

**Claims on Equity: Voting and Liquidity Differentials, Cash
flow Preferences and Financing Rights**

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Claims on Equity: Voting Differentials, Cash flow Preferences, Liquidity and Financing Rights

In conventional valuation, we assume that all equity claims are identical and divide the value of equity by the number of claims (shares) to get the value per claim (share). In practice, though, claims on equity can vary on a number of dimensions. First, the claim can be a direct and perpetual one (standard equity) or it can be contingent on the value changing (equity option). Second, some equity investors have preferential claims on the cash flows—dividends in some cases and cash flows in liquidation in other cases. Third, some equity claims have superior control rights over other claims: this can take the form of differential voting rights in some cases and a bigger role in board composition and management in others. In some instances, the power is triggered by a control event such as an acquisition or an initial public offering. Fourth, some equity investors are provided with special rights to protect their interests when the firm acts in later periods. These can include disproportionate rights in subsequent financing decisions – the right to partake in the financing at a fixed price, for instance, or veto rights over new financing – as well as redemption rights, where they can reclaim the capital that they have invested. Finally, equity claims can vary in terms of liquidity, with some claims being more marketable than others. All of these differences can affect value, resulting in some equity claims having higher value than others.

Much of the literature and theory in discounted cash flow valuation is directed at estimating the value of the enterprise, and from that value, deriving the value of equity. Thus, there has been a great deal of research on how best to estimate expected cash flows, discount rates and growth rates and the effects of these estimates on aggregate value. Once we derive equity value, though, we generally pay short shrift to assessing the value of individual equity claims (shares in the case of a publicly traded firm), because we implicitly assume that all equity investors have identical claims.

In the real world, equity claims can be differentiated on at least five dimensions. First, some equity claims are contingent claims whereas others are direct ones: management options and warrants are examples of the former, with common stock being an example of the latter. Second, there are differences in cash flow claims across equity investors, with some investors entitled to higher dividends and cash flows in liquidation. Third, some equity investors get more control rights than other equity investors, either because they have all of or a disproportionately large portion of the voting rights, or because they get special powers in the face of an acquisition or an initial public offering. Fourth, some equity investors have rights designed to protect their interests, and can include the right to partake in subsequent financing at preferential terms as well as the redemption rights, allowing them to reclaim capital. Finally, there can be wide differences in liquidity across different equity claims, which, in turn, can affect their values. As equity claims splinter, deciding how much of the equity value to assign to individual claims becomes more problematic. In this paper, we will consider the empirical evidence on the values of these differential claims and lay out a theoretical framework for assessing the values of the claims. In effect, we will lay the groundwork for determining how the overall value of equity should be allocated to the multiple claimholders on that equity.

Direct versus Contingent Claims

A standard equity claim entitles the owner to a share of the residual cashflows of a business. The claim holder is thus involved in both the upside and the downside that represent risk in an investment. A contingent equity claim is more selective, insofar as it

allows the holder to share in the upside, if there is upside, but not in the downside. In this section, we will explore the nature of these contingent claims, how best to value them and how their presence affects conventional equity claim holders.

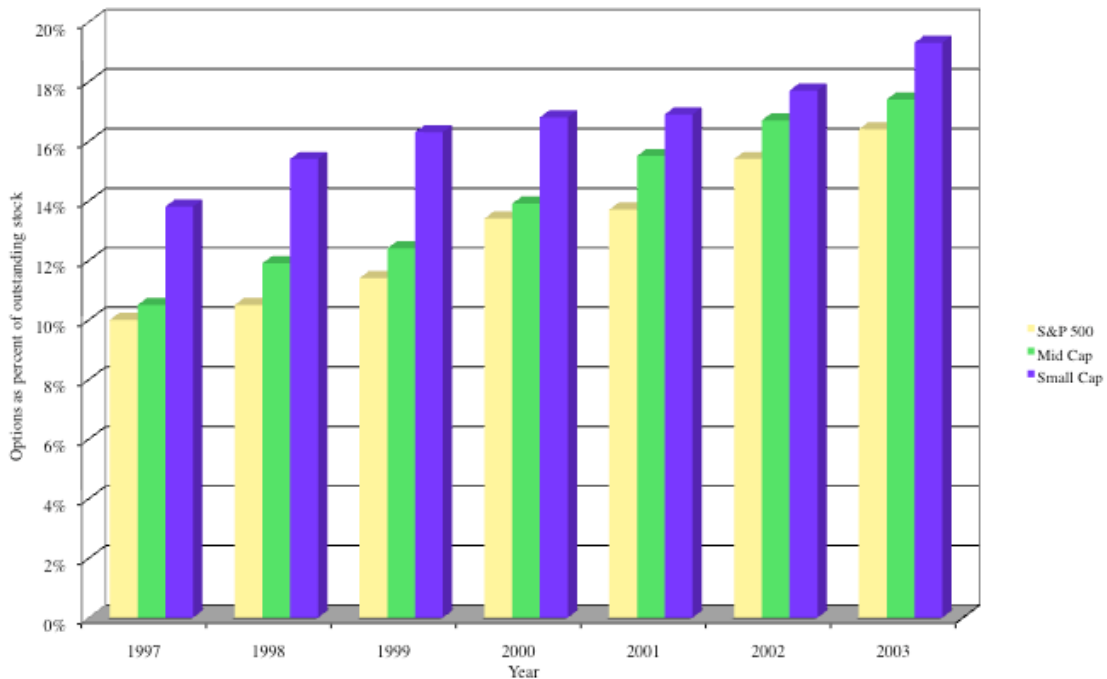
Prevalence of Contingent Equity Claims

Contingent claims on equity have been publicly available and traded for much of financial market history. However the preponderance of these claims were not created by the firms themselves but by outside entities (banks, brokers or option exchanges) and had little or no effect on how the equity within the firms was allocated across claimholders. The first crack in this process occurred in the 1970s and 1980s as some publicly traded firms in Europe turned to issuing long term equity options – warrants – as a way of raising new equity. While U.S. firms were reluctant to jump on the “warrant” bandwagon, because of the fear of dilution, they were drawn to issue equity options in three different contexts:

1. As an inducement to “early” equity investors: Many small companies, especially in the technology arena, gave venture capitalists and others who invested in their equity early in the life cycle, the rights to make additional equity investments at pre-set terms. In effect, they granted call options to these equity investors that could be exercised if the company succeeded (often by going public or being acquired).
2. Convertible securities: Many companies issue convertible bonds and convertible preferred stock, in effect combining a call option on the equity with a conventional bond or preferred stock.
3. Management compensation: The explosion in the use of equity options as compensation for managers starting in the 1980s but building up with the technology boom in the 1990s is well chronicled. With some firms, option grants to managers amounted to 15% or greater of the outstanding common stock, thus creating the need for analysts to be more careful about how they allocated equity value across claimholders.

The magnitude of the “option overhang” varies widely across companies. In figure 1, we look at options outstanding at publicly traded companies as a percentage of the outstanding stock:

Figure 1: Option Overhang at US Companies



The question that we then face is a simple one. Given that there are two claims on equity at most publicly traded firms, one from common stockholders (or conventional equity) and one from option holders (managers, convertible security holders), what is the best process for allocating equity value across the claims?

Value Effects of Claims

Why does the granting of options affect value per share? Note that not all options do. In fact, options issued and listed by the options exchanges have no effect on the value per share of the firms on which they are issued. The options issued by firms do have an effect on value per share, since there is a chance that they will be exercised in the near or far future. Given that these options offer the right to individuals to buy stock at a fixed price, they will be exercised only if the stock price rises above that exercise price. When firms grant options to employees, it is existing stockholders who pay for these options. Consequently, the question is not whether options affect the value of common stock, but how they affect value.

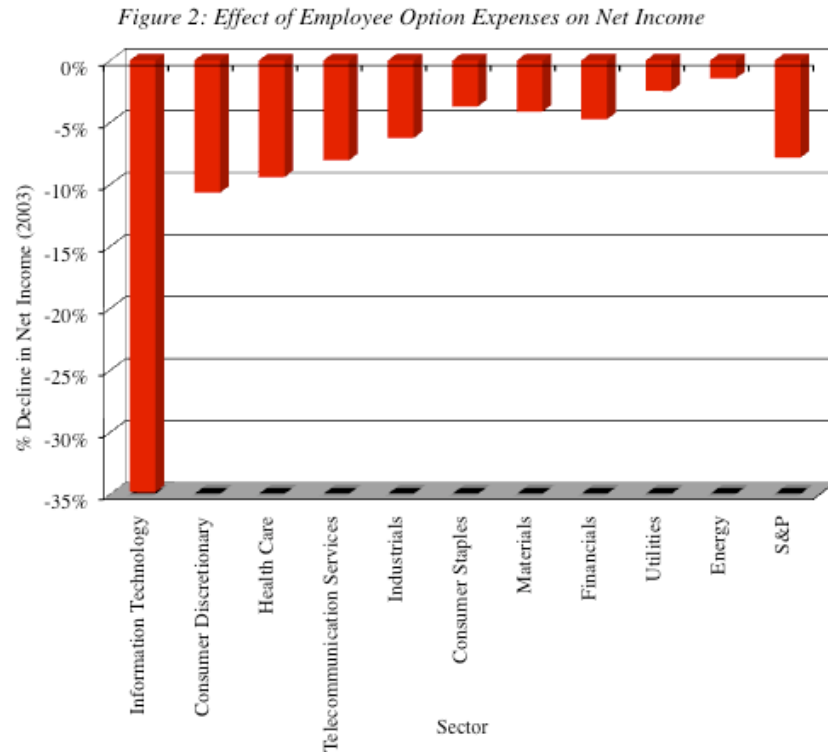
In this section, we will consider three levels at which options affect equity value per share. The first and narrowest measure is the effect that granting options in the

current year will have on the current earnings of a firm. The second is the potential dilution effect created not just by options issued in the current year but by the cumulative options outstanding at the firm; the exercise of options will increase the number of shares at some future date, but expectations of that happening will affect the value per share today. The third is and broadest measure looks at the effect that the continued granting of options will have on expected future earnings and thus on value per share.

1. Earnings Effect: The earnings effect of option grants varies across firms. In a study of the S&P 500 and the NASDAQ 100 firms, Bear Stearns estimated the effect of employee options being treated as expenses on the earnings of individual firms.¹ On average, they estimated that earnings would decline 8% at S&P 500 companies if option grants were treated as expenses and by 25% at NASDAQ 100 companies.² They also estimated the earnings effect of option expensing on each of the 600 companies. Figure 2 summarizes the effect on net income of considering share-based employee compensation as an expense on firms in different sectors of the S&P 500:

¹ *2004 Earnings Impact of Stock Options on the S&P 500 and NASDAQ 100 Earnings*, Bear Stearns Equity Research publication, March 21, 2005.

² The Bear Stearns study looks at the effect of forcing option expensing on all companies and comes up with a 5% drop in net income at S&P 500 companies and 22% at technology companies. However, it also notes that some companies had already switched to expensing options in 2003. The numbers we report include the option expenses at those companies as well and are thus larger.



Source: Bear Stearns (2005)

The effect was greatest at technology companies, where the cumulative cost of share-based compensation would have amounted to \$15.43 billion in 2004, representing 32% of the unadjusted net income (prior to expensing share-based compensation) of \$ 48.53 billion.

2. Dilution Effect: While option grants in the current year reduce earnings for the year, the value of equity per share in a company is weighed down by the cumulative effect of options that have been granted over time that are still outstanding. While some of these options may be out-of-the-money, there is still a probability that they will be exercised in the future, thus increasing the number of shares outstanding. This potential dilution effect from options outstanding will reduce the value of equity per share, and will do so more at firms that have more options outstanding (as a percent of outstanding shares) than at firms with less. Figure 2, reported earlier, noted the differences in the option overhang at firms in old economy, new economy, financial service and utility companies.

Analysts and accountants have tried to grapple with the potential value loss from dilution by using fully diluted (where all options are treated as outstanding shares) or partially diluted (where only in—the-money options are considered) numbers of shares

when computing the earnings per share. These measures do not reflect or even attempt to measure the probabilities that options will be exercised and thus provide only a very rough proxy for the dilution effect.

There are some who argue that there does not have to be a dilution effect from option exercise. Many firms, they note, repurchase stock and set them aside to cover option exercise rather than issuing new shares. That is true but such actions still affect value per share by affecting expected cash flows. In the absence of these options, the stockholders of these firms would have been able to lay claim to much larger cash flows each year (even though they might not have received them as dividends).

3. Future Earnings Effect: Looking at options granted in the current year (and the effect on earnings) and cumulative options (and the dilution effect) allows analysts to consider the effect of past option grants on value. However, most firms that grant options will continue to use them in the future, thus affecting future earnings. The expected option grants are employee compensation and will increase operating expenses in future years and reduce operating income. The value of a firm today is the present value of expected cash flows, and these will be much lower for a firm that is expected to be more generous with its option grants.

Accounting standards have finally come to grips with the effect of granting options on current earnings (see FAS 123R) and analysts do attempt to capture the dilution effect, albeit sloppily, with diluted share numbers. Analysts, though, are still haphazard about dealing with expected future option grants. While some try to forecast the magnitude of these grants, most valuations either completely ignore them or build them in implicitly by forecasting out a current income number that incorporates option expenses.³

Allocating Equity Value across Different Types of Claims

As we noted in the last section, the value per share is weighed down by the cumulative effect of all options outstanding. There are four approaches that are used to

³ For example, assume that we are valuing Coca Cola, a company which has been expensing employee options since 2003. If we use earnings in 2004 as our base year and apply an expected growth rate to it, we

incorporate that effect of options that are already outstanding into the value per share. The first is to adjust the number of shares outstanding to reflect options outstanding. The second is to try to forecast out when the options will be exercised and the effect on share numbers in future years. The third, called the treasury stock approach, is an extension of the first approach. In addition to using diluted shares, this approach also adjusts the value of the equity to reflect the expected proceeds from the option exercise. The last approach values the options outstanding at fair value rather than at exercise value, and subtracts this from the overall value of equity to arrive at the value of equity in common stock. We believe that the last approach is the only one that completely incorporates the effect of existing options into value per share.

1. Use fully diluted number of shares to estimate per-share value

The simplest way to incorporate the effect of outstanding options on value per share is to divide the estimated value of equity from a discounted cash flow model by the number of shares that will be outstanding if all options are exercised today – the fully diluted number of shares. While this approach has the virtue of simplicity, it will lead to too low of an estimate of value per share for three reasons:

- It considers all options outstanding, not just ones that are in the money and vested. To be fair, there are variants of this approach where the shares outstanding are adjusted to reflect only in-the-money and vested options.
- It does not incorporate the expected proceeds from exercise, which will comprise a cash inflow to the firm.
- Finally, this approach does not build in the time premium on the options into the valuation.

Illustration 1: Fully Diluted Approach to estimating Value per Share

To apply the fully diluted approach to estimate the per share value, we will value two publicly traded companies with significant option overhangs – Cisco and Google and one hypothetical private business, a small candy company called Kristin Kandy, in 2005. In Table 1 we summarize the equity values we estimated for the companies, using

are assuming that option expenses will continue as a line item into the future but that it will remain at the same percentage of revenues it was in 2004.

conventional discounted cash flow models, and then adjust for value per share using fully diluted shares.⁴

Table 1: Fully Diluted Approach to Estimating Value per Share

	<i>Cisco</i>	<i>Google</i>	<i>Kristin Kandy</i> <i>(private)</i>
Value of Equity (in millions)	\$ 65,622	\$ 32,187	1,800
Primary Shares (in millions)	6,487	277.78	270
Options outstanding	1436	25.61	30
Fully Diluted Shares	7,923	303.39	300
Value per share (Primary)	\$ 10.12	\$ 115.87	\$6.67
Value per share (fully diluted)	\$ 8.28	\$ 106.09	\$ 6.00

The value per share, using the fully diluted approach, is significantly lower than the value per share, using the primary shares outstanding. This value, however, ignores both the proceeds from the exercise of the options as well as the time value inherent in the options. At Cisco, for example, a significant number of the options issued in past years are out-of-the-money and may never be exercised.

A modified version of this approach counts only in-the-money options when computing diluted shares. With Kristin Kandy, we dispense with this approach since there is no publicly accessible market price for a private business. With Cisco and Google, this approach yields the following values per share:

	<i>Cisco</i>	<i>Google</i>
Value of Equity (in millions)	\$ 65,622	\$ 32,187
Primary Shares (in millions)	6,487	277.78
In-the-money options	591	25.61
Partially Diluted Shares	7,076	303.39
Value per share (partially diluted)	\$ 9.27	\$ 106.09

For Google, there is no effect from the adjustment since all their options are in-the-money. For Cisco, only 591 million shares are in-the-money (based upon the stock price of \$17.67 at the time of the analysis). In fact, counting only vested in-the-money options

⁴ These were conventional discounted cash flow valuations. Details of the valuations can be obtained on my web site (<http://www.damodaran.com>).

at Cisco would reduce the number of options considered to 441 million options and increase the value per share a little more.

II. Estimate expected option exercises in the future and build in expected dilution

In this approach, we forecast when in the future options will be exercised and build in the expected cash outflows associated with the exercise, by assuming that the firm will go out and buy back stock to cover the exercise. The biggest limitation of this approach is that it requires estimates of what the stock price will be in the future and when options will be exercised on the stock. Given that our objective is to examine whether the price today is correct, forecasting future prices to estimate the current value per share seems circular. In general, this approach is neither practical nor is it particularly useful in coming up with reasonable estimates of value.

III. Treasury Stock Approach

This approach is a variant of the fully diluted approach. Here, the number of shares is adjusted to reflect options that are outstanding, but the expected proceeds from the exercise (the product of the exercise price and the number of options) are added to the value of equity. The limitations of this approach are that it does not consider the time premium on the options and there is no effective way of dealing with vesting. Generally, this approach, by under estimating the value of options granted, will over estimate the value of equity per share.

The biggest advantage of this approach is that it does not require a value per share (or stock price) to incorporate the option value into per-share value. As we will see with the last (and recommended) approach, there is a circularity that is created when the stock price is an input into the process of estimating option value which, in turn, is needed to obtain the value per share.

Illustration 2: Treasury Stock Approach

In Table 2, we re-estimated the value per share is estimated using the treasury stock approach for Cisco, Google and Kristin Kandy:

Table 2: Value of Equity per Share: Treasury Stock Approach

	<i>Cisco</i>	<i>Google</i>	<i>Kristin Kandy</i>

Number of options outstanding	1436	25.61	30
Average exercise price	\$ 25.02	\$24.41	\$4.00
Proceeds from Exercise	\$ 35,928	\$ 625	\$150.00
Value of Equity	\$ 65,622	\$ 32,187	\$1,800
+ Proceeds from Exercise	\$ 35,928	\$ 625	\$ 120
Total Value	\$101,550	\$32,812	\$1,920
Fully Diluted number of shares	7923	303.39	300
Value per share	\$12.82	\$108.15	\$6.40

Note that the value per share using this approach is higher than the value per share using the fully diluted approach for all three companies. The difference is greatest for Cisco because the average exercise price is high, relative to the current stock price. For Google, the effect is much smaller since the exercise price is well below the current stock price (of almost \$300). The estimated value per share still ignores the time value of the options.

As with the diluted approach, there are modified versions of this approach where only in-the-money options are considered. This will reduce the value per share for Cisco considerably since the average exercise price for the in-the-money options is much lower than the weighted average exercise price of \$25.02.

IV. Valuing Options

The correct approach to dealing with options is to estimate the value of the options today, given today's value per share and the time premium on the option. Once this value has been estimated, it is subtracted from the estimated equity value, and divided by the number of shares outstanding to arrive at value per share.

Value of Equity per share = (Estimated Value of Equity – Value of Employee Options outstanding) / Primary number of shares outstanding

In this section, we will consider both the measurement issues associated with valuing employee options and the models that have been developed to value them.

Measurement Issues

In valuing employee options, however, there are five measurement issues that we have to confront. One relates to the fact that not all of the options outstanding are vested, and that some of the non-vested options might never become vested. The second centers on the illiquidity of employee options. As a result, employee options are often exercised

before maturity, making them less valuable than otherwise similar traded options that are marketable. The third relates to the stock price to use in valuing these options. While conventional option pricing models are built around using the current market price as a key input, we do come up with estimates of value per share when we value companies, and these estimates can be very different from current stock prices. We have to consider whether we want to use our estimates of value per share, rather than the market prices, to preserve valuation consistency. The fourth issue is taxation. As we noted earlier in the section on accounting for options, firms are allowed to deduct the difference between the stock and the exercise price of an option at exercise and there is potential tax saving at the time of option exercise. The final issue relates to options granted at private firms or firms on the verge of a public offering. Key inputs to the option-pricing model, including the stock price and the variance, cannot be obtained for these firms, but the options have to be valued nevertheless.

a. Vesting

As noted earlier in the paper, firms granting employee options usually require that the employee receiving the options stay with the firm for a specified period, for the option to be vested. Consequently, when we examine the options outstanding at a firm, we are looking at a mix of vested and non-vested options. The non-vested options should be worth less than the vested options, but the probability of vesting will depend upon how in-the-money the options are and the period left for an employee to vest. There have been attempts⁵ to develop option pricing models that allow for the possibility that employees may leave a firm before vesting and forfeit the value of their options. Carpenter (1998) developed a simple extension of the standard option pricing model to allow for early exercise and forfeiture, and used it to value executive options.⁶ Since the new accounting standards governing employee options require firms to estimate forfeiture rates at the time of the grant, there will undoubtedly be attempts to build new models for vesting and forfeiture.

⁵ Cuny, C. and P. Jorion, 1995, *Valuing Executive Stock Options with Endogenous Departure*, Journal of Accounting and Economics, v20, 193-205.. They examine the valuation of options when there is the possibility of forfeiture.

⁶ Carpenter, J.N. (1998), *The exercise and valuation of executive stock options*, Journal of Financial

b. Illiquidity

Employees who are compensated with options can become wealthy on paper but may not be able to cash in on their implicit wealth because the options cannot be traded. In addition, it is often infeasible or illegal to hedge these options. The effect of this illiquidity on option value has been both widely studied and well debated. In particular, the illiquidity of these options may induce employees to exercise options early and give up the time premiums on these options.

While some have argued that early exercise is irrational, there are clearly good reasons for early exercise. Huddart (1994) shows that early exercise is in fact optimal for a risk-averse investor.⁷ Lambert, Larcker, and Verrecchia (1991) and Hemmer, Matsunaga, and Shevlin (1994), show that restrictions on short selling and hedging option positions can lead to early exercise.⁸ Brooks, Chance and Cline (2005) argue that private information may also cause early exercise: the managers who hold employee options often have the information to make a judgment on whether their stock is over valued or not. If it is over valued, in their estimation, early exercise becomes more likely.⁹

The empirical evidence is also clearly supportive of the early exercise theory. In a comprehensive study of 262,931 option exercises of employee options between 1996 and 2003 by U.S. companies, Brooks, Chance and Cline (cited above) note that 92.3% exercise early. On average, they find that exercise takes place 2.69 years after vesting, with 4.71 years left to expiration. Put another way, an employee option with a stated maturity of 10 years is usually exercised in 5.29 years. Bettis, Bizjak and Lemmon (2003) also find significant variation in exercise policies across firms, with employees in riskier firms exercising their options almost one and a half years earlier than employees in more stable firms.¹⁰ The implications for option valuation are straightforward. Using the stated

Economics, v 48, 127–158.

⁷ Huddart, S. 1994. *Employee Stock Options*. Journal of Accounting and Economics, v18, 207-231.

⁸ Lambert, R., D. Larcker, and R. Verrecchia. 1991. *Portfolio Considerations in Valuing Executive Compensation*. Journal of Accounting Research, 129-149; Hemmer, T., S. Matsunaga, and T. Shevlin. 1994. *Estimating the 'Fair Value' of Employee Stock Options with Expected Early Exercise*. Accounting Horizons, v8, no. 4 (December): 23-42.

⁹ Brooks, R., D. Chance and B.N. Cline, *Private Information and the Exercise of Executive Stock Options*, Working Paper, SSRN.

¹⁰ Bettis, J.C., J.M. Bizjak and M.L. Lemmon, 2003, *The Cost of Employee Stock Options*, Working Paper, SSRN.

maturity in option pricing models, which is what we do for most marketable options, will overstate the value of employee options.

c. Which stock price?

The answer to this question may seem obvious. If the stock is traded, and we can obtain a stock price, it would seem that we should be using the current stock price to value options. However, we are valuing these options to arrive at a value per share that we will then compare to the market price to decide whether a stock is under or over valued. For instance, we may conclude that a stock with a price of \$ 25 per share is really worth only \$12 per share. Using the current market price to arrive at the value of the options and then using this option value to estimate an entirely different value per share seems inconsistent.

There is a solution. You can value the options using the estimated value per share. This creates circular reasoning in our valuation. In other words, we need the option value to estimate value per share, and the value per share to estimate the option value. We can estimate the value per share using the treasury stock approach, and we can then converge on the proper value per share by iterating.¹¹

There is another related issue. When options are exercised, they increase the number of shares outstanding, and thus have an effect on the stock price. In conventional option pricing models, the exercise of the option does not affect the stock price. These models have to be adapted to allow for the dilutive effect of option exercise.

d. Taxation

When options are exercised, the firm can deduct the difference between the stock price at the time and the exercise price as an employee expense, for tax purposes. This potential tax benefit reduces the drain on value created by having options outstanding. To provide an illustration of the magnitude of the tax benefit, Cisco claimed a tax deduction of \$2.5 billion for option exercise in 2000, almost entirely offsetting its operating income of \$2.67 billion that year and effectively paying little in taxes. There are three ways in which we can account for this tax deductibility in valuing employee:

1. Reduce tax rates on operating income to reflect employee option deductions: To compute free cashflow to the firm, we use after-tax operating income. If a firm has substantial numbers of options outstanding, we could use a much lower tax rate in the near years of the forecasts to reflect tax deductions from employee options.¹² This will increase cash flows in those years (and consequently value). We would move the tax rates towards statutory tax rates as we approach terminal value, since the option exercise tax savings will fade over time.

2. Tax Effect the exercise value of options; A simpler way to estimate the tax benefit is to multiply the difference between the stock price today and the exercise price by the tax rate; clearly, this would make sense only if the options are in-the-money. While this does not allow for the expected price appreciation over time, it has the benefit of simplicity.

3. Tax Effect the fair value of options: An alternative way of estimating the tax benefit is to compute the after-tax value of the options:

After-tax Value of Options = Value from option pricing model (1- tax rate)

This approach is also straightforward and allows us to consider the tax benefits from option exercise in valuation. One of the advantages of this approach is that it can be used to consider the potential tax benefit even when options are out of the money.

Now that the accounting rules have changed to force option expensing, it seems to us only a matter of time before the tax rules change as well to match. If that does happen, we will be able to expense option grants in the periods that they are made and we will no longer need to tax effect the existing options (since the tax savings would have accrued when the options were granted).

e. Non-traded Firms

A couple of key inputs to the option pricing model – the current price per share and the variance in stock prices – cannot be obtained if a firm is not publicly traded. There are two choices in this scenario. One is to revert to the treasury stock approach to

¹¹ The value per share, obtained using the treasury stock approach, will become the stock price in the option pricing model. The option value that results from using this price is used to compute a new value per share which is fed back into the option pricing model and so on.

¹² Edwards, C., J. R. Graham, M.H. Lang and D. Shackelford, *Employee Stock Options and Taxes*, Working Paper, SSRN. In this paper, they estimate the tax rates for firms with substantial employee options

estimate the value of the options outstanding and abandon the option pricing models. The other is to stay with the option pricing models and to use the value per share, from the discounted cash flow model. The variance of similar firms that are publicly traded can be used to estimate the value of the options. There is, however, a related issue in that in addition to the option being illiquid (an issue that we examined earlier), the underlying asset (equity in a private business) is also illiquid. The most straightforward way to deal with this asset illiquidity is to adjust the estimated value of equity per share for its illiquidity. The process of how best to make this adjustment will be examined both later in this paper and in a more detailed fashion in a companion paper on illiquidity.¹³

Option Pricing Models

With all of these issues affecting valuation, how do we adapt conventional option pricing models to value employee options? This question has been addressed both by option pricing theorists and by FASB, in its attempts to give guidance to firms that have to value these options for expensing.

Black Scholes and Modifications

The conventional Black Scholes model is designed to value European options on traded assets and does not explicitly factor in the dilution inherent in employee options or the illiquidity/vesting issues specific to these options. However, adaptations of the model provide reasonable estimates of value:

1. Build in expected dilution into the stock price: One of the inputs into the Black Scholes is the current stock price. To the extent that the exercise of options increases the number of shares outstanding (at a price less than the current stock price), the stock price will drop on exercise. A simple adjustment to the stock price can incorporate this effect:

$$\text{Adjusted Stock Price} = \text{Current Stock Price} \left[\frac{n_{\text{shares outstanding}}}{(n_{\text{shares outstanding}} + n_{\text{options}})} \right]$$

The resulting lower adjusted stock price will also reduce the option value.

outstanding and note that it is well below the marginal tax rate. For Dell, they estimate a tax rate of 20%, as a result of option expensing, as opposed to the marginal tax rate of 35%.

¹³ Damodaran, A., 2005, *Marketability and Value: Measuring the Illiquidity Discount*, Working Paper, SSRN.

2. Reduce the life of the option to reflect illiquidity and early exercise: Earlier in this paper, we noted that employees often exercise options well before maturity because these options are illiquid. Typically, options are exercised about half way through their stated lives. Using a reduced life for the option will reduce its value.

3. Adjust option value for probability of vesting: The vesting adjustment can be made in the process of calculating of the option value. If we can assess the probability of vesting, multiplying this probability by the option value will yield an expected value for the option.

While purist would still resist, the model has provided remarkably resilient even in environments where its basic assumptions are violated.

There are numerous variants of the Black-Scholes model that have been developed for employee options. Two examples are listed below:

1. The FASB Model: While FASB does not propose a specific model, they recommend that employee options be valued assuming a forfeiture rate for employees (based upon the firm's history) and using a shorter life than the stated maturity (allowing for the early exercise option). To make both estimates, they recommend using historical data.
2. The Bulow-Shoven Model: The Bulow-Shoven model starts off with the premise that long-term employee options are not long term at all. The model proposes a technique that begins by treating all employee stock options as if they have a 90 day life, in estimating an initial value using a Black-Scholes model. However, as employees continue working for the firm day to day, quarter to quarter, they are granted 90-day extensions on the term of their options and these extensions are valued as options and treated as expenses in subsequent periods.¹⁴

These variations yield lower values for employee options than using the unadjusted Black Scholes models.

Binomial Models

The possibility of early exercise and non-vesting, which is substantial in employee options, leads many practitioners to argue for the use of Binomial lattice models to value employee options. Unlike the Black-Scholes, these models not only can

¹⁴ Bulow, J. and J..B. Shoven, 2004, *Accounting for Stock Options*, Working Paper, SSRN.

model for early exercise, but can be modified to allow for other special features specific to employee options, including vesting. In addition, binomial models allow for more flexibility on inputs, with volatility changing from period to period rather than remaining constant (which is the assumption in the Black-Scholes model). The limitation of the binomial models is that they are more information intensive, requiring the user to input prices at each branch of the binomial model. In any realistic version of the model, where the time intervals are short, this could translate into hundreds of potential prices.

It is true that we can derive binomial trees from standard deviations and thus avoid the estimation problems associated with developing these trees, but the resulting values tend to be close to Black-Scholes model values. In other words, to get the full benefits of the binomial model, we have to go through the exercise of developing the pricing tree. The initial version of FAS 123R did require firms to use binomial models to value employee options. The final version wisely left the model choice decision to the firm.

The primary benefit of binomial models comes from the flexibility that they offer users to model the interaction between the stock price and early exercise. One example is the Hull-White Model, which proposes reducing the life used to value employee options to a more realistic level.¹⁵ This model take into account the employee exit rate during the vesting period (thus taking into account the probability that options will end up unvested and worthless) and the expected life of the option after they get vested. To estimate the latter, the model assumes that there will be exercise if the stock price reaches a pre-specified multiple of the exercise price, thus making exercise an endogenous component of the model, rather than an exogenous component. The resulting option values are usually lower than those estimated using the Black-Scholes model.

Simulation Models

The third choice for valuing employee options is Monte Carlo simulation models. These models begin with a distribution for stock prices and a pre-specified exercise strategy. The stock prices are then simulated to arrive at the probabilities that employee options will be exercised and an expected value for the options based upon the exercise.

¹⁵ J. Hull and A. White, *How to Value Employee Stock Options*, Financial Analysts Journal 60 (1) (2004),

The advantage of simulations is that they offer the most flexibility for building in the conditions that may affect the value of employee options. In particular, the interplay between vesting, the stock price and early exercise can all be built into the simulation rather than specified as assumptions. The disadvantage is that simulations require far more information than other models.

Market Prices

All of the models proposed to value employee options can be contested as hypothetical and unrealistic. In fact, there is a reasonable argument that what we would really want to use to value employee options are market prices for these options. While this may seem unrealistic, Cisco proposed a novel solution to the employee option valuation problem, by creating a "market instrument" that would parallel employee options. Buyers of the new instruments, called *employee stock option reference securities*, or ESORs, would not be able to transfer them and would have options that would vest over five years. Both provisions are similar to those in employee stock options. Cisco argued that the market prices for these securities should be used to value employee options. In September 2005, the SEC rejected the Cisco proposal, arguing that investors in companies would not value employee options at the same level as employees would. They did leave the door open to a market based solution at a future date.

How much does the model matter?

How much does the model used to value employee options matter? Put another way, are there significant differences in values when we use alternative models to value employee options? For the most part, the biggest single component determining employee option value is the life of the option. Using the stated life of employee options in the Black-Scholes models yields too high a value for these options. If we use an expected life for the option (which takes into account early exercise and vesting probabilities), the values that we arrive at are not dissimilar using different models. Ammann and Seiz (2003) show that the employee option pricing models in use (the

binomial, Black Scholes with adjusted life and Hull White) all yield similar values.¹⁶ As a consequence, they argue we should steer away from models that require difficult to estimate inputs (such as risk aversion coefficients) and towards simpler models.

Illustration 3: Option Value Approach

In Table 3, we begin by estimating the value of the options outstanding at Cisco and Google, using the Black-Scholes model, adjusted for dilution and using half the stated maturity (to allow for early exercise) and using the stock price as an input to the option pricing model. For Kristin Kandy, we have no stock price available and use the fully diluted value per share (\$6.00) from table 1 as the input to the model; this value is based upon a total value of equity of \$1.8 million that is already adjusted for the illiquidity of the asset.¹⁷ To estimate the value of the options, we first estimate the standard deviation in stock prices¹⁸ over the previous 2 years. Weekly stock prices are used to make this estimate, and this estimate is annualized¹⁹. All options, vested as well as non-vested, are valued and there is no adjustment for non-vesting.

Table 3: Estimated Value of Options Outstanding

<i>Option Pricing Model</i>	<i>Cisco</i>	<i>Google</i>	<i>Kristin Kandy</i>
Number of Options Outstanding	1436	25.61	30.00
Average Exercise Price	\$ 25.02	\$24.41	4.00
Estimated Standard Deviation (Volatility)	45%	55%	30% ^a
Average stated maturity	5.17	9.00	6.00
Maturity adjusted for early exercise	2.58	4.50	3.00
Stock Price at time of analysis	\$17.67	\$295.97	\$6.00 ^b
Value per option	\$ 2.27	\$ 274.27	\$2.32
Value of options outstanding	\$ 3,257	\$ 7.023	\$69.51

¹⁶ Ammann, M., and R. Seiz, 2003, *Does the Model Matter? A Valuation Analysis of Employee Stock Options*, Working Paper, SSRN.

¹⁷ The value that we obtained for equity, using a discounted cash flow model, was \$2.1 million. We reduced this by 14.29%, to reflect illiquidity, to arrive at our estimate of value of equity in the firm.

¹⁸ The variance estimate is actually on the natural log of the stock prices. This allows us to cling to at least the possibility of a normal distribution. Neither stock prices nor stock returns can be normally distributed since prices cannot fall below zero and returns cannot be lower than -100%.

¹⁹ All of the inputs to the Black Scholes model have to be in annual terms. To annualize a weekly variance, we multiply by 52.

Tax Rate	36.80%	35.00%	35.00%
After-tax Value of options outstanding	\$ 2,058	\$ 4,565	\$45.18

^a This is the average standard deviation of publicly traded candy companies.

^b This is the estimated value of equity per share, adjusted for illiquidity.

In estimating the after-tax value of the options at these companies, we have used the marginal tax rate of 35%. Since the tax law allows for tax deductions only at exercise and only for the exercise value, we are potentially overstating the possible tax benefits (and understating the costs).

The value per share is computed in Table 4 by subtracting the value of the options outstanding from the value of equity and then dividing by the primary number of shares outstanding:

Table 4: Value of Equity per Share

	<i>Cisco</i>	<i>Google</i>	<i>Kristin Kandy</i>
Value of Equity	\$65,622	\$ 32,187	\$1,800
- Value of Options outstanding	\$ 2,058	\$ 4,565	\$ 45
Value of Equity in shares outstanding	\$ 63,564	\$ 27.622	\$1,755
Primary shares outstanding	6487	277.78	270
Value per Share	\$ 9.80	\$99.44	\$6.50

The inconsistency averred to earlier is clear when we compare the value per share that we have estimated in this table to the price per share that we used in the previous one to estimate the value of the options. For instance, Google's value per share is \$99.44, whereas the price per share used in the option valuation is \$ 295.97. If we choose to iterate, we would revalue the options using the estimated value of \$99.44, which would lower the value of the options and increase the value per share, leading to a second iteration and a third one and so on. The values converge to yield a consistent estimate. The consistent estimates of value are provided in Table 5:

Table 5: Consistent Estimates of Value per Share

	<i>Cisco</i>	<i>Google</i>	<i>Kristin Kandy</i>
Value of Options (with current stock price)	\$ 2,058	\$ 4,565	\$45
Value per share	\$9.80	\$99.44	\$6.50
Value of Options (with iterated value)	\$ 332	\$1,501	\$50
Value per share	\$ 10.07	\$ 110.47	\$6.47

For both publicly traded firms, the estimated after-tax value of the options drops dramatically, leading to an increase in value per share. For Kristin Kandy, the effect is much smaller simply because we used the estimated value of equity per share in the first round, albeit not one adjusted for the value of the equity options.

Cash Flow Claims

In conventional valuation, we assume that all equity claimholders have identical claims on the cash flows that are paid out to equity. In the most common scenario, these cash flows are dividends and all equity shares receive the same dividends per share. There are scenarios where different equity claims may be entitled not only to different amounts of dividends but one set of equity investors has priority in receiving dividends. There is also the possibility that one class of equity investors gets first claim on the cash flows, if the firm is liquidated. In this section, we will examine the prevalence of differences in cash flow claims and how best to value these claims.

Prevalence of differential cash flow claims

Differences in cash flow claims across equity claims are not uncommon, especially as we widen the net and look at different categories of equity. In many publicly traded firms, this takes the form of preferred shares that are entitled to a fixed dividend and common shares that can receive (but are not entitled to) a residual dividend. In private firms, the variations across different equity classes can be even wider, with differences not only in the amount and predictability of dividends but also in the priority of claims to the cash flows if the firm is liquidated.

Preferred Shares versus Common Shares

With publicly traded companies in the United States, preferred shares have characteristics that clearly set them apart from common shares. The dividend payment on the shares are set at the time of the issue and the firm commits to meeting that dividend commitment, if it has the earnings to do so. In the case of cumulative preferred stock, the firm agrees to make up for the failure to pay preferred dividends in one period in subsequent periods. Generally, preferred stockholders are not entitled to any additional

dividends over and above the committed amount. There are no equivalent dividend commitments with common stock and failure to pay dividends in a period usually does not create a liability for the following periods. In 2008, about 10% of publicly traded companies in the United States had preferred stock outstanding, with two groups of companies making the bulk of the offerings. The first are financial service companies, which tend to issue plain vanilla preferred stock (with a fixed dividend and relatively few additional features); the advantage of preferred stock for these firms is that it is counted as equity for regulatory capital. The second are young, growth companies, which are far more likely to issue convertible preferred stock, where the conversion is to common stock. In private businesses, there are more variants on preferred stock. In some cases, preferred stockholders are entitled not only to the promised dividends but can share (at some pre-arranged rate) in the profits above the preferred dividend, thus giving them a stake in the common equity as well.

Outside the United States, preferred stock generally continues to get priority in terms of dividends, but there are wide differences in how these priorities are structured. In some countries, for instance, preferred stockholders are entitled to a fixed proportion of the earnings as dividend, rather than a fixed dividend. In other countries, preferred stockholders receive a higher dividend rate than common stockholders; thus, if common stockholders receive a 2% dividend yield, preferred stockholders will be entitled to a 3% dividend yield.

Liquidation Cash Preferences

In publicly traded firms, preferred stockholders get first claim (over common equity investors) in the event of liquidation, at least up to the face value of the preferred stock. Debt holders get prior claims over both preferred and common equity claim holders to the proceeds from liquidation. In general, debt holders are not entitled to any cash flows in excess of the amount they are owed and preferred stockholder claims are restricted to the book capital supplied by them to the firm.

In private businesses, there are two types of liquidation claims that are attached to preferred shares:

- a. Non-participating preferred stockholders are entitled to receive proceeds from liquidation prior to common stockholders, but only up to the fixed amount of their book value. They do not share in upside beyond this preference, but in some cases, they have the right to convert their preferred shares into common stock, if such a conversion will yield larger proceeds.
- b. Participating preferred stockholders are entitled to liquidation proceeds prior to common stockholders up to the fixed preference amount but then share any additional proceeds proportionately with common stockholders. In some cases, the returns (and the share of the proceeds) is constrained to not exceed a pre-specified amount (say 2 times the original amount invested by the preferred stockholders) and in other cases, it is unrestricted.

Note that liquidation, in this context, is not just in the event of distress but can also cover transactions such as sale of the business or just its assets to another entity – public or private. As a result, it is entirely possible that the liquidation cash flow provides significant upside to investors in the firm.

Valuing differential cash flow claims

When different classes of equity have different claims on the cash flows, we would expect their values to reflect these claims. Other things remaining equal, prior claims on cash flows should be worth more than later claims on cash flows. In this section, we look at how best to value cash flow claims, by first looking at claims on on-going cash flows (dividends or variants) and then considering claims on liquidation cash flows.

Dividends

As described in the last section, there are three types of prior dividend claims embedded in equity. The first is fixed absolute dividends, with no appreciation or upside prospects, that can be either cumulative or non-cumulative. The second is a fixed dividend, not in absolute terms but as a proportion of earnings, which can also be cumulative or non-cumulative. The third is a fixed dividend, in conjunction with a share of any excess cash flows.

Fixed absolute dividends (absolute)

Consider the simplest case first: an absolute dividend that is set at the time the equity claim is issued. This is often the case with preferred stock issued by publicly traded company, where the company commits to a fixed dollar dividend at the time of the issue. Since the dividends are fixed, valuing these shares is more akin to valuing a bond than it is to valuing equity. In effect, we can discount the promised dividends back at a rate that reflects the risk that the commitment will not be met. If the commitment is perpetual, i.e., the equity claim has no finite maturity, this can be written as:

$$\text{Value of fixed-dividend equity claim} = \frac{\text{Promised Dividend}}{r_{\text{preferred stock}}}$$

where $r_{\text{preferred stock}}$ is the risk-adjusted discount rate. If the equity claim does have a finite maturity, the equation can be modified to reflect the dividends as an annuity, with the face value of the equity claim representing the final cash flow.

The only estimation question that we then face in valuing this security is how best to estimate the discount rate. If we accept the premise that this is similar to a debt issue, we can approach this question in much the same way that we estimate the pre-tax cost of debt. In effect, the risk-adjusted rate for fixed preferred dividend-paying equity would be:

$$\text{Risk-adjusted Discount Rate} = \text{Riskfree rate} + \text{Spread capturing default risk (of defaulting on dividend payment)}$$

The default spread can be estimated in one of two ways. If there are publicly traded preferred stock of equivalent risk to the one being valued, we can use the preferred dividend yield on these shares as the risk-adjusted discount rate.

Preferred dividend yield = Annualized Preferred dividend/ Current Preferred price
Just as we use ratings to find bonds of equivalent risk, we can use ratings to find preferred stock of equivalent risk; there are fewer rated preferred stock than bonds, though. The other is to extend an approach that is often used to estimate the cost of debt for non-rated companies, where we estimate a synthetic rating for a company based upon its financial ratios and use that rating to come up with a default spread. In fact, one ratio that is widely used for synthetic bond ratings is the interest coverage ratio:

$$\text{Interest coverage ratio} = \text{Operating income} / \text{Interest expenses}$$

This ratio can be adapted to incorporate preferred dividends in the denominator (treated like interest expenses).

$$\text{Preferred coverage ratio} = \text{Operating income} / (\text{Interest expenses} + \text{Preferred dividends})$$

The resulting number should yield a synthetic rating for preferred stock, which, in turn, can be used to estimate the default spread and the risk-adjusted cost of preferred stock. The resulting number should be higher than the pre-tax cost of debt, because preferred dividends are paid after interest expenses, but lower than the cost of equity, because preferred stockholders get their dividends before common stockholders.

$$\text{Pre-tax cost of debt} < r_{\text{Preferred dividends}} < \text{Cost of equity}$$

The question of whether the dividend is cumulative or non-cumulative can be examined in this context as well, with the rate on cumulative preferred stock being lower than the rate on non-cumulative preferred stock.

Illustration 4: Valuing fixed-dividend preferred stock

In June 2008, General Electric had perpetual preferred stock outstanding, with a preferred dividend rate of 6.95% on a face value of \$ 100. Standard and Poor's gave a rating of AA to the preferred shares and the average dividend yield on preferred shares with a AA rating in June 2008 was 6.45%. The fixed-dividend preferred stock can be valued as follows:

$$\text{Value} = \text{Preferred dividend} / r_{\text{Preferred Dividends}} = 6.95 / .0645 = \$107.75$$

The presence of a rating made the computation a fairly straightforward one.

We face a more difficult time valuing preferred stock, when the stock is not rated. In June 2008, Entertainment Property Tryst had perpetual preferred outstanding, with a dividend set at 7.75% of the face value of \$ 100. The trust reported operating income of \$171.4 million, interest expenses of \$66.9 million and total preferred dividends of \$24.9 million. We estimated both the conventional interest coverage ratio and the modified interest coverage ratio (treating preferred dividends as interest) as well as the synthetic ratings (and default spreads) for each ratio:

<i>Measure of coverage</i>	<i>Value</i>	<i>Synthetic Rating</i>	<i>Default Spread</i>	<i>Risk-adjusted rate</i>
EBIT/ Interest expense	2.55	B+	4.50%	8.50%
EBIT/ (Interest expense + Preferred dividends)	1.86	B-	6.50%	10.50%

The risk-adjusted rate is obtained by adding the default spread to the riskfree rate at the time of the analysis (4%). To value the preferred stock, we use the risk-adjusted rate of 10.50%:

$$\text{Value of preferred stock} = 7.75 / .105 = \$73.81$$

Fixed dividends (proportional)

In some companies, preferred shareholders are entitled to a fixed percentage of the earnings reported by a firm, rather than fixed dividends. Thus, the owner of preferred shares in the company may be entitled to receive 35% of the net income of the company as dividends. Consequently, the dividends paid on these shares fluctuate with the earnings of the company. Since cash flows to common equity investors also vary with earnings, valuing proportional dividend equity claims is closer to valuing common equity than debt.

$$\text{Value of proportional-dividend claim} = \sum_{t=1}^{t=\infty} \frac{E(\text{Earnings}_t)(\text{Dividend ratio})}{(1 + r_{\text{Pref Dividend}})}$$

This is, in effect, the dividend discount model applied to preferred stock, with the only practical difference being that the dividend ratio is a fixed number (based upon the claim) and not an expected value, as is the case with common stock. As with fixed dividend preferred, the challenge is in estimating the discount rate for the preferred dividend shares. Since these claims share more risk characteristics with common equity claims than debt, it is logical to start with the cost of equity for common equity. Since the preferred dividend claims have to be met first, we would expect the discount rate for these claims to be lower than the cost of equity, because the cash flows are more assured. The question of how much lower, though, will depend upon how much common equity investors depend upon dividends for returns and the magnitude of other cash flow claims on earnings, including reinvestment needs. As a limiting case, consider a firm with no reinvestment needs and common equity investors who have no interest in cash dividends. For this firm, there is no cost created by paying 35% of earnings as dividends and the cost of equity, computed using any of the standard risk and return models, for common stock can be applied to the preferred dividends as well. At the other extreme, consider a firm with substantial reinvestment needs and common equity investors who value cash dividends. In this firm, the common dividends will be much more volatile than preferred

dividends and the cost of equity for common dividends should be higher than the cost of equity for preferred dividends.²⁰

In the latter case, we will be faced with the question of best to adjust the cost of equity for preferred stock. One is to adjust the parameters of the risk and return model for estimating the cost of equity. Thus, if the capital asset pricing model is used to estimate the cost of equity, the beta for common stock may be set at 1.20 whereas the beta for proportional preferred stock will be lower (say 1.00). These numbers can be obtained by running regressions against market indices for each class of shares.

Illustration 5: Valuing fixed-dividend (proportional) preferred stock

Consider LaTouro Inc, a company with two classes of equity – class A is entitled to 40% of the earnings per share as dividends and class B stock is conventional common stock, with no dividend entitlement - there are 20 million shares of each class outstanding. The firm is expected to have net income of \$ 50 million in the next year and you expect earnings to grow 4% a year in perpetuity; the firm has a return on equity of 8%. The beta for class B shares is 1.0 and the beta for class A shares is 0.9; the riskfree rate is 4% and the risk premium is 5%.

Let us first value the stock that is entitled to 40% of earnings as dividends. The cost of equity, based upon the beta for this class, is 8.5%:

$$\text{Cost of equity} = 4\% + 0.9 (5\%) = 8.5\%$$

The value of equity can then be obtained using expected earnings and fixed payout ratio:

$$\text{Expected Earnings per share (across all shares)} = 50 / 40 = \$ 1.25 / \text{share}$$

$$\text{Value of class A/ share} = \frac{\text{Earnings next year} * \text{Fixed payout ratio}}{\text{Cost of equity}_{\text{Fixed dividend shares}} - g} = \frac{1.25 (.40)}{.085 - .04} = \$11.11$$

To value the class B equity, with no dividend claim, we value the aggregate equity in the firm, using the details provided:

$$\text{Cost of equity} = 4\% + 1 (5\%) = 9\%$$

$$\text{Equity reinvestment rate} = g / \text{ROE} = .04 / .08 = 50\%$$

²⁰ As a general proposition, the smaller the dividend advantage enjoyed by preferred stockholders and the less commitment there is associated with the claim, the smaller the difference between the two costs of equity. As a limiting case, preferred stockholders whose only advantage is that they receive a slightly higher dividend rate than common stockholders, with no commitment to make up for lost dividends, bear very similar risk to common stockholders and should be given the same cost of equity.

$$\begin{aligned} \text{Value of equity (aggregate)} &= \frac{\text{Earnings next year} * (1 - \text{Equity Reinvestment Rate})}{\text{Cost of equity}_{\text{Standard Equity}} - g} \\ &= 50 (1 - .50) / (.09 - .04) = \$500 \text{ million} \end{aligned}$$

Value of equity in class B shares = Value of equity (aggregate) – Value of equity in class A shares = 500 – 20 (11.11) = \$277.88

Value per class B share = 277.88/20 = \$13.89

At first sight, it may seem surprising that the class A shares with prior claims to dividends would be worth less than the class B shares but the fact that the dividend payout ratio is fixed at 40% has two effects on the value of class A shares. The positive effect is that it reduces risk and lowers the cost of equity but this effect is outweighed (in this case) by the negative effect of capping the dividends at 40% of earnings. Since this firm, based on its fundamentals, can pay 50% of its earnings as dividends, the class B shareholders gain at the expense of the class A shareholders.

Participation Claims

Preferred stockholders, in addition to being entitled to fixed dividends (absolute or proportional), sometimes can claim a pro-rata share of any additional dividends paid to common stockholders as well. Consider a firm with \$ 100 million in earnings and 20 millions shares apiece of both common and preferred stock. Assume that preferred stockholders are entitled to \$ 1 per share in fixed dividends; the total preferred dividend paid by the firm will be \$ 20 million, leaving behind a residual amount of \$ 80 million. Assume further that the firm decides to pay half of this amount as dividends. With a participation right, preferred stockholders will receive an additional dollar in dividends to match the common stockholders dividends.

The most direct way to value participation claims is to consider them to be common equity claims, rather than preferred claims. Once we make this judgment, the valuation of the participation claims parallels the valuation of common equity, using the dividend discount model:

$$\text{Value of participation claims} = \sum_{t=1}^{\infty} \frac{E(\text{Earnings}_t)E(\text{Dividend ratio}_t)}{(1 + r_{\text{Common Equity}})}$$

Note that the participation claim is in addition to the fixed dividend claim, and the value of the preferred stock will therefore be the consolidated sum of the two claims:

Value of Preferred stock = Value of fixed dividend claim + Value of participation claim

While participating preferred shares seem to offer the best of both worlds, there is one consideration that needs to be kept in mind. If common equity investors decide on dividends, they are likely to adjust the dividends paid to reflect the participation claim. For instance, the common equity investors may decide to not pay dividends and hold the cash in the firm in the above example. It is true that they will then have to give up their dividends as well, but they will be compensated with a higher share price (reflecting the cash holdings of the firm) and more price appreciation.

Illustration 6: Valuing participating preferred stock

Revisiting illustration 5, assume that class A shareholders in LaTouro, in addition to being entitled to 40% of the earnings per share as a dividend, can share equally with class B shareholders in the remaining dividends that will be paid out by the firm. To value class A shares now, we will need to value the participation claim that these shareholders have in the dividends of the firm:

Expected Earnings next year = \$ 50 million

Earnings per share = \$ 1.25

Fixed dividend per share (class A shares) = \$ 0.50

Total fixed dividend paid out = \$0.50 (20) = \$ 10 million

Total Potential dividends = Expected Earnings (1- Equity Reinvestment Rate) = 50 (1-.5)
= \$ 25 million

Thus, the firm can pay an additional \$ 15 million in dividends; on a per share basis across all remaining shares, this works out to \$ 0.375 a share (\$15/40). To value this claim, we use the cost of equity in common stock:

$$\text{Value of residual claim} = \frac{\text{Expected Dividend per share}}{\text{Cost of equity}_{\text{Standard Equity}} - g} = \frac{\$0.375}{(.09 - .04)} = \$7.50$$

This is the value of the participation claim for class A shareholders, which augments the value of the fixed claim, leading to a value of equity per share of \$18.61.

$$\begin{aligned} \text{Value of class A share} &= \text{Value of fixed dividend claim} + \text{Value of participation claim} \\ &= \$11.11 + \$ 7.50 = \$18.61 \end{aligned}$$

Note that the first part of the value (\$11.11) was estimated using the lower discount rate of 8.5%, whereas the participation claim was valued using the higher discount rate of 9%, reflecting its greater risk.

To value the class B claim, we subtract the total value of class A claims from the aggregate value of equity for the firm:

$$\begin{aligned}\text{Value of class B claims} &= \text{Aggregate value of equity} - \text{Value of class A claims} \\ &= 500 - 20(18.61) = \$128.8 \text{ million}\end{aligned}$$

$$\text{Value per class B share} = 128.8/20 = \$ 6.40/\text{share}$$

Clearly, class B shareholders lose in this scenario. They also have every incentive to not pay out the residual dividends of \$ 15 million. By withholding the dividends, they will reduce the value of class A shares and increase the value of their own.

Liquidation Cash Flows

As we noted at the start of this section, preferred stockholders also get first claim on the cash flows of the firm in liquidation. Sometimes, these claims are constrained to the original investment made by the preferred stockholders. In others, they are augmented to include a share of the excess proceeds. Unlike dividends, which represent an on-going claim, liquidation is a one-time event and the valuation approaches that we use reflect the difference. In the first approach, we try to incorporate the likelihood of and the expected cash flows from liquidation into a discounted cash flow model and arrive at a value today. In the second approach, we value the liquidation preferences as options, and allocate value across different claim holders.

a. Scenario Analysis

The value of a claim, in a discounted cash flow model, is the present value of the expected cash flows to the claim. In most conventional discounted cash flow models, we assume the firm to be a going concern with a perpetual life and value securities accordingly. Thus, the value of equity in a publicly traded firm is computed as the present value of dividends or free cash flows to equity forever. Since liquidation is an event that terminates the cash flows, incorporating any cash flow preferences in liquidation into a discounted cash flow model requires us to also estimate the likelihood of the liquidation event each period and the cash flows to each claimholder.

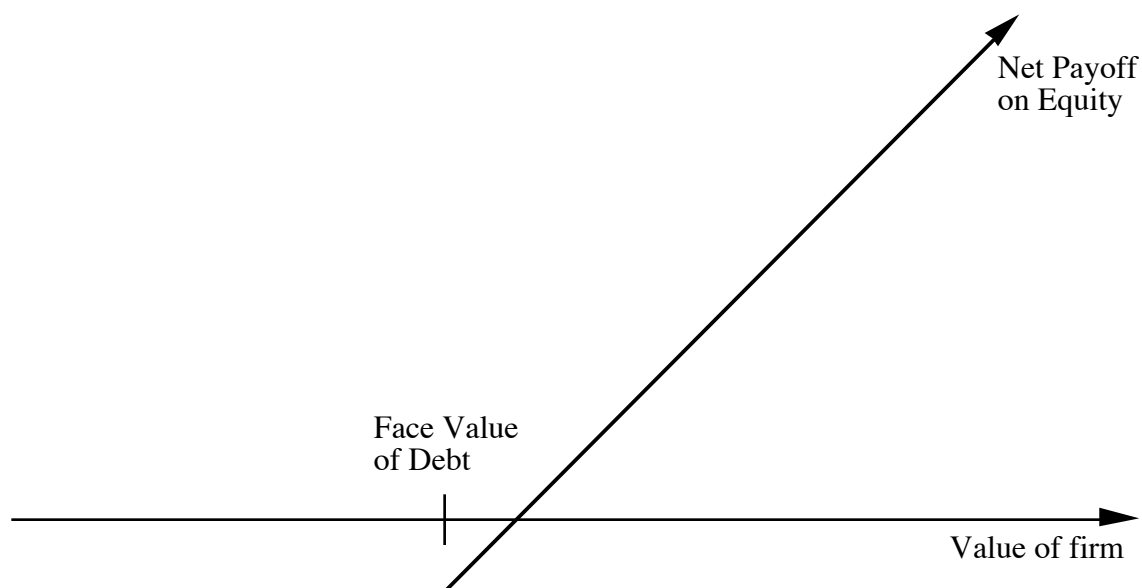
The simplest way to do this is to create two scenarios. In the first, you value the claims assuming the firm is a going concern; in effect, you assume that the cash flows (dividends or free cashflows) continue forever and compute the present value. In the second, you assume that the firm will be liquidated at a specific point in time (say 5 years from now) and compute the value of the claims on the firm based upon the cashflows each period during the time period and the cash flows in liquidation. Once the claims have been valued under both scenarios, you estimate the probability of each scenario (going concern and liquidation) and compute an expected value. The approach is predicated on the assumption that liquidation will occur only at the specified point in time and that the probability of it occurring can be estimated with reasonable ease.

A more complete and complicated approach to bringing in liquidation cash flows is to allow for the possibility of liquidation in each time period and to incorporate the effect of liquidation into your expected cash flows. In effect, with a 5-year time horizon for expected cash flows, you are assuming that liquidation can occur in any of the five intermediate years and that there will be no subsequent cashflows, if liquidation does occur. Thus, we have to estimate the probabilities of liquidation in each year, rather than the one consolidated probability that we estimated in the previous approach. We also have to modify the discounted cash flow valuation, allowing for liquidation in any time period. Adapting the scenario approach, we could estimate six scenarios for the firm – a going concern scenario, and five liquidation scenarios – one for each time period. The final value for the firm will be an expected value across all six scenarios.

b. Option Pricing Approaches

When there are multiple claims on the cashflows from liquidation, with different preferences, we can model the residual claims as options. Consider the simplest case of a firm that has only two claims – conventional debt and standard equity. In liquidation, the proceeds are first used to meet the claims of the debt holders and whatever cash flow is left over goes to equity investors.

Figure 2: Equity as a Liquidation Option



In this case, the cashflow from liquidating a firm's assets is the determining value and the claim from debt holders (usually the face value of debt) becomes the strike price. Since equity investors control the process only until the debt comes due, the life of the option is set to the maturity of the debt.

When we add equity claims (such as preferred stock) that also have priority in payment in the event of liquidation, we can modify the option pricing approach to value each layer. The sequence is as follows:

Step 1: Estimate the liquidation value of the assets of the firm. If the assets are separable and marketable (for example, real estate holdings), the easiest way to estimate the liquidation value is to aggregate the market values of the assets, net of any illiquidity costs. While some analysts use the discounted cash flow value that they have estimated for the firm as the liquidation value, there may be a potential problem. A portion of the discounted cash flow value of a going concern reflects growth assets – investments that you expect the firm to make in the future. In the event of liquidation, the buyer of the firm may be unwilling to pay for such growth assets. Consequently, it may be more reasonable to estimate a discounted cash flow valuation of just assets in place, i.e., value the firm as a mature firm, with no value added by new investments. If there are liquidity concerns, the discounted cash flow value will have to be reduced to reflect them.

Step 2: Value the standard equity in the firm as an option, since it is the only claim that has no priority in the cashflows and is entitled to the residual cash flow. To make this

estimate, consider the aggregated claims of all prior claimholders (including debt and preferred stock) as the strike price of the option and the liquidation value from step 1 as the equivalent of the stock price.

Step 3: Now consider the claimant just ahead of standard equity and value the claim against using an option pricing model, with two key differences. The first is that the strike price will now be adjusted down to reflect the remaining claimholders and thus will be smaller than the strike price in step 2. The second is that option value may have to be adjusted if the liquidation claim has a cap; preferred stockholders, for instance, may receive cash flows only up to their original investment in the firm.

Step 4: Move sequentially through the remaining claimants, repeating step 3, until you get to the very first claimant (in terms of priority). The value of that first claim will not be estimated using option pricing models, but instead will reflect the difference between the overall value of the assets and the values of inferior claims, estimated as equity values.

Illustration 7: Valuing claims on liquidation cash flows

Assume that you have are valuing a firm with three securities – conventional common stock, preferred stock and debt, with the following claims in liquidation

Security	Priority of claim on cash flow	Claim amount
Debt (10 year)	First claim on cash flow	\$ 100 million
Preferred Equity	Claim after debt, up to face value	\$ 150 million
Common Equity	After debt and preferred stoc	Residual

Assume that the firm expects after-tax operating income of \$ 25 million next year and has a cost of capital of 12.5%; the firm has an expected growth rate of 4% a year, and expects to generate a 20% return on capital on new investments.

A conventional discounted cash flow valuation of the firm yields a value of \$ 235 million. This is obtained by first backing into a reinvestment rate, from the growth rate and return on capital estimates:

$$\text{Reinvestment Rate} = \text{Expected growth rate} / \text{ROC} = 4\% / 20\% = 20\%$$

$$\begin{aligned} \text{Free Cashflow to the firm} &= \text{After-tax Operating Income} (1 - \text{Reinvestment Rate}) \\ &= 25 (1 - .20) = \$ 20 \text{ million} \end{aligned}$$

$$\begin{aligned} \text{Value of firm today} &= \text{Free Cashflow to firm next year} / (\text{Cost of capital} - g) \\ &= 20 / (.125 - .04) = \$235 \text{ million} \end{aligned}$$

In valuing equity as an option, we will assume that the firm is unlikely to receive the value of its growth opportunities in liquidation and that the discount on asset value it will face is 10%. To estimate liquidation value, therefore, we make the following assumptions:

$$\begin{aligned} \text{Value of assets in place} &= \text{After-tax operating income} / \text{Cost of capital} \\ &= 25 / .125 = \$ 200 \text{ million} \end{aligned}$$

$$\begin{aligned} \text{Estimated liquidation value} &= \text{Value of assets in place} (1 - \text{Liquidation Discount}) \\ &= 200 (1 - .10) = \$ 180 \text{ million} \end{aligned}$$

Assume further that the standard deviation in this estimated value is 25% (on an annualized basis).

To value conventional equity as a call option, we use the following inputs:

$$\text{Value of the underlying asset} = \text{Liquidation Value} = \$ 180 \text{ million}$$

$$\text{Strike price} = \text{Debt claim} + \text{Preferred claim} = \$ 250 \text{ million}$$

$$\text{Option life} = \text{Debt maturity} = 10 \text{ years}$$

$$\text{Standard deviation in liquidation value} = 25\%$$

If the ten-year riskfree rate is 4%, we can estimate the value of equity as a call option to be \$59.8 million. Moving to the next claimant, the preferred stock, we have to value two options, one with a strike price of \$ 100 million and one with a strike price of \$ 250 million (using the same inputs for the underlying asset value and maturity, used above). The value of the liquidation claim on preferred stock is the difference between the two values:²¹

$$\text{Value of option with a strike price of } \$ 100 \text{ million} = \$117.1 \text{ million}$$

$$\text{Value of option with a strike price of } \$ 250 \text{ million} = \$ 59.8 \text{ million}$$

$$\text{Value of liquidation claim on preferred stock} = \$ 58.3 \text{ million}$$

To value of the liquidation claim on debt can then be computed as the difference between the liquidation value and the values computed for equity and preferred stock:

$$\begin{aligned} \text{Value of liquidation claim for debt} &= \text{Liquidation Value} - \text{Value of preferred} - \text{Value of} \\ \text{common equity} &= 180 - 58.3 - 59.8 = \$62.9 \text{ million} \end{aligned}$$

²¹ In effect, we are valuing a capped call. By subtracting the value of the call with a strike price of \$250 million, we are taking into consideration the effect of the cap on proceeds.

Control Claims

Generally, there are two characteristics that differentiate equity from debt. The first is the fact that equity investors have a residual claim on the cash flows. The second is that equity investors control the firm in the sense that they hire the managers and through them make the decisions on investments, financing and dividends that determine firm value. While we generally do not separate out and value control in a firm, we may have to do so, if all equity claim holders do not have the same control rights. In this section, we will begin by first examining what the expected value of control in a firm is and then move on to explore the nature of differences in control rights and how best to value these differences.

The Expected value of Control

The value of controlling a firm derives from the fact that you believe that you or someone else would operate the firm differently from the way it is operated currently. In fact, the expected value of control is the product of two variables: the change in value from changing the way a firm is operated and the probability that this change will occur. We will begin this section by considering the dimensions on which management decisions can affect the value of the firm and how to measure the effect of the change. We will follow up by considering the probability that existing management policies can be changed.

1. The Value of Control

The value of a business is determined by decisions on made by the managers of that business on where to invest its resources, how to fund these investments and how much cash to return to the owners of the business. Consequently, when we value a business, we make implicit or explicit assumptions about both who will run that business and how they will run it. In other words, the value of a business will be much lower if we assume that it is run by incompetent managers rather than by competent ones. When valuing an existing company, private or public, where there is already a management in place, we are faced with a choice.

If we consider value to be the end result of the investment, financing and dividend decisions made by a firm, the value of firm will be a function of how optimal (or sub-optimal) we consider a firm's management to be. If we estimate a value for the firm, assuming that existing management practices continue, and call this a status quo value and re-estimate the value of the same firm, assuming that it is optimally managed, and call this estimate the optimal value, the value of changing management can be written as:

$$\text{Value of management change} = \text{Optimal firm value} - \text{Status quo value}$$

The value of changing management will be a direct consequence of how much we can improve the way the firm is run. The value of changing management will be zero in a firm that is already optimally managed and substantial for a firm that is badly managed.

Considering the determinants of value, it should also be quite clear that the pathway to value enhancement will vary for different firms. Sub-optimal management can manifest itself in different ways for different firms. For firms where existing assets are poorly managed, the increase in value will be primarily from managing those assets more efficiently – higher cash flows from these assets and efficiency growth. For firms where investment policy is sound but financing policy is not, the increase in value will come from changing the mix of debt and equity and a lower cost of capital. We examine the determinants of value much more extensively in a companion paper on valuation and value enhancement.²² Table 6 considers potential problems in existing management, fixes to these problems and the value consequences:

Table 6: Ways of Increasing Value

Potential Problem	Manifestations	Possible fixes	Value Consequence
Existing assets are poorly managed	Operating margins are lower than peer group and return on capital is lower than the cost of capital	Manage existing assets better. This may require divesting some poorly performing assets.	Higher operating margin and return on capital on existing assets -> Higher operating income Efficiency growth -> in near term as return on capital improves
Management is under investing (It is too conservative in exploiting growth opportunities)	Low reinvestment rate and high return on capital in high growth period	Reinvest more in new investments, even if it means lower return on capital (albeit > cost of capital)	Higher growth rate and higher reinvestment rate during high growth period -> Higher value because growth is value creating.
Management is over	High reinvestment rate and	Reduce	Lower growth rate and

²² Damodaran, A., 1999, Value Enhancement: Back to the Future, Contemporary Finance Digest.

investing (It is investing in value destroying new investments)	return on capital that is lower than cost of capital	reinvestment rate until marginal return on capital is at least equal to cost of capital	lower reinvestment rate during high growth period - > Higher value because growth is no longer value destroying
Management is not exploiting possible strategic advantages	Short or non-existent high growth period with low or no excess returns.	Build on competitive advantages	Longer high growth period, with larger excess returns - > Higher value
Management is too conservative in its use of debt	Debt ratio is lower than optimal (or industry average)	Increase debt financing	Higher debt ratio and lower cost of capital -> Higher firm value
Management is over using debt	Debt ratio is higher than optimal	Reduce debt financing	Lower debt ratio and lower cost of capital -> Higher firm value
Management is using wrong type of financing	Cost of debt is higher than it should be, given the firm's earning power	Match debt up to assets, using swaps, derivatives or refinancing	Lower cost of debt and cost of capital -> Higher firm value
Management holds excess cash and is not trusted by the market with the cash.	Cash and marketable securities are a large percent of firm value; Firm has poor track record on investments.	Return cash to stockholders, either as dividends or stock buybacks	Firm value is reduced by cash paid out, but stockholders gain because the cash was discounted in the firm's hands.
Management has made investments in unrelated companies.	Substantial cross holdings in other companies that are being undervalued by the market.	As a first step, try to be more transparent about cross holdings. If that is not sufficient, divest cross holdings	Firm value is reduced by divested cross holdings but increased by cash received from divestitures. When cross holdings are under valued, the latter should exceed the former.

Illustration 8: The Value of Changing Management – Nova Chemicals

Nova Chemicals is a publicly traded chemical firm. In the most recent financial year, the firm reported after-tax operating income of \$ 12 million on revenues of \$ 200 million; the firm also reported capital expenditures of \$ 9 million, depreciation of \$ 4 million and an increase in working capital of \$ 1 million. The capital invested in the firm (book value) at the beginning of the year was \$ 150 million and the firm was entirely equity financed, with a cost of equity (and capital) of 12%. Under existing management, the firm is expected to maintain its existing reinvestment rate and generate its prevailing return on capital on both existing and new investments for the foreseeable future. To value the firm, let us consider the inputs:

Reinvestment Rate = (Capital Expenditures – Depreciation + Change in Working capital)/ After-tax operating income = $(9-4+1)/12 = 50\%$

Return on capital = After-tax operating income/ Capital invested = $12/150 = 8\%$

Cost of capital = Cost of equity = 12%

The expected growth rate in operating income can be computed from the reinvestment rate and return on capital:

$$\text{Expected growth rate in operating income} = 50\% * 8\% = 4\%$$

The status-quo value of the firm can be estimated, allowing for these inputs in perpetuity:

$$\begin{aligned} \text{Value of the firm} &= \frac{\text{After - tax Operating Income} (1 - \text{Reinvestment Rate}) (1 + g)}{(\text{Cost of capital} - g)} \\ &= 12 (1 - .5) (1.04) / (.12 - .04) = \$ 78 \text{ million} \end{aligned}$$

You believe that new management can turn the firm around by doing the following:

- a. With cost-cutting and more efficient operations, the firm can increase its after-tax operating margin to 7.5% of revenues (from the existing 6%), while keeping capital invested unchanged.
- b. You believe that being more selective in your new investments will lower the reinvestment rate to 40%, but it will increase the return on capital on new investments to 10%.
- c. By using a mix of 80% equity and 20% debt, the cost of capital for the firm can be lowered to 10%.

With these changes in place, let us consider the new parameters for the valuation:

$$\text{New After-tax Operating Income} = \text{New Margin} * \text{Revenue} = .075 * 200 = \$ 15 \text{ million}$$

Note that this will increase the return on capital on existing assets to 10% (15/150).

$$\text{Reinvestment Rate} = 33\%$$

$$\text{Return on capital on new investments} = 12\%$$

$$\text{Cost of capital} = 10\%$$

The expected growth rate in operating income can be computed from the reinvestment rate and return on capital:

$$\text{Expected growth rate in operating income} = 40\% * 10\% = 4\%$$

The optimal value of the firm can be estimated, allowing for these inputs in perpetuity:

$$\begin{aligned} \text{Value of the firm} &= \frac{\text{After - tax Operating Income} (1 - \text{Reinvestment Rate}) (1 + g)}{(\text{Cost of capital} - g)} \\ &= 15 (1 - .40) (1.04) / (.10 - .04) = \$ 156 \text{ million} \end{aligned}$$

$$\text{Value of changing management} = \$156 \text{ million} - \$78 \text{ million} = \$ 78 \text{ million}$$

In the case of Nova Chemicals, the value of changing management is substantial.

2. The Probability of Changing Management

While the value of changing management in a badly managed firm can be substantial, the increased value will be created only if management policies are changed. While this change can sometimes be accomplished by convincing existing managers to modify their ways, all too often it requires replacing the managers themselves. If the likelihood of management change happening is low, the expected value of control will also be low. In this section, we first consider the mechanisms for changing management, and then some of the factors that determine the likelihood of management change.

Mechanisms for Changing Management

It is difficult to change the way a company is run, but in general, there are four ways in which it can be done. The first is a variation of moral or at least economic suasion, where one or more large institutional investors introduce shareholder proposals designed primarily to improve corporate governance, holding the threat of more extreme action over the heads of managers. The second is a proxy contest, where incumbent managers are challenged by an investor who is unhappy with the way the firm is run, for proxy votes; with sufficient votes, the investor can get representation on the board and may be able to change management policy. The third is to try to replace the existing managers in the firm with more competent managers; in publicly traded firms, this will require a board of directors that is willing to challenge management. The fourth and most extreme is a hostile acquisition of the firm by an investor or another firm; the incumbent management is usually replaced after the acquisition and management policy is revamped.

Estimating the probability of management change

In one of the first papers to assess the likelihood of takeovers by comparing target firms in acquisitions to firms that were not targets, Palepu (1986) noted that target firms in takeovers were smaller than non-target firms and invested inefficiently.²³ In a later

²³ Palepu, K.G. 1986, *Predicting Take-Over Targets : A Methodological and Empirical Analysis*. Journal of Accounting and Economics, v8 , 3-35.

paper, North (2001) concluded that firms with low insider/managerial ownership were more likely to be targeted in acquisitions²⁴. Neither paper specifically focused on hostile acquisitions, though. Nuttall (1999) found that target firms in hostile acquisitions tended to trade at lower price to book ratios than other firms and Weir (1997) added to this finding by noting that target firms in hostile acquisitions also earned lower returns on invested capital²⁵. Finally, Pinkowitz (2003) finds no evidence to support the conventional wisdom that firms with substantial cash balances are more likely to become targets of hostile acquisitions.²⁶ In summary, then, target firms in hostile acquisitions tend to be smaller, trade at lower multiples of book value and earn relatively low returns on their investments.²⁷

While many CEO changes are either voluntary (retirement or job switching), some CEOs are forced out by the board. In recent years, researchers have examined when forced CEO turnover is most likely to occur.

- The first factor is stock price and earnings performance, with forced turnover more likely in firms that have performed poorly relative to their peer group and to expectations.²⁸ One manifestation of poor management is overpaying on acquisitions, and there is evidence that CEOs of acquiring firms that pay too much on acquisitions are far more likely to be replaced than CEOs who do not do such acquisitions.²⁹

²⁴ North, D.S. 2001, *The Role of Managerial Incentives in Corporate Acquisitions: the 1990s Evidence*, Journal of Corporate Finance, 7, 125-149.

²⁵ Nuttall, R. 1999, *Take-Over Likelihood Models for UK Quoted Companies*. Nuffield College working paper, Oxford University; Weir, C. *Corporate Governance, Performance and Take-Overs: An Empirical Analysis of UK Mergers*. Applied Economics, 29 (1997), 1465-1475.

²⁶ Pinkowitz, L., 2003, *The Market for Corporate Control and Corporate Cash Holdings*, Working Paper, SSRN. His study of hostile acquisitions between 1985 and 1994 concludes that firms with large cash balances are less (not more) likely to be targets of hostile acquisitions.

²⁷ In a contrary finding, Franks and Mayer (1996) find no evidence of poor performance in target firms in hostile acquisitions in the UK, Franks, J. and C. Mayer, 1996, *Hostile Takeovers and the Correction of Management Failure*, Journal of Financial Economics, v40, 163-181.

²⁸ Warner, J., R. Watts and K. Wruck, 1988, *Stock Prices and Top Management Changes*, Journal of Financial Economics, v20, 461-492; Murphy, K and J. Zimmerman, 1993, *Financial Performance surrounding CEO Turnover*, Journal of Accounting and Economics, v16, 273-316; Puffer, S. and J.B. Weintrop, 1991, *Corporate Performance and CEO Turnover: The Role of Performance Expectations*, Administrative Science Quarterly, v36, 1-19;

²⁹ Lehn, K. and M. Zhao, 2004, *CEO Turnover after Acquisitions: Do Bad Bidders get fired?* Working Paper, University of Pittsburgh.

- The second factor is the structure of the board, with forced CEO changes more likely to occur when the board is small³⁰, is composed of outsiders³¹ and when the CEO is not also the chairman of the board of directors.³²
- The third and related factor is the ownership structure; forced CEO changes are more common in companies with high institutional and low insider holdings.³³ They also seem to occur more frequently in firms that are more dependent upon equity markets for new capital.³⁴
- The fifth factor is industry structure, with CEOs more likely to be replaced in competitive industries.³⁵
- Finally, the age of the CEO has an effect on the probability of CEO change, with older CEOs more likely to be replaced than younger CEOs.

In summary, firms where you see forced CEO change share some characteristics with firms that are targets of hostile acquisitions – they are poorly managed and run – but they tend to have much more effective boards of directors and more activist investors who are able to change management without turning over the firm to a hostile acquirer.

While the determinants of management change can be listed, it is far more difficult to quantitatively estimate the probability that it will occur. One statistical approach that is promising is a logit or probit, where we assess the probability of management change by contrasting the characteristics of firms where management has changed in the past with firms where that has not occurred. Researchers have applied this technique to look at both acquisitions and forced CEO change. In the context of hostile acquisitions, Morck, Schleifer and Vishny (1988) noted that the probability of being the

³⁰ Faleye, O., 2003, *Are large boards poor monitors? Evidence from CEO turnover*, Working Paper, SSRN. Using a proportional hazard model, he finds that every additional director on the board reduces the probability of a forced CEO change by 13%.

³¹ Weisbach, M., 1988, *Outside Directors and CEO Turnover*, Journal of Financial Economics, v20, 431-460.

³² Goyal, V.K., and C.W. Park, 2001, *Board Leadership Structure and CEO Turnover*, Journal of Corporate Finance, v8, 49--66.

³³ Dennis, D.J., D.K. Dennis and A. Sarin, 1997, *Ownership Structure and Top Executive Turnover*, Journal of Financial Economics, v45, 193-221.

³⁴ Hillier, D., S. Linn and P. McColgan, 2003, *Equity Issuance, Corporate Governance Reform and CEO Turnover in the UK*, Working Paper, SSRN. They find that CEO are more likely to be forced out just before new equity issues or placings.

³⁵ DeFondt, M.L. and C.W. Park, 1999, *The effect of competition on CEO Turnover*, Journal of Accounting and Economics, v27, 35-56.

target of a hostile acquisition decreased with firm size, insider holdings and the ratio of a market value to replacement cost (Tobin's Q) for the industry in which the firm operated:³⁶

$$\text{Probability}_{\text{hostile takeover}} = 0.563 - 0.184 \ln \text{MVAL} - 0.737 \text{FAMILY} - 1.33 \text{INSHOLD} - 0.872 \text{Industry Q} - 0.701 \text{DQ}$$

MVAL = Market Capitalization (in millions)

FAMILY = 1 if founding family part of management; 0 otherwise;

INSHOLD = Proportion of equity owned by incumbent managers

IndustryQ = Industry average of Market value of firm/ Replacement Cost

DQ = Differential between target firm and industry average on Tobin's Q

Kaplan and Minton (2006) looked at CEO turnover between 1992 and 2005 to estimate the probability of external CEO change as a function of a firm's characteristics:³⁷

$$\text{Probability}_{\text{CEO change}} = 0.1179 - 0.0003 \text{S\&P 500} - 0.0033 \text{IndvsS\&P} - 0.0645 \text{StkRet} - 0.1535 \text{ChgROC} + 0.1488 \text{CEOAge} \quad R^2 = 7.45\%$$

S&P 500 = Return on S&P 500 during the year

IndvsS&P = Industry Average Return – S&P 500 Return

StkRet = Return on Stock – Industry average return

ChROC = Change in Return on Capital

CEOAge = 1 if CEO is older than 60; 0 otherwise

In effect, the probability of a CEO change increases with poor stock price performance (at the overall market level, at the industry level and at the company level), decreases with improvements with return on capital and is higher at firms with older CEOs. Thus, the probability of a management change next year at a firm that has a 65-year old CEO, has reported an increase in return on capital of 3% over the last year and has delivered a stock return of 10% (while the sector delivered a return of 15% and the S&P 500 had a return of 5%) can be estimated as follows:

³⁶ R. Morck, A. Schleifer and R. Vishny, 1988, *Characteristics of targets of hostile and friendly takeovers*. In: A. Auerbach, Editor, *Corporate takeovers: Causes and consequences*, The University of Chicago Press, Chicago.

³⁷ Kaplan, S.N., and B.A. Minton, 2006, *How has CEO Turnover Changed? Increasingly Performance Sensitive Boards and Increasingly Uneasy CEOs*, NBER Working Paper.

Probability_{CEO change} = 0.1179 – 0.0003 (.05)– 0.0033 (.15-.05) -0.0645 (.10-.15) -0.1535 (.03) +0.1488 (1) = 0.2650 or 26.50%

There are two notes of caution that should be added in the use of these statistical equations. The first is that the definition of management change used in these studies may not coincide with our definition, which requires that the way the firm is run is changed. Thus, those analyses that look at only hostile acquisitions may be looking at too narrow a sub-set of firms, since the way a firm is managed can be changed without an acquisition. At the other extreme, a change in CEO may not necessarily imply changes in the way the company is run. The other is the low predictive power of these regressions, captured by the low R-squared, results in estimates of the probability of change that are inherently noisy.

Differences in control rights

Consider the conventional publicly traded firm, which raises equity by issuing common shares to the public. Each share is endowed with a voting right and is thus entitled to an equal claim of the overall value of the firm. There are, however, cases where some classes of equity investors have superior control rights over others. In its most common form, this takes the form of higher voting rights for some classes of common shares than others, but there are other possibilities as well. For instance, a class of equity shareholders may be given special rights only in the event of a hostile takeover or public offering, giving them either the power to effectively veto the event or modify it to their benefit.

I. Voting versus Non-voting shares

The most common manifestation of control differences across equity claims arises when voting rights differ across classes of equity. In its most extreme form, one class of shares has no voting rights and the other class is endowed with all the voting rights. In variants, voting rights are not completely removed from one class of shares but are lower than on the other class.

Prevalence of voting right differences

The prevalence of differences in voting rights varies widely across markets, and within some markets, widely across time. Common shares with different voting rights are relatively scarce in the United States, though many firms issue non-voting preferred shares with fixed dividends.³⁸ While the Securities Exchange Commission does not disallow different voting class shares, the New York Stock Exchange did impose restrictions on these shares from 1926 to 1986. In recent years, there have been a few high profile companies like Viacom and Google that have issued classes of equity with different voting rights, with founders/managers holding on to the bulk of the voting right shares, but they still remain the exception rather than the rule. Gompers and Ishii (2008) conclude that about 6% of all firms, representing 8% of overall market cap, in the United States have dual class shares and that the voting rights in the superior voting shares are typically ten times the voting rights in inferior voting shares. Furthermore, they find that dual class share structures are concentrated in a few sectors, especially the media and publishing business.³⁹ For instance, the New York Times, Washington Post and Dow Jones all had dual class share structures entering 2008.

In Latin America, most companies have a two tiered equity structure with voting shares (common) and non-voting shares with priority in dividend claims (preferred). Unlike preferred stock in the US, the latter have variable dividends and are more akin to non-voting common shares than preferred shares. In Asia, the practice varies widely across countries. In India and Japan, company law prohibits the issuance of shares with different voting classes, but the practice is common in both China and Korea. In China, many companies have three classes of shares – common shares with traditional voting rights, state shares with enhanced voting rights and restricted institutional shares without voting rights. In much of continental Europe, companies have shares with different voting rights, but the practice is unusual in the United Kingdom. In Canada, about 15 to 20% of

³⁸ Dual class shares, with different voting rights, were not allowed in the United States between 1924 and 1986. In fact, the only companies with dual class shares were companies like Ford that were given an exemption in the early years from the application of the law. For an a very early study of differential voting rights in the United States, see Stevens, W.H.S., 1926, *Stockholders' Voting Rights and the Centralization of Voting Control*, *The Quarterly Journal of Economics*, v40, 353-392.

³⁹ Gompers, P.A. and J. Ishii, 2008, *Extreme Governance: An Analysis of Dual-Class Firms in the United States*, Working Paper, SSRN.

companies have dual class shares and the Toronto stock exchange allows five classes of shares to be listed on the exchange – non-voting shares (NV), multiple-voting shares (V), subordinate-voting shares (SV), limited-voting shares (LV) and restricted-voting shares (RV).

In recent years, activist institutional investors have challenged the practice, arguing that it strips stockholders of the power to push through changes in management, when companies are poorly run. More generally, many countries have strengthened the protection of stockholders with limited or no voting rights (minority stockholders) in an attempt to attract more foreign institutional investment into their markets.

Empirical Evidence on voting shares

The empirical work on the premium that investors are willing to pay for voting rights can be categorized into three groups. The first group of studies looks at the premium that voting shares trade at, relative to non-voting shares, and look at the average premium and differences in the cross section. The second group of studies examines the impact of corporate governance events – changes in rules relating to minority stockholders and hostile acquisitions– on the voting share premium. The third group of studies evaluates the effect of the creation or reunifications of dual class share structures and the impact that they have on overall equity value.

i. Voting versus Non-voting Shares – The Cross Sectional Evidence

As we noted in the last section, shares with different voting rights are unusual in the United States, especially among larger market capitalization companies. Notwithstanding this fact, the earliest studies of voting share premiums were done with companies with different voting share classes in the United States. Lease, McConnell and Mikkelsen (1983) found that voting shares in that market trade, on average, at a relatively small premium of 5-10% over non-voting shares.⁴⁰ They also found extended periods where the voting share premium disappeared or voting shares traded at a discount to non-voting shares, a surprising finding that can be explained partially by the relative illiquidity of voting shares (since only a small percentage is available for public trading).

⁴⁰ Lease, R.C., J.J. McConnell and W.H. Mikkelsen 1983, *The market value of control in publicly-traded corporations*, Journal of Financial Economics, v11, 439-471.

Reilly (2003) updated this study to look at 28 companies with voting and non-voting shares in 1994 and 1999 and concluded that the median voting share premium increased from 2% in 1994 to 2.8% in 1999.⁴¹ The small premium commanded by voting shares was confirmed by Zingales in a study in 1995.⁴² Studies in recent years have expanded the analysis of voting share premiums to other markets, where differential voting rights are more common. Premiums of a magnitude similar to those found in the United States (5-10%) were found in the United Kingdom and Canada. Much larger premiums are reported in Latin America (50-100%), Israel (75%) and Italy (80%). In a comparative study of voting premiums across 661 companies in 18 countries, Nenova (2003) finds that the median value of control block votes varies widely across the countries, ranging from less than 1% in the US to 25% or greater in France, Italy, Korea and Australia. She concludes that the legal environment is the key factor in explaining differences across countries and that the voting premium is smaller in countries with better legal protection for minority and non-voting stockholders and larger for countries without such protection.⁴³

Some of the studies mentioned above also hypothesize (and test) for why voting premiums may vary within the same market. Zingales, in a study of Italian stocks, concludes that some of the differences in voting premiums across Italian shares can be explained by the proportion of shares that are voting shares (lower proportions translate into larger premiums per share) and the dividend privileges of non-voting shares (the greater the privileges, the smaller the premium).⁴⁴ However, he also concludes a large proportion of the differences in voting share premiums cannot be explained by these variables, and given the low likelihood of hostile takeovers, he attributes the differences to private benefits that accrue to voting shareholders. Caprio and Croci (2007) also examine the voting share premiums in Italian stocks between 1974 and 2003 and note

⁴¹ Reilly, R.F., 2005, *Quantifying the Valuation Discount for Lack of Voting Rights and Premium*, American Bankruptcy Institute Journal.

⁴² Zingales, L., 1995, *What determines the value of corporate votes?*, Quarterly Journal of Economics, v4, 1047-1073.

⁴³ Nenova, T., 2003, *The value of corporate voting rights and control: A cross-country analysis*, Journal of Financial Economics, v68, 325-351

⁴⁴ Zingales, L. (1994), *The value of the voting right: A study of the Milan Stock Exchange experience*, The Review of Financial Studies, 1, 125-148.

that the premium is greater in family controlled firms, a finding that they attribute to the fact that families value control more highly and are more prone to take advantage of non-voting shareholders.⁴⁵ Chung and Kim (1995) examine voting and non-voting shares in Korea and conclude that while the overall value of voting rights is about 10% of value, the premium on voting shares is a decreasing function of the proportion of shares that are voting shares and the overall market value of the firm.⁴⁶

Hauser and Lauterback (2004) examined 84 dual-class reunifications in the Israel, where superior vote shareholders gave up their superior voting status for compensation, with the intent of estimating how much the voting rights were worth. They estimated the price of 1% of the voting power to be about 0.2% and noted that the price of a vote increased with percentage vote lost by the majority stockholders, declined as institutional holdings increased and was larger for family-controlled firms.⁴⁷

A factor to consider when looking at voting and non-voting shares is that foreign investors in some markets are restricted from investing in voting shares. This was the case, for instance, in Norway, prior to 1995, where foreign investors were restricted from buying voting shares. In this period, non-voting shares traded at a premium to voting shares, reflecting the increased demand from foreign investors. When the law was changed in 1995 to remove the restrictions on foreign investors, Odegaard (2000) noted that voting shares began trading at a premium over non-voting shares.⁴⁸

ii. Effects of Changing Corporate Governance

If the price difference between voting and non-voting shares is a function of the expected value of control, any event that changes the expected value of control should also have an impact on the pricing of voting claims. In an attempt to isolate the effect of control on voting share premiums, Linciano examined the effects of changes in takeover law and corporate governance on Italian voting and non-voting shares. A “mandatory

⁴⁵ Caprio, L. and E. Croci, 2007, *The Determinants of Voting Premium in Italy: The Evidence from 1974 to 2003*, Working Paper, University of Lugano.

⁴⁶ Chung, K.H. and J. Kim, 1995, *Corporate ownership and the value of a vote in an emerging market*, *Journal of Corporate Finance*, v5, .35–54

⁴⁷ Hauser, S. and B. Lauterbach, 2004, *The Value of Voting Rights to Majority Stockholders: Evidence from Dual-Class Stock Unifications*, *Review of Financial Studies*, v17, 1167-1184.

⁴⁸ Odegaard, B. A., 2000, *Price Differences between Equity Classes. Corporate Control, Foreign Ownership or Liquidity? Evidence from Norway*, Working paper, Norwegian School of Management.

bid” rule, introduced in 1992 in Italy, allowed small voting shareholders to receive the same price in an acquisition as large voting shareholders but did not extend to non-voting shareholders. Not surprisingly, the premium on voting shares increased marginally (about 2%) after this rule. A subsequent corporate governance reform law in 1997, which increased the power of non-voting shareholders, decreased the premium by about 7%. Nenova (2001) reports similar results from Brazil, where decreased protection for minority stockholders in a 1997 law doubled the premium on voting shares and a subsequent reform of the law in 1999 reversed both effects.⁴⁹

In some cases, the event may not explicitly change any rules or laws but may alter perceptions about how much voting rights matter. The most obvious example of such an event is a firm with dual class shares that is targeted for an acquisition. Not surprisingly, the premiums for superior voting-rights shares increase, when an acquisition is imminent.⁵⁰ The effects on voting share premiums, though, may extend beyond the targeted stock to other companies. For instance, InBev’s acquisition of Ambev, the Brazilian beverage company, in 2002, laid bare the power that voting shareholders in Brazilian companies had, relative to non-voting shareholders, and increased the premium on voting shares across all Brazilian companies.

iii. Dual Class Share Reunification and Creation

The expected value of control is the product of the probability of control changing the value of changing management at a firm:

Expected value of control = Probability of management changing * (Optimal value of firm – Status Quo value)

When a publicly traded company with a single class of shares creates classes of shares with different voting rights, it is affecting the expected value of control. Since the class of shares with superior voting rights is usually held by insiders and incumbent management,

⁴⁹ Nenova, T., 2001, *Control values and Changes in Corporate Law in Brazil*, Working paper, SSRN.

⁵⁰ Bergstrom, C. and K. Rydqvist, 1992, *Differentiated Bids for Voting and Restricted Voting Shares in Public Tender Offers*, Journal of Banking and Finance, 16,97-114; Rydqvist, K., 1996, *Takeover bids and the relative prices of shares that differ in their voting rights*, Journal of Banking and Finance, v20, 1407-1425. The first paper report a premium of 27% for shares with superior voting rights, on average, across 40 Swedish firms with dual class shares. The second paper compares the average voting share premium for dual class shares prior to acquisitions to the acquisition premium for 25 Swedish firms and chronicles an increase in the average premium from 4% to 29%.

the probability of management changing decreases as a result of the action.⁵¹ On the other hand, there are some who argue that removing the need to keep “short term” investors happy allows the firm to make long-term decisions that increase the “status quo” value. The net effect on overall equity value, of creating dual class shares, will be negative if the probability of management changing decreases significantly but the status quo value does not change much. It will be positive if the increase in status quo value (because of better management) dominates the effect on the probability of management changing.

There are a number of studies that have examined the consequences of both the creation and the elimination of dual class share structures, which cast light on whether the net effect of dual class shares is positive or negative. In the most comprehensive of these studies, Studies that have looked at publicly traded companies that switch from a single class to a dual class structure arrive at mixed conclusions about the wealth effect, with Jarrell and Poulsen (1988) finding a decline in aggregate equity value, Partch (1987) concluding that the action was close to value neutral and Millon-Cornett and Vetsuypens (1989) documenting a mild positive effect.⁵² Studies that have focused on dual class firms that choose to unify dual class shares into a single class seem to be more in agreement about the consequences for value. Dittman and Ulbricht (2003) study 31 German companies with dual class shares that changed to a single class and concluded that the reunification increased aggregate market value of equity at these firms by roughly 10%.⁵³ Pajuste (2005) looked 493 firms from seven European countries that went from dual class to single class shares and concluded that the aggregated market value of these firms increased after the unification, bringing their market to book ratios up to par with the industry averages; revenues and measures of operating efficiency also posted improvements after the unification. Gompers and Ishii, referenced earlier in this paper, come to the same conclusion, albeit through a different method. Using the ratio of market

⁵¹ Gompers and Ishii (see earlier reference) in an examination of dual class shares in the United States, note that insiders control about 60% of the voting rights at these companies.

⁵² Jarrell, Gregg A., and Annette B. Poulsen, 1988, *Dual-class recapitalizations as anti-takeover mechanisms: the recent evidence*, Journal of Financial Economics 20, 129-152; Partch, Megan, 1987, *The creation of a class of limited voting common stock and shareholder wealth*, Journal of Financial Economics 18, 313-39; Millon-Cornett, Marcia, and Michael Vetsuypens, 1989, *Voting rights and shareholder wealth: the issuance of limited voting common stock*, Managerial and Decision Economics 10 (3), 175-88.

⁵³ Dittmann, Ingolf, and Niels Ulbricht, 2003, *When do firms abolish dual-class stocks?* Unpublished Working Paper.

value to replacement cost (Tobin's Q) as the measure of value, they find that firms that separate voting rights from cash flow rights (dual class firms) trade at lower multiples of replacement costs than the rest of the market.⁵⁴

Looking across the studies, we would conclude the following. When firms first create dual class shares, either in initial public offerings or in subsequent actions, the potential benefits from improved management may be perceived to be as large or larger than the costs created by weaker management oversight. Over time, though, the costs imposed by a dual class structure seem to outweigh the potential benefits. The management that was viewed as superior at the time of the dual class shares issue may very well find their reputations damaged after a few years of sub-par performance.

Valuing voting right differentials

If different classes of equity have different voting rights, how do we value the difference? In general, there are two ways we can go about this estimation. In the first, we use the empirical findings on the voting share premium in markets to arrive at a reasonable value for voting rights. In the second, we argue that the voting right premium is an extension of the expected value of control and that estimating that value should allow us to quantify the premium.

a. Market estimates

When we presented the evidence on voting share premiums across markets, we noted that the premium varies widely across markets, with relatively small premiums in some markets like the United States and larger premiums in other markets like Italy. When faced with the task of apportioning equity value across classes of equity with different voting rights, we could make the assumption that the median premium across all voting shares is the best estimate of the premium for any specific company. Thus, we would assume a premium of about 2-5% for voting shares in a US company and allocate equity value accordingly.

There are, however, three problems with this practice. The first is that the studies that report a median premium across voting class shares also note that there are wide

⁵⁴ They find that firm value increases as insider cash flow rights increase but decreases as insider voting rights increase.

differences across companies and across time, in the risk premium. The second is that this practice assumes that one class or shares is voting and the other is non-voting, and does not allow for variations in voting rights differences across companies. Thus, it is not clear how we can use the “median premium” if one class of shares has three times the voting rights of the other, rather than the ten times the voting rights, more typical of other companies in the studies. The third is that in most countries, differences in voting rights also go with differences in dividends and liquidity. In fact, non-voting shares receive higher dividends and are more liquid than voting shares, and these preferences may push the premium down on voting shares; in some cases, they may lead to the latter trading at a discount.

It is true that as the data on voting and non-voting (or inferior voting) shares gets richer, we can deal, at least partially, with each of these issues. The Zingales study of Italian stocks, for instance, related differences in voting share premiums to the proportion of shares that were voting shares and dividend privileges on the non-voting shares. Other studies of the voting share premium try to adjust for differences in liquidity and other firm-specific measures. We could use these expanded analyses to estimate voting right premiums, on a firm-specific basis. Note, though, that many of these studies also find that large portions of the differences in voting share premiums across companies cannot be explained by the variables used, making any estimate that emerges a fairly noisy one.

b. Discounted Cash flow estimates

To link the premium on voting shares to the expected value of control, let us begin with an extreme and very simplistic example. Assume that you have a company with n_v voting and n_{nv} non-voting shares and that the voting shareholders have complete and total control of the business. Thus they are free to ignore the views of non-voting shares in the event of a hostile takeover and negotiate the best deal that they can for themselves with the acquirer.⁵⁵ Assume further that this firm has a status quo value of V_b and an optimal value of V_a and that the likelihood of management changing in this firm is π . Since the non-voting shares have absolutely no say in whether the management can be changed, the value per non-voting share will be based purely upon the status quo value:

Value per non-voting share = $V_b / (n_v + n_{nv})$

The voting shares will trade at a premium that reflects the expected value of control:

Value per voting share = $V_b / (n_v + n_{nv}) + (V_a - V_b) \pi / n_v$

The premium on voting shares should therefore be a function of the probability that there will be a change in management at that firm (π) and the value of changing management ($V_a - V_b$).

To the extent that non-voting shareholders are protected or can extract some of the expected value of control, the difference between voting and non-voting shares will be lower. It is possible, for instance, for non-voting shares to gain some of the value of control if it is accomplished by changing managers, rather than by a hostile takeover. In that case, the value of the firm will increase and all shareholders will benefit.

If the primary reason for the voting share premium is the expected value of control, there are several conclusions that follow:

a. *The difference between voting and non-voting shares should go to zero if there is no chance of changing management/control.* This will clearly be a function of the concentration of ownership of the voting shares. If there are relatively few voting shares, held entirely by insiders, the probability of management change may very well be close to zero and voting shares should trade at the same price as non-voting shares. If, on the other hand, a significant percentage of voting shares is held by the public, the probability of management change should be higher and the voting shares should reflect this premium.

b. *Other things remaining equal, voting shares should trade at a larger premium on non-voting shares at badly managed firms than well-managed firms.* Since the expected value of control is close to zero in well-managed firms, voting shares and non-voting shares should trade at roughly the same price in these firms. In a badly managed firm, the expected value of control is likely to be higher, as should the voting share premium.

c. *Other things remaining equal, the smaller the number of voting shares relative to non-voting shares, the higher the premium on voting shares should be.* Since the expected value of control is divided by the number of voting shares to get the premium, the smaller

⁵⁵ In reality, even non-voting shareholders are provided at least partial protection in the event of a takeover

that number, the greater the value attached to each share. This has to be weighed off against the reality that when the number of voting shares is small, it is more likely to be held entirely by incumbent managers and insiders, thus reducing the likelihood of management change.

d. Other things remaining equal, the greater the percentage of voting shares that are available for trading by the general public (float), the higher the premium on voting shares should be. When voting shares are entirely or predominantly held by managers and insiders, the probability of control changing is small and so is the expected value of control. In contrast, if voting shares are widely held, the probability of control changing should be higher, translating into a higher voting share premium.

e. Any event that illustrates the power of voting shares relative to non-voting shares is likely to affect the premium at which all voting shares trade. The expected value of control is a function of perceptions that management at these firms can be changed. In a market where incumbent managers are entrenched, voting shares may not trade at a premium because investors assess no value to control. A hostile acquisition in this market or a regulatory change providing protection to non-voting shareholders can increase the expected value of control for all companies and, with it, the voting share premium.

In summary, then, we would expect the voting share premium to be highest in badly managed firms where voting shares are dispersed among the public. We would expect it to be smallest in well-managed firms and in firms where the voting shares are concentrated in the hands of insiders and management. To the extent that the presence of voting and non-voting shares reduces the probability of management changing, we would expect the value of equity in the aggregate to be lower for a firm with two classes of shares than for a firm with one class of shares.

Illustration 9A: Valuing voting and non-voting shares – Nova Chemicals

To value voting and non-voting shares, we will return to Nova Chemicals, the company for which we assessed status quo and optimal values in illustration 8.

Status Quo Value = \$78 million

Optimal Value = \$156 million

and will share in some of the benefits.

Voting versus Non-voting shares: Assume that Nova Chemicals has 10 million voting shares and 10 million non-voting shares and that the probability of changing management, with the existing share structure, is only 10%. To estimate the value per non-voting share, we will consider only the status quo value:

$$\begin{aligned}\text{Value per share} &= \text{Status Quo value} / (\# \text{ of Voting shares} + \# \text{ of Non-voting shares}) \\ &= 78 / (10+10) = \$3.90\end{aligned}$$

The value per voting share can now be estimated, by first computing the expected value of control and then considering the number of voting shares:

$$\text{Expected value of control} = \text{Probability of changing management (Optimal – Status Quo Value)} = 0.10 * (156-78) = \$7.80 \text{ million}$$

$$\begin{aligned}\text{Voting share premium} &= \text{Expected Value of control} / \# \text{ of Voting shares} \\ &= \$7.80 \text{ million} / 10 \text{ million} = \$0.78\end{aligned}$$

$$\text{Value per voting share} = \$3.90 + \$0.78 = \$4.68$$

The value per voting share will be a function of the number of voting shares, relative to non-voting shares. For instance, if there were 5 million voting shares and 15 million non-voting shares in Nova, the value per voting share would be \$5.46.

$$\text{Voting share premium} = \$7.80/5 = \$1.56$$

$$\text{Value per voting share} = \$3.90 + \$1.56 = \$5.46$$

Shares with different voting rights: It is simple to modify this approach to capture variants on voting right differentials. For instance, assume that Nova had two classes of shares and that class A shares had ten times the voting rights of class B shares; there are ten million of each class. Using the same probability of management change as in the previous example, we first estimate the value per voting right:

$$\text{Number of voting rights} = \text{Voting rights per class A share} * \text{Class A shares} + \text{Voting rights per class B share} * \text{Class B shares} = 10 * 10 + 1 * 10 = 110 \text{ million voting rights}$$

$$\text{Expected value of control} = \$7.80$$

$$\text{Value per voting right} = \$7.80/110 = \$0.0709$$

$$\begin{aligned}\text{Value per class A share} &= \text{Status quo value per share} + \text{Value of voting rights} \\ &= \$78/20 + 10 * \$0.0709 = \$4.61\end{aligned}$$

$$\begin{aligned}\text{Value per class B share} &= \text{Status quo value per share} + \text{Value of voting rights} \\ &= \$78/20 + 1 * \$0.0709 = \$3.97\end{aligned}$$

In both cases, the voting share premium will increase as a function of the probability of management changing and the effect on value of a management change. At one extreme, where the incumbent managers use voting shares to completely entrench themselves, the probability of management change will go to zero and with it the voting share premium. At the other extreme, in an acquisition, the probability of management change may approach 100% and, in the absence of protection for minority stockholders, the premium on voting shares will approach 100%, with voting shares trading at \$7.80 per share, while non-voting shares stay at \$3.90 a share.

Illustration 9B: Valuing voting and non-voting shares - Embraer

For a real world example, we will consider Embraer, the Brazilian aerospace company. As is typical of most Brazilian companies, the company has common (voting) shares and preferred (non-voting shares).

We valued the company twice, first under the status quo and next under optimal management. With existing management in place, we estimated a value of 12.5 billion \$R for the equity; this was based upon the assumption that the company would continue to maintain its conservative (low debt) financing policy and high returns on investments (albeit with a low reinvestment rate) at least for the near term. We then revalued the firm at 14.7 billion \$R, assuming that the firm would be more aggressive both in its use of debt and in its reinvestment policy.

There are 242.5 million voting shares and 476.7 non-voting shares in the company and the probability of management change is relatively low, partly because the bulk of the voting shares are held by insiders⁵⁶ and partly because the Brazilian government has significant influence in the company.⁵⁷ Assuming a probability of 20% that management will change, we estimated the value per non-voting and voting share:

$$\begin{aligned} \text{Value per non-voting share} &= \text{Status Quo Value} / (\# \text{ voting shares} + \# \text{ non-voting shares}) \\ &= 12,500 / (242.5 + 476.7) = 17.38 \text{ \$R/ share} \end{aligned}$$

⁵⁶ Of the 242.5 million voting shares, 80% is equally held by four entities – Cia Bozano, Previ, Sistel and the European Group. Effectively, they control the company.

⁵⁷ The Brazilian government owns only 0.8% of the voting shares but a significant portion of Embraer's customer financing is provided by the Brazilian development bank (BNDES), which also owns 9.6% of the non-voting shares.

Value per voting share = Status Quo value/share + Probability of management change * (Optimal value – Status Quo Value) = 17.38 + 0.2* (14,700-12,500)/242.5 = 19.19 \$R/share

With our assumptions, the voting shares should trade at a premium of 10.4% over the non-voting shares. If the probability of management change increases, we would expect the premium to increase.

It should be noted that the non-voting shares in Embraer do have some advantages that may offset some or all of the control premium. Non-voting shares have a prior claim to dividends over voting shares and they also pay higher dividends. In addition, a higher percentage of the non-voting shares are available to the public and traded, thus leading to higher liquidity; only 19% of the voting shares are traded whereas 90% of the non-voting shares are traded either on the Bovespa (34%) or on the New York Stock Exchange (56%).

II. Golden Shares

When government-owned telecommunication, power and infrastructure companies in Europe and Latin America were privatized in the 1980s and 1990s, the governments often sought to retain the power to make big decisions within these companies, in the public interest. In some cases, this was accomplished by the retention of a significant portion of the common equity but that reduced the proceeds from privatization. In other cases, governments reserved a special class of shares for themselves called golden shares that allowed them to have veto power over significant decisions at the firm, including preventing takeovers deemed not in the public interest, the acquisition or disposal of assets and the composition of the board of directors.

While the golden shares themselves are not traded, their presence will affect other shareholders. To the extent that golden shares make acquisitions less likely, for example, they should lower the expected value of control at a firm and thus the price at which the shares of the firm trade at. In other words, equity in a firm where the government has a golden share should trade at a discount to equity in an otherwise similar firm without a golden share. But how would we measure this discount? If we are valuing equity using a discounted cash flow model, it may already be reflected in the value. The presence of a

golden share and the protection it offers incumbent management may manifest itself as lower returns on existing assets or a sub-optimal capital structure or dividend policy. To the extent that these choices affect our forecasts for the future – lower returns on capital translate into lower expected growth rates and sub-optimal debt policy creates a higher cost of capital – the discounted cash flow value for the firm will be depressed. Applying a further discount to this value, using the golden share as the rationale, would be double counting. If valuing equity with a relative valuation model, there may be a need to explicitly consider the presence of the golden share. In the best case scenario, some of the firms that you are comparing your firm to will have golden shares and some not. If so, we can try to estimate how much of a discount (if any) the market is applying to the golden share companies.

In recent years, the use of golden shares to stock acquisitions has been challenged at least in the European Union. The European Commission took France to the European Court of Justice for its use of a golden share in preventing foreign entities from acquiring Elf Aquitaine, arguing that it breached common market laws allowing for the free movement of capital across markets.

III. Event-contingent Control

With golden shares, we noted that the holders of the shares (usually the government) often get special powers if a specified event (hostile acquisition, major investment) is imminent. Expanding on this notion, there are classes of equity in some firms that have similar event-contingent powers. These powers can be classified loosely into two groups – veto powers, where the equity class has the right to prevent the event from occurring, if it feels that its interests are not being served, and protective powers, where the equity class obtains special protection against its value or ownership claim being diluted.

Veto power, i.e. the power to say no to an event occurring, does protect the rights of the equity claim endowed with the power, but it does so at the expense of overall firm value. By reducing the probability of a specific event (acquisition, initial public offering) that may increase overall firm value at the expense of a specific claim on equity, it will reduce the expected value of the business and thus the value of all claims on the business.

For instance, assume that the status quo value of a firm is \$ 10 million, the value to an acquirer is \$ 15 million and that the probability of an acquisition is 40%. If the firm has only class of shares outstanding and there are ten million shares, the value per share can be estimated as follows:

$$\begin{aligned} \text{Value per share} &= \frac{\text{Status Quo Value} (1 - \text{Prob}_{\text{Acq}}) + \text{Acquisition Value} (\text{Prob}_{\text{Acq}})}{\text{Number of shares}} \\ &= \frac{10 (1 - .4) + 15 (.4)}{10} = \$1.20/\text{share} \end{aligned}$$

Assume now that there are two classes of equity, 5 million class A shares with no special rights and 5 million class B shares with veto rights over acquisitions. As a consequence, the probability of an acquisition drops to 20%. The estimated value of equity per share will reflect this change:

$$\text{Value per generic share} = \frac{10 (1 - .2) + 15 (.2)}{10} = \$1.10/\text{share}$$

Note that the class B shareholders are costing the firm a million dollars in value. It is possible that they could negotiate to give up their veto rights for approximately that amount. Consequently, the value per class B share can be computed as follows:

$$\begin{aligned} \text{Value per Class B share} &= \text{Value per share} + \frac{\text{Value Loss}}{\# \text{ Class B shares}} = \$1.10 + \frac{(\$12 - \$11)}{5} \\ &= \$1.30 \text{ per share} \end{aligned}$$

The veto power that the class B shareholders have will allow them to have a higher value than the class A shareholders, but they can monetize this value only if they are willing to give up their veto power.

Protective rights can be more complicated to value, because the right extends beyond the power to say no. In effect, the equity claimholders who have the right receive cash flows to compensate for the loss of value from the event. It is more akin to an option, providing protection against negative consequences, and can be valued as such.

Financing Claims

As firms grow, it is common for them to seek out fresh financing from both debt and equity to fund their growth. As the firm raises new debt and equity, existing equity and debt investors may find themselves losing control of the firm and perhaps even value

to the new investors in the firm. It is for this reason that some equity claimholders demand and receive special rights that are triggered when a firm seeks new financing.

Prevalence of Financing Claims

Young firms are more likely to seek out new financing, often in large amounts and at unpredictable points to fund growth. Consequently, rights triggered by new financing are most common early in a firm's life cycle, especially when the equity in the firm is held by venture capitalists. They are also more common in private firms, where new equity raised by a firm can differ from old equity, than in publicly traded firms, where equity is more standardized. In general, there are two classes of rights associated with financing:

- a. Participation rights: Participation rights allow existing stockholders (preferred or common) to purchase a portion of any new security offering by the firm, in proportion to their existing claims on the firm. Thus, if preferred shareholders represent 20% of the overall capital of the firm, they are entitled to buy 20% of any new offering at the same terms as new investors.
- b. First refusal and Co-sale rights: These rights allow preferred shareholders to limit the sale of common stock by the founders/ key managers of the firm. First refusal rights give preferred shareholders the right to buy common shares that founders try to sell, at the price offered by a third party. Co-sale rights allow preferred shareholders to sell their shares at the same time that common stockholders do, in proportion to their ownership. Thus, if common stockholders try to sell 10,000 of their shares to a third party, and preferred stock amounts to 20% of the overall capital, preferred stockholders can demand 2,000 of their preferred shares (20% of the shares being sold) be substituted for the common stock.

In general, financing claims can be categorized into claims that allow its holders to partake in any new financing that the firm may do in the future or claims that are designed to stop the new financing, if they feel that their interests will be hurt. Thus, they represent a special case of the event-contingent control claims that we discussed in the last section. Consequently, we can consider the first set of claims to be protective claims and the second set to be veto power claims.

Valuing Financing Claims

A claim that allows existing investors to partake in additional financing in later time periods at the same price as new investors are paying is a protective claim. In effect, you are protecting yourself against deals where new investors are given disproportionate value or rights that may reduce your claim on the firm. A claim that allows existing investors to partake in additional financing at a preferential price (relative to new investors) is an option. Thus, a contract that allows existing equity investors to invest in additional shares of the firm, if the firm issue shares to new investors, at a fixed price (say \$10) derives its value from the fact that existing equity investors will exercise this option only if new investors are paying more than \$ 10 to buy shares in the company.

A claim that allows existing investors veto power over new financing, i.e., the power to stop new financing, if they feel that their interests will be hurt, affects the value of the firm. The value of this protective claim is likely to be higher in small firms that repeatedly tap investors for new equity and offer wildly divergent terms on each deal. As with other veto power claims, the effect here is to reduce the overall value of the firm, by making new financing (that the firm may need for expansion) less likely, and consequently reduces the value of all equity claims on the firm.

Liquidity Differences

One reason firms go public is to increase the liquidity of their holdings. In effect, investors can cash out their holding much more easily in a publicly traded company than in a private business. In some publicly traded companies, with multiple classes of shares, some classes of equity may be more liquid than others. In its most extreme form, one class of equity may be traded and the other not at all, leading us to question how we would incorporate these liquidity differences into the values that we attach to each class of equity.

Prevalence of liquidity differences across equity classes

When publicly traded firms have different classes of equity, there will almost certainly be differences in liquidity across the classes. Some of the differences will arise from the fact that there are more shares outstanding of one class of shares than another.

More importantly, though, the differences arise because one class of shares is held by investors who do not trade or trade very infrequently. This is especially the case with voting and non-voting shares, where the former are held by insiders, who control the firm, and are reluctant to let the shares out of their hands. Consequently, non-voting shares tend to be more liquid than voting shares, when both exist. Gompers and Ishii (2008), in a comprehensive survey of dual class shares in the United States, find that when such structures exist, the voting shares do not trade at all in 85% of the cases. In these companies, the only shares that trade are the non-voting shares. In the remaining 15% of the sample with dual class shares, both classes of shares trade, but the non-voting shares are always more liquid and have higher float.

With private businesses, where none of the classes of equity are traded, this may seem like a non-issue since all classes are illiquid. However, even with these businesses, it may be easier to sell one class of equity than another. Ironically, the equity with the voting claims may be easier to sell than the equity without those claims, thus making it the more liquid claim.

Valuing/ Costing illiquidity: Theory

The notion that investors will pay less for illiquid assets than for otherwise similar liquid assets is neither new nor revolutionary. Over the last two decades researchers have examined the effect of illiquidity on price using three different approaches. In the first, the value of an asset is reduced by the present value of expected future transactions costs, thus creating a discount on value. In the second, the required rate of return on an asset is adjusted to reflect its illiquidity, with higher required rates of return (and lower values) for less liquid assets. In the third, the loss of liquidity is valued as an option, where the holder of the illiquid asset is assumed to lose the option to sell the asset when it has a high price. All three arrive at the conclusion that an illiquid asset should trade at a lower price than an otherwise similar liquid asset.

a. An Illiquidity Discount on Value

Assume that you are an investor trying to determine how much you should pay for an asset. In making this determination, you have to consider the cashflows that the asset

will generate for you and how risky these cashflows are to arrive at an estimate of intrinsic value. You will also have to consider how much it will cost you to sell this asset when you decide to divest it in the future. In fact, if the investor buying it from you builds in a similar estimate of transactions cost she will face when she sells it, the value of the asset today should reflect the expected value of all future transactions cost to all future holders of the asset. This is the argument that Amihud and Mendelson used in 1986, when they suggested that the price of an asset would embed the present value of the costs associated with expected transactions costs in the future.⁵⁸ In their model, the bid-ask spread is used as the measure of transactions costs and small spreads can translate into big illiquidity discounts on value. The magnitude of the discount will be a function of investor holding periods and turnover ratios, with shorter holding periods and higher turnover associated with bigger discounts. Vayanos (1998) argues that the effect of changes in transactions costs on asset prices is much smaller than estimated by Amihud and Mendelson because investors adjust holding periods to reflect transactions costs. In fact, he argues that the price of a stock can actually increase as its transactions costs increase, especially for more frequently traded stocks; the increase in holding periods can offset the transactions costs increase.⁵⁹

Jarrow and Subramanian (2001) present an alternate model for estimating the illiquidity discount on value.⁶⁰ They model the discount as the difference between the market value of an asset and its value when liquidated and argue that the discount should be larger when there are execution lags in liquidation. They derive optimal trading rules and the magnitude of the illiquidity discount for investors with power utility functions. Lo, Mamaysky and Wang (2001) assume fixed transactions costs and conclude, like Amihud and Mendelson, that small trading costs can create significant illiquidity discounts and that these discounts are influenced heavily by the risk aversion of investors.⁶¹

⁵⁸ Amihud, Y. and H. Mendelson, 1986, *Asset Pricing and the Bid-ask Spread*, Journal of Financial Economics, v 17, 223-250.

⁵⁹ Vayanos, D., 1998, *Transactions Costs and Asset Prices: A Dynamic Equilibrium Model*, Journal of Financial Economics, v11, 1-58.

⁶⁰ Jarrow, R. and A. Subramanian, 2001, *The Liquidity Discount*, Mathematical Finance, v11, 447-474.

⁶¹ Lo, A.W., H. Mamaysky and J. Wang, 2001, *Asset Prices and Trading Volume under Fixed Transactions Costs*, Working Paper, Yale International Center of Finance.

In summary, the papers that develop theoretical models for illiquidity discounts all link them to expected transactions costs on assets but require investor holding periods as an input for estimating the magnitude of the discount. The discount for any given set of transaction costs will be smaller if investors have long time horizons than if they have short time horizons.

b. Illiquidity and Discount Rates

In conventional asset pricing models, the required rate of return for an asset is a function of its exposure to market risk. Thus, in the CAPM, the cost of equity is a function of the beta of an asset, whereas in the APM or multi-factor model, the cost of equity is determined by the asset's exposure to multiple sources of market risk. There is little in these models that allow for illiquidity. Consequently, the required rate of return will be the same for liquid and illiquid assets with similar market risk exposure. In recent years, there have been attempts to expand these models to allow for illiquidity risk in one of two ways. The first are theoretical models that build in a market premium for illiquidity that affects all assets and measures of illiquidity for individual assets. Differences in the latter will cause required rates of return to vary across companies with different degrees of liquidity. The second are purely empirical multi-factor models that attempt to explain differences in returns across stocks over long time periods, with a measure of illiquidity such as trading volume or the bid-ask spread considered one of the factors.

The earliest theoretical discussions of how best to incorporate illiquidity into asset pricing models occurred in the 1970s. Mayers (1972, 1973, 1976) extended the capital asset pricing model to consider non-traded assets as well as human capital.⁶² The resulting models did not make explicit adjustments for illiquidity, though. In a more recent attempt to incorporate illiquidity into expected return models, Acharya and Pedersen (2005) examine how assets are priced with liquidity risk and make a critical

⁶² Mayers, D., 1972, *Nonmarketable assets and capital market equilibrium under uncertainty*, in M.C. Jensen, *Studies in the Theory of Capital Markets* (Praeger, New York, NY); Mayers, D., 1973, *Nonmarketable assets and the determination of capital asset prices in the absence of a riskless asset*, *Journal of Business*, v46, 258-267; Mayers, D., 1976, *Nonmarketable assets, market segmentation and the level of asset prices*, *Journal of Financial and Quantitative Analysis*, v11, 1-12.

point.⁶³ It is not just how illiquid an asset is that matters but when it is illiquid. In particular, an asset that is illiquid when the market itself is illiquid (which usually coincides with down markets and economic recessions) should be viewed much more negatively (with a resulting higher expected return) than an asset that is illiquid when the market is liquid. Thus the liquidity beta of an asset will reflect the covariance of the asset's liquidity with market liquidity. Acharya and Pedersen estimate that illiquid stocks have annualized risk premiums about 1.1% higher than liquid stocks, and that 80% of this premium can be explained by the covariance between a stock's illiquidity and overall market illiquidity. Pastor and Stambaugh (2002) also concluded that it is not a stock's liquidity per se that matters but its relationship to overall market liquidity. Over the 34-year period that they examined stock returns, they concluded that stocks whose returns are more sensitive to market liquidity have annual returns that are 7.5% higher than stocks whose returns have low sensitivity to market liquidity, after adjusting for the standard size, value and momentum factors.⁶⁴

The difficulties associated with modeling liquidity and arriving at usable models have lead many researchers to consider more practical ways of incorporating illiquidity into expected returns. Building on the work done on multi-factor models in the 1980s and proxy models the 1990s, they looked for ways of measuring liquidity and including these measures in models that explained differences in stock returns over long time periods. Amihud and Mendelson (1989) examined whether adding bid-ask spreads to betas helped better explain differences in returns across stocks in the U.S.⁶⁵ In their sample of NYSE stocks from 1961-1980, they concluded that every 1% increase in the bid-ask spread (as a percent of the stock price) increased the annual expected return by 0.24-0.26%. Eleswarapu (1997) confirmed this finding by showing a positive relationship between returns and spreads for Nasdaq stocks.⁶⁶ Other studies have used trading volume,

⁶³ Acharya, V. and L.H. Pedersen, 2005, *Asset Pricing with Liquidity Risk*, Journal of Financial Economics, v77, 375-410.

⁶⁴ Pastor, L. and R. Stambaugh, 2003, *Liquidity Risk and Stock Market Returns*, Journal of Political Economy, v111, 642-685.

⁶⁵ Amihud, Y. and . Mendelson, 1989, *The Effects of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns*, Journal of Finance, v 44, 479-486.

⁶⁶ Eleswarapu, V.R. 1997, *Cost of transacting and expected returns in the NASDAQ Market*, Journal of Finance, 52 (5), 2113-2127. There are other studies that find a weaker or no relationship between stock returns and bid-ask spreads. Chalmers and Kadec use the amortized spread and find no relationship

turnover ratios (dollar trading volume/ market value of equity) and illiquidity ratios as proxies for illiquidity with consistent results. Brennan and Subrahmanyam(1996) break transactions costs down into fixed and variable costs and find evidence of a significant effect on returns due to the variable cost of trading after controlling for factors such as firm size and the market to book ratio.⁶⁷ Brennan, Chordia and Subrahmanyam (1998) find that dollar trading volume and stock returns are negatively correlated, after adjusting for other sources of market risk.⁶⁸ Datar, Nair and Radcliffe (1998) use the turnover ratio as a proxy for liquidity. After controlling for size and the market to book ratio, they conclude that liquidity plays a significant role in explaining differences in returns, with more illiquid stocks (in the 90th percentile of the turnover ratio) having annual returns that are about 3.25% higher than liquid stocks (in the 10th percentile of the turnover ratio). In addition, they conclude that every 1% increase in the turnover ratio reduces annual returns by approximately 0.54%.⁶⁹ Amihud (2002) developed a measure of illiquidity by dividing the absolute price change by the average daily trading volume for the stock to estimate an illiquidity ratio and concluded that stock returns are positively correlated with this measure.⁷⁰ Nguyen, Mishra and Prakash (2005) conclude that stocks with higher turnover ratios do have lower expected returns. They also find that market capitalization and price to book ratios, two widely used proxies that have been shown to explain differences in stock returns, do not proxy for illiquidity.⁷¹

In summary, both the theoretical models and the empirical results suggest that we should adjust discount rates for illiquidity, with the former focusing on systematic liquidity as the key factor and the latter using proxies such as bid-ask spreads and

between spreads and returns for NYSE stocks. Chalmers, J.M.R. and G.B. Kadlec, 1998, *An Empirical examination of the Amortized Spread*, Journal of Financial Economics, 48 (2), 159-188.

⁶⁷ Brennan, M. J. and A. Subrahmayam, 1996, *Market microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns*, Journal of Financial Economics, v41, 441-464.

⁶⁸ Brennan, M. J., T. Chordia, and A. Subrahmanyam, 1998. *Alternative factor specifications, security characteristics and the cross-section of expected stock returns*, Journal of Financial Economics, 49, 345–373.

⁶⁹ Datar, V.T., N. Y. Naik and R. Radcliffe, 1998, *Liquidity and stock returns: An alternative test,*”*Journal of Financial Markets* 1, 203-219.

⁷⁰ Amihud, Y., 2002, *Illiquidity and stock returns: Cross-section and time-series effects*, Journal of Financial Markets v5, 31-56.

⁷¹ Nguyen, D., S. Mishra and A.J. Prakash, 2005, *On Compensation for Illiquidity in Asset Pricing: An Empirical Evaluation using the Three-factor Model and the Three-moment CAPM*, Working Paper, SSRN.

turnover ratios to measure liquidity. Both approaches also seem to indicate that the adjustment will vary across time and will be dependent upon a market wide demand for liquidity. Thus, for any given level of illiquidity, the expected premium added on to discount rates will be much greater in periods when the market values liquidity more and smaller in periods when it values it less.

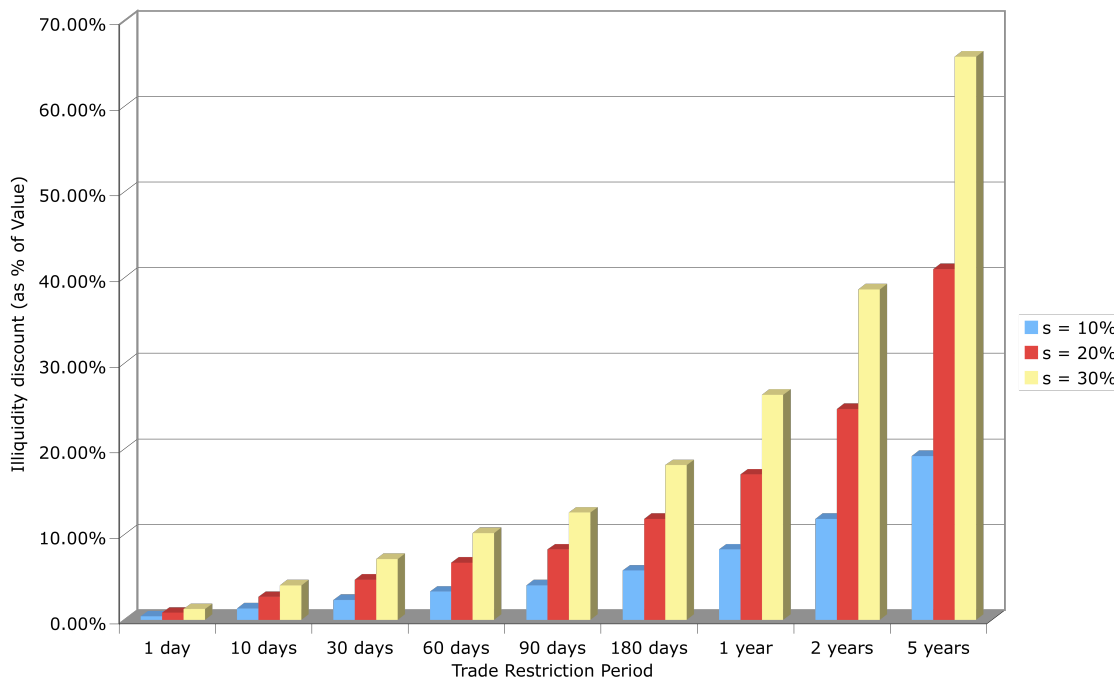
c. Illiquidity as an Option

What is the value of liquidity? Put differently, when does an investor feel the loss of liquidity most strongly when holding an asset? There are some who would argue that the value of liquidity lies in being able to sell an asset, when it is most overpriced; the cost of illiquidity is not being able to do this. In the special case, where the owner of an asset has the information to know when this overpricing occurs, the value of illiquidity can be considered an option.

Longstaff (1995) presents an upper bound for the option by considering an investor with perfect market timing abilities who owns an asset on which she is not allowed to trade for a period (t). In the absence of trading restrictions, this investor would sell at the maximum price that an asset reaches during the time period and the value of the look-back option estimated using this maximum price should be the outer bound for the value of illiquidity.⁷² Using this approach, Longstaff estimates how much marketability would be worth as a percent of the value of an asset for different illiquidity periods and asset volatilities. The results are graphed in figure 3:

⁷² Longstaff, F.A., 1995, *How much can marketability affect security values?* Journal of Finance, v 50, 1767-1774.

Figure 3: Upper bounds on Marketability Discount - Option Pricing Model



It is worth emphasizing that these are upper bounds on the value of illiquidity since it is based upon the assumption of a perfect market timer. To the extent that investors are unsure about when an asset has reached its maximum price, the value of illiquidity will be lower than these estimates. The more general lessons will still apply. The cost of illiquidity, stated as a percent of firm value, will be greater for more volatile assets and will increase with the length of the period for which trading is restricted.

Estimating the Impact of Illiquidity on Value

Much of the work on costing illiquidity has been done in private company appraisal, where the task is to estimate the discount to be attached to a private business, because it is an illiquid asset. The question we face in this paper, though, is a narrower one. Once the value of equity is estimated in a private or a public business, how do we allocate that equity value across different equity claims, if some claims are more liquid than other claims. There are three approaches that can be used to quantify the effect of illiquidity and they mirror the three theoretical approaches in the last section. In the first, we ignore illiquidity while estimating value and then estimate a discount to this value,

where we reflect the illiquidity of the asset. In the second, we discount the expected cash flows to the less liquid equity claim at a higher discount rate, with the adjustment to the discount rate a function of the illiquidity of the claim. In the third, we adapt the option-based approach, described in the last section, to estimate the illiquidity discount for less liquid equity claims.

a. Illiquidity Discounts

In the earlier section on valuing voting versus non-voting shares, we referenced the fairly common practice in valuation of using the median premium on voting shares, from empirical studies of large numbers of voting and non-voting shares, as the premium for any given voting share class. In a similar vein, we could study more liquid versus less liquid equities, issued by the same firm, and estimate the discount that the market attaches to the less liquid securities. This discount can then be applied when valuing less liquid equity claims.

The most extreme version of liquidity differences arises when one class of shares trades and the other does not. This is, in fact, the case with restricted stock, where publicly traded companies are allowed to bypass the SEC registration process and issue shares directly to investors, who, in turn, are restricted from trading on these shares for one year following the issue. Since we can observe the price at which the regular shares of the company trade at and the price at which the restricted stock is placed with investors, we are in the unique position of being able to isolate the liquidity discount on restricted shares. Several papers have examined restricted stock issues over time and have concluded that the discount ranges from 20 to 40%. In a paper taking a closer look at this discount, Bajaj, Dennis, Ferris and Sarin (2001) argued that there is sampling bias induced by the fact that companies that tend to use restricted stock are smaller, more troubled firms. To counter this bias, they looked at 88 private placements from 1990 to 1997 and report median discounts of 9.85% for registered private placements and 28.13% for restricted stocks. After controlling for differences across the firms making these issues, they attribute only 7.23% to the marketability discount.⁷³

⁷³ Bajaj, M., D.J. Dennis, S.P.Ferris and A.Sarin, 2001, *Firm Value and Marketability Discounts*, Journal of Corporate Law, v27.

Studies that look at multiple classes of shares issued by the same firm also try to isolate the effect of illiquidity on value. One candidate for study is the Chinese market, where most companies have Restricted Institutional Shares (RIS) which are almost completely illiquid⁷⁴ and common shares which are traded on the exchange. Chen and Xiong (2001) compare the market prices of the traded common stock in 258 Chinese companies with the auction and private placement prices of the RIS shares and conclude that the discount on the latter is 78% for auctions and almost 86% for private placements.⁷⁵ This astoundingly high discount, which they attributed to illiquidity, does vary across firms, with smaller discounts at larger, less volatile firms. In a different vein, researchers have compared the stock prices of Class A and Class B shares of Chinese companies. The former are open only to Chinese investors, whereas the latter can be bought by both domestic and foreign investors. While they both offer the same claims on the cashflows, Class B shares trade at a significant discount on Class A shares. The differences, though, seem to be only partially attributable to differences in liquidity and seem more due to differential information.⁷⁶

The wide ranges of discounts reported in studies of illiquid securities, relative to liquid securities, brings home the difficulty and dangers of using a median discount. When faced with two classes of shares, one liquid and the other illiquid, do we apply the 7% discount (that Bajaj et al find), the 25-30% discount from restricted stock studies or the even larger discounts reported in emerging markets? Furthermore, how do we differentiate between an equity class that is not traded from one that is lightly traded? A compromise solution exists. Rather than use just the median discount from these studies, we could look at determinants of these discounts. Silber (1989), for instance, related the restricted stock discount to the magnitude of revenues at the firm, whether the firm was

⁷⁴ Restricted Institutional Shares have to be transacted through private placements. Starting in August 2000, the Chinese Government has also allowed for auctions of these shares, where it is presumably a little easier to find a potential buyer.

⁷⁵ Chen, Z. and P. Xiong, 2001, *Discounts on Illiquid Stocks: Evidence from China*, Working Paper, Yale University.

⁷⁶ Wang, S.S. and L. Jiang, 2003, *Location of Trade, Ownership Restrictions, and Market Illiquidity: Examining Chinese A- and H-Shares*, Working Paper, Hong Kong Polytechnic University. Chan, K., A.J. Menkveld and Z. Yang, 2002, *Evidence of the Foreign Share Discount Puzzle in China: Liquidity or Information Asymmetry*, Working Paper, National Center for Economic Research, Tsinghua University.

healthy or not (positive/ negative earnings) and trading volume to arrive at a cross sectional regression of the discount against these determinants:

$$\text{LN(RPRS)} = 4.33 + 0.036 \ln(\text{REV}) - 0.142 \text{LN(RBRT)} + 0.174 \text{DERN} + 0.332 \text{DCUST}$$

where,

RPRS = Restricted Stock Price/ Unrestricted stock price = 1 – illiquidity discount

REV = Revenues of the private firm (in millions of dollars)

RBRT = Restricted Block relative to Total Common Stock (in %)

DERN = 1 if earnings are positive; 0 if earnings are negative;

DCUST = 1 if there is a customer relationship with the investor; 0 otherwise;

The illiquidity discount tends to be smaller for firms with higher revenues, decreases as the block offering decreases and is lower when earnings are positive and when the investor has a customer relationship with the firm. We could plug in the specific characteristics of the security that we are valuing into this regression and obtain a modified discount. The problem with this regression is that the sample size will be small.

If we accept the proposition that all securities are illiquid and that the bid-ask spread is a measure of the illiquidity, we can expand the sample substantially. Damodaran (2002) regressed the bid-ask spread (as a percent of the price) against company-specific and security-specific characteristics including the level of revenues, profitability and security trading volume.

$$\text{Spread} = 0.145 - 0.0022 \ln(\text{Annual Revenues}) - 0.015 (\text{DERN}) - 0.016 (\text{Cash/Firm Value}) - 0.11 (\text{\$ Monthly trading volume/ Firm Value})$$

Entering the trading volume for a security into this regression will allow us to quantify the impact of differential liquidity on the bid-ask spread, and the spread itself can then be used to estimate an illiquidity discount.

Illustration 10: Estimating the illiquidity discount for equity claims

Consider again the example of Nova Chemicals, with two classes of shares – voting and non-voting shares. In illustration 9, we estimated the values of each class of shares:

Value per voting share = \$4.68

Value per non-voting share = \$3.9

Now assume that the voting shares are not traded, while the non-voting shares are liquid and traded, with a bid-ask spread of \$0.10 a share and a monthly dollar trading volume of \$ 10 million. We can assess the illiquidity discount for class A shares in a number of different ways:

1. The simplest way to assess the spread is to discount the value per share by the median discount for illiquid shares, estimated by looking at restricted stock. Using the 25-30% discount from these studies would yield a value per share for class A shares:

$$\begin{aligned} \text{Value per class A share} &= \text{Estimated value per share (1- Median discount)} \\ &= \$4.68 (1-.25) = \$3.5 \end{aligned}$$

2. A more sophisticated variant would be to adjust the bid-ask spread for the traded class for the lack of liquidity on the non-traded stock. In this case, the bid-ask spread as a percent of the value per share of class B stock is:

$$\begin{aligned} \text{Bid-ask spread on traded shares} &= \text{Bid ask spread} // \text{ Estimated value per share} \\ &= \$0.10 / \$3.90 = 2.65\% \end{aligned}$$

Using the regression for bid-ask spreads listed above, we can make the adjustment for the lack of trading in class A stock.

$$\begin{aligned} \text{Estimated bid-ask spread for class A shares} &= \text{Bid-ask spread on class B stock} + 0.11 \\ &(\text{Monthly trading volume on class B stock} / \text{ Firm Value}) = 0.0265 + 0.11 (100/78) = \\ &16.75\% \end{aligned}$$

Note that the only adjustment we are making is for the trading volume, since both classes of shares are on the same company, and the other coefficients on the regression will be the same for each. This would suggest that the illiquidity discount on class A shares should be about 16.75%, resulting a value per share of \$3.90.⁷⁷

$$\begin{aligned} \text{Value per class A share} &= \text{Estimated value per share (1- Adjusted discount)} \\ &= \$4.68 (1-.1675) = \$3.90 \end{aligned}$$

b. Liquidity Adjusted Discount Rates

Rather than wait until the value has been estimated and then adjust for illiquidity, we can try to adjust the discount rate that we use for a set of expected cashflows for the

⁷⁷ This is based on the assumption that holders of the non-traded stock will transact only once and hold the stock in perpetuity. As the holding period decreases, the illiquidity discount will increase.

illiquidity of the asset generating these cash flows. In effect, we add an illiquidity premium to the discount rate that we estimate from conventional risk and return models. The practical question then becomes how best to estimate this illiquidity premium. The research that we cited in the last section provides some guidance in this process. Some of the studies relate the returns earned on publicly traded securities to measures of how liquid these securities are, using measures such as turnover ratio and trading volume. Generically, the return on a security can then be written as:

$$\text{Return} = a + b (\text{Risk Measure}) + c (\text{Liquidity Measure})$$

If a company has two classes of equity – one less liquid than the other – we could assume that they share the same underlying business risk but will still have different expected returns, because of differences in liquidity.

Once the discount rate is adjusted for illiquidity, the effects of illiquidity can be incorporated into the value today. More liquid assets should be worth more than less liquid assets, with the same expected cashflows.

c. Option-Based Discount

In an earlier section, we examined an option-pricing based approach, which allowed you to estimate an upper bound for the illiquidity discount, by assuming an investor with perfect market timing skills. There have been attempts to extend option pricing models to valuing illiquidity, with mixed results. In one widely used variation, liquidity is modeled as a put option for the period when an investor is restricted from trading. Thus, the illiquidity discount on value for an asset where the owner is restricted from trading for 2 years will be modeled as a 2-year at-the-money put option.⁷⁸ There are several flaws, both intuitive and conceptual, with this approach. The first is that liquidity does not give you the right to sell a stock at today's market price anytime over the next 2 years. What it does give you is the right to sell at the prevailing market price anytime over the next 2 years.⁷⁹ The second (and smaller) problem is that option pricing models

⁷⁸ In a 1993 study, David Chaffe used this approach to estimate illiquidity discounts ranging from 28-49% for an asset, using the Black Scholes option pricing model and volatilities ranging from 60 to 90% for the underlying asset.

⁷⁹ There is a simple way to illustrate that this put option has nothing to do with liquidity. Assume that you own stock in a liquid, publicly traded company and that the current stock price is \$ 50. A 2-year put option on this stock with a strike price of \$ 50 will have substantial value, even though the underlying stock is

are based upon continuous price movements and arbitrage and it is difficult to see how these assumptions will hold up for an illiquid asset.

The value of liquidity ultimately has to derive from the investor being able to sell at some pre-determined price during the non-trading period rather than being forced to hold until the end of the period. The look-back option approach that assumes a perfect market timer, explained earlier in the paper, assumes that the sale would have occurred at the high price and allows us to estimate an upper bound on the value. Can we use option pricing models to value illiquidity without assuming perfect market timing. Consider one alternative. Assume that you have a disciplined investor who always sells investments, when the price rises 25% above the original buying price. Not being able to trade on this investment for a period (say, 2 years) undercuts this discipline and it can be argued that the value of illiquidity is the produce of the value of the put option (estimated using a strike price set 25% above the purchase price and a 2 year life) and the probability that the stock price will rise 25% or more over the next 2 years.

If you decide to apply option pricing models to value illiquidity in private businesses, the value of the underlying asset (which is the private business) and the standard deviation in that value will be required inputs. While estimating them for a private business is more difficult to do than for a publicly traded firm, we can always use industry averages.

The Composite Effect

We have discussed differences in the nature of equity claims, the control rights that they possess, preferential claims on cash flows and liquidity differences, as separate rights, when valuing them. In practice, the claims are intermingled, with preferential cash flow claims often accompanied by inferior voting rights and higher liquidity. We examine how to value equity claims that have intermingled rights – stronger rights on some dimensions and weaker rights in others.

completely liquid. The value has nothing to do with liquidity but is a price you are willing to pay for insurance.

Potential Issues

When control rights, cash flow rights and liquidity differences are combined in the same security, there are two challenges we face. The first is examining whether the presence of one claim – say additional voting rights – affects the value of another claim—for instance, inferior cash flow rights. In other words, we are testing to see whether the value of a combination of rights and preferences will be worth a different amount than the values of the individual rights aggregated. The second is an agency issue that arises when we combined different powers in the same equity claim and the claimholders has the power to affect the value through their actions. Equity investors with superior control claims – higher voting rights – may be able to affect dividend policy and thus alter the value of cash flow claims.

a. Interaction between components

While we have hitherto considered contingent claims, control claims, cash flow claims and liquidity differences separately, they are often interlinked in practice. In general, superior claims on one issue are combined with inferior claims on others in a security. For instance, preferred stock combines superior claims on cash flows – first claim on higher dividends – with inferior claims on control – preferred stockholders have no voting rights. Since we are valuing the security and not the individual claims, we have to consider whether combining the claims augments or reduces the value of individual claims.

The empirical evidence is supportive of the proposition that there is interaction between preferential dividends, voting rights and liquidity. Cox and Roden (2001) examined 98 firms with two classes of shares with different voting rights and, in some cases, differential dividends and concluded that the voting right premium decreases as the differential dividend to the non-voting shares increases.⁸⁰ In other words, committing to pay higher dividends to the non-voting shareholders reduces the value of superior voting rights. While the average premium for voting over non-voting shares in the sample is 11.4%, the premium drops to 6.5% for firms that promise at least an equal dividend to

⁸⁰ Cox, S.R., and D. M. Roden, 2002, *The source of value of voting rights and related dividend promises*, Journal of Corporate Finance, v8, 337-351.2

non-voting shareholders and to 3.8% for firms where non-voting shares are promised a higher dividend. Beiner and Gibson (1999) present a model that links voting rights and liquidity, where they argue that voting shareholders, by virtue of the control privileges, will be less desirous of liquidity (and will charge a lower cost for illiquidity) than non-voting shareholders.⁸¹

b. Agency problems

Security holders with superior control claims – higher voting rights or the power to veto or modify major decisions made by the firm – can affect the value of other claimholders through their claims. Common stockholders, usually the holders of voting rights, can affect the value of preferred stockholders, through their investment and financing decisions. Investing in riskier assets or increasing debt can reduce the value of preferred stock by putting their dividends at risk, if dividends are fixed in absolute terms, or altering the dividend amount, if dividends are a proportion of earnings.

There is both anecdotal and empirical evidence that separating voting rights from cash flow rights, which is the natural result of having classes of equity with different voting rights, gives rise to problems that manifest themselves in decisions that benefit shareholders with superior voting rights. In one well publicized case, Conrad Black, the controlling stockholder at Hollinger International, a publicly traded Canadian firm, was accused of conducting related party transactions for personal benefit at corporate expense.⁸² There are several studies that document that firm value or stock returns are lower as corporate insiders control more voting rights relative to cash-flow rights.⁸³

⁸¹ Beiner, N. and R. Gibson, 1999, *A Theoretical Analysis of the Liquidity Risk Premium embedded in the Prices of Voting and Non-voting stocks*, Journal of Corporate Finance, v5, 209-225.

⁸² At Hollinger International, Conrad Black controlled 73% of the voting rights by owning all of the company's class B shares (about 30% of overall equity). He was accused of extracting management fees, consulting payments and personal dividends that amounted to 95% of the company's net income between 1997 and 2003.

⁸³ Claessens, S., S. Djankov, Joseph P.H. Fan, and Larry H.P. Lang, 2002, *Disentangling the incentive and entrenchment effects of large shareholdings*, Journal of Finance 58, 81-112; Lemmon, Michael L., and Karl V. Lins, 2003, *Ownership structure, corporate governance, and firm value: Evidence from the East Asian financial crisis*, Journal of Finance 58, 1445-1468; Gompers, Paul A., Joy Ishii, and Andrew Metrick, 2006. *Extreme governance: An analysis of Dual Class firms in the United States*, Working paper.

Masulis, Wang and Xie (2006) examine the specific agency problems associated with having dual class shares and document four findings:⁸⁴

1. When dual class companies are acquirers of other companies, the more cash flow rights deviate from voting rights, the more likely it is that they will experience negative abnormal returns from the acquisitions and the less likely they are to withdraw acquisitions in the face of negative market reactions. The implication is that insiders who control the voting rights are willing to make value-destroying acquisitions that provide benefits to them.
2. CEO pay increases as cash flow rights deviate from voting rights, suggesting that managers put private benefits over stockholder interests in companies with dual class shares.
3. Cash in the hands of dual-class companies is valued less as the divergence between cash flow rights and voting rights widens. This would suggest that investors in these companies worry more about cash being redirected to either poor investments or to enrich voting class shareholders.
4. Capital expenditures at companies with dual class shares contribute less to value, as the divergence between cash flow rights and voting rights widens.

In summary, companies with significant differences in voting rights across equity claims are viewed as more likely to make decisions that hurt overall firm value, while enriching the voting class shareholders. Gompers and Ishii (2008), in their examination of dual class shares, found that firms with dual class shares had higher financial leverage than other firms, which they attributed to the unwillingness of insiders who own the voting class shares to issue additional shares. As a final note, Francis, Schillper and Vincent (2005) examined companies with dual class shares and noted that they were more likely to pay dividends than other companies: 59% of dual class share companies paid dividends versus 28% of single class share companies. They also concluded that the

⁸⁴ Masulis, R.W., C. Wang and F. Xie, 2006, *Agency Problems at Dual Class Companies*, Working Paper, SSRN.

market values of these companies were more closely tied to dividends than earnings, which they attributed to the separation of cash flow rights from voting rights.⁸⁵

A Framework for valuing differential equity claims

Now that we have examined the effects of differences in equity claims on each dimension and laid out strategies and alternative approaches for valuing each claim, we can develop a general framework for valuing securities that combine these claims. In particular, we will consider which approach to pick for each claim and the sequence of valuing the claims.

Choosing an Approach

There are three basic approaches that we have used to value differential claims in equity. In the first, we stayed within the discounted cash flow framework and tried to value the effect of a differential claim. For example, we valued shares with superior dividend rights by discounting the expected dividends at a lower discount rate, reflecting the priority of dividend claims. The second includes probabilistic approaches – scenarios and simulations – to estimate the value of a claim. Thus, we valued cash flow preferences in liquidation by estimating the probability that the firm would face liquidation and the expected cash flows as a consequence. The third is option-pricing models, where we model the superior claim as a contingent cash flow and value the claims as options. This approach was used to value conversion privileges embedded in preferred stock and even illiquidity in a security.

Since more than one approach can sometimes be used to value the same claim – we used both option pricing models and scenario analysis to value liquidation cash flow claims – and the approaches usually yield different estimates of value, it is worth setting up a template that allows us to choose the “best” approach for valuing each claim. In making this judgment, we have to consider the assumptions that we need to make within each approach (and the likelihood that those assumptions will be violated in practice) and the ease with which the inputs to each approach can be obtained in each case. As a

⁸⁵ Francis, J., K. Schipper and L. Vincent, 2005, *Earnings and Dividend Informativeness When Cash Flow Rights Are Separated from Voting Rights*. *Journal of Accounting and Economics*, 39(2):329-360.

consequence, we will categorize all the claims that we have considered in this paper into three groups:

- a. Claims on the ongoing business: These claims entitle the holder to advantages on a continuous business as long as it stays in operation. Examples would be superior dividend claims and additional voting rights, both of which give the holder advantages for the life of the firm. For these claims, the best approach to use is discounted cash flow valuation, which values the business as an ongoing concern.
- b. Event-contingent claims: Claims that are triggered by the occurrence of a specific event are usually most easily valued using a probabilistic approach. Thus, a first claim on liquidation cash flows or a right that is triggered by an initial public offering can be valued by first estimating the probability of the event in question and then the value increase that arises as a consequence.
- c. Value-contingent claims: Claims that are contingent on value increasing or dropping below a pre-specified trigger value are best valued using an option-pricing model. The obvious example for this is a conversion option in preferred stock, where preferred stockholders get the right to convert to common stock if the common stock price exceeds a strike price.

The line between event contingent and value contingent claims is often a fine one, since an event contingent claim can sometimes be tied to value. For instance, if liquidation is a consequence only of distress, we could model it as a function of the liquidation value of the assets and the face value of fixed claims against the firm (debt, for instance), converting it into a value contingent claim. If on the other hand, liquidation covers a wider array of possible outcomes including being acquired and going public, the probabilistic approach will hold.

All three approaches described above can be called intrinsic approaches, because we are trying to estimate it on a security specific basis. With many claims, there is an alternate approach that we can call the market approach, where we can estimate the value of a differential claim by looking at what similar traded claims in the market go for. Thus, we estimate the premium for a voting share over a non-voting share by examining the premiums at which publicly traded voting shares trade at, relative to non-voting shares. When should you use the market approach as opposed to an intrinsic approach? Part of

answer lies in your view of markets and the purpose of the valuation. If you trust markets to be right on average and/or you are valuing a security for issue to the market, there is good reason to go with the market approach. If you mistrust markets and/or are interested purely in intrinsic value, any of the intrinsic approaches will dominate. The other part of the answer lies in the number and homogeneity of publicly traded firms. In other words, if there are hundreds of companies with traded voting and non-voting shares, and differences in voting rights are uniform across securities, you have a much better basis for using the market average premium. If on the other hand, differences in voting rights vary widely across securities, there are relatively few traded securities and there are other variables that are difficult to control for (liquidity, cash flow preferences), market approaches will either need to be modified substantially or used with caution.

To close this part of the discussion, table 7 summarizes the differential equity claims that we have considered in this paper, the approach that we believe makes the most sense for each and market estimates that may be available for each.

Table 7: Differential Equity Claims and Valuation Approaches

<i>Differential Claim</i>	<i>“Best” Intrinsic Approach</i>	<i>Market Approach</i>
Direct versus Contingent Claim	Option Pricing Model	Does not work
Preferential Dividends	DCF model (with lower discount rate, for equity with first claim on dividends)	Price/Dividend ratio for similar securities (equivalent dividends and risk)
Liquidation Cash Flow preferences (where liquidation is triggered by acquisition or IPO)	Probabilistic Model (Liquidation versus Going concern scenarios, Simulations)	Does not work
Liquidation Cash Flow preferences (where liquidation occurs because of distress, i.e, failure to meet obligations)	Option Pricing Model (with obligated cash flow as strike price and asset value in liquidation as determinant)	Does not work
Differential voting rights	DCF model (status quo versus optimal value & expected value of control)	Median (or adjusted) premium for traded voting versus non-voting shares
Event-triggered veto rights (power to stop event from happening)	DCF Model (with lower value reflecting effect of veto power)	Does not work
Event-triggered protective	Option Pricing Model (with	Does not work

rights (Participation rights)	inputs determined by the participation right)	
Differential liquidity	Option Pricing Model (with value a function of volatility in value and pre-specified trigger point for selling)	Median (or adjusted) discount for illiquid securities relative to liquid securities.

Sequence in Valuation

As we noted in the last section, equity classes in the same company can vary on multiple dimensions, with superior rights on some intermingled with inferior rights on claims. Since it is difficult to value them all together, we would suggest the following sequence for the valuation:

1. Conventional DCF valuation of the firm and aggregate equity: Value the business and aggregate equity in the firm, paying no heed to the fact that there are multiple equity claim holders. Thus, the standard textbook sequence of estimating cash flows, discount rates and growth rates is followed, with aggregate numbers used rather than per share numbers.
2. Value equity options held by managers and investors: These options represent a claim on the aggregate equity of the firm, and by extension, reduce the value of equity held by other claimholders. To value these options, we will need a price for the underlying shares: if the stock is publicly traded, the market price can be used, but if not, the estimated value per share (from a subsequent step) will have to be used.
3. Value ongoing claims, extending the discounted cash approach: For those claims, like voting rights and cash flow claims, where discounted cash flow models can be extended to arrive at an estimate of value, make that estimate. Consequently, to value voting rights, we would estimate the status quo and optimal values, and derive the value of a voting right. With preferential dividends, we would use a lower discount rate when estimate the cash flows to these prior claim holders to arrive at the value premium.
4. Value event-contingent claims, using a probabilistic approach: If any of the equity claims has event-contingent rights, these rights have to be valued at this stage by

estimating the probability of the event and the cash flow consequence of the right. This will reallocate equity value to the claimholder that possesses the right from claimholders who do not.

5. Value value-contingent claims across equity claims, using option pricing approaches: While options on equity are valued in step 2, this step is designed to bring into account options that one class of equity may have on another equity class. This will often require that output from the prior steps be used as input to the option-pricing model. For instance, to value option to convert preferred to common shares, we will need the value per common share and the value per preferred share, estimated in step 2.⁸⁶
6. Check values for each claim against market estimates, if available: To the extent that we may or may not have access to or believe in the market approach, this is an optional step. If there are large numbers of publicly traded securities, comparable to the one you are valuing, prudence demands that you check your estimates against the market estimates and have an explanation available for large differences (if any).
7. Estimate the consolidated value of each equity claim and check against aggregate: Once the values per share have been estimated for each equity class, the consolidated values of each class can be estimated. It should be checked against aggregate equity value in the first steps. Consistency demands that the two be equal, and if there are differences, the reasons for the differences should be made explicit.
8. Adjust the value of each equity claim, for illiquidity, if necessary: As a last step in the process, you should adjust the value of equity claims that are illiquid. Thus, if one class of equity is traded and the other is not, you would apply a illiquidity discount to the latter value to reflect the illiquidity.

Note that this sequence is not meant to be rigid and that deviations from it are reasonable, for individual companies.

⁸⁶ A standard problem with options issued by firms is that the exercise of the options itself can affect the value of equity, which in turn is affected by the existence of the options. The expected dilution created by exercise can be built into option pricing models. See Damodaran (2002) for more details.

Caveats and Concerns

Valuing equity claims that differ on multiple dimensions is messy but we can reduce the potential for problems by doing the following:

- a. Do not use the discount rate as the vehicle to load up your concerns about liquidity and control: While it is tempting to use higher discount rates on illiquid securities and stocks with fewer voting rights, it is also dangerous for two reasons. The first is that the discount rate adjustment, once made, is usually not revisited in the course of the valuation. This effectively assumes that the problem remains the same over time and that the cost it imposes are unchanged across the estimation period. The second is that it is far easier to double count an item with discount rate adjustments. It is not uncommon in valuations to see a proxy for liquidity (say the size of the company or trading float) used to adjust discount rates upwards (and value downwards) and then to see an illiquidity discount attached to this value, resulting in a double counting of the cost.
- b. Be wary about using option pricing models: Option pricing models are powerful and very useful tools in valuation but they can be misused. In particular, option pricing models are appropriate for rights that are contingent on value, where the possessor of the right has some degree of exclusivity. Rights that are contingent on events happening (new financing, initial public offerings) cannot be valued as options, unless the event itself can be made contingent on value; for instance, you could assume that the firm will make an initial public offering only if its value exceeds a certain pre-specified amount. There are measurement issues that also contaminate the values we derive from option pricing models. With management options, we noted that using the stated maturity of the option can overstate value because of the illiquidity of the options. With private businesses, deriving the variance of the asset value can be problematic, in the absence of traded prices.
- c. Market based estimates are noisy: As we have noted through the paper, it is possible to get market-based estimates of voting share premiums and illiquidity discounts, by looking at traded securities with these characteristics. While these numbers are useful starting points for analysis, they cannot be applied easily to individual securities. That is because there are no truly similar securities in the

market; each firm is a special case, with characteristics that separate it from the general population, resulting in different values. As a consequence, even the most comprehensive studies, which use large sample sizes and maintain statistical rigor, yield “noisy” estimates (i.e. an average voting share premium, with a large standard error).

Illustration 11: Valuing equity claims with multiple differences

Tolema Inc. is a small, publicly traded company with two classes of equity, 20 million class A shares and 60 million class B shares, with the following characteristics:

Voting rights and Control: Class A shares have ten times the voting rights as class B shares.

Liquidity: Class B shares are publicly traded (with a bid-ask spread of \$0.20) but class A shareholders are held by insiders/incumbent managers and are not publicly traded.

Cashflows: Class B shares are entitled to a preferred dividend of \$0.10 a share and a proportionate share of dividends with class A shareholders.

Contingent claim: Class B shareholders have the right to convert 1.5 shares of class B shares into one share of class A shares any time over the next 5 years.

Step 1: Discounted Cashflow valuation

Under its existing management, Tolema is expected to generate \$ 80 million in after-tax operating income next year on capital invested of \$ 1 billion. The cost of capital for the firm, under its existing debt policy, is 10%; the cost of aggregate equity is 12%. The firm expects to maintain its existing return on capital on new investments and reinvest 25% of its after-tax operating income back into the business. For the status-quo valuation, we use these inputs:

Return on capital on existing (and new) investments = $80/1000 = 8\%$

Expected growth rate in operating income = Return on capital * Reinvestment rate
 $= 8\% * 25\% = 2\%$

Value of firm = $\frac{\text{After - tax Operating Income next year (1 - Reinvestment Rate)}}{(\text{Cost of capital} - g)}$
 $= 80 (1 - .25) / (.10 - .02) = \$ 750 \text{ million}$

However, you believe that the firm can be run more efficiently, with better management. In particular, you feel that the following changes can be made to the firm:

- a. By cutting costs and improving existing operations, the after-tax operating income next year can be increased to \$ 100 million, thus bringing the return on capital on existing investments to 10%.
- b. The cost of capital can be reduced to 9%, by using a different financing mix, and that the return on capital on new investments can be improved to 12%, if the firm were more selective about its choices (while keeping the reinvestment rate at 25%).

The “optimal” value of the firm, with these changes in place is \$1.25 billion.

$$\begin{aligned}\text{Expected growth rate in operating income} &= \text{Return on capital} * \text{Reinvestment rate} \\ &= 12\% * 25\% = 3\%\end{aligned}$$

$$\begin{aligned}\text{Value of firm} &= \frac{\text{After - tax Operating Income next year} (1 - \text{Reinvestment Rate})}{(\text{Cost of capital} - g)} \\ &= 100 (1 - .25) / (.09 - .03) = \$1.25 \text{ billion}\end{aligned}$$

Given the current shareholding structure at the firm, where insiders control the voting shares, there is only a 20% probability of management changing at the firm. Consequently, the expected value of the firm is \$850 million.

$$\text{Expected value} = \text{Status Quo Value} (1 - \text{Probability of management change}) + \text{Optimal value} (\text{Probability of management change}) = 750 (.8) + 1250 (.2) = \$ 850 \text{ million}$$

Subtracting out the existing debt outstanding at the firm of \$ 250 million results in a value of equity of \$ 600 million.

Step 3: DCF valuation of differential voting rights and dividends

The class B shares are entitled to a dividend of \$ 0.10 per share. This dividend is paid after interest expenses, but before making discretionary capital expenditures and paying common dividends. Given the cost of equity for the firm is 12% and the pre-tax cost of debt is 6%, we estimated a discount rate for the fixed dividend of 8%, higher than the pre-tax cost of debt, since it is made after interest payments, but much lower than the overall cost of equity.

$$\text{Value of fixed dividends} = \frac{\text{Fixed Dividend}}{\text{Risk - adjusted Discount Rate}} = \frac{\$0.10}{0.08} = \$1.25$$

The total value of the fixed dividends across 60 million class B shares is therefore \$ 75 million.

Since both classes of shares receive a proportionate share of the remaining dividends, we will allocate the remaining status quo equity value equally among all equity claimholders:

Status Quo Equity Value = Status Quo Firm Value - Debt = 750– 250 =	500 million
- Value of fixed dividends across Class B shares	- 75 million
Remaining Status Quo Value =	= 425 million
/ Number of class A and class B shares outstanding	/ 80 million
Remaining Status Quo Value per share	= \$5.31 / share
Value of Class B share with fixed dividend	= \$6.56/ share
Value of Class A share (before valuing voting rights)	= \$5.31/share

To value control, we will first isolate the expected value of control and then estimate the value per voting right:

Optimal Value of the firm (from step 1)	= \$1,250 million
- Status Quo Value of the firm (from step 1)	= \$ 750 million
Value of management change	= \$ 500 million
X Probability of management change	X 20%
Expected value of control	= \$ 100 million
/ Number of voting rights across class A and B shares	= 260 million
Value per voting right	= \$0.3846

(Number of voting rights = # Class A shares * 10 + # Class B shares * 1 = 20*10 + 60*1)

Given that class A shares have 10 voting rights per share and class B shares have only 1 voting right, we can estimate the value per share with the voting rights.

Value per Class A share = \$5.31 + 10 (0.3846) = \$ 9.16/share

Value per Class B share = \$6.56 + 1 (0.3846) = \$6.95/share

Steps 4 &5: Valuation of event-contingent and value-contingent rights:

To value the option to convert 1.5 class B shares into one class A share anytime over the next 5 years, we have to estimate the variance in the ratio over time. Given that class A shares are not traded, we cannot use past stock prices to make this judgment. However, the primary reason for the difference is the expected value of control and the volatility in that value over time, caused both by changes in the probability of management changing and the value of making that change. Using a sample of

companies with dual class shares (with voting right differences similar to Tolema), where both classes of shares trade, we estimated the variance in the price ratio (for non-voting versus voting shares) of 0.04. Stating the option inputs in terms of a single class B share, we arrive at the following:

$$S = \text{Estimated value of Class B share} / \text{Estimated value of Class A share (from step 2)} \\ = \$6.95 / \$9.16 = 0.76$$

$$K = 0.667 \text{ (Each share of class B stock} = 2/3 \text{ of class A share)}$$

$$T = 5 \text{ years}$$

$$\sigma^2 = 0.04$$

Type of option = Put (Class B shareholders will convert if the ratio falls below 0.50)

Using a five-year riskless rate of 4% yields a value for the put option of 0.0381.⁸⁷ Since this value is stated as a proportion of the current value of class A shares, the dollar value per option can be computed as follows:

$$\text{\$ Value of option} = \text{Value of Option} * \text{Current value of Class B share} \\ = \$9.16 * 0.0381 = \$ 0.35$$

$$\text{Value of class B share with option} = \text{Estimated value per B share} + \text{Value of option} \\ = \$6.95 + \$ 0.35 = \$ 7.30$$

Class A shareholders are giving up these options and we have to reduce the value of class A shares by the aggregate value of the options. Since there are three times as many class B shares as class A shares, each class A shareholder, in effect, is giving up three options.

$$\text{Value of class A share with option} = \text{Estimated value per A share} - \text{Value of option} * \\ \text{Number of options} = \$9.16 - 3 (\$ 0.35) = \$ 8.11$$

Step 6 & 7: Check estimated values of equity per share against aggregate equity value

At this stage in the estimation process, we have derived a value of \$600 million for aggregate equity and values per share of \$8.48 and \$7.63 for class A and class B shareholders respectively. To ensure that our estimates are internally consistent, we value all class A and class B shares:

$$\text{Value of Class A shares} = 20 \text{ million} * \$8.11 = \$162.24 \text{ million}$$

$$\text{Value of Class B shares} = 60 \text{ million} * \$7.30 = \$437.76 \text{ million}$$

⁸⁷ We used a Black-Scholes model, with an adjustment for the dividend yield.

Aggregate value of equity = \$600.00 million

This matches up to the derived value of equity.

Based on our estimates, class A shares should have a value roughly 11% higher than class B shares. Looking at other companies with dual-class shares, with similar voting right differences, the median voting share premium was smaller (about 6%), but the estimate of 11% falls well within a reasonable range and can be justified by the larger expected value of control in Tolema Inc.

Step 8: Adjust the values per share for illiquidity

While class B shares are traded (with a monthly trading volume of \$ 100 million), class A shares are not and are therefore more illiquid. Note that the current bid-ask spread on class B shares (\$0.20) reflects the fact that even publicly traded shares have an illiquidity cost, but we would expect this cost to be higher for class A shares. To estimate a synthetic bid-ask spread for class A shares, we draw on the bid-ask spread regression presented earlier in the paper that related the spread to monthly trading volume:

Spread on Class B shares = $\$0.20 / \$7.30 = 2.74\%$

Synthetic Spread on Class A shares = Spread for class B shares + 0.11 (Monthly trading volume/ Estimated value of Class B shares) = $2.74\% + 0.11 (100/\$437.76) = 0.0525$ or 5.25%

This would be the illiquidity discount, if investors bought and held each class in perpetuity. To the extent that they have shorter time horizons, the effective discount will be larger. Annualizing the spread cost over the holding period, and discounting back at the cost of equity should yield a simple approximation:

Illiquidity Discount = (Spread/ Holding Period)/ Cost of equity

Assuming a 4-year time horizon investors in class A shares and a 3-year time horizon for class B shares translates into illiquidity discounts of 10.95% and 7.61% for class A and B shares respectively:

Illiquidity Discount_{Class A shares} = $(0.0525/4)/.12 = 10.95\%$

Illiquidity Discount_{Class B shares} = $(.0274/3)/.12 = 7.61\%$

The final estimates of value per share are as follows:

Value per Class A share = $\$8.11 (1-.1095) = \7.22

Value per Class B shares = \$ 7.30 (1-.0761) = \$6.74

As a final note, we are aware of the circularity involved in each step in the process. For instance, the value of the option to convert class B shares to class A shares should use the liquidity adjusted values from this step. In fact, we could build in an iterative process to reflect the circularity and it does shift the estimates of value per share to \$7.57 and \$6.60 for class A and B shares respectively. The drop in value for class B shares can be attributed to the fact that the option to convert to class A shares becomes less valuable once we consider the fact that class A shares are illiquid.

Conclusion

In conventional valuation, we assume that all equity investors have identical claims on the firm and divide the value of equity equally among claim holders. In the real world, there can be differences across equity claim holders on at least five dimensions. The first is whether the claim is a direct claim on cash flows and value, as is the case with conventional common stock, or a contingent claim, where the equity claim has value only if the value is greater than or less than a pre-specified price. The second difference is on cash flow claims, with some investors laying first claim on cash flows than others. This is often the case with preferred stock and common stock. The third set of differences lie in control claims, with differences in voting rights across different claimholders being the most common form. In some cases, the differences in control claims are dormant under the status quo but rise to the surface when a pre-specified event – liquidation, acquisition or initial public offering – occurs. In its fourth dimension, equity claims can vary depending upon how much power they have (or not have) in the face of new financing to either protect their interests or to veto the new financing. Finally, equity claims can vary in terms of liquidity.

While we presented multiple approaches to valuing individual claims, we also created a framework for analyzing securities that represent bundles of superior claims on some dimensions and inferior claims on others. We argue that differential claims that can be traced back to ongoing cash flows – voting rights and dividend preferences, for instance – are best valued using discounted cash flow models. Once we have arrived at

the values per share of different equity classes, we can first incorporate the value of event-specific claims and contingent claims.