The Cost of Distress: Survival, Truncation Risk and Valuation

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Traditional valuation techniques- both DCF and relative - short change the effects of financial distress on value. In most valuations, we ignore distress entirely and make implicit assumptions that are often unrealistic about the consequences of a firm being unable to meet its financial obligations. Even those valuations that purport to consider the effect of distress do so incompletely. In this paper, we begin by considering how distress is dealt with in traditional discounted cash flow models, and when these models value distress correctly. We then look at ways in which we can incorporate the effects of distress into value in discounted cashflow models. We conclude by looking at the effect of distress on relative valuations, and ways of incorporating its effect into relative value.

In both discounted cash flow and relative valuation, we implicitly assume that the firms that we are valuing are going concerns and that any financial distress that they are exposed to is temporary. After all, a significant chunk of value in every discounted cash flow valuation comes from the terminal value, usually well in the future. But what if the distress is not temporary and there is a very real chance that the firm will not survive to get to the terminal value? In this paper, we will argue that we tend to over value firms such as these in traditional valuation models, largely because is difficult to capture fully the effect of such distress in the expected cash flows and the discount rate. The degree to which traditional valuation models misvalue distressed firms will vary, depending upon the care with which expected cash flows are estimated, the ease with which these firms can access external capital market and the consequences of distress.

In this paper, we will begin by looking at the underlying assumptions of discounted cash flow valuation, why DCF models do not explicitly consider the possibility of distress and when analysts can get away with ignoring distress. We will follow up by considering ways in which we can adjust discounted cashflow models to explicitly allow for the possibility of distress. In the next part of the paper, we consider how distress is considered (or as is more often, ignored) in relative valuation and ways of adjusting multiples for the possibility of failure. We will close the paper by looking at why equity in deeply distressed firms may continue to have value because of the limited liability feature of publicly traded equity.

The Possibility and Consequences of Financial Distress

Growth is not inevitable and firms may not remain as going concerns. In fact, even large publicly traded firms sometimes become distressed for one reason or the other and the consequences for value can be serious. In this section, we will consider first how common it is for firms to become distressed and follow up by looking at the consequences of distress.

The Possibility of Distress

Financial distress is far more common in the real world that most of us assume it to be. In fact, even casual empirical observation suggests that a very large number of

firms, especially smaller and higher growth, will not survive and will go out of business. Some will fail because they borrow money to fund their operations and then are unable to make these debt payments. Other will fail because they do not have the cash to cover their operating needs.

To get a measure at the probability of distress, we have to begin by defining distress. If we define it as companies that enter chapter 11, relatively few publicly traded firms at any point in time can be considered distressed. If we define it more broadly as firms that are having trouble making interest payments and meeting other contractual commitments, distress is much more common. Kahl (2001) examined all publicly traded firms in the US between 1980 and 1983 and found that 1346 firms had trouble making their interest expenses from operating income in at least one year and that 151 firms could be considered distressed, in the sense that they were renegotiating with lenders to restructure debt.[1] Following up on these firms, he finds that while less than a half of these firms declare chapter 11, only a third of these firms survive as independent companies and that the rest either get either acquired or liquidated.

The Consequences of Distress

What are the consequences of financial failure? Firms that are unable to make their debt payments have to liquidate their assets, often at bargain basement prices, and use the cash to pay off debt. If there is any cash left over, which is highly unlikely, it will be paid out to equity investors. Firms that are unable to make their operating payments also have to offer themselves to the highest bidder, and the proceeds will be distributed to the equity investors. In effect, these "liquidation costs" can be considered the direct costs of bankruptcy.

In fact, the costs of distress stretch far beyond the conventional costs of bankruptcy and liquidation. The perception of distress can do serious damage to a firm's operations, as employees, customers, suppliers and lenders react. Firms that are viewed as distressed lose customers (and sales), have higher employee turnover and have to accept much tighter restrictions from suppliers than healthy firms. These indirect bankruptcy

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[1] Kahl, M., 2001, Financial Distress as a Selection Mechanism, SSRN Working Paper.

costs can be catastrophic for many firms and essentially make the perception of distress into a reality. The magnitude of these costs has been examined in studies and can range from 10-25% of firm value.[2]

In summary, then, the possibility and costs of distress are far to substantial to be ignored in valuation. The question then becomes not whether we should adjust firm value for the potential for distress but how best to make this adjustment.

Discounted Cash flow Valuation

Consider how we value a firm in a discounted cash flow world. We begin by projecting expected cash flows for a period, we estimate a terminal value at the end of the period that captures what we believe the firm will be worth at that point in time and we then discount the cash flows back at a discount rate that reflects the riskiness of the firm's cash flows. This approach is an extraordinarily flexible one and can be stretched to value firms ranging from those with predictable earnings and little growth to those in high growth with negative earnings and cash flows. Implicit in this approach, though, is the assumption that a firm is a going concern, with potentially an infinite life. The terminal value is usually estimated by assuming that earnings grow at a constant rate forever (a perpetual growth rate). Even when the terminal value is estimated using a multiple of revenues or earnings, this multiple is derived by looking at publicly traded firms (usually healthy ones)..

Distress in Discounted Cashflow Valuation

Given the likelihood and consequences of distress, it seems foolhardy to assume that we can ignore this possibility when valuing a firm, and particularly so, when we are valuing firms in poor health and with substantial debt obligations. So, what you might wonder, are the arguments offered by proponents of discounted cash flow valuation for not explicitly considering the possibility of firms failing? We will consider five reasons

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[2] For an examination of the theory behind indirect bankruptcy costs, see Opler, T. and S. Titman, 1994, Financial Distress and Corporate Performance. Journal of Finance 49, 1015-1040. For an estimate on how large these indirect bankruptcy costs are in the real world, see Andrade, G. and S. Kaplan, 1998, How Costly is Financial (not Economic) Distress? Evidence from Highly Leveraged Transactions that Become Distressed. Journal of Finance. 53, 1443-1493. They look at highly levered transactions that subsequently became distressed snd conclude that the magnitude of these costs ranges from 10% to 23% of firm value.

often provided by for this oversight. The first two reasons are offered by analysts who believe that there is no need to consider distress explicitly, and the last three reasons by those who believe that discounted cashflow valuations already incorporate the effect of distress.

1. We value only large, publicly traded firms and distress is very unlikely for these firms.

It is true that the likelihood of distress is lower for larger, more established firms, but experience suggests that even these firms can become distressed. The last few months of 2001 saw the astonishing demise of Enron, a firm that had a market capitalization in excess of $ 70 billion just a few months previously At the end of 2001, analysts were openly discussing the possibility that large firms like Kmart and Lucent Technologies would be unable to make their debt payments and may have to declare bankruptcy. In 2006, the same talk could be heard about GM and Delta Airlines. The other problem with this argument, even if we accept the premise, is that smaller, high growth firms are traded and need to be valued just as much as larger firms. In fact, we could argue that the need for valuation is greater for smaller firms, where the uncertainty and the possibility of pricing errors are greater.

2. We assume that access to capital is unconstrained

In valuation, as in much of corporate finance, we assume that a firm with good investments has access to capital markets and can raise the funds it needs to meet its financing and investment needs. Thus, firms with great growth potential will never be forced out of business because they will be able to raise capital (more likely equity than debt) to keep going. In buoyant and developed financial markets, this assumption is not outlandish. Consider, for instance, the ease with which new economy companies with negative earnings and few if any assets were able to raise new equity in the late 1990s. However, even in a market as open and accessible as the United States, access to capital dried up as investors drew back in 2000 and 2001. In summary, then, we may have been able to get away with the assumption that firms with valuable assets will not be forced into a distress sale in 1998 and 1999, but that assumption would have been untenable in 2001.

3. We adjust the discount rate for the possibility of distress

The discount rate is the vehicle we use to adjust for risk in discounted cash flow valuation. Riskier firms have higher costs of equity, higher costs of debt and usually have higher costs of capital than safer firms. A reasonable extension of this argument would be that a firm with a greater possibility of distress should have a higher cost of capital and thus a lower firm value. The argument has merit up to a point. The cost of capital for a distressed firm, estimated correctly, should be higher than the cost of capital for a safer firm. If the distress is caused by high financial leverage, the cost of equity should be much higher. Since the cost of debt is based upon current borrowing rates, it should also climb as the firm becomes more exposed to the risk of bankruptcy and the effect will be exacerbated if the tax advantage of borrowing also dissipates (as a result of operating losses). Ultimately though, the adjustment to value that results from using a higher discount rate is only a partial one. The firm is still assumed to generate cash flows in perpetuity, though the present value is lower. A significant portion of the firm's current value still comes from the terminal value. In other words, the biggest risk of distress that is the loss of all future cash flows is not adequately captured in value.

4. We adjust the expected cash flows for the possibility of distress

To better understand this adjustment, it is worth reviewing what the expected cash flows in a discounted cash flow valuation are supposed to measure. The expected cash flow in a year should be the probability-weighted estimate of the cash flows under all scenarios for the firm, ranging from the best to the worst case. In other words, if there is a 30% chance that a firm will not survive the next year, the expected cash flow should reflect both this probability and the resulting cash flow. In practice, we tend to be far sloppier in our estimation of expected cash flows. In fact, it is not uncommon to use an exogenous estimate of the expected growth rate (from analyst estimates) on the current year's earnings or revenues to generate future values. Alternatively, we often map out an optimistic path to profitability for unprofitable firms and use this path as the basis for estimating expected cash flows. We could estimate the expected cash flows under all scenarios and use the expected values in our valuation. Thus, the expected cash flows would be much lower for a firm with a significant probability of distress. Note, though, that contrary to conventional wisdom, this is not a risk adjustment. We are doing what we should have been doing in the first place and estimating the expected cash flows

correctly. If we wanted to risk-adjust the cash flows, we would have to adjust the expected cash flows even further downwards using a certainty equivalent.[3] If we do this, though, the discount rate used would have to be the riskfree rate and not the risk-adjusted cost of capital. As a practical matter, it is very difficult to adjust expected cash flows for the possibility of distress. Not only do we need to estimate the probability of distress each year, we have to keep track of the cumulative probability of distress as well. This is because a firm that becomes distressed in year 3 loses its cash flows not just in that year but also in all subsequent years.

5. We assume that even in distress, the firm will be able to receive the present value of expected cash flows from its assets as proceeds from the sale.

The problem with distress, from a DCF standpoint, is not that the firm ceases to exist but that all cash flows beyond that point in time are lost. Thus, a firm with great products and potentially a huge market may never see this promise converted into cash flows because it goes bankrupt early in its life. If we assume that this firm can sell itself to the highest bidder for a distress sale value that is equal to the present value of expected future cash flows, however, distress does not have to be considered explicitly. This is a daunting assumption because we are not only assuming that a firm in distress has the bargaining power to demand fair market value for its assets, but we are also assuming that it can do this not only with assets in place (investments it has already made and products that it has produced) but with growth assets (products that it may have been able to produce in the future).

In summary, the failure to explicitly consider distress in discounted cash flow valuation will not have a material impact in value if any the following conditions hold:

1. There is no possibility of bankruptcy, either because of the firm's size and standing or because of a government guarantee.

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[3] A certainty equivalent cashflow replaces an uncertain cash flow with an equivalent riskless cashflow. Thus, an expected cashflow of $ 125 million will be replaced by a riskless cashflow of $ 100 million. The more uncertain the cash flow, the greater the downward adjustment.

2. Easy access to capital markets allows firms with good investments to raise debt or equity capital to sustain themselves through bad times, thus ensuring that these firms will never be forced into a distress sale.
3. We use expected cash flows that incorporate the likelihood of distress and a discount rate that is adjusted for the higher risk associated with distress. In addition, we have to assume that the firm will receive sale proceeds that are equal to the present value of expected future cash flows as a going concern in the event of a distress sale.

If these conditions do not hold, and it is easy to make an argument that they will not for some firms at some points in time, discounted cash flow valuations will overstate firm value.

Adapting Discounted Cash flow Valuation to Distress Situations

When will the failure to consider distress in discounted cash flow valuation have a material impact on value? If the likelihood of distress is high, access to capital is constrained (by internal or external factors) and distress sale proceeds are significantly lower than going concern values, discounted cash flow valuations will overstate firm and equity value for distressed firms, even if the cash flows and the discount rates are correctly estimated. In this section, we will consider several ways of incorporating the effects of distress into the estimated value.

Simulations

In traditional valuation, we estimate expected values for each of the input variables. For instance, in valuing a firm, we may assume an expected growth rate in revenues of 30% a year and that the expected operating margin will be 10%. In reality, each of these variables has a distribution of values, which we condense into an expected value. Simulations attempt to utilize the information in the entire distribution, rather than just the expected value, to arrive at a value. By looking at the entire distribution, simulations provide us with an opportunity to deal explicitly with distress.

Before we begin running the simulations, we will have to decide the circumstances which will constitute distress and what will happen in the event of distress.

For example, we may determine that cumulative operating losses of more than $ 1 billion over three years will push the firm into distress and that it will sell its assets for 25% of book value in that event. The parameters for distress will vary not only across firms, based upon their size and asset characteristics, but also on the state of financial markets and the overall economy. A firm that has three bad years in a row in a healthy economy with rising equity markets may be less exposed to default than a similar firm in the middle of a recession. The steps in the simulation are as follows:

Step 1: The first step involves choosing those variables whose expected values will be replaced by distributions. While there may be uncertainty associated with every variable in valuation, only the most critical variables might be chosen at this stage. For instance, revenue growth and operating margins may be the key variables that we choose to build distributions for.

Step 2: We choose a probability distribution for each of the variables. There are a number of choices here, ranging from discrete probability distributions (probabilities are assigned to specific outcomes) to continuous distributions (the normal, lognormal or exponential distribution). In making this choice, the following factors should be considered:

- The range of feasible outcomes for the variable; (e.g., the revenues cannot be less than zero, ruling out any distribution that requires the variable to take on large negative values, such as the normal distribution).
- The experience of the company on this variable. Data on a variable, such as operating margins historically, may help us determine the type of distribution that best describes it.

While no distribution will provide a perfect fit, the distribution that best fits the data should be used.

Step 3: Next, the parameters of the distribution chosen for each variable are estimated. The number of parameters will vary from distribution to distribution; for instance, the mean and the variance have to be estimated for the normal distribution, while the uniform distribution requires estimates of the minimum and maximum values for the variable.

Step 4: One outcome is drawn from each distribution; the variable is assumed to take on that value for that particular simulation. To make the analysis richer, we can repeat this process each year and allow for correlation across variables and across time.[4]

Step 5: The expected cash flows are estimated based upon the outcomes drawn in step 4. If the firm meets the criteria for a going concern, defined before the simulation, we will then discount the cash flows to arrive at a conventional estimate of discounted cash flow value. If it fails to meet the criteria, we will value it as a distressed firm.

Step 6: Steps 4 and 5 are repeated until a sufficient number of simulations have been conducted. In general, the more complex the distribution (in terms of the number of values the variable can take on and the number of parameters needed to define the distribution) and the greater the number of variables, the larger this number will be.

Step 7: Each simulation will generate a value, going concern or distressed as the case may be, for the firm. The average across all simulated values will be the value of the firm. We should also be able to assess the probability of default from the simulation and the effect of distress on value.

The primary limitation of simulation analysis is the information that is required for it to work. In practice, it is difficult to choose both the right distribution to describe a variable and the parameters of that distribution. When these choices are made carelessly or randomly, the output from the simulation may look impressive but actually conveys no valuable information.

Modified Discounted Cash flow Valuation

We can adapt discounted cash flow valuation to reflect some or most of the effects of distress on value. To do this, we will to bring in the effects of distress into both expected cash flows and discount rates.

1. Estimating Expected Cash flows: To consider the effects of distress into a discounted cash flow valuation, we have to incorporate the probability that a firm will not survive

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[4] For example, you may increase the likelihood of the earnings being low if the earnings in previous years were low and the likelihood of negative margins if revenue growth is low.

into the expected cash flows. In its most complete form, this would require that we consider all possible scenarios, ranging from the most optimistic to the most pessimistic, assign probabilities to each scenario and cash flows under each scenario, and estimate the expected cash flows each year.

$$\text{Expected cash flow} = \sum_{j=1}^{j=n} \pi_{jt} \, (\text{Cashflow}_{jt})$$

where π_{jt} is the probability of scenario j in period t and Cashflow_{jt} is the cashflow under that scenario and in that period. These inputs have to be estimated each year, since the probabilities and the cash flows are likely to change from year to year.

A shortcut, albeit an approximate one, would require estimates for only two scenarios – the going concern scenario and the distress scenario. For the going concern scenario, we could use the expected growth rates and cash flows estimated under the assumption that the firm will be nursed back to health. Under the distress scenario, we would assume that the firm will be liquidated for its distress sale proceeds. Our expected cash flow for each year then would be:

$$\text{Expected cash flow}_t = (\text{Cashflow}_{\text{Going concern, t}}) * \pi_{\text{Going concern, t}} + (\text{Cashflow}_{\text{Distress, t}}) * (1 - \pi_{\text{Going concern, t}})$$

where $\pi_{\text{Going concern, t}}$ is the cumulative probability that the firm will continue as a going concern through period t. The probabilities of distress will have to be estimated for each year and the cumulative probability of surviving as a going concern can then be written as follows:

$$\text{Cumulative probability of survival - period t} = \pi_t = \prod_{n=1}^{n=t} (1 - \pi_{distress,n})$$

where $\pi_{\text{distress, t}}$ is the probability that the firm will become distressed in period t. For example, if a firm has 20% chance of distress in year 1 and a 10% chance of distress in year 2, the cumulative probability of surviving as a going concern over two years can be written as:

Cumulative probability of survival over 2 years = (1- .20) (1 - .10) = .72 or 72%

2. Estimating Discount Rates: In conventional valuation, we often estimate the cost of equity using a regression beta and the cost of debt by looking at the market interest rates on publicly traded bonds issued by the firm. For firms with a significant probability of

distress, these approaches can lead to inconsistent estimates. Consider first the use of regression betas. Since regression betas are based upon past prices over long periods (two to five years, for instance), and distress occurs over shorter periods, we will find that these betas will understate the true risk in the distressed firm.[5] With the interest rates on corporate bonds, we run into a different problem. The yields to maturity on the corporate bonds of firms that are viewed as distressed reach extremely high levels, largely because the interest rates are computed based upon promised cash flows (coupons and face value) rather than expected cash flows. The presumption in a going concern valuation is that the promised cash flows have to be made for the firm to remain a going concern, and it is thus appropriate to base the cost of debt on promised rather than expected cash flows. For a firm with a significant likelihood of distress, this presumption is clearly unfounded.

What are the estimation choices for distressed firms? To estimate the cost of equity, we have two options that provide more reasonable estimates than regression betas:

a. *CAPM Betas adjusted for distress*: Instead of using regression betas, we could use the bottom-up unlevered beta (the weighted average of unlevered betas of the businesses that the firm operates in) and the current market debt to equity ratio of the firm. Since distressed firms often have high debt to equity ratios, brought about largely as a consequence of dropping stock prices, this will lead to levered betas that are significantly higher than regression betas[6]. If we couple this with the reality that most distressed firms are in no position to get any tax advantages from debt, the levered beta will become even higher.

Levered beta = Bottom-up Unlevered beta (1 + (1- tax rate) (Debt to Equity ratio))

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[5] As an extreme example, consider estimating a beta for Enron at the end of 2001. The beta estimate from Bloomberg, using 2 years of data, was 1.45. Over three-quarters of this period, Enron was viewed (rightly or wrongly) as a healthy firm with positive earnings. It is only in the last part of the regression period that you see the effects of distress on stock prices and the debt to equity ratio of the firm.

[6] For more on bottom-up betas, refer to Damodaran (2000).

Note, though, that it is reasonable to re-estimate debt to equity ratios and tax rates for future years based upon our expectations for the firm and adjust the beta to reflect these changes.[7]

b. *Distress factor Models*: In addition to the standard factor for market risk, we could add a separate distress factor to the cost of equity. In effect, this would make the cost of equity for distressed firms much higher than healthy firms in the same business. In fact, some have attributed the higher returns that Fama and French (1992) show are earned by firms with low price to book ratios to distress; low price to book stocks, they argue, are more likely to be distressed.[8] Other studies, however, contest this notion by noting that portfolios of distressed firms have earned lower returns than portfolios of healthy firms historically.[9]

To estimate the cost of debt for a distressed firm, we would recommend using the interest rate based upon the firm's bond rating. While this will still yield a high cost of debt, it will be more reasonable than the yield to maturity when default is viewed as imminent.[10] To compute the cost of capital, we need to estimate the weights on debt on equity. In the initial year, we should use the current market debt to capital ratio (which may be very high for a distressed firm). As we make our forecasts for future years and build in our expectations of improvements in profitability, we should adjust the debt ratio towards

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[7] There are other variations on this leverage adjustment. Some analysts, for instance, prefer a more complete version that allows debt to carry systematic risk and have a beta. Others prefer to eliminate the tax adjustment. Still others argue for other ways of adjusting betas for distress risk.

[8] Fama, Eugene F, and Kenneth R. French, 1992, The cross section of expected stock returns, *Journal of Finance,* 47, 427-465. The argument that the return premium earned by low price to book stocks is due to distress is contested in other studies.

[9] Dichev Ilya D., 1998, Is the Risk of Bankruptcy a Systematic Risk ? *Journal of Finance,* 53, 1131-1147; Campbell, J.Y., J. Hilscher and J. Szilagyi, 2005, In Search of Distress Risk, SSRN Working Paper.

[10] The yields to maturity on bonds issued by companies where there is a significant probability of distress will be stratospheric, because they are based upon the promised cash flows on the bond, rather than expected cashflows.

more reasonable levels. The conventional practice of using target debt ratios for the entire valuation period (which reflect industry averages or the optimal mix) can lead to misleading estimates of value for firms that are significantly over levered.

Limitations of Approach

The biggest roadblock to using this approach is that even in its limited form, it is difficult to estimate the cumulative probabilities of distress (and survival) each year for the forecast period. Consequently, the expected cash flows may not incorporate the effects of distress completely. In addition, it is difficult to bring both the going concern and the distressed firm assumptions into the same model. We attempt to do so using probabilities, but the two approaches make different and sometimes contradictory assumptions about how markets operate and how distressed firms evolve over time.

Dealing with Distress Separately

An alternative to the modified discounted cash flow model presented in the last section is to separate the going concern assumptions and the value that emerges from it from the effects of distress. To value the effects of distress, we estimate the cumulative probability that the firm will become distressed over the forecast period, and the proceeds that we estimate we will get from the distress sale. The value of the firm can then be written as:

Firm Value = Going concern value * $(1-\pi_{Distress})$ + Distress sale value * $\pi_{Distress}$

where $\pi_{distress}$ is the cumulative probability of distress over the valuation period. In addition to making valuation simpler, it also allows us to make consistent assumptions within each valuation.

You may wonder about the differences between this approach and the far more conventional one of estimating liquidation value for deeply distressed firms. You can consider the distress sale value to be a version of liquidation value, and if you assume that the probability of distress is one, the firm value will, in fact, converge on liquidation value. The advantage of this approach is that it allows us to consider the possibility that even distressed firms have a chance (albeit small) of becoming going concerns.

Going Concern DCF

To value a firm as a going concern, we consider only those scenarios where the firm survives. The expected cash flow is estimated only across these scenarios and thus should be higher than the expected cash flow estimated in the modified discounted cash flow model. When estimating discount rates, we make the assumption that debt ratios will, in fact, decrease over time, if the firm is over levered, and that the firm will derive tax benefits from debt as it turns the corner on profitability. This is consistent with the assumption that the firm will remain a going concern. Most discounted cash flow valuations that we observe in practice are going concern valuations, though they may not come with the tag attached.

A less precise albeit easier alternative is to value the company as if it were a healthy company today. This would require estimating the cashflows that the firm would have generated if it were a healthy firm, a task most easily accomplished by replacing the firm's operating margin by the average operating margin of healthy firms in the business. The cost of capital for the distressed firm can be set to the average cost of capital for the industry and the value of the firm can be computed. The danger with this approach is that it will overstate firm value by assuming that the return to financial health is both painless and imminent.

Estimating the Probability of Distress

A key input to this approach is the estimate of the cumulative probability of distress over the valuation period. In this section, we will consider three ways in which we can estimate this probability. The first is a statistical approach, where we relate the probability of distress to a firm's observable characteristics – firm size, leverage and profitability, for instance – by contrasting firms that have gone bankrupt in prior years with firms that did not. The second is a less data intensive approach, where we use the bond rating for a firm, and the empirical default rates of firms in that rating class to estimate the probability of distress. The third is to use the prices of corporate bonds issued by the firm to back out the probability of distress.

a. Statistical Approaches

The fact that hundreds of firms go bankrupt every year provides us with a rich database that can be examined to evaluate both why bankruptcy occurs and how to predict the likelihood of future bankruptcy. One of the earliest studies that used this approach was by Altman (1968), where he used linear discriminant analysis to arrive at a measure that he called the Z score. In this first paper, that he has since updated several times, the Z score was a function of five ratios:

Z = 0.012 (Working capital/ Total Assets) + 0.014 (Retained Earnings/ Total Assets) + 0.033 (EBIT/ Total Assets) + 0.006 (Market value of equity/ Book value of total liabilities) + 0.999 (Sales/ Total Assets)

Altman argued that we could compute the Z scores for firms and use them to forecast which firms would go bankrupt, and he provided evidence to back up his claim. Since his study, both academics and practitioners have developed their own versions of these credit scores.[11]

Notwithstanding its usefulness in predicting bankruptcy, linear discriminant analysis does not provide a probability of bankruptcy. To arrive at such an estimate, we use a close variant – a probit. In a probit, we begin with the same data that was used in linear discriminant analysis, a sample of firms that survived a specific period and firms that did not. We develop an indicator variable that takes on a value of zero or one, as follows:

Distress Dummy	= 0	for any firm that survived the period
	= 1	for any firm that went bankrupt during the period

We then consider information that would have been available at the beginning of the period that may have allowed us to separate the firms that went bankrupt from the firms that did not. For instance, we could look at the debt to capital ratios, cash balances and operating margins of all of the firms in the sample at the start of the period – we would expect firms with high debt to capital ratios, low cash balances and negative margins to be more likely to go bankrupt. Finally, using the dummy variable as our dependent

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[11] Altman, E.I., 1968, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," Journal of Finance; For a more updated version of the Altman Z-score and its relationship to default probabilities take a look at Altman, E.I., 1993, Corporate Financial Distress and Bankruptcy, 2nd

variable and the financial ratios (debt to capital and operating margin) as independent variables, we look for a relationship:

Distress Dummy = a + b (Debt to Capital) + c (Cash Balance/ Value) + d (Operating Margin)

If the relationship is statistically and economically significant, we have the basis for estimating probabilities of bankruptcy.[12]

One advantage of this approach is that it can be extended to cover the likelihood of distress at firms without significant debt. For instance, we could relate the likelihood of distress at young, technology firms to the cash-burn ratio, which measures how much cash a firm has on hand relative to its operating cash needs.[13]

b. Based upon Bond Rating

Many firms, especially in the United States, have bonds that are rated for default risk by the ratings agencies. These bond ratings not only convey information about default risk (or at least the ratings agency's perception of default risk) but they come with a rich history. Since bonds have been rated for decades, we can look at the default experience of bonds in each ratings class. Assuming that the ratings agencies have not significantly altered their ratings standards, we can use these default probabilities as inputs into discounted cash flow valuation models. Altman and Kishore (2001) has estimated the cumulative probabilities of default for bonds in different ratings classes over five and ten-year periods and the estimates are reproduced in Table 1 below[14]:

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ed., John Wiley & Sons, New York.

[12] This looks like a multiple regression. In fact, a probit is a more sophisticated version of this regression with constraints built in ensuring that the probabilities do not exceed one or become negative.

[13] Cash Burn Ratio = Cash Balance/ EBITDA. With negative EBITDA, this yields a measure of the time that it will take the firm to burn through its cash balance.

[14] Altman, E.I. and V.Kishore, 1999, *The Default Experience of U.S. Bond*s, Working Paper, Salomon Center.

Table 1: Bond Rating and Probability of Default – 1971 - 2001

Rating	*Cumulative Probability of Distress*	
	5 years	*10 years*
AAA	0.03%	0.03%
AA	0.18%	0.25%
A+	0.19%	0.40%
A	0.20%	0.56%
A-	1.35%	2.42%
BBB	2.50%	4.27%
BB	9.27%	16.89%
B+	16.15%	24.82%
B	24.04%	32.75%
B-	31.10%	42.12%
CCC	39.15%	51.38%
CC	48.22%	60.40%
C+	59.36%	69.41%
C	69.65%	77.44%
C-	80.00%	87.16%

As elaboration, the cumulative default probability for a BB rated bond over ten years is 16.89%.[15]

What are the limitations of this approach? The first is that we are delegating the responsibility of estimating default probabilities to the ratings agencies and we assume that they do it well. The second is that we are assuming that the ratings standards do not shift over time. The third is that the table measures the likelihood of default on a bond, but it does not indicate whether the defaulting firm goes out of business. Many firms continue to operate as going concerns after default.

We can illustrate the use of this approach with Global Crossing. At the end of 2001, Global Crossing had been assigned a bond rating of CCC by S&P. Based upon this bond rating and the history of defaults between 1971 and 2001, we would have estimated a cumulative probability of bankruptcy of 51.38% over 10 years for the firm.

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[15] Altman estimates the probability of default only for AAA, AA, A, BBB, BB, B and CCC bonds. We interpolated to get the rest of the table.

c. Based upon Bond Price

The conventional approach to valuing bonds discounts promised cash flows back at a cost of debt that incorporates a default spread to come up with a price. Consider an alternative approach. We could discount the expected cash flows on the bond, which would be lower than the promised cash flows because of the possibility of default, at the riskfree rate to price the bond. If we assume that a constant annual probability of default, we can write the bond price as follows for a bond with fixed coupon maturing in N years.

$$\text{Bond Price} = \sum_{t=1}^{t=N} \frac{\text{Coupon } (1-\pi_{Distress})^t}{(1+\text{Riskfree Rate})^t} + \frac{\text{Face Value of Bond}(1-\pi_{Distress})^N}{(1+\text{Riskfree Rate})^N}$$

This equation can now be used, in conjunction with the price on a traded corporate bond to back out the probability of default. We are solving for an annualized probability of default over the life of the bond, and ignoring the reality that the annualized probability of default will be higher in the earlier years and decline in the later years.

While this approach has the attraction of being a simple one, we would hasten to add the following caveats in using it. First, note that we not only need to find a straight bond issued by the company – special features such as convertibility will render the approach unusable – but the bond price has to be available. If the corporate bond issue is privately placed, this will not be feasible. Second, the probabilities that are estimated may be different for different bonds issued by the same firm. Some of these differences can be traced to the assumption we have made that the annual probability of default remains constant and others can be traced to the mis-pricing of bonds. Third, as with the previous approach, failure to make debt payments does not always result in the cessation of operations. Finally, we are assuming that the coupon is either fully paid or not at all; if there is a partial payment of either the coupon or the face value in default, we will over estimate the probabilities of default using this approach.

Illustration 1: Estimating the probability of bankruptcy using bond price: Global Crossing

In late 2001, Global Crossing had a 12% coupon bond with 8 years to maturity trading at $ 653. To estimate the probability of default (with a treasury bond rate of 5% used as the riskfree rate):

$$653 = \sum_{t=1}^{t=8} \frac{120(1-\pi_{Distress})^t}{(1.05)^t} + \frac{1000(1-\pi_{Distress})^8}{(1.05)^8}$$

Solving for the probability of bankruptcy[16], we get

$$\pi_{Distress} = \text{Annual probability of default} = 13.53\%$$

To estimate the cumulative probability of distress over 10 years:

Cumulative probability of surviving 10 years = $(1 - .1353)^{10} = 23.37\%$

Cumulative probability of distress over 10 years = 1 - .2337 = .7663 or 76.63%

Estimating Distress Sale Proceeds

Once we have estimated the probability that the firm will be unable to make its debt payments and will cease to exist, we have to consider the logical follow-up question. What happens then? As noted earlier in the paper, it is not distress per se that is the problem but the fact that firms in distress have to sell their assets for less than the present value of the expected future cash flows from existing assets and expected future investments. Often, they may be unable to claim even the present value of the cash flows generated even by existing investments. Consequently, a key input that we need to estimate is the expected proceeds in the event of a distress sale. We have three choices:

a. Estimate the present value of the expected cash flows in a discounted cash flow model, and assume that the distress sale will generate only a percentage (less than 100%) of this value. Thus, if the discounted cash flow valuation yields $ 5 billion as the value of the assets, we may assume that the value will only be $ 3 billion in the event of a distress sale.

b. Estimate the present value of expected cash flows only from existing investments as the distress sale value. Essentially, we are assuming that a buyer will not pay for future investments in a distress sale. In practical terms, we would estimate the distress sale value by considering the cash flows from assets in place as a perpetuity (with no growth).

c. The most practical way of estimating distress sale proceeds is to consider the distress sale proceeds as a percent of book value of assets, based upon the experience of other distressed firms. Thus, the fact that distressed telecomm

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[16] With a 10-year bond, it is a process of trial and error to estimate this value. The solver function in excel accomplishes the same in far less time.

companies are able to sell their assets for 20% of book value would indicate that the distress sale proceeds would be 20% of the book value of the assets of the firm.

Note that many of the issues that come up when estimating distress sale proceeds – the need to sell at below fair value, the urgency of the need to sell – are issues that are relevant when estimating liquidation value.

Illustration 2: Estimating Distress Sale Proceeds in January 2002: Global Crossing

To estimate the expected proceeds in the event of a distress sale, we considered several factors. First, the sluggish growth in the economy in January 2002 clearly did not bode well for any firm trying to sell its assets in a liquidation. Second, the fact that a large number of telecomm firms were in distress and looking for potential buyers at that time was also likely to weigh down the proceeds in a sale. In fact, PSInet, another telecomm firm that had recently been forced into a distress sale, was able to receive less than 10% of its book value in the sale. For Global Crossing, we asssumed that the distress sale proceeds would be 15% of the book value of the non-cash assets.

Book value of non-cash assets	= $14,531 million
Distress sale value = 15% of book value = .15*14531	= $ 2,180 million

Since the firm had debt outstanding with a face value of $7,647 million, the equity investors would receive nothing in the event of a distress sale, even if we considered the cash and marketable securities of $2,260 million that the firm had on its books.

Illustration 3: Valuing Global Crossing with Distress valued separately

To value Global Crossing with distress valued separately, we began with a going concern valuation of Global Crossing and then consider the distress sale proceeds:

1. Valuing Global Crossing as a going concern

Global Crossing provided managed data and voice products over a fiber optic network. Over its three-year history, the firm had increased revenues from $420 million in 1998 to $4,040 million in 2001, but it had gone from an operating income of $120 million in 1998 to an operating loss of $1,895 million in 2001[17]. In addition, the firm was

[17] While the financial statements for 2000 had not been released, the trailing 12-month numbers were used for most of the inputs in November 2001

capital intensive and reported substantial capital expenditures ($4,289 million) and depreciation ($1,436 million) in 2000.

In making the valuation, we assumed that there would be no revenue growth in the first year (to reflect a slowing economy) and that revenue growth would be brisk for the following 4 years and then taper off to a stable growth rate of 5% in the terminal phase, that EBITDA as a percent of sales would move from the current level (of about –10%) to a industry average of 30%. by the end of the tenth year and that capital expenditures would be ratcheted down over the next two years to maintenance levels. Table 2 summarizes our assumptions on revenue growth, EBITDA/Sales and reinvestment needs over the next 10 years.

Table 2: Assumptions for Valuation

	Revenue	*EBITDA/ Revenues*	*Growth rate in Capital Spending*	*Growth rate in Depreciation*	*Working capital as % of Revenue*
1	0.00%	-3%	-20%	10%	3.00%
2	40.00%	0.00%	-50%	10%	3.00%
3	30.00%	5.00%	-30%	10%	3.00%
4	20.00%	10.00%	5%	10%	3.00%
5	10.00%	15.00%	5%	-50%	3.00%
6	10.00%	18.00%	5%	-30%	3.00%
7	10.00%	21.00%	5%	5%	3.00%
8	8.00%	24.00%	5%	5%	3.00%
9	6.00%	27.00%	5%	5%	3.00%
10	5.00%	30.00%	5%	5%	3.00%

For both revenue growth and improvement in EBITDA margins, we assumed that the larger changes occured in the earlier years. Note that the changes in depreciation lag the changes in capital spending – the capital spending is cut first and depreciation drops later. Finally, we assumed that the firm would need to set aside 3% of the revenue change each year into working capital based upon the industry averages.

With these forecasts, we estimated revenues, operating income and after-tax operating income each year for the high growth period in Table 3. To estimate taxes, we considered the net operating losses carried forward into 2001 of $2,075 million and add on the additional losses that we expected in the first few years of the projection.

Table 3: Expected after-tax operating income to firm: Global Crossing

Year	*Revenues*	*EBITDA*	*Depreciation*	*EBIT*	*NOL at beginning of year*	*Taxes*	*EBIT (1-t)*

1	$3,804	-$95	$1,580	-$1,675	$2,075	0	-$1,675
2	$5,326	$0	$1,738	-$1,738	$3,750	$0	-$1,738
3	$6,923	$346	$1,911	-$1,565	$5,487	$0	-$1,565
4	$8,308	$831	$2,102	-$1,272	$7,052	$0	-$1,272
5	$9,139	$1,371	$1,051	$320	$8,324	$0	$320
6	$10,053	$1,809	$736	$1,074	$8,004	$0	$1,074
7	$11,058	$2,322	$773	$1,550	$6,931	$0	$1,550
8	$11,942	$2,508	$811	$1,697	$5,381	$0	$1,697
9	$12,659	$3,038	$852	$2,186	$3,685	$0	$2,186
10	$13,292	$3,589	$894	$2,694	$1,498	$419	$2,276
Terminal	$13,957	$4,187	$939	$3,248	$0	$1,137	$2,111

The accumulated losses over the first nine years shield the firm from paying taxes until the tenth year. After that point, we assumed a marginal tax rate of 35%.[18]

Finally, we estimated free cash flows to the firm with our assumptions about capital expenditures and working capital (in Table 4).

Table 4: Expected free cashflows to the firm: Global Crossing

Year	*EBIT (1-t)*	*Capital Expenditures*	*Depreciation*	*Change in working capital*	*FCFF*
1	-$1,675	$3,431	$1,580	$0	-$3,526
2	-$1,738	$1,716	$1,738	$46	-$1,761
3	-$1,565	$1,201	$1,911	$48	-$903
4	-$1,272	$1,261	$2,102	$42	-$472
5	$320	$1,324	$1,051	$25	$22
6	$1,074	$1,390	$736	$27	$392
7	$1,550	$1,460	$773	$30	$832
8	$1,697	$1,533	$811	$27	$949
9	$2,186	$1,609	$852	$21	$1,407
10	$2,276	$1,690	$894	$19	$1,461
Terminal	$2,111	$2,353	$939	$20	$677

The firm uses debt liberally to fund these investments and had book value of debt outstanding of $7,647 million at the end of 2001. We estimated a market value for the debt of $4,923 million.[19] Based upon its market capitalization (for equity) of $1,649 million at the time of this valuation, we estimated a market debt to capital ratio for the firm.

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[18] The tax rate in year 10 is less than 35% because of the net operating losses carried forward from the previous year.

[19] To estimate the market value, we discounted the face value of debt and the interest payments back at the estimated pre-tax cost of debt of 12.80%.

$$\text{Debt to capital} = \frac{4923}{4923+1649} = 74.91\%$$

$$\text{Equity to capital} = \frac{1649}{4923+1649} = 25.09\%$$

To estimate the bottom-up beta, we began with an unlevered beta of 0.7527 (based upon all publicly traded telecomm services firms) and estimated the levered beta for the firm:

Levered beta = Unlevered beta (1 + (1- tax rate) (Debt/Equity))

= 0.7527 (1 + (1-0) (4923/1649)) = 3.00

Using a bottom-up beta of 3.00 for the equity and a cost of debt of 12.80% based upon the then current rating for the firm(CCC), we estimated a cost of capital for the next 5 years. (The riskfree rate was 4.8% and the risk premium used was 4%.)

Cost of equity = 4.8% + 3 (4%) = 16.80%

After-tax cost of debt = 12.8% (1-0) = 12.8% (The firm does not pay taxes)

Cost of capital = 16.80% (0.2509) + 12.8% (0.7491) = 13.80%

In stable growth, after year 10, we assumed that the beta would decrease to 1.00 and that the pre-tax cost of debt would decrease to 8%. The adjustment occurs in linear increments from years 6 through 10 as shown in Table 5:

Table 5: Cost of capital – Global Crossing

Year	*1-5*	*6*	*7*	*8*	*9*	*10*	*Terminal*
Tax Rate		0%	0%	0%	0%	16%	35%
Beta	3.00	2.60	2.20	1.80	1.40	1.00	1.00
Cost of Equity	16.80%	15.20%	13.60%	12.00%	10.40%	8.80%	8.80%
Cost of Debt	12.80%	11.84%	10.88%	9.92%	8.96%	6.76%	5.20%
Debt Ratio	74.91%	67.93%	60.95%	53.96%	46.98%	40.00%	40.00%
Cost of Capital	13.80%	12.92%	11.94%	10.88%	9.72%	7.98%	7.36%

To estimate the reinvestment rate in the terminal year, we assumed that Global Crossing would earn its cost of capital of 7.36% in perpetuity after year 10, and that the expected growth rate would be 5%. This yields a reinvestment rate of 67.93%.

$$\text{Reinvestment rate in stable growth} = \frac{5\%}{7.36\%} = 67.93\%$$

$$= EBIT_{11}(1 - t)(1 - \text{reinvestment rate})$$

$$\text{Expected FCFF in terminal year} = 2111(1 - 0.35)(1 - 0.6793)$$

$$= \$677 \text{ million}$$

$$\text{Terminal value} = \frac{FCFF_{11}}{\text{Cost of capital} - g} = \frac{677}{0.0736 - 0.05} = \$\ 28{,}683 \text{ million}$$

Discounting the operating cashflows and the terminal value back to the present, we arrived at an estimate of the value of the operating assets of $5,530 million. Note, though, that almost of this value came from our presumption that Global Crossing would not only survive but become profitable, which is the source of the large terminal value. Adding back the cash and marketable securities held by the firm ($2,260 million) and subtracting out the value of debt ($4,923 million) and the estimated value of management options outstanding ($14.3 million)[20], we arrive at a value of equity of $2.852 billion. Dividing by the number of shares outstanding results in a value per share of $3.22.

Value of the operating assets of the firm =	$5,529.92
+ Cash and Marketable Securities =	$2,260.00
- Market Value of Debt =	$4,922.75
Market Value of Equity =	$2,867.17
Value of Options Outstanding =	$14.31
Value of Equity in Common Stock =	$2,852.86
Value of Equity per Share =	$3.22

Valued as a going concern, we would have assigned a value of $3.22 per share to Global Crossing's equity.

Dealing with Distress

In illustration 1 we estimated the cumulative probability of distress for Global Crossing to be 76.63% over the next 10 years, and in illustration 2, we estimated the distress sale proceeds to be 15% of book value, based upon how much the assets of other bankruptcy telecomm firms were receiving in the market place currently. Combining

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[20] The Black-Scholes model was used to estimate the value of the options outstanding. In fact, these options had lost a substantial portion of value because of the drop in the stock price.

these two inputs, we arrive at an estimate of an expected value for the operating assets with distress built into the assumptions:

Expected Value of Operating Assets = 5530 (1 - .7663) + 2180 (.2337) = $2,962.90 million

If we add back the cash and marketable securities and net out the debt, we arrive at an adjusted value of equity for the firm.

Value of the firm =	$2,962.90
+ Cash and Marketable Securities =	$2,260.00
- Market Value of Debt =	$4,922.75
Market Value of Equity =	$300.15
Value of Options Outstanding (See option worksheet) =	$14.31
Value of Equity in Common Stock =	$285.84
Value of Equity per Share =	$0.32

One limitation of this approach is that it does not consider the fact that equity has limited liability. In other words, if distress occurs and the value of the operating assets is less than the debt outstanding (as is inevitable), equity investors will get nothing from their investment but will not be required to make up the difference. We can estimate a more realistic value of equity by taking a weighted average of equity per share:

Value of equity = $3.22 (1- .7663) + $ 0.00 (.7663) = $0.75

One way to read this difference is to consider the first estimate of value ($0.32) as the value without limited liability and the second estimate ($0.75) as the value to equity investors of limited liability.

Adjusted Present Value (APV)

In the adjusted present value (APV) approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. To do this, we assume that the primary benefit of borrowing is a tax benefit and that the most significant cost of borrowing is the added risk of bankruptcy. With distressed firms, the advantage of separating the value impact of debt from the value of the operating assets is that more attention can be paid to the cost and probability of distress.

Reviewing the steps in the APV approach, we estimate the value of the firm in three steps. We begin by estimating the value of the firm with no leverage, by discounting the expected free cash flow to the firm at the unlevered cost of equity. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

$$\text{Value of Unlevered Firm} = \frac{FCFF_0(1+g)}{\rho_u - g}$$

where $FCFF_0$ is the current after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity and g is the expected growth rate. In the more general case, we can value the firm using any set of growth assumptions we believe are reasonable for the firm.

We then consider the present value of the interest tax savings generated by borrowing a given amount of money. This tax benefit is a function of the tax rate of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

$$\begin{aligned}\text{Value of Tax Benefits} &= \frac{(\text{Tax Rate})(\text{Cost of Debt})(\text{Debt})}{\text{Cost of Debt}} \\ &= (\text{Tax Rate})(\text{Debt}) \\ &= t_c D\end{aligned}$$

For a distressed firm, this value will be depressed if the firm has substantial operating losses and does not expect to get tax benefits for the foreseeable future.

The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. This requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

$$\begin{aligned}\text{PV of Expected Bankruptcy cost} &= (\text{Probability of Bankruptcy})(\text{PV of Bankruptcy Cost}) \\ &= \pi_a BC\end{aligned}$$

We can use the approaches described in the last section to arrive at an estimate of the probability of bankruptcy. We can also consider the difference between the value of a firm as a going concern and the distress sale value as the cost of bankruptcy. Thus, if the present value of expected cash flows is $ 5 billion – the going concern value – and the

distress sale proceeds is expected to be only 25% of the book value of $ 4 billion, the bankruptcy cost is $ 4 billion.

Expected bankruptcy cost = $ 5 billion - .25 (4 billion) = $ 4 billion

Again, with distressed firms, the present value of expected bankruptcy costs is likely to be a large number. The combination of low tax benefits and large bankruptcy costs is likely to reduce firm value.

Almeida and Philippon (2005) suggest a variation of the adjusted present value model, arguing that the conventional measure of distress costs understates its magnitude because it does not factor in the reality that distress costs are often systematic (market and economy driven).[21] They present two ways of adjusting distress cost value to reflect this systematic risk. In the first, they derive probabilities of default from corporate bond spreads, akin to what we did earlier in illustration 1. In the second, they derive the risk adjustment from historical data on distress probabilities and asset-pricing models. They conclude that the expected bankruptcy costs are substantial and have a large impact on value.

Illustration 4: Valuing Global Crossing: Adjusted Present Value

To value Global Crossing on an adjusted present value basis, we would first need to value the firm as an unlevered entity. We can do this by using the unlevered cost of equity as the cost of capital.

Unlevered beta for Global Crossing[22] = 0.7527

Using the riskfree rate of 4.8% and the market risk premium of 4%,

Unlevered cost of equity for Global Crossing = 4.8% + 0.7527 (4%) = 7.81%

We use this cost of equity as the cost of capital and discount the expected free cashflows to the firm shown earlier in Table 4. Table 6 summarizes the present value of the cashflows at the unlevered cost of equity. (Note that the terminal value is left unchanged. We will continue to assume that the firm will earn its cost of capital on investments after year 10)

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[21] Almeida, H. and T. Philippon, 2005, The Risk-adjusted Cost of Financial Distress, SSRN Working Paper.

[22] We used the unlevered beta of telecomm services firms as the unlevered beta for Global Crossing.

Table 6: Present Value of FCFF at Unlevered Cost of Equity

Year	*FCFF*	*Terminal Value*	*PV at 7.81%*
1	-$3,526		-$3,270.85
2	-$1,761		-$1,515.31
3	-$903		-$720.38
4	-$472		-$349.17
5	$22		$15.02
6	$392		$249.55
7	$832		$491.64
8	$949		$519.81
9	$1,407		$715.26
10	$1,461	$28,683	$14,210.82
Total			$10,346.39

The unlevered firm value is $10,346 million. To this we should add the expected tax benefits of debt. Since the firm is losing money and has substantial net operating losses, the expected tax benefits accrue almost entirely after year 10. Consequently, we discount back the expected tax benefits after year 10 at 7.81%[23]. To estimate the bankruptcy cost, we consider the difference between the going concern value of $14,211 million and the distress sale proceeds estimate of $2,180 million (estimated in illustration 2) as the bankruptcy cost. Multiplying this by the probability of bankruptcy estimated in illustration 1 yields the expected cost of bankruptcy:

Expected tax benefits = $.35*10,346/1.0781^{10}$= $1,707 million

Adjusted Present Value of Global Crossing's assets = Unlevered firm value + Present value of tax benefits – Expected bankruptcy costs = 10,346 + 1,707 – 0.7663 (14,211 – 2,180) = $ 2,834 million

[23] This is conservative. The tax benefits may be higher because of the accumulated net operating losses.

Adding back the cash and marketable securities and subtracting out debt yields a value of equity for Global Crossing:

APV of Global Crossing Assets =		$ 2,834 million
+ Cash & Marketable Securities =		$ 2,260 million
- Market value of Debt	=	$ 4,923 million
Value of Equity	=	$171 million

Value per share = $171 million/ 886.47 = $0.19

There were 886.47 million shares outstanding in December 2001.

Relative Valuation

Most valuations in practice, including those of distressed firms, are relative valuations. In particular, firms are valued using multiples and groups of comparable firms. An open question then becomes whether the effects of distress are reflected in relative valuations and, if not, how best to do so.

Distress in Relative Valuation

It is not clear how distress is incorporated into an estimate of relative value. Consider how relative valuation is most often done. We choose a group of firms that we believe are comparable to the firm that we are valuing. Usually, we pick firms in the same business that our firm is in. We then standardize prices by computing a multiple – price earnings, price to book, enterprise value to sales or enterprise value to EBITDA. Finally, we examine how our firm measures up on this multiple, relative to the comparable firms. While this time honored approach is used for distressed firms as well, the issues listed below generally are unique to distressed firms:

1. Revenue and EBITDA multiples are used more often to value distressed firms than healthy firms. The reasons are pragmatic. Multiple such as price earnings or price to book value often cannot even be computed for a distressed firm. Analysts therefore move up the income statement looking for a positive number. For firms that make heavy infrastructure investments, where depreciation and amortization is a significant charge against operating income and there are substantial interest expenses, the EBITDA is often

positive while net income is negative. For some firms, even EBITDA is negative and revenue multiples are only multiples that yield positive values.

2. Analysts who are aware of the possibility of distress often consider them subjectively when they compare the multiple for the firm they are analyzing to the industry average. For example, assume that the average telecomm firm trades at 2 times revenues and that the firm we are analyzing trades at 1.25 times revenues. Assume also that the firm has substantially higher default risk than the average telecomm firm. We may conclude that the firm is not undervalued even though it trades at a significant discount on the average, because of the potential for default. The perils of subjective adjustment are obvious. Barring the most egregious misvaluations, analysts will find a way to justify their prior biases about firms.

Adapting Relative Valuation to Distress

Is there a way in which relative valuation can be adapted to cover distressed firms? We believe so, though the adjustments tend to be much more approximate than those described in the discounted cash flow section. We consider two ways of building distress explicitly into relative valuations. In the first, we compare a distressed company's valuation to the valuations of other distressed companies. In the second, we use healthy companies as comparable companies, but find a way to adjust for the distress that the firm we are valuing is facing.

Choosing the Comparables

To value a distressed firm, we can find a group of distressed firms in the same business and look at how much the market is willing to pay for them. For instance, we could value a troubled telecomm firm by looking at the enterprise value to sales (or book capital) multiples at which other troubled telecomm firms trade. While there is promise in this approach, it works only if a large number of firms in a sector slip into financial trouble at the same time. In addition, by categorizing firms as distressed or not distressed firms, we run the risk of lumping together firms that are distressed to different degrees.

One possible way to expand this approach is to look at distressed firms across the whole market, rather than just the sector in which the firm operates. This will allow for a

larger sample though there is the possible disadvantage that a troubled grocery store may be in a better position (in terms of generating distress sale proceeds) than a troubled technology company.

Illustration 5: Choosing distressed comparables

To value Global Crossing, we considered only telecomm service firm with negative operating income and high leverage (market debt to capital ratios that exceed 75%). We measured book capital as the sum of the book values of equity and debt at the end of the most recent financial year. Our objective was to arrive at a sample of telecomm firms that have a significant likelihood of distress. Table 7 summarizes the enterprise value/ book capital ratios for these firms:

Table 7: Distressed Telecomm Firms

Company Name	*Value to Book Capital*	*EBIT*	*Market Debt to Capital Ratio*
SAVVIS Communications Corp	0.80	-83.67	75.20%
Talk America Holdings Inc	0.74	-38.39	76.56%
Choice One Comm. Inc	0.92	-154.36	76.58%
FiberNet Telecom Group Inc	1.10	-19.32	77.74%
Level 3 Communic.	0.78	-761.01	78.89%
Global Light Telecom.	0.98	-32.21	79.84%
Korea Thrunet Co. Ltd Cl A	1.06	-114.28	80.15%
Williams Communications Grp	0.98	-264.23	80.18%
RCN Corp.	1.09	-332.00	88.72%
GT Group Telecom Inc Cl B	0.59	-79.11	88.83%
Metromedia Fiber 'A'	0.59	-150.13	91.30%
Global Crossing Ltd.	0.50	-15.16	92.75%
Focal Communications Corp	0.98	-11.12	94.12%
Adelphia Business Solutions	1.05	-108.56	95.74%
Allied Riser Communications	0.42	-127.01	95.85%
CoreComm Ltd	0.94	-134.07	96.04%
Bell Canada Intl	0.84	-51.69	96.42%
Globix Corp.	1.06	-59.35	96.94%
United Pan Europe Communicatio	1.01	-240.61	97.27%
Average	0.87		

Global Crossing trades at 50% of book capital invested, significantly lower than the average ratio across these distressed firms. We could view this as indicative of the fact that Global Crossing is under valued on a relative basis, though that conclusion would be

justified only if we assume that the firms are exposed to equal degrees to financial distress.

Adjusting the Multiple

A second possibility is to look for objective ways of adjusting the multiple for distress. Consider one possible solution. We could examine the multiple of revenues or operating income at which firms in different ratings classes trade at to get a measure of the discount (if any) that is being applied by the market for the degree of distress to which a firm is exposed. If there are enough firms in the sector that we are analyzing in each ratings class, we could do this on a sector basis. If there are not, we could look at the multiple across the entire market and examine differences across bond rating classes.

Illustration 6: Adjusted Multiple: Global Crossing

Looking at all telecomm firms, and categorizing them based upon bond ratings, we were able to estimate the value to book ratios at the end of 2001 by bond rating class:

Bond Rating	*Value to Book Capital Ratio*
A	1.70
BBB	1.61
BB	1.18
B	1.06
CCC	0.88
CC	0.61

The differences between ratings classes provide us with an indication of the discount that we would apply when valuing distressed firms. For instance, Global Crossing with its CCC ratings should have a multiple that is roughly half that of a healthy A rated firm in the same sector.

Considering the Possibility of Distress Explicitly

One of the adaptations that we suggested for discounted cash flow valuation was an explicit assessment of default risk and a firm value that was a weighted estimate of a going concern value and a distress sale value. For a distressed firm in a sector where the average firm is healthy, this approach offers promise. We can estimate the value of the distressed firm using the comparable firms and consider it the going concern value. For instance, if healthy firms in the business trade at 2 times revenues, we would multiple the

firm's revenues by 2 to arrive at the going concern value. We could then estimate the firm value as follows:

Firm Value = Going concern relative value * (1-$\pi_{Distress}$)+ Distress sale value * $\pi_{Distress}$

The probability of distress and the distress sale value would be estimated just as they were in the last section. This approach makes the most sense when valuing a firm that is distressed in a sector containing mostly healthy firms, since the prior two approaches could not be used here.

In some cases, we may have to use forecasted values for revenues and operating income to arrive at the going concern value, especially if current revenues and operating income are adversely impacted by the overhang of distress.

Illustration 7: Forward Multiples and Distress

Consider the forecasts of revenues and EBITDA made in Table 3 for Global Crossing. While the firm is losing a substantial amount of money currently, we are forecasting a return to financial health. In year 5, for instance, Global Crossing is expected to have an EBITDA of $1,371 million on revenues of $9,139 million. Using the average enterprise value/EBITDA multiple of 7.2 at which healthy telecomm firms[24] trade, we can estimate an expected enterprise value in year 5.

Expected Enterprise value in year 5 = $EBITDA_5$ * EV/ $EBITDA_{Current\ for\ healthy\ telecomm\ firms}$

= 1,371 * 7.2 = $9,871 million

We can estimate the present value of this estimated value by discounting back at Global Crossing's cost of capital.

Enterprise value today = $9871/1.138^5$ = $5,172 million

This, of course, is based upon the assumption that Global Crossing will become a healthy firm. Using the probability of survival (23.37%) and distress (76.63%) estimated earlier, we can value Global Crossing's operating assets today:

Estimated Enterprise Value

= Going Concern Value ($\pi_{Going\ Concern}$) + Distres Sale Value (1 - $\pi_{Going\ Concern}$)

= 5172 (.2337) + 2180 (.7663) = $2,879 million

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[24] We considered only firms with positive operating income and low debt to capital ratios (less than 30%) as healthy firms.

Note that the estimate of the distress sale value of $2,180 million was made earlier in illustration 2. Adding back the cash balance of the firm ($ 2,260 million) and subtracting out debt ($4923 million) yields a value for the equity:

Enterprise Value	= $ 2,879 million
+ Cash & Marketable Securities	= $ 2,260 million
- Debt	= $4,923 million
Value of Equity	= $216 million
Value per share = $216/ 886.47	=$ 0.24

From Firm to Equity Value in Distressed Firms

In conventional valuation, we subtract the market value of the debt from firm value to arrive at equity value. When valuing distressed firms, we have to consider two specific issues. The first is the shifting debt load at these firms, since these firms are often in the process of restructuring and renegotiating debt, can make identifying the dollar debt due sat a point in time a hazardous exercise. The second is that equity in distressed firms may sometimes take on option characteristics and trade at a premium on what discounted cash flow valuations would suggest is the value.

The Shifting Debt Load

In addition to having a substantial amount of debt, distressed firms often have very complicated debt structures. Not only do they owe money to a number of different creditors, but the debt itself often is usually complex – convertible, callable and filled with special features demanded by the creditors for their own protection. In addition, distressed firms are often in the process of negotiating with debt holders, trying to convince them to change the terms of the debt and, in some cases, convert their debt into equity. Consequently, the value of the debt can change dramatically from day to day, thus affecting the value of equity, even if the enterprise value does not.

When estimating the value of debt in a distressed firm, we should consider doing the following:

- Rather than relying on the last available financial statements for the available debt, we should try to obtain an updated estimate of the outstanding debt. This

may be difficult to do when the debt negotiations are private (between the distressed firm and the lenders.

- We should update the estimated market value of debt frequently, since the default risk of distressed firms can change substantially from period to period. Even if the debt is not traded, it is never appropriate with distressed firms to use the book value of debt as a proxy for the market value of debt. Instead, we should estimate the market value of debt, treating book debt like a corporate bond.
- When confronted with convertible debt, we should strip the conversion option from the debt and treat it as equity. Again, a simple way to do this is to value the convertible debt as if it were straight debt – this will yield the debt portion of the convertible debt – and consider the difference between the market value of the convertible debt and the straight debt portion as equity.

In general, valuing a distressed firm is far easier than valuing equity in the same firm, largely because the debt outstanding will vary over time.

Equity as an Option

In most publicly traded firms, equity has two features. The first is that the equity investors run the firm and can choose to liquidate its assets and pay off other claim holders at any time. The second is that the liability of equity investors in some private firms and almost all publicly traded firms is restricted to their equity investments in these firms. This combination of the option to liquidate and limited liability allows equity to have the features of a call option. In firms with substantial debt and a significant potential for bankruptcy, the option value of equity may be in excess of the discounted cash flow value of equity.

The Payoff on Equity as an Option

The equity in a firm is a residual claim, that is, equity holders lay claim to all cash flows left after other financial claimholders (debt, preferred stock, etc.) have been satisfied. If a firm is liquidated, the same principle applies; equity investors receive the cash that is left in the firm after all outstanding debt and other financial claims have been paid off. With limited liability, if the value of the firm is less than the value of the

outstanding debt, equity investors cannot lose more than their investment in the firm. The payoff to equity investors on liquidation can therefore be written as:

Payoff to equity on liquidation = V - D if V > D

= 0 if V ≤ D

where

V = Liquidation Value of the firm

D = Face Value of the outstanding debt and other external claims

Equity can thus be viewed as a call option on the firm, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercise price) be paid off. The firm is the underlying asset and the option expires when the debt comes due. The payoffs are shown in Figure 1.

Figure 1: Payoff on Equity as Option on a Firm

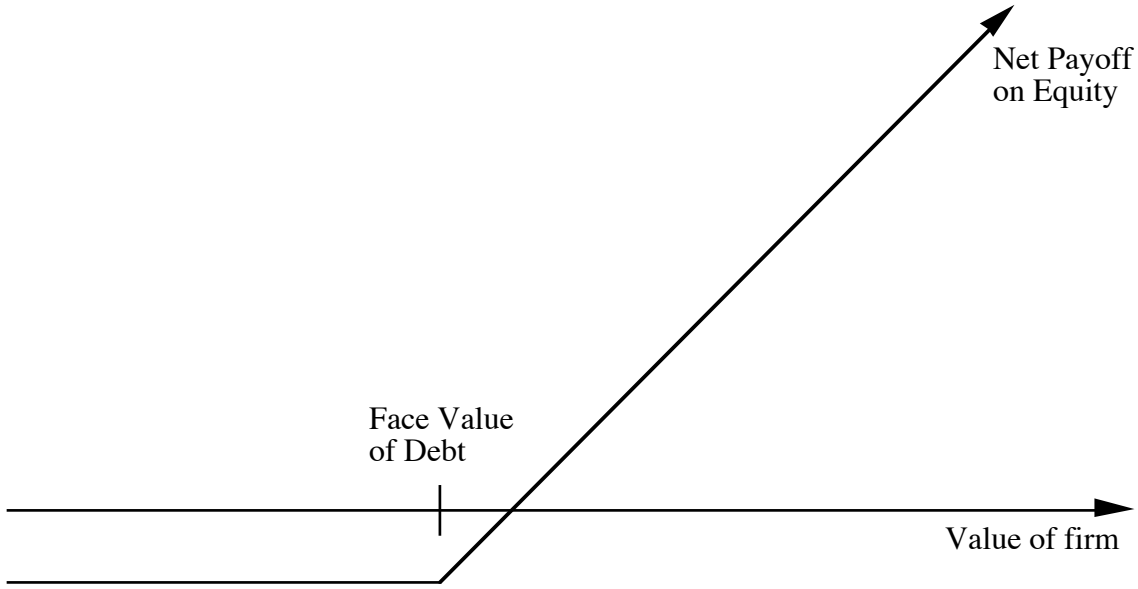


Illustration 8: Valuing Equity as an Option

Assume that we are valuing the equity in a firm whose assets are currently valued at $100 million; the standard deviation in this asset value is 40%. The face value of debt is $80 million (it is zero coupon debt with 10 years left to maturity). The 10-year treasury bond rate is 10%. We can value equity as a call option on the firm, using the following inputs for the option pricing model.

Value of the underlying asset = S = Value of the firm = $ 100 million

Exercise price = K = Face Value of outstanding debt = $ 80 million

Life of the option = t = Life of zero-coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based upon these inputs, the Black-Scholes model provides the following value for the call.

$d_1 = 1.5994$ $N(d_1) = 0.9451$

$d_2 = 0.3345$ $N(d_2) = 0.6310$

Value of the call = $100 (.9451) – 80 $e^{-(.10)(10)}$ (.6310) = $75.94 million

Since the call value represents the value of equity and the firm value is $100 million, the estimated value of the outstanding debt can be calculated.

Value of the outstanding debt = $100 - $75.94 = $24.06 million

Since the debt is a 10-year zero coupon bond, the market interest rate on the bond can be calculated.

Interest rate on debt $= \left(\frac{\$80}{\$24.06}\right)^{\frac{1}{10}} - 1 = 12.77\%$

Thus, the default spread on this bond should be 2.77%.

Implications of viewing Equity as an Option

When the equity in a firm takes on the characteristics of a call option, we have to change the way we think about its value and what determines its value. In this section, we will consider a number of potential implications for equity investors and bondholders in the firm.

When will equity be worthless?

In discounted cash flow valuation, we argue that equity is worthless if what we own (the value of the firm) is less than what we owe. The first implication of viewing equity as a call option is that equity will have value, even if the value of the firm falls well below the face value of the outstanding debt. While the firm will be viewed as troubled by investors, accountants and analysts, its equity is not worthless. In fact, just as deep out-of-the-money traded call options command value because of the possibility that the value of the underlying asset may increase above the strike price in the remaining lifetime of the option, equity commands value because of the time premium on the option

(the time until the bonds mature and come due) and the possibility that the value of the assets may increase above the face value of the bonds before they come due.

Illustration 9: Firm Value and Equity Value

Revisiting the preceding example, assume that the value of the firm drops to $50 million, below the face value of the outstanding debt ($80 million). Assume that all the other inputs remain unchanged. The parameters of equity as a call option are as follows:

Value of the underlying asset = S = Value of the firm = $ 50 million

Exercise price = K = Face Value of outstanding debt = $ 80 million

Life of the option = t = Life of zero-coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based upon these inputs, the Black-Scholes model provides the following value for the call.

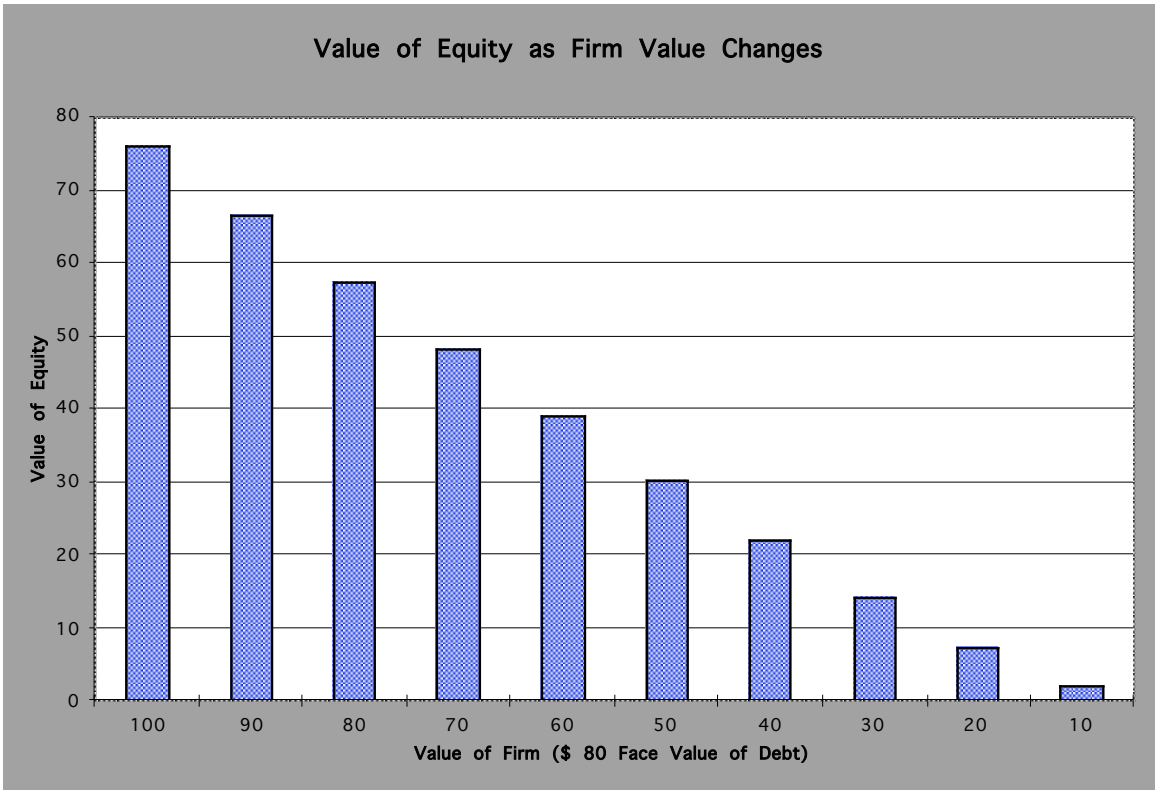
$d_1 = 1.0515$ $N(d_1) = 0.8534$

$d_2 = -0.2135$ $N(d_2) = 0.4155$

Value of the call (equity) = $50 (0.8534) - 80 \exp^{(-0.10)(10)} (0.4155) = \$30.44$ million

Value of the bond= $50 - $30.44 = $19.56 million

As we can see, the equity in this firm retains value, because of the option characteristics of equity. In fact, equity continues to have value in this example even if the firm value drops to $10 million or below, as shown in Figure 2.



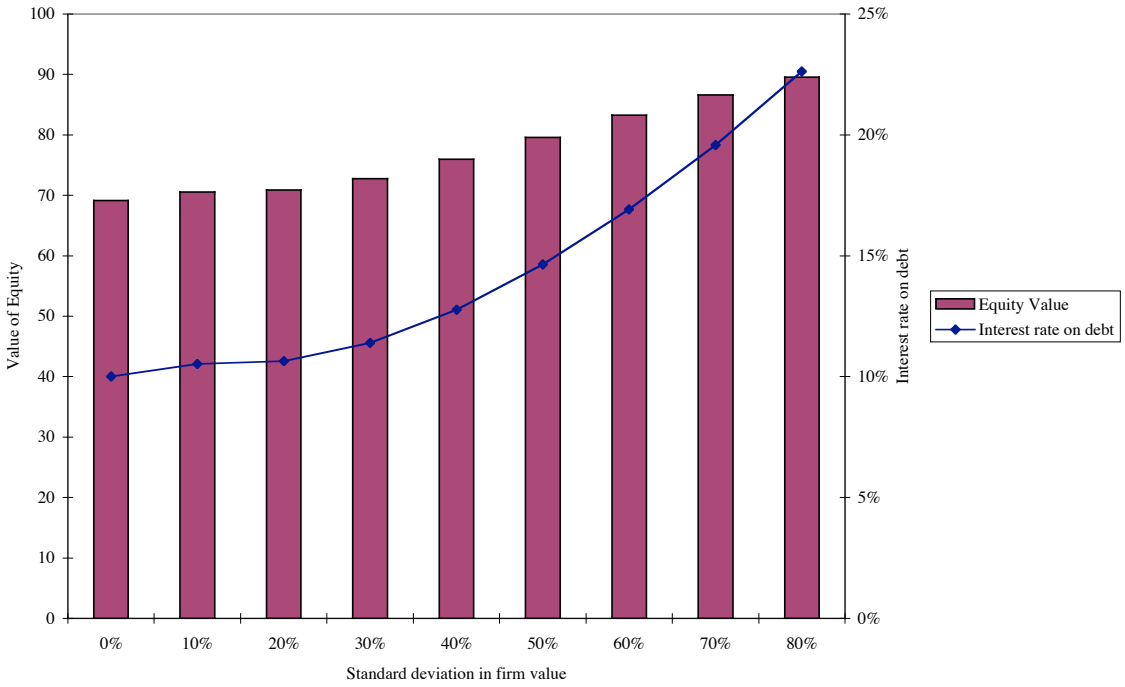
Increasing Risk can increase Equity Value

In traditional discounted cash flow valuation, higher risk almost always translates into lower value for equity investors. When equity takes on the characteristics of a call option, we should not expect this relationship to continue to hold. Risk can become our ally, when we are equity investors in a troubled firm. In essence, we have little to lose and much to gain from swings in firm value.

Illustration 10: Equity Value and Volatility

Let us revisit the valuation in Illustration 8. The value of the equity is a function of the variance in firm value, which we assumed to be 40%. If we change this variance, holding all else constant, the value of the equity will change as evidenced in Figure 3.

Figure 3: Equity Value and Standard Deviation in Firm Value



Note that the value of equity increases, if we hold firm value constant, as the standard deviation increases. The interest rate on debt also increases as the standard deviation increases.

Probability of Default and Default Spreads

One of the more interesting pieces of output from the option pricing model is the risk-neutral probability of default that we can obtain for the firm. In the Black-Scholes model, we can estimate this value from N(d2), which is the risk-neutral probability that S>K, which in this model is the probability that the value of the firm's asset will exceed the face value of the debt.

$$\text{Risk-neutral probability of default} = 1 - N(d2)$$

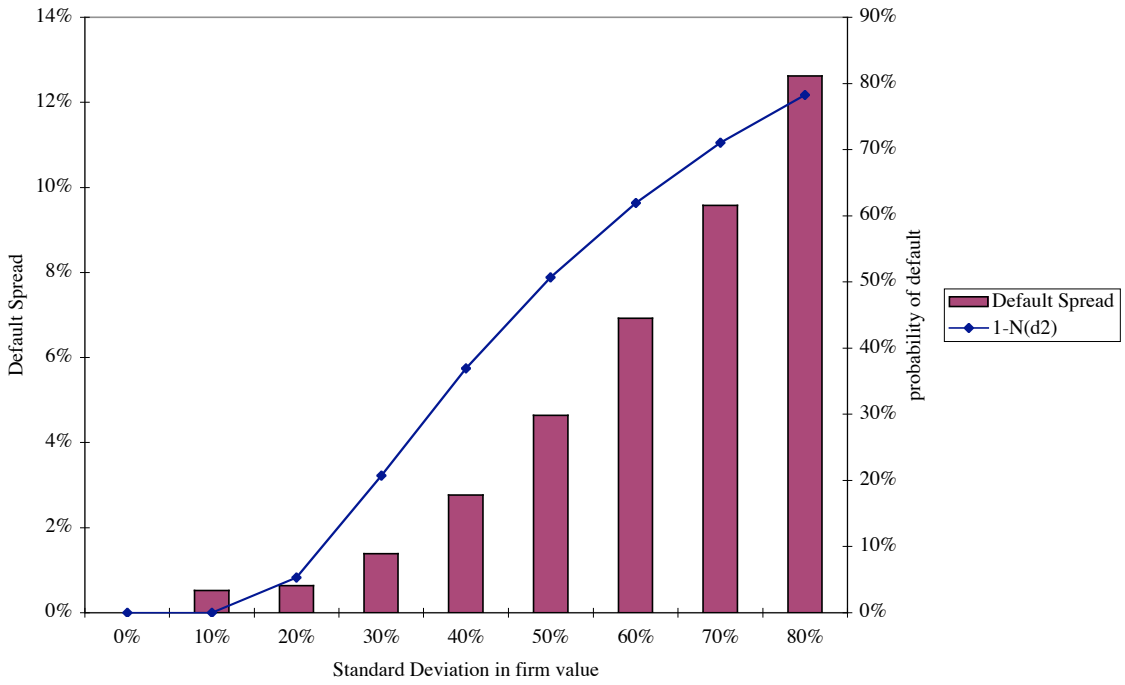
In addition, the interest rate from the debt allows us to estimate the appropriate default spread to charge on bonds.

You can see the potential in applying this model to bank loan portfolios to extract both the probability of default and to measure whether you are charging an interest rate that is high enough on the debt. In fact, there are commercial services that use fairly sophisticated option pricing models to estimate both values for firms.

Illustration 11: Probabilities of default and Default Spreads

We return to Illustration 8 and estimate the probability of default as N(d2) and the default spread, measured as the difference between the interest rate on a firm's debt and the riskfree rate, as a function of the variance. These values are graphed in Figure 4.

Figure 4: Risk Neutral Probability of default and Default spreads



Note that the probability of default climbs very quickly as the standard deviation in firm value increases and the default spread follows it along.

Estimating the Value of Equity as an Option

The examples we have used thus far to illustrate the application of option pricing to value equity have included some simplifying assumptions. Among them are the following.

1. There are only two claimholders in the firm - debt and equity.
2. There is only one issue of debt outstanding and it can be retired at face value.
3. The debt has a zero coupon and no special features (convertibility, put clauses, etc.)
4. The value of the firm and the variance in that value can be estimated.

Each of these assumptions is made for a reason. First, by restricting the claimholders to just debt and equity, we make the problem more tractable; introducing other claimholders such as preferred stock makes it more difficult to arrive at a result, albeit not impossible.

Second, by assuming only one zero-coupon debt issue that can be retired at face value any time prior to maturity, we align the features of the debt more closely to the features of the strike price on a standard option. Third, if the debt is coupon debt, or more than one debt issue is outstanding, the equity investors can be forced to exercise (liquidate the firm) at these earlier coupon dates if they do not have the cash flows to meet their coupon obligations.

Finally, knowing the value of the firm and the variance in that value makes the option pricing possible, but it also raises an interesting question about the usefulness of option pricing in equity valuation. If the bonds of the firm are publicly traded, the market value of the debt can be subtracted from the value of the firm to obtain the value of equity much more directly. The option pricing approach does have its advantages, however. Specifically, when the debt of a firm is not publicly traded, option pricing theory can provide an estimate of value for the equity in the firm. Even when the debt is publicly traded, the bonds may not be correctly valued and the option pricing framework can be useful in evaluating the values of debt and equity. Finally, relating the values of debt and equity to the variance in firm value provides some insight into the redistributive effects of actions taken by the firm.

Inputs for Valuing Equity as an Option

Since most firms do not fall into the neat framework developed above (such as having only one zero-coupon bond outstanding), we have to make some compromises to use this model in valuation.

Value of the Firm

We can obtain the value of the firm in one of four ways. In the first, we cumulate the market values of outstanding debt and equity, assuming that all debt and equity are traded, to obtain firm value. The option pricing model then reallocates the firm value between debt and equity. This approach, while simple, is internally inconsistent. We start with one set of market values for debt and equity and, using the option pricing model, end up with entirely different values for each.

In the second, we estimate the market values of the assets of the firm by discounting expected cash flows at the cost of capital. The one consideration that we need to keep in mind is that the value of the firm in an option pricing model should be the

value obtained on liquidation. This may be less than the total firm value, which includes expected future investments and it may also be reduced to reflect the cost of liquidation. If we estimate the firm value using a discounted cash flow model, then this would suggest that only existing investments[25] should be considered while estimating firm value. The biggest problem with this approach is that financial distress can affect operating income and thus the value that we obtain by using current operating income may be too low.

In the third approach, we estimate a multiple of revenues by looking at healthy firms in the same business and apply this multiple to the revenues of the firm we are valuing. Implicitly, we are assuming that a potential buyer, in the event of liquidation, will pay this value.

We can use the fourth approach for firms that have separable assets that are individually traded. Here, we cumulate the value of the market values of the assets to arrive at firm value. For example, we can value a troubled real estate firm that owns five properties by valuing each property separately and then aggregating the values.

Variance in Firm value

We can obtain the variance in firm value directly if both stocks and bonds in the firm are traded. Defining σ_e^2 as the variance in the stock price and σ_d^2 as the variance in the bond price, w_e as the market-value weight of equity and w_d as the market-value weight of debt, we can write the variance in firm value as:[26]

$$\sigma^2_{firm} = w_e^2\sigma_e^2 + w_d^2\sigma_d^2 + 2w_e w_d \rho_{ed}\sigma_e\sigma_d$$

where ρ_{ed} is the correlation between the stock and the bond prices. When the bonds of the firm are not traded, we can use the variance of similarly rated bonds as the estimate of σ_d^2 and the correlation between similarly rated bonds and the firm's stock as the estimate of ρ_{ed}.

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[25] Technically, this can be done by putting the firm into stable growth and valuing it as a stable growth firm, where reinvestments are used to either preserve or augment existing assets.

[26] This is an extension of the variance formula for a two-asset portfolio.

When companies get into financial trouble, this approach can yield misleading results as both its stock prices and its bond prices become more volatile. An alternative that often yields more reliable estimates is to use the average variance in firm value for other firms in the sector. Thus, the value of equity in a deeply troubled steel company can be estimated using the average variance in firm value of all traded steel companies.

Maturity of the Debt

Most firms have more than one debt issue on their books and much of the debt comes with coupons. Since the option pricing model allows for only one input for the time to expiration, we have to convert these multiple bonds issues and coupon payments into one equivalent zero-coupon bond.

- One solution, which takes into account both the coupon payments and the maturity of the bonds, is to estimate the duration of each debt issue and calculate a face-value-weighted average of the durations of the different issues. This value-weighted duration is then used as a measure of the time to expiration of the option.
- An approximation is to use the face-value weighted maturity of the debt converted to the maturity of the zero-coupon bond in the option pricing model.

Face Value of Debt

When a distressed firm has multiple debt issues outstanding, we have three choices when it comes to what we use as the face value of debt:

- We could add up the principal due on all of the debt of the firm and consider it to be the face value of the hypothetical zero coupon bond that we assume that the firm has issued. The limitation of this approach is that it will understate what the firm will truly have to pay out over the life of the debt, since there will be coupon payments and interest payments during the period.
- At the other extreme, we could add the expected interest and coupon payments that will come due on the debt to the principal payments to come up with a cumulated face value of debt. Since the interest payments occur in the near years and the principal payments are due only when the debt comes due, we are mixing cash flows up at different points in time when we do this. This is, however, the simplest approach of dealing with intermediate interest payments coming due.

- We can consider only the principal due on the debt as the face value of the debt and the interest payments each year, specified as a percent of firm value, can take the place of the dividend yield in the option pricing model. In effect, each year that the firm remains in existence, we would expect to see the value of the firm decline by the expected payments on the debt.

Illustration 12: Valuing Equity as an option – Eurotunnel in 1997

Eurotunnel was the firm that was created to build and ultimately profit from the tunnel under the English Channel, linking England and France. While the tunnel was readied for operations in the early 1990s, it was never a commercial success and reported significant losses each year after opening. In early 1998, Eurotunnel had a book value of equity of -£117 million, and in 1997, the firm had reported earnings before interest and taxes of -£3.45 million and net income of -£611 million on revenues of £456 million. By any measure, it was a firm in financial trouble.

Much of the financing for the tunnel had come from debt and, at the end of 1997, Eurotunnel had debt obligations in excess of £5,000 million, raised from a variety of bond issues and bank debt. Adding the expected interest payments and coupon payments on the debt brings the total obligations of the firm up to £8,865 million. Table 8 summarizes the outstanding debt at the firm, with our estimates of the expected duration for each class of debt.

Table 8: Debt Breakdown for Eurotunnel

Debt Type	*Face Value (including cumulated coupons)*	*Duration*
Short term	£ 935	0.50
10 year	£ 2435	6.7
20 year	£ 3555	12.6
Longer	£ 1940	18.2
Total	£8,865 mil	10.93 years

The firm's only significant asset is its ownership of the tunnel and we estimated the value of this asset from its expected cash flows and the appropriate cost of capital. The assumptions we made were as follows.

1. Revenues will grow 10% a year for the next 5 years and 3% a year in perpetuity after that.
2. The cost of goods sold which was 72% of revenues in 1997 will drop to 60% of revenues by 2002 in linear increments and stay at that level.

3. Capital spending and depreciation will grow 3% a year for the next 5 years. Note that the net capital expenditure is negative for each of these years – we are assuming that the firm will be able to not make significant reinvestments for the next 5 years. Beyond year 5, capital expenditures will offset depreciation.
4. There are no working capital requirements.
5. The debt ratio, which was 95.35% at the end of 1997, will drop to 70% by 2002. The cost of debt is 10% for the next 5 years and 8% after that.
6. The beta for the stock will be 2.00 for the next five years, and drop to 0.8 thereafter (as the leverage decreases).

The long-term bond rate at the time of the valuation was 6% and the tax rate was 35%. Based on these assumptions, we estimated the cash flows in Table 9.

Table 9: Estimated FCFF: Eurotunnel

	1	*2*	*3*	*4*	*5*	*Terminal Year*
Revenues	£501.60	£551.76	£606.94	£667.63	£734.39	£756.42
- COGS	£361.15	£380.71	£400.58	£420.61	£440.64	£453.85
- Depreciation	£141.11	£145.34	£149.70	£154.19	£158.82	£163.59
EBIT	(£0.66)	£25.70	£56.65	£92.83	£134.94	£138.98
- EBIT*t	£0.00	£9.00	£19.83	£32.49	£47.23	£48.64
EBIT (1-t)	(£0.66)	£16.71	£36.83	£60.34	£87.71	£90.34
+ Depreciation	£141.11	£145.34	£149.70	£154.19	£158.82	£163.59
- Capital Spending	£46.35	£47.74	£49.17	£50.65	£52.17	£163.59
- Chg. Working Capital	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00
Free CF to Firm	£94.10	£114.31	£137.36	£163.89	£194.36	£90.34
Terminal Value					£2,402.66	
Present Value	£87.95	£99.86	£112.16	£125.08	£1,852.67	
Value of firm =	£2,277.73					

The value of the assets of the firm is £2,278 million.

The final input we estimated was the standard deviation in firm value. Since there are no directly comparable firms, we estimated the standard deviations in Eurotunnel stock and debt using the data over the previous years.

Standard deviation in Eurotunnel stock price (ln) = 41%

Standard deviation in Eurotunnel bond price (ln) = 17%

We also estimated a correlation of 0.50 between Eurotunnel stock and bond prices and the average market debt to capital ratio during the two-year period was 85%. Combining these inputs, we estimated the standard deviation in firm value to be:

$$\sigma^2_{\text{firm}} = (0.15)^2(0.41)^2 + (0.85)^2(0.17)^2 + 2(0.15)(0.85)(0.5)(0.41)(0.17) = 0.0335$$

In summary, the inputs to the option pricing model were as follows.

Value of the underlying asset = S = Value of the firm = £2,278 million

Exercise price = K = Face Value of outstanding debt = £8,865 mil

Life of the option = t = Weighted average duration of debt = 10.93 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.0335

Riskless rate = r = Treasury bond rate corresponding to option life = 6%

Based upon these inputs, we estimate the following value for the call:

$d_1 = -0.8582$ $\quad N(d_1) = 0.1955$

$d_2 = -1.4637$ $\quad N(d_2) = 0.0717$

$$\text{Value of the call} = 2,278(0.1955) - 8,865e^{(-0.06)(10.93)}(0.0717) = \$116 \text{ million}$$

Eurotunnel's equity was trading at £150 million in 1997.

The option pricing framework, in addition to yielding a value for Eurotunnel equity, yields some valuable insight into the drivers of value for this equity. While it is certainly important that the firm try to bring costs under control and increase operating margins, the two most critical variables determining equity value are the duration of the debt and the variance in firm value. Any action that increases (decreases) the debt duration will have a positive (negative) effect on equity value. For instance, when the French government put pressure on the bankers who had lent money to Eurotunnel to ease restrictions and allow the firm more time to repay its debt, equity investors benefited as their options became more long term. Similarly, an action that increases the volatility of expected firm value will increase the value of the option.

Conclusion

Distressed firms, i.e., firms with negative earnings that are exposed to substantial likelihood of failure, present a challenge to analysts valuing them because so much of conventional valuation is built on the presumption that firms are going concerns. In this paper, we have examined how both discounted cash flow and relative valuation deal (sometimes partially and sometimes not at all) with distress. With discounted cash flow valuation, we suggested four ways in which we can incorporate distress into value – simulations that allow for the possibility that a firm will have to be liquidated, modified discounted cash flow models, where the expected cash flows and discount rates are adjusted to reflect the likelihood of default, separate valuations of the firm as a going concern and in distress and adjusted present value models. With relative valuation, we can adjust the multiples for distress or use other distressed firms as the comparable firms.

In the last part of the paper, we examine two issues that may come up when going from firm value to equity value. The first relates to the shifting debt load at these firms, as the terms of debt get renegotiated and debt sometimes becomes equity. The second comes from the option characteristics exhibited by equity, especially in firms with significant financial leverage and potential for bankruptcy.