Danger and Opportunity: Ruminations on Risk

Aswath Damodaran

www.damodaran.com

Here is a good definition of risk…

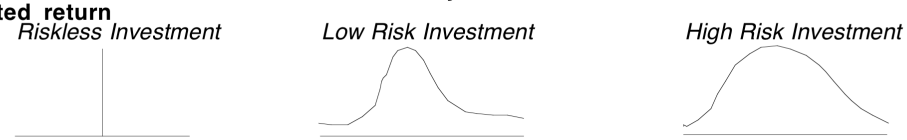
- Risk, in traditional terms, is viewed as a 'negative'. Webster's dictionary, for instance, defines risk as "exposing to danger or hazard". The Chinese symbols for crisis, reproduced below, give a much better description of risk.

- The first symbol is the symbol for "danger", while the second is the symbol for "opportunity", making risk a mix of danger and opportunity.

Risk and Return Models in Finance

Step 1: Defining Risk

The risk in an investment can be measured by the variance in actual returns around an expected return



Step 2: Differentiating between Rewarded and Unrewarded Risk

Risk that is specific to investment (Firm Specific)
Can be diversified away in a diversified portfolio
1. each investment is a small proportion of portfolio
2. risk averages out across investments in portfolio

Risk that affects all investments (Market Risk)
Cannot be diversified away since most assets are affected by it.

The marginal investor is assumed to hold a "diversified" portfolio. Thus, only market risk will be rewarded and priced.

Step 3: Measuring Market Risk

The CAPM	**The APM**	**Multi-Factor Models**	**Proxy Models**
If there is 1. no private information 2. no transactions cost the optimal diversified portfolio includes every traded asset. Everyone will hold this market portfolio **Market Risk = Risk added by any investment to the market portfolio:**	If there are no arbitrage opportunities then the market risk of any asset must be captured by betas relative to factors that affect all investments. **Market Risk = Risk exposures of any asset to market factors**	Since market risk affects most or all investments, it must come from macro economic factors. **Market Risk = Risk exposures of any asset to macro economic factors.**	In an efficient market, differences in returns across long periods must be due to market risk differences. Looking for variables correlated with returns should then give us proxies for this risk. **Market Risk = Captured by the Proxy Variable(s)**
Beta of asset relative to Market portfolio (from a regression)	Betas of asset relative to unspecified market factors (from a factor analysis)	Betas of assets relative to specified macro economic factors (from a regression)	Equation relating returns to proxy variables (from a regression)

The Mean Variance Framework… Convenience at a price…

- In the mean variance framework, there are only two dimensions on which we measure investments – expected returns (good… opportunity) and variance/ standard deviation (bad… danger).
- There are two ways in which we can justify a mean-variance framework:
  - Investors have utility functions for wealth that are derive their value from only expected return and standard deviation.
  - Returns are normally distributed… or log returns are normally distributed… or some version of returns is normally distributed.…
- If both these assumptions do not hold up, the mean variance framework breaks down. In effect, investors will no longer view investing as a trade off between expected return and standard deviation. Two examples:
  - Skewness preference models: Investors will pay extra for the possibility of large payoffs, even if such a possibility is low (small firms, distressed stocks)
  - Power distributions, which allow for much higher probabilities of large jumps in either direction. That "jump" probability adds to risk and affects expected returns.

Stand alone risk versus risk added to a portfolio.. The Markowitz Revolution..

- The key insight: The risk in an investment is not the risk of it standing alone but the risk that it adds to your overall investment portfolio:
- If you live in a mean-variance world, the risk in an investment becomes the risk added to what you have in your portfolio.
- The marginal investors in an investment are those investors who set prices at the margin, i.e., own enough of the investment and trade on that holding to make a difference. If we assume that the marginal investors (those investors who at the margin are most likely to be setting prices for stocks) are diversified, the risk of an investment becomes the risk added to a diversified portfolio (measured as the covariance of returns on the investment with the returns on the portfolio).
- Conclusion: If the marginal investors are diversified, the "rewarded" risk in an investment should be the risk that cannot be diversified away.

The CAPM…Sharpe and Lintner to the rescue..

- Now assume that we add two items to the mix:
  - A riskless asset with an E(R) = R_r and $\sigma_f = 0$
  - No transactions costs and no private information
- We obtain the following:
  - There is no logical limit to diversification and we all end up holding every single traded asset, held in proportion to market value. This supremely diversified portfolio is the market portfolio, with E(R) = E(R_m) and a variance of σ^2_m.
  - The risk in an individual investment (j) then becomes the risk added to this market portfolio and is measured as follows:

    Risk added to market portfolio by asset j = $\rho_{jm}\sigma_m\sigma_j$
  - Scaling the risk of an individual investment to the risk of the market, we get:

$$\text{Scaled Risk} = \frac{\rho_{jm}\sigma_m\sigma_j}{\sigma^2_m} = \frac{\rho_{jm}\sigma_j}{\sigma_m} = Beta_j$$

A closer look at Betas…

- One way to interpret the beta is as a relative standard deviation measure:

$$\frac{\rho_{jm}\sigma_j}{\sigma_m}$$

$\rho_{jm}\sigma_j$ ← Non-diversifiable portion of standard deviation in investment j

σ_m ← Standard deviation of market portfolio

In the CAPM, the expected return on an individual investment can then be written in beta units as follows:

Expected Return in beta units = $R_f + \text{Beta}_j\,(E(R_m) - R_f)$

- We can rewrite the same equation in standard deviation units:

Expected Return in std deviation units = $R_f + \rho_{jm}\sigma_j \left[\frac{E(R_m) - R_f}{\sigma_m}\right]$

$\rho_{jm}\sigma_j$: # units of market standard deviation in investment j

$\left[\frac{E(R_m) - R_f}{\sigma_m}\right]$: Price per unit of market standard deviation

An undiversified investor in a diversified world..

- Now assume that you introduce a rebel into this CAPM world, who instead of holding the market portfolio, chooses to invest all of his wealth in his own private business j (with an E(R)= E(R_j) and standard deviation of σ_j). His portfolio is composed only of this investment and the standard deviation is:

  Standard deviation in investment j = σ_j

- Since he operates in a CAPM world, the price of risk is set by the market at

$$\frac{E(R_m) - R_f}{\sigma_m}$$

- If he wants to earn a return that is comparable to what he would made as a diversified investor:

$$E(R) = R_f + \sigma_j * \frac{E(R_m) - R_f}{\sigma_m}$$

$$= R_f + \frac{\sigma_j}{\sigma_m}(E(R_m) - R_f)$$

Contrasting Views of Risk

	CAPM follower	CAPM Rebel
Holds	Market Portfolio	Just investment j
Risk in investment j	$\frac{\rho_{jm}\sigma_j}{\sigma_m} = Beta$	$\frac{\sigma_j}{\sigma_m}$ = Total Beta
Expected Return	R_f + $Beta_j$ (E(R_m) – R_f)	R_f + Total $Beta_j$ (E (R_m) – R_f)

$$\text{Total Beta} = \frac{\text{Market Beta}}{\rho_{jm}}$$

And Consequences..

- A diversified investor will see less risk in the same investment than an undiversified investor looking at that investment.
- If these investors have to face the same market price per risk, the diversified investor will demand a lower expected return (and discount rate) for the same investment as an undiversified investor.
- If the investors have the same expectations of cash flows from the asset, the diversified investor will pay a higher price for the same asset than an undiversified investor.

Implication 1: When selling a private business or asset, the best potential buyer, other things remaining equal, will be a diversified investor or an entity with diversified investors (a publicly traded firm).

Implication 2: Private business owners who are fully invested in their own businesses are holding on to these businesses at a discount, especially if going public or selling to a publicly traded company is an option.

A diversification continuum..

- Assume that you have a private business operating in a sector, where publicly traded companies have an average beta of 1 and where the average correlation of firms with the market is 0.25. Consider the cost of equity at three stages in the process (Riskfree rate = 4%; ERP = 5%):

Stage 1: The nascent business, with a private owner, who is fully invested in that business.

Perceived Beta = 1/ 0.25 = 4

Cost of Equity = 4% + 4 (5%) = 24%

Stage 2: Angel financing provided by specialized venture capitalist, who holds multiple investments, in high technology companies. (Correlation of portfolio with market is 0.5)

Perceived Beta = 1/0.5 = 2

Cost of Equity = 4% + 2 (5%) = 14%

Stage 3: Public offering, where investors are retail and institutional investors, with diversified portfolios:

Perceived Beta = 1

Cost of Equity = 4% + 1 (5%) = 9%

To value this company…

Assume that this company will be fully owned by its current owner for two years, will access the technology venture capitalist at the start of year 3 and that is expected to either go public or be sold to a publicly traded firm at the end of year 5.

Growth rate 2% forever after year 5

	1	2	3	4	5	Terminal year
E(Cash flow)	$100	$125	$150	$165	$170	$175
Market beta	1	1	1	1	1	1
Correlation	0.25	0.25	0.5	0.5	0.5	1
Beta used	4	4	2	2	2	1
Cost of equity	24.00%	24.00%	14.00%	14.00%	14.00%	9.00%
Terminal value					$2,500	
Cumulated COE	1.2400	1.5376	1.7529	1.9983	2.2780	2.4830
PV	$80.65	$81.30	$85.57	$82.57	$1,172.07	

175/ (.09-.02)

Value of firm	$1,502

(Correct value, using changing costs of equity)

Value of firm	$1,221

(using 24% as cost of equity forever. You will undervalue firm)

Value of firm	$2,165

(Using 9% as cost of equity forever. You will overvalue firm)

Here are reasons why you might not like total beta…

- You do not like the mean-variance framework.
  - Build models that carry other measures of opportunity (skewness or the possibility of high payoffs) and danger (kurtosis, or dangers of jumps).
- You do not believe that the market price for risk is set by diversified investors. In other words, you believe that the bulk of the trading done in markets is by undiversified investors and the bulk of the assets are held by these investors.
  - The relationship between expected return and risk may no longer be linear.
  - Relate expected returns to micro factors (not macro factors). Proxy models that are built only on firm-specific characteristics (such as market cap) do this.
- You do not like the assumptions of the CAPM, i.e., no transactions costs and no private information.
  - Multifactor models that try to capture the risks that the CAPM does not capture, but only macro risks.
- You do not believe that private business owners set expected returns based upon traded assets in the market place.

Even if you do believe in total beta… notes of caution…

- Total beta should provide little explanatory power for expected returns at publicly traded firms, especially those that are widely held by institutions and have large market cap.
- It is not the appropriate measure of risk if an asset is being valued to a potential buyer, who is partially or mostly diversified. Thus, when valuing a private business for sale to a publicly traded company or even to a partially diversified investor, it is not appropriate to use total beta (and cost of equity).
- If asked to assess fair value, where fair value is the value to the best potential buyer of a business, using total beta is unlikely to provide the answer, unless you happen to be in a business where all of the potential buyers are undiversified.

And consider the alternative…

- The alternative to the total beta model seems to be the build up model, where you start with the CAPM and proceed to add premiums for various aspects of private firms that you feel are being ignored, including:
  - Size: Private firms are smaller than public firms – hence, a small firm premium
  - Illiquidity: Private firms are not traded – hence, an illiquidity discount
  - Sector premiums: In some cases, sector premiums are added on.
- Build up models are recipes for disaster and here is why:
  - Dependence on historical data
  - Double counting or triple counting of risk
  - Internal inconsistencies

Does it actually work?

- With publicly traded companies, here are some implications that are testable:
  - While betas should explain differences in returns across larger, widely held, publicly traded companies, standard deviations should explain differences in returns across smaller, closely held companies. (The investors in the latter are likely to be less diversified).
  - It follows then that when we look at excess returns earned by companies, when those excess returns are measured relative to the CAPM, smaller, closely held firms should earn excess returns.
- With private businesses, here are some implications:
  - The transaction prices and multiples paid for private businesses in sectors where there are diversified buyers (publicly traded companies) should be higher than the transaction prices and multiples paid in sectors where these buyers do not exist.
  - As venture capitalists and private equity investors become more diversified, the values of private businesses that depend on these investors for capital should increase.

The bottom line..

- The degree to which the buyer of an asset is diversified will affect the risk he or she perceives in that asset and by extension the value.
- If you stay within the parameters of the CAPM and assume that private business owners operate at the margin, are completely undiversified and have to take market prices set by the public market, total beta is a defensible measure..
- If you abandon those assumptions, then the task becomes more arduous. However, build-up models whose sole objective seems to arrive at a high enough rate (which you can still legally defend) are not the alternative.
- Finally, the diversification discount is separate from the illiquidity discount. In other words, it is perfectly logical to use a higher discount rate to capture the absence of diversification and also apply an illiquidity discount to value. The same cannot be said of build up models.

Dealing with Risk in Analysis

Ways of dealing with risk in analysis

- Risk Adjusted Value
  - Estimate expected cash flows and adjust the discount rate for risk
  - Use certainty equivalent cash flows and use the riskfree rate as the discount rate
  - Hybrid approaches
- Probabilistic Approaches
  - Sensitivity Analysis
  - Decision Trees
  - Simulations
- Value at Risk (VAR) and variants
- Real Options

I. Risk Adjusted Value

- The value of a risky asset can be estimated by discounting the expected cash flows on the asset over its life at a risk-adjusted discount rate:

$$\text{Value of asset} = \frac{E(CF_1)}{(1+r)} + \frac{E(CF_2)}{(1+r)^2} + \frac{E(CF_3)}{(1+r)^3} ..... + \frac{E(CF_n)}{(1+r)^n}$$

where the asset has a n-year life, $E(CF_t)$ is the expected cash flow in period t and r is a discount rate that reflects the risk of the cash flows.

- Alternatively, we can replace the expected cash flows with the guaranteed cash flows we would have accepted as an alternative (certainty equivalents) and discount these at the riskfree rate:

$$\text{Value of asset} = \frac{CE(CF_1)}{(1+r_f)} + \frac{CE(CF_2)}{(1+r_f)^2} + \frac{CE(CF_3)}{(1+r_f)^3} ..... + \frac{CE(CF_n)}{(1+r_f)^n}$$

where $CE(CF_t)$ is the certainty equivalent of $E(CF_t)$ and r_f is the riskfree rate.

a. Risk Adjusted Discount Rates

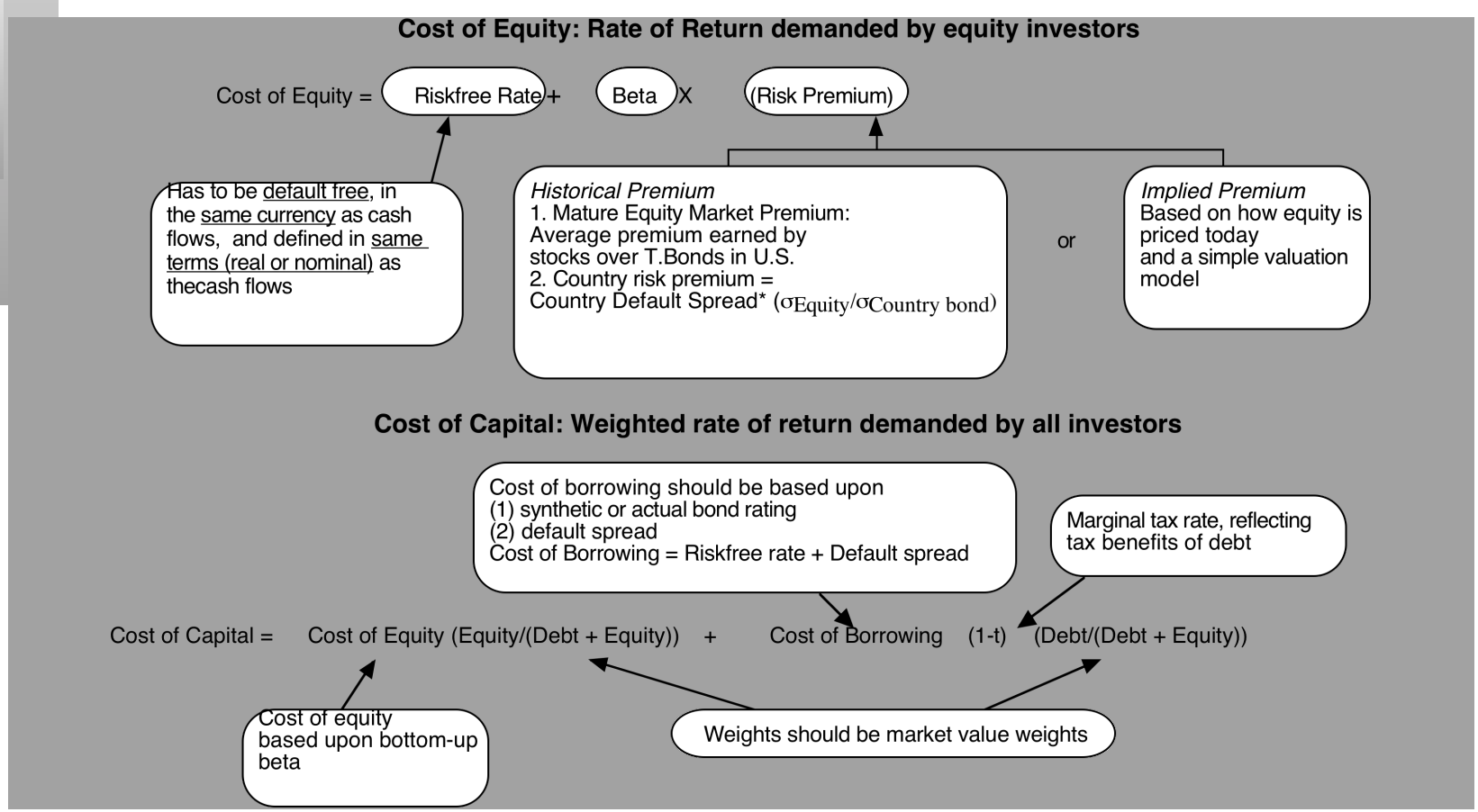
Step 1: Estimate the expected cash flows from a project/asset/business. If there is risk in the asset, this will require use to consider/estimate cash flows under different scenarios, attach probabilities to these scenarios and estimate an expected value across scenarios. In most cases, though, it takes the form of a base case set of estimates that capture the range of possible outcomes.

Step 2: Estimate a risk-adjusted discount rate. While there are a number of details that go into this estimate, you can think of a risk-adjusted discount rate as composed of two components

Risk adjusted rate = Riskfree Rate + Risk Premium

Step 3: Take the present value of the cash flows at the risk adjusted discount rate.

A primer on risk adjusted discount rates

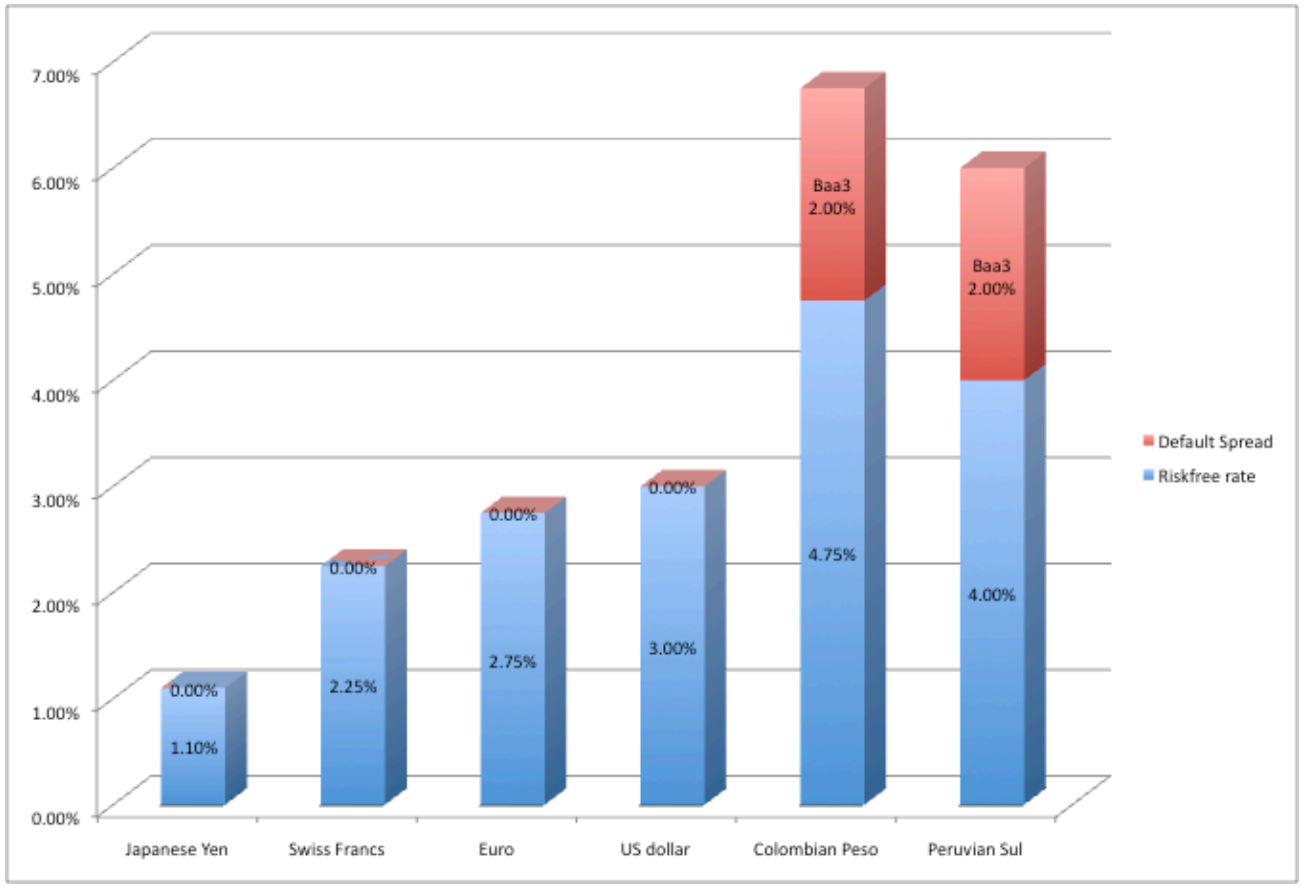


i. A Riskfree Rate

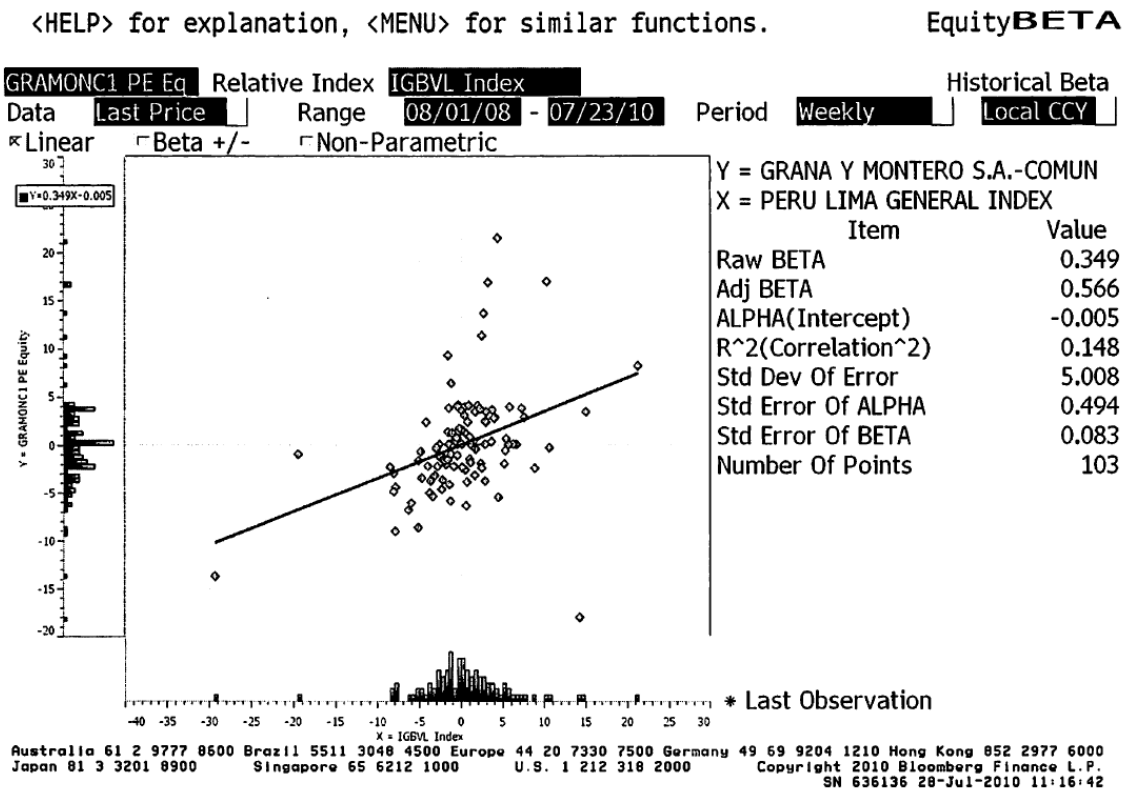
- On a riskfree asset, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have
  - No default risk
  - No reinvestment risk

1. Time horizon matters: Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time.
2. Not all government securities are riskfree: Some governments face default risk and the rates on bonds issued by them will not be riskfree.

Comparing Riskfree Rates



ii. Beta Estimation: A regression is not the answer…



One solution: Estimate sector (bottom up) betas
Disney's bottom up beta

- Estimate the bottom up unlevered beta for Disney's operating assets.

Business	Revenues in 2008	EV/Sales	Estimated Value	Firm Value Proportion	Unlevered beta
Media Networks	$16,116	2.13	$34,327.78	58.92%	0.7056
Parks and Resorts	$11,504	1.51	$17,408.14	29.88%	0.5849
Studio Entertainment	$7,348	0.78	$5,754.86	9.88%	1.3027
Consumer Products	$2,875	0.27	$768.20	1.32%	1.0690
Disney	$37,843		$58,258.99	100.00%	0.7333

Step 1: Start with Disney's revenues by business.

Step 2: Estimate the value as a multiple of revenues by looking at what the market value of publicly traded firms in each business is, relative to revenues.

$$\text{EV/Sales} = \frac{\text{Mkt Equity} + \text{Debt} - \text{Cash}}{\text{Revenues}}$$

Step 3: Multiply the revenues in step 1 by the industry average multiple in step 2.

- Disney has a cash balance of $3,795 million. If we wanted a beta for all of Disney's assets (and not just the operating assets), we would compute a weighted average:

$$\text{Beta for Disney's assets} = 0.7333\left(\frac{58,259}{(58,259+3,795)}\right) + 0\left(\frac{3,795}{(58,259+3,795)}\right) = 0.6885$$

Disney's Cost of Equity

- Step 1: Allocate debt across businesses

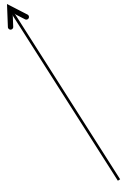
	Start with this(1)	From comparable firms(2)		As % (3)	Adjust to Disney's debt (3)*16,682	EV - Allocated Debt	Allocated Debt/ Estimated Equity
Business	Estumated Value	D/E Ratio of comps	Estimated debt	Proportions	Allocated Debt	Estimated Equity	D/E Ratio
Media Networks	$34,328	38.71%	$9,581	51.44%	$8,582	$25,746	33.33%
Parks and Resorts	$17,408	65.10%	$6,864	36.86%	$6,148	$11,260	54.61%
Studio Entertainment	$5,755	53.89%	$2,015	10.82%	$1,805	$3,950	45.70%
Consumer Products	$768	27.21%	$164	0.88%	$147	$621	23.70%
For example.			$18,624	100.00%	$16,682		
Media Networks	*$34,328*	*38.71%*	*34,328*(.3871/1.3871)*	*9581/18624*	*.5144*16,682*	*34328-8582*	*8582/25746*

- Step 2: Compute levered betas and costs of equity for Disney's operating businesses.

Business	Unlevered Beta	D/E Ratio	Levered Beta	Cost of Equity
Media Networks	0.7056	33.33%	0.8514	8.61%
Parks and Resorts	0.5849	54.61%	0.7829	8.20%
Studio Entertainment	1.3027	45.70%	1.6718	13.53%
Consumer Products	1.0690	23.70%	1.2261	10.86%
Disney	0.7333	36.91%	0.9011	8.91%

- Step 2a: Compute the cost of equity for all of Disney's assets:

$$\text{Equity Beta}_{\text{Disney as company}} = 0.6885 \ \ (1 + (1 - 0.38)(0.3691)) = 0.8460$$



Riskfree Rate = 3.5%
Risk Premium = 6%

iii. And equity risk premiums matter..

	Arithmetic Average		Geometric Average	
	Stocks – T. Bills	Stocks – T. Bonds	Stocks – T. Bills	Stocks – T. Bonds
1928-2009	7.53%	6.03%	5.56%	4.29%
	(2.28%)	(2.40%)		
1960-2009	5.48%	3.78%	4.09%	2.74%
	(2.42%)	(2.71%)		←
2000-2009	-1.59%	-5.47%	-3.68%	-7.22%
	(6.73%)	(9.22%)		

Historical premium

In 2010, the actual cash returned to stockholders was 40.38. That was down about 40% from 2008 levels.

Analysts expect earnings to grow 21% in 2010, resulting in a compounded annual growth rate of 7.2% over the next 5 years. We will assume that dividends & buybacks will keep pace.

After year 5, we will assume that earnings on the index will grow at 3.84%, the same rate as the entire economy (= riskfree rate).

43.29 46.40 49.74 53.32 57.16

$$1115.10 = \frac{43.29}{(1+r)} + \frac{46.40}{(1+r)^2} + \frac{49.74}{(1+r)^3} + \frac{53.32}{(1+r)^4} + \frac{57.16}{(1+r)^5} + \frac{57.16(1.0384)}{(r-.0384)(1+r)^5}$$

January 1, 2010
S&P 500 is at 1115.10
Adjusted Dividends & Buybacks for 2008 = 40.38

Expected Return on Stocks (1/1/10) = 8.20%
T.Bond rate on 1/1/10 = 3.84 %
Equity Risk Premium = 8.20% - 3.84% = 4.36%

Additional country risk?

- Even if we accept the proposition that an equity risk premium of about 4.5% is reasonable for a mature market, you would expect a larger risk premium when investing in an emerging market.
- Consider Peru. There is clearly more risk investing in Peruvian equities than there is in investing in a mature market. To estimate the additional risk premium that should be charged, we follow a 3-step process:
  - Step 1: Obtain a measure of country risk for Peru. For instance, the sovereign rating for Peru is Baa3 and the default spread associated with that rating in early 2010 was 2%,
  - Step 2: Estimate how much riskier equities are, relative to bonds. The standard deviation in weekly returns over the last 2 years for Peruvian equities was 26% and the standard deviation in the bond was 13%.
  - Step 3: Additional risk premium for Peru = 2% (26/13) = 3%
  - Step 4: Total equity risk premium for Peru = 4.5%+3%=7.5%

An example: Rio Disney
Expected Cash flow in US $ (in April 2009)

	0	1	2	3	4	5	6	7	8	9	10
Operating Income		-$50	-$150	-$84	$106	$315	$389	$467	$551	$641	$658
Taxes		-$19	-$57	-$32	$40	$120	$148	$178	$209	$244	$250
Operating Income after Taxes		-$31	-$93	-$52	$66	$196	$241	$290	$341	$397	$408
+ Depreciation & Amortization		$50	$425	$469	$444	$372	$367	$364	$364	$366	$368
- Capital Expenditures	$2,500	$1,000	$1,188	$752	$276	$258	$285	$314	$330	$347	$350
- Change in Working Capital	$0	$0	$63	$25	$38	$31	$16	$17	$19	$21	$5
Cash flow to Firm	-$2,500	-$981	-$918	-$360	$196	$279	$307	$323	$357	$395	$422
+ Pre-Project Investment	500										
- Pre-project Deprecn * t		$19	$19	$19	$19	$19	$19	$19	$19	$19	$19
+ Fixed G&A (1-t)		$0	$78	$109	$155	$194	$213	$234	$258	$284	$289
Incremental Cash flow to Firm	-$2,000	-$1,000	-$859	-$270	$332	$454	$501	$538	$596	$660	$692

Rio Disney: Risk Adjusted Discount Rate

- Since the cash flows were estimated in US dollars, the riskfree rate is the US treasury bond rate of 3.5% (at the time of the analysis.
- The beta for the theme park business is 0.7829. This was estimated by looking at publicly traded theme park companies.
- The risk premium is composed of two parts, a mature market premium of 6% and an additional risk premium of 3.95% for Brazil.

  Country risk premium for Brazil = 3.95%

  Cost of Equity in US$= 3.5% + 0.7829 (6%+3.95%) = 11.29%
- Using this estimate of the cost of equity, we use Disney's theme park debt ratio of 35.32% and its after-tax cost of debt of 3.72%, we can estimate the cost of capital for the project:

  Cost of Capital in US$ = 11.29% (0.6468) + 3.72% (0.3532) = 8.62%

Rio Disney: Risk Adjusted Value
Risk Adjusted Discount Rates

Discounted at Rio Disney cost of capital of 8.62%

Year	Annual Cashflow	Terminal Value	Present Value
0	-$2,000		-$2,000
1	-$1,000		-$921
2	-$860		-$729
3	-$270		-$211
4	$332		$239
5	$453		$300
6	$502		$305
7	$538		$302
8	$596		$307
9	$660		$313
10	$692	$10,669	$4,970
Net Present Value =			$2,877

b. Certainty Equivalents

Step 1: Compute your expected cash flows in each time period. This will require you to consider (at least implicitly) the range of possible outcomes (from euphoric to catastrophic) and the possibilities of each outcome. Note that this is not a risk adjusted cash flow.

Step 1: Convert your expected cash flow to a certainty equivalent. There are three ways you can do this:

a. Compute certainty equivalents, using utility functions (forget this)

b. Convert your expected cash flow to a certainty equivalent

$$\text{Certainty Equivalent CF} = \frac{(1+\text{Riskfree rate})^t}{(1+\text{Risk adjusted Discount Rate})^t} E(CF_t)$$

c. Subjectively estimate a haircut to the expected cash flows

Step 2: Discount the certainty equivalent cash flows at the riskfree rate.

Rio Disney: Risk Adjusted Value
Certainty Equivalent Cash flows

CF_t* 1.035^t/1.0862^t **Discount at 3.5%**

Year	Annual Cashflow	Terminal Value	Certainty Equivalent	Present Valiue
0	-$2,000		-$2,000	-$2,000
1	-$1,000		-$953	-$921
2	-$860		-$780	-$729
3	-$270		-$234	-$211
4	$332		$274	$239
5	$453		$356	$300
6	$502		$375	$305
7	$538		$384	$302
8	$596		$405	$307
9	$660		$427	$313
10	$692	$10,669	$7,011	$4,970
				$2,877

II. Probabilistic Approaches

- The essence of risk that you are unclear about what the outcomes will be from an investment. In the risk adjusted cash flow approach, we make the adjustment by either raising discount rates or lowering cash flows.
- In probabilistic approaches, we deal with uncertainty more explicitly by
  - Asking what if questions about key inputs and looking at the impact on value (Sensitivity Analysis)
  - Looking at the cash flows/value under different scenarios for the future (Scenario Analysis)
  - Using probability distributions for key inputs, rather than expected values, and computing value as a distribution as well (Simulations)

a. Sensitivity Analysis and What-if Questions…

- The NPV, IRR and accounting returns for an investment will change as we change the values that we use for different variables.
- One way of analyzing uncertainty is to check to see how sensitive the decision measure (NPV, IRR..) is to changes in key assumptions. While this has become easier and easier to do over time, there are caveats that we would offer.

Caveat 1: When analyzing the effects of changing a variable, we often hold all else constant. In the real world, variables move together.

Caveat 2: The objective in sensitivity analysis is that we make better decisions, not churn out more tables and numbers.

Corollary 1: Less is more. Not everything is worth varying…

Corollary 2: A picture is worth a thousand numbers (and tables).

b. Scenario Analysis

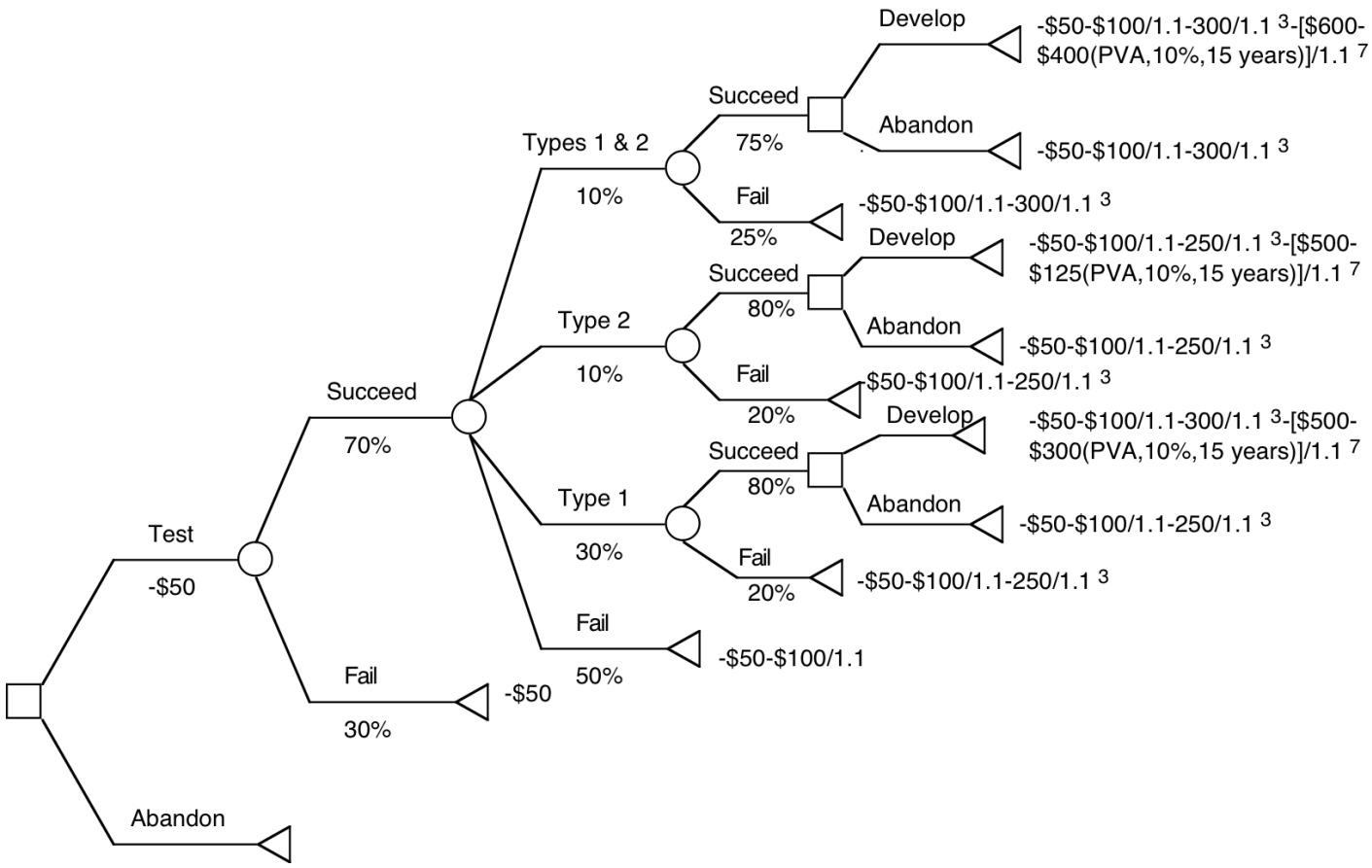
- Scenario analysis is best employed when the outcomes of a project are a function of the macro economic environment and/or competitive responses.
- As an example, assume that Boeing is considering the introduction of a new large capacity airplane, capable of carrying 650 passengers, called the *Super Jumbo*, to replace the Boeing 747. The cash flows will depend upon two major "uncontrollable" factors:
  - The growth in the long-haul, international market, relative to the domestic market. Arguably, a strong Asian economy will play a significant role in fueling this growth, since a large proportion of it will have to come from an increase in flights from Europe and North America to Asia.
  - The likelihood that Airbus, Boeing's primary competitor, will come out with a larger version of its largest capacity airplane, the A-300, over the period of the analysis.

The scenarios…

Number of planes sold under each scenario (and probability of each scenario)

	Airbus Large Jet	Airbus A-300	Airbus abandons large capacity airplane
High Growth in Asia	120 (12.5%)	150 (12.5%)	200 (0%)
Average Growth in Asia	100 (15%)	135 (25%)	160 (10%)
Low Growth in Asia	75 (5%)	110 (10%)	120 (10%)

c. Decision Trees

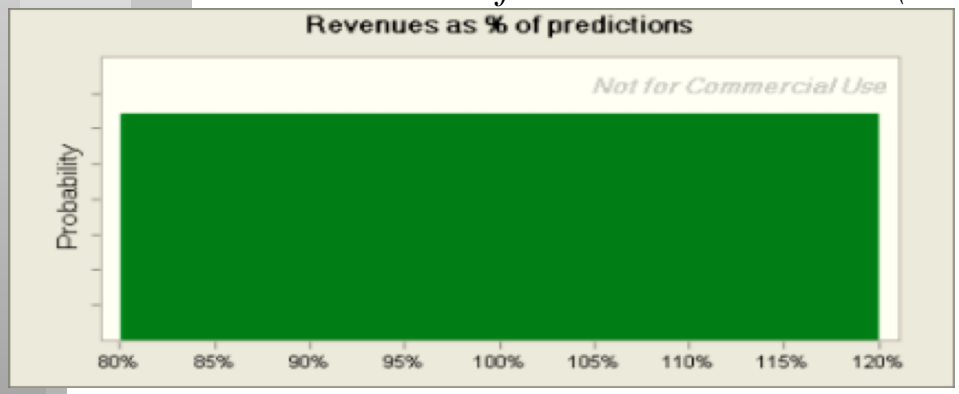


Another use for decision trees: Dealing with discrete risk…

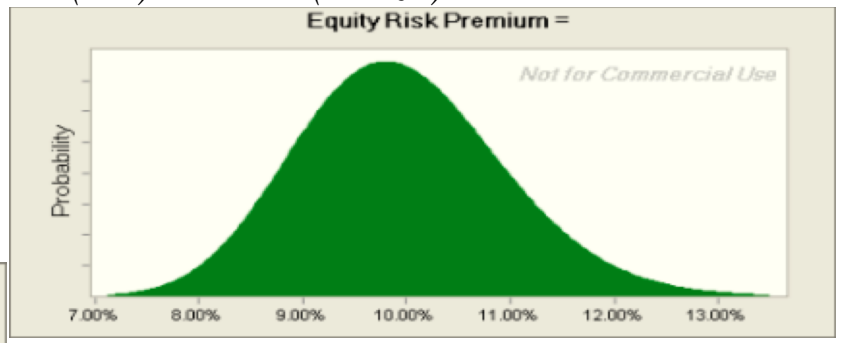
- Assume that you are valuing Gazprom, the Russian oil company and have estimated a value of US $180 billion for the operating assets. The firm has $30 billion in debt outstanding. What is the value of equity in the firm?
- Now assume that the firm has 15 billion shares outstanding. Estimate the value of equity per share.
- The Russian government owns 42% of the outstanding shares. Would that change your estimate of value of equity per share?

d. Simulations
The Disney Theme Park

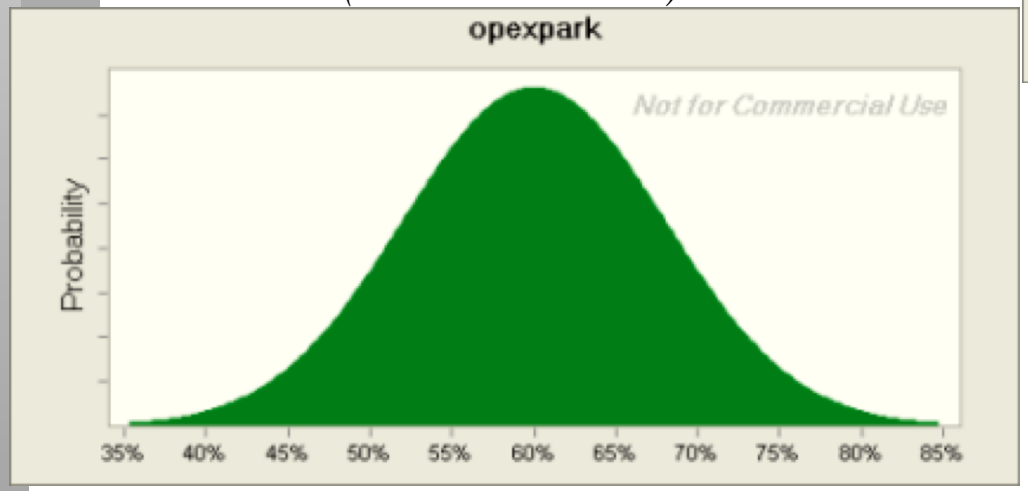
Actual Revenues as % of Forecasted Revenues (Base case = 100%)



Equity Risk Premium (Base Case = 6% (US)+ 3.95% (Brazil) = 9.95%

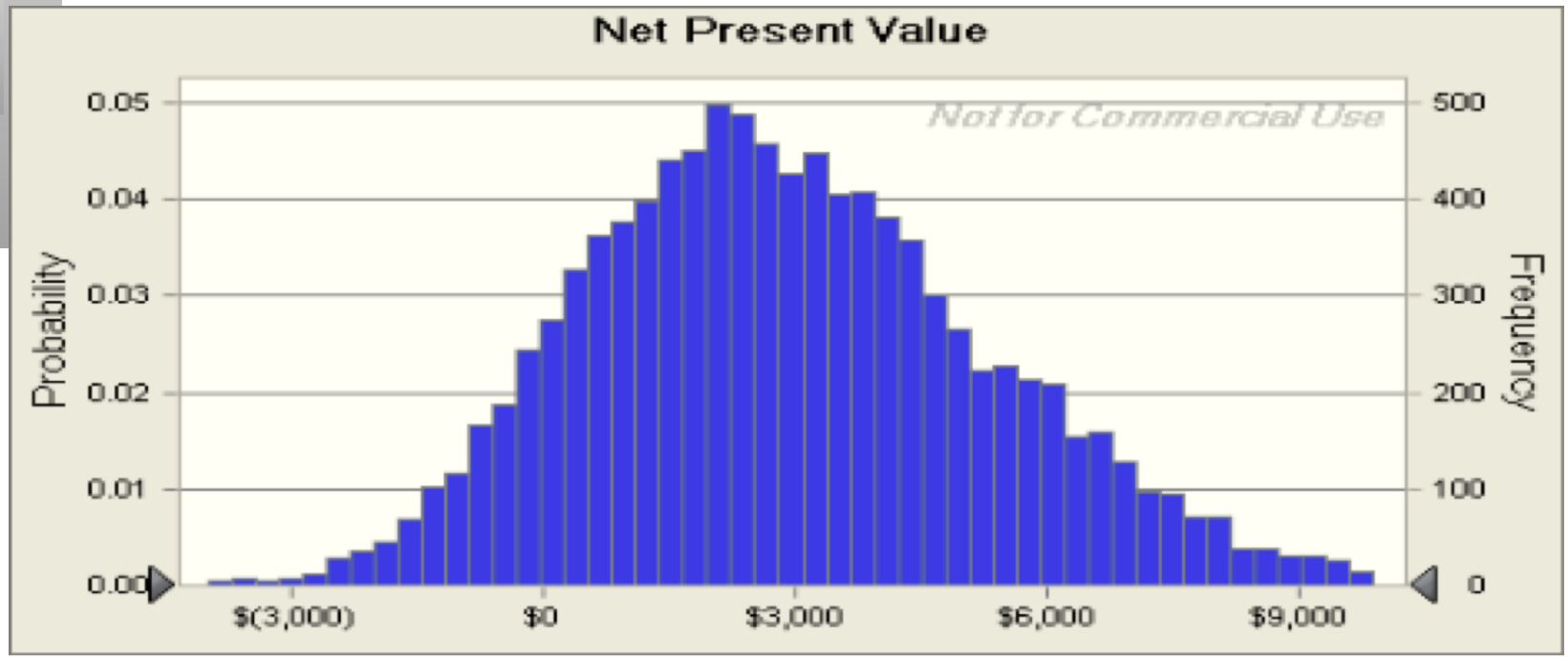


Operating Expenses at Parks as % of Revenues (Base Case = 60%)



The resulting outcome…

Average = $2.95 billion
Median = $2.73 billion



NPV ranges from -$4 billion to +$14 billion. NPV is negative 12% of the time.

Using simulations in valuation: The Exxon Mobil valuation

- You are valuing Exxon Mobil, using data from the most recent fiscal year (2008). The following provides the key numbers:

Revenues	$477 billion
EBIT (1-t)	$ 58 billion
Net Cap Ex	$ 3 billion
Chg WC	$ 1 billion
FCFF	$ 54 billion

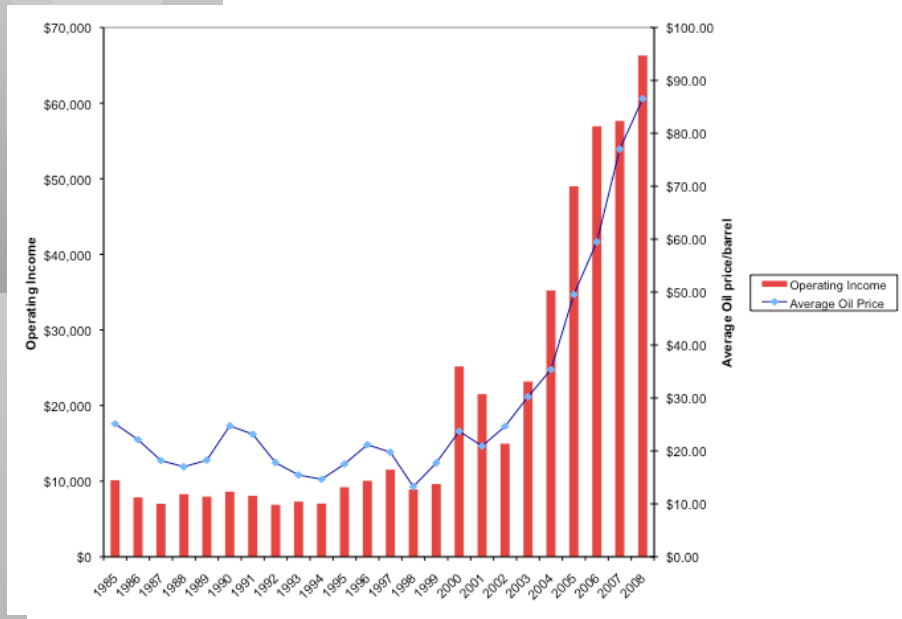
- The cost of capital for the firm is 8% and you use a very conservative stable growth rate of 2% to value the firm. The market cap for the firm is $330 billion and it has $ 10 billion in debt outstanding.

  a. How under or over valued is the equity in the firm?

  b. Would you buy the stock based on this valuation? Why or why not?

And one possible response…

Step 1: Look at history



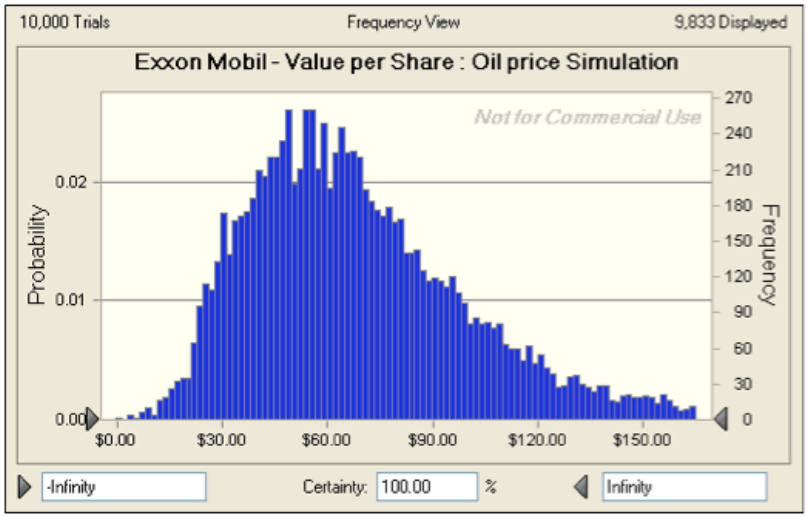
Step 2: Look for relationship

Regression of Exxon income against oil price

Op Inc = -6,934 + 911 (Price per barrel of oil)

R squared = 94%

Step 3: Run simulation



Choosing a Probabilistic Approach

Discrete/Continuous	*Correlated/Independent*	*Sequential/Concurrent*	*Risk Approach*
Discrete	Independent	Sequential	Decision Tree
Discrete	Correlated	Concurrent	Scenario Analysis
Continuous	Either	Either	Simulations

III. Value at Risk (VaR)

- Value at Risk measures the potential loss in value of a risky asset or portfolio over a defined period for a given confidence interval. Thus, if the VaR on an asset is $ 100 million at a one-week, 95% confidence level, there is a only a 5% chance that the value of the asset will drop more than $ 100 million over any given week.
- There are three key elements of VaR – a specified level of loss in value, a fixed time period over which risk is assessed and a confidence interval. The VaR can be specified for an individual asset, a portfolio of assets or for an entire firm
- VaR has been used most widely at financial service firms, where the risk profile is constantly shifting and a big loss over a short period can be catastrophic (partly because the firms have relatively small equity, relative to the bets that they make, and partly because of regulatory constraints)

IV. Real Options

- One of the limitations of traditional investment analysis is that it is static and does not do a good job of capturing the options embedded in investment.
  - The first of these options is the option to delay taking a project, when a firm has exclusive rights to it, until a later date.
  - The second of these options is taking one project may allow us to take advantage of other opportunities (projects) in the future
  - The last option that is embedded in projects is the option to abandon a project, if the cash flows do not measure up.
- Unlike other risk adjustment approaches in finance, which tend to just penalize investments for risk, real options explicitly brings in the upside of risk into the analysis.

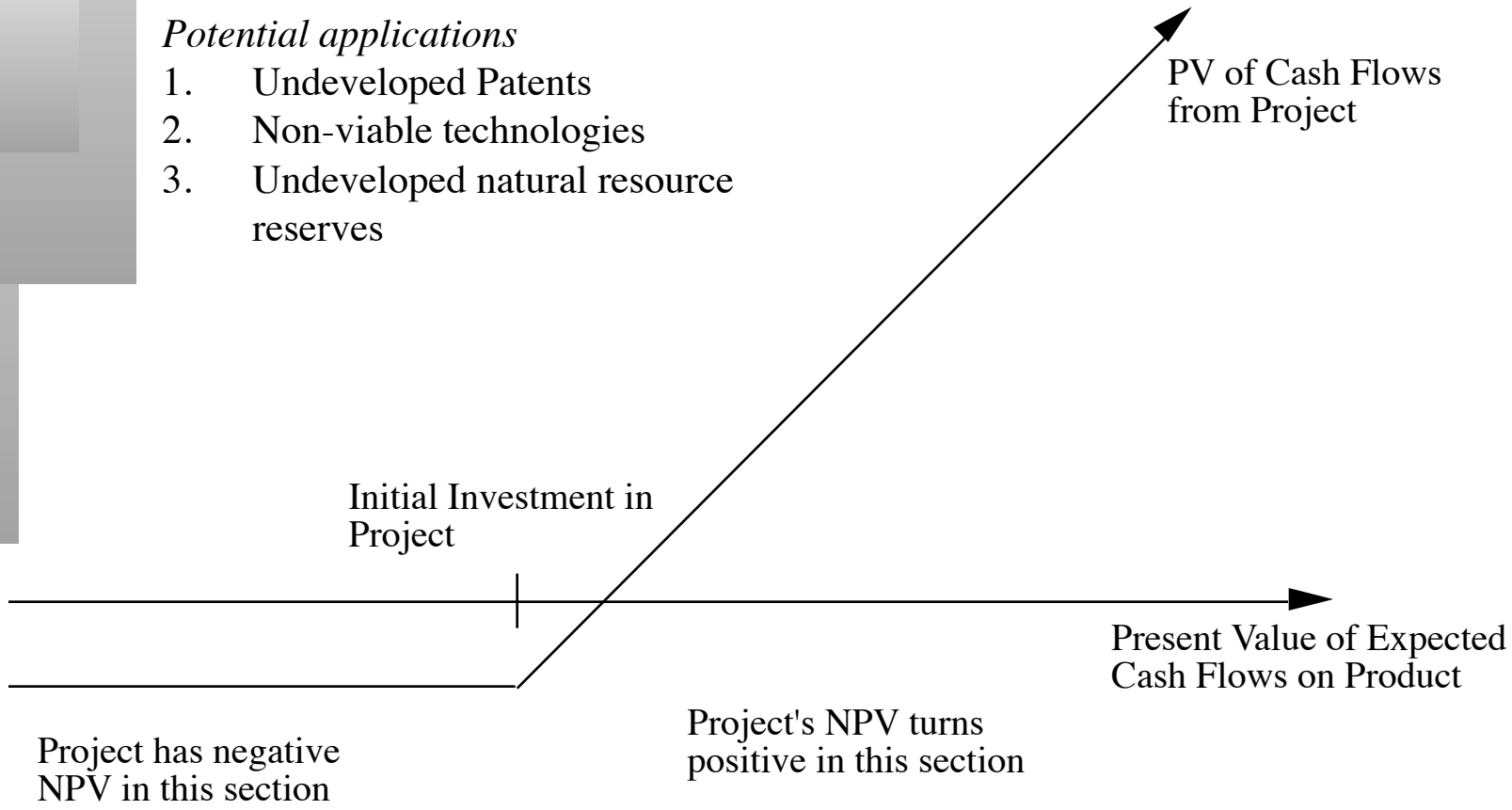
The Option to Delay

- When a firm has exclusive rights to a project or product for a specific period, it can delay taking this project or product until a later date.
- A traditional investment analysis just answers the question of whether the project is a "good" one if taken today.
- Thus, the fact that a project does not pass muster today (because its NPV is negative, or its IRR is less than its hurdle rate) does not mean that the rights to this project are not valuable.

Valuing the Option to Delay a Project

Potential applications

1. Undeveloped Patents
2. Non-viable technologies
3. Undeveloped natural resource reserves

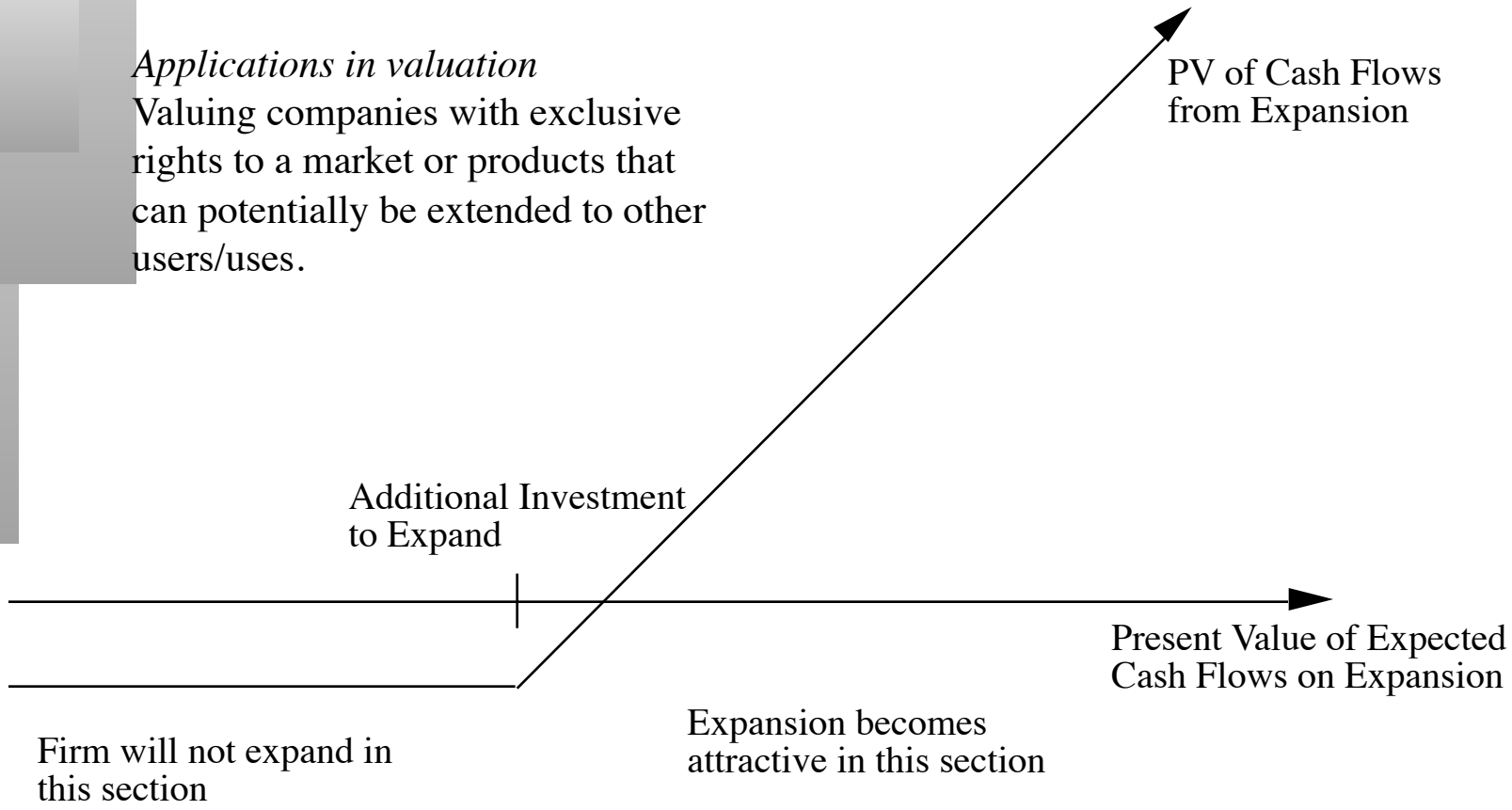


The Option to Expand/Take Other Projects

- Taking a project today may allow a firm to consider and take other valuable projects in the future.
- Thus, even though a project may have a negative NPV, it may be a project worth taking if the option it provides the firm (to take other projects in the future) has a more-than-compensating value.
- These are the options that firms often call "strategic options" and use as a rationale for taking on "negative NPV" or even "negative return" projects.

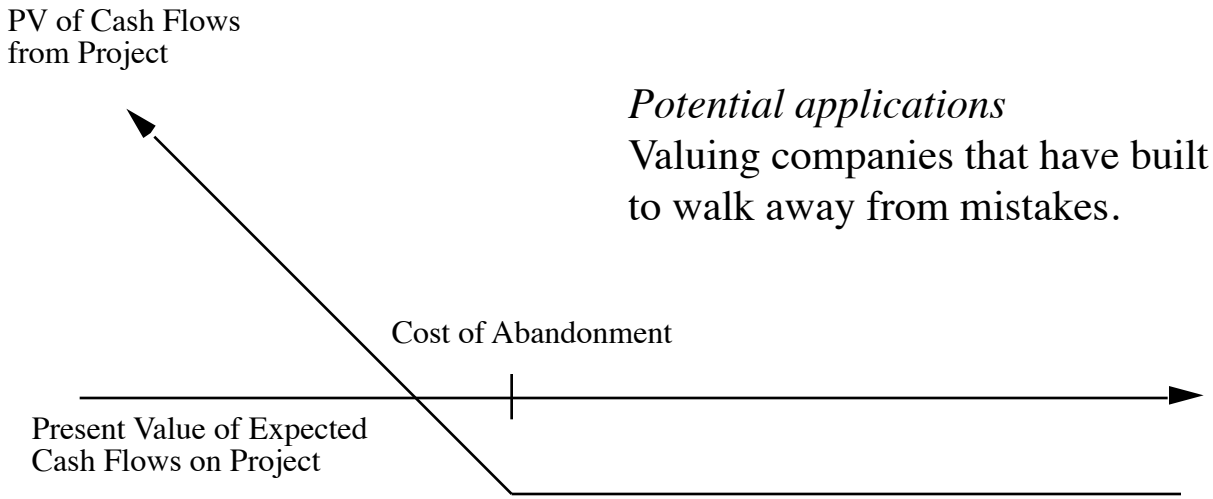
The Option to Expand

Applications in valuation
Valuing companies with exclusive rights to a market or products that can potentially be extended to other users/uses.



The Option to Abandon

- A firm may sometimes have the option to abandon a project, if the cash flows do not measure up to expectations.
- If abandoning the project allows the firm to save itself from further losses, this option can make a project more valuable.



Potential applications
Valuing companies that have built in flexibility to walk away from mistakes.