to forecast the effects of such growth on working capital needs, and building these effects into the cash flows.

Dealing with Negative or Abnormally Low Earnings



Normalizing Earnings: Amazon

Year	Revenues	Operating Margin	EBIT	
Tr12m	$1,117	-36.71%	-$410	
1	$2,793	-13.35%	-$373	
2	$5,585	-1.68%	-$94	
3	$9,774	4.16%	$407	
4	$14,661	7.08%	$1,038	
5	$19,059	8.54%	$1,628	
6	$23,862	9.27%	$2,212	
7	$28,729	9.64%	$2,768	
8	$33,211	9.82%	$3,261	
9	$36,798	9.91%	$3,646	
10	$39,006	9.95%	$3,883	
TY(11)	$41,346	10.00%	$4,135	Industry Average

Estimating FCFF: Titan Cement

- EBIT = 232 million Euros
- Tax rate = 25.47%
- Net Capital expenditures = Cap Ex - Depreciation = 109.5 - 60.3 = 49.2 million
- Change in Working Capital = +51.80 million

Estimating FCFF

Current EBIT * (1 - tax rate) =	232 (1-.2547) =	172.8	Million
- (Capital Spending - Depreciation)	49.2		
- Change in Working Capital		51.8	
Current FCFF		71.8	Million Euros

Estimating FCFF: Amazon.com

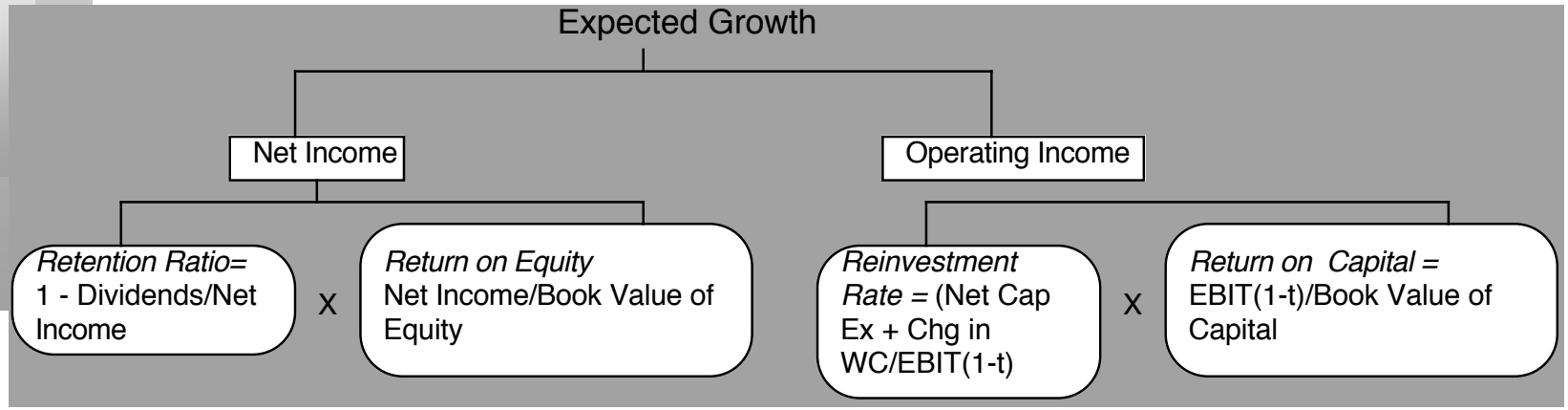
- EBIT (Trailing 1999) = -$ 410 million
- Tax rate used = 0% (Assumed Effective = Marginal)
- Capital spending (Trailing 1999) = $ 243 million
- Depreciation (Trailing 1999) = $ 31 million
- Non-cash Working capital Change (1999) = - 80 million
- Estimating FCFF (1999)

Current EBIT * (1 - tax rate) = - 410 (1-0)	= - $410 million
- (Capital Spending - Depreciation)	= $212 million
- Change in Working Capital	= -$ 80 million
Current FCFF	= - $542 million

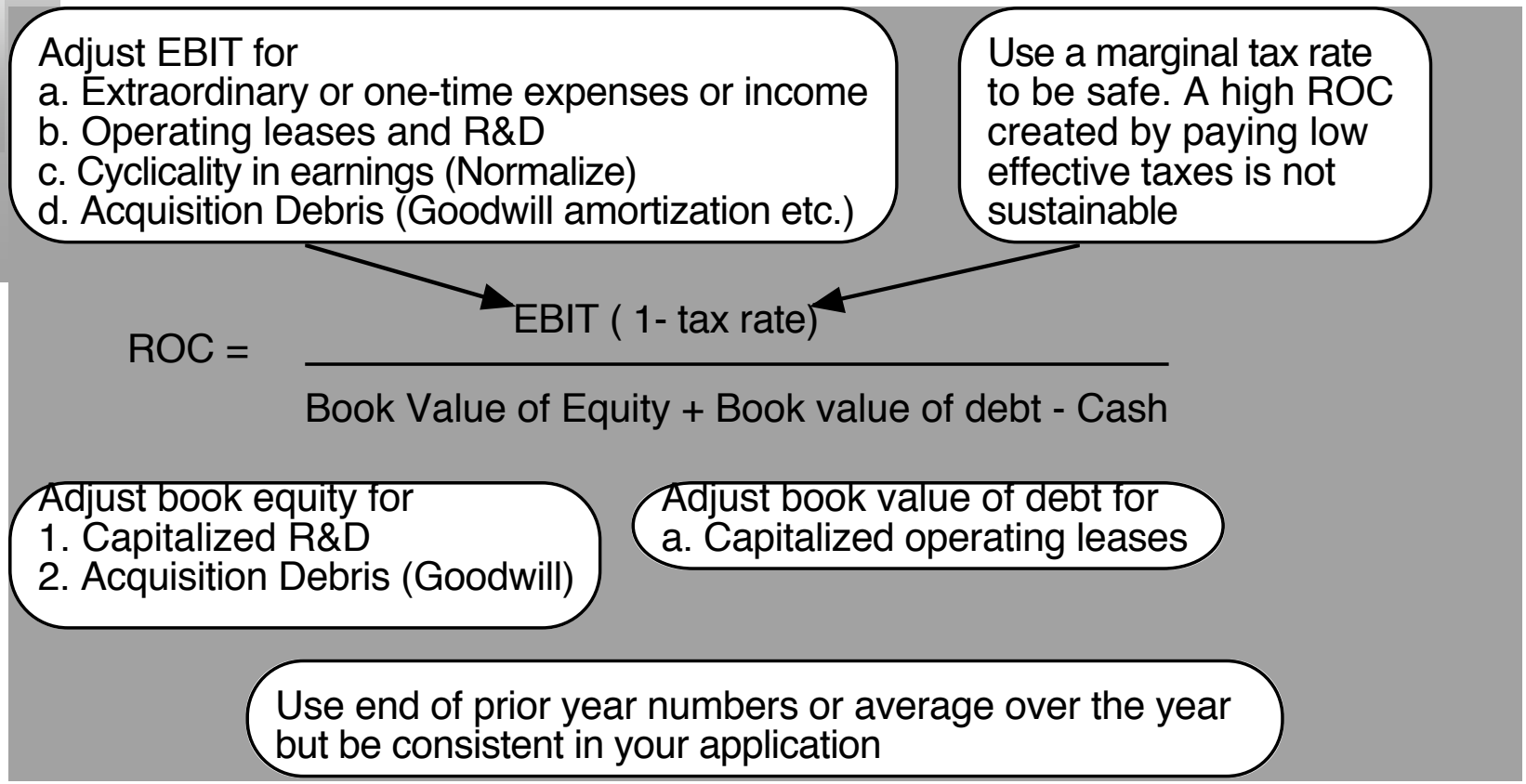
Growth in Earnings

- Look at the past
  - The historical growth in earnings per share is usually a good starting point for growth estimation
- Look at what others are estimating
  - Analysts estimate growth in earnings per share for many firms. It is useful to know what their estimates are.
- Look at fundamentals
  - Ultimately, all growth in earnings can be traced to two fundamentals - how much the firm is investing in new projects, and what returns these projects are making for the firm.

Fundamental Growth when Returns are stable



Measuring Return on Capital (Equity)



Normalizing Reinvestment: Titan Cement

	2000	*2001*	*2002*	*2003*	*2004*	*Total*
Cp Ex	$50.54	$81.00	$113.30	$102.30	$109.50	$456.64
Depreciation	$39.26	$40.87	$80.94	$73.70	$60.30	$295.07
EBIT	$162.78	$186.39	$200.60	$222.00	$231.80	
EBIT(1-t)	$121.32	$138.92	$149.51	$154.42	$172.76	$736.92
Net Cap Ex as % of EBIT(1-t)	9.30%	28.89%	21.64%	18.52%	28.48%	**21.92%**
Revenues	622.7	982.9	1036.1	1035.7	1104.4	4781.8
Non-cashh Current assets	248.55	342.95	352.93	$402.10	$398.90	
Non-debt current liabilities	133.33	177.15	194.57	255	190	
Non-cash WC	115.22	165.8	158.36	147.1	208.9	795.38
as % of revenues	18.50%	16.87%	15.28%	14.20%	18.92%	**16.63%**

Expected Growth Estimate: Titan Cement

- Normalized Change in working capital = (Working capital as percent of revenues) * Change in revenues in 2004 = .1663 (1104.4-1035.7) = 11.4 mil Euros
- Normalized Net Cap Ex = Net Cap ex as % of EBIT(1-t) * EBIT (1-t) in 2004 = .2192*(232 (1-.2547)) = 37.90 million Euros
- Normalized reinvestment rate = (11.4+37.9)/(232(1-..2547)) = 28.54%
- Return on capital = 232 (1-.2547)/ (499+399) = 19.25%
  - The book value of debt and equity from last year was used.
- Expected growth rate = .2854*.1925= 5.49%

Fundamental Growth when return on equity (capital) is changing

- When the return on equity or capital is changing, there will be a second component to growth, positive if the return is increasing and negative if the return is decreasing.
- If ROC_t is the return on capital in period t and ROC_{t+1} is the return on capital in period t+1, the expected growth rate in operating income will be:

$$\text{Expected Growth Rate} = ROC_{t+1} * \text{Reinvestment rate} + (ROC_{t+1} - ROC_t) / ROC_t$$

An example: Motorola

- Motorola's current return on capital is 12.18% and its reinvestment rate is 52.99%.
- We expect Motorola's return on capital to rise to 17.22% over the next 5 years (which is half way towards the industry average)

  Expected Growth Rate

  $= ROC_{New\ Investments} * Reinvestment\ Rate_{current} + \{[1+(ROC_{In\ 5\ years} - ROC_{Current})/ROC_{Current}]^{1/5} - 1\}$

  $= .1722 * .5299 + \{[1+(.1722-.1218)/.1218]^{1/5} - 1\}$

  $= .174$ or 17.40%
- One way to think about this is to decompose Motorola's expected growth into
  - Growth from new investments: .1722*5299= 9.12%
  - Growth from more efficiently using existing investments: 17.40%-9.12%=8.28%

Revenue Growth and Operating Margins

- With negative operating income and a negative return on capital, the fundamental growth equation is of little use for Amazon.com
- For Amazon, the effect of reinvestment shows up in revenue growth rates and changes in expected operating margins:

Expected Revenue Growth in $ = Reinvestment (in $ terms) * (Sales/ Capital)

- The effect on expected margins is more subtle. Amazon's reinvestments (especially in acquisitions) may help create barriers to entry and other competitive advantages that will ultimately translate into high operating margins and high profits.

Growth in Revenues, Earnings and Reinvestment: Amazon

Year	Revenue Growth	Chg in Revenue	Reinvestment	Chg Rev/ Chg Reinvestment	ROC
1	150.00%	$1,676	$559	3.00	-76.62%
2	100.00%	$2,793	$931	3.00	-8.96%
3	75.00%	$4,189	$1,396	3.00	20.59%
4	50.00%	$4,887	$1,629	3.00	25.82%
5	30.00%	$4,398	$1,466	3.00	21.16%
6	25.20%	$4,803	$1,601	3.00	22.23%
7	20.40%	$4,868	$1,623	3.00	22.30%
8	15.60%	$4,482	$1,494	3.00	21.87%
9	10.80%	$3,587	$1,196	3.00	21.19%
10	6.00%	$2,208	$736	3.00	20.39%

Assume that firm can earn high returns because of established economies of scale.

III. The Tail that wags the dog… Terminal Value

Getting Closure in Valuation

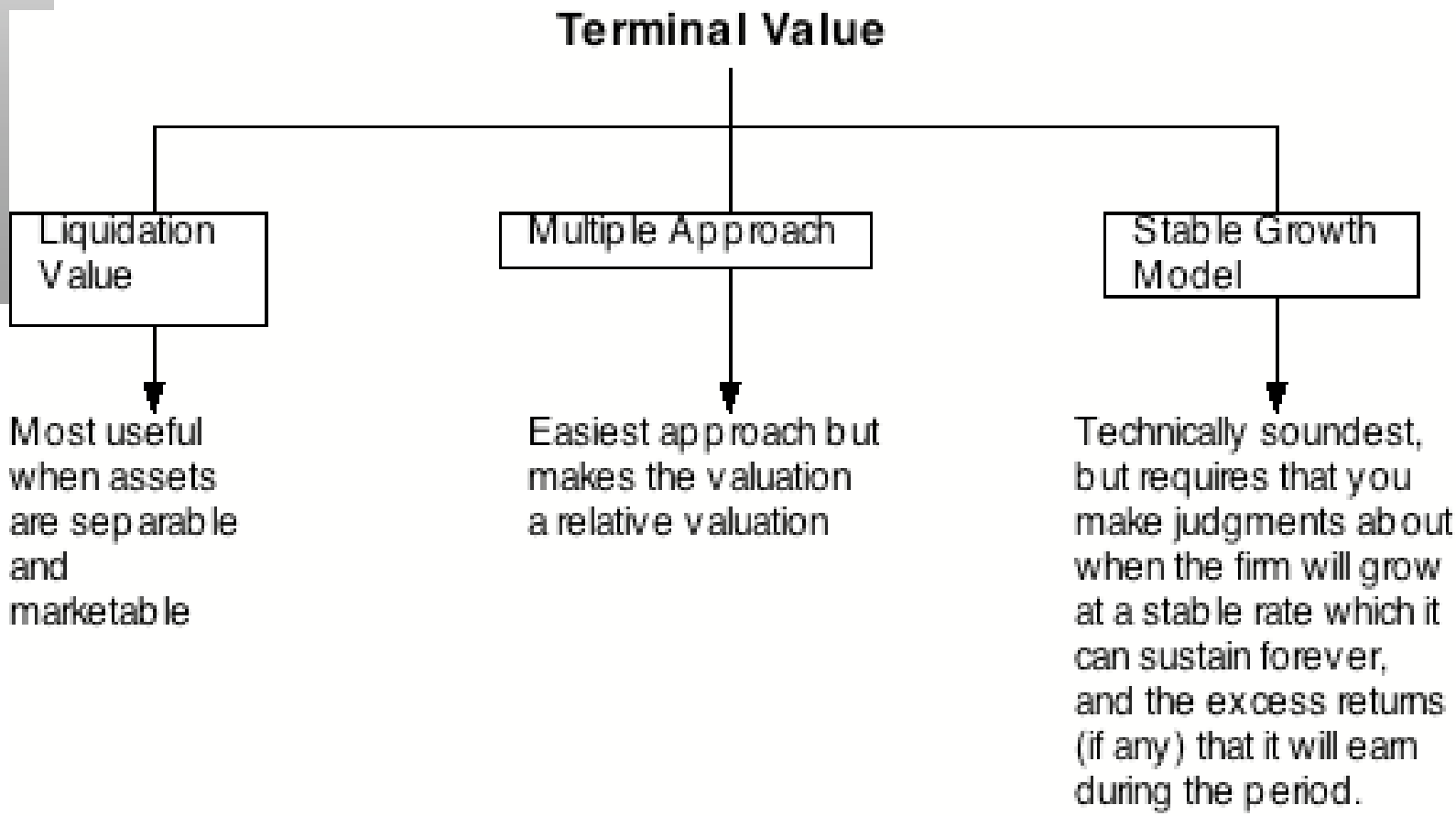
- A publicly traded firm potentially has an infinite life. The value is therefore the present value of cash flows forever.

$$\text{Value} = \sum_{t=1}^{t=\infty} \frac{CF_t}{(1+r)^t}$$

- Since we cannot estimate cash flows forever, we estimate cash flows for a "growth period" and then estimate a terminal value, to capture the value at the end of the period:

$$\text{Value} = \sum_{t=1}^{t=N} \frac{CF_t}{(1+r)^t} + \frac{\text{Terminal Value}}{(1+r)^N}$$

Ways of Estimating Terminal Value



Stable Growth and Terminal Value

- When a firm's cash flows grow at a "constant" rate forever, the present value of those cash flows can be written as:

  Value = Expected Cash Flow Next Period / (r - g)

  where,

  r = Discount rate (Cost of Equity or Cost of Capital)

  g = Expected growth rate

- This "constant" growth rate is called a stable growth rate and cannot be higher than the growth rate of the economy in which the firm operates.
- While companies can maintain high growth rates for extended periods, they will all approach "stable growth" at some point in time.

Limits on Stable Growth

- The stable growth rate cannot exceed the growth rate of the economy but it can be set lower.
  - If you assume that the economy is composed of high growth and stable growth firms, the growth rate of the latter will probably be lower than the growth rate of the economy.
  - The stable growth rate can be negative. The terminal value will be lower and you are assuming that your firm will disappear over time.
  - If you use nominal cashflows and discount rates, the growth rate should be nominal in the currency in which the valuation is denominated.
- One simple proxy for the nominal growth rate of the economy is the riskfree rate.

Stable Growth and Excess Returns

- Strange though this may seem, the terminal value is not as much a function of stable growth as it is a function of what you assume about excess returns in stable growth.
- In the scenario where you assume that a firm earns a return on capital equal to its cost of capital in stable growth, the terminal value will not change as the growth rate changes.
- If you assume that your firm will earn positive (negative) excess returns in perpetuity, the terminal value will increase (decrease) as the stable growth rate increases.

Getting to Stable Growth: High Growth Patterns

- A key assumption in all discounted cash flow models is the period of high growth, and the pattern of growth during that period. In general, we can make one of three assumptions:
  - there is no high growth, in which case the firm is already in stable growth
  - there will be high growth for a period, at the end of which the growth rate will drop to the stable growth rate (2-stage)
  - there will be high growth for a period, at the end of which the growth rate will decline gradually to a stable growth rate(3-stage)
  - Each year will have different margins and different growth rates (n stage)
- Concurrently, you will have to make assumptions about excess returns. In general, the excess returns will be large and positive in the high growth period and decrease as you approach stable growth (the rate of decrease is often titled the fade factor).

Determinants of Growth Patterns

- Size of the firm
  - Success usually makes a firm larger. As firms become larger, it becomes much more difficult for them to maintain high growth rates
- Current growth rate
  - While past growth is not always a reliable indicator of future growth, there is a correlation between current growth and future growth. Thus, a firm growing at 30% currently probably has higher growth and a longer expected growth period than one growing 10% a year now.
- Barriers to entry and differential advantages
  - Ultimately, high growth comes from high project returns, which, in turn, comes from barriers to entry and differential advantages.
  - The question of how long growth will last and how high it will be can therefore be framed as a question about what the barriers to entry are, how long they will stay up and how strong they will remain.

Stable Growth Characteristics

- In stable growth, firms should have the characteristics of other stable growth firms. In particular,
  - The risk of the firm, as measured by beta and ratings, should reflect that of a stable growth firm.
    - Beta should move towards one
    - The cost of debt should reflect the safety of stable firms (BBB or higher)
  - The debt ratio of the firm might increase to reflect the larger and more stable earnings of these firms.
    - The debt ratio of the firm might moved to the optimal or an industry average
    - If the managers of the firm are deeply averse to debt, this may never happen
  - The reinvestment rate of the firm should reflect the expected growth rate and the firm's return on capital
    - Reinvestment Rate = Expected Growth Rate / Return on Capital

To fade or not to fade…

- If you have a firm with high growth, high risk and high excess returns that you expect to become a firm with stable growth, average risk and no excess returns, the question becomes whether you should fade these numbers over a transition period and if so at what rate. Here are some basic propositions:
  - The value of your firm is affected far more by your starting and ending numbers on each of these values than by your choice of a fade factor.
  - It makes more sense to have fade factors when you have very large differences between high and stable growth periods.
  - The fade factor should be a firm-specific choice. A constant fade factor cannot and should not be imposed across all firms.

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    - Beta should move towards one
    - The cost of debt should reflect the safety of stable firms (BBB or higher)
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  - The reinvestment rate of the firm should reflect the expected growth rate and the firm's return on capital
    - Reinvestment Rate = Expected Growth Rate / Return on Capital

Titan and Amazon.com: Stable Growth Inputs

	High Growth	*Stable Growth*
■ Titan Cement		
• Beta	0.93	1.00
• Debt Ratio	17.6%	17.6%
• Return on Capital	19.25%	6.57%
• Cost of Capital	6.78%	6.57%
• Expected Growth Rate	5.49%	3.41%
• Reinvestment Rate	28.54%	3.41%6.57% = 51.93%
■ Amazon.com		
• Beta	1.60	1.00
• Debt Ratio	1.20%	15%
• Return on Capital	Negative	20%
• Expected Growth Rate	NMF	6%
• Reinvestment Rate	>100%	6%/20% = 30%

IV. Loose Ends in Valuation: From firm value to value of equity per share

But what comes next?

Value of Operating Assets	Since this is a discounted cashflow valuation, should there be a real option premium?
+ Cash and Marketable Securities	Operating versus Non-opeating cash Should cash be discounted for earning a low return?
+ Value of Cross Holdings	How do you value cross holdings in other companies? What if the cross holdings are in private businesses?
+ Value of Other Assets	What about other valuable assets? How do you consider under utlilized assets?
Value of Firm	Should you discount this value for opacity or complexity? How about a premium for synergy? What about a premium for intangibles (brand name)?
- Value of Debt	What should be counted in debt? Should you subtract book or market value of debt? What about other obligations (pension fund and health care? What about contingent liabilities? What about minority interests?
= Value of Equity	Should there be a premium/discount for control? Should there be a discount for distress
- Value of Equity Options	What equity options should be valued here (vested versus non-vested)? How do you value equity options?
= Value of Common Stock	Should you divide by primary or diluted shares?
/ Number of shares	
= Value per share	Should there be a discount for illiquidity/ marketability? Should there be a discount for minority interests?

1. The Value of Cash

- The simplest and most direct way of dealing with cash and marketable securities is to keep it out of the valuation - the cash flows should be before interest income from cash and securities, and the discount rate should not be contaminated by the inclusion of cash. (Use betas of the operating assets alone to estimate the cost of equity).
- Once the operating assets have been valued, you should add back the value of cash and marketable securities.
- In many equity valuations, the interest income from cash is included in the cashflows. The discount rate has to be adjusted then for the presence of cash. (The beta used will be weighted down by the cash holdings). Unless cash remains a fixed percentage of overall value over time, these valuations will tend to break down.

Should you ever discount cash for its low returns?

- There are some analysts who argue that companies with a lot of cash on their balance sheets should be penalized by having the excess cash discounted to reflect the fact that it earns a low return.
  - Excess cash is usually defined as holding cash that is greater than what the firm needs for operations.
  - A low return is defined as a return lower than what the firm earns on its non-cash investments.
- This is the wrong reason for discounting cash. If the cash is invested in riskless securities, it should earn a low rate of return. As long as the return is high enough, given the riskless nature of the investment, cash does not destroy value.
- There is a right reason, though, that may apply to some companies… Managers can do stupid things with cash (overpriced acquisitions, pie-in-the-sky projects….) and you have to discount for this possibility.

2. Dealing with Holdings in Other firms

- Holdings in other firms can be categorized into
  - Minority passive holdings, in which case only the dividend from the holdings is shown in the balance sheet
  - Minority active holdings, in which case the share of equity income is shown in the income statements
  - Majority active holdings, in which case the financial statements are consolidated.
- We tend to be sloppy in practice in dealing with cross holdings. After valuing the operating assets of a firm, using consolidated statements, it is common to add on the balance sheet value of minority holdings (which are in book value terms) and subtract out the minority interests (again in book value terms), representing the portion of the consolidated company that does not belong to the parent company.

How to value holdings in other firms in a full information environment..

Fin Statement	*Valuing*	*What to do…*
Not consolidated	Equity	Value equity in subsidiary and take share of holding.
Not consolidated	Firm	Value subsidiary as a firm and add portion of firm value. Add portion of debt in subsidiary to the debt in estimating equity value.
Consolidated	Firm	Strip operating income of subsidiary and value subsidiary separately. Add portion of this value to value of parent firm.

Two compromise solutions…

- The market value solution: When the subsidiaries are publicly traded, you could use their traded market capitalizations to estimate the values of the cross holdings. You do risk carrying into your valuation any mistakes that the market may be making in valuation.
- The relative value solution: When there are too many cross holdings to value separately or when there is insufficient information provided on cross holdings, you can convert the book values of holdings that you have on the balance sheet (for both minority holdings and minority interests in majority holdings) by using the average price to book value ratio of the sector in which the subsidiaries operate.

Titan's Cash and Cross Holdings

- Titan has a majority interest in another company and the financial statements of that company are consolidated with those of Titan. The minority interests (representing the equity in the subsidiary that does not belong to Titan) are shown on the balance sheet at 25.50 million Euros.
- Estimated market value of minority interests = Book value of minority interest * P/BV of sector that subsidiary belongs to = 25.50 * 1.80 = 45.90 million

Present Value of FCFF in high growth phase =		$595.37
Present Value of Terminal Value of Firm =	$2,302.05	
Value of operating assets of the firm =		$2,897.42
Value of Cash, Marketable Securities & Non-operating assets =	$76.80	
Value of Firm =		$2,974.22
Market Value of outstanding debt =		$414.25
Value of Minority Interests in Consolidated Company =	$45.90	
Market Value of Equity =		$2,514.07

3. Other Assets that have not been counted yet..

- **Unutilized assets**: If you have assets or property that are not being utilized (vacant land, for example), you have not valued it yet. You can assess a market value for these assets and add them on to the value of the firm.
- **Overfunded pension plans**: If you have a defined benefit plan and your assets exceed your expected liabilities, you could consider the over funding with two caveats:
  - Collective bargaining agreements may prevent you from laying claim to these excess assets.
  - There are tax consequences. Often, withdrawals from pension plans get taxed at much higher rates.

  Do not double count an asset. If you count the income from an asset in your cashflows, you cannot count the market value of the asset in your value.

4. A Discount for Complexity: An Experiment

	Company A	*Company B*
Operating Income	$ 1 billion	$ 1 billion
Tax rate	40%	40%
ROIC	10%	10%
Expected Growth	5%	5%
Cost of capital	8%	8%
Business Mix	Single Business	Multiple Businesses
Holdings	Simple	Complex
Accounting	Transparent	Opaque

- *Which firm would you value more highly?*

Measuring Complexity: Volume of Data in Financial Statements

Company	*Number of pages in last 10Q*	*Number of pages in last 10K*
General Electric	65	410
Microsoft	63	218
Wal-mart	38	244
Exxon Mobil	86	332
Pfizer	171	460
Citigroup	252	1026
Intel	69	215
AIG	164	720
Johnson & Johnson	63	218
IBM	85	353

Measuring Complexity: A Complexity Score

Item	*Factors*	*Follow-up Question*	*Answer*	*Complexity score*
Operating Income	1. Multiple Businesses	Number of businesses (with more than 10% of revenues) =	2	4
	2. One-time income and expenses	Percent of operating income =	20%	1
	3. Income from unspecified sources	Percent of operating income =	15%	0.75
	4. Items in income statement that are volatile	Percent of operating income =	5%	0.25
Tax Rate	1. Income from multiple locales	Percent of revenues from non-domestic locales =	100%	3
	2. Different tax and reporting books	Yes or No	Yes	3
	3. Headquarters in tax havens	Yes or No	Yes	3
	4. Volatile effective tax rate	Yes or No	Yes	2
Capital Expenditures	1. Volatile capital expenditures	Yes or No	Yes	2
	2. Frequent and large acquisitions	Yes or No	Yes	4
	3. Stock payment for acquisitions and investments	Yes or No	Yes	4
Working capital	1. Unspecified current assets and current liabilities	Yes or No	Yes	3
	2. Volatile working capital items	Yes or No	Yes	2
Expected Growth rate	1. Off-balance sheet assets and liabilities (operating leases and R&D)	Yes or No	Yes	3
	2. Substantial stock buybacks	Yes or No	Yes	3
	3. Changing return on capital over time	Is your return on capital volatile?	Yes	5
	4. Unsustainably high return	Is your firm's ROC much higher than industry average?	Yes	5
Cost of capital	1. Multiple businesses	Number of businesses (more than 10% of revenues) =	2	2
	2. Operations in emerging markets	Percent of revenues=	30%	1.5
	3. Is the debt market traded?	Yes or No	Yes	0
	4. Does the company have a rating?	Yes or No	Yes	0
	5. Does the company have off-balance sheet debt?	Yes or No	No	0
		Complexity Score =		**51.5**

Dealing with Complexity

In Discounted Cashflow Valuation

- The Aggressive Analyst: Trust the firm to tell the truth and value the firm based upon the firm's statements about their value.
- The Conservative Analyst: Don't value what you cannot see.
- The Compromise: Adjust the value for complexity
  - Adjust cash flows for complexity
  - Adjust the discount rate for complexity
  - Adjust the expected growth rate/ length of growth period
  - Value the firm and then discount value for complexity

In relative valuation

In a relative valuation, you may be able to assess the price that the market is charging for complexity:

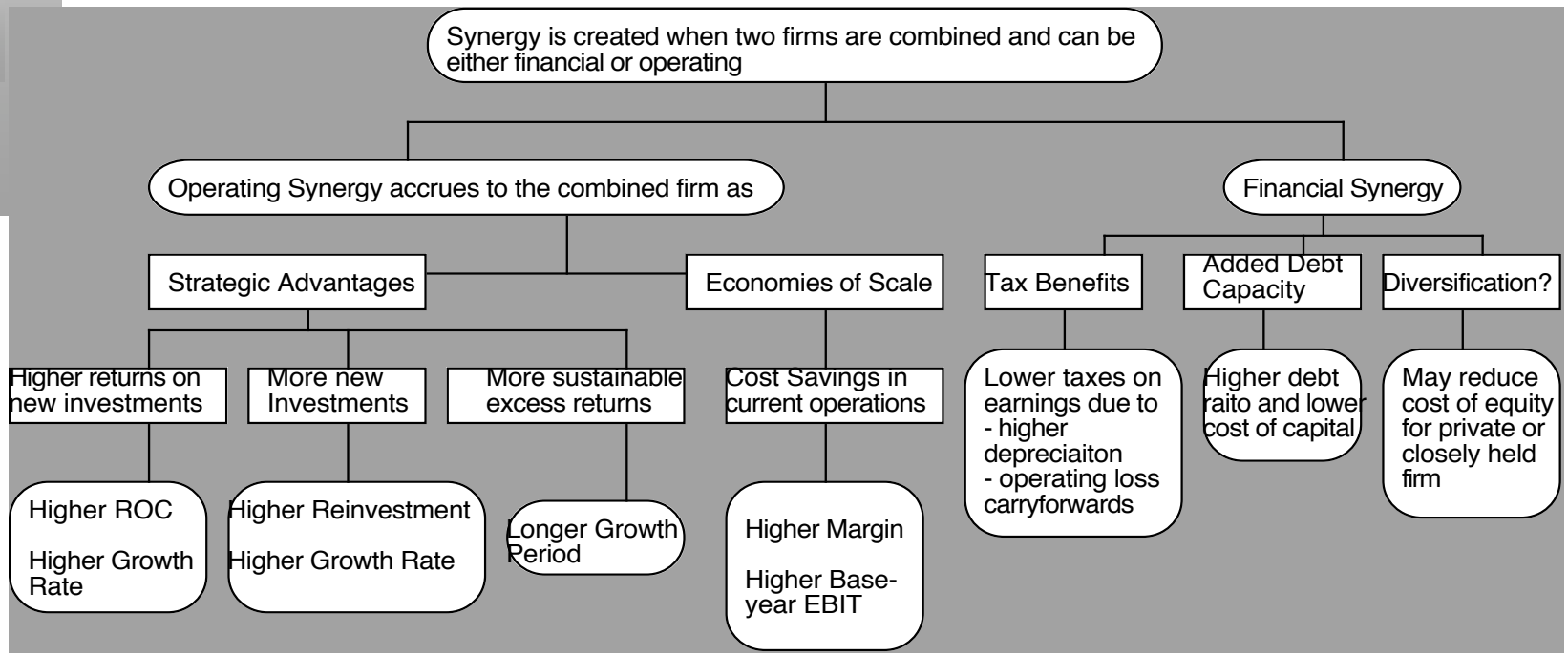
With the hundred largest market cap firms, for instance:

PBV = 0.65 + 15.31 ROE – 0.55 Beta + 3.04 Expected growth rate – 0.003 # Pages in 10K

4. The Value of Synergy

- Synergy can be valued. In fact, if you want to pay for it, it should be valued.
- To value synergy, you need to answer two questions:
  (a) What **form** is the synergy expected to take? Will it **reduce costs** as a percentage of sales and increase profit margins (as is the case when there are economies of scale)? Will it **increase future growth** (as is the case when there is increased market power)?)
  (b) **When can the synergy be reasonably expected to start** affecting cashflows? (Will the gains from synergy show up instantaneously after the takeover? If it will take time, when can the gains be expected to start showing up?)
- If you cannot answer these questions, you need to go back to the drawing board…

Sources of Synergy



Valuing Synergy

(1) the firms involved in the merger are **valued independently**, by discounting expected cash flows to each firm at the weighted average cost of capital for that firm.

(2) the **value of the combined firm, with no synergy**, is obtained by adding the values obtained for each firm in the first step.

(3) The **effects of synergy are built into expected growth rates and cashflows**, and the combined firm is re-valued with synergy.

Value of Synergy = Value of the combined firm, with synergy - Value of the combined firm, without synergy

Valuing Synergy: P&G + Gillette

	P&G	Gillette	Piglet: No Synergy	Piglet: Synergy	
Free Cashflow to Equity	$5,864.74	$1,547.50	$7,412.24	$7,569.73	Annual operating expenses reduced by $250 million
Growth rate for first 5 years	12%	10%	11.58%	12.50%	Slighly higher growth rate
Growth rate after five years	4%	4%	4.00%	4.00%	
Beta	0.90	0.80	0.88	0.88	
Cost of Equity	7.90%	7.50%	7.81%	7.81%	Value of synergy
Value of Equity	$221,292	$59,878	$281,170	$298,355	**$17,185**

5. Brand name, great management, superb product …

- There is often a temptation to add on premiums for intangibles. Among them are
  - Brand name
  - Great management
  - Loyal workforce
  - Technological prowess
- If your discounted cashflow valuation is done right, your inputs should already reflect these strengths.
- If you add a premium, you will be double counting the strength.

Valuing Brand Name

	Coca Cola	Generic Cola Company
AT Operating Margin	**18.56%**	**7.50%**
Sales/BV of Capital	1.67	1.67
ROC	31.02%	12.53%
Reinvestment Rate	65.00% (19.35%)	65.00% (47.90%)
Expected Growth	20.16%	8.15%
Length	10 years	10 yea
Cost of Equity	12.33%	12.33%
E/(D+E)	97.65%	97.65%
AT Cost of Debt	4.16%	4.16%
D/(D+E)	2.35%	2.35%
Cost of Capital	12.13%	12.13%
Value	**$115**	**$13**

6. Be circumspect about defining debt for cost of capital purposes…

- **General Rule**: Debt generally has the following characteristics:
  - Commitment to make fixed payments in the future
  - The fixed payments are tax deductible
  - Failure to make the payments can lead to either default or loss of control of the firm to the party to whom payments are due.
- Defined as such, debt should include
  - All interest bearing liabilities, short term as well as long term
  - All leases, operating as well as capital
- Debt should not include
  - Accounts payable or supplier credit

Book Value or Market Value

- For some firms that are in financial trouble, the book value of debt can be substantially higher than the market value of debt. Analysts worry that subtracting out the market value of debt in this case can yield too high a value for equity.
- A discounted cashflow valuation is designed to value a going concern. In a going concern, it is the market value of debt that should count, even if it is much lower than book value.
- In a liquidation valuation, you can subtract out the book value of debt from the liquidation value of the assets.

Converting book debt into market debt,,,,,

But you should consider other potential liabilities when getting to equity value

- If you have under funded pension fund or health care plans, you should consider the under funding at this stage in getting to the value of equity.
  - If you do so, you should not double count by also including a cash flow line item reflecting cash you would need to set aside to meet the unfunded obligation.
  - You should not be counting these items as debt in your cost of capital calculations….
- If you have contingent liabilities - for example, a potential liability from a lawsuit that has not been decided - you should consider the expected value of these contingent liabilities
  - Value of contingent liability = Probability that the liability will occur * Expected value of liability

7. The Value of Control

- The value of the control premium that will be paid to acquire a block of equity will depend upon two factors -
  - **Probability that control of firm will change**: This refers to the probability that incumbent management will be replaced. this can be either through acquisition or through existing stockholders exercising their muscle.
  - **Value of Gaining Control of the Company:** The value of gaining control of a company arises from two sources - the increase in value that can be wrought by changes in the way the company is managed and run, and the side benefits and perquisites of being in control

  Value of Gaining Control = Present Value (Value of Company with change in control - Value of company without change in control) + Side Benefits of Control

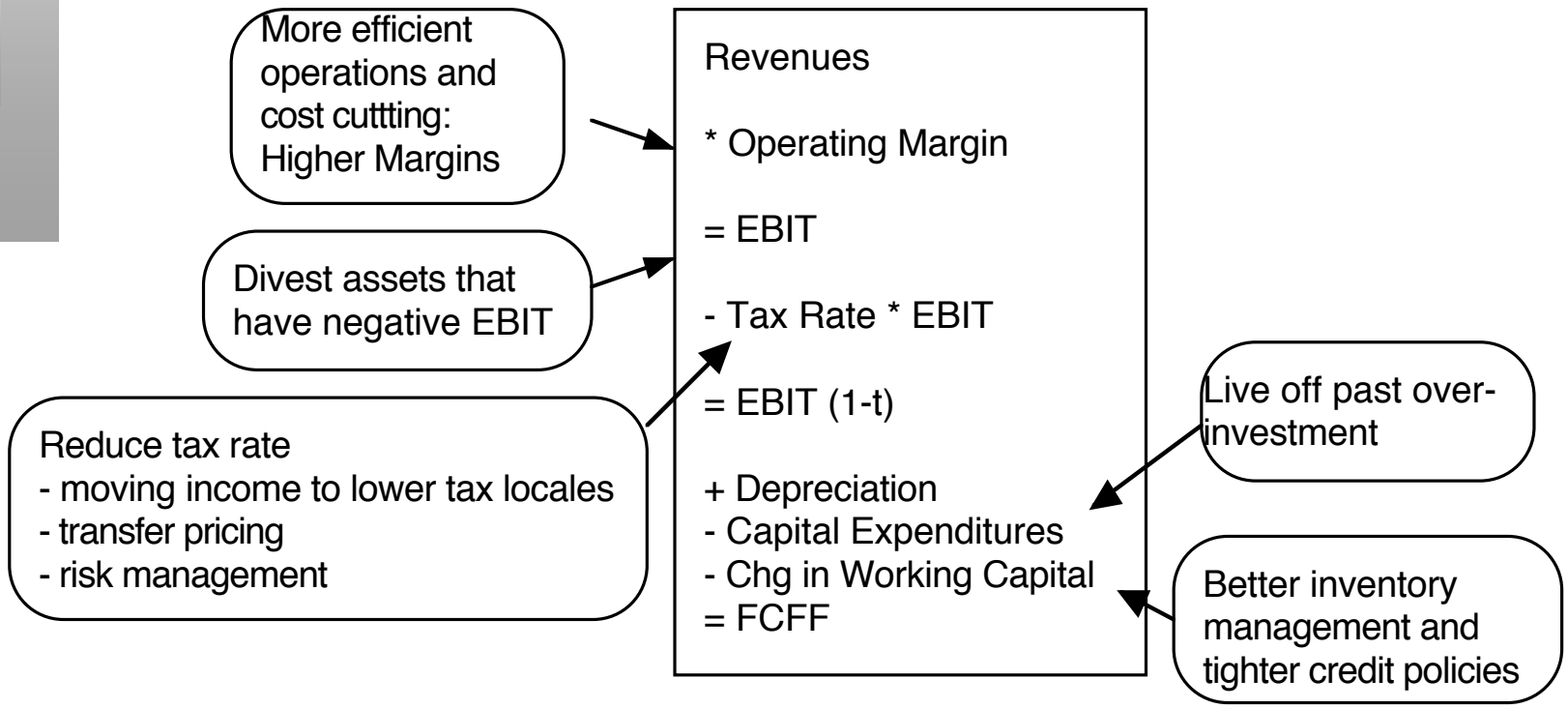
Where control matters…

- In publicly traded firms, control is a factor
  - In the pricing of every publicly traded firm, since a portion of every stock can be attributed to the market's views about control.
  - In acquisitions, it will determine how much you pay as a premium for a firm to control the way it is run.
  - When shares have voting and non-voting shares, the value of control will determine the price difference.
- In private firms, control usually becomes an issue when you consider how much to pay for a private firm.
  - You may pay a premium for a badly managed private firm because you think you could run it better.
  - The value of control is directly related to the discount you would attach to a minority holding (<50%) as opposed to a majority holding.
  - The value of control also becomes a factor in how much of an ownership stake you will demand in exchange for a private equity investment.

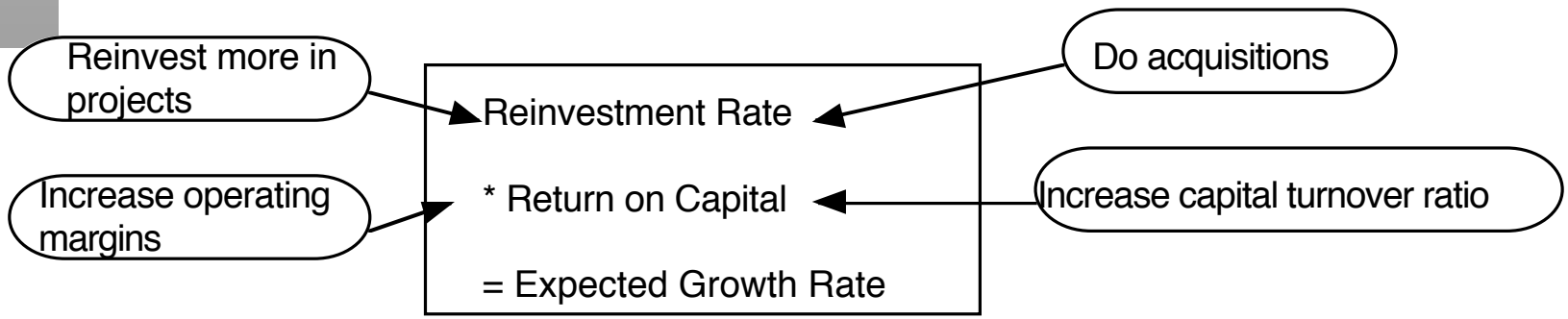
Value of Gaining Control.. You could enhance a firm's value by…

- Using the DCF framework, there are four basic ways in which the value of a firm can be enhanced:
  - The cash flows from existing assets to the firm can be increased, by either
    - increasing after-tax earnings from assets in place or
    - reducing reinvestment needs (net capital expenditures or working capital)
  - The expected growth rate in these cash flows can be increased by either
    - Increasing the rate of reinvestment in the firm
    - Improving the return on capital on those reinvestments
  - The length of the high growth period can be extended to allow for more years of high growth.
  - The cost of capital can be reduced by
    - Reducing the operating risk in investments/assets
    - Changing the financial mix
    - Changing the financing composition

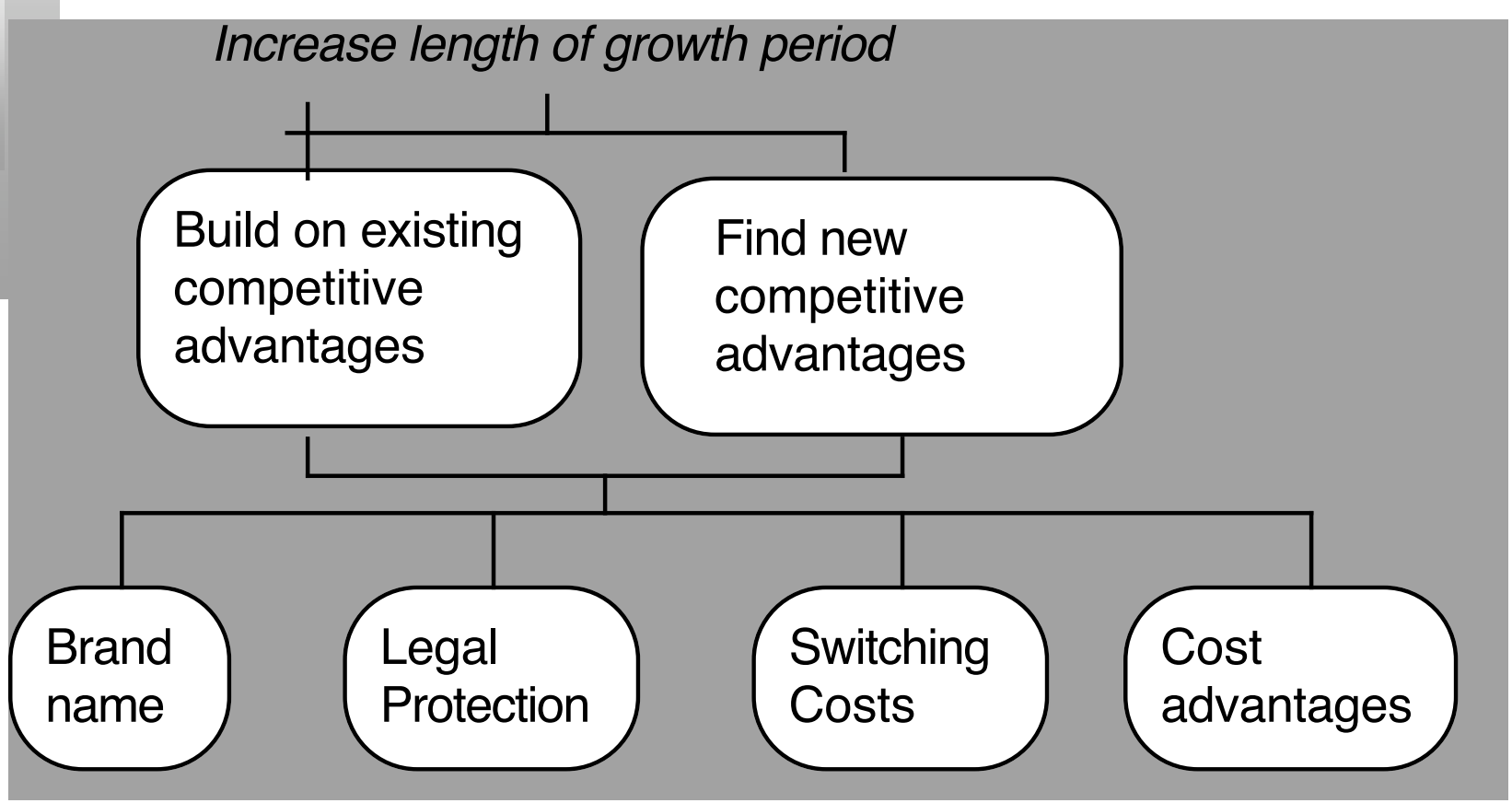
I. Ways of Increasing Cash Flows from Assets in Place



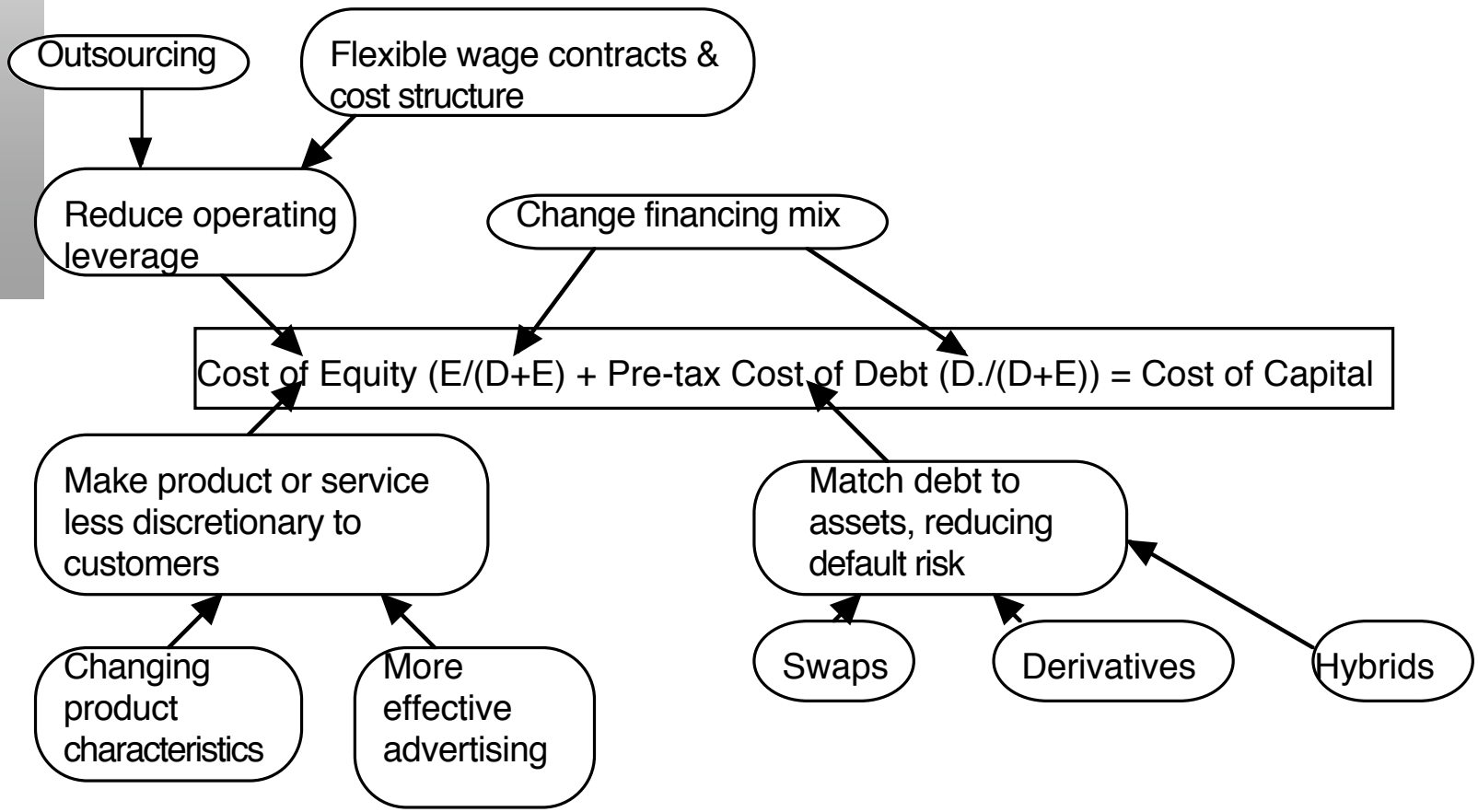
II. Value Enhancement through Growth



III. Building Competitive Advantages: Increase length of the growth period



IV. Reducing Cost of Capital



Titan : Optimal Capital Structure

Debt Ratio	Beta	Cost of Equity	Bond Rating	Interest rate on debt	Tax Rate	Cost of Debt (after-tax)	WACC	Firm Value (G)
0%	0.80	6.97%	AAA	3.76%	25.47%	2.80%	6.97%	$2,146
10%	0.86	7.26%	AAA	3.76%	25.47%	2.80%	6.82%	$2,239
20%	0.95	7.63%	AA	3.91%	25.47%	2.91%	6.69%	$2,324
30%	1.05	8.11%	A-	4.41%	25.47%	3.29%	6.66%	$2,344
40%	1.19	8.74%	BB	5.91%	25.47%	4.40%	7.00%	$2,125
50%	1.39	9.62%	B	7.41%	25.47%	5.52%	7.57%	$1,840
60%	1.71	11.04%	CC	13.41%	23.78%	10.22%	10.55%	$1,066
70%	2.28	13.58%	CC	13.41%	20.38%	10.68%	11.55%	$929
80%	3.49	18.99%	C	15.41%	15.52%	13.02%	14.21%	$687
90%	6.99	34.58%	C	15.41%	13.80%	13.28%	15.41%	$612

Titan Cements: Restructured

Reinvest more in slightly less attractive projects

Return on Capital
18%

Reinvestment Rate
50%

Current Cashflow to Firm
EBIT(1-t) : 173
- Nt CpX 49
- Chg WC 52
= FCFF 72
Reinvestment Rate = 101/173
=58.5%

Expected Growth in EBIT (1-t)
.50*.18=.09
9%

Stable Growth
g = 3.41%; Beta = 1.00;
Country Premium= 0%
Cost of capital = 5.97%
ROC= 5.97%; Tax rate=33%
Reinvestment Rate=51.9%

Terminal Value5= 106.0/(.0597-.0341) = 4137

Year	1	2	3	4	5	Term Yr
EBIT	€ 252.66	€ 275.40	€ 300.19	€ 327.20	€ 356.65	368.8
EBIT(1-t)	€ 188.31	€ 205.26	€ 223.73	€ 243.87	€ 265.81	247.1
- Reinvestment	€ 94.15	€ 102.63	€ 111.86	€ 121.93	€ 132.91	141.1
= FCFF	€ 94.15	€ 102.63	€ 111.86	€ 121.93	€ 132.91	106.0

Op. Assets	3,468
+ Cash:	77
- Debt	411
- Minor. Int.	46
=Equity	3,088
-Options	0
Value/Share	$40.33

Discount at Cost of Capital (WACC) = 8.11% (.70) + 3.29% (0.30) = 6.6%

Use a higher debt raito

Cost of Equity
8.11%

Cost of Debt
(3.41%+.74%+.26%)(1-.2547)
= 3.29%

Weights
E = 70% D = 30%

Riskfree Rate:
Euro riskfree rate = 3.41%

+

Beta
1.05

Unlevered Beta for Sectors: 0.80

Firm's D/E Ratio: 42%

x

Risk Premium
4.46%

Mature risk premium
4%

Country Equity Prem
0.46%

The Value of Control in a publicly traded firm..

- If the value of a firm run optimally is significantly higher than the value of the firm with the status quo (or incumbent management), you can write the value that you should be willing to pay as:

  Value of control = Value of firm optimally run - Value of firm with status quo

  Value of control at Titan Cements = 40.33 Euros per share - 32.84 Euros per share = 7.49 Euros per share

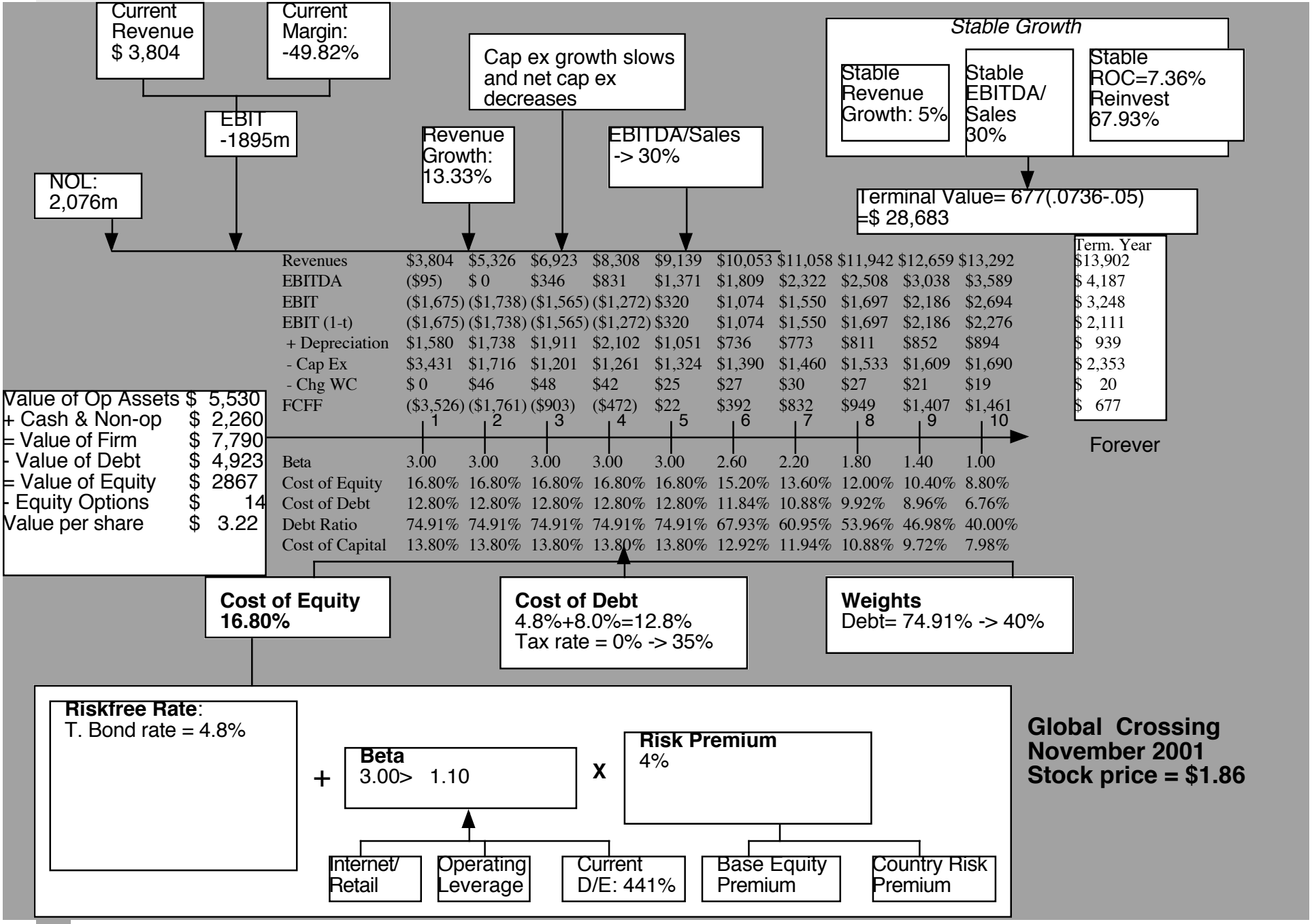
- Implications:
  - In an acquisition, this is the most that you would be willing to pay as a premium (assuming no other synergy)
  - As a stockholder, you will be willing to pay a value between 32.84 and 40.33, depending upon your views on whether control will change.
  - If there are voting and non-voting shares, the difference in prices between the two should reflect the value of control.

Minority and Majority interests in a private firm

- When you get a controlling interest in a private firm (generally >51%, but could be less…), you would be willing to pay the appropriate proportion of the optimal value of the firm.
- When you buy a minority interest in a firm, you will be willing to pay the appropriate fraction of the status quo value of the firm.
- For badly managed firms, there can be a significant difference in value between 51% of a firm and 49% of the same firm. This is the minority discount.
- If you own a private firm and you are trying to get a private equity or venture capital investor to invest in your firm, it may be in your best interests to offer them a share of control in the firm even though they may have well below 51%.

8. Distress and the Going Concern Assumption

- Traditional valuation techniques are built on the assumption of a going concern, i.e., a firm that has continuing operations and there is no significant threat to these operations.
  - In discounted cashflow valuation, this going concern assumption finds its place most prominently in the terminal value calculation, which usually is based upon an infinite life and ever-growing cashflows.
  - In relative valuation, this going concern assumption often shows up implicitly because a firm is valued based upon how other firms - most of which are healthy - are priced by the market today.
- When there is a significant likelihood that a firm will not survive the immediate future (next few years), traditional valuation models may yield an over-optimistic estimate of value.


Current Revenue $ 3,804
Current Margin: -49.82%
EBIT -1895m
NOL: 2,076m
Revenue Growth: 13.33%
Cap ex growth slows and net cap ex decreases
EBITDA/Sales -> 30%
Stable Growth
Stable Revenue Growth: 5%
Stable EBITDA/ Sales 30%
Stable ROC=7.36% Reinvest 67.93%
Terminal Value= 677(.0736-.05) =$ 28,683
Term. Year
Forever
Revenues $3,804 $5,326 $6,923 $8,308 $9,139 $10,053 $11,058 $11,942 $12,659 $13,292 $13,902
EBITDA ($95) $ 0 $346 $831 $1,371 $1,809 $2,322 $2,508 $3,038 $3,589 $ 4,187
EBIT ($1,675) ($1,738) ($1,565) ($1,272) $320 $1,074 $1,550 $1,697 $2,186 $2,694 $ 3,248
EBIT (1-t) ($1,675) ($1,738) ($1,565) ($1,272) $320 $1,074 $1,550 $1,697 $2,186 $2,276 $ 2,111
+ Depreciation $1,580 $1,738 $1,911 $2,102 $1,051 $736 $773 $811 $852 $894 $ 939
- Cap Ex $3,431 $1,716 $1,201 $1,261 $1,324 $1,390 $1,460 $1,533 $1,609 $1,690 $ 2,353
- Chg WC $ 0 $46 $48 $42 $25 $27 $30 $27 $21 $19 $ 20
FCFF ($3,526) ($1,761) ($903) ($472) $22 $392 $832 $949 $1,407 $1,461 $ 677
1 2 3 4 5 6 7 8 9 10
Beta 3.00 3.00 3.00 3.00 3.00 2.60 2.20 1.80 1.40 1.00
Cost of Equity 16.80% 16.80% 16.80% 16.80% 16.80% 15.20% 13.60% 12.00% 10.40% 8.80%
Cost of Debt 12.80% 12.80% 12.80% 12.80% 12.80% 11.84% 10.88% 9.92% 8.96% 6.76%
Debt Ratio 74.91% 74.91% 74.91% 74.91% 74.91% 67.93% 60.95% 53.96% 46.98% 40.00%
Cost of Capital 13.80% 13.80% 13.80% 13.80% 13.80% 12.92% 11.94% 10.88% 9.72% 7.98%
Value of Op Assets $ 5,530
+ Cash & Non-op $ 2,260
= Value of Firm $ 7,790
- Value of Debt $ 4,923
= Value of Equity $ 2867
- Equity Options $ 14
Value per share $ 3.22
Cost of Equity 16.80%
Cost of Debt 4.8%+8.0%=12.8% Tax rate = 0% -> 35%
Weights Debt= 74.91% -> 40%
Riskfree Rate: T. Bond rate = 4.8%
+
Beta 3.00> 1.10
X
Risk Premium 4%
Internet/ Retail
Operating Leverage
Current D/E: 441%
Base Equity Premium
Country Risk Premium
Global Crossing November 2001 Stock price = $1.86

Valuing Global Crossing with Distress

- Probability of distress
  - Price of 8 year, 12% bond issued by Global Crossing = $ 653

$$653 = \sum_{t=1}^{t=8} \frac{120(1-\pi_{Distress})^t}{(1.05)^t} + \frac{1000(1-\pi_{Distress})^8}{(1.05)^8}$$

  - Probability of distress = 15.53% a year
  - Cumulative probability of survival over 10 years = $(1- .1353)^{10} = 23.37\%$
- Distress sale value of equity
  - Book value of capital = $14,531 million
  - Distress sale value = 15% of book value = .15*14531 = $2,180 million
  - Book value of debt = $7,647 million
  - Distress sale value of equity = $ 0
- Distress adjusted value of equity
  - Value of Global Crossing = $3.22 (.2337) + $0.00 (.7663) = $0.75

9. Equity Value and Per Share Value

- The conventional way of getting from equity value to per share value is to divide the equity value by the number of shares outstanding. This approach assumes, however, that common stock is the only equity claim on the firm.
- In many firms, there are other equity claims as well including:
  - warrants, that are publicly traded
  - management and employee options, that have been granted, but do not trade
  - conversion options in convertible bonds
  - contingent value rights, that are also publicly traded.
- The value of these non-stock equity claims has to be subtracted from the value of equity before dividing by the number of shares outstanding.

Amazon: Estimating the Value of Equity Options

- Details of options outstanding
  - Average strike price of options outstanding = $ 13.375
  - Average maturity of options outstanding = 8.4 years
  - Standard deviation in ln(stock price) = 50.00%
  - Annualized dividend yield on stock = 0.00%
  - Treasury bond rate = 6.50%
  - Number of options outstanding = 38 million
  - Number of shares outstanding = 340.79 million
- Value of options outstanding (using dilution-adjusted Black-Scholes model)
  - Value of equity options = $ 2,892 million

10. Analyzing the Effect of Illiquidity on Value

- Investments which are less liquid should trade for less than otherwise similar investments which are more liquid.
- The size of the illiquidity discount should depend upon
  - *Type of Assets owned by the Firm*: The more liquid the assets owned by the firm, the lower should be the liquidity discount for the firm
  - *Size of the Firm*: The larger the firm, the smaller should be size of the liquidity discount.
  - *Health of the Firm*: Stock in healthier firms should sell for a smaller discount than stock in troubled firms.
  - *Cash Flow Generating Capacity*: Securities in firms which are generating large amounts of cash from operations should sell for a smaller discounts than securities in firms which do not generate large cash flows.
  - *Size of the Block*: The liquidity discount should increase with the size of the portion of the firm being sold.

Illiquidity Discount: Restricted Stock Studies

- Restricted securities are securities issued by a company, but not registered with the SEC, that can be sold through private placements to investors, but cannot be resold in the open market for a two-year holding period, and limited amounts can be sold after that. Studies of restricted stock over time have concluded that the discount is between 25 and 35%. Many practitioners use this as the illiquidity discount for all private firms.
- A more nuanced used of restricted stock studies is to relate the discount to fundamental characteristics of the company - level of revenues, health of the company etc.. And to adjust the discount for any firm to reflect its characteristics:
  - The discount will be smaller for larger firms
  - The discount will be smaller for healthier firms

Illiquidity Discounts from Bid-Ask Spreads

- Using data from the end of 2000, for instance, we regressed the bid-ask spread against annual revenues, a dummy variable for positive earnings (DERN: 0 if negative and 1 if positive), cash as a percent of firm value and trading volume.

Spread = 0.145 – 0.0022 ln (Annual Revenues) -0.015 (DERN) – 0.016 (Cash/Firm Value) – 0.11 ($ Monthly trading volume/ Firm Value)

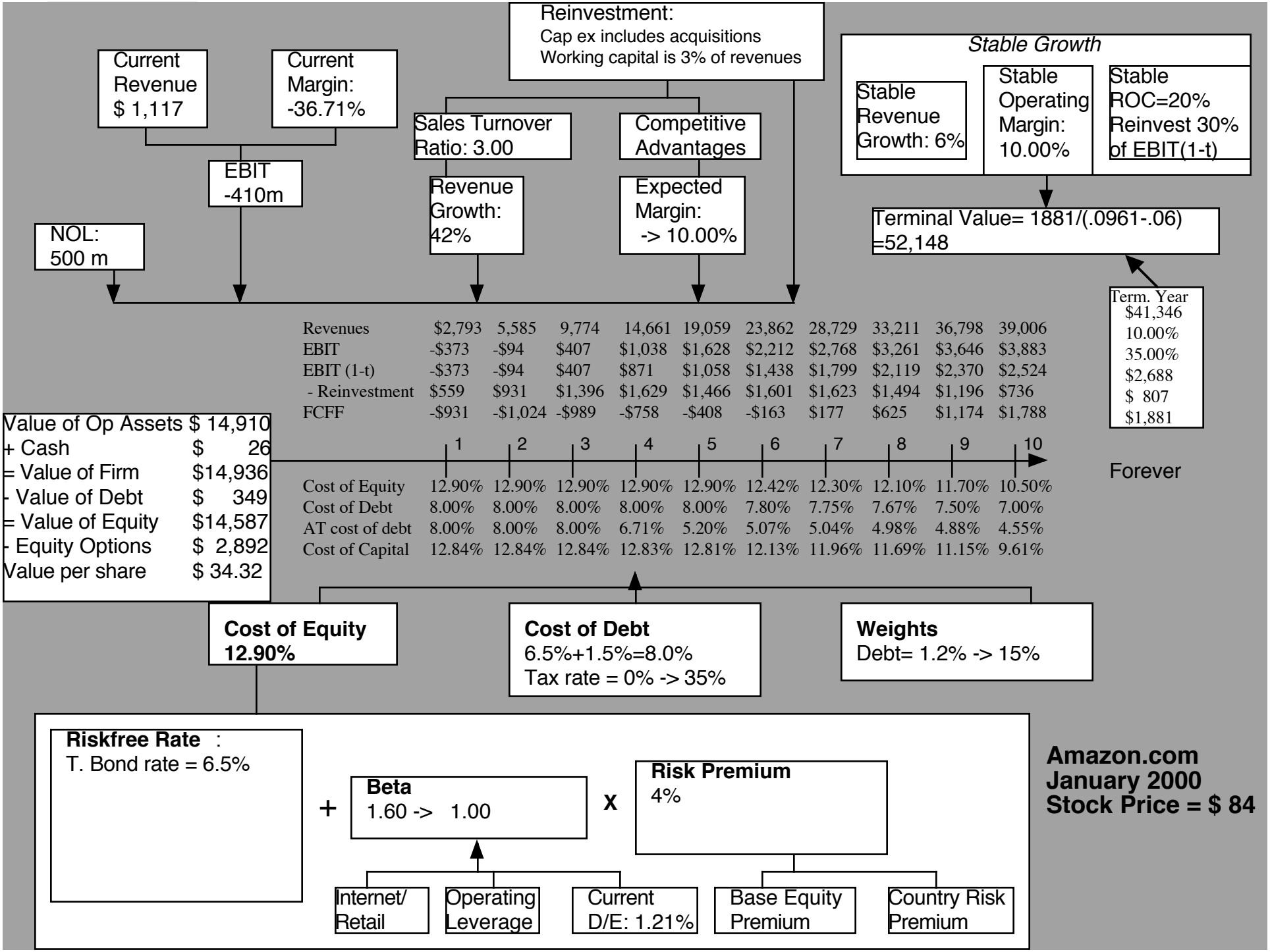
- We could substitute in the revenues of Kristin Kandy ($5 million), the fact that it has positive earnings and the cash as a percent of revenues held by the firm (8%):

Spread = 0.145 – 0.0022 ln (Annual Revenues) -0.015 (DERN) – 0.016 (Cash/Firm Value) – 0.11 ($ Monthly trading volume/ Firm Value)

= 0.145 – 0.0022 ln (5) -0.015 (1) – 0.016 (.08) – 0.11 (0) = .12.52%

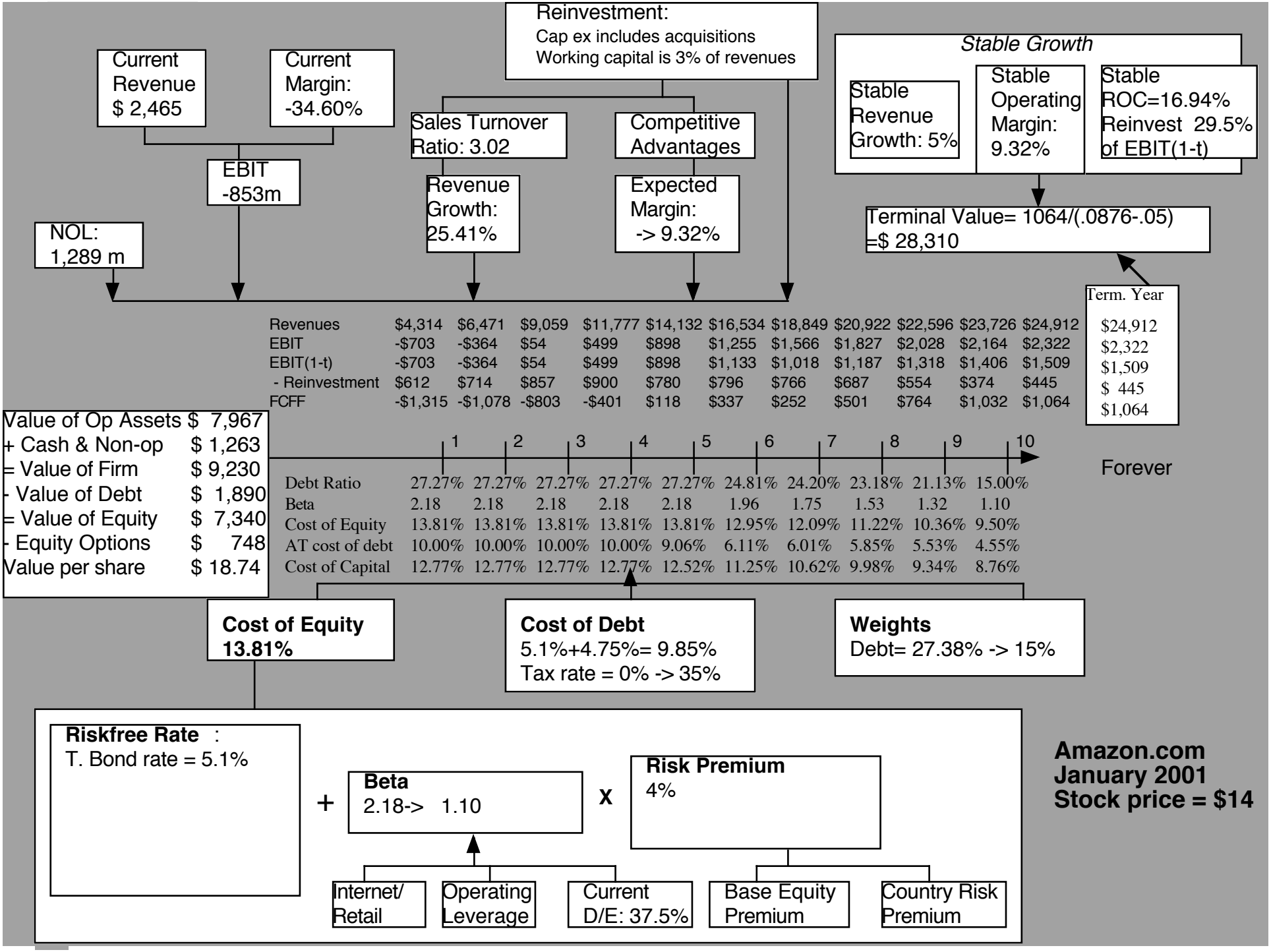
- Based on this approach, we would estimate an illiquidity discount of 12.52% for Kristin Kandy.

V. Value, Price and Information: Closing the Deal

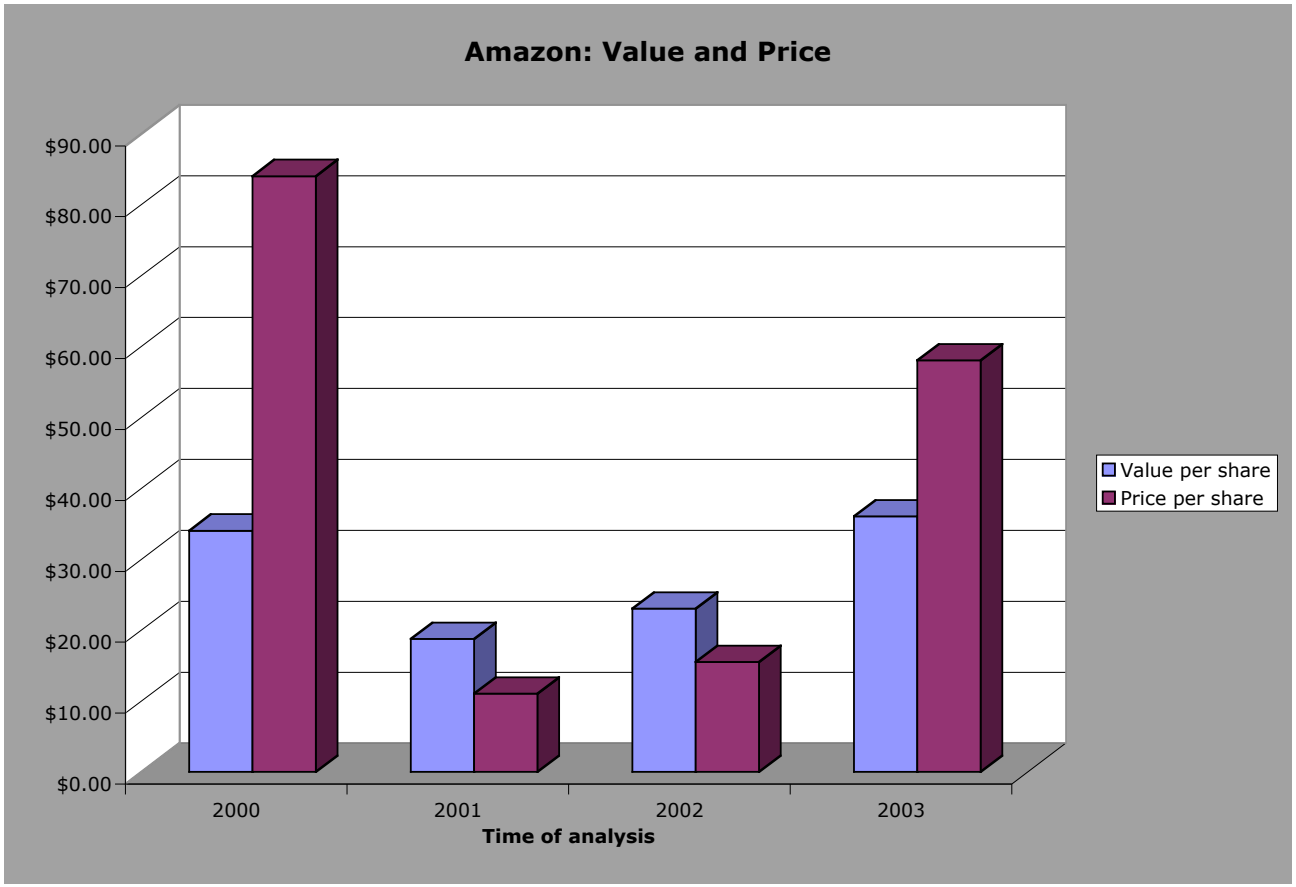

Current Revenue $ 1,117
Current Margin: -36.71%
EBIT -410m
NOL: 500 m
Reinvestment:
Cap ex includes acquisitions
Working capital is 3% of revenues
Sales Turnover Ratio: 3.00
Competitive Advantages
Revenue Growth: 42%
Expected Margin: -> 10.00%
Stable Growth
Stable Revenue Growth: 6%
Stable Operating Margin: 10.00%
Stable ROC=20% Reinvest 30% of EBIT(1-t)
Terminal Value= 1881/(.0961-.06) =52,148
Term. Year
$41,346
10.00%
35.00%
$2,688
$ 807
$1,881
Revenues $2,793 5,585 9,774 14,661 19,059 23,862 28,729 33,211 36,798 39,006
EBIT -$373 -$94 $407 $1,038 $1,628 $2,212 $2,768 $3,261 $3,646 $3,883
EBIT (1-t) -$373 -$94 $407 $871 $1,058 $1,438 $1,799 $2,119 $2,370 $2,524
- Reinvestment $559 $931 $1,396 $1,629 $1,466 $1,601 $1,623 $1,494 $1,196 $736
FCFF -$931 -$1,024 -$989 -$758 -$408 -$163 $177 $625 $1,174 $1,788
1 2 3 4 5 6 7 8 9 10
Forever
Cost of Equity 12.90% 12.90% 12.90% 12.90% 12.90% 12.42% 12.30% 12.10% 11.70% 10.50%
Cost of Debt 8.00% 8.00% 8.00% 8.00% 8.00% 7.80% 7.75% 7.67% 7.50% 7.00%
AT cost of debt 8.00% 8.00% 8.00% 6.71% 5.20% 5.07% 5.04% 4.98% 4.88% 4.55%
Cost of Capital 12.84% 12.84% 12.84% 12.83% 12.81% 12.13% 11.96% 11.69% 11.15% 9.61%
Value of Op Assets $ 14,910
+ Cash $ 26
= Value of Firm $14,936
- Value of Debt $ 349
= Value of Equity $14,587
- Equity Options $ 2,892
Value per share $ 34.32
Cost of Equity 12.90%
Cost of Debt 6.5%+1.5%=8.0% Tax rate = 0% -> 35%
Weights Debt= 1.2% -> 15%
Riskfree Rate : T. Bond rate = 6.5%
+
Beta 1.60 -> 1.00
x
Risk Premium 4%
Internet/ Retail
Operating Leverage
Current D/E: 1.21%
Base Equity Premium
Country Risk Premium
Amazon.com
January 2000
Stock Price = $ 84

Amazon.com: Break Even at $84?

	6%	8%	10%	12%	14%
30%	$ (1.94)	$ 2.95	$ 7.84	$ 12.71	$ 17.57
35%	$ 1.41	$ 8.37	$ 15.33	$ 22.27	$ 29.21
40%	$ 6.10	$ 15.93	$ 25.74	$ 35.54	$ 45.34
45%	$ 12.59	$ 26.34	$ 40.05	$ 53.77	$ 67.48
50%	$ 21.47	$ 40.50	$ 59.52	$ 78.53	$ 97.54
55%	$ 33.47	$ 59.60	$ 85.72	$ 111.84	$ 137.95
60%	$ 49.53	$ 85.10	$ 120.66	$ 156.22	$ 191.77


Reinvestment:
Cap ex includes acquisitions
Working capital is 3% of revenues
Current Revenue $ 2,465
Current Margin: -34.60%
EBIT -853m
NOL: 1,289 m
Sales Turnover Ratio: 3.02
Revenue Growth: 25.41%
Competitive Advantages
Expected Margin: -> 9.32%
Stable Growth
Stable Revenue Growth: 5%
Stable Operating Margin: 9.32%
Stable ROC=16.94% Reinvest 29.5% of EBIT(1-t)
Terminal Value= 1064/(.0876-.05) =$ 28,310
Term. Year
$24,912
$2,322
$1,509
$ 445
$1,064
Revenues $4,314 $6,471 $9,059 $11,777 $14,132 $16,534 $18,849 $20,922 $22,596 $23,726 $24,912
EBIT -$703 -$364 $54 $499 $898 $1,255 $1,566 $1,827 $2,028 $2,164 $2,322
EBIT(1-t) -$703 -$364 $54 $499 $898 $1,133 $1,018 $1,187 $1,318 $1,406 $1,509
- Reinvestment $612 $714 $857 $900 $780 $796 $766 $687 $554 $374 $445
FCFF -$1,315 -$1,078 -$803 -$401 $118 $337 $252 $501 $764 $1,032 $1,064
1 2 3 4 5 6 7 8 9 10
Forever
Value of Op Assets $ 7,967
+ Cash & Non-op $ 1,263
= Value of Firm $ 9,230
- Value of Debt $ 1,890
= Value of Equity $ 7,340
- Equity Options $ 748
Value per share $ 18.74
Debt Ratio 27.27% 27.27% 27.27% 27.27% 27.27% 24.81% 24.20% 23.18% 21.13% 15.00%
Beta 2.18 2.18 2.18 2.18 2.18 1.96 1.75 1.53 1.32 1.10
Cost of Equity 13.81% 13.81% 13.81% 13.81% 13.81% 12.95% 12.09% 11.22% 10.36% 9.50%
AT cost of debt 10.00% 10.00% 10.00% 10.00% 9.06% 6.11% 6.01% 5.85% 5.53% 4.55%
Cost of Capital 12.77% 12.77% 12.77% 12.77% 12.52% 11.25% 10.62% 9.98% 9.34% 8.76%
Cost of Equity 13.81%
Cost of Debt 5.1%+4.75%= 9.85% Tax rate = 0% -> 35%
Weights Debt= 27.38% -> 15%
Riskfree Rate : T. Bond rate = 5.1%
+
Beta 2.18-> 1.10
X
Risk Premium 4%
Internet/ Retail
Operating Leverage
Current D/E: 37.5%
Base Equity Premium
Country Risk Premium
Amazon.com
January 2001
Stock price = $14

Amazon over time…



Relative Valuation

Aswath Damodaran

What is relative valuation?

- In relative valuation, the value of an asset is compared to the values assessed by the market for similar or comparable assets.
- To do relative valuation then,
  - we need to identify comparable assets and obtain market values for these assets
  - convert these market values into standardized values, since the absolute prices cannot be compared This process of standardizing creates price multiples.
  - compare the standardized value or multiple for the asset being analyzed to the standardized values for comparable asset, controlling for any differences between the firms that might affect the multiple, to judge whether the asset is under or over valued

Relative valuation is pervasive…

- Most valuations on Wall Street are relative valuations.
  - Almost 85% of equity research reports are based upon a multiple and comparables.
  - More than 50% of all acquisition valuations are based upon multiples
  - Rules of thumb based on multiples are not only common but are often the basis for final valuation judgments.
- While there are more discounted cashflow valuations in consulting and corporate finance, they are often relative valuations masquerading as discounted cash flow valuations.
  - The objective in many discounted cashflow valuations is to back into a number that has been obtained by using a multiple.
  - The terminal value in a significant number of discounted cashflow valuations is estimated using a multiple.

Why relative valuation?

"If you think I'm crazy, you should see the guy who lives across the hall"

Jerry Seinfeld talking about Kramer in a Seinfeld episode

" A little inaccuracy sometimes saves tons of explanation"

H.H. Munro

" If you are going to screw up, make sure that you have lots of company"

Ex-portfolio manager

So, you believe only in intrinsic value? Here is why you should still care about relative value

- Even if you are a true believer in discounted cashflow valuation, presenting your findings on a relative valuation basis will make it more likely that your findings/recommendations will reach a receptive audience.
- In some cases, relative valuation can help find weak spots in discounted cash flow valuations and fix them.
- The problem with multiples is not in their use but in their abuse. If we can find ways to frame multiples right, we should be able to use them better.

Standardizing Value

- You can standardize either the equity value of an asset or the value of the asset itself, which goes in the numerator.
- You can standardize by dividing by the
  - Earnings of the asset
    - Price/Earnings Ratio (PE) and variants (PEG and Relative PE)
    - Value/EBIT
    - Value/EBITDA
    - Value/Cash Flow
  - Book value of the asset
    - Price/Book Value(of Equity) (PBV)
    - Value/ Book Value of Assets
    - Value/Replacement Cost (Tobin's Q)
  - Revenues generated by the asset
    - Price/Sales per Share (PS)
    - Value/Sales
  - Asset or Industry Specific Variable (Price/kwh, Price per ton of steel ....)

The Four Steps to Understanding Multiples

- Define the multiple
  - In use, the same multiple can be defined in different ways by different users. When comparing and using multiples, estimated by someone else, it is critical that we understand how the multiples have been estimated
- Describe the multiple
  - Too many people who use a multiple have no idea what its cross sectional distribution is. If you do not know what the cross sectional distribution of a multiple is, it is difficult to look at a number and pass judgment on whether it is too high or low.
- Analyze the multiple
  - It is critical that we understand the fundamentals that drive each multiple, and the nature of the relationship between the multiple and each variable.
- Apply the multiple
  - Defining the comparable universe and controlling for differences is far more difficult in practice than it is in theory.

Definitional Tests

- Is the multiple consistently defined?
  - **Proposition 1: Both the value (the numerator) and the standardizing variable (the denominator) should be to the same claimholders in the firm. In other words, the value of equity should be divided by equity earnings or equity book value, and firm value should be divided by firm earnings or book value.**
- Is the multiple uniformly estimated?
  - The variables used in defining the multiple should be estimated uniformly across assets in the "comparable firm" list.
  - If earnings-based multiples are used, the accounting rules to measure earnings should be applied consistently across assets. The same rule applies with book-value based multiples.

Descriptive Tests

- What is the average and standard deviation for this multiple, across the universe (market)?
- What is the median for this multiple?
  - The median for this multiple is often a more reliable comparison point.
- How large are the outliers to the distribution, and how do we deal with the outliers?
  - Throwing out the outliers may seem like an obvious solution, but if the outliers all lie on one side of the distribution (they usually are large positive numbers), this can lead to a biased estimate.
- Are there cases where the multiple cannot be estimated? Will ignoring these cases lead to a biased estimate of the multiple?
- How has this multiple changed over time?

Analytical Tests

- What are the fundamentals that determine and drive these multiples?
  - Proposition 2: Embedded in every multiple are all of the variables that drive every discounted cash flow valuation - growth, risk and cash flow patterns.
  - In fact, using a simple discounted cash flow model and basic algebra should yield the fundamentals that drive a multiple
- How do changes in these fundamentals change the multiple?
  - The relationship between a fundamental (like growth) and a multiple (such as PE) is seldom linear. For example, if firm A has twice the growth rate of firm B, it will generally not trade at twice its PE ratio
  - **Proposition 3: It is impossible to properly compare firms on a multiple, if we do not know the nature of the relationship between fundamentals and the multiple.**

Application Tests

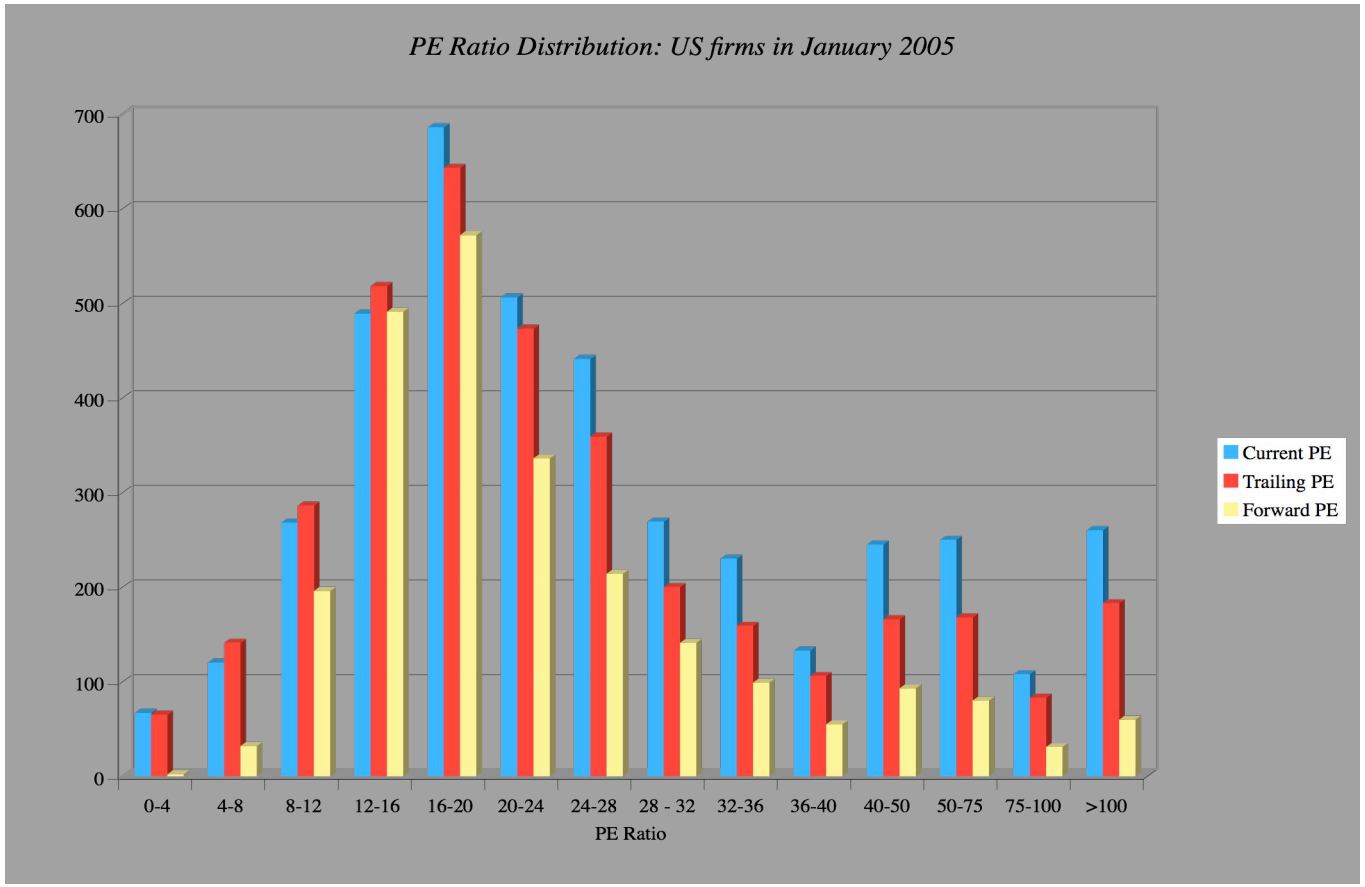
- Given the firm that we are valuing, what is a "comparable" firm?
  - While traditional analysis is built on the premise that firms in the same sector are comparable firms, valuation theory would suggest that a comparable firm is one which is similar to the one being analyzed in terms of fundamentals.
  - **Proposition 4: There is no reason why a firm cannot be compared with another firm in a very different business, if the two firms have the same risk, growth and cash flow characteristics.**
- Given the comparable firms, how do we adjust for differences across firms on the fundamentals?
  - **Proposition 5: It is impossible to find an exactly identical firm to the one you are valuing.**

Price Earnings Ratio: Definition

PE = Market Price per Share / Earnings per Share

- There are a number of variants on the basic PE ratio in use. They are based upon how the price and the earnings are defined.
- Price: is usually the current price
  is sometimes the average price for the year
- EPS: earnings per share in most recent financial year
  earnings per share in trailing 12 months (Trailing PE)
  forecasted earnings per share next year (Forward PE)
  forecasted earnings per share in future year

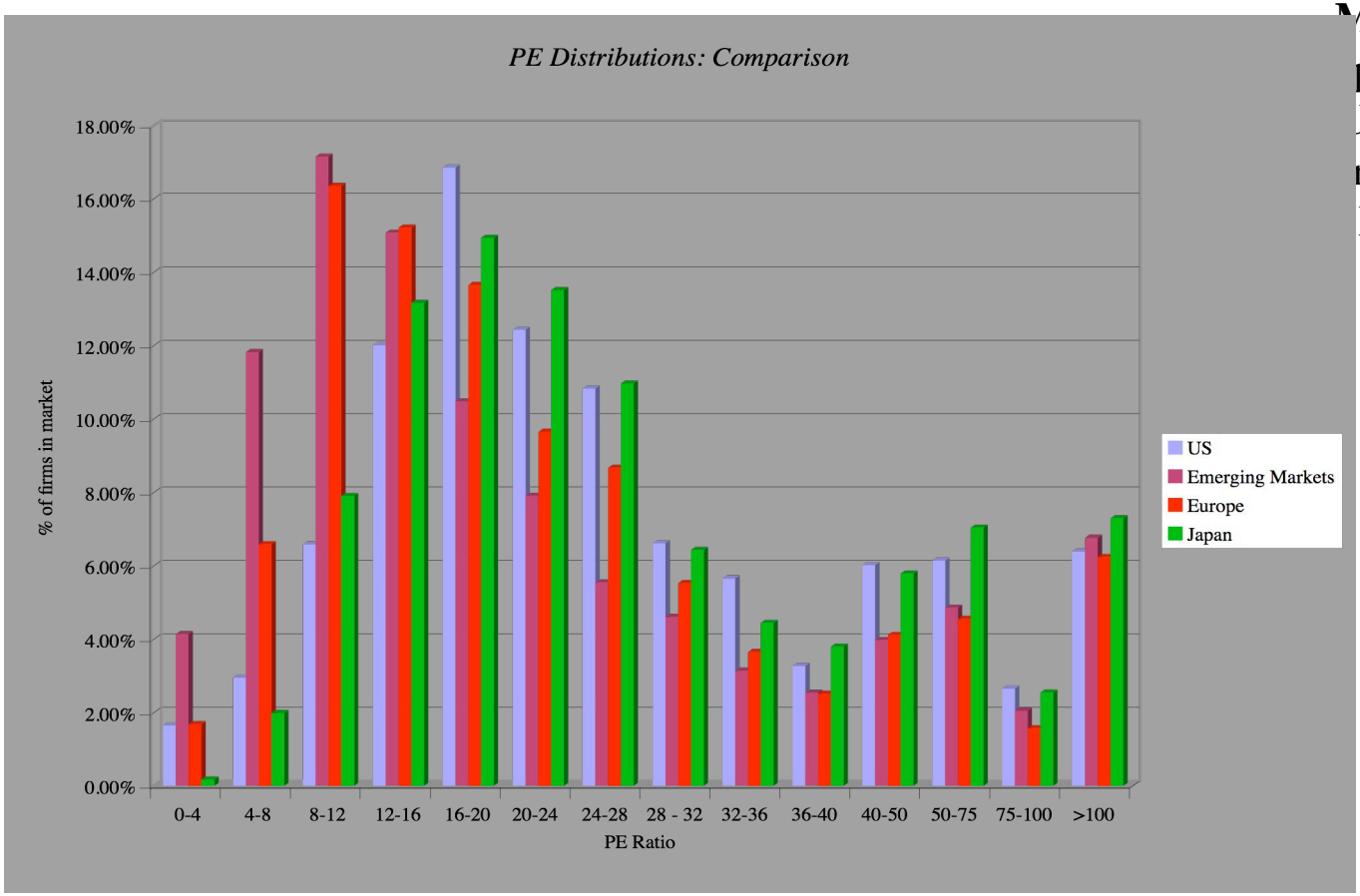
Looking at the distribution…



PE: Deciphering the Distribution

	Current PE	*Trailing PE*	*Forward PE*
Mean	48.12	42.86	28.53
Standard Error	3.69	3.39	0.98
Median	23.21	20.65	19.21
Kurtosis	1214.98	1428.36	157.28
Skewness	31.75	32.86	10.85
Minimum	1.15	1.31	1.40
Maximum	10081.26	9713	1017.00
Number of firms	4072	3637	2402.00
Largest(500)	58.90	44.72	29.31
Smallest(500)	12.65	11.11	14.54

Comparing PE Ratios: US, Europe, Japan and Emerging Markets



Median PE

pan = 23.45

JS = 23.21

rope = 18.79

Mkts = 16.18

PE Ratio: Understanding the Fundamentals

- To understand the fundamentals, start with a basic equity discounted cash flow model.
- With the dividend discount model,

$$P_0 = \frac{DPS_1}{r - g_n}$$

- Dividing both sides by the earnings per share,

$$\frac{P_0}{EPS_0} = PE = \frac{\text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

- If this had been a FCFE Model,

$$P_0 = \frac{FCFE_1}{r - g_n}$$

$$\frac{P_0}{EPS_0} = PE = \frac{(FCFE/Earnings) * (1 + g_n)}{r - g_n}$$

PE Ratio and Fundamentals

- **Proposition: Other things held equal, higher growth firms will have higher PE ratios than lower growth firms.**
- **Proposition: Other things held equal, higher risk firms will have lower PE ratios than lower risk firms**
- **Proposition: Other things held equal, firms with lower reinvestment needs will have higher PE ratios than firms with higher reinvestment rates.**
- Of course, other things are difficult to hold equal since high growth firms, tend to have risk and high reinvestment rats.

Using the Fundamental Model to Estimate PE For a High Growth Firm

- The price-earnings ratio for a high growth firm can also be related to fundamentals. In the special case of the two-stage dividend discount model, this relationship can be made explicit fairly simply:

$$P_0 = \frac{EPS_0 * \text{Payout Ratio} * (1+g) * \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)}{r - g} + \frac{EPS_0 * \text{Payout Ratio}_n * (1+g)^n * (1+g_n)}{(r - g_n)(1+r)^n}$$

  - For a firm that does not pay what it can afford to in dividends, substitute FCFE/Earnings for the payout ratio.

- Dividing both sides by the earnings per share:

$$\frac{P_0}{EPS_0} = \frac{\text{Payout Ratio} * (1+g) * \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)}{r - g} + \frac{\text{Payout Ratio}_n * (1+g)^n * (1+g_n)}{(r - g_n)(1+r)^n}$$

Expanding the Model

- In this model, the PE ratio for a high growth firm is a function of growth, risk and payout, exactly the same variables that it was a function of for the stable growth firm.
- The only difference is that these inputs have to be estimated for two phases - the high growth phase and the stable growth phase.
- Expanding to more than two phases, say the three stage model, will mean that risk, growth and cash flow patterns in each stage.

A Simple Example

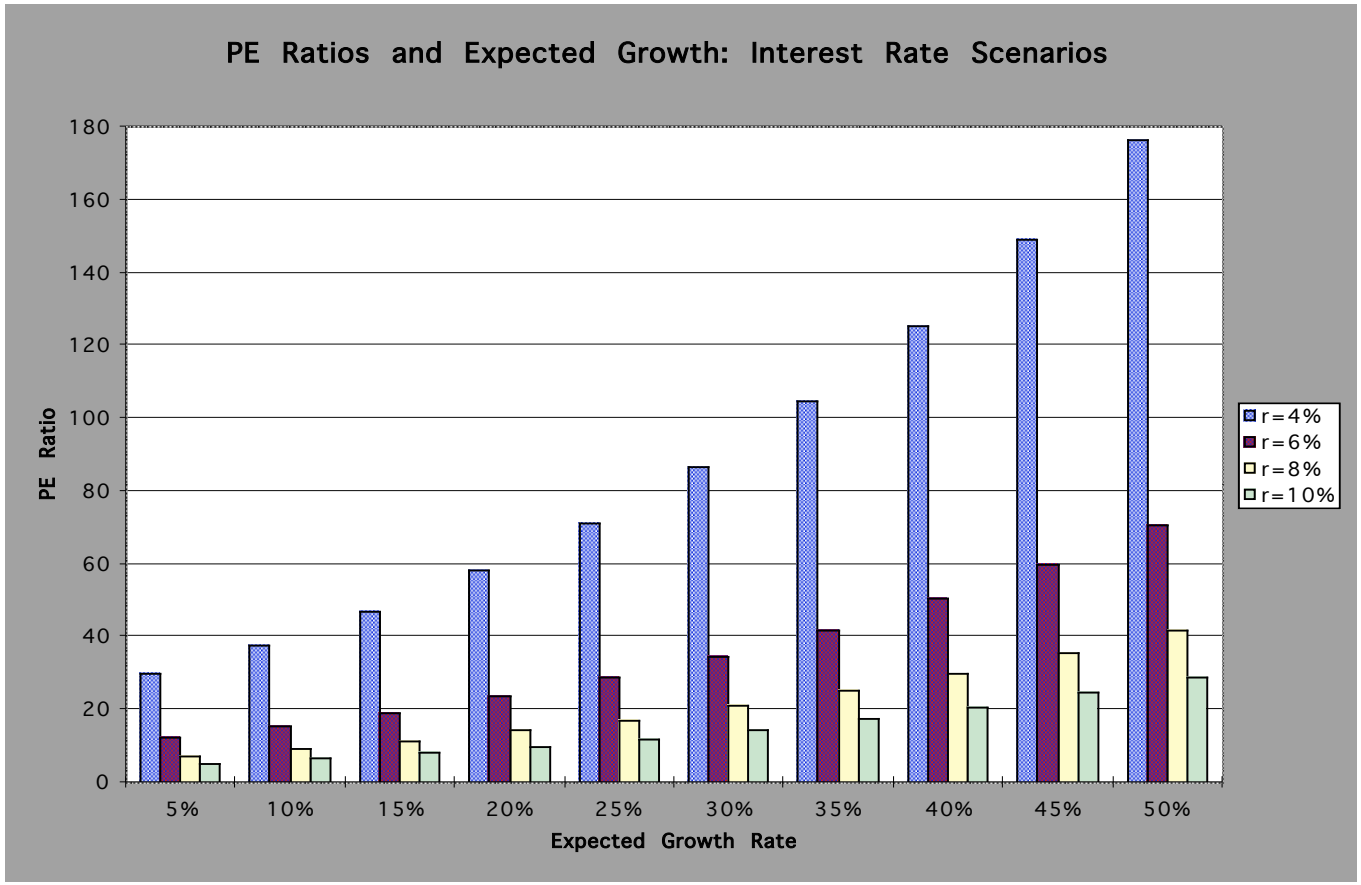
- Assume that you have been asked to estimate the PE ratio for a firm which has the following characteristics:

Variable	High Growth Phase	Stable Growth Phase
Expected Growth Rate	25%	8%
Payout Ratio	20%	50%
Beta	1.00	1.00
Number of years	5 years	Forever after year 5

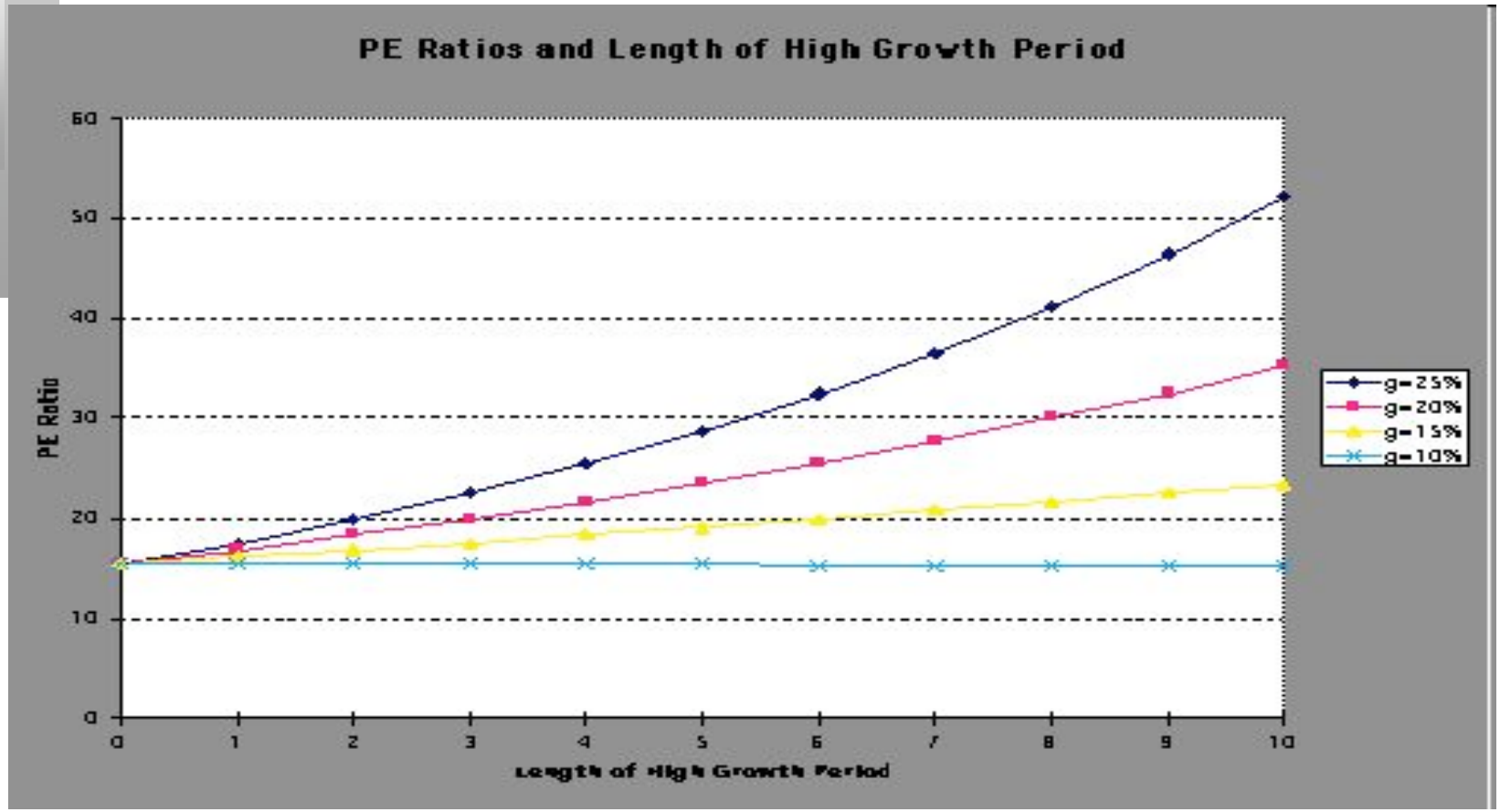
- Riskfree rate = T.Bond Rate = 6%
- Required rate of return = 6% + 1(5.5%)= 11.5%

$$PE = \frac{0.2 * (1.25) * \left(1 - \frac{(1.25)^5}{(1.115)^5}\right)}{(.115 - .25)} + \frac{0.5 * (1.25)^5 * (1.08)}{(.115 - .08)(1.115)^5} = 28.75$$

PE and Growth: Firm grows at x% for 5 years, 8% thereafter

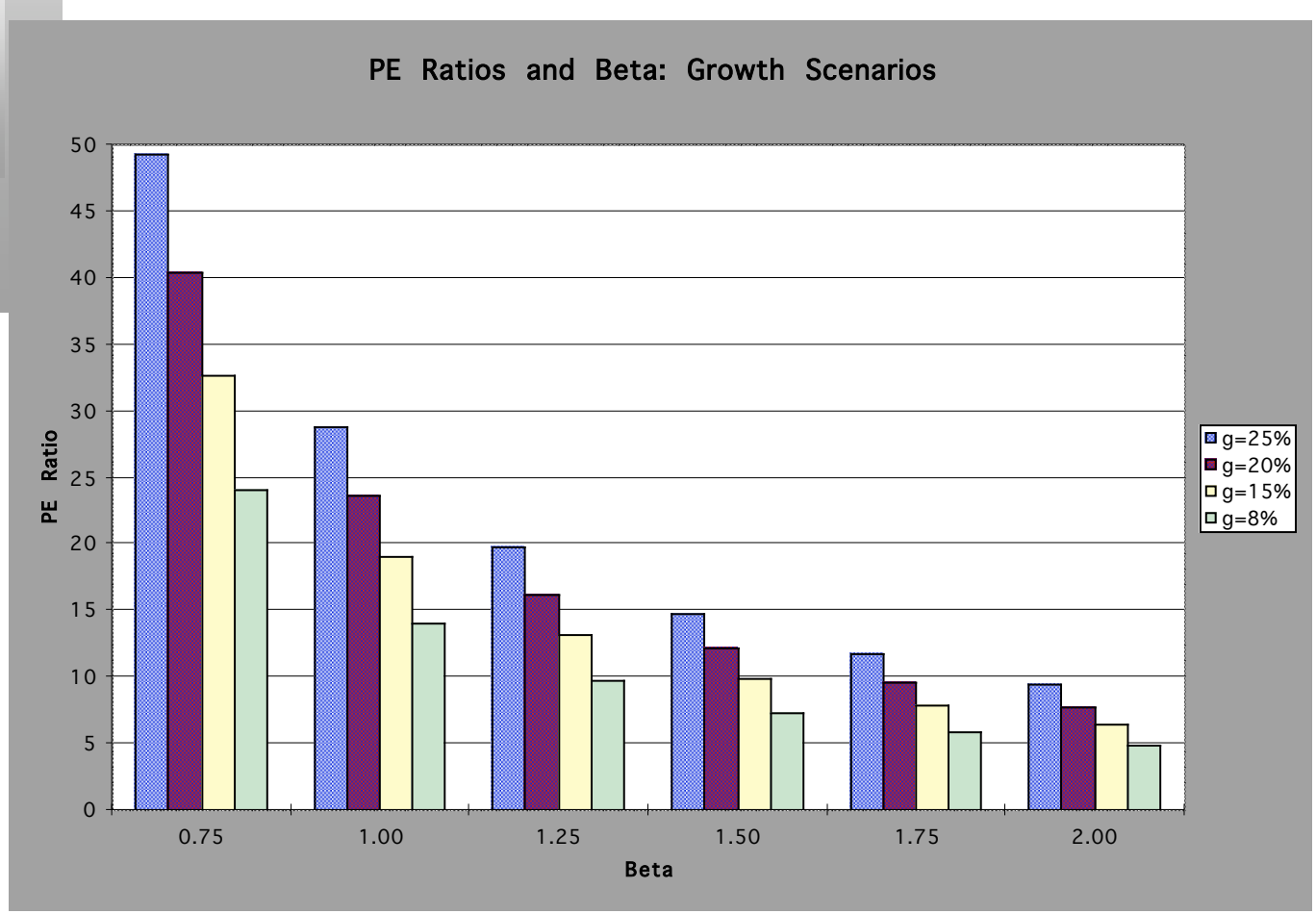


PE Ratios and Length of High Growth: 25% growth for n years; 8% thereafter

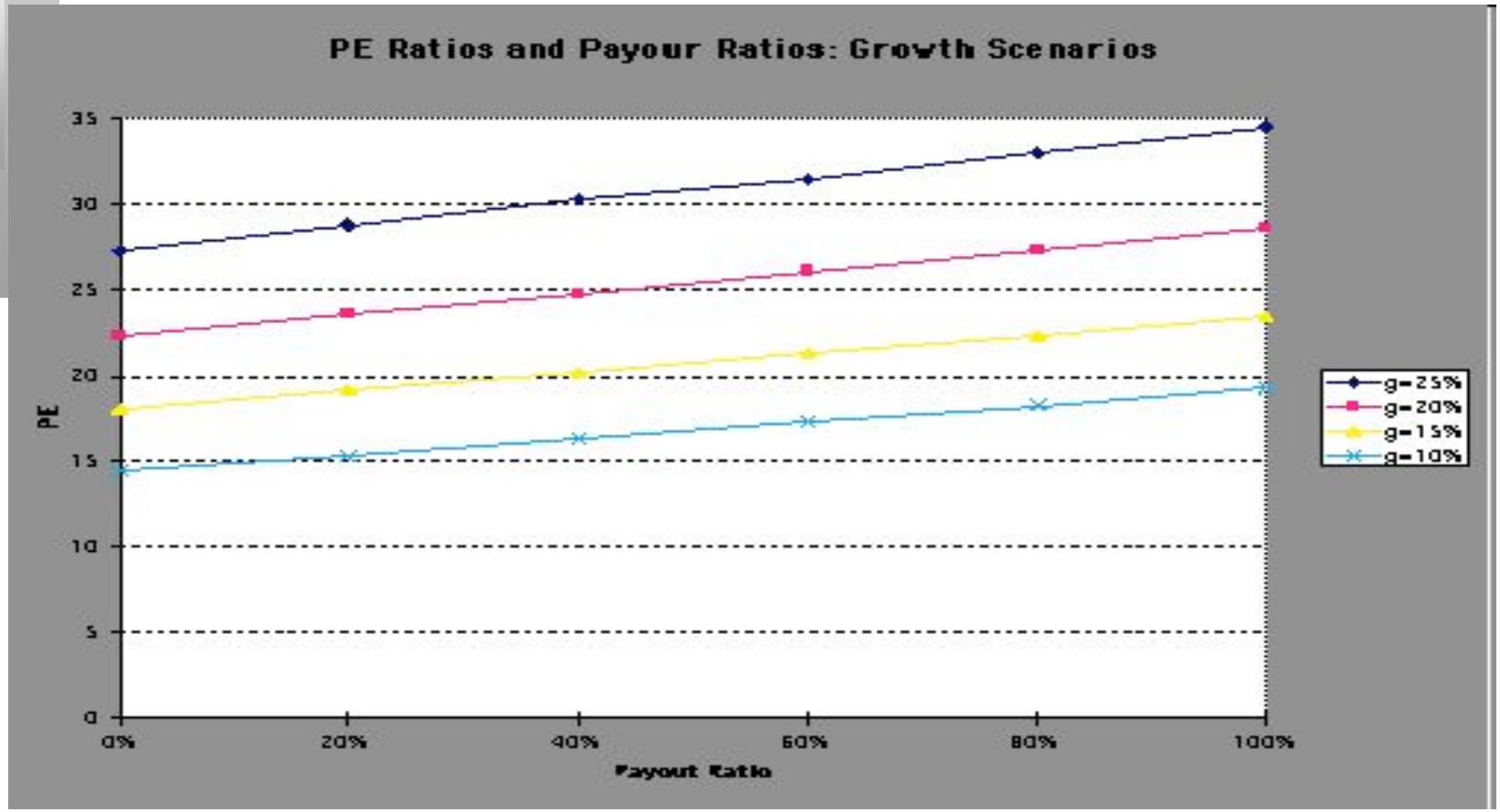


PE and Risk: Effects of Changing Betas on PE Ratio:

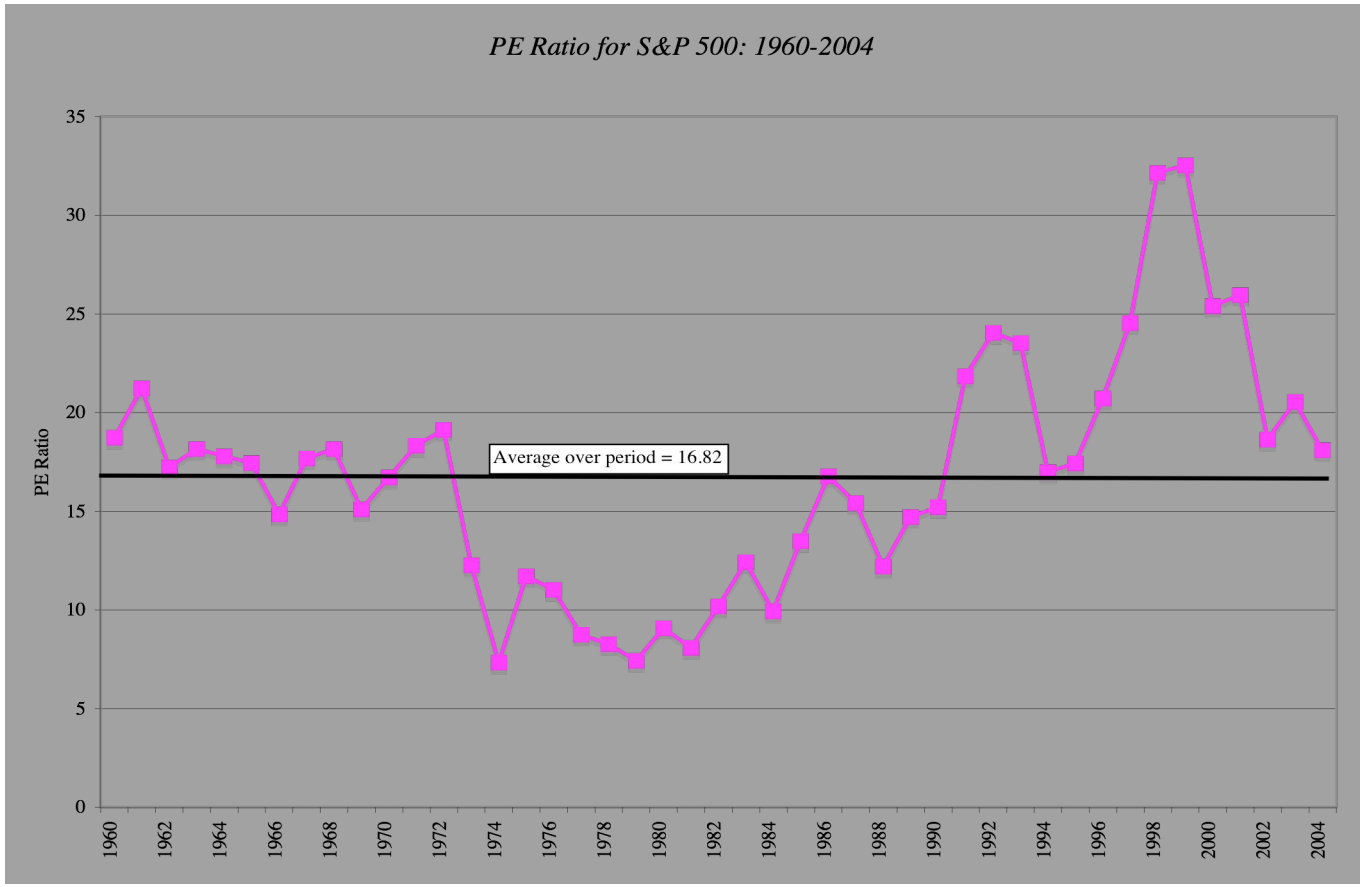
Firm with x% growth for 5 years; 8% thereafter



PE and Payout



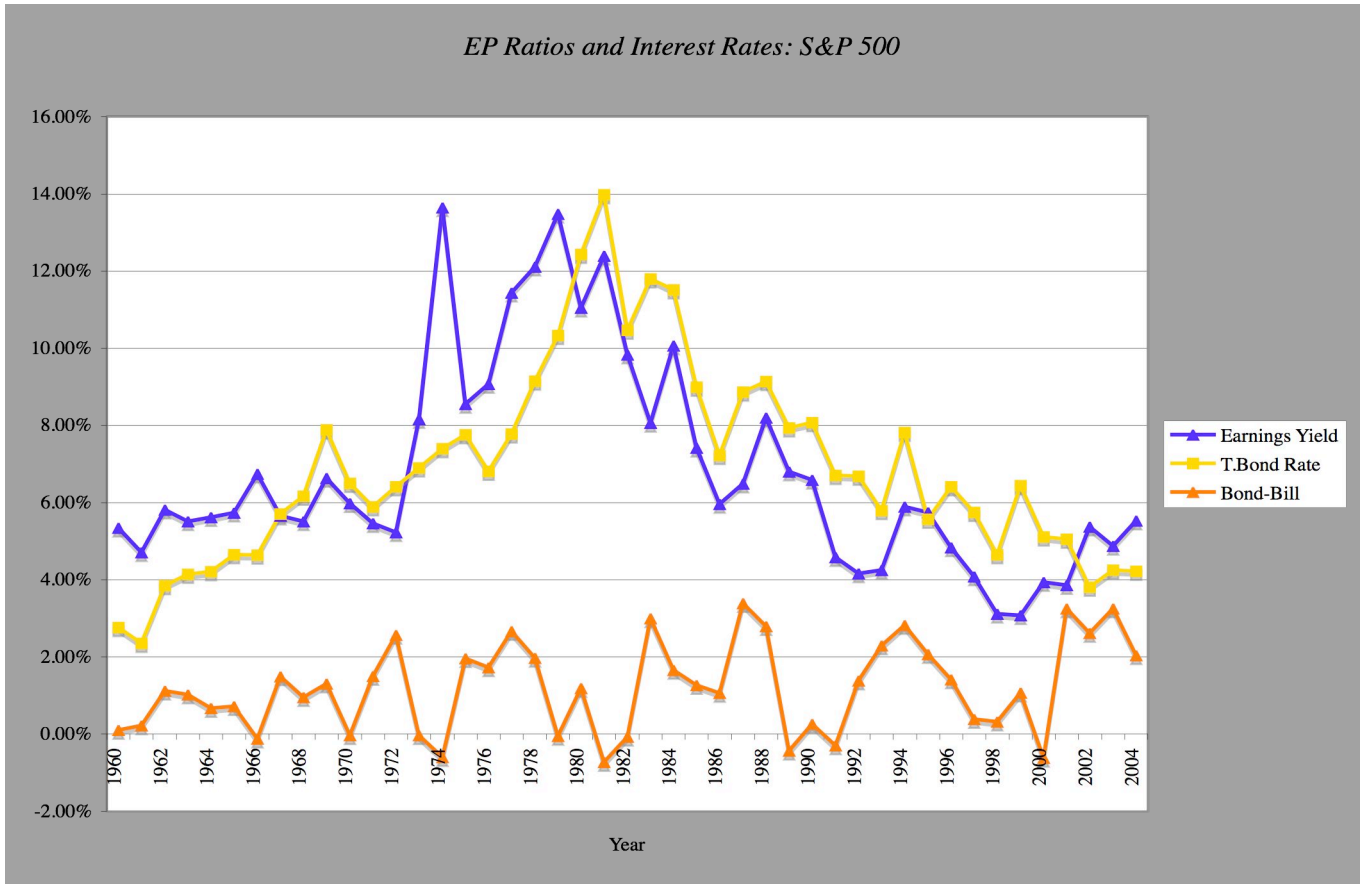
IV. Comparisons of PE across time: PE Ratio for the S&P 500



Is low (high) PE cheap (expensive)?

- A market strategist argues that stocks are over priced because the PE ratio today is too high relative to the average PE ratio across time. Do you agree?
  - Yes
  - No
- If you do not agree, what factors might explain the higher PE ratio today?

E/P Ratios , T.Bond Rates and Term Structure



Regression Results

- There is a strong positive relationship between E/P ratios and T.Bond rates, as evidenced by the correlation of 0.70 between the two variables.,
- In addition, there is evidence that the term structure also affects the PE ratio.
- In the following regression, using 1960-2004 data, we regress E/P ratios against the level of T.Bond rates and a term structure variable (T.Bond - T.Bill rate)

E/P = 2.07% + 0.746 T.Bond Rate - 0.323 (T.Bond Rate-T.Bill Rate)
(2.31) (6.51) (-1.28)

R squared = 51.11%

II. Comparing PE Ratios across a Sector

Company Name	PE	Growth
PT Indosat ADR	7.8	0.06
Telebras ADR	8.9	0.075
Telecom Corporation of New Zealand ADR	11.2	0.11
Telecom Argentina Stet - France Telecom SA ADR B	12.5	0.08
Hellenic Telecommunication Organization SA ADR	12.8	0.12
Telecomunicaciones de Chile ADR	16.6	0.08
Swisscom AG ADR	18.3	0.11
Asia Satellite Telecom Holdings ADR	19.6	0.16
Portugal Telecom SA ADR	20.8	0.13
Telefonos de Mexico ADR L	21.1	0.14
Matav RT ADR	21.5	0.22
Telstra ADR	21.7	0.12
Gilat Communications	22.7	0.31
Deutsche Telekom AG ADR	24.6	0.11
British Telecommunications PLC ADR	25.7	0.07
Tele Danmark AS ADR	27	0.09
Telekomunikasi Indonesia ADR	28.4	0.32
Cable & Wireless PLC ADR	29.8	0.14
APT Satellite Holdings ADR	31	0.33
Telefonica SA ADR	32.5	0.18
Royal KPN NV ADR	35.7	0.13
Telecom Italia SPA ADR	42.2	0.14
Nippon Telegraph & Telephone ADR	44.3	0.2
France Telecom SA ADR	45.2	0.19
Korea Telecom ADR	71.3	0.44

PE, Growth and Risk

Dependent variable is: PE

R squared = 66.2% R squared (adjusted) = 63.1%

Variable	Coefficient	SE	t-ratio	prob
Constant	13.1151	3.471	3.78	0.0010
Growth rate	121.223	19.27	6.29	≤ 0.0001
Emerging Market	-13.8531	3.606	-3.84	0.0009

Emerging Market is a dummy: 1 if emerging market
0 if not

Is Telebras under valued?

- Predicted PE = 13.12 + 121.22 (.075) - 13.85 (1) = 8.35
- At an actual price to earnings ratio of 8.9, Telebras is slightly overvalued.

- What about Hellenic Telecom?
  - If viewed as a developed market telecom

  13.12 + 121.22 (0.12) -13.85 (0) = 27.66

  It is dramatically undervalued at 12.8 times earnings
  - If viewed as an emerging market telecom

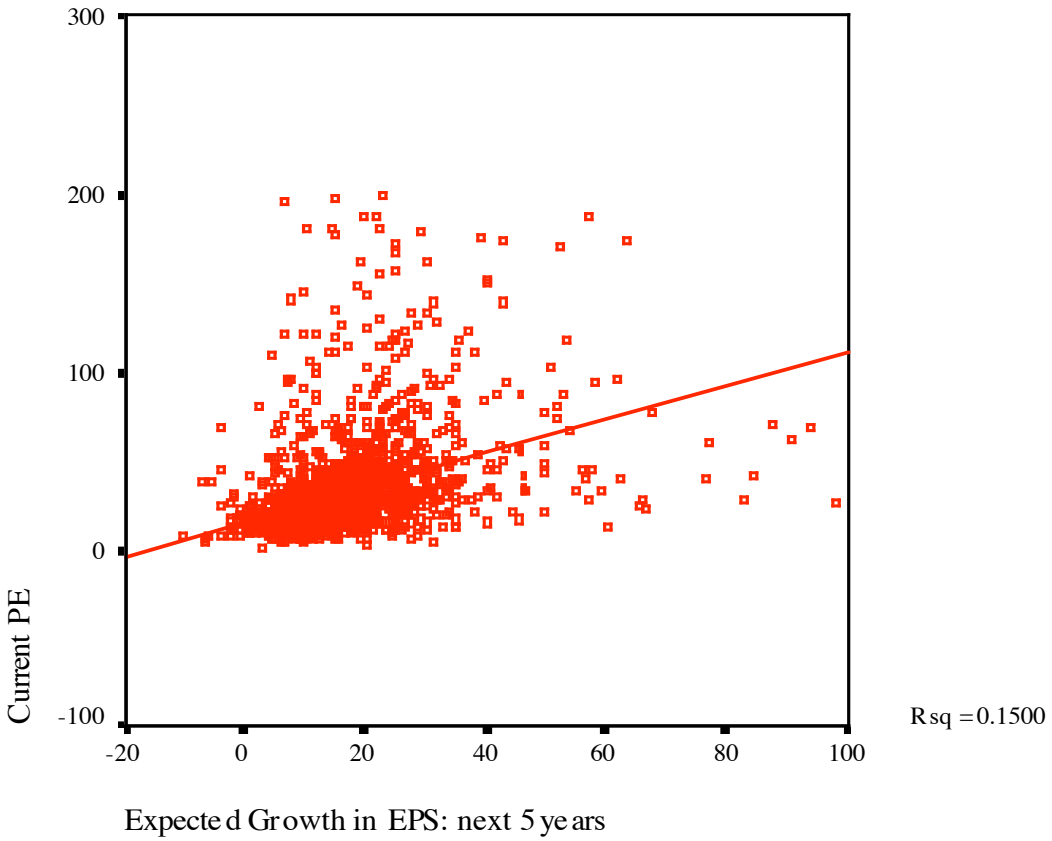
  13.12 + 121.22 (0.12) -13.85 (1) = 13.81

  It is close to fairly valued

Using the entire crosssection: A regression approach

- In contrast to the 'comparable firm' approach, the information in the entire cross-section of firms can be used to predict PE ratios.
- The simplest way of summarizing this information is with a multiple regression, with the PE ratio as the dependent variable, and proxies for risk, growth and payout forming the independent variables.

PE versus Growth



PE Ratio: Standard Regression for US stocks - January 2005

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.487[a]	.238	.236	1498.825 106505 786 000

a. Predictors: (Constant), Payout Ratio, 3-yr Regression Beta, Expected Growth in EPS: next 5 years

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	14.781	.979		15.099	.000	12.861	16.701
	Expected Growth in EPS: next 5 years	.914	.040	.470	23.117	.000	.837	.992
	3-yr Regression Beta	.220	.641	.007	.343	.732	-1.038	1.477
	Payout Ratio	-4.892 E-02	.015	-.062	-3.165	.002	-.079	-.019

a. Dependent Variable: Current PE

b. Weighted Least Squares Regression - Weighted by Market Cap

Problems with the regression methodology

- The basic regression assumes a linear relationship between PE ratios and the financial proxies, and that might not be appropriate.
- The basic relationship between PE ratios and financial variables itself might not be stable, and if it shifts from year to year, the predictions from the model may not be reliable.
- The independent variables are correlated with each other. For example, high growth firms tend to have high risk. This multi-collinearity makes the coefficients of the regressions unreliable and may explain the large changes in these coefficients from period to period.

The Multicollinearity Problem

Correlations

		Expected Growth in EPS: next 5 years	3-yr Regression Beta	Payout Ratio
Expected Growth in EPS: next 5 years	Pearson Correlation	1	.238**	-.191**
	Sig. (2-tailed)	.	.000	.000
	N	2509	2509	2087
3-yr Regression Beta	Pearson Correlation	.238**	1	-.084**
	Sig. (2-tailed)	.000	.	.000
	N	2509	7024	3979
Payout Ratio	Pearson Correlation	-.191**	-.084**	1
	Sig. (2-tailed)	.000	.000	.
	N	2087	3979	3979

**. Correlation is significant at the 0.01 level (2-tailed).

Using the PE ratio regression

- Assume that you were given the following information for Dell. The firm has an expected growth rate of 10%, a beta of 1.20 and pays no dividends. Based upon the regression, estimate the predicted PE ratio for Dell.

  Predicted PE =

- Dell is actually trading at 22 times earnings. What does the predicted PE tell you?

The value of growth

Time Period	Value of extra 1% of growth	Equity Risk Premium
January 2005	0.914	3.65%
January 2004	0.812	3.69%
July 2003	1.228	3.88%
January 2003	2.621	4.10%
July 2002	0.859	4.35%
January 2002	1.003	3.62%
July 2001	1.251	3.05%
January 2001	1.457	2.75%
July 2000	1.761	2.20%
January 2000	2.105	2.05%

The value of growth is in terms of additional PE…

Fundamentals hold in every market: PE ratio regression for Japan

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.575[a]	.330	.325	19198.600156508 5

a. Predictors: (Constant), Estimated Growth in earnings per share, BETA, Payout Ratio

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-8.110	4.207		-1.928	.055
	Payout Ratio	.528	.064	.345	8.227	.000
	BETA	14.605	3.417	.177	4.274	.000
	Estimated Growth in earnings per share	.799	.083	.399	9.658	.000

a. Dependent Variable: PE

b. Weighted Least Squares Regression - Weighted by Market Capitalization

Value/Earnings and Value/Cashflow Ratios

- While Price earnings ratios look at the market value of equity relative to earnings to equity investors, Value earnings ratios look at the market value of the firm relative to operating earnings. Value to cash flow ratios modify the earnings number to make it a cash flow number.
- The form of value to cash flow ratios that has the closest parallels in DCF valuation is the value to Free Cash Flow to the Firm, which is defined as:

$$\text{Value/FCFF} = \frac{\text{(Market Value of Equity + Market Value of Debt-Cash)}}{\text{EBIT (1-t) - (Cap Ex - Deprecn) - Chg in WC}}$$

- Consistency Tests:
  - If the numerator is net of cash (or if net debt is used, then the interest income from the cash should not be in denominator
  - The interest expenses added back to get to EBIT should correspond to the debt in the numerator. If only long term debt is considered, only long term interest should be added back.

Value of Firm/FCFF: Determinants

- Reverting back to a two-stage FCFF DCF model, we get:

$$V_0 = \frac{FCFF_0 (1+g)\left(1 - \frac{(1+g)^n}{(1+WACC)^n}\right)}{WACC - g} + \frac{FCFF_0 (1+g)^n (1+g_n)}{(WACC - g_n)(1+WACC)^n}$$

- V_0 = Value of the firm (today)
- $FCFF_0$ = Free Cashflow to the firm in current year
- g = Expected growth rate in FCFF in extraordinary growth period (first n years)
- WACC = Weighted average cost of capital
- g_n = Expected growth rate in FCFF in stable growth period (after n years)

Value Multiples

- Dividing both sides by the FCFF yields,

$$\frac{V_0}{FCFF_0} = \frac{(1+g)\left(1-\frac{(1+g)^n}{(1+WACC)^n}\right)}{WACC-g} + \frac{(1+g)^n(1+g_n)}{(WACC-g_n)(1+WACC)^n}$$

- The value/FCFF multiples is a function of
  - the cost of capital
  - the expected growth

Value/FCFF Multiples and the Alternatives

- Assume that you have computed the value of a firm, using discounted cash flow models. Rank the following multiples in the order of magnitude from lowest to highest?
  - Value/EBIT
  - Value/EBIT(1-t)
  - Value/FCFF
  - Value/EBITDA
- What assumption(s) would you need to make for the Value/EBIT(1-t) ratio to be equal to the Value/FCFF multiple?

Illustration: Using Value/FCFF Approaches to value a firm: MCI Communications

- MCI Communications had earnings before interest and taxes of $3356 million in 1994 (Its net income after taxes was $855 million).
- It had capital expenditures of $2500 million in 1994 and depreciation of $1100 million; Working capital increased by $250 million.
- It expects free cashflows to the firm to grow 15% a year for the next five years and 5% a year after that.
- The cost of capital is 10.50% for the next five years and 10% after that.
- The company faces a tax rate of 36%.

$$\frac{V_0}{FCFF_0} = \frac{(1.15)\left(1 - \frac{(1.15)^5}{(1.105)^5}\right)}{.105 - .15} + \frac{(1.15)^5(1.05)}{(.10 - .05)(1.105)^5} = 31.28$$

Multiple Magic

- In this case of MCI there is a big difference between the FCFF and short cut measures. For instance the following table illustrates the appropriate multiple using short cut measures, and the amount you would overpay by if you used the FCFF multiple.

Free Cash Flow to the Firm

= EBIT (1-t) - Net Cap Ex - Change in Working Capital

= 3356 (1 - 0.36) + 1100 - 2500 - 250 = $ 498 million

	$ Value	*Correct Multiple*
FCFF	$498	31.28382355
EBIT (1-t)	$2,148	7.251163362
EBIT $ 3,356		4.640744552
EBITDA	$4,456	3.49513885

Reasons for Increased Use of Value/EBITDA

1. The multiple can be computed even for firms that are reporting net losses, since earnings before interest, taxes and depreciation are usually positive.
2. For firms in certain industries, such as cellular, which require a substantial investment in infrastructure and long gestation periods, this multiple seems to be more appropriate than the price/earnings ratio.
3. In leveraged buyouts, where the key factor is cash generated by the firm prior to all discretionary expenditures, the EBITDA is the measure of cash flows from operations that can be used to support debt payment at least in the short term.
4. By looking at cashflows prior to capital expenditures, it may provide a better estimate of "optimal value", especially if the capital expenditures are unwise or earn substandard returns.
5. By looking at the value of the firm and cashflows to the firm it allows for comparisons across firms with different financial leverage.

Value/EBITDA Multiple

- The Classic Definition

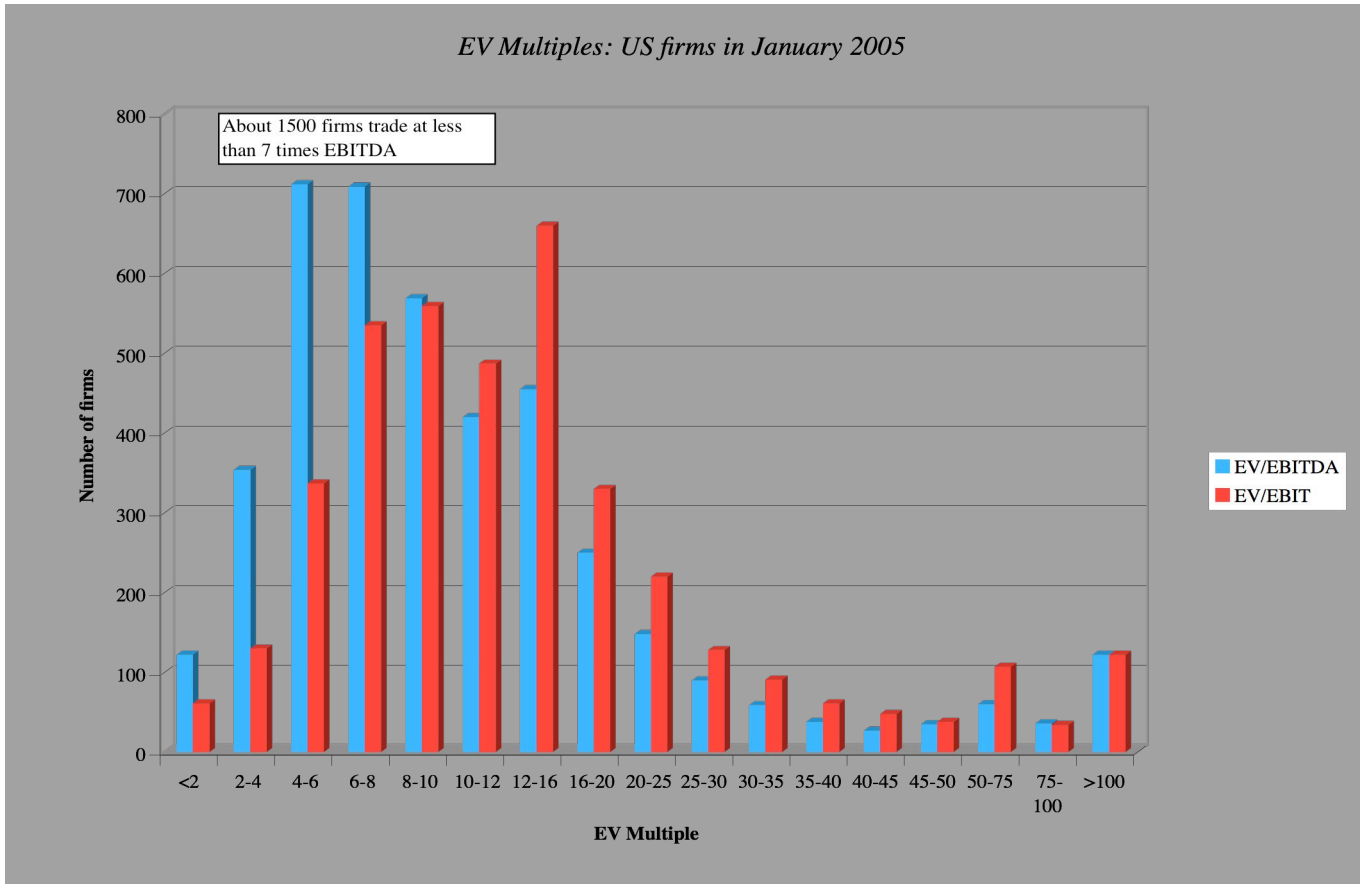
$$\frac{\text{Value}}{\text{EBITDA}} = \frac{\text{Market Value of Equity} + \text{Market Value of Debt}}{\text{Earnings before Interest, Taxes and Depreciation}}$$

- The No-Cash Version

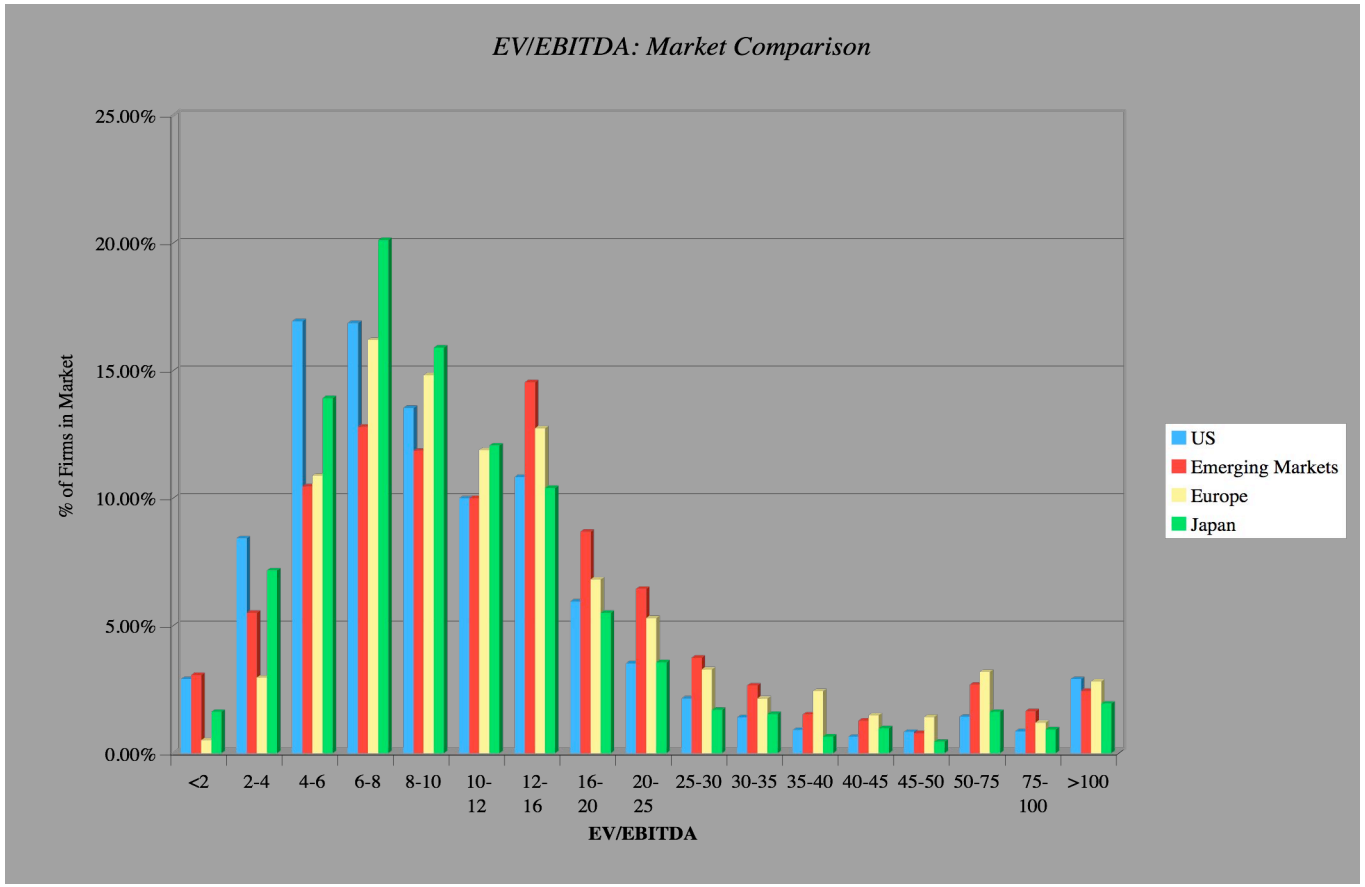
$$\frac{\text{Enterprise Value}}{\text{EBITDA}} = \frac{\text{Market Value of Equity} + \text{Market Value of Debt} - \text{Cash}}{\text{Earnings before Interest, Taxes and Depreciation}}$$

- When cash and marketable securities are netted out of value, none of the income from the cash and securities should be reflected in the denominator.

Enterprise Value/EBITDA Distribution - US



Value/EBITDA Multiple: Europe, Japan and Emerging Markets in January 2005



The Determinants of Value/EBITDA Multiples: Linkage to DCF Valuation

- Firm value can be written as:

$$V_0 = \frac{FCFF_1}{WACC - g}$$

- The numerator can be written as follows:

FCFF = EBIT (1-t) - (Cex - Depr) - Δ Working Capital
= (EBITDA - Depr) (1-t) - (Cex - Depr) - Δ Working Capital
= EBITDA (1-t) + Depr (t) - Cex - Δ Working Capital

From Firm Value to EBITDA Multiples

- Now the Value of the firm can be rewritten as,

$$\text{Value} = \frac{\text{EBITDA}\,(1-t) + \text{Depr}\,(t) - \text{Cex} - \Delta\,\text{Working Capital}}{\text{WACC} - g}$$

- Dividing both sides of the equation by EBITDA,

$$\frac{\text{Value}}{\text{EBITDA}} = \frac{(1-t)}{\text{WACC} - g} + \frac{\text{Depr}\,(t)/\text{EBITDA}}{\text{WACC} - g} - \frac{\text{CEx/EBITDA}}{\text{WACC} - g} - \frac{\Delta\,\text{Working Capital/EBITDA}}{\text{WACC} - g}$$

A Simple Example

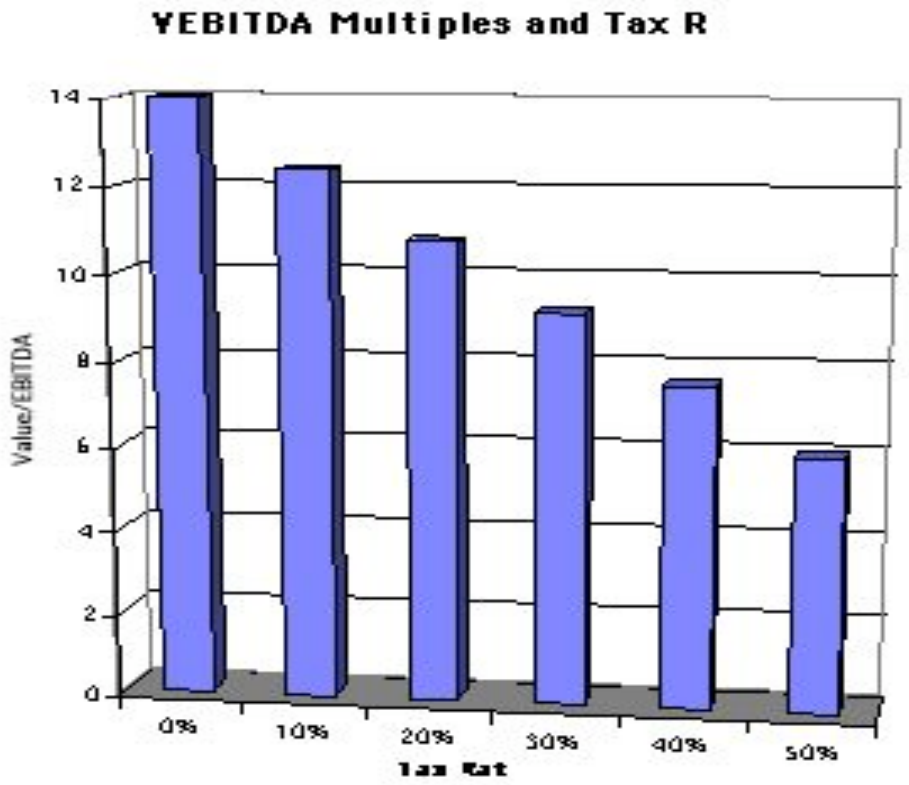
- Consider a firm with the following characteristics:
  - Tax Rate = 36%
  - Capital Expenditures/EBITDA = 30%
  - Depreciation/EBITDA = 20%
  - Cost of Capital = 10%
  - The firm has no working capital requirements
  - The firm is in stable growth and is expected to grow 5% a year forever.

Calculating Value/EBITDA Multiple

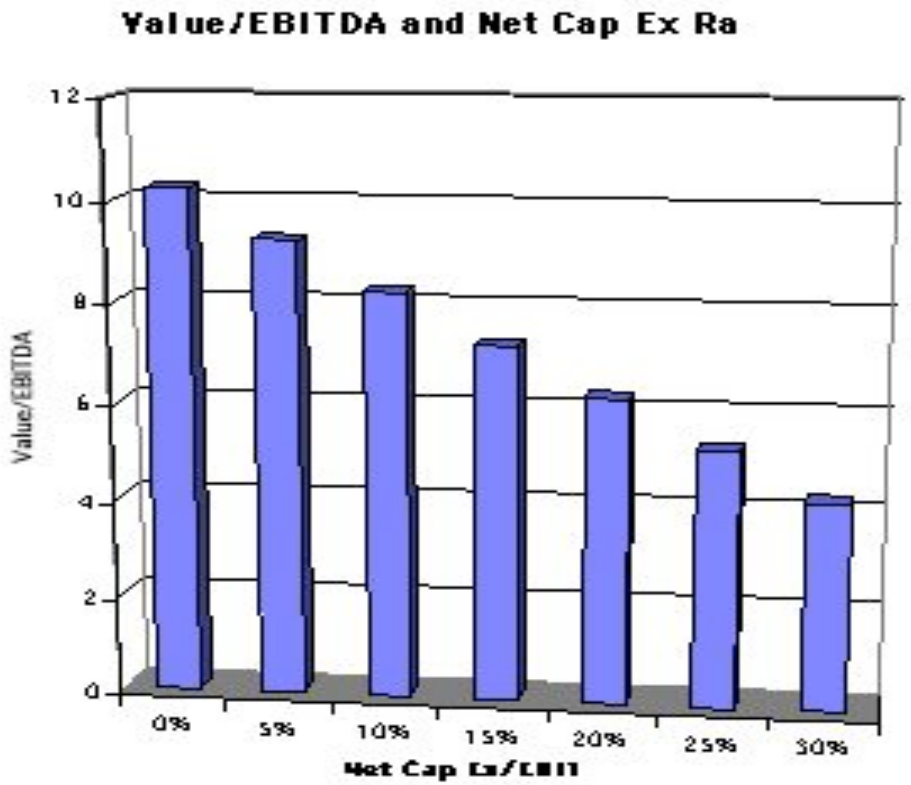
- In this case, the Value/EBITDA multiple for this firm can be estimated as follows:

$$\frac{\text{Value}}{\text{EBITDA}} = \frac{(1-.36)}{.10-.05} + \frac{(0.2)(.36)}{.10-.05} - \frac{0.3}{.10-.05} - \frac{0}{.10-.05} = 8.24$$

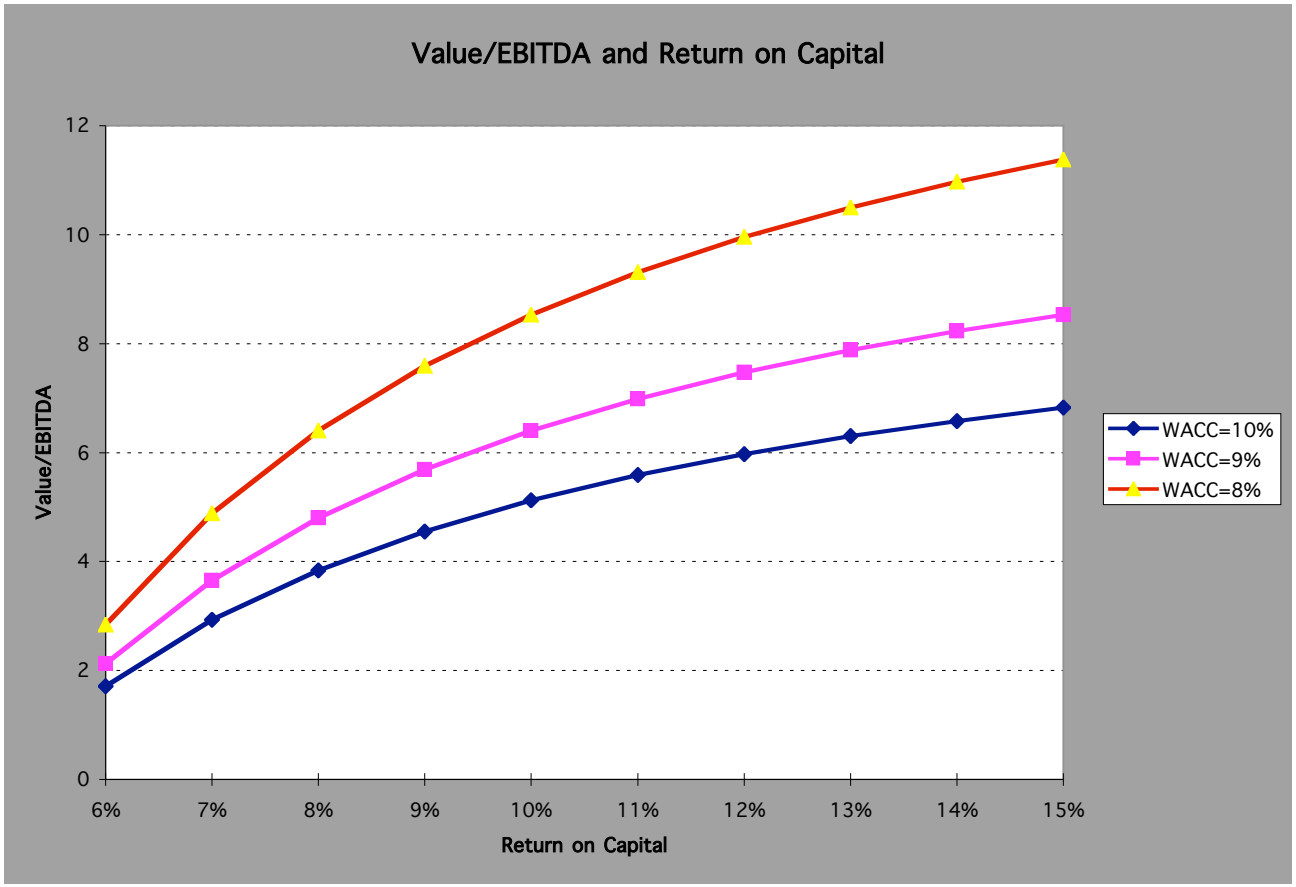
Value/EBITDA Multiples and Taxes



Value/EBITDA and Net Cap Ex



Value/EBITDA Multiples and Return on Capital



Value/EBITDA Multiple: Trucking Companies

Company Name	*Value*	*EBITDA*	*Value/EBITDA*
KLLM Trans. Svcs.	$ 114.32	$ 48.81	2.34
Ryder System	$ 5,158.04	$ 1,838.26	2.81
Rollins Truck Leasing	$ 1,368.35	$ 447.67	3.06
Cannon Express Inc.	$ 83.57	$ 27.05	3.09
Hunt (J.B.)	$ 982.67	$ 310.22	3.17
Yellow Corp.	$ 931.47	$ 292.82	3.18
Roadway Express	$ 554.96	$ 169.38	3.28
Marten Transport Ltd.	$ 116.93	$ 35.62	3.28
Kenan Transport Co.	$ 67.66	$ 19.44	3.48
M.S. Carriers	$ 344.93	$ 97.85	3.53
Old Dominion Freight	$ 170.42	$ 45.13	3.78
Trimac Ltd	$ 661.18	$ 174.28	3.79
Matlack Systems	$ 112.42	$ 28.94	3.88
XTRA Corp.	$ 1,708.57	$ 427.30	4.00
Covenant Transport Inc	$ 259.16	$ 64.35	4.03
Builders Transport	$ 221.09	$ 51.44	4.30
Werner Enterprises	$ 844.39	$ 196.15	4.30
Landstar Sys.	$ 422.79	$ 95.20	4.44
AMERCO	$ 1,632.30	$ 345.78	4.72
USA Truck	$ 141.77	$ 29.93	4.74
Frozen Food Express	$ 164.17	$ 34.10	4.81
Arnold Inds.	$ 472.27	$ 96.88	4.87
Greyhound Lines Inc.	$ 437.71	$ 89.61	4.88
USFreightways	$ 983.86	$ 198.91	4.95
Golden Eagle Group Inc.	$ 12.50	$ 2.33	5.37
Arkansas Best	$ 578.78	$ 107.15	5.40
Airlease Ltd.	$ 73.64	$ 13.48	5.46
Celadon Group	$ 182.30	$ 32.72	5.57
Amer. Freightways	$ 716.15	$ 120.94	5.92
Transfinancial Holdings	$ 56.92	$ 8.79	6.47
Vitran Corp. 'A'	$ 140.68	$ 21.51	6.54
Interpool Inc.	$ 1,002.20	$ 151.18	6.63
Intrenet Inc.	$ 70.23	$ 10.38	6.77
Swift Transportation	$ 835.58	$ 121.34	6.89
Landair Services	$ 212.95	$ 30.38	7.01
CNF Transportation	$ 2,700.69	$ 366.99	7.36
Budget Group Inc	$ 1,247.30	$ 166.71	7.48
Caliber System	$ 2,514.99	$ 333.13	7.55
Knight Transportation Inc	$ 269.01	$ 28.20	9.54
Heartland Express	$ 727.50	$ 64.62	11.26
Greyhound CDA Transn Corp	$ 83.25	$ 6.99	11.91
Mark VII	$ 160.45	$ 12.96	12.38
Coach USA Inc	$ 678.38	$ 51.76	13.11
US 1 Inds Inc.	$ 5.60	$ (0.17)	NA
Average			**5.61**

A Test on EBITDA

- Ryder System looks very cheap on a Value/EBITDA multiple basis, relative to the rest of the sector. What explanation (other than misvaluation) might there be for this difference?

US Market: Cross Sectional Regression
January 2005

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.618[a]	.382	.380	800.039502101183000

a. Predictors: (Constant), Market Debt to Capital, Eff Tax Rate, Reinvestment Rate, Expected Growth in Revenues: next 5 years

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients			95% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	8.554	.843		10.152	.000	6.901	10.207
	Expected Growth in Revenues: next 5 years	1.016	.045	.552	22.699	.000	.928	1.104
	Eff Tax Rate	-.150	.020	-.164	-7.503	.000	-.190	-.111
	Reinvestment Rate	-1.884E-02	.005	-.079	-3.532	.000	-.029	-.008
	Market Debt to Capital	-6.642E-02	.015	-.108	-4.414	.000	-.096	-.037

a. Dependent Variable: EV/EBITDA

b. Weighted Least Squares Regression - Weighted by Market Cap

Europe: Cross Sectional Regression
January 2005

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.551[a]	.304	.303	1618.393594200679000

a. Predictors: (Constant), Market Debt to Capital, Reinvestment Rate, TAX_RATE

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	16.797	1.066		15.755	.000
	TAX_RATE	-.356	.027	-.207	-13.147	.000
	Reinvestment Rate	6.093E-04	.001	.017	1.106	.269
	Market Debt to Capital	.518	.017	.490	31.223	.000

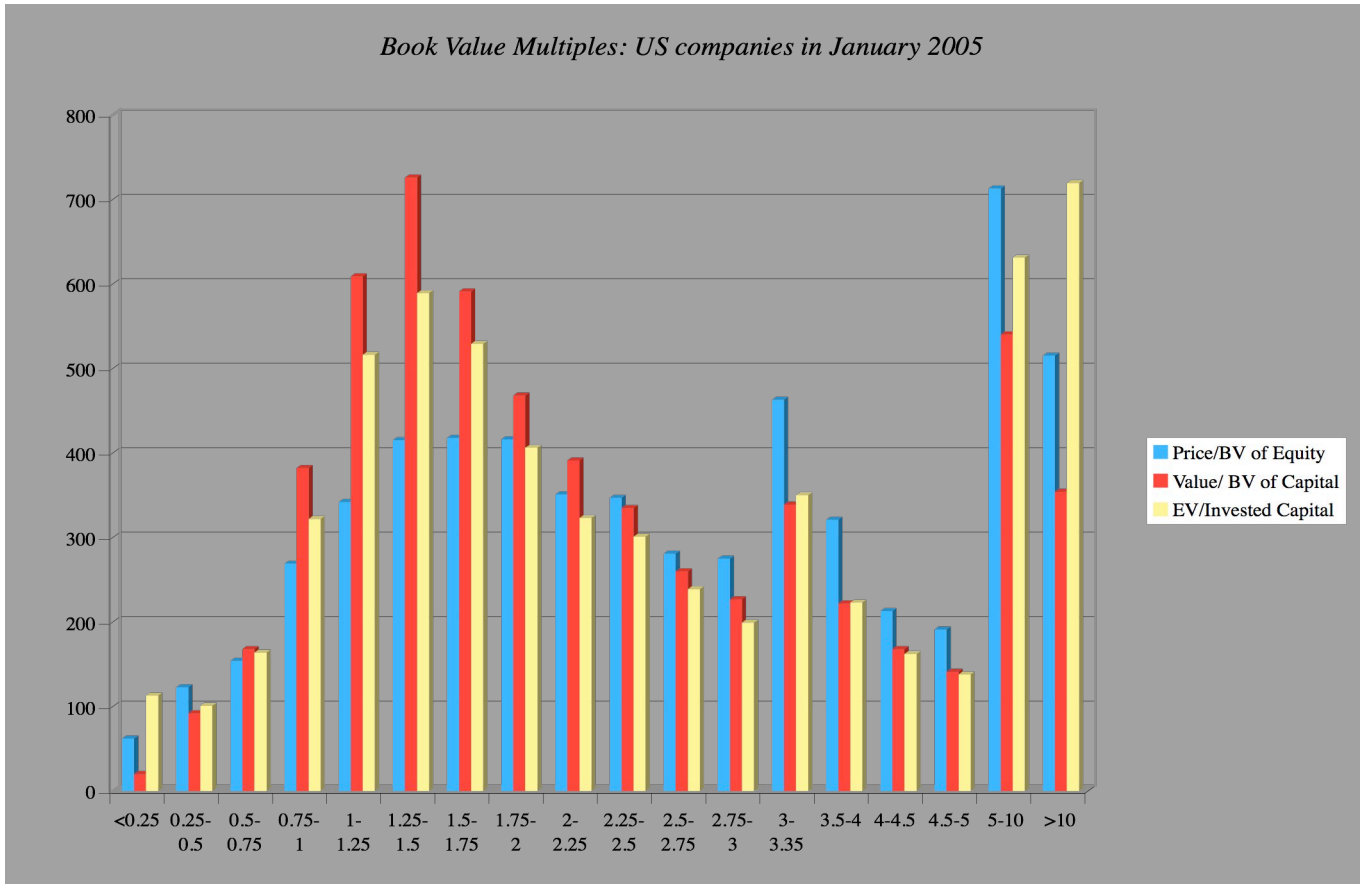
a. Dependent Variable: EV/EBITDA

b. Weighted Least Squares Regression - Weighted by Market Capitalization

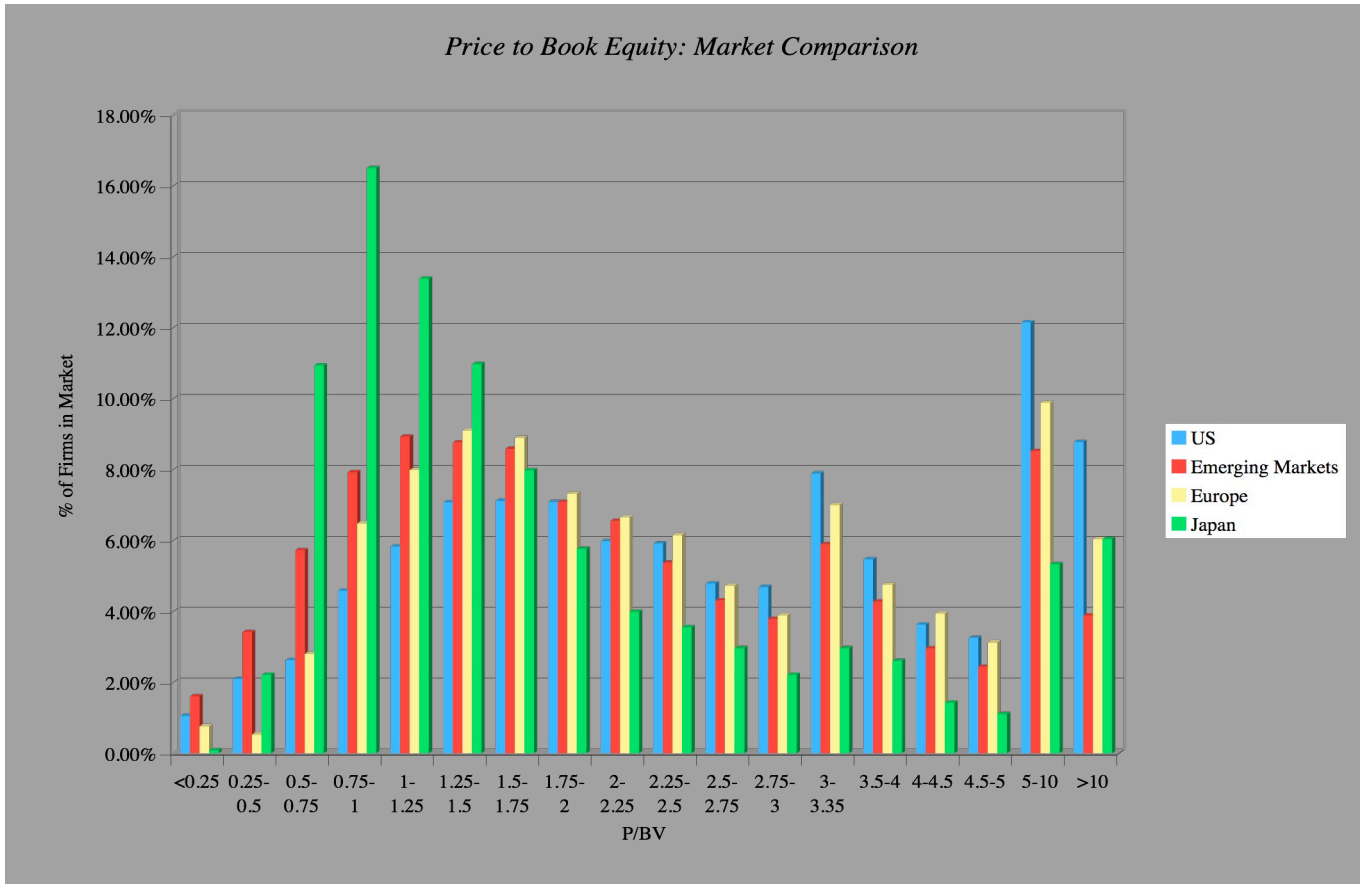
Price-Book Value Ratio: Definition

- The price/book value ratio is the ratio of the market value of equity to the book value of equity, i.e., the measure of shareholders' equity in the balance sheet.
- Price/Book Value = $\frac{\text{Market Value of Equity}}{\text{Book Value of Equity}}$
- Consistency Tests:
  - If the market value of equity refers to the market value of equity of common stock outstanding, the book value of common equity should be used in the denominator.
  - If there is more that one class of common stock outstanding, the market values of all classes (even the non-traded classes) needs to be factored in.

Book Value Multiples: US stocks



Price to Book: Europe, Japan and Emerging Markets



Price Book Value Ratio: Stable Growth Firm

- Going back to a simple dividend discount model,

$$P_0 = \frac{DPS_1}{r - g_n}$$

- Defining the return on equity (ROE) = EPS_0 / Book Value of Equity, the value of equity can be written as:

$$P_0 = \frac{BV_0 * ROE * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

$$\frac{P_0}{BV_0} = PBV = \frac{ROE * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

- If the return on equity is based upon expected earnings in the next time period, this can be simplified to,

$$\frac{P_0}{BV_0} = PBV = \frac{ROE * \text{Payout Ratio}}{r - g_n}$$

PBV/ROE: European Banks

Bank	*Symbol*	*PBV*	*ROE*
Banca di Roma SpA	BAHQE	0.60	4.15%
Commerzbank AG	COHSO	0.74	5.49%
Bayerische Hypo und Vereinsbank AG	BAXWW	0.82	5.39%
Intesa Bci SpA	BAEWF	1.12	7.81%
Natexis Banques Populaires	NABQE	1.12	7.38%
Almanij NV Algemene Mij voor Nijver	ALPK	1.17	8.78%
Credit Industriel et Commercial	CIECM	1.20	9.46%
Credit Lyonnais SA	CREV	1.20	6.86%
BNL Banca Nazionale del Lavoro SpA	BAEXC	1.22	12.43%
Banca Monte dei Paschi di Siena SpA	MOGG	1.34	10.86%
Deutsche Bank AG	**DEMX**	**1.36**	**17.33%**
Skandinaviska Enskilda Banken	SKHS	1.39	16.33%
Nordea Bank AB	NORDEA	1.40	13.69%
DNB Holding ASA	**DNHLD**	**1.42**	**16.78%**
ForeningsSparbanken AB	FOLG	1.61	18.69%
Danske Bank AS	DANKAS	1.66	19.09%
Credit Suisse Group	***CRGAL***	***1.68***	***14.34%***
KBC Bankverzekeringsholding	KBCBA	1.69	30.85%
Societe Generale	SODI	1.73	17.55%
Santander Central Hispano SA	***BAZAB***	***1.83***	***11.01%***
National Bank of Greece SA	NAGT	1.87	26.19%
San Paolo IMI SpA	SAOEL	1.88	16.57%
BNP Paribas	BNPRB	2.00	18.68%
Svenska Handelsbanken AB	SVKE	2.12	21.82%
UBS AG	UBQH	2.15	16.64%
Banco Bilbao Vizcaya Argentaria SA	BBFUG	2.18	22.94%
ABN Amro Holding NV	ABTS	2.21	24.21%
UniCredito Italiano SpA	UNCZA	2.25	15.90%
Rolo Banca 1473 SpA	ROGMBA	2.37	16.67%
Dexia	DECCT	2.76	14.99%
Average		1.60	14.96%

PBV versus ROE regression

- Regressing PBV ratios against ROE for banks yields the following regression:

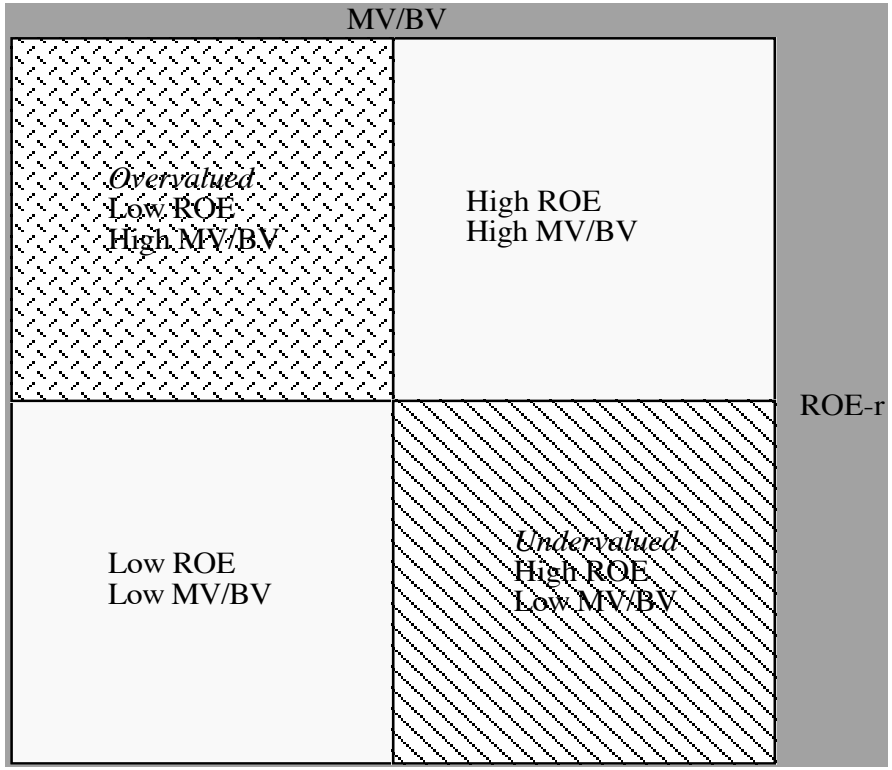
$$PBV = 0.81 + 5.32\ (ROE) \qquad R^2 = 46\%$$

- For every 1% increase in ROE, the PBV ratio should increase by 0.0532.

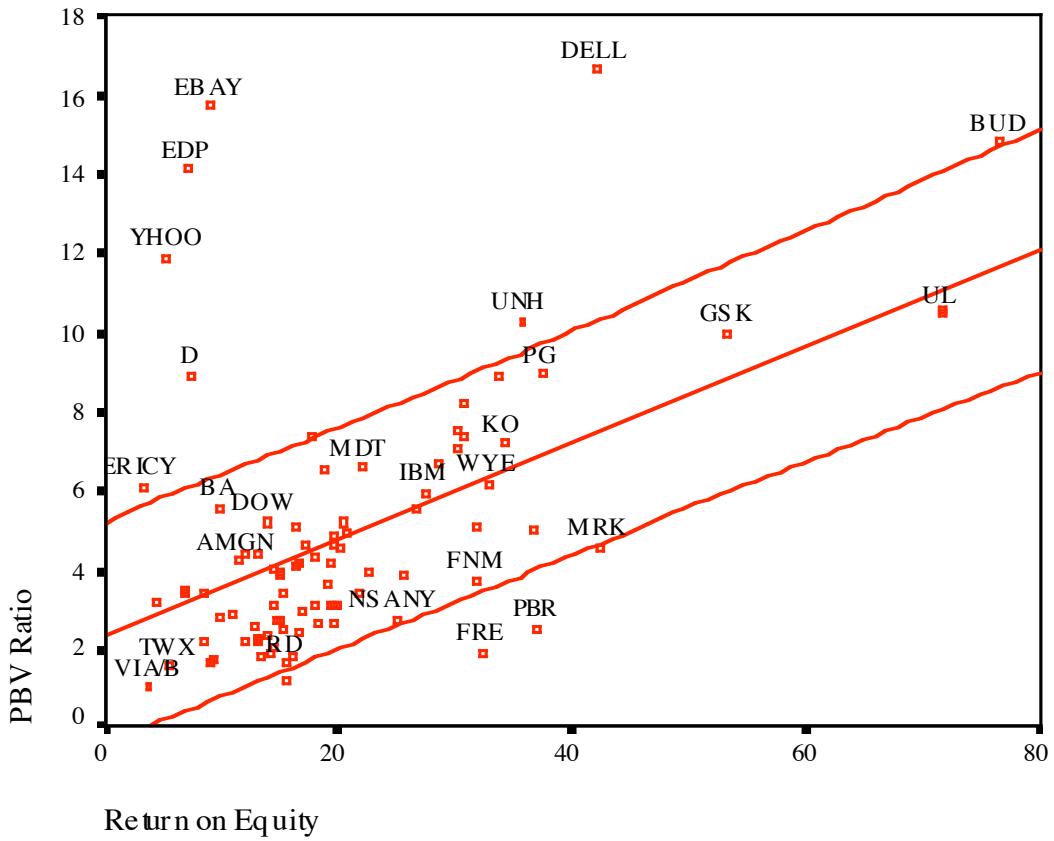
Under and Over Valued Banks?

Bank	*Actual*	*Predicted*	*Under or Over*
Banca di Roma SpA	0.60	1.03	-41.33%
Commerzbank AG	0.74	1.10	-32.86%
Bayerische Hypo und Vereinsbank AG	0.82	1.09	-24.92%
Intesa Bci SpA	1.12	1.22	-8.51%
Natexis Banques Populaires	1.12	1.20	-6.30%
Almanij NV Algemene Mij voor Nijver	1.17	1.27	-7.82%
Credit Industriel et Commercial	1.20	1.31	-8.30%
Credit Lyonnais SA	1.20	1.17	2.61%
BNL Banca Nazionale del Lavoro SpA	1.22	1.47	-16.71%
Banca Monte dei Paschi di Siena SpA	1.34	1.39	-3.38%
Deutsche Bank AG	1.36	1.73	-21.40%
Skandinaviska Enskilda Banken	1.39	1.68	-17.32%
Nordea Bank AB	1.40	1.54	-9.02%
DNB Holding ASA	1.42	1.70	-16.72%
ForeningsSparbanken AB	1.61	1.80	-10.66%
Danske Bank AS	1.66	1.82	-9.01%
Credit Suisse Group	1.68	1.57	7.20%
KBC Bankverzekeringsholding	1.69	2.45	-30.89%
Societe Generale	1.73	1.74	-0.42%
Santander Central Hispano SA	1.83	1.39	31.37%
National Bank of Greece SA	1.87	2.20	-15.06%
San Paolo IMI SpA	1.88	1.69	11.15%
BNP Paribas	2.00	1.80	11.07%
Svenska Handelsbanken AB	2.12	1.97	7.70%
UBS AG	2.15	1.69	27.17%
Banco Bilbao Vizcaya Argentaria SA	2.18	2.03	7.66%
ABN Amro Holding NV	2.21	2.10	5.23%
UniCredito Italiano SpA	2.25	1.65	36.23%
Rolo Banca 1473 SpA	2.37	1.69	39.74%
Dexia	2.76	1.61	72.04%

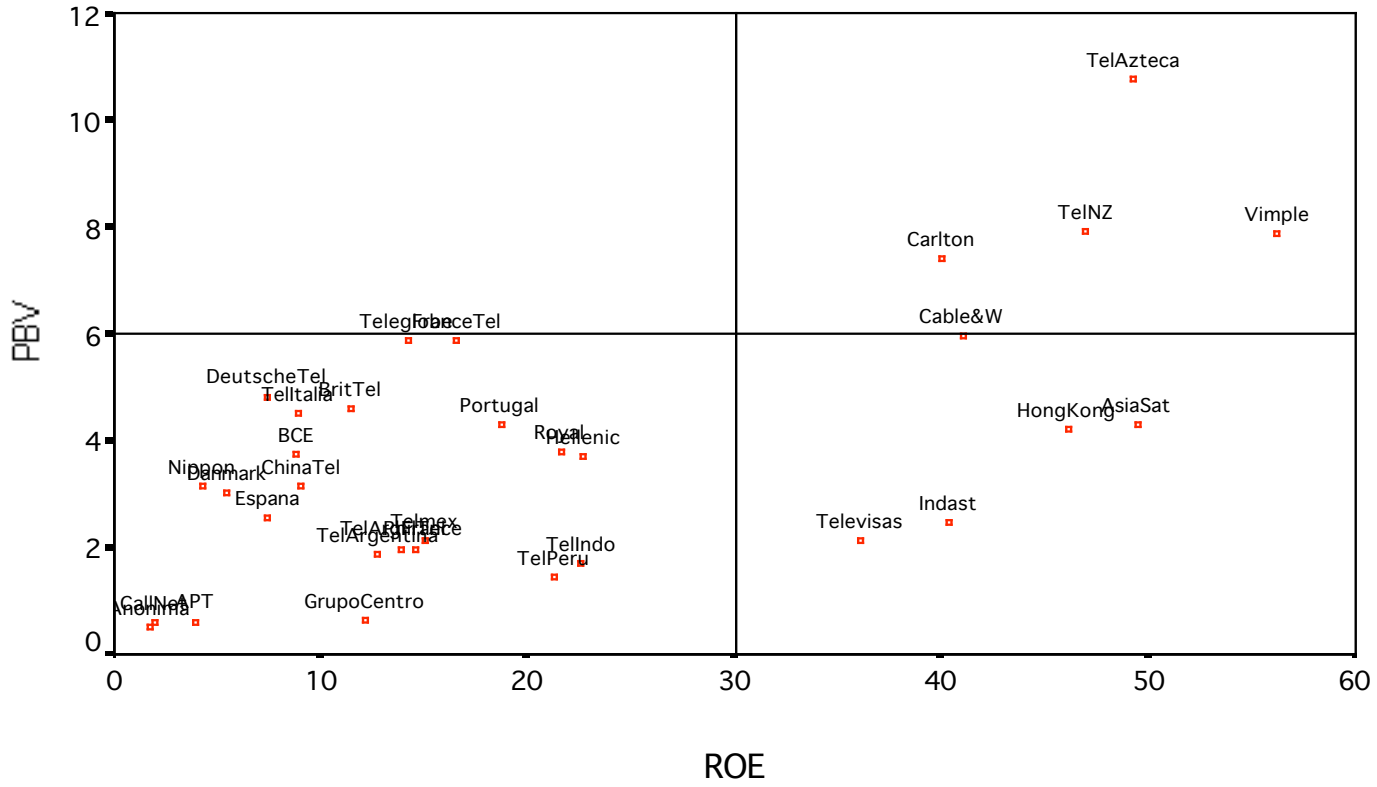
Looking for undervalued securities - PBV Ratios and ROE : The Valuation Matrix



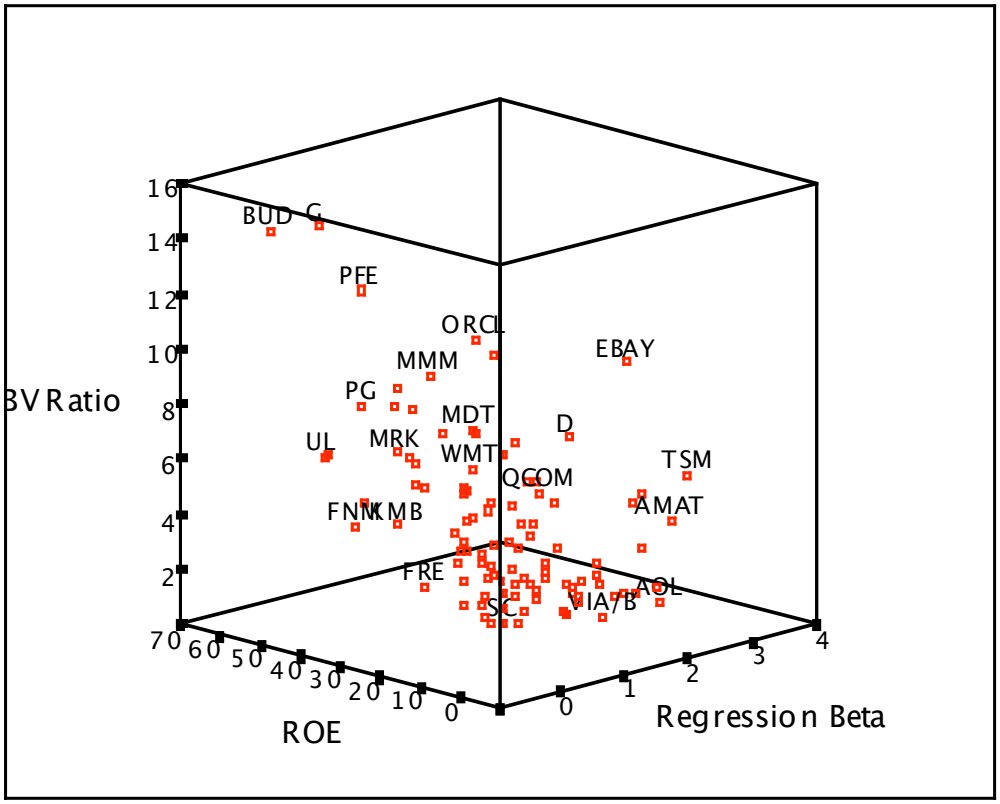
Price to Book vs ROE: Largest Market Cap Firms in the United States: January 2005



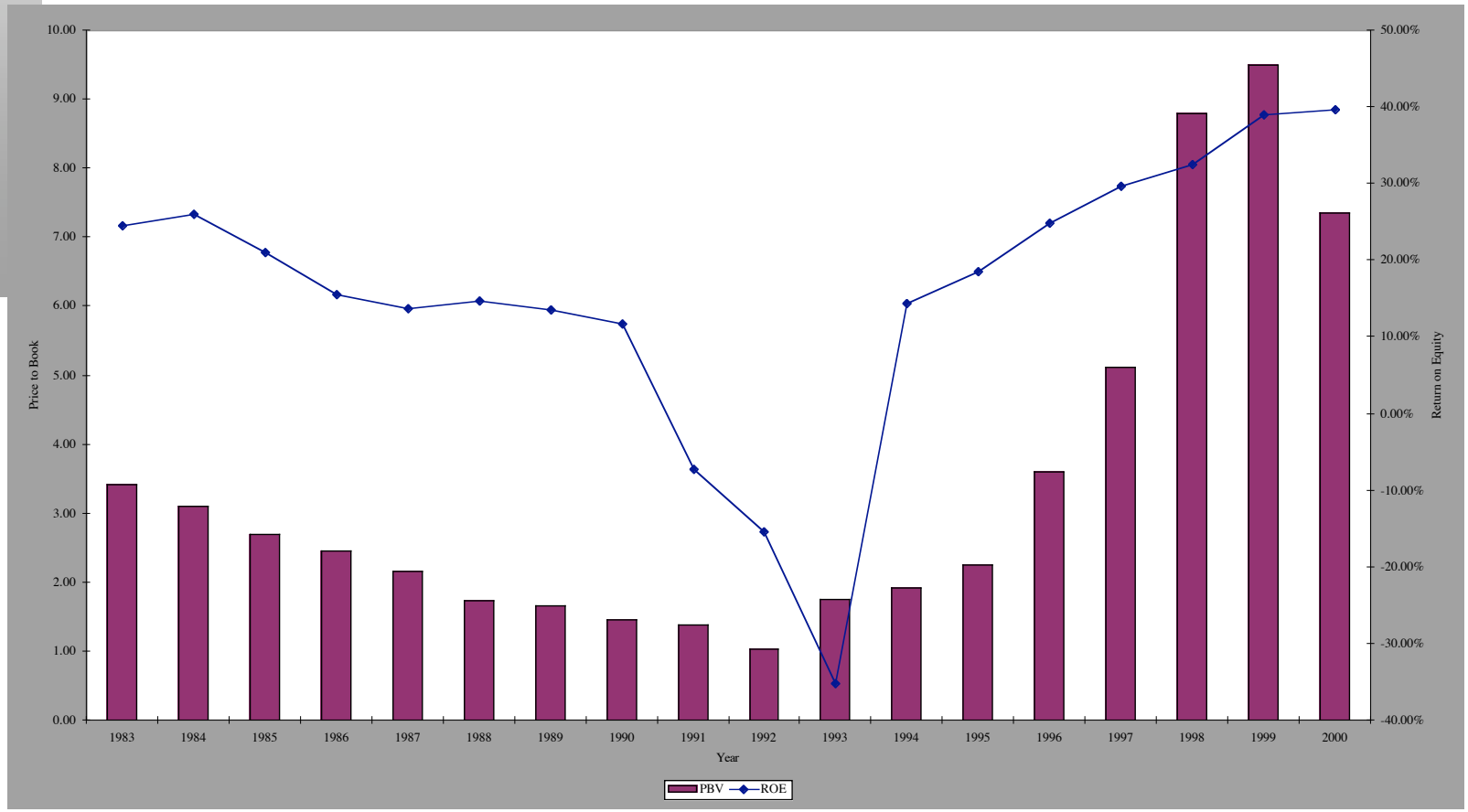
PBV Matrix: Telecom Companies



PBV, ROE and Risk: Large Cap US firms



IBM: The Rise and Fall and Rise Again



PBV Ratio Regression: US
January 2005

Model Summary

Model	R	R Square[a]	Adjusted R Square	Std. Error of the Estimate
1	.909[b]	.826	.826	201.88912643130000

a. For regression through the origin (the no-intercept model), R Square measures the proportion of the variability in the dependent variable about the origin explained by regression. This CANNOT be compared to R Square for models which include an intercept.

b. Predictors: Return on Equity, Expected Growth in EPS: next 5 years, Payout Ratio, 3-yr Regression Beta

Coefficients[a,b,c]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	Expected Growth in EPS: next 5 years	9.835E-02	.005	.310	21.120	.000	.089	.107
	3-yr Regression Beta	-.297	.078	-.058	-3.800	.000	-.450	-.144
	Payout Ratio	-1.349E-02	.002	-.097	-7.438	.000	-.017	-.010
	Return on Equity	.202	.003	.800	58.317	.000	.196	.209

a. Dependent Variable: PBV Ratio

b. Linear Regression through the Origin

c. Weighted Least Squares Regression - Weighted by Market Cap

PBV Regression: Emerging Markets
January 2005

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.525[a]	.276	.272	2.525

a. Predictors: (Constant), ROE, Payout Ratio, IBES Est 5 year growth, BETA

Coefficients[a]

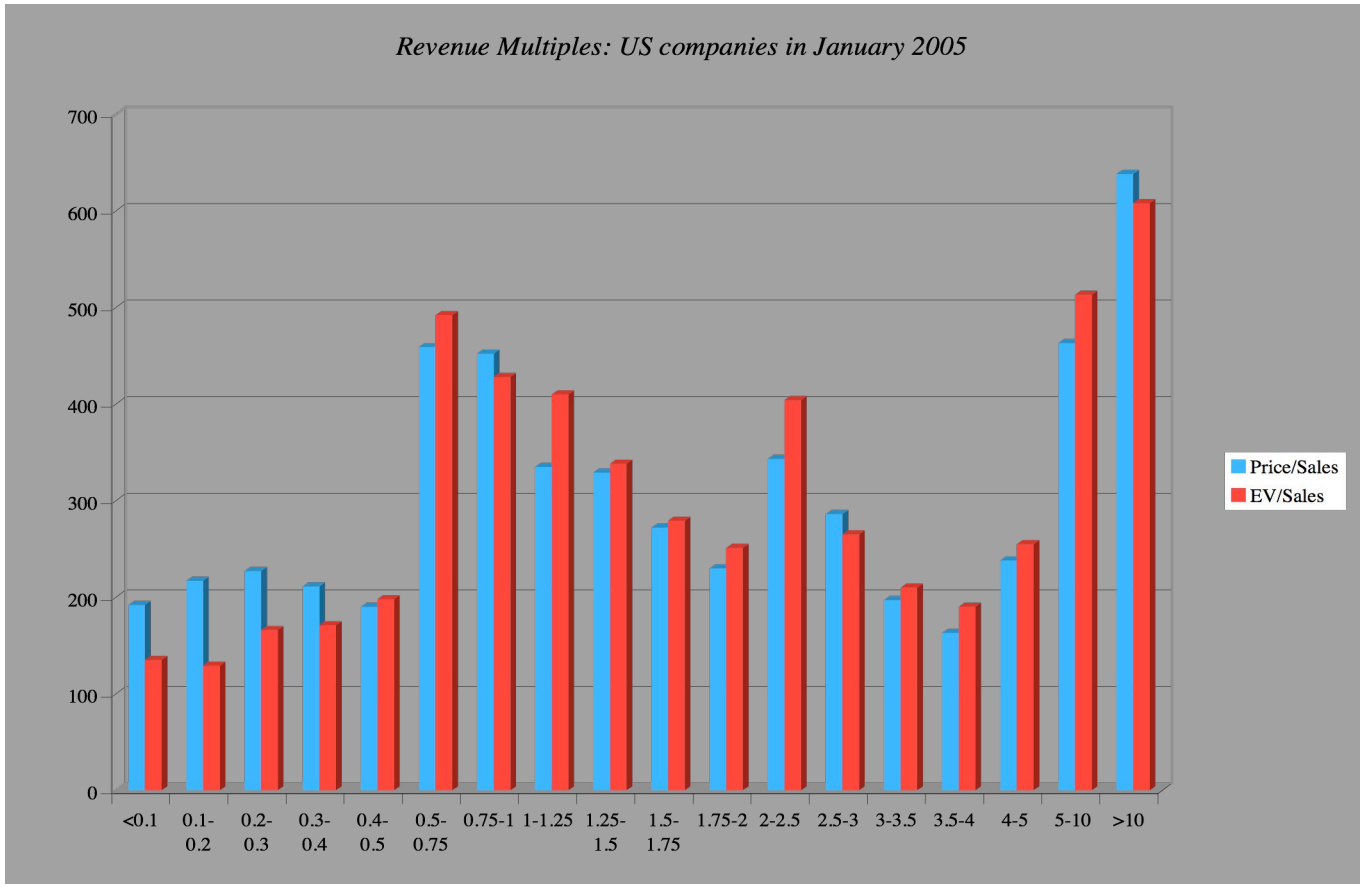
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.337	.383		3.490	.001
	IBES Est 5 year growth	2.286E-02	.004	.162	5.267	.000
	BETA	-.839	.333	-.078	-2.519	.012
	Payout Ratio	5.396E-04	.001	.023	.766	.444
	ROE	.109	.007	.502	16.292	.000

a. Dependent Variable: PBV

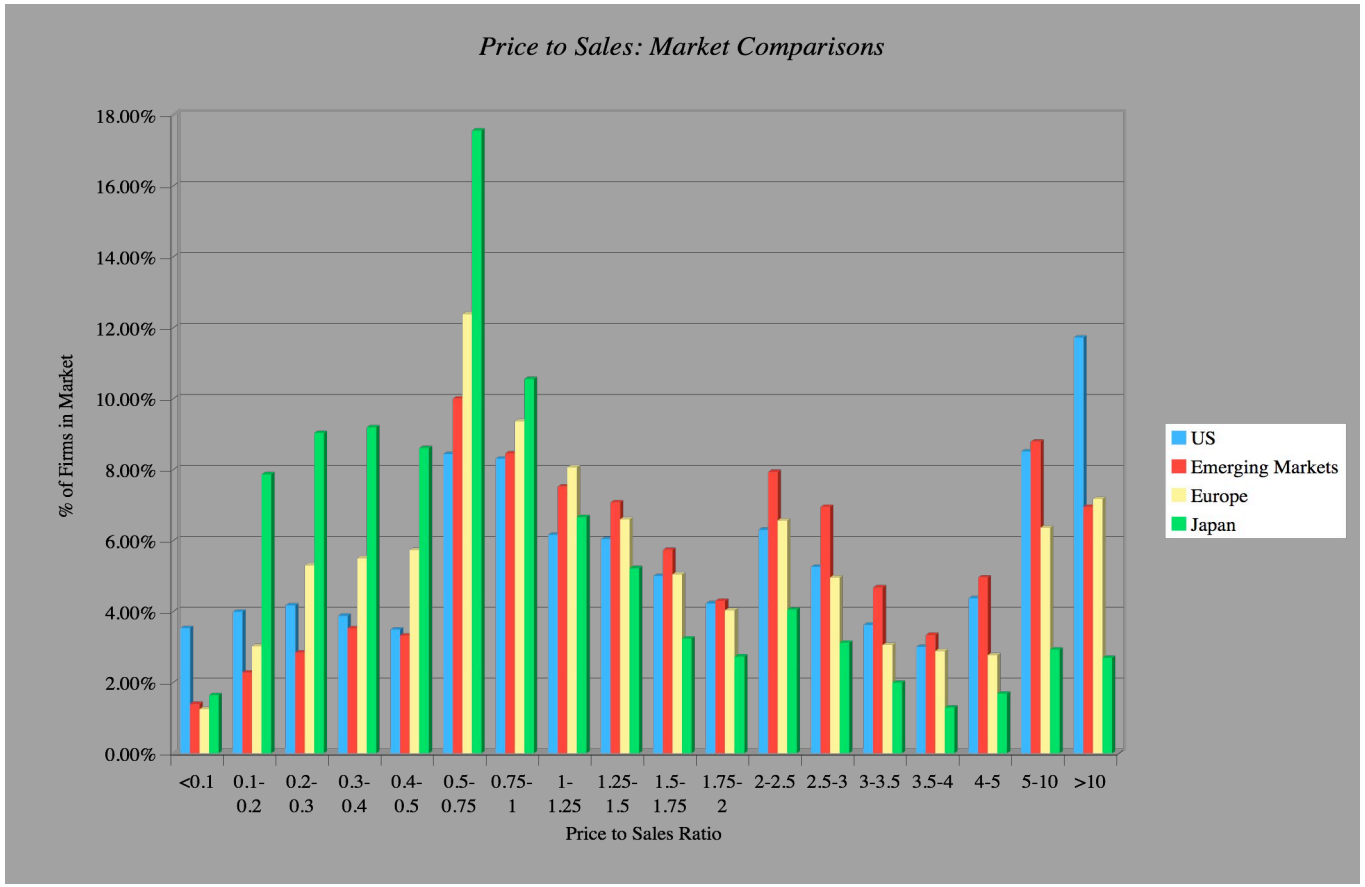
Price Sales Ratio: Definition

- The price/sales ratio is the ratio of the market value of equity to the sales.
- Price/ Sales= $\frac{\text{Market Value of Equity}}{\text{Total Revenues}}$
- Consistency Tests
  - The price/sales ratio is internally inconsistent, since the market value of equity is divided by the total revenues of the firm.

Price/Sales Ratio: US stocks



Price to Sales: Europe, Japan and Emerging Markets



Price/Sales Ratio: Determinants

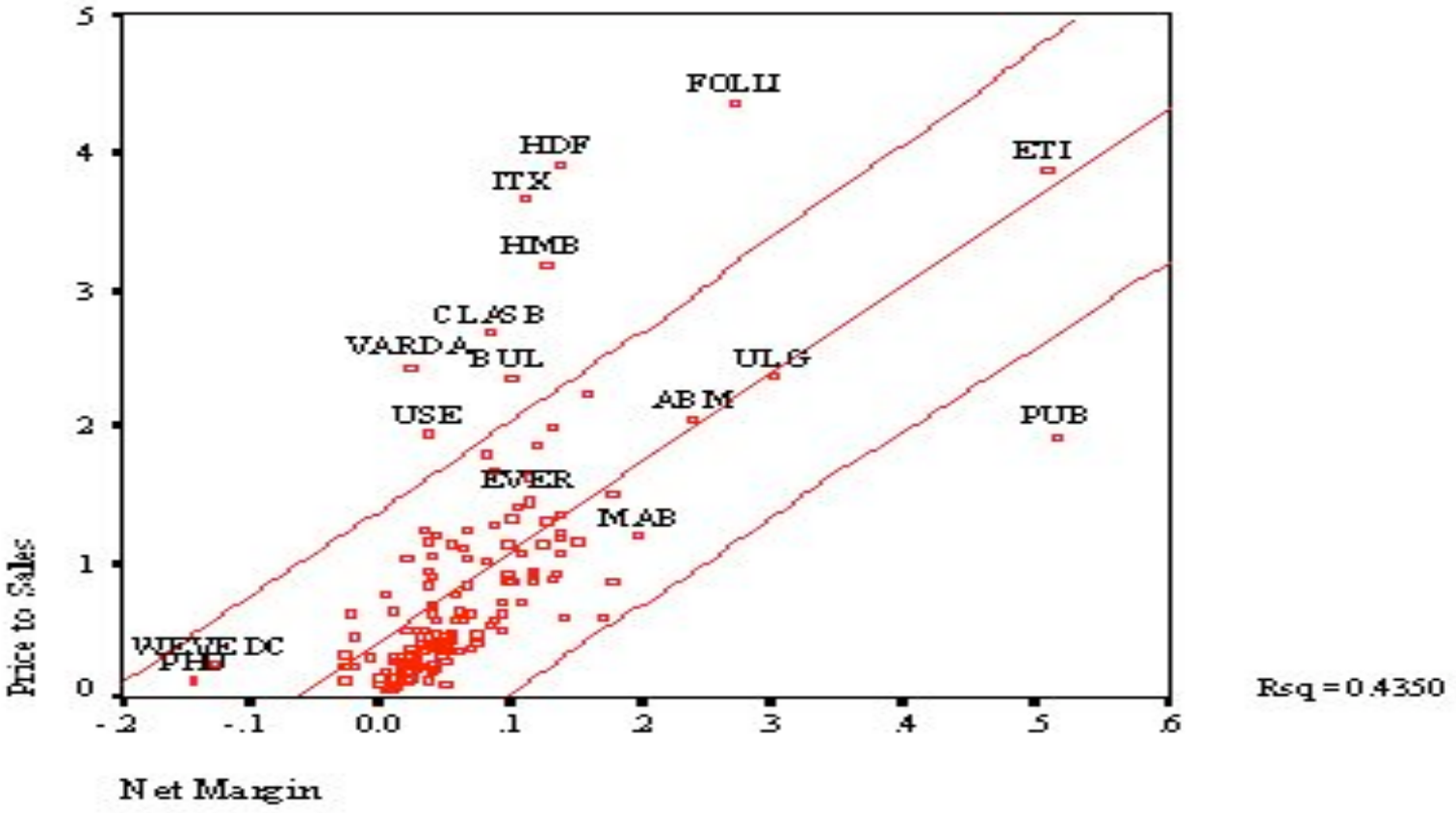
- The price/sales ratio of a stable growth firm can be estimated beginning with a 2-stage equity valuation model:

$$P_0 = \frac{DPS_1}{r - g_n}$$

- Dividing both sides by the sales per share:

$$\frac{P_0}{Sales_0} = PS = \frac{\text{Net Profit Margin} * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

PS/Margins: European Retailers - September 2003



Regression Results: PS Ratios and Margins

- Regressing PS ratios against net margins,

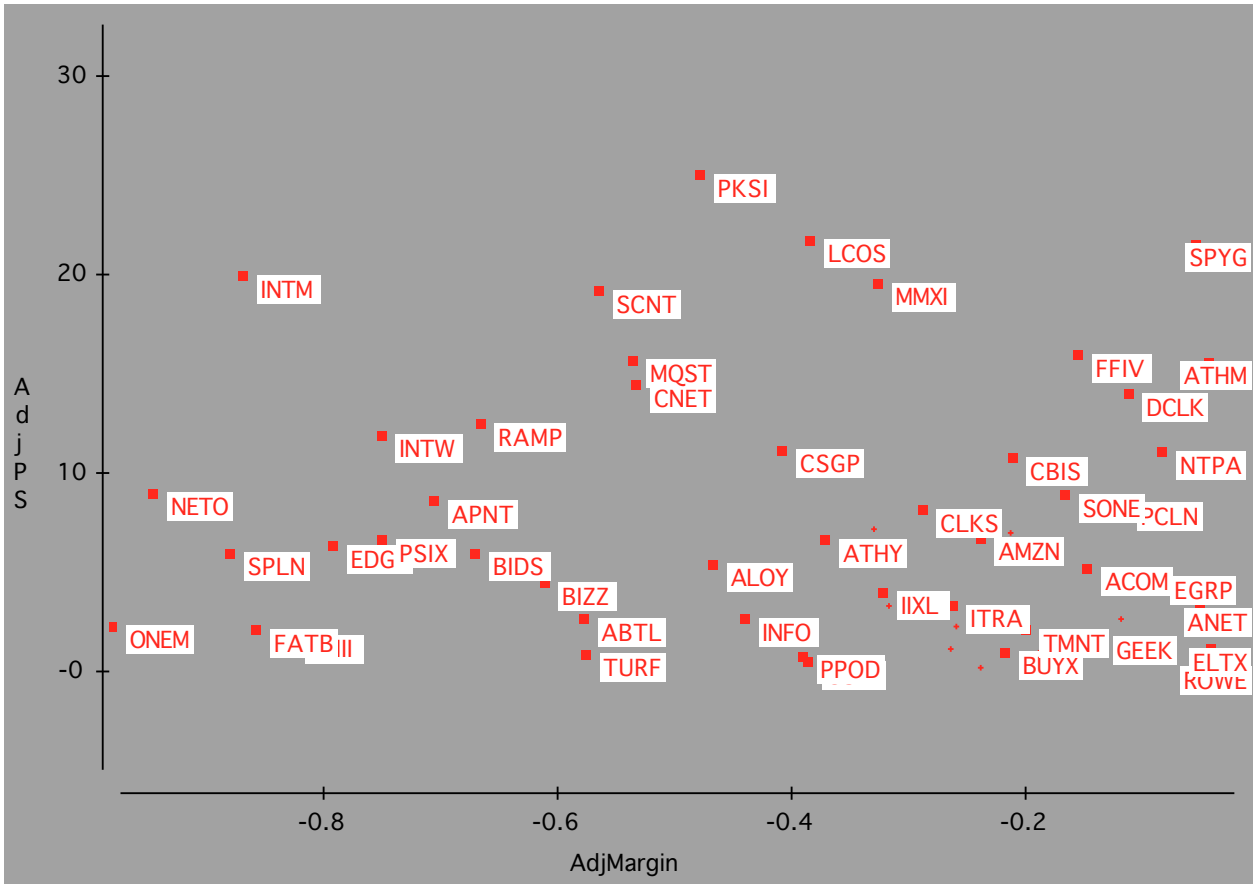
  PS = -.39 + 0.6548 (Net Margin) R^2 = 43.5%

- Thus, a 1% increase in the margin results in an increase of 0.6548 in the price sales ratios.
- The regression also allows us to get predicted PS ratios for these firms

Current versus Predicted Margins

- One of the limitations of the analysis we did in these last few pages is the focus on current margins. Stocks are priced based upon expected margins rather than current margins.
- For most firms, current margins and predicted margins are highly correlated, making the analysis still relevant.
- For firms where current margins have little or no correlation with expected margins, regressions of price to sales ratios against current margins (or price to book against current return on equity) will not provide much explanatory power.
- In these cases, it makes more sense to run the regression using either predicted margins or some proxy for predicted margins.

A Case Study: The Internet Stocks



PS Ratios and Margins are not highly correlated

- Regressing PS ratios against current margins yields the following

  PS = 81.36 - 7.54(Net Margin) R^2 = 0.04

  (0.49)

- This is not surprising. These firms are priced based upon expected margins, rather than current margins.

Solution 1: Use proxies for survival and growth: Amazon in early 2000

- Hypothesizing that firms with higher revenue growth and higher cash balances should have a greater chance of surviving and becoming profitable, we ran the following regression: (The level of revenues was used to control for size)

PS = 30.61 - 2.77 ln(Rev) + 6.42 (Rev Growth) + 5.11 (Cash/Rev)

(0.66) (2.63) (3.49)

R squared = 31.8%

Predicted PS = 30.61 - 2.77(7.1039) + 6.42(1.9946) + 5.11 (.3069) = 30.42

Actual PS = 25.63

Stock is undervalued, relative to other internet stocks.

Solution 2: Use forward multiples

- Global Crossing lost $1.9 billion in 2001 and is expected to continue to lose money for the next 3 years. In a discounted cashflow valuation (see notes on DCF valuation) of Global Crossing, we estimated an expected EBITDA for Global Crossing in five years of $ 1,371 million.
- The average enterprise value/ EBITDA multiple for healthy telecomm firms is 7.2 currently.
- Applying this multiple to Global Crossing's EBITDA in year 5, yields a value in year 5 of
  - Enterprise Value in year 5 = 1371 * 7.2 = $9,871 million
  - Enterprise Value today = $ 9,871 million/ 1.138^5 = $5,172 million

  (The cost of capital for Global Crossing is 13.80%)
  - The probability that Global Crossing will not make it as a going concern is 77%.
  - Expected Enterprise value today = 0.23 (5172) = $1,190 million

PS Regression: United States - January 2005

Model Summary

Model	R	R Square[a]	Adjusted R Square	Std. Error of the Estimate
1	.861[b]	.742	.741	1.940824345822851

a. For regression through the origin (the no-intercept model), R Square measures the proportion of the variability in the dependent variable about the origin explained by regression. This CANNOT be compared to R Square for models which include an intercept.

b. Predictors: Net Margin, Payout Ratio, 3-yr Regression Beta, Expected Growth in EPS: next 5 years

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	Expected Growth in EPS: next 5 years	5.164E-02	.004	.265	14.501	.000	.045	.059
	3-yr Regression Beta	7.050E-02	.054	.023	1.304	.192	-.036	.177
	Payout Ratio	-6.877E-03	.001	-.063	-4.734	.000	-.010	-.004
	Net Margin	.219	.005	.683	43.066	.000	.209	.229

a. Dependent Variable: PS Ratio

b. Linear Regression through the Origin

PS Regression: Emerging Markets in January 2005

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.713[a]	.509	.506	2.237

a. Predictors: (Constant), Net Margin, BETA, Payout Ratio, IBES Est 5 year growth

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.306E-02	.317		.168	.867
	IBES Est 5 year growth	2.315E-02	.004	.152	5.989	.000
	BETA	-.307	.292	-.027	-1.051	.294
	Payout Ratio	6.706E-04	.001	.027	1.077	.282
	Net Margin	.155	.005	.718	28.459	.000

a. Dependent Variable: PS

Choosing Between the Multiples

- As presented in this section, there are dozens of multiples that can be potentially used to value an individual firm.
- In addition, relative valuation can be relative to a sector (or comparable firms) or to the entire market (using the regressions, for instance)
- Since there can be only one final estimate of value, there are three choices at this stage:
  - Use a simple average of the valuations obtained using a number of different multiples
  - Use a weighted average of the valuations obtained using a nmber of different multiples
  - Choose one of the multiples and base your valuation on that multiple

Picking one Multiple

- This is usually the best way to approach this issue. While a range of values can be obtained from a number of multiples, the "best estimate" value is obtained using one multiple.
- The multiple that is used can be chosen in one of two ways:
  - Use the multiple that best fits your objective. Thus, if you want the company to be undervalued, you pick the multiple that yields the highest value.
  - Use the multiple that has the highest R-squared in the sector when regressed against fundamentals. Thus, if you have tried PE, PBV, PS, etc. and run regressions of these multiples against fundamentals, use the multiple that works best at explaining differences across firms in that sector.
  - Use the multiple that seems to make the most sense for that sector, given how value is measured and created.

A More Intuitive Approach

- Managers in every sector tend to focus on specific variables when analyzing strategy and performance. The multiple used will generally reflect this focus. Consider three examples.
  - In retailing: The focus is usually on same store sales (turnover) and profit margins. Not surprisingly, the revenue multiple is most common in this sector.
  - In financial services: The emphasis is usually on return on equity. Book Equity is often viewed as a scarce resource, since capital ratios are based upon it. Price to book ratios dominate.
  - In technology: Growth is usually the dominant theme. PEG ratios were invented in this sector.

In Practice…

- As a general rule of thumb, the following table provides a way of picking a multiple for a sector

Sector	*Multiple Used*	*Rationale*
Cyclical Manufacturing	PE, Relative PE	Often with normalized earnings
High Tech, High Growth	PEG	Big differences in growth across firms
High Growth/No Earnings	PS, VS	Assume future margins will be good
Heavy Infrastructure	VEBITDA	Firms in sector have losses in early years and reported earnings can vary depending on depreciation method
REITa	P/CF	Generally no cap ex investments from equity earnings
Financial Services	PBV	Book value often marked to market
Retailing	PS	If leverage is similar across firms
	VS	If leverage is different

Reviewing: The Four Steps to Understanding Multiples

- Define the multiple
  - Check for consistency
  - Make sure that they are estimated uniformly
- Describe the multiple
  - Multiples have skewed distributions: The averages are seldom good indicators of typical multiples
  - Check for bias, if the multiple cannot be estimated
- Analyze the multiple
  - Identify the companion variable that drives the multiple
  - Examine the nature of the relationship
- Apply the multiple

Real Options: Fact and Fantasy

Aswath Damodaran

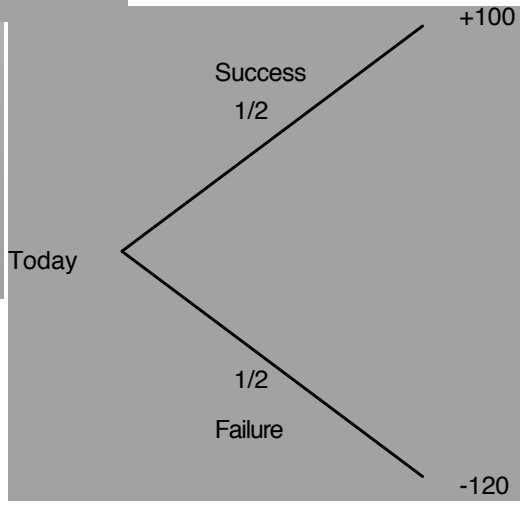
Underlying Theme: Searching for an Elusive Premium

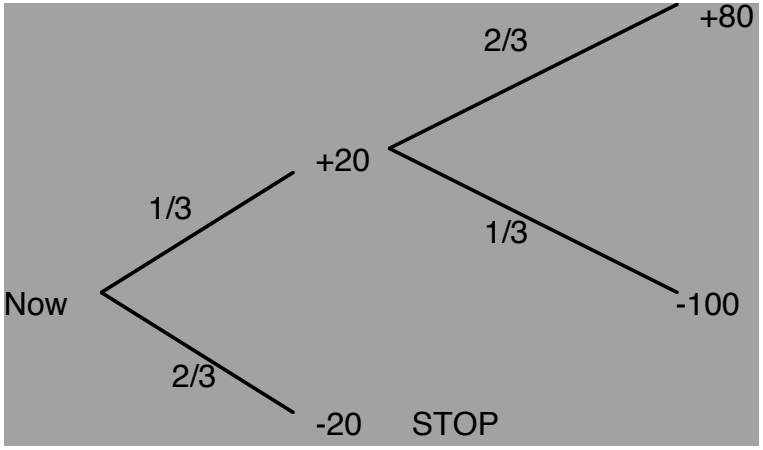
- Traditional discounted cashflow models under estimate the value of investments, where there are options embedded in the investments to
  - Delay or defer making the investment (delay)
  - Adjust or alter production schedules as price changes (flexibility)
  - Expand into new markets or products at later stages in the process, based upon observing favorable outcomes at the early stages (expansion)
  - Stop production or abandon investments if the outcomes are unfavorable at early stages (abandonment)
- Put another way, real option advocates believe that you should be paying a premium on discounted cashflow value estimates.

A Real Option Premium

- In the last few years, there are some who have argued that discounted cashflow valuations under valued some companies and that a real option premium should be tacked on to DCF valuations. To understanding its moorings, compare the two trees below:

A bad investment…….. Becomes a good one..





1. Learn at relatively low cost
2. Make better decisions based on learning

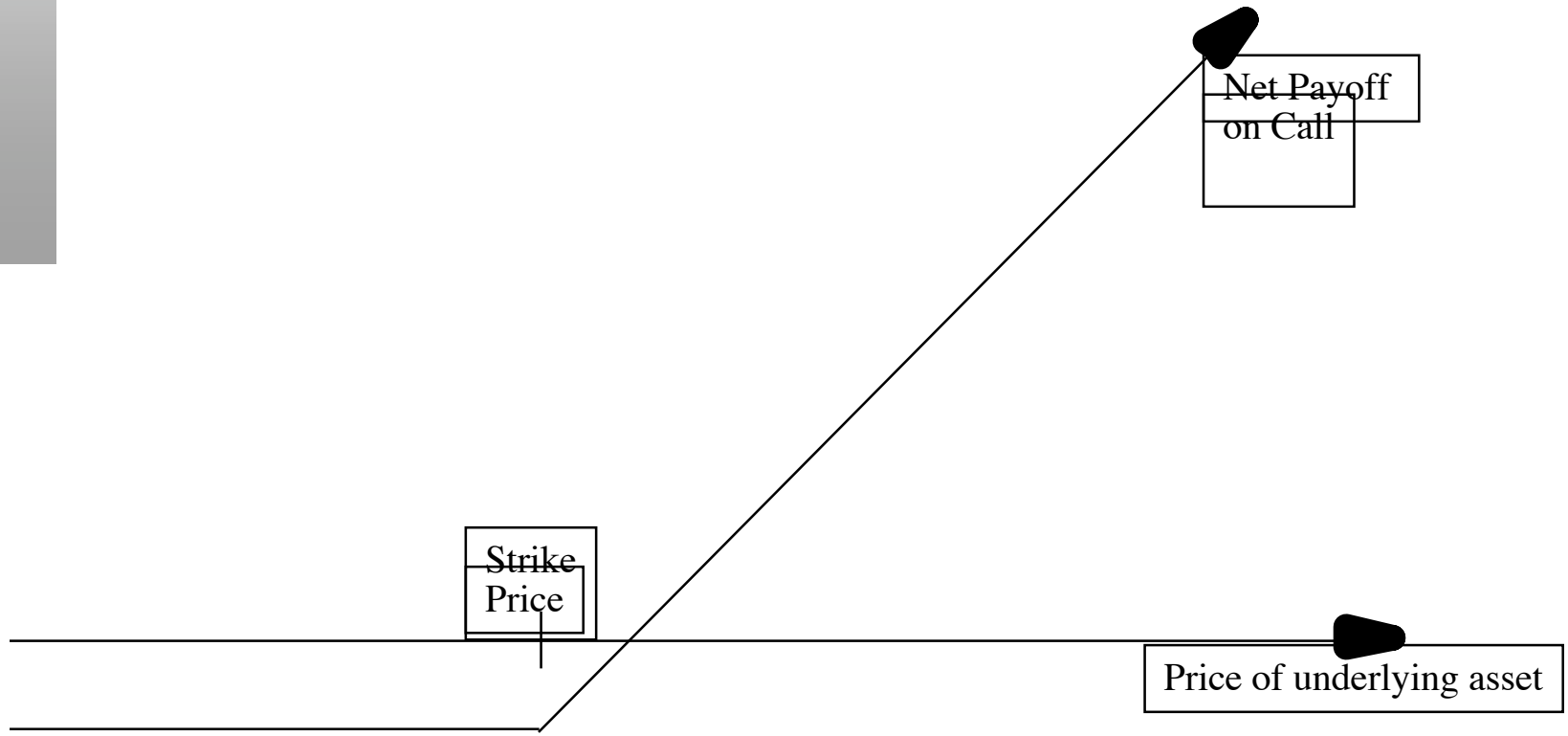
Three Basic Questions

- When is there a real option embedded in a decision or an asset?
- When does that real option have significant economic value?
- Can that value be estimated using an option pricing model?

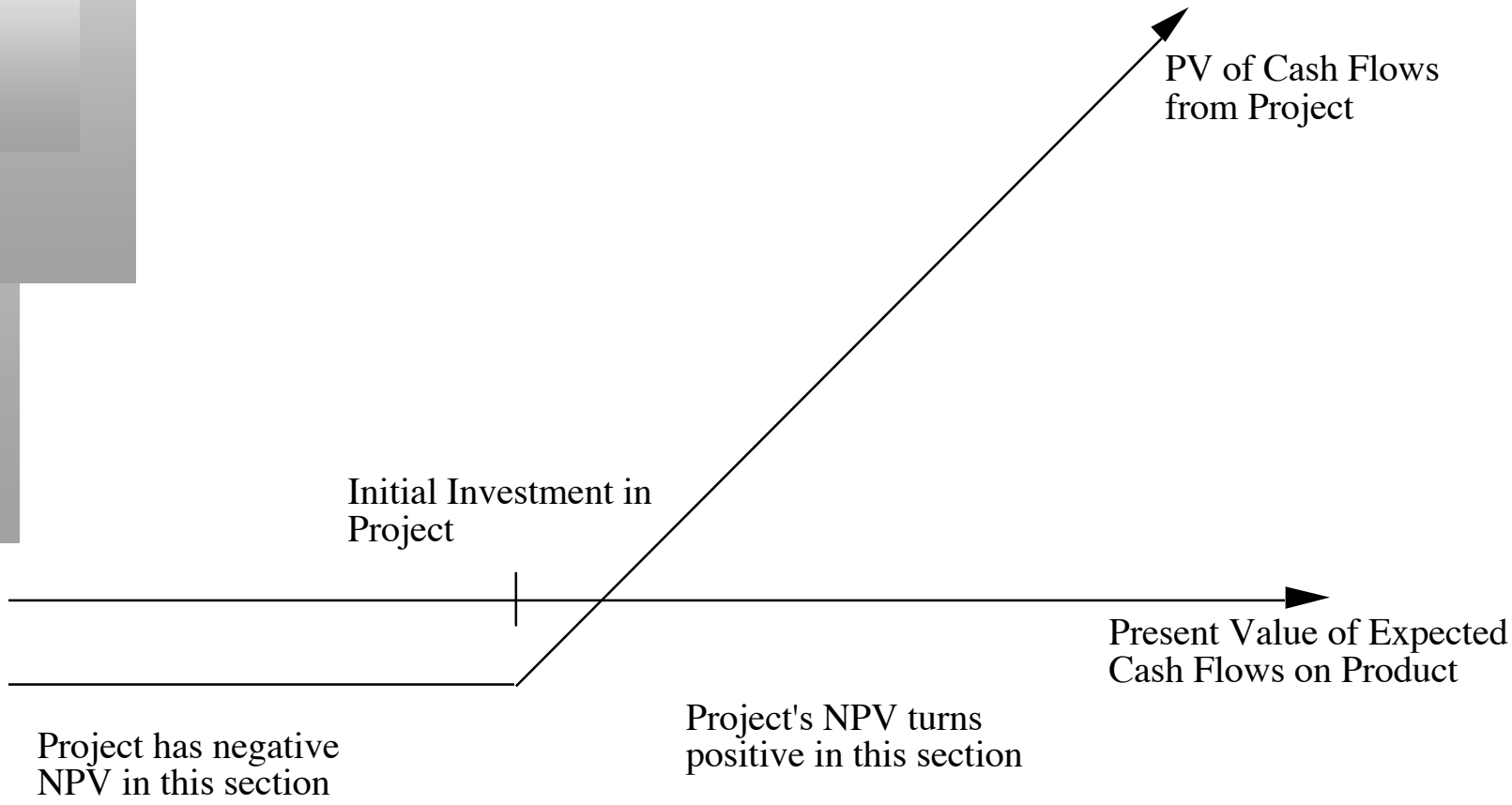
When is there an option embedded in an action?

- An option provides the holder with the **right** to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- There has to be a clearly defined underlying asset whose value changes over time in unpredictable ways.
- The payoffs on this asset (real option) have to be contingent on an specified event occurring within a finite period.

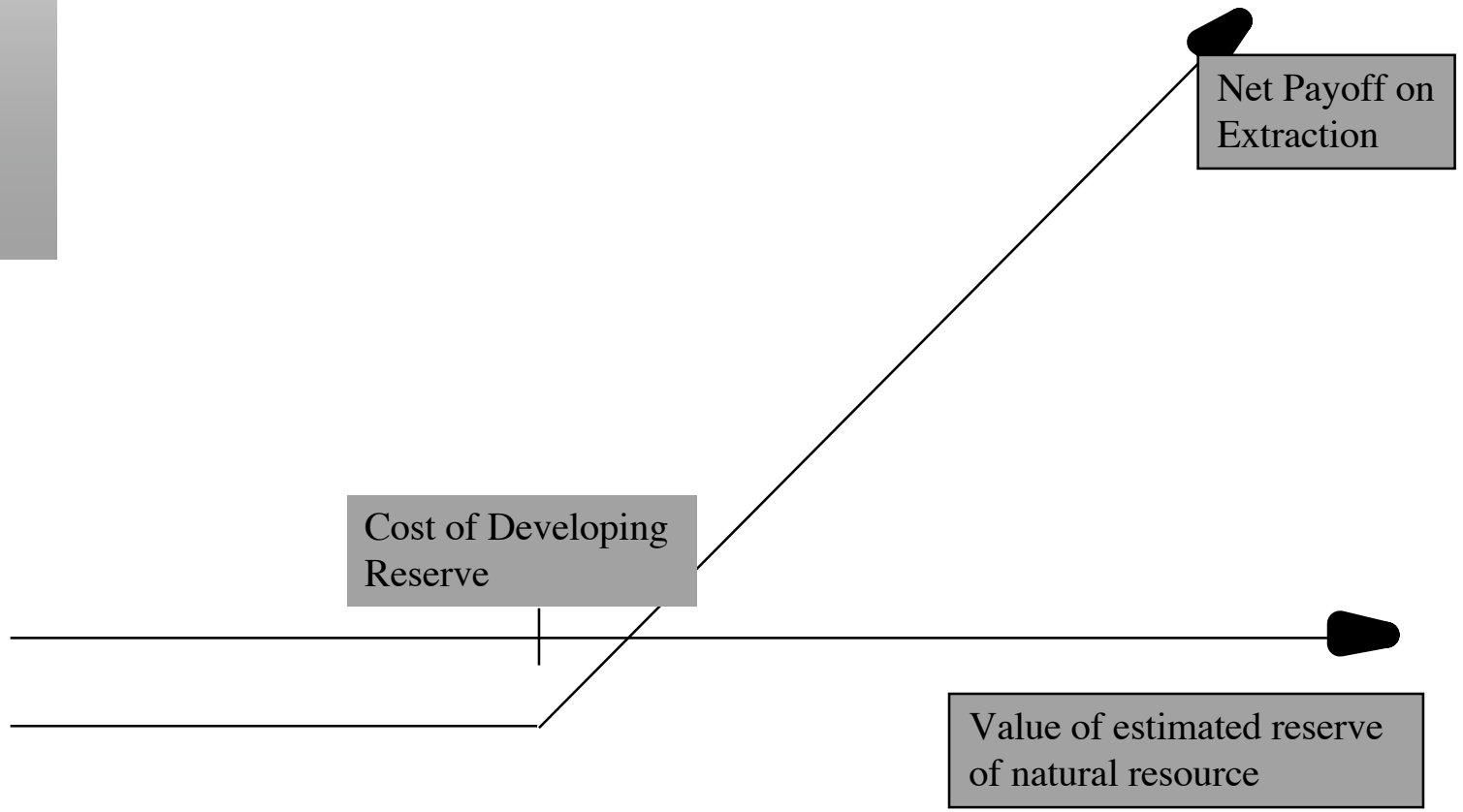
Payoff Diagram on a Call



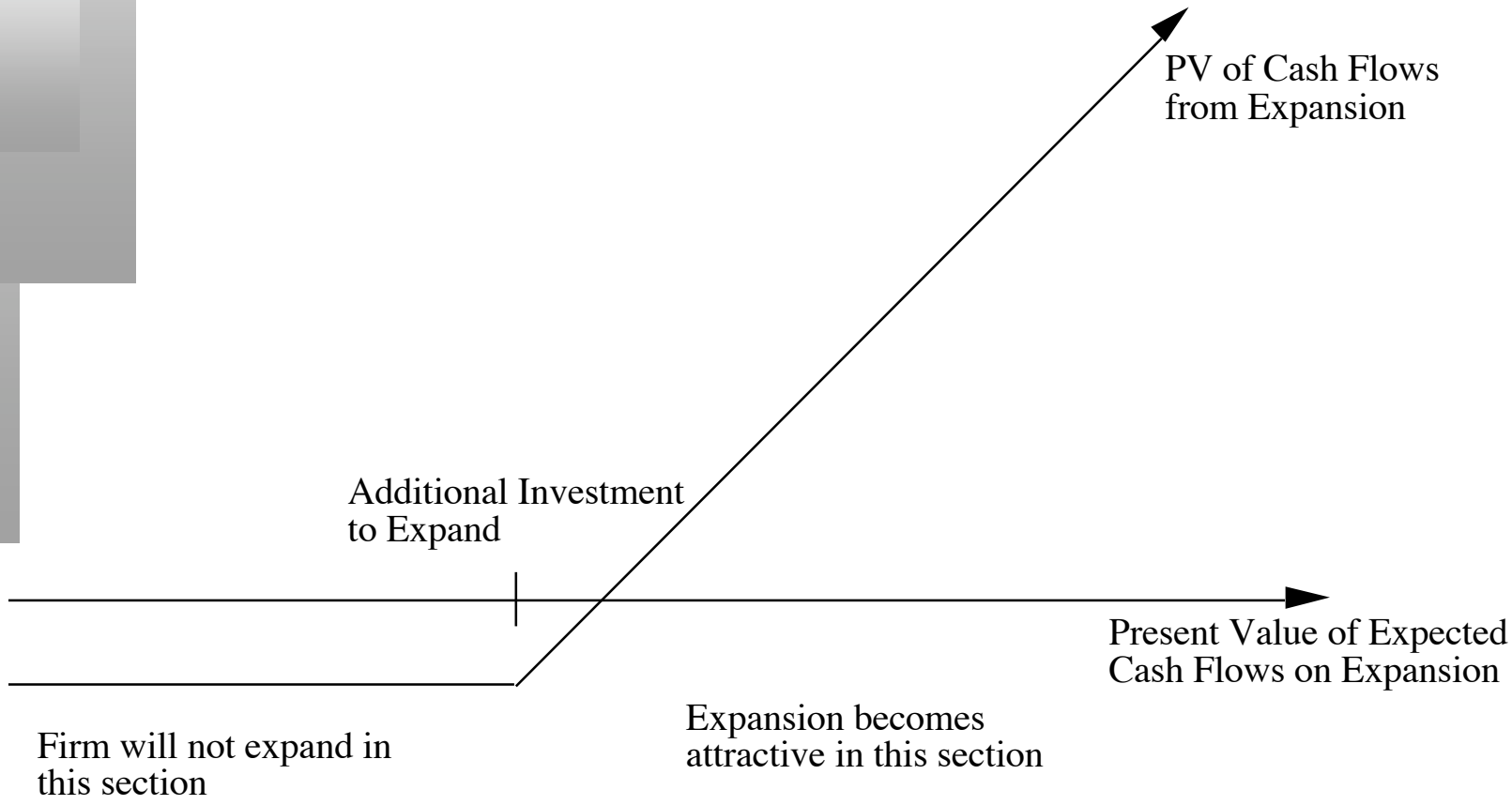
Example 1: Product Patent as an Option



Example 2: Undeveloped Oil Reserve as an option



Example 3: Expansion of existing project as an option



When does the option have significant economic value?

- For an option to have significant economic value, there has to be a restriction on competition in the event of the contingency. In a perfectly competitive product market, no contingency, no matter how positive, will generate positive net present value.
- At the limit, real options are most valuable when you have exclusivity - you and only you can take advantage of the contingency. They become less valuable as the barriers to competition become less steep.

Exclusivity: Putting Real Options to the Test

- Product Options: Patent on a drug
  - Patents restrict competitors from developing similar products
  - Patents do not restrict competitors from developing other products to treat the same disease.
- Natural Resource options: An undeveloped oil reserve or gold mine.
  - Natural resource reserves are limited.
  - It takes time and resources to develop new reserves
- Growth Options: Expansion into a new product or market
  - Barriers may range from strong (exclusive licenses granted by the government - as in telecom businesses) to weaker (brand name, knowledge of the market) to weakest (first mover).

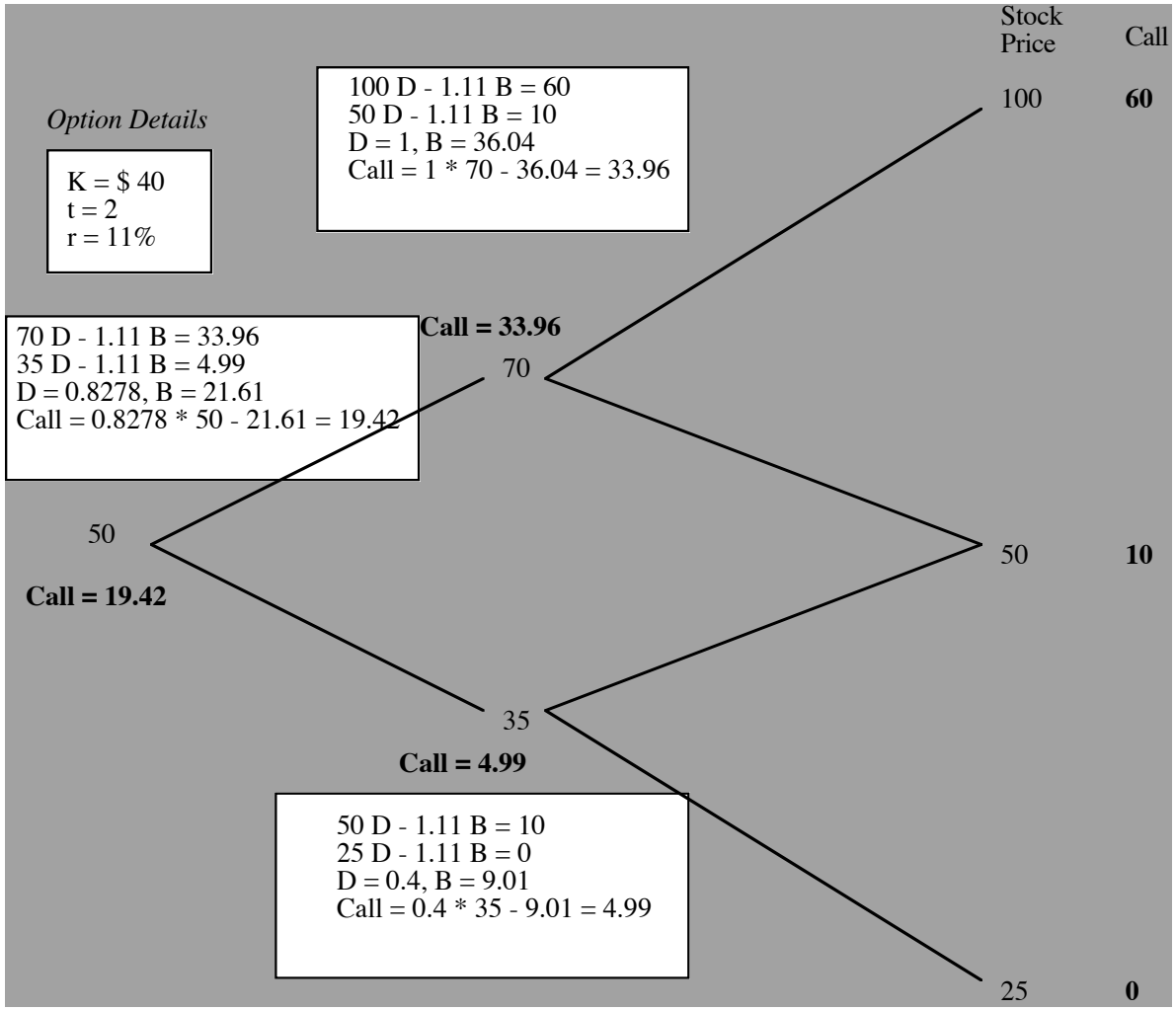
Determinants of option value

- Variables Relating to Underlying Asset
  - Value of Underlying Asset; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
  - Variance in that value; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
  - Expected dividends on the asset, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- Variables Relating to Option
  - Strike Price of Options; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
  - Life of the Option; both calls and puts benefit from a longer life.
- Level of Interest Rates; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

The Building Blocks for Option Pricing Models: Arbitrage and Replication

- The objective in creating a replicating portfolio is to use a combination of riskfree borrowing/lending and the underlying asset to create the same cashflows as the option being valued.
  - Call = Borrowing + Buying Δ of the Underlying Stock
  - Put = Selling Short Δ on Underlying Asset + Lending
  - The number of shares bought or sold is called the **option delta**.
- The principles of arbitrage then apply, and the value of the option has to be equal to the value of the replicating portfolio.

The Binomial Option Pricing Model



The Limiting Distributions….

- As the time interval is shortened, the limiting distribution, as t -> 0, can take one of two forms.
  - If as t -> 0, **price changes become smaller**, the limiting distribution is the normal distribution and the **price process is a continuous one**.
  - If as t->0, **price changes remain large**, the limiting distribution is the poisson distribution, i.e., a **distribution that allows for price jumps**.
- **The Black-Scholes model** applies when the **limiting distribution is the normal distribution** , and explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

The Black Scholes Model

Value of call = $S\, N(d_1) - K\, e^{-rt}\, N(d_2)$

where,

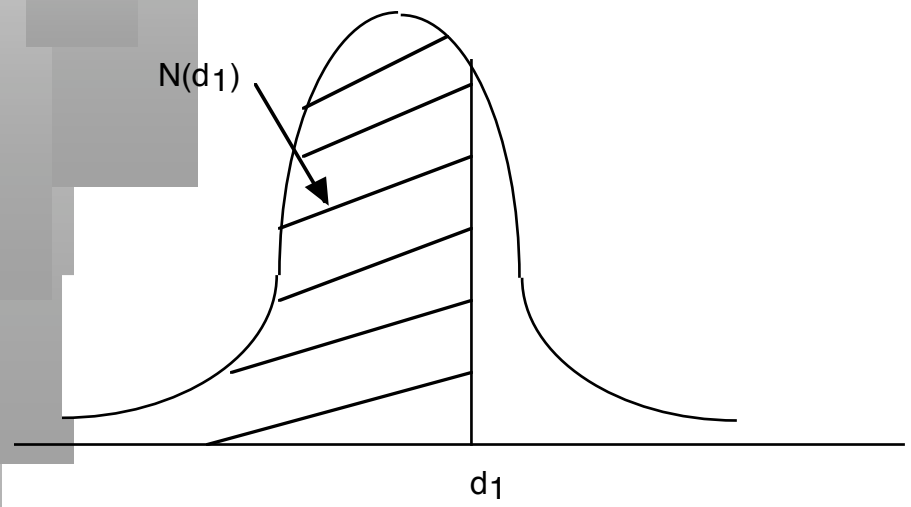
$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

- $d_2 = d_1 - \sigma \sqrt{t}$

■ The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to

- Buy N(d1) shares of stock; N(d1) is called the option delta
- Borrow $K\, e^{-rt}\, N(d_2)$

The Normal Distribution



d	N(d)	d	N(d)	d	N(d)
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

When can you use option pricing models to value real options?

- The notion of a replicating portfolio that drives option pricing models makes them most suited for valuing real options where
  - The underlying asset is traded - this yield not only observable prices and volatility as inputs to option pricing models but allows for the possibility of creating replicating portfolios
  - An active marketplace exists for the option itself.
  - The cost of exercising the option is known with some degree of certainty.
- When option pricing models are used to value real assets, we have to accept the fact that
  - The value estimates that emerge will be far more imprecise.
  - The value can deviate much more dramatically from market price because of the difficulty of arbitrage.

Valuing a Product Patent as an option: Avonex

- Biogen, a bio-technology firm, has a patent on Avonex, a drug to treat multiple sclerosis, for the next 17 years, and it plans to produce and sell the drug by itself. The key inputs on the drug are as follows:

  PV of Cash Flows from Introducing the Drug Now = S = $ 3.422 billion

  PV of Cost of Developing Drug for Commercial Use = K = $ 2.875 billion

  Patent Life = t = 17 years Riskless Rate = r = 6.7% (17-year T.Bond rate)

  Variance in Expected Present Values = σ^2 = 0.224 (Industry average firm variance for bio-tech firms)

  Expected Cost of Delay = y = 1/17 = 5.89%

  d1 = 1.1362 N(d1) = 0.8720

  d2 = -0.8512 N(d2) = 0.2076

Call Value= 3,422 $\exp^{(-0.0589)(17)}$ (0.8720) - 2,875 ($\exp^{(-0.067)(17)}$ (0.2076)= $ 907 million

Valuing an Oil Reserve

- Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil, where the cost of developing the reserve is $ 600 million today.
- The firm has the rights to exploit this reserve for the next twenty years and the marginal value per barrel of oil is $12 per barrel currently (Price per barrel - marginal cost per barrel). There is a 2 year lag between the decision to exploit the reserve and oil extraction.
- Once developed, the net production revenue each year will be 5% of the value of the reserves.
- The riskless rate is 8% and the variance in ln(oil prices) is 0.03.

Valuing an oil reserve as a real option

- Current Value of the asset = S = Value of the developed reserve discounted back the length of the development lag at the dividend yield = \$12 * 50 /$(1.05)^2$ = \$ 544.22
- (If development is started today, the oil will not be available for sale until two years from now. The estimated opportunity cost of this delay is the lost production revenue over the delay period. Hence, the discounting of the reserve back at the dividend yield)
- Exercise Price = Present Value of development cost = \$12 * 50 = \$600 million
- Time to expiration on the option = 20 years
- Variance in the value of the underlying asset = 0.03
- Riskless rate =8%
- Dividend Yield = Net production revenue / Value of reserve = 5%

Valuing the Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:

  d1 = 1.0359 N(d1) = 0.8498

  d2 = 0.2613 N(d2) = 0.6030

- Call Value= 544 .22 $\exp^{(-0.05)(20)}$ (0.8498) -600 ($\exp^{(-0.08)(20)}$ (0.6030)= $ 97.08 million
- This oil reserve, though not viable at current prices, still is a valuable property because of its potential to create value if oil prices go up.
- Extending this concept, the value of an oil company can be written as the sum of three values:

Value of oil company = Value of developed reserves (DCF valuation)
+ Value of undeveloped reserves (Valued as option)

An Example of an Expansion Option

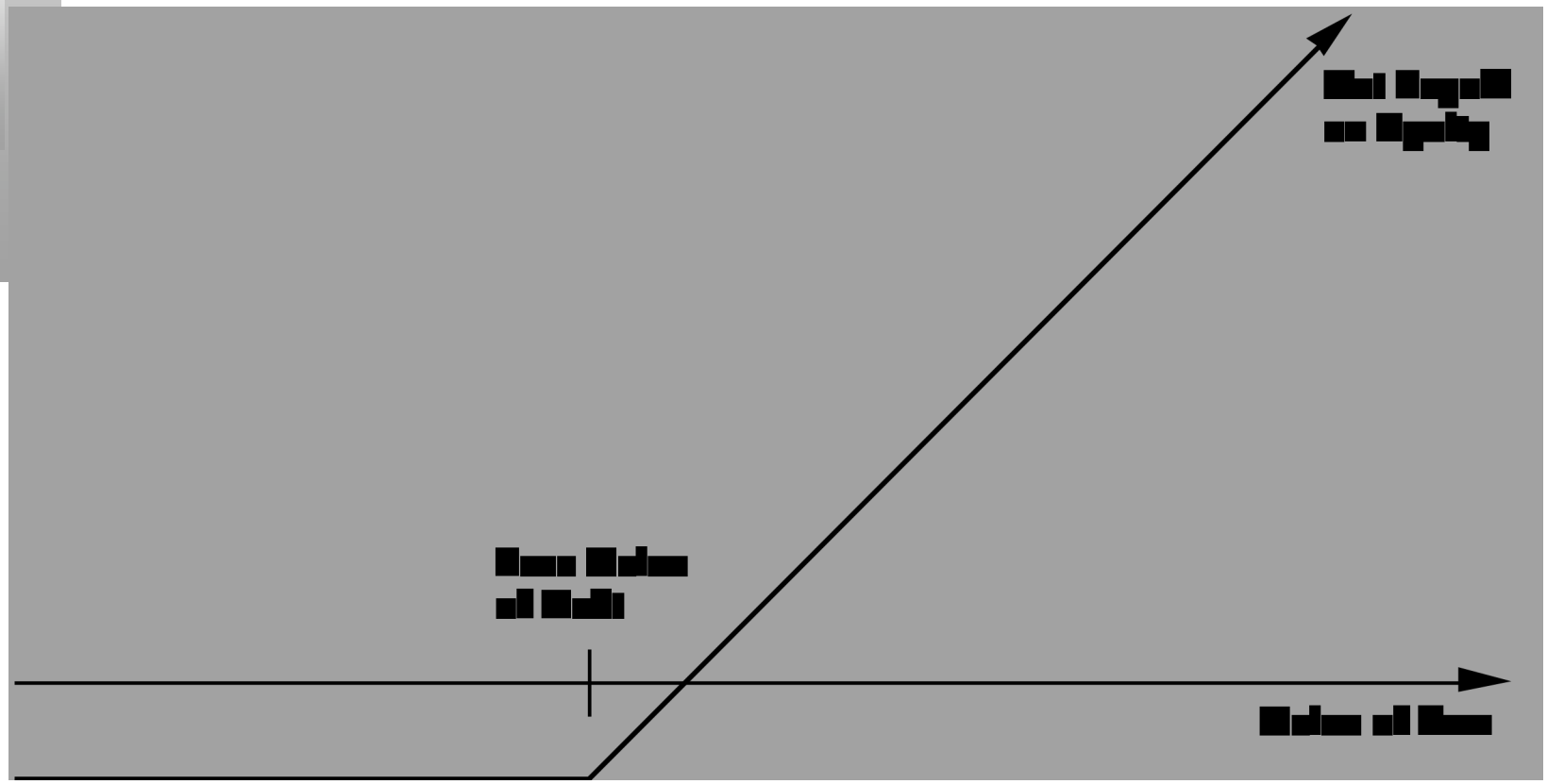
- Ambev is considering introducing a soft drink to the U.S. market. The drink will initially be introduced only in the metropolitan areas of the U.S. and the cost of this "limited introduction" is $ 500 million.
- A financial analysis of the cash flows from this investment suggests that the present value of the cash flows from this investment to Ambev will be only $ 400 million. Thus, by itself, the new investment has a **negative NPV of $ 100 million**.
- If the initial introduction works out well, Ambev **could go ahead with a full-scale introduction to the entire market** with **an additional investment of $ 1 billion** any time over the next 5 years. While the current expectation is that the cash flows from having this investment is only $ 750 million, there is considerable uncertainty about both the potential for the drink, leading to significant variance in this estimate.

Valuing the Expansion Option

- Value of the Underlying Asset (S) = PV of Cash Flows from Expansion to entire U.S. market, if done now =$ 750 Million
- Strike Price (K) = Cost of Expansion into entire U.S market = $ 1000 Million
- We estimate the standard deviation in the estimate of the project value by using the annualized standard deviation in firm value of publicly traded firms in the beverage markets, which is approximately 34.25%.
  - Standard Deviation in Underlying Asset's Value = 34.25%
- Time to expiration = Period for which expansion option applies = 5 years

Call Value= $ 234 Million

One final example: Equity as a Liquidatiion Option



Application to valuation: A simple example

- Assume that you have a firm whose assets are currently valued at $100 million and that the standard deviation in this asset value is 40%.
- Further, assume that the face value of debt is $80 million (It is zero coupon debt with 10 years left to maturity).
- If the ten-year treasury bond rate is 10%,
  - how much is the equity worth?
  - What should the interest rate on debt be?

Valuing Equity as a Call Option

- Inputs to option pricing model
  - Value of the underlying asset = S = Value of the firm = $ 100 million
  - Exercise price = K = Face Value of outstanding debt = $ 80 million
  - Life of the option = t = Life of zero-coupon debt = 10 years
  - Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16
  - Riskless rate = r = Treasury bond rate corresponding to option life = 10%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:
  - d1 = 1.5994 N(d1) = 0.9451
  - d2 = 0.3345 N(d2) = 0.6310
- Value of the call = 100 (0.9451) - 80 $\exp^{(-0.10)(10)}$ (0.6310) = $75.94 million
- Value of the outstanding debt = $100 - $75.94 = $24.06 million
- Interest rate on debt = ($ 80 / $24.06)$^{1/10}$ -1 = 12.77%

The Effect of Catastrophic Drops in Value

- Assume now that a catastrophe wipes out half the value of this firm (the value drops to $ 50 million), while the face value of the debt remains at $ 80 million. What will happen to the equity value of this firm?
  - It will drop in value to $ 25.94 million [$ 50 million - market value of debt from previous page]
  - It will be worth nothing since debt outstanding > Firm Value
  - It will be worth more than $ 25.94 million

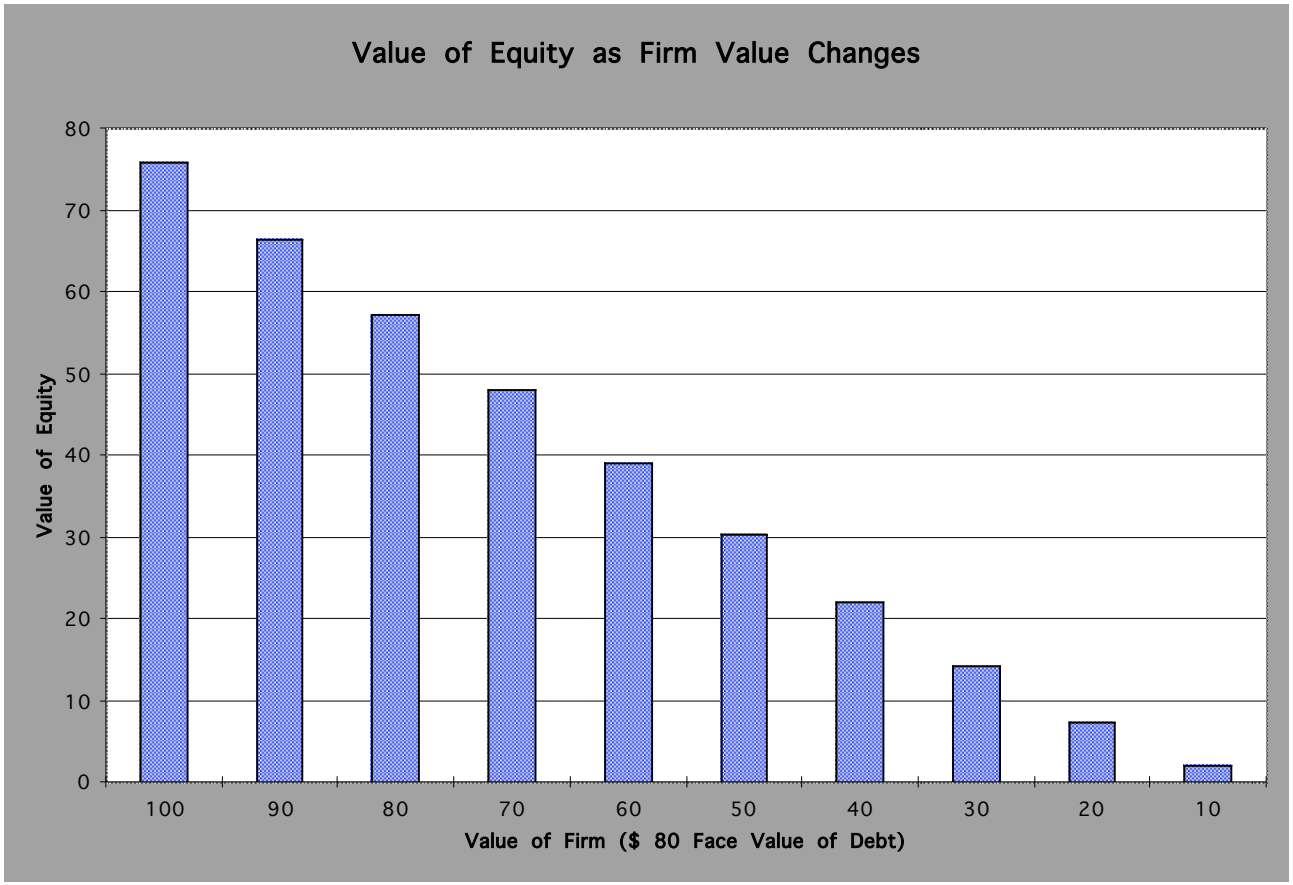
Valuing Equity in the Troubled Firm

- Value of the underlying asset = S = Value of the firm = $ 50 million
- Exercise price = K = Face Value of outstanding debt = $ 80 million
- Life of the option = t = Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16
- Riskless rate = r = Treasury bond rate corresponding to option life = 10%

The Value of Equity as an Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
  - d1 = 1.0515 N(d1) = 0.8534
  - d2 = -0.2135 N(d2) = 0.4155
- Value of the call = 50 (0.8534) - 80 $\exp^{(-0.10)(10)}$ (0.4155) = $30.44 million
- Value of the bond= $50 - $30.44 = $19.56 million
- The equity in this firm drops by, because of the option characteristics of equity.
- This might explain why stock in firms, which are in Chapter 11 and essentially bankrupt, still has value.

Equity value persists ..



Obtaining option pricing inputs in the real worlds

Input	Estimation Process
Value of the Firm	• Cumulate market values of equity and debt (or) • Value the assets in place using FCFF and WACC (or) • Use cumulated market value of assets, if traded.
Variance in Firm Value	• If stocks and bonds are traded, $\sigma^2_{firm} = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2 w_e w_d \rho_{ed} \sigma_e \sigma_d$ where σ_e^2 = variance in the stock price w_e = MV weight of Equity σ_d^2 = the variance in the bond price w_d = MV weight of debt • If not traded, use variances of similarly rated bonds. • Use average firm value variance from the industry in which company operates.
Value of the Debt	• If the debt is short term, you can use only the face or book value of the debt. • If the debt is long term and coupon bearing, add the cumulated nominal value of these coupons to the face value of the debt.
Maturity of the Debt	• Face value weighted duration of bonds outstanding (or) • If not available, use weighted maturity

Valuing Equity as an option - Eurotunnel in early 1998

- Eurotunnel has been a financial disaster since its opening
  - In 1997, Eurotunnel had earnings before interest and taxes of -£56 million and net income of -£685 million
  - At the end of 1997, its book value of equity was -£117 million
- It had £8,865 million in face value of debt outstanding
  - The weighted average duration of this debt was 10.93 years

Debt Type	Face Value	Duration
Short term	935	0.50
10 year	2435	6.7
20 year	3555	12.6
Longer	1940	18.2
Total	*£8,865 mil*	*10.93 years*

The Basic DCF Valuation

- The value of the firm estimated using projected cashflows to the firm, discounted at the weighted average cost of capital was £2,312 million.
- This was based upon the following assumptions –
  - Revenues will grow 5% a year in perpetuity.
  - The COGS which is currently 85% of revenues will drop to 65% of revenues in yr 5 and stay at that level.
  - Capital spending and depreciation will grow 5% a year in perpetuity.
  - There are no working capital requirements.
  - The debt ratio, which is currently 95.35%, will drop to 70% after year 5. The cost of debt is 10% in high growth period and 8% after that.
  - The beta for the stock will be 1.10 for the next five years, and drop to 0.8 after the next 5 years.
  - The long term bond rate is 6%.

Other Inputs

- The stock has been traded on the London Exchange, and the annualized std deviation based upon ln (prices) is 41%.
- There are Eurotunnel bonds, that have been traded; the annualized std deviation in ln(price) for the bonds is 17%.
  - The correlation between stock price and bond price changes has been 0.5. The proportion of debt in the capital structure during the period (1992-1996) was 85%.
  - Annualized variance in firm value

  $= (0.15)^2 (0.41)^2 + (0.85)^2 (0.17)^2 + 2 (0.15) (0.85)(0.5)(0.41)(0.17) = 0.0335$
- The 15-year bond rate is 6%. (I used a bond with a duration of roughly 11 years to match the life of my option)

Valuing Eurotunnel Equity and Debt

- Inputs to Model
  - Value of the underlying asset = S = Value of the firm = £2,312 million
  - Exercise price = K = Face Value of outstanding debt = £8,865 million
  - Life of the option = t = Weighted average duration of debt = 10.93 years
  - Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.0335
  - Riskless rate = r = Treasury bond rate corresponding to option life = 6%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:

  d1 = -0.8337 N(d1) = 0.2023

  d2 = -1.4392 N(d2) = 0.0751
- Value of the call = 2312 (0.2023) - 8,865 $\exp^{(-0.06)(10.93)}$ (0.0751) = £122 million
- Appropriate interest rate on debt = $(8865/2190)^{(1/10.93)}$-1= 13.65%

Back to Lemmings...