

## I. THE STABLE GROWTH DDM: GORDON GROWTH MODEL

### The Model:

$$\text{Value of Stock} = \text{DPS}_1 / (r - g)$$

where  $\text{DPS}_1$  = Expected Dividends one year from now

$r$  = Required rate of return for equity investors

$g$  = Annual Growth rate in dividends *forever*

### A BASIC PREMISE

- This infinite growth rate cannot exceed the growth rate for the overall economy (GNP) by more than a small amount (1-2%)

#### *Estimate for the US*

*Upper end: Long term inflation rate (5%) + Growth rate in real GNP (3%) = 8%*

*Lower end: Long term inflation rate (3%) + Growth rate in real GNP (2%) = 5%*

- *If the company is a multinational, the real growth rate will be the growth rate of the world economy, which is about one percent higher.*

- *The inflation rate used should be consistent with the currency being used in the valuation.*

### **WORKS BEST FOR:**

- firms with stable growth rates
- firms which pay out dividends that are high and approximate FCFE.
- firms with stable leverage.

### **Some obvious candidates for the Gordon Growth Model**

- Regulated Companies, such as utilities, because
  - their growth rates are constrained by geography and population to be close to the growth rate in the economy in which they operate.
  - they pay high dividends, largely again as a function of history
  - they have stable leverage (usually high)
- Large financial service companies, because
  - their size makes it unlikely that they will generate extraordinary growth
  - Free cash flows to equity are difficult to compute
  - they pay large dividends
  - they generally do not have much leeway in terms of changing leverage
- Real estate investment trusts, because

- they have to pay out 95% of their earnings as dividends
- they are constrained in terms of investment policy and cannot grow at high rates.

## Applications: To stocks

*Illustration 1: To a utility: Con Edison - Electrical Utility (North East United States)*

*Rationale for using the model*

- The firm is in stable growth; based upon size and the area that it serves. Its rates are also regulated; It is unlikely that the regulators will allow profits to grow at extraordinary rates.
- The beta is 0.75 and has been stable over time.
- The firm is in stable leverage.
- The firm pays out dividends that are roughly equal to FCFE.

Average Annual FCFE between 1991 and 1995 = \$480 million

Average Annual Dividends between 1991 and 1995 = \$ 461 million

Dividends as % of FCFE = 96.04%

*Background Information*

Earnings per share in 1995 = \$ 2.95

Dividend Payout Ratio in 1995 = 69.15%

Dividends per share in 1995 = \$2.04

Expected Growth Rate in Earnings and Dividends = 5%

Con Ed Beta = 0.75

Cost of Equity =  $6\% + 0.75 * 5.5\% = 10.13\%$

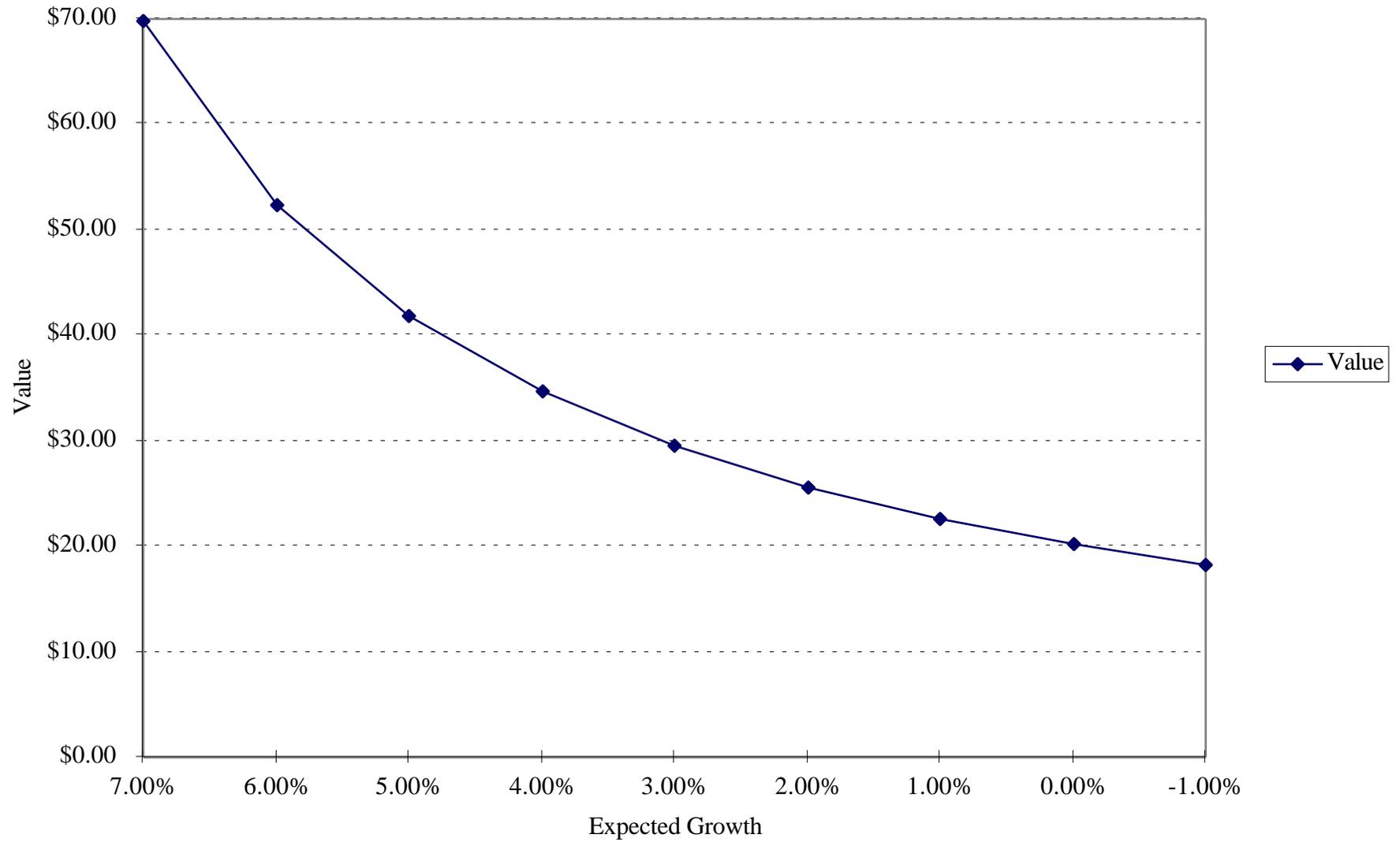
Value of Equity =  $\$2.04 * 1.05 / (.1013 -.05) = \$ 41.80$

Con Ed was trading for \$ 30 on the day of this analysis. (January 1996)

*What growth rate would Con Ed have to attain to justify the current stock price?*

The following table estimates value as a function of the expected growth rate (assuming a beta of 0.75 and current dividends per share of \$2.04).

**Con Ed: Value versus Expected Growth**



Solving for the expected growth rate that provides the current price,

$$\$30.00 = 2.04 (1+g) / (.1013-g)$$

Solving for g,

$$g = (.1013*30-2.04) / (30.00+2.04) = 3.12 \%$$

The growth rate in earnings and dividends would have to be 3.12% a year to justify the stock price of \$30.00.

*Illustration 2: To a financial service firm: J.P. Morgan*

*A Rationale for using the Gordon Growth Model*

- As a financial service firm in an extremely competitive environment, it is unlikely that J.P. Morgan's earnings are going to grow much faster than the economy over the long term. Allowing for expansion, the expected growth rate used is 7%.
- As a financial service firm, free cash flows to equity are difficult to estimate. Hence, the dependence on dividends.
- The leverage of financial service firms is high and unlikely to change over time.

*Background Information*

Current Earnings per share = \$ 6.30

Current Dividend Payout Ratio = 47.62%

Dividends per share = \$ 3.00

Expected Growth Rate in Earnings and Dividends = 7%

Stock Beta = 1.15

Cost of Equity =  $6\% + 1.15 * 5.5\% = 12.33\%$

Value of Equity =  $\$3.00 * 1.07 / (.1233 -.07) = \$ 60.23$

J.P. Morgan was trading for \$ 80 on the day of this analysis. (January 1996)

*Notes of Concern*

- The beta is high for a stable growth firm. It reflects the additional risk that many financial service firms have encountered and exposed themselves to in the last few years.

*Illustration 3: To the overall market: S&P 500 Index on January 1, 1997*

- The average dividend yield (Dividends/ Price) for stocks in the index at the end of 1995 was 1.92%.
- The level of the index on January 1, 1997 was 753.79.
- Using the T.Bond rate of 7.00% and an expected growth rate in the nominal GNP of 6%, the level of the index can be obtained from the

Gordon Growth model:

Dividends per share in year 0 =  $1.95\%$  of 753.79 = \$ 14.70

Infinite growth rate = 6%

Required return of return for equity investors =  $7.00\% + 1 * 5.5\% = 12.50\%$

Intrinsic Value of the market =  $14.70 * 1.06 / (.125 - .06) = 239.72$

### **Scary! So what are we missing?**

- Maybe dividend yields do not reflect the capacity of firms to buy back stock. With stock buybacks, this measure rises to about 3% of the overall index.
- The stocks in the index are expected to see their earnings grow faster than the economy for the next two or three years. (Estimates - 10-12%)
- The risk premium may be trending down, reflecting
  - the greater willingness of investors to go with the flow - take losses and continue in the market, rather than panic
  - the greater flow of cash into pension funds
  - other paradigm shifts
- With a 3% dividend yield, 12% growth next year in earnings and a 3.5% premium, you can arrive at an index level of 562.83.

**Implied Risk Premium: What would the risk premium have to be to justify the level of the index today?**

### What is wrong with this valuation? DDM Stable

- |  | <i>Solution</i>                        |
|--|--|
| • If you get a low value from this model, it may be because              |  |
| - the firm's dividend payout ratio may be low for a stable firms (< 40%) | Try using the FCFE Stable Model        |
| - the beta is high for a stable firm                                     | Use a beta closer to one               |
| • If you get too high a value, it is because                             |  |
| - the expected growth rate is too high for a stable firm                 | Use a growth rate closer to GNP growth |

## II. TWO-STAGE GROWTH MODEL WITH INFINITE GROWTH RATE AT END

### The Model:

- The model is based upon two stages of growth, an extraordinary growth phase that lasts n years, and a stable growth phase that lasts forever after that:

Extraordinary growth rate: g% each year for n years                      Stable growth: g<sub>n</sub> forever



- Value of the Stock = PV of Dividends during extraordinary phase + PV of terminal price

$$P_0 = \sum_{t=1}^{t=n} \frac{DPS_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \quad \text{where } P_n = \frac{EPS_n * (1+g_n) * \text{New payout ratio}}{r - g_n}$$

where

DPS<sub>t</sub> = Expected dividends per share in year t

r = Required rate of return: Cost of equity (may be different for high growth and stable growth phases)

P<sub>n</sub> = Price at the end of year n

g<sub>n</sub> = Growth rate forever after year n

- In the case where the extraordinary growth rate (g) and payout ratio are the same for the first n years, this formula can be simplified as follows:

$$P_0 = \frac{DPS_0 * (1+g) * \left[ 1 - \frac{(1+g)^n}{(1+k_e)^n} \right]}{k_e - g} + \frac{DPS_{n+1}}{(k_{e,n} - g_n)(1+k_e)^n}$$

where  $k_e$  = Cost of Equity during high growth phase

$k_{e,n}$  = Cost of Equity during the stable growth phase

This simplifies calculations because it does not require the estimation of dividends each year for the first n years.

### Calculating the terminal price

- The growth rate for the Gordon Growth Rate model (within 2% of growth rate in nominal GNP) apply here as well.
- The payout ratio has to be consistent with the estimated growth rate. If the growth rate is expected to drop significantly after year n, the payout ratio should be higher. This can be estimated in one of two ways –

- from fundamentals*

$$\text{Stable Period Payout ratio} = 1 - b = 1 - \left( \frac{g}{ROC + D/E (ROC - i (1-t))} \right) = 1 - g/ROE$$

where the inputs for this equation will be for the stable growth period.

- from other stable firms*

Stable Period Payout Ratio = Average Payout Ratio for other stable firms (40-70% depending on industry: See Industry Average Table)

**Works best for:**

- firms where the growth rate is not yet stable, but is moderating
- firms which pay out dividends that roughly approximate FCFE (or) FCFE cannot be estimated easily.

*Illustration 4: Valuing a firm with the two-stage dividend discount model::American Express*

*A Rationale for using the Model*

- *Why two-stage?* While American Express is a large financial service firm in a competitive market place, normally not a candidate for above-stable growth, it has gone through an extended period of depressed earnings . It is expected that the recovery in earnings will create higher growth over the next five years.
- *Why dividends?* As a financial service firm, free cash flows to equity are difficult to estimate.
- Leverage is stable.

*Background Information*

- Current Earnings / Dividends
  - Earnings per share in 1995 = \$3.10
  - Dividends per share in 1995 = \$ 0.90
- Inputs for the High Growth Period

Length of the High Growth Period = 5 years

Beta during High Growth Period = 1.45

Cost of Equity during High Growth Period =  $6.0\% + 1.45 (5.5\%) = 13.98\%$

- Return on Assets during high growth period = 14.56% (this was the 1995 return on assets)
- Dividend Payout Ratio = 29.03%
- Debt/Equity Ratio = 100% (slightly higher than the current debt/equity ratio of 92.14%)
- Interest rate on debt = 8.50% (Tax Rate = 36%)

Expected Growth Rate =  $b ( ROC + D/E (ROC - i (1-t)) = 0.7097 (14.56\% + 1 (14.56\% - 8.50\% (1-.36))) = 16.81\%$

- Inputs for the Stable Growth
  - Expected Growth Rate = 6%
  - Beta during stable growth phase = 1.10 : Cost of Equity =  $6.00\% + 1.1 (5.5\%) = 12.05\%$
  - The ROC is expected to drop to 12.50%; D/E Ratio and interest rate are assumed to remain unchanged during the stable growth period.

Stable Payout Ratio =  $1 - g / ( ROC + D/E (ROC - i (1-t)) = 1 - .06 / (12.50\% + 1 (12.50\% - 8.50\% (1-.36))) = 69.33\%$

*Estimating the value:*

- The first component of value is the present value of the expected dividends during the high growth period. Based upon the current earnings (\$3.10), the expected growth rate (16.81%) and the expected dividend payout ratio (29.03%), the expected dividends can be computed for each year in the high growth period.

Year	EPS	DPS	Present Value
1	\$3.62	\$1.05	\$0.92
2	\$4.23	\$1.23	\$0.95
3	\$4.94	\$1.43	\$0.97
4	\$5.77	\$1.68	\$0.99
5	\$6.74	\$1.96	\$1.02

Cumulative Present Value of Dividends (@13.98%) = \$0.92 + \$ 0.95+ \$ 0.97 + \$ 0.99 + \$ 1.02 = \$ 4.85
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The present value of the dividends can also be computed in short hand using the following computation:

$$\text{PV of Dividends} = \frac{\$0.90 * (1.1681)^0 * 1 - \frac{(1.1681)^5}{(1.1398)^5}}{.1398 - .1681} = \$4.85$$

The price at the end of the high growth phase (end of year 5), can be estimated using the constant growth model.

$$\text{Terminal price} = \text{Expected Dividends per share}_{n+1} / (r - g_n)$$

$$\text{Expected Earnings per share}_6 = 3.10 * 1.1681^5 * 1.06 = \$ 7.15$$

$$\text{Expected Dividends per share}_6 = \$7.15 * 0.6933 = \$ 4.95$$

$$\text{Terminal price} = \$ 4.95 / (.1205 - .06) = \$ 81.87$$

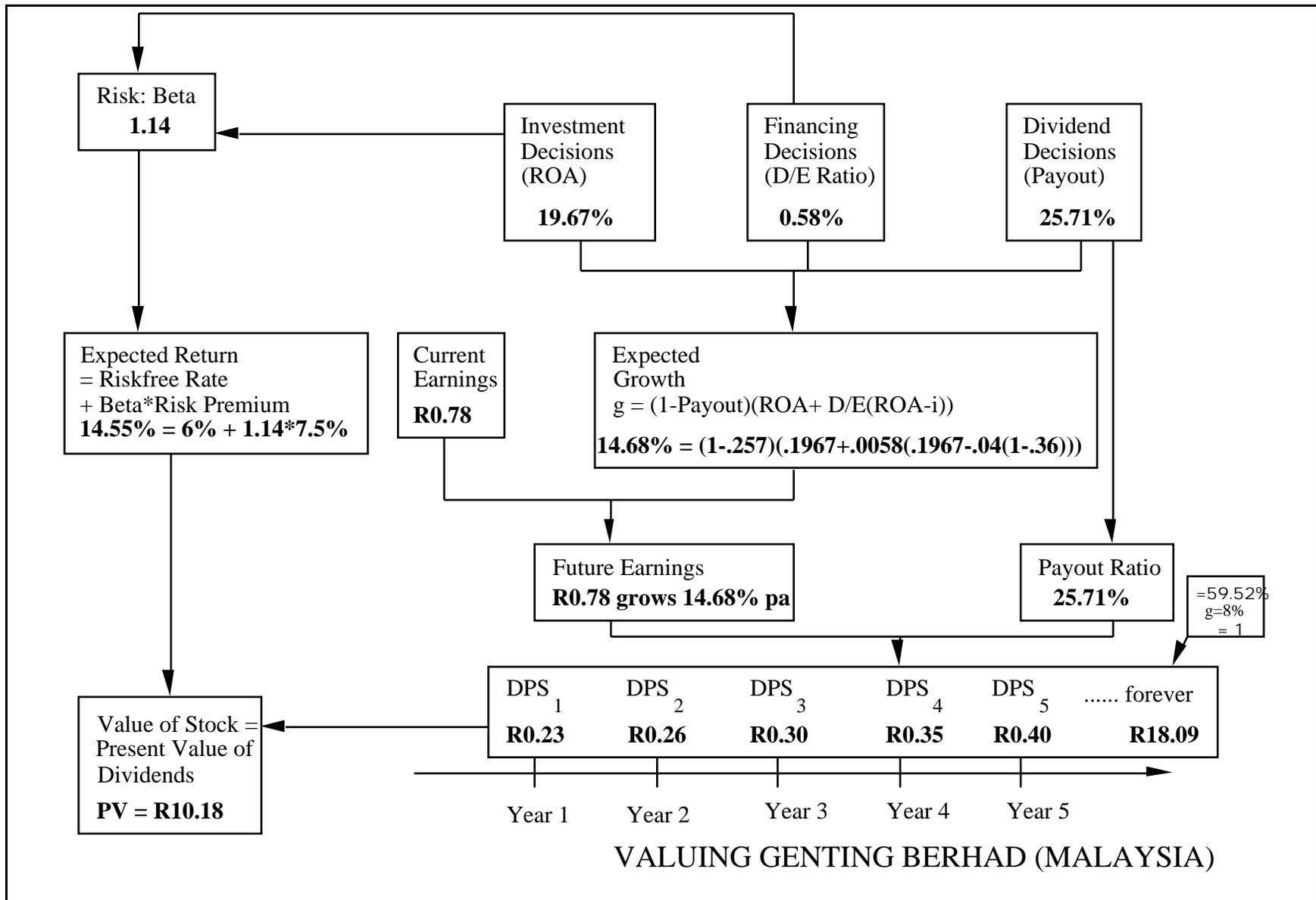
The present value of the terminal price can be then written as -

$$\text{PV of Terminal Price} = \frac{\$81.87}{(1.1398)^5} = \$42.57$$

The cumulated present value of dividends and the terminal price can then be calculated as follows:

$$P_0 = \frac{\$0.90 * (1.1681)^* \left[ 1 - \frac{(1.1681)^5}{(1.1398)^5} \right]}{.1398 - .1681} + \frac{\$81.87}{(1.1398)^5} = \$4.85 + \$42.57 = \$47.42$$

American Express was trading at \$40.00 in February 1996, at the time of this analysis.



### What is wrong with this valuation? DDM 2 Stage

- If you get an extremely low value from the 2-stage DDM, the likely culprits are
  - the stable period payout ratio is too low for a stable firm (< 40%)  
If using fundamentals, use a higher ROC  
If entering directly, enter a higher payout
  - the beta in the stable period is too high for a stable firm  
Use a beta closer to one.
  - the use of the two-stage model when the three-stage model is more appropriate  
Use a three-stage model
- If you get an extremely high value,
  - the growth rate in the stable growth period is too high for stable firm  
Use a growth rate closer to GNP growth

### THE VALUE OF GROWTH

$$P_0 = \underbrace{\left\{ \left[ \sum_{t=1}^{t=n} \frac{DPS_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \right] - \frac{DPS_0 * (1+g_n)}{(r-g_n)} \right\}}_{\text{Extraordinary Growth}} + \underbrace{\left\{ \frac{DPS_0 * (1+g_n)}{(r-g_n)} - \frac{DPS_0}{r} \right\}}_{\text{Stable Growth}} + \underbrace{\frac{DPS_0}{r}}_{\text{Assets in place}}$$

where

$DPS_t$  = Expected dividends per share in year t

r = Required rate of return

$P_n$  = Price at the end of year n

$g_n$  = Growth rate forever after year n

Value of extraordinary growth = Value of the firm with extraordinary growth in first n years - Value of the firm as a stable growth firm<sup>1</sup>

Value of stable growth = Value of the firm as a stable growth firm - Value of firm with no growth

Assets in place = Value of firm with no growth

#### *Illustration 5: An Illustration of the value of growth: American Express*

Consider the example of American Express in February 1996,

<sup>1</sup> The payout ratio used to calculate the value of the firm as a stable firm can be either the current payout ratio, if it is reasonable, or the new payout ratio calculated using the fundamental growth formula.

$$\begin{aligned} \text{Value of the assets in place} &= \text{Current EPS} * \text{Payout ratio} / r \\ &= \$3.10 * 0.2903 / .1205 = \$ 7.47 \end{aligned}$$

- The discount rate from the stable growth phase is used for this calculation.

$$\begin{aligned} \text{Value of stable growth} &= \text{Current EPS} * \text{Payout ratio} * (1+g_n)/(r-g_n) - \$ 7.47 \\ &= (\$3.10 * 0.2903 * 1.06)/(.1205 - .06) - \$ 7.47 = \$ 8.30 \end{aligned}$$

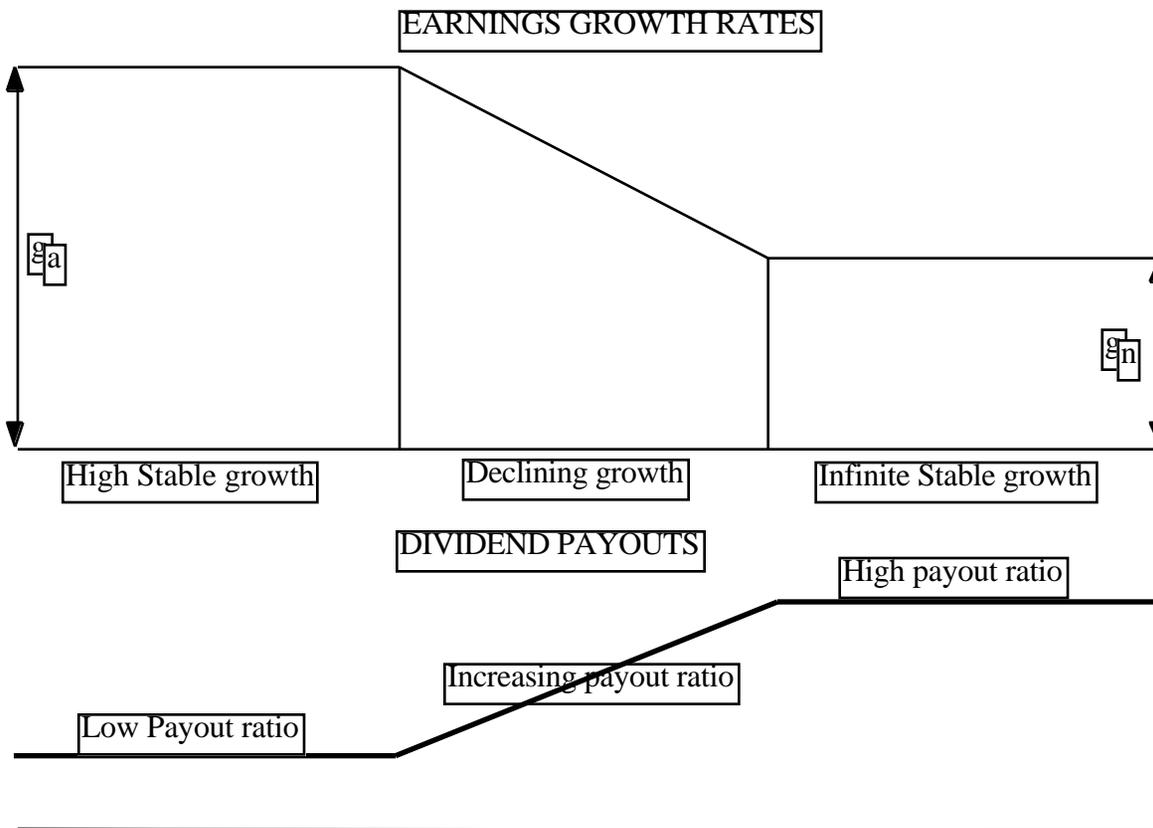
$$\text{Value of extraordinary growth} = \$ 47.42 - (7.47+8.30) = \$ 31.65$$

#### *The Determinants of the Value of Growth*

- Length of the high growth period
- Extent of the extraordinary growth
- Costs of higher growth , i.e., how much risk is added and how much cash is drained as a consequence.

### III. THREE-STAGE DIVIDEND DISCOUNT MODEL

*The Model*



- The value of the stock is then the present value of expected dividends during the high growth and the transitional periods, and of the terminal price at the start of the final stable growth phase.

$$P_0 = \frac{\sum_{t=1}^{t=n1} \text{EPS}_0 * (1+g_a)^t * a}{(1+k_e)^t} + \frac{\sum_{t=n1+1}^{t=n2} \text{DPS}_t}{(1+k_e)^t} + \frac{\text{EPS}_{n2} * (1+g_n)^n}{(k_{e,n} - g_n)(1+k_e)^n}$$

High growth phase
Transition
Stable growth phase

where,

$\text{EPS}_t$  = Earnings per share in year t

$\text{DPS}_t$  = Dividends per share in year t

$g_a$  = Growth rate in high growth phase (lasts n1 periods)

$g_n$  = Growth rate in stable phase

$a$  = Payout ratio in high growth phase

$n$  = Payout ratio in stable growth phase

$k_e$  = Required rate of return on equity: Can vary across periods;  $k_e$  is the cost of equity during the high growth and transition period and  $k_{e,n}$  is the cost of equity during the stable growth period.

*Works best for:*

It is best suited for firms which are

- paying out and plan to continue paying dividends which are roughly equal to FCFE
- growing at
  - an extraordinary rate now and are expected to maintain this rate for an initial period,
  - after which the differential advantage of the firm is expected to deplete leading to gradual declines in the growth rate
  - to a stable growth rate.
- in stable leverage

*Illustration 6 : Valuing with the Three-stage DDM model: The Home Depot*

*A Rationale for using the Three-Stage Dividend Discount Mode;*

- *Why three-stage?* The Home Depot is still in very high growth. Analysts project that its earnings per share will grow at 36% for the next five years.
- *Why dividends?* The firm has had a track record of paying out dividends that roughly approximate FCFE
- The financial leverage is stable.

*Background Information*

- Current Earnings / Dividends
  - Earnings per share in 1994 = \$ 1.55
  - Dividends per share in 1994 = \$ 0.19
- Inputs for the High Growth Period
  - Length of the High Growth Period = 5 years
  - Expected growth rate = 36.00% (Based upon analyst projections)
  - Beta during High Growth Period = 1.60
  - Cost of Equity during High Growth Period =  $7.5\% + 1.60(5.5\%) = 16.30\%$
  - Dividend Payout Ratio = 12.03% (based on existing payout ratio)
- Inputs for the transition period
  - Length of the transition period = 5 years
  - Growth rate in earnings will decline from 36% in year 5 to 6% in year 10 in linear increments.
  - Payout ratio will increase from 12.03% to 60% over the same period in linear increments.
  - Beta will drop from 1.60 to 1.00 over the same period in linear increments.
- Inputs for the Stable Growth

- Expected Growth Rate = 6%
- Beta during stable growth phase = 1.00 : Cost of Equity = 7.50% + 1.0 (5.5%) = 13.00%
- Payout Ratio = 60%

*Estimating the Value*

- These inputs are used to estimate expected earnings per share, dividends per share and costs of equity for both the high growth and stable periods. The present values are also shown.

Period	EPS	Payout Ratio	DPS	Cost of Equity	Present Value
1	\$1.81	12.03%	\$0.22	16.30%	\$0.19
2	\$2.46	12.03%	\$0.30	16.30%	\$0.22
3	\$3.35	12.03%	\$0.40	16.30%	\$0.25
4	\$4.55	12.03%	\$0.55	16.30%	\$0.30
5	\$6.19	12.03%	\$0.74	16.30%	\$0.35
6	\$8.04	21.62%	\$1.74	15.64%	\$0.71
7	\$9.97	31.22%	\$3.11	14.98%	\$1.10
8	\$11.77	40.81%	\$4.80	14.32%	\$1.49
9	\$13.18	50.41%	\$6.64	13.66%	\$1.81

10	\$13.97	60.00%	\$8.38	13.00%	\$2.02
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(Note: Since the costs of equity change each year, the present value has to be calculated using the cumulated cost of equity. Thus, in year 7, the present value of dividends is -

$$\text{PV of year 7 dividend} = \$3.11 / \{(1.1630)^5 (1.1564) (1.1498)\} = \$1.10$$

The terminal price at the end of year 10 can be calculated based upon the earnings per share in year 11, the stable growth rate of 6%, a cost of equity of 13.00% (based upon the beta of 1) and the payout ratio of 60.00% -

$$\text{Terminal price} = \$ 13.97 * 1.06 * 0.60 / (.13 - .06) = \$ 126.96$$

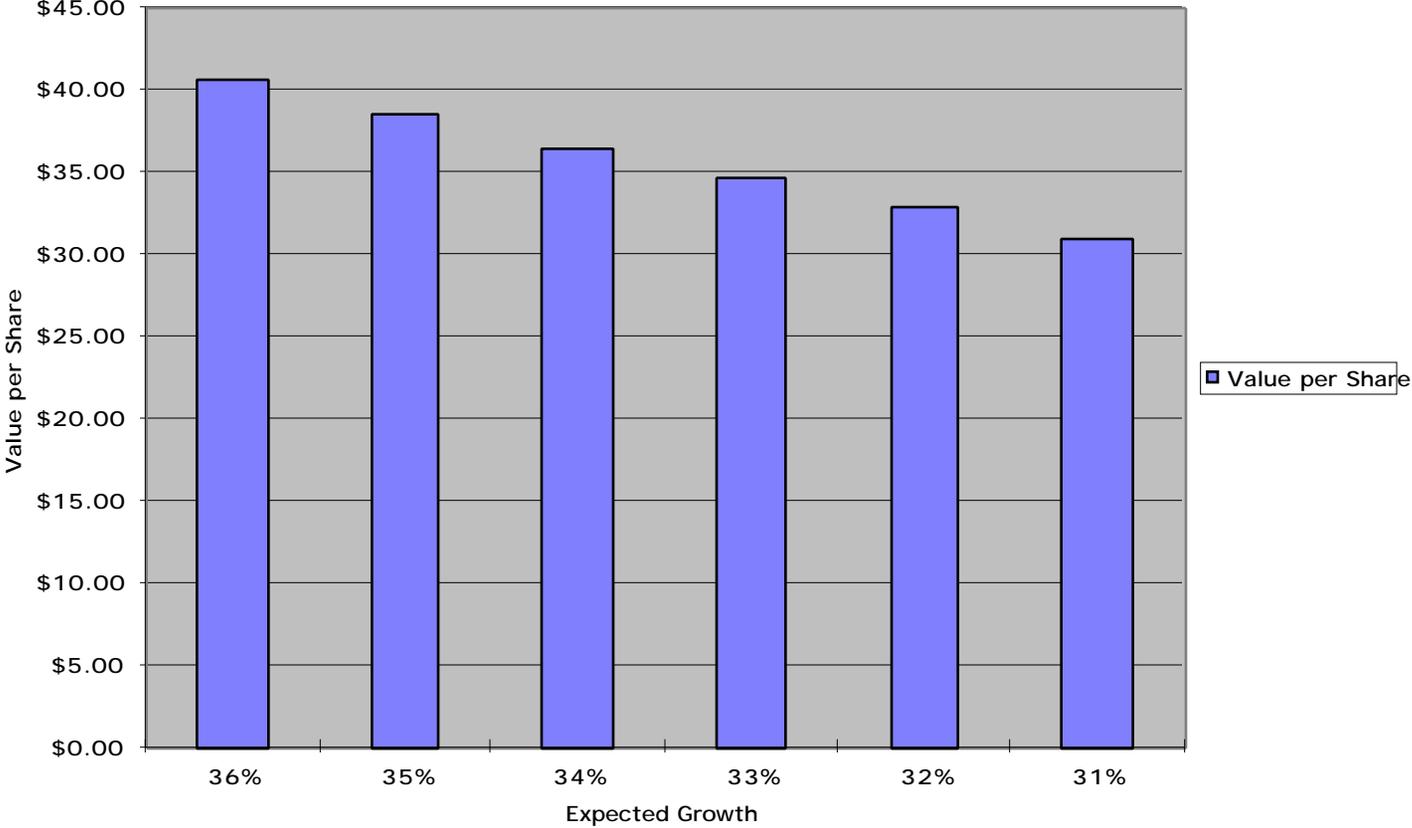
The components of value are as follows:

Present Value of dividends in high growth phase:	\$ 1.31
Present Value of dividends in transition phase:	\$ 7.12
Present Value of terminal price at end of transition:	\$ 30.57
Value of Home Depot Stock :	\$39.00

Home Depot was trading at \$45, in February 1995.

*Sensitivity to Growth Rates*

### VALUE VS. EXPECTED GROWTH



### What is wrong with this model? (3 stage DDM)

- If you are getting too low a value from this model,
  - the stable period payout ratio is too low for a stable firm (< 40%)  
If using fundamentals, use a higher ROC  
If entering directly, enter a higher payout
  - the beta in the stable period is too high for a stable firm  
Use a beta closer to one.
- If you get an extremely high value,
  - the growth rate in the stable growth period is too high for stable firm  
Use a growth rate closer to GNP growth
  - the period of growth (high + transition) is too high  
Use shorter high growth & transition periods

