

The uValue Companion: A Handbook on Valuation

CHAPTER 1

APPROACHES TO VALUATION

In general terms, there are three approaches to valuation. The first, discounted cashflow valuation, relates the value of an asset to the present value of expected future cashflows on that asset. The second, relative valuation, estimates the value of an asset by looking at the pricing of 'comparable' assets relative to a common variable like earnings, cashflows, book value or sales. The third, contingent claim valuation, uses option pricing models to measure the value of assets that share option characteristics.

Discounted Cashflow Valuation

In discounted cashflows valuation, the value of an asset is the present value of the expected cashflows on the asset, discounted back at a rate that reflects the riskiness of these cashflows. This approach gets the most play in classrooms and comes with the best theoretical credentials.

Basis for Approach

We buy most assets because we expect them to generate cash flows for us in the future. In discounted cash flow valuation, we begin with a simple proposition. The value of an asset is not what someone perceives it to be worth but it is a function of the expected cash flows on that asset. Put simply, assets with high and predictable cash flows should have higher values than assets with low and volatile cash flows. In discounted cash flow valuation, we estimate the value of an asset as the present value of the expected cash flows on it.

$$\text{Value of asset} = \frac{E(CF_1)}{(1+r)} + \frac{E(CF_2)}{(1+r)^2} + \frac{E(CF_3)}{(1+r)^3} \dots + \frac{E(CF_n)}{(1+r)^n}$$

where,

n = Life of the asset

E(CF_t) = Expected cashflow in period t

r = Discount rate reflecting the riskiness of the estimated cashflows

The cashflows will vary from asset to asset -- dividends for stocks, coupons (interest) and the face value for bonds and after-tax cashflows for a business. The discount rate will be

a function of the riskiness of the estimated cashflows, with higher rates for riskier assets and lower rates for safer ones.

Using discounted cash flow models is in some sense an act of faith. We believe that every asset has an intrinsic value and we try to estimate that intrinsic value by looking at an asset's fundamentals. What is intrinsic value? Consider it the value that would be attached to an asset by an all-knowing analyst with access to all information available right now and a perfect valuation model. No such analyst exists, of course, but we all aspire to be as close as we can to this perfect analyst. The problem lies in the fact that none of us ever gets to see what the true intrinsic value of an asset is and we therefore have no way of knowing whether our discounted cash flow valuations are close to the mark or not.

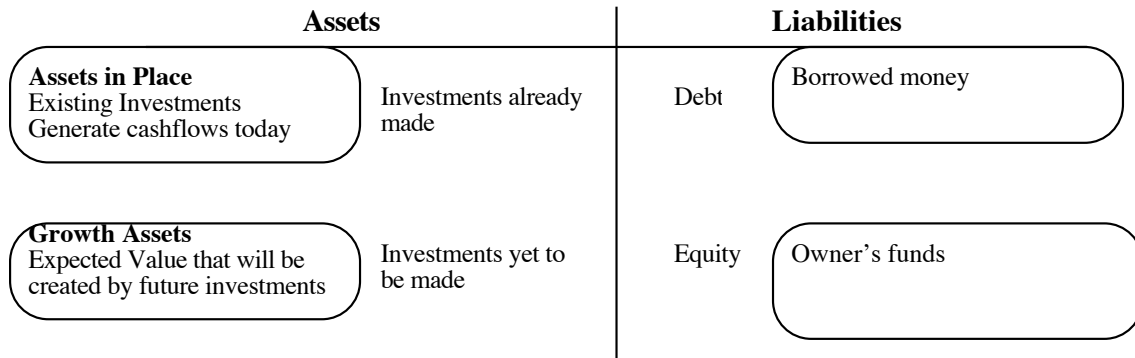
Classifying Discounted Cash Flow Models

There are three distinct ways in which we can categorize discounted cash flow models. In the first, we differentiate between valuing a business as a going concern as opposed to a collection of assets. In the second, we draw a distinction between valuing the equity in a business and valuing the business itself. In the third, we lay out three different and equivalent ways of doing discounted cash flow valuation – the expected cash flow approach, a value based upon excess returns and adjusted present value.

a. Going Concern versus Asset Valuation

The value of an asset in the discounted cash flow framework is the present value of the expected cash flows on that asset. Extending this proposition to valuing a business, it can be argued that the value of a business is the sum of the values of the individual assets owned by the business. While this may be technically right, there is a key difference between valuing a collection of assets and a business. A business or a company is an on-going entity with assets that it already owns and assets it expects to invest in the future. This can be best seen when we look at the financial balance sheet (as opposed to an accounting balance sheet) for an ongoing company in figure 1.1:

Figure 1.1: A Simple View of a Firm



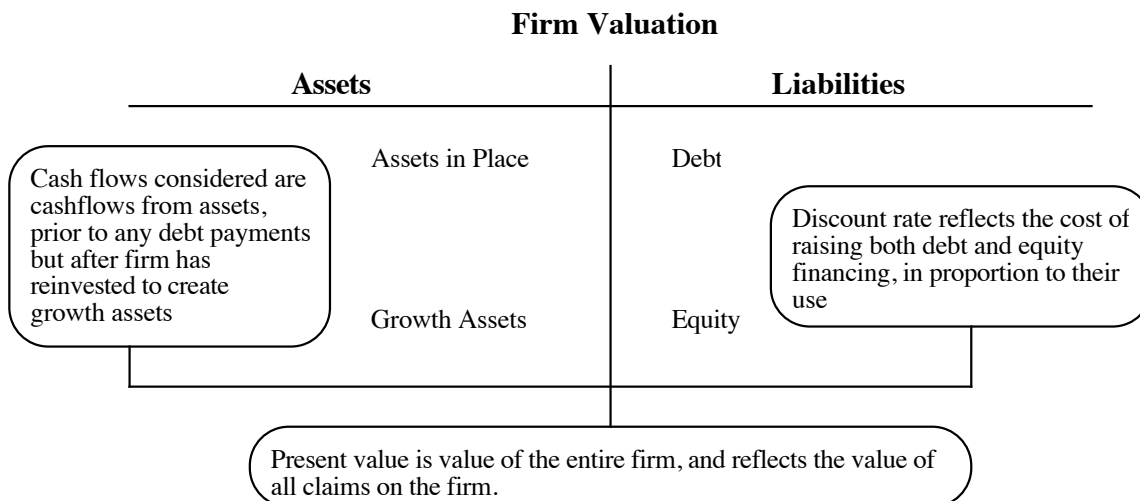
Note that investments that have already been made are categorized as assets in place, but investments that we expect the business to make in the future are growth assets.

A financial balance sheet provides a good framework to draw out the differences between valuing a business as a going concern and valuing it as a collection of assets. In a going concern valuation, we have to make our best judgments not only on existing investments but also on expected future investments and their profitability. While this may seem to be foolhardy, a large proportion of the market value of growth companies comes from their growth assets. In an asset-based valuation, we focus primarily on the assets in place and estimate the value of each asset separately. Adding the asset values together yields the value of the business. For companies with lucrative growth opportunities, asset-based valuations will yield lower values than going concern valuations.

One special case of asset-based valuation is liquidation valuation, where we value assets based upon the presumption that they have to be sold now. In theory, this should be equal to the value obtained from discounted cash flow valuations of individual assets but the urgency associated with liquidating assets quickly may result in a discount on the value. How large the discount will be will depend upon the number of potential buyers for the assets, the asset characteristics and the state of the economy.

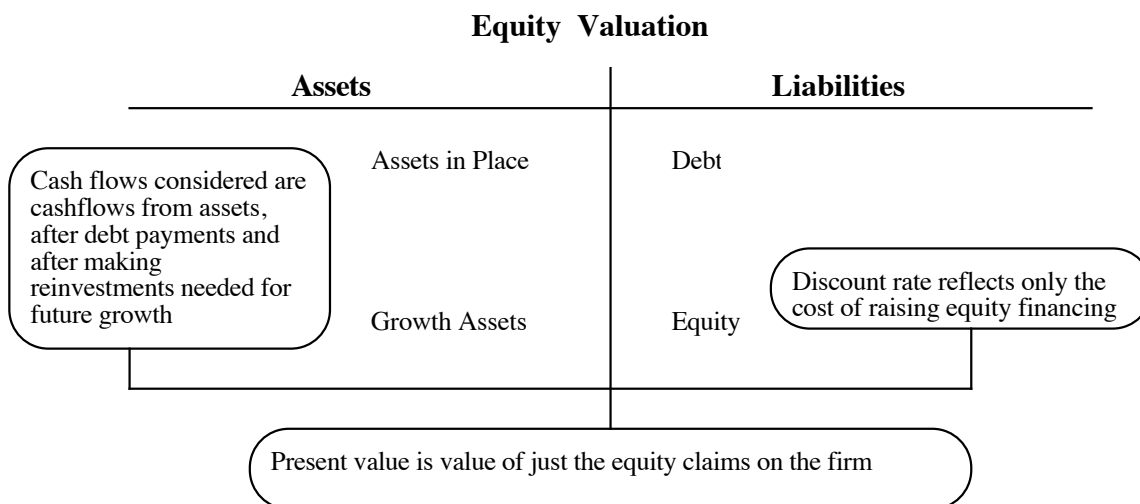
b. Equity Valuation versus Firm Valuation

There are two ways in which we can approach discounted cash flow valuation. The first is to value the entire business, with both assets-in-place and growth assets; this is often termed firm or enterprise valuation.



The cash flows before debt payments and after reinvestment needs are called free cash flows to the firm, and the discount rate that reflects the composite cost of financing from all sources of capital is called the cost of capital.

The second way is to just value the equity stake in the business, and this is called equity valuation.



The cash flows after debt payments and reinvestment needs are called free cash flows to equity, and the discount rate that reflects just the cost of equity financing is the cost of equity.

Note also that we can always get from the former (firm value) to the latter (equity value) by netting out the value of all non-equity claims from firm value. Done right, the value of equity should be the same whether it is valued directly (by discounting cash

flows to equity at the cost of equity) or indirectly (by valuing the firm and subtracting out the value of all non-equity claims).

c. Variations on DCF Models

The model that we have presented in this section, where expected cash flows are discounted back at a risk-adjusted discount rate, is the most commonly used discounted cash flow approach but there are two widely used variants. In the first, we separate the cash flows into excess return cash flows and normal return cash flows. Earning the risk-adjusted required return (cost of capital or equity) is considered a normal return cash flow but any cash flows above or below this number are categorized as excess returns; excess returns can therefore be either positive or negative. With the *excess return valuation* framework, the value of a business can be written as the sum of two components:

$$\text{Value of business} = \text{Capital Invested in firm today} + \text{Present value of excess return cash flows from both existing and future projects}$$

If we make the assumption that the accounting measure of capital invested (book value of capital) is a good measure of capital invested in assets today, this approach implies that firms that earn positive excess return cash flows will trade at market values higher than their book values and that the reverse will be true for firms that earn negative excess return cash flows.

In the second variation, called the *adjusted present value (APV) approach*, we separate the effects on value of debt financing from the value of the assets of a business. In general, using debt to fund a firm's operations creates tax benefits (because interest expenses are tax deductible) on the plus side and increases bankruptcy risk (and expected bankruptcy costs) on the minus side. In the APV approach, the value of a firm can be written as follows:

$$\text{Value of business} = \text{Value of business with 100\% equity financing} + \text{Present value of Expected Tax Benefits of Debt} - \text{Expected Bankruptcy Costs}$$

In contrast to the conventional approach, where the effects of debt financing are captured in the discount rate, the APV approach attempts to estimate the expected dollar value of debt benefits and costs separately from the value of the operating assets.

While proponents of each approach like to claim that their approach is the best and most precise, we will show later in the book that the three approaches yield the same estimates of value, if we make consistent assumptions.

Inputs to Discounted Cash Flow Models

There are three inputs that are required to value any asset in this model - the *expected cash flow*, the *timing* of the cash flow and the *discount rate* that is appropriate given the riskiness of these cash flows.

a. Discount Rates

In valuation, we begin with the fundamental notion that the discount rate used on a cash flow should reflect its riskiness, with higher risk cash flows having higher discount rates. There are two ways of viewing risk. The first is purely in terms of the likelihood that an entity will default on a commitment to make a payment, such as interest or principal due, and this is called default risk. When looking at debt, the cost of debt is the rate that reflects this default risk.

The second way of viewing risk is in terms of the variation of actual returns around expected returns. The actual returns on a risky investment can be very different from expected returns; the greater the variation, the greater the risk. When looking at equity, we tend to use measures of risk based upon return variance. There are some basic points on which these models agree. The first is that risk in an investment has to be perceived through the eyes of the marginal investor in that investment, and this marginal investor is assumed to be well diversified across multiple investments. Therefore, the risk in an investment that should determine discount rates is the *non-diversifiable or market risk* of that investment. The second is that the expected return on any investment can be obtained starting with the expected return on a riskless investment, and adding to it a premium to reflect the amount of market risk in that investment. This expected return yields the cost of equity.

The *cost of capital* can be obtained by taking an average of the cost of equity, estimated as above, and the after-tax cost of borrowing, based upon default risk, and weighting by the proportions used by each. We will argue that the weights used, when valuing an on-going business, should be based upon the market values of debt and equity.

While there are some analysts who use book value weights, doing so violates a basic principle of valuation, which is that at a fair value¹, one should be indifferent between buying and selling an asset.

b. Expected Cash Flows

In the strictest sense, the only cash flow an equity investor gets out of a publicly traded firm is the dividend; models that use the dividends as cash flows are called *dividend discount models*. A broader definition of cash flows to equity would be the cash flows left over after the cash flow claims of non-equity investors in the firm have been met (interest and principal payments to debt holders and preferred dividends) and after enough of these cash flows has been reinvested into the firm to sustain the projected growth in cash flows. This is the free cash flow to equity (FCFE), and models that use these cash flows are called *FCFE discount models*.

The cashflow to the firm is the cumulated cash flow to all claimholders in the firm. One way to obtain this cashflow is to add the free cash flows to equity to the cash flows to lenders (debt) and preferred stockholders. A far simpler way of obtaining the same number is to estimate the cash flows prior to debt and preferred dividend payments, by subtracting from the after-tax operating income the net investment needs to sustain growth. This cash flow is called the free cash flow to the firm (FCFF) and the models that use these cash flows are called *FCFF models*.

c. Expected Growth

It is while estimating the expected growth in cash flows in the future that analysts confront uncertainty most directly. There are three generic ways of estimating growth. One is to look at a company's past and use the historical growth rate posted by that company. The peril is that past growth may provide little indication of future growth. The second is to obtain estimates of growth from more informed sources. For some analysts,

¹ When book value weights are used, the costs of capital tend to be much lower for many U.S. firms, since book equity is lower than market equity. This then pushes up the value for these firms. While this may make it attractive to the sellers of these firms, very few buyers would be willing to pay this price for the firm, since it would require that the debt that they use in their financing will have to be based upon the book value, often requiring tripling or quadrupling the dollar debt in the firm.

this translates into using the estimates provided by a company's management whereas for others it takes the form of using consensus estimates of growth made by others who follow the firm. The bias associated with both these sources should raise questions about the resulting valuations.

There is a third way to estimate growth, where the expected growth rate is tied to two variables that are determined by the firm being valued - how much of the earnings are reinvested back into the firm and how well those earnings are reinvested. In the equity valuation model, this expected growth rate is a product of the retention ratio, i.e. the proportion of net income not paid out to stockholders, and the return on equity on the projects taken with that money. In the firm valuation model, the expected growth rate is a product of the reinvestment rate, which is the proportion of after-tax operating income that goes into net new investments and the return on capital earned on these investments. The advantages of using these fundamental growth rates are two fold. The first is that the resulting valuations will be internally consistent and companies that are assumed to have high growth are required to pay for the growth with more reinvestment. The second is that it lays the foundation for considering how firms can make themselves more valuable to their investors.

DCF Valuation: Pluses and Minuses

To true believers, discounted cash flow valuation is the only way to approach valuation, but the benefits may be more nuanced than they are willing to admit. On the plus side, discounted cash flow valuation, done right, requires analysts to understand the businesses that they are valuing and ask searching questions about the sustainability of cash flows and risk. Discounted cash flow valuation is tailor made for those who buy into the Warren Buffett adage that what we are buying are not stocks but the underlying businesses. In addition, discounted cash flow valuations is inherently contrarian in the sense that it forces analysts to look for the fundamentals that drive value rather than what market perceptions are. Consequently, if stock prices rise (fall) disproportionately relative to the underlying earnings and cash flows, discounted cash flows models are likely to find stocks to be over valued (under valued).

There are, however, limitations with discounted cash flow valuation. In the hands of sloppy analysts, discounted cash flow valuations can be manipulated to generate estimates of value that have no relationship to intrinsic value. We also need substantially more information to value a company with discounted cash flow models, since we have to estimate cashflows, growth rates and discount rates. Finally, discounted cash flow models may very well find every stock in a sector or even a market to be over valued, if market perceptions have run ahead of fundamentals. For portfolio managers and equity research analysts, who are required to find equities to buy even in the most over valued markets, this creates a conundrum. They can go with their discounted cash flow valuations and conclude that everything is overvalued, which may put them out of business, or they can find an alternate approach that is more sensitive to market moods. It should come as no surprise that many choose the latter.

Relative Valuation

While the focus in classrooms and academic discussions remains on discounted cash flow valuation, the reality is that most assets are valued on a relative basis. In relative valuation, we value an asset by looking at how the market prices similar assets. Thus, when determining what to pay for a house, we look at what similar houses in the neighborhood sold for rather than doing an intrinsic valuation. Extending this analogy to stocks, investors often decide whether a stock is cheap or expensive by comparing its pricing to that of similar stocks (usually in its peer group). In this section, we will consider the basis for relative valuation, ways in which it can be used and its advantages and disadvantages.

Basis for approach

In relative valuation, the value of an asset is derived from the pricing of 'comparable' assets, standardized using a common variable. Included in this description are two key components of relative valuation. The first is the notion of comparable or similar assets. From a valuation standpoint, this would imply assets with similar cash flows, risk and growth potential. In practice, it is usually taken to mean other companies that are in the same business as the company being valued. The other is a standardized price. After all, the price per share of a company is in some sense arbitrary since it is a

function of the number of shares outstanding; a two for one stock split would halve the price. Dividing the price or market value by some measure that is related to that value will yield a standardized price. When valuing stocks, this essentially translates into using multiples where we divide the market value by earnings, book value or revenues to arrive at an estimate of standardized value. We can then compare these numbers across companies.

The simplest and most direct applications of relative valuations are with real assets where it is easy to find similar assets or even identical ones. The asking price for a Mickey Mantle rookie baseball card or a 1965 Ford Mustang is relatively easy to estimate given that there are other Mickey Mantle cards and 1965 Ford Mustangs out there and that the prices at which they have been bought and sold can be obtained. With equity valuation, relative valuation becomes more complicated by two realities. The first is the absence of similar assets, requiring us to stretch the definition of comparable to include companies that are different from the one that we are valuing. After all, what company in the world is remotely similar to Microsoft or GE? The other is that different ways of standardizing prices (different multiples) can yield different values for the same company.

Harking back to our earlier discussion of discounted cash flow valuation, we argued that discounted cash flow valuation was a search (albeit unfulfilled) for intrinsic value. In relative valuation, we have given up on estimating intrinsic value and essentially put our trust in markets getting it right, at least on average.

Variations on Relative Valuation

In relative valuation, the value of an asset is based upon how similar assets are priced. In practice, there are three variations on relative valuation, with the differences primarily in how we define comparable firms and control for differences across firms:

a. *Direct comparison*: In this approach, analysts try to find one or two companies that look almost exactly like the company they are trying to value and estimate the value based upon how these “similar” companies are priced. The key part in this analysis is identifying these similar companies and getting their market values.

b. Peer Group Average: In the second, analysts compare how their company is priced (using a multiple) with how the peer group is priced (using the average for that multiple). Thus, a stock is considered cheap if it trade at 12 times earnings and the average price earnings ratio for the sector is 15. Implicit in this approach is the assumption that while companies may vary widely across a sector, the average for the sector is representative for a typical company.

c. Peer group average adjusted for differences: Recognizing that there can be wide differences between the company being valued and other companies in the comparable firm group, analysts sometimes try to control for differences between companies. In many cases, the control is subjective: a company with higher expected growth than the industry will trade at a higher multiple of earnings than the industry average but how much higher is left unspecified. In a few cases, analysts explicitly try to control for differences between companies by either adjusting the multiple being used or by using statistical techniques. As an example of the former, consider PEG ratios. These ratios are computed by dividing PE ratios by expected growth rates, thus controlling (at least in theory) for differences in growth and allowing analysts to compare companies with different growth rates. For statistical controls, we can use a multiple regression where we can regress the multiple that we are using against the fundamentals that we believe cause that multiple to vary across companies. The resulting regression can be used to estimate the value of an individual company. In fact, we will argue later in this book that statistical techniques are powerful enough to allow us to expand the comparable firm sample to include the entire market.

Applicability of multiples and limitations

The allure of multiples is that they are simple and easy to relate to. They can be used to obtain estimates of value quickly for firms and assets, and are particularly useful when there are a large number of comparable firms being traded on financial markets, and the market is, on average, pricing these firms correctly. In fact, relative valuation is tailor made for analysts and portfolio managers who not only have to find under valued equities in any market, no matter how overvalued, but also get judged on a relative basis. An analyst who picks stocks based upon their PE ratios, relative to the sectors they

operate in, will always find under valued stocks in any market; if entire sectors are over valued and his stocks decline, he will still look good on a relative basis since his stocks will decline less than comparable stocks (assuming the relative valuation is right).

By the same token, they are also easy to misuse and manipulate, especially when comparable firms are used. Given that no two firms are exactly similar in terms of risk and growth, the definition of 'comparable' firms is a subjective one. Consequently, a biased analyst can choose a group of comparable firms to confirm his or her biases about a firm's value. While this potential for bias exists with discounted cashflow valuation as well, the analyst in DCF valuation is forced to be much more explicit about the assumptions which determine the final value. With multiples, these assumptions are often left unstated.

The other problem with using multiples based upon comparable firms is that it builds in errors (over valuation or under valuation) that the market might be making in valuing these firms. If, for instance, we find a company to be under valued because it trades at 15 times earnings and comparable companies trade at 25 times earnings, we may still lose on the investment if the entire sector is over valued. In relative valuation, all that we can claim is that a stock looks cheap or expensive relative to the group we compared it to, rather than make an absolute judgment about value. Ultimately, relative valuation judgments depend upon how well we have picked the comparable companies and how how good a job the market has done in pricing them.

Contingent Claim Valuation

There is little in either discounted cashflow or relative valuation that can be considered new and revolutionary. In recent years, though, analysts have increasingly used option-pricing models, developed to value listed options, to value assets, businesses and equity stakes in businesses. These applications are often categorized loosely as real options, but as we will see later in this book, they have to be used with caution.

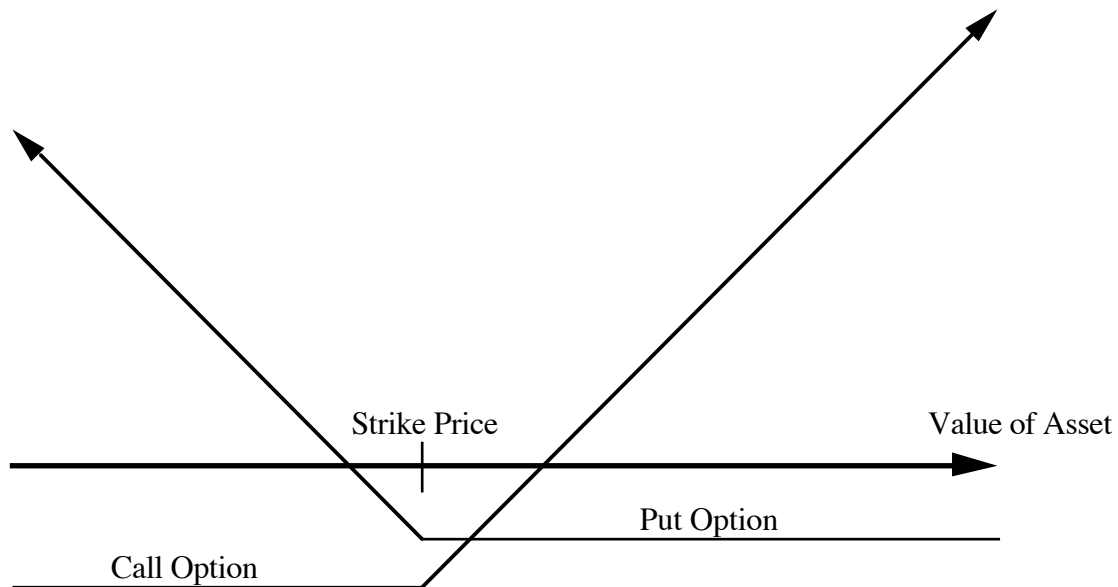
Basis for Approach

A contingent claim or option is an asset which pays off only under certain contingencies - if the value of the underlying asset exceeds a pre-specified value for a call option, or is less than a pre-specified value for a put option. Much work has been done in

the last few decades in developing models that value options, and these option-pricing models can be used to value any assets that have option-like features.

Figure 1.2 illustrates the payoffs on call and put options as a function of the value of the underlying asset:

Figure 1.2: Payoffs on Options as a Function of the Underlying Asset's Value



An option can be valued as a function of the following variables - the current value and the variance in value of the underlying asset, the strike price and the time to expiration of the option and the riskless interest rate. This was first established by Black and Scholes (1972) and has been extended and refined subsequently in numerous variants.² While the Black-Scholes option-pricing model ignored dividends and assumed that options would not be exercised early, it can be modified to allow for both. A discrete-time variant, the Binomial option-pricing model, has also been developed to price options.

An asset can be valued as a call option if the payoffs on it are a function of the value of an underlying asset; if that value exceeds a pre-specified level, the asset is worth the difference; if not, it is worth nothing. It can be valued as a put option if it gains value as the value of the underlying asset drops below a pre-specified level, and if it is worth

² Black, F. and M. Scholes, 1972, *The Valuation of Option Contracts and a Test of Market Efficiency*, *Journal of Finance*, v27, 399-417.

nothing when the underlying asset's value exceeds that specified level. There are many assets that generally are not viewed as options but still share several option characteristics. A patent can be analyzed as a call option on a product, with the investment outlay needed to get the project going considered the strike price and the patent life becoming the life of the option. An undeveloped oil reserve or gold mine provides its owner with a call option to develop the reserve or mine, if oil or gold prices increase.

The essence of the real options argument is that discounted cash flow models understate the value of assets with option characteristics. The understatement occurs because DCF models value assets based upon a set of expected cash flows and do not fully consider the possibility that firms can learn from real time developments and respond to that learning. For example, an oil company can observe what the oil price is each year and adjust its development of new reserves and production in existing reserves accordingly rather than be locked into a fixed production schedule. As a result, there should be an option premium added on to the discounted cash flow value of the oil reserves. It is this premium on value that makes real options so alluring and so potentially dangerous.

Applicability and Limitations

Using option-pricing models in valuation does have its advantages. First, there are some assets that cannot be valued with conventional valuation models because their value derives almost entirely from their option characteristics. For example, a biotechnology firm with a single promising patent for a blockbuster cancer drug wending its way through the FDA approval process cannot be easily valued using discounted cash flow or relative valuation models. It can, however, be valued as an option. The same can be said about equity in a money losing company with substantial debt; most investors buying this stock are buying it for the same reasons they buy deep out-of-the-money options. Second, option-pricing models do yield more realistic estimates of value for assets where there is a significant benefit obtained from learning and flexibility. Discounted cash flow models will understate the values of natural resource companies, where the observed price of the natural resource is a key factor in decision making. Third, option-pricing models do

highlight a very important aspect of risk. While risk is considered almost always in negative terms in discounted cash flow and relative valuation (with higher risk reducing value), the value of options increases as volatility increases. For some assets, at least, risk can be an ally and can be exploited to generate additional value.

This is not to suggest that using real options models is an unalloyed good. Using real options arguments to justify paying premiums on discounted cash flow valuations, when the options argument does not hold, can result in overpayment. While we do not disagree with the notion that firms can learn by observing what happens over time, this learning has value only if it has some degree of exclusivity. We will argue later in this book that it is usually inappropriate to attach an option premium to value if the learning is not exclusive and competitors can adapt their behavior as well. There are also limitations in using option pricing models to value long-term options on non-traded assets. The assumptions made about constant variance and dividend yields, which are not seriously contested for short term options, are much more difficult to defend when options have long lifetimes. When the underlying asset is not traded, the inputs for the value of the underlying asset and the variance in that value cannot be extracted from financial markets and have to be estimated. Thus the final values obtained from these applications of option pricing models have much more estimation error associated with them than the values obtained in their more standard applications (to value short term traded options).

CHAPTER 2

VALUING BUSINESSES ACROSS SECTORS AND THE LIFE CYCLE

While the fundamentals of valuation are straightforward, the challenges we face in valuing companies shift as these firms move through the life cycle from idea businesses, often privately owned, to young growth companies, either public or on the verge of going public, to mature companies, with diverse product lines and serving different markets, to companies in decline, marking time until they are liquidated. At each stage, we are called upon to estimate the same inputs – cash flows, growth rates and discount rates – but with varying amounts of information and different degrees of precision. All too often, when confronted with significant uncertainty or limited information, we will be tempted by the dark side of valuation, where first principles are abandoned, new paradigms are created and common sense is the casualty.

Foundations of Value

Without delving into the estimation details, we can use the intrinsic value equation of the business to list the four broad questions that we need to answer, in order to value any business: What are the cash flows that will be generated by the existing investments of the company? How much value, if any, will be added by future growth? How risky are the expected cashflows from both existing and growth investments and what is the cost of funding them? When will the firm become a stable growth firm, allowing us to estimate a terminal value?

What are the cash flows generated by existing assets?

If a firm has significant investments that it has already made, the first inputs into valuation are the cash flows from these existing assets. In practical terms, this requires estimating how much the firm generated in earnings and cashflows from these assets in the most recent period, how much growth (if any) is expected in these earnings/cashflows over time and how long the assets will continue to generate cash flows. While data that allows us to answer all of these questions may be available in current financial statements, they might not be conclusive. In particular, cash flows can be difficult to obtain if the existing assets are still not fully operational (infrastructure investments that

have been made, but are not in full production mode) or if they are not being efficiently utilized. There can also be estimation issues when the firm in question is in a volatile business, where earnings on existing assets can rise and fall as a result of macroeconomic forces.

How much value will be added by future investments (growth)?

For some companies, the bulk of value will be derived from investments that you expect them to make in the future. To estimate the value added by these investments, you have to make judgments on two variables. The first is the magnitude of these new investments, relative to the size of the firm. In other words, the value added can be very different, if you assume that a firm reinvests 80% of its earnings back into new investments than if you assume that it reinvests 20%. The second is the quality of the new investments, measured in terms of excess returns, i.e., the returns the firm makes on the investments over and above the cost of funding those investments. Investing in new assets that generate returns of 15%, when the cost of capital of 10%, will add value, but investing in new assets that generate returns of 10%, with the same cost of capital, will not. In other, words, it is growth with excess returns that creates value, not growth per se.

Since growth assets rest entirely on expectations and perception, we can make two statements about them. One is that valuing growth assets will generally pose more challenges than valuing existing assets; historical or financial statement information is less likely to provide conclusive results. The other is that there will be far more volatility in the value of growth assets than in the value of existing assets, both over time and across different people valuing the same firm. Analysts are likely to not only differ more on the inputs into growth asset value – the magnitude and quality of new investments – but will also change their own estimates more over time, as new information comes out about the firm. A poor earnings announcement by a growth company may alter the value of its existing assets just a little but can dramatically shift expectations about the value of growth assets.

How risky are the cashflows and what are the consequences for discount rates?

Neither the cash flows from existing assets, nor the cash flows from growth investments, is guaranteed. When valuing these cash flows, we have to consider risk

somewhere and the discount rate is usually the vehicle that we use to convey the concerns that we may have about uncertainty in the future. In practical terms, we use higher discount rates to discount riskier cash flows, and thus give them a lower value than more predictable cash flows. While this a common sense notion, there are issues that we run into in putting this into practice, when valuing firms:

1. Dependence on the past: The risk that we are concerned about is entirely in the future, but our estimates of risk are usually based upon data from the past – historical prices, earnings and cash flows. While this dependence upon historical data is understandable, it can give rise to problems when that data is unavailable, unreliable or shifting.
2. Diverse risk investments: When valuing firms, we generally estimate one discount rate for its aggregate cash flows, partly because of the way we estimate risk parameters and partly for convenience. Firms do generate cashflows from multiple assets, in different locations, with varying amounts of risk, and the discount rates we use should be different for each set of cash flows.
3. Changes in risk over time: In most valuations, we estimate one discount rate and we leave it unchanged over time, again partly for ease and partly because we feel uncomfortable changing discount rates over time. When valuing a firm, though, it is entirely possible, and indeed likely, that the risk of the firm will change over time as its asset mix changes and it matures. In fact, if we accept the earlier proposition that the cash flows from growth assets are more difficult to predict than cash flows from existing assets, we should expect the discount rate used on the cumulative expected cash flows of a growth firm to decrease as its growth rate declines over time.

When will the firm become mature?

The question of when a firm will become a mature firm is relevant because it determines the length of the high growth period and the value that we attach to the firm at the end of the period (the terminal value). It is question that may be easy to answer for a few firms, including larger and more stable firms that are either already mature businesses or close to it, or firms that derive their growth from a single competitive

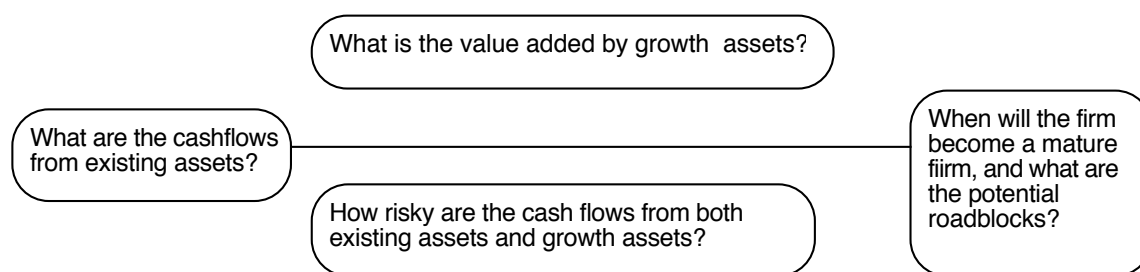
advantage with an expiration date (for instance, a patent). For most firms, the conclusion will be murky for two reasons:

1. Making a judgment about when a firm will become mature requires us to look at the sector in which the firm operates, the state of its competitors and what they will do in the future. For firms in sectors that are evolving, with new entrants and existing competitors exiting, this will be difficult to do.
2. While we are sanguine about mapping out pathways to the terminal value in discounted cash flow models and generally assume that every firm makes it to stable growth and goes on, the real world delivers surprises along the way that may impede these paths. After all, most firms do not make it to the steady state that we aspire, and instead get acquired, restructured or go bankrupt well before the terminal year.

In summary, not only is estimating when a firm will become mature difficult to do, but considering whether a firm will make it as a going concern for a valuation is just as important.

Pulling together all four questions, we get the framework for valuing any business in figure 2.1:

Figure 2.1: The fundamental questions in Valuation



While these questions may not change as we value individual firms, the ease with which we can answer them can change, not only as we look across firms at a point in time, but across time, even for the same firm. To get from the value of the business to the value of the equity in the business may seem like a simple exercise: subtracting out the debt outstanding, but the process can be complicated if the debt is not clearly defined or contingent on an external event (a claim in a lawsuit). Once we have the value of equity, getting the value of a unit claim in equity (per share value) can be difficult if different equity claims have different voting rights, cash flow claims or liquidity.

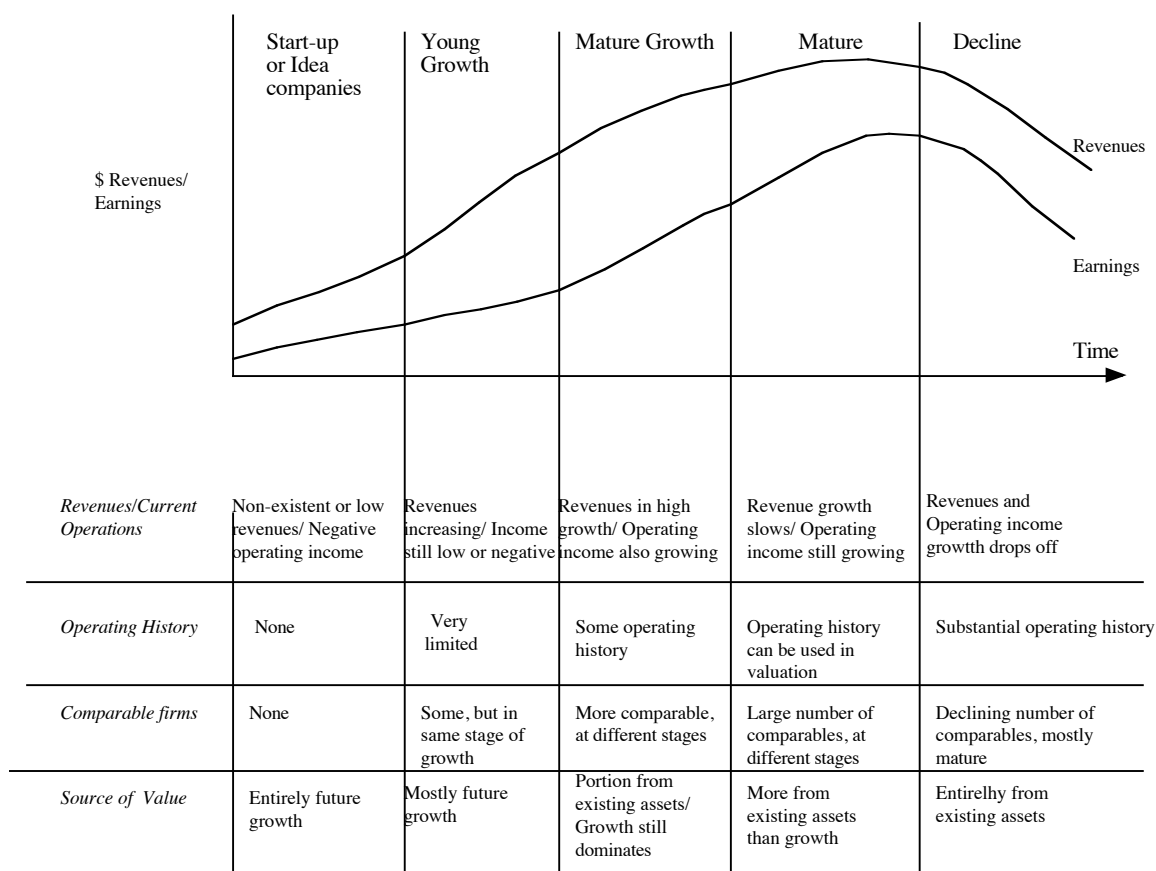
Valuation across the Life Cycle

While the inputs into valuation are the same for all businesses, the challenges we face in making the estimates can vary significantly across firms. In this section, we first break firms down into four groups based upon where they are in the life cycle and then explore the estimation issues we run into with firms in each stage.

The Business Life Cycle

Firms pass through a life cycle, starting as young, idea companies and working their way to high growth, maturity and eventual decline. Since the difficulties associated with estimating valuation inputs vary as firms go through the life cycle, it is useful to start with the five phases that we divide the life cycle into and consider the challenges in each phase separately in figure 2.2:

Figure 2.2: Valuation Issues across the Life Cycle



Note that the time spent in each phase can vary widely across firms, with some like Google and Amazon, speeding through the early phases and quickly become growth

companies whereas others make the adjustment much more gradually. Many growth companies have only a few years of growth before they become mature businesses, whereas a few, like Coca Cola, IBM and Walmart, are able to stretch their growth periods to last decades. At each phase in the cycle, there are companies that never make it through, either because they run out of cash and access to capital or have trouble making debt payments.

Early in the life cycle: Young companies

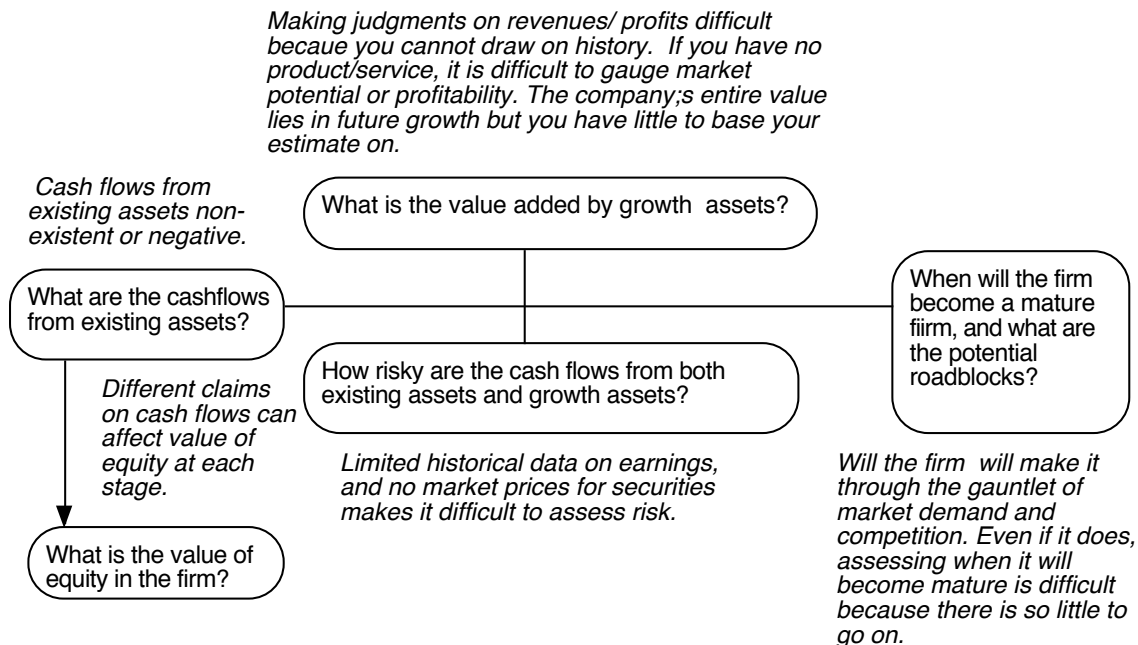
Every business starts with an idea; the idea germinates in a market need that an entrepreneur sees (or thinks he sees) and a way of filling that need. While most ideas go nowhere, some individuals take the next step of investing in the idea. The capital to finance the investment usually comes from personal funds (from savings, friends and family), and in the best-case scenario yields a commercial product or service. Assuming that the product or service finds a ready market, the business will usually need to access more capital, supplied usually by venture capitalists, who provide funds in return for a share of the equity in the business. Building on the most optimistic assumptions again, success for the investors in the business ultimately is manifested as a public offering to the market or sale to larger entity.

At each stage in the process, we need estimates of value. At the idea stage, the value may never be put down on paper but it is the potential for this value that induces the entrepreneur to invest both time and money in developing the idea. At subsequent stages of the capital raising process, the valuations become more explicit because they determine what the entrepreneur will have to give up as a share of ownership in return for external funding. At the time of the public offering, the valuation is key to determining the offering price.

Using the template for valuation that we developed in the last section, it is easy to see why young companies also create the most daunting challenges for valuation. There are few or no existing assets and almost all of the value comes from expectations of future growth. The current financial statements of the firm provide no clues about the potential margins and returns that will be generated by the future, and there is little historical data that can be used to develop risk measures. To cap the estimation problem,

many young firms will not make it to stable growth and estimating when it will happen for firms that survive is difficult to do. In addition, these firms are often dependent upon one or a few key people for their success, and losing them can have significant effects on value. A final valuation challenge we face with valuing equity in young companies is that different equity investors have different claims on the cash flows: the investors with the first claims on the cash flows should have the more valuable claims. Figure 2.3 summarizes these valuation challenges:

Figure 2.3: Estimation Issues - Young and Start-up Companies



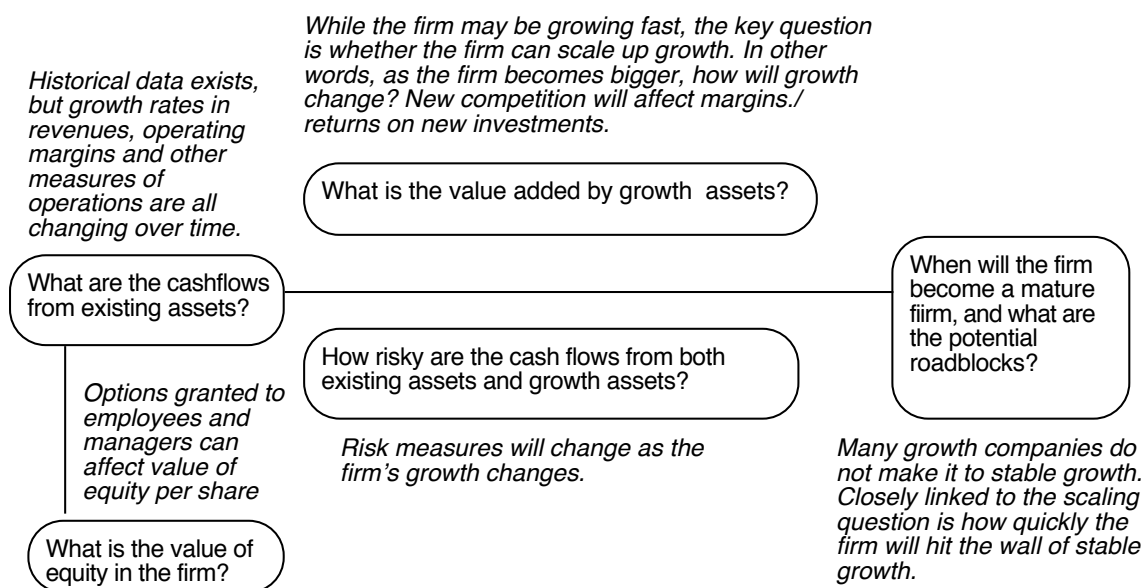
Given these problems, it is not surprising that analysts often fall back on simplistic measures of value, guesstimates or on rules of thumb to value young companies.

The Growth Phase: Growth companies

Some idea companies make it through the test of competition to become young growth companies. Their products or services have found a market niche and many of these companies make the transition to the public market, though a few remain private. Revenue growth is usually high but the costs associated with building up market share can result in losses and negative cash flows, at least early in the growth cycle. As revenue growth persists, earnings turn positive and often grow exponentially in the first few years.

Valuing young growth companies is a little easier than valuing start-up or idea companies because the markets for products and services are more clearly established and the current financial statements provides some clues to future profitability. There are four key estimation issues that can still create valuation uncertainty. The first is how well the revenue growth that the company is reporting will scale up; in other words, how quickly will revenue growth decline as the firm gets bigger? The answer will differ across companies, and will be a function of both the company's competitive advantages and the market that it serves. The second is determining how profit margins will evolve over time, as revenues grow. The third is making reasonable assumptions about reinvestment to sustain revenue growth, with concurrent judgments about the returns on investment in the business. The fourth is that as revenue growth and profit margins change over time, the risk of the firm will also shift, with the requirement that we estimate how risk will evolve in the future. The final issue that we face when valuing equity in growth companies is valuing options that the firm may granted to employees over time and the effect these grants have on value per share. Figure 2,4 captures the estimation issues that we face in valuing growth companies.

Figure 2.4: Estimation Issues - Growth Companies



As firms move through the growth cycle, from young growth to more established growth, some of these questions become easier to answer. The proportion of firm value that

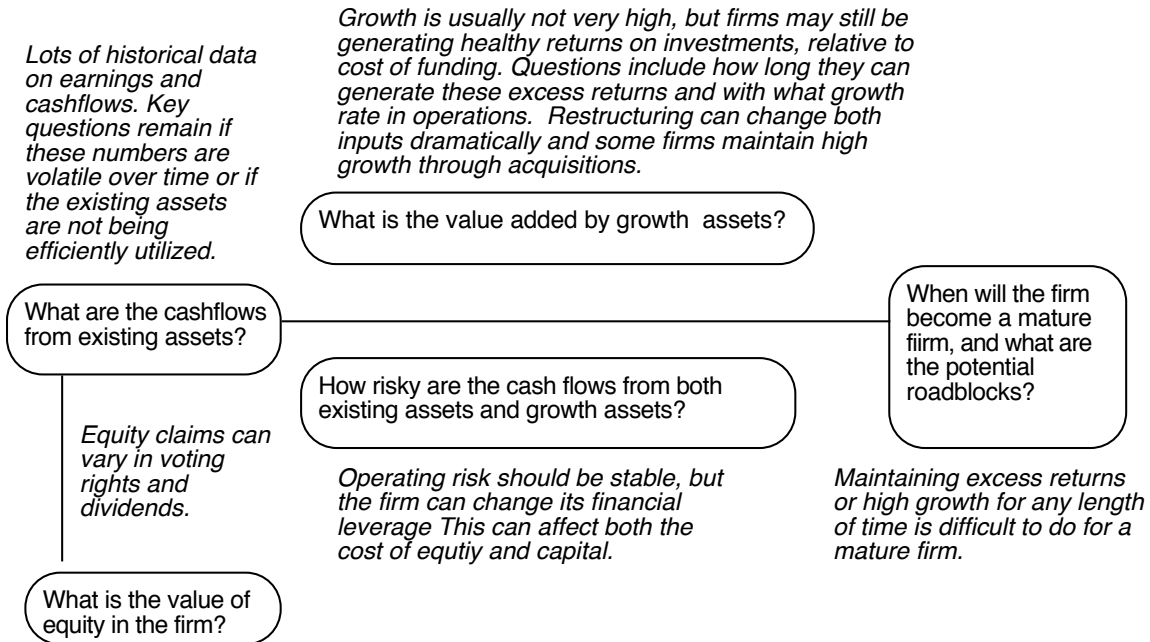
comes from growth assets declines as existing assets become more profitable and also account for a larger chunk of overall value.

Maturity – A mixed blessing: Mature firms

Even the best of growth companies reach a point where size works against them. Their growth rates in revenues and earnings converge on the growth rate of the economy. In this phase, the bulk of a firm's value comes from existing investments, and financial statements become more informative. Revenue growth is steady and profit margins have settled into a pattern, making it easier to forecast earnings and cashflows.

While estimation does become simpler with these companies, there are potential problems that analysts have to consider. The first is that the results from operations (including revenues and earnings) reflect how well the firm is utilizing its existing assets. Changes in operating efficiency can have large impacts on earnings and cash flows, even in the near term. The second is that mature firms sometimes turn to acquisitions to recreate growth potential, and predicting the magnitude and consequences of acquisitions is much more difficult to do than estimating growth from organic or internal investments. The third is that mature firms are more likely to look to financial restructuring to increase their value; the mix of debt and equity used to fund the business may change overnight and assets (such as accounts receivable) may be securitized. The final issue is that mature companies sometimes have equity claims with differences in voting right and control claims, and hence different values. Figure 2.5 frames the estimation challenges at mature companies.

Figure 2.5: Estimation Issues - Mature Companies



Not surprisingly, mature firms are usually targeted in hostile acquisitions and leveraged buyouts, where the buyer believes that changing the way the firm is run can result in significant increases in value.

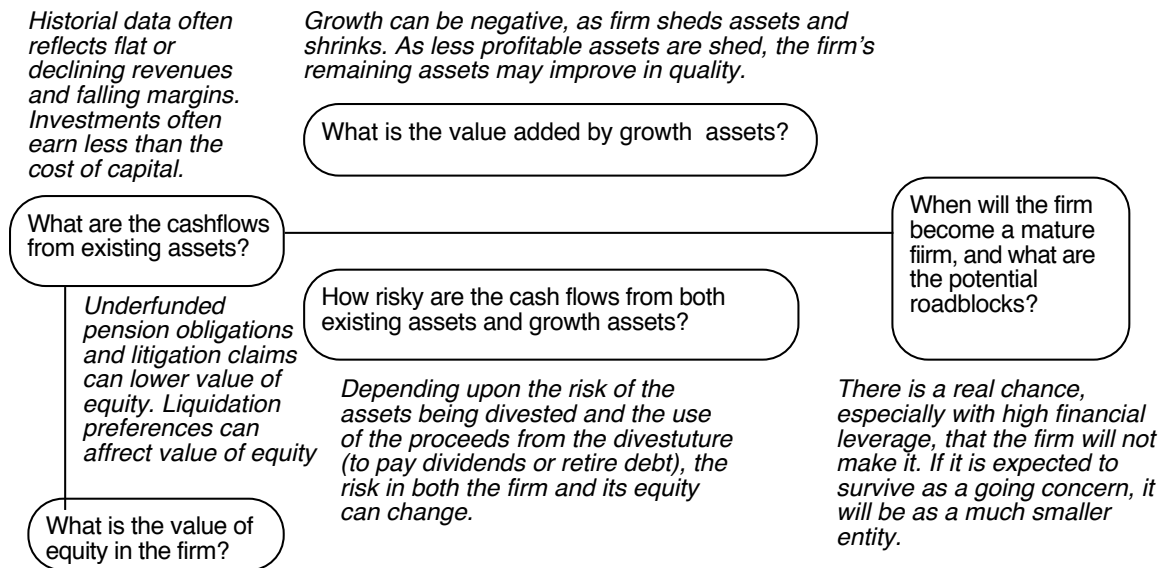
Winding down: Dealing with decline

Most firms reach a point in the life cycle, where their existing markets are shrinking and becoming less profitable, and the forecast for the future is more of the same. Under these circumstances, these firms react by selling assets and returning cash to investors. Put another way, these firms derive their value entirely from existing assets and that value is expected to shrink over time.

Valuing declining companies requires making judgments about the assets that will be divested over time and the profitability of the assets that will be left in the firm. Judgments about how much cash will be received in these divestitures and how that cash will be utilized (pay dividends, buy back shares, retire debt) can influence the value attached to the firm. There is another concern that overhangs this valuation. Some firms in decline that have significant debt obligations can become distressed, a problem not specific to declining firms but more common with them. Finally, the equity values in

declining firms can be affected significantly by the presence of underfunded pension obligations and the overhand of litigation costs than other firms. In figure 2.6, we look at these questions:

Figure 2.6: Estimation Issues - Declining Companies



Valuing firms in decline poses a special challenge for analysts who are used to conventional valuation models that adopt a growth-oriented view of the future. In other words, assuming that current earnings will grow at healthy rates for the future or forever will result in estimates of value for these firms that are way too high.

Valuation across the business spectrum

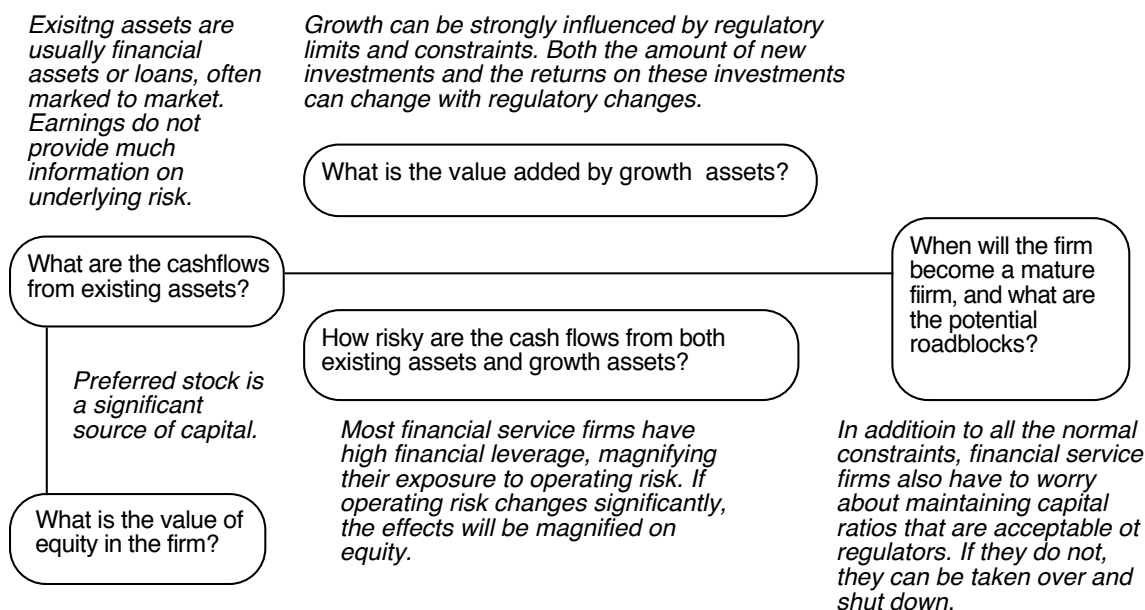
In the last section, we considered the different issues we fact in estimating cashflows, growth rates, risk and maturity across the business life cycle. In this section, we consider how firms in some businesses are more difficult to value than others. We consider five groups of companies – financial service firms such as banks, investment banks and insurance companies, cyclical and commodity businesses, businesses with intangible assets (human capital, patents, technology), emerging market companies that face significant political risk and multi-business, global companies. With each group, we examine what it is about the firms within the group that generate valuation problems.

Financial Service firms

While financial service firms have historically been viewed as stable investments that are relatively simple to value, financial crises bring out the dangers in this assumption. In 2008, for instance, the equity values at most banks swing wildly, and the equity at many others including Lehman, Bear Stearns and Fortis lost all value. It was a wake-up call to analysts who had used fairly simplistic models to value these banks and had missed the brewing problems.

So what are the potential problems with valuing financial service firms? We can frame them in terms of the four basic inputs into the valuation process. The existing assets of banks are primarily financial assets, with a good portion being traded in markets. While accounting rules require that these assets be marked to market, they are not always consistently applied across different classes of assets. Since the risk in these assets can vary widely across firms, and information about this risk is not always forthcoming, accounting errors feed into valuation errors. The risk is magnified by the high financial leverage at banks and investment banks, and it is not uncommon to see banks have debt to equity ratios of 30 to 1 or higher, allowing them to leverage up the profitability of their operations. Financial service firms are, for the most part, regulated, and regulatory rules can affect growth potential. The regulatory restrictions on book equity capital as a ratio of loans, at a bank, influences how quickly the bank can expand over time and how profitable that expansion will be. Changes in regulatory rules will therefore have big effects on growth and value, with more lenient (stricter) rules resulting in more (less) value from growth assets. Finally, since the damage created by a troubled bank or investment bank can be extensive, it is also likely that problems at these entities will evoke much swifter reactions from authorities than at other firms. A troubled bank will be quickly taken over to protect depositors, lenders and customers, but the equity in the banks will be wiped out in the process. As a final point, getting to the value of equity per share for a financial service firm can be complicated by the presence of preferred stock, which shares characteristics with both debt and equity. Figure 2.7 summarizes the valuation issues at financial service firms:

Figure 2.7: Estimation Issues - Financial Service Firms



Analysts who value banks go through cycles. In good times, they tend to under estimate the risk of financial crises and extrapolate from current profitability to arrive at higher values for financial service firms. In crises, they lose perspective and mark down the values of healthy banks and unhealthy banks, without much discrimination.

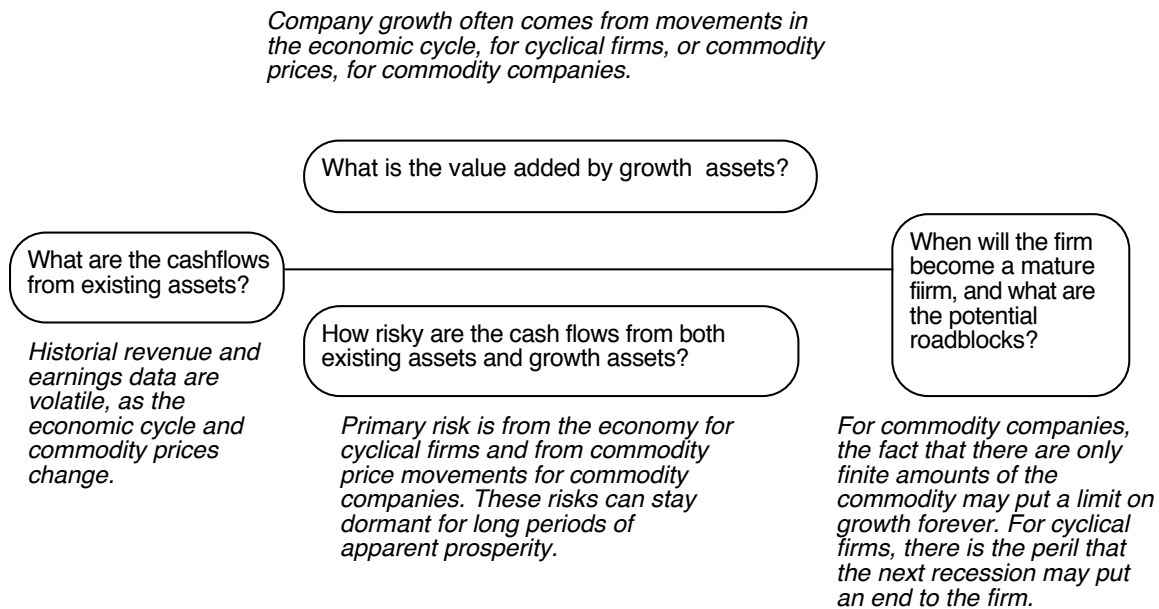
Cyclical and Commodity Companies

If we define a mature company as one that delivers predictable earnings and revenues, period after period, cyclical and commodity companies will never be mature, since even the largest, most established of them have volatile earnings. The earnings volatility has little to do with the company and is more reflective of variability in the underlying economy (for cyclical firms) or the base commodity (for a commodity company).

The biggest issue with valuing cyclical and commodity companies lies in the base year numbers that are used in valuation. If we do what we do with most other companies and use the current year as the base year, we risk building into our valuations the vagaries of the economy or commodity prices in that year. As an illustration, valuing oil companies, using earnings from 2007 as a base year, will inevitably result in too high a value; the spike in oil prices that year contributed to the profitability of almost all oil

companies, small and large, efficient and inefficient. Similarly valuing housing companies, using earnings and other numbers from 2008, when the economy was drastically slowing down, will result in values that are too low. The uncertainty we feel about base year earnings also percolates into other parts of the valuation. Estimates of growth at cyclical and commodity companies depend more on our views on overall economic growth and the future of commodity prices than they do on the investments made at individual companies. Similarly, risk that lies dormant when the economy is doing well and commodity prices are rising can manifest itself suddenly when the cycle turns. Finally, for highly levered cyclical and commodity companies, especially when the debt was accumulated during earnings upswings, a reversal of fortune can very quickly put the firm at risk. In addition, for companies like oil companies, the fact that natural resources are finite – there is only so much oil under the ground – can put a crimp on what we assume about what happens to the firm in stable growth. Figure 2.8 lists the estimation questions:

Figure 2.8: Estimation Issues - Cyclical and Commodity Companies



When valuing cyclical and commodity companies, analysts often make implicit assumptions about the economy and commodity prices by extrapolating past earnings and growth rates. Many of these implicit assumptions turn out to be unrealistic and the valuations that lead from them are equally flawed.

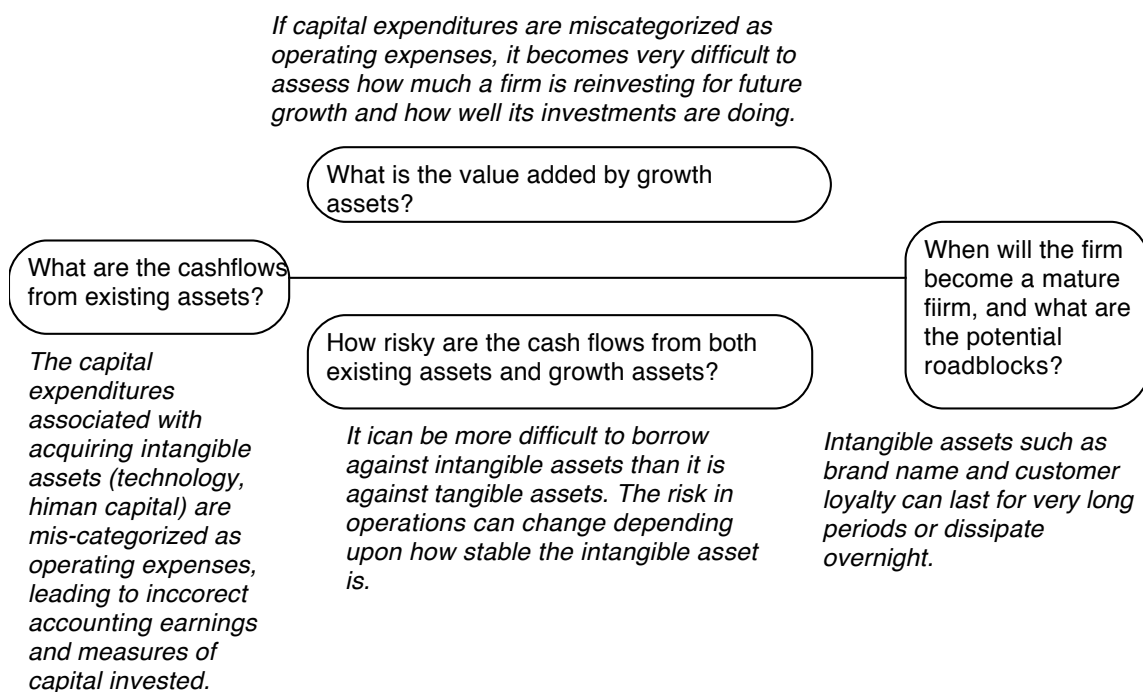
Businesses with Intangible Assets

In the last two decades, we have seen mature economies, such as the US and Western Europe, shift away from manufacturing to service and technology businesses. In the process, we have come to recognize how little of the value at many of our largest companies today comes not from physical assets (like land, machinery and factories) and how much of the value comes from intangible assets. Intangible assets range the spectrum, from brand name at Coca Cola to technological know-how at Google and human capital at firms like McKinsey. As accountants grapple with how best to deal with these intangible assets, we face similar challenges when valuing them.

Let us state at the outset that there should be no reason why the tools that we have developed over time for physical assets cannot be applied to intangible assets. The value of a brand name or patent should be the present value of the cash flows from that asset, discounted back at an appropriate risk adjusted rate. The problem that we face is that the accounting standards for firms with intangible assets are not entirely consistent with the standards for firms with physical assets. An automobile company that invests in a new plant/factory is allowed to treat that expenditure as a capital expenditure, record the item as an asset and depreciate it over its life. A technology firm that invests in research and development, with the hope of generating new patents, is required to expense the entire expenditure, record no assets and cannot amortize or depreciate the item. The same can be said of a consumer product company that expends millions on advertising with the intent of building up a brand name. The consequences for estimating the basic inputs for valuation are profound. For existing assets, the accounting treatment of intangible assets makes both current earnings and book value unreliable, since the former is net of R&D and the latter does not include investments in the firm's biggest assets. Since reinvestment and accounting return numbers are flawed for the same reasons, assessing expected growth becomes more difficult. Since lenders tend to be wary about lending to firms with intangible assets, they tend to be funded predominantly with equity, and the risk of equity can change quickly over a firm's life cycle. Finally, estimating when a firm with intangible assets gets to steady state can be complex. On the one hand, easy entry into and exit from the business and rapid changes in technology can cause growth rates to drop quickly at some firms. On the other hand, the long life of some competitive

advantages like brand name and the ease with which firms can scale up (they do not need heavy infrastructure or physical investments) can allow other firms to maintain high growth, with excess returns, for decades. The problems that we face in valuing companies with intangible assets are shown in figure 2.9:

Figure 2.9: Estimation Issues - Intangible Assets



Analysts when faced with valuing firms with intangible assets tend to use the accounting earnings and book values at these firms, without correcting for the miscategorization of capital expenditures. Any analyst who compares the PE ratio for Microsoft to the PE ratio for GE is guilty of this error. In addition, there is also the temptation, when doing valuations, to add arbitrary premiums to estimated value to reflect the value of intangibles. Thus, adding a 30% premium to the value estimate of Coca Cola is not a sensible way of capturing the value of a brand name.

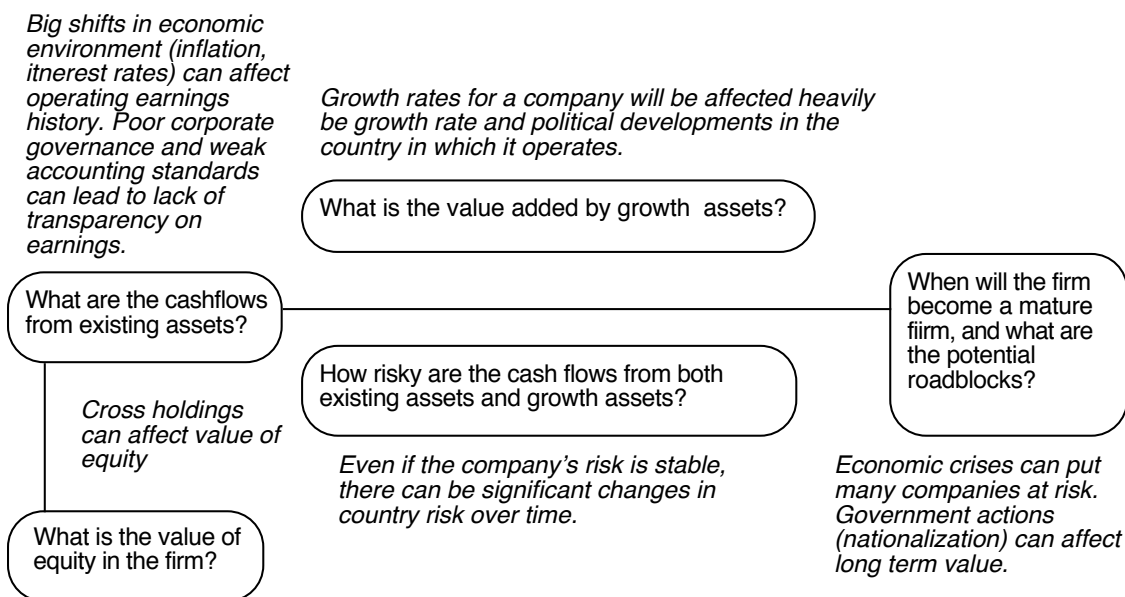
Emerging Market Companies

In the last decade, the economies that have grown the fastest have been in Asia and Latin America. With that growth, we have also seen an explosion of listings in

financial markets in these emerging economies and increased interest in valuing companies in these markets.

In valuing emerging market companies, the overriding concern that analysts have is that the risk of the countries that these companies operate in often overwhelms the risk in the companies themselves. Investing in a stable company in Argentina will still expose you to considerable risk, as country risk swings back and forth. While the inputs to valuing emerging market companies are familiar – cashflows from existing and growth assets, risk and getting to stable growth – country risk creates estimation issues with each input. Variations in accounting standards and corporate governance rules across emerging markets often result in lack of transparency when it comes to current earnings and investments, making it difficult to assess the value of existing assets. Expectations of future growth rest almost as much on how the emerging market that the company is located will evolve, as they do on the company's own prospects. Put another way, it is difficult for even the best-run emerging market company to grow, if the market it operates in is in crisis. In a similar vein, the overlay of country risk on company risk indicates that we have to confront and measure both, if we want to value emerging market companies. Finally, in addition to economic crises that visit emerging markets at regular intervals, putting all companies at risk, there is also the added risk that companies can be nationalized or appropriated by the government. The challenges associated with valuing emerging market companies are captured in figure 2.10:

Figure 2.10: Estimation Issues - Emerging Market Companies



Analysts who value emerging market companies develop their own coping mechanisms for dealing with the overhang of country, with some mechanisms being healthier than others. In its most unhealthy form, analysts avoid even dealing with the risk, switching to more stable currencies for their valuations and adopting very simplistic measures of country risk (such as adding a fixed premium to every company in a market). In other cases, their pre-occupation with country risk leads them to double count and triple count the risk and not pay sufficient attention to the company being valued.

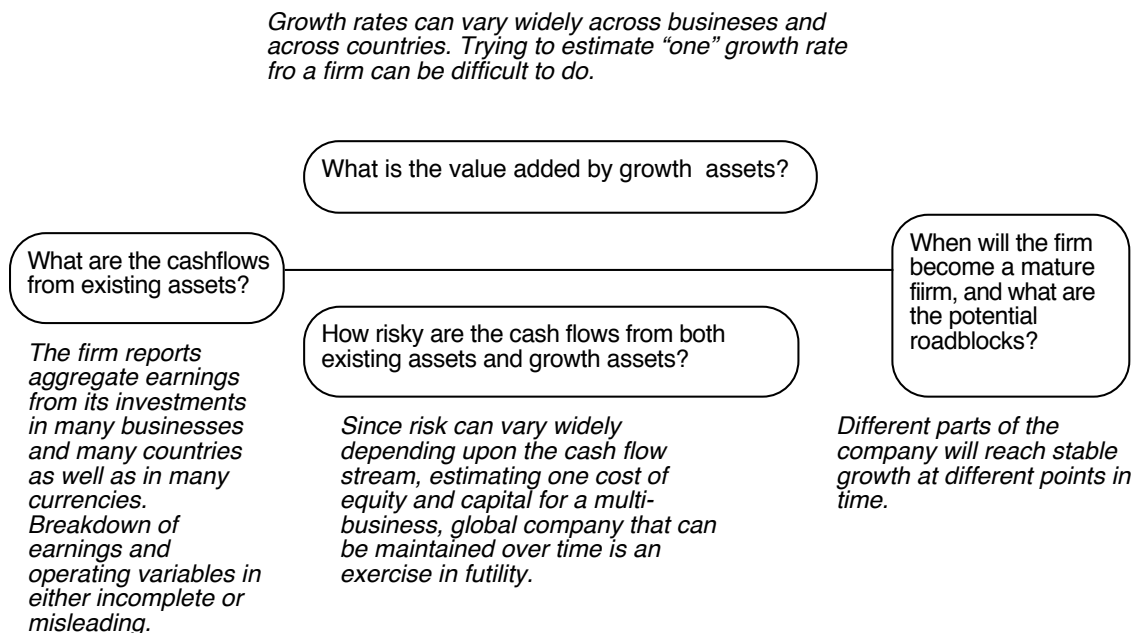
Multi-business and Global companies

As investors globalize their portfolios, companies are also becoming increasingly globalized, with many of the largest ones operating in multiple businesses. Given that these businesses have very different risk and operating characteristics, valuing the multi-business, global company can be a challenge even to the best-prepared analyst.

The conventional approach to valuing a company has generally been to work with the consolidated earnings and cashflows of the business, and discount those cash flows using an aggregated risk measure for the company that reflects its mix of businesses. While this approach works well for firms in one or few lines of business, it becomes increasingly difficult as companies spread their operations across multiple businesses in multiple markets. Consider a firm like General Electric, a conglomerate that operates in

dozens of businesses and in almost every country on the globe. The financial statements of the company reflect its aggregated operations, across its different businesses and geographic locations. Attaching a value to existing assets becomes difficult to do, since these assets vary widely in terms of risk and return generating capacity. While GE may break down earnings for its different business lines, those numbers are contaminated by the accounting allocation of centralized costs and intra-business transactions. The expected growth rates can be very different for different parts of the business, not only in terms of magnitude but also in quality. Furthermore, as the firm grows at different rates in different businesses, its overall risk will change to reflect the new business weights, adding another problem to valuation. Finally, different pieces of the company may approach stable growth at different points in time, making it difficult to stop and assess the terminal value. Figure 2.11 summarizes the estimation questions that we have to answer for complex companies.

Figure 2.11: Estimation Issues - Multi-business and Global Businesses



Analysts who value multi-business and global companies often draw on the averaging argument to justify not knowing as much as they should about individual businesses. Higher growth (risk) in some businesses will be offset by lower growth (risk) in other businesses, they argue, thus justifying their overall estimates of growth and risk. They

underestimate the dangers of the unknown. All too often, with companies like these, what you do not know is more likely to contain bad news than good news.

Conclusion

Some companies are easier to value than others. When we have to leave the comfort zone of companies with solid earnings and predictable futures, we invariably stray into the dark side of valuation, where we invent new principles, violate established ones and come up with unsustainable values for businesses.

In this chapter, we laid out the four inputs that we have to estimate to value any company – the expected cash from investments that the business has already made (existing assets), the value that will be added by new investments (growth assets), the risk in these cash flows and the point in time where we expect the firm to become a mature firm. The estimation challenges we face will vary widely across companies and we consider how estimation issues vary across the life cycle of a firm. For young and start-up firms, the absence of historical data and the dependence on growth assets makes estimating future cash flows and risk particularly difficult to do. With growth firms, the question shifts to whether growth rates can be maintained and, if so, for how long, as firms scale up. With mature firms, the big issue in valuation shifts to whether existing assets are being efficiently utilized and whether the financial mix used by the firm makes sense; restructuring the firm to make it better run may dramatically alter value. For declining firms, estimating revenues and margins as assets get divested, while also considering the possibility of default can be tricky. The estimation challenges we face can also be different for different subsets of companies – cyclical and commodity companies have volatile operating results, companies with intangible assets have earnings that are skewed by how accountants treat investments in these assets, and the risk in emerging market and global companies can be difficult to assess. Finally, valuing any company can become more difficult in economies where the fundamentals – riskfree rates, risk premiums and economic growth – are volatile.

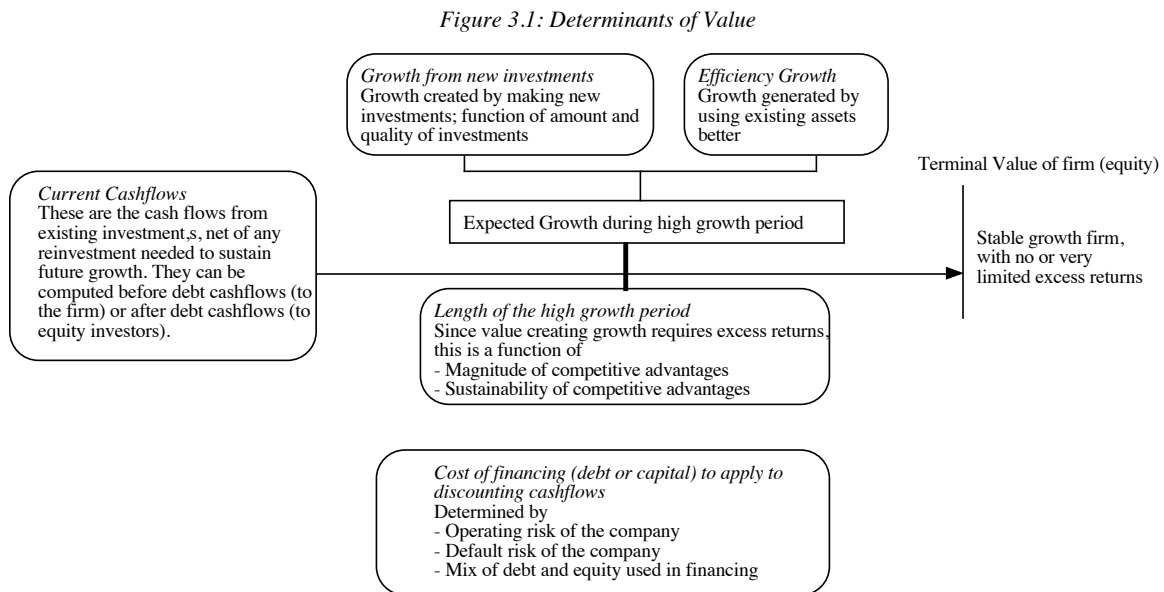
CHAPTER 3

INTRINSIC VALUATION: THE COST OF CAPITAL (WACC) APPROACH

In discounted cashflows valuation, the value of an asset is the present value of the expected cashflows on the asset, discounted back at a rate that reflects the riskiness of these cashflows. In chapter 1, we noted the distinction between valuing a business and valuing equity in the business. In this section, we will focus on how best to value a business, using the cost of capital approach.

Determinants of Value

While you can choose to value just the equity or the entire business, there are four basic inputs that you need for a value estimate, though how you define the inputs will be different depending upon whether you do firm or equity valuation. Figure 3.1 summarizes the determinants of value.



The first input is the cashflow from existing assets, defined either as pre-debt (and to the firm) or as post-debt (and to equity) earnings, net of reinvestment to generate future growth. With equity cashflows, we can use an even stricter definition of cash flow and consider only dividends paid. The second input is growth, with growth in operating income being the key input when valuing the entire business and growth in equity income (net income or earnings per share) becoming the focus when valuing equity. The third

input is the discount rate, defined as the cost of the overall capital of the firm, when valuing the business, and as cost of equity, when valuing equity. The final input, allowing for closure, is the terminal value, defined as the estimated value of firm (equity) at the end of the forecast period in firm (equity) valuation.

For the rest of this section, we will focus on estimating the inputs into firm valuation models, starting with the cashflows, moving on to risk (and discount rates) and then closing with a discussion of how best to estimate the growth rate for the high growth period and the value at the end of that period.

Cash Flows

When valuing a business, as opposed to just the equity stake in the business, we begin by separating the operating assets of the business from its cash and marketable securities and estimating the cash flows from those operating assets. This operating cash flow, which is generally called the free cash flow to the firm (FCFF) should be both after taxes and after all reinvestment needs have been met. Since a firm raises capital from debt and equity investors, the cash flow to the firm should be before interest and principal payments on debt.

To estimate the free cash flow to the firm, we start with after-tax operating earnings and to estimate the cash flows to the firm prior to debt payments but after reinvestment needs have been met:

Step 1: To estimate the operating earnings after taxes, we start with the reported earnings before interest and taxes (EBIT) and estimate the taxes we would have paid if this income had been the taxable income.

After-tax operating income = EBIT (1- tax rate)

There are two points to note about this measure. The first is that the tax rate used to compute after-tax income can be much lower than the statutory tax rate in the near term, but that gap will narrow over time, as the company matures. The second is that the actual taxes paid can and often is much lower than the hypothetical tax that we are estimating in the computation. This will especially be true for firms with substantial interest expenses, since these expenses can reduce taxable income and taxes. We are intentionally ignoring

the tax benefits accruing from interest expenses, when computing cash flows, because we will consider these same tax benefits, when computing the discount rate (cost of capital).

Step 2: To estimate reinvestment, we will break it down into two parts:

- a. *Reinvestment in long-lived assets* is measured as the difference between capital expenditures (the amount invested in long lived assets during the period) and depreciation (the accounting expense generated by capital expenditures in prior periods). We net the latter because it is not a cash expense and hence can be added back to net income.
- b. *Reinvestment in short-lived assets* is measured by the change in non-cash working capital. In effect, increases in inventory and accounts receivable represent cash tied up in assets that do not generate returns – wasting assets. The reason we do not consider cash in the computation is because we assume that companies with large cash balances generally invest them in low-risk, marketable securities like commercial paper and treasury bills; these investments earn a low but a fair rate of return and are therefore not wasting assets.³ To the extent that they are offset by the use of supplier credit and accounts payable, the effect on cash flows can be muted. The overall change in non-cash working capital (Δ NWC) therefore is investment in short-term assets.

In summary, the free cash flow to the firm can be written as:

$$\text{FCFF} = \text{EBIT} (1-t) - (\text{Capital Expenditures} - \text{Depreciation}) - \Delta \text{NWC}$$

Reinvestment reduces the free cash flow, but it provides a payoff in terms of future growth. We will come back and consider whether the net effect is positive or negative after we consider how best to estimate growth.

Another way of presenting the same equation is to cumulate the net capital expenditures and working capital change into one number, and state it as a percentage of the after-tax operating income. This ratio of reinvestment to after-tax operating income is called the reinvestment rate, and the free cash flow to the firm can be written as:

³ Note that we do not make the distinction between operating and non-operating cash that some analysts do (they proceed to include operating cash in working capital). Our distinction is between wasting cash (which would include currency or cash earning below-market rate returns) and non-wasting cash. We are assuming that the former will be a small or negligible number at a publicly traded company.

$$\text{Reinvestment Rate} = \frac{(\text{Capital Expenditures} - \text{Depreciation} + \Delta \text{ Working Capital})}{\text{After - tax Operating Income}}$$

$$\text{Free Cash Flow to the Firm} = \text{EBIT} (1-t) (1 - \text{Reinvestment Rate})$$

Note that the reinvestment rate can exceed 100%⁴, if the firm has substantial reinvestment needs. The reinvestment rate can also be less than zero, for firms that are divesting assets and shrinking capital.

A few final thoughts about free cash flow to the firm are worth noting before we move on to discount rates. First, the free cash flow to the firm can be negative, just as the FCFE can, but debt cash flows can no longer be the culprit; even highly levered firms that are paying down debt will report positive FCFE while also registering negative FCFE. If the FCFE is negative, the firm will be raising fresh capital, with the mix of debt and equity being determined by the mix used to compute the cost of capital. Second, the cash flow to the firm is the basis for all cash distributions made by the firm to its investors; dividends, stock buybacks, interest payments and debt repayments all have to be made out of these cash flows.

Risk

Cash flows that are riskier should be assessed a lower value than more stable cashflows, but how do we measure risk and reflect it in value? In conventional discounted cash flow valuation models, the discount rate becomes the vehicle for conveying our concerns about risk. We use higher discount rates on riskier cash flows and lower discount rates on safer cash flows. In this section, we will begin by contrasting how the risk in equity can vary from the risk in a business, and then consider the mechanics of estimating the cost of equity and capital.

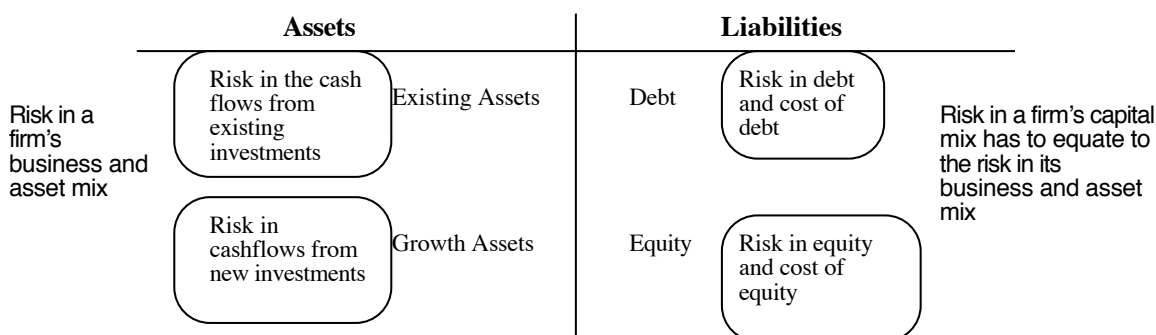
Business Risk versus Equity Risk

Before we delve into the details of risk measurement and discount rates, we should draw a contrast between two different ways of thinking about risk that relate back to the financial balance sheet that we presented in chapter 1. In the first, we think about

⁴ In practical terms, this firm will have to raise external financing, either from debt or equity or both, to cover the excess reinvestment.

the risk in a firm's operations or assets, i.e., the risk in the business. In the second, we look at the risk in the equity investment in this business. Figure 3.2 captures the differences between the two measures:

Figure 3.2: Risk in Business versus Risk in Equity



As with any other aspect of the balance sheet, this one has to balance as well, with the weighted risk in the assets being equal to the weighted risk in the ingredients to capital – debt and equity. Note that the risk in the equity investment in a business is partly determined by the risk of the business the firm is in and partly by its choice on how much debt to use to fund that business. The equity in a safe business can be rendered risky, if the firm uses enough debt to fund that business.

In discount rate terms, the risk in the equity in a business is measured with the cost of equity, whereas the risk in the business is captured in the cost of capital. The latter will be a weighted average of the cost of equity and the cost of debt, with the weights reflecting the proportional use of each source of funding.

Measuring Equity Risk and the Cost of Equity

Measuring the risk in equity investments and converting that risk measure into a cost of equity is rendered difficult by two factors. The first is that equity has an implicit cost, which is unobservable, unlike debt, which comes with an explicit cost in the form of an interest rate. The second is that risk in the eyes of the beholder and different equity investors in the same business can have very different perceptions of risk in that business and demand different expected returns as a consequence.

The Diversified Marginal Investor

If there were only one equity investor in a company, estimating equity risk and the cost of equity would be a far simpler exercise. We would measure the risk of investing in equity in that company to the investor and assess a reasonable rate of return, given that risk. In a publicly traded company, we run into the practical problem that the equity investors number in the hundreds, if not the thousands, and that they not only vary in size, from small to large investors, but also in risk aversion. So, whose perspective should we take when measuring risk and cost of equity? In corporate finance and valuation, we develop the notion of the *marginal investor*, i.e., the investor most likely to influence the market price of publicly traded equity. The marginal investor in a publicly traded stock has to own enough stock in the company to make a difference and be willing to trade on that stock. The common theme shared by risk and return models in finance is that the marginal investor is diversified, and we measure the risk in an investment as the risk added to a diversified portfolio. Put another way, it is only that portion of the risk in an investment that is attributable to the broader market or economy, and hence not diversifiable, that should be built into expected returns.

Models for Expected Return (Cost of Equity)

It is on the issue of how best to measure this non-diversifiable risk that the different risk and return models in finance part ways. Let us consider the alternatives:

- In the *capital asset pricing model (CAPM)*, this risk is captured in the *beta* that we assign an asset/business, with that number carrying the burden of measuring exposure to all of the components of market risk. The expected return on an investment can then be specified as a function of three variables – the riskfree rate, the beta of the investment and the equity risk premium (the premium demanded for investing in the average risk investment):

$$\text{Expected Return} = \text{Riskfree Rate} + \text{Beta}_{\text{Investment}} (\text{Equity Risk Premium})$$

The riskfree rate and equity risk premium are the same for all investments in a market but the beta will capture the market risk exposure of the investment; a beta of one represents an average risk investment, and betas above (below) one indicate investments that are riskier (safer) than the average risk investment in the market.

- In the arbitrage pricing and multi-factor models, we allow for multiple sources of non-diversifiable (or market) risk and estimate betas against each one. The expected return on an investment can be written as a function of the multiple betas (relative to each market risk factor) and the risk premium for that factor. If there are k factors in the model with β_{ji} and Risk Premium $_j$ representing the beta and risk premium of factor j , the expected return on the investment can be written as:

$$\text{Expected Return} = \text{Riskfree Rate} + \sum_{j=1}^{j=k} \beta_j (\text{Risk Premium}_j)$$

Note that the capital asset pricing model can be written as a special case of these multi-factor models, with a single factor (the market) replacing the multiple factors.

- The final class of models can be categorized as proxy models. In these models, we essentially give up on measuring risk directly and instead look at historical data for clues on what types of investments (stocks) have earned high returns in the past, and then use the common characteristic(s) that they share as a measure of risk. For instance, researchers have found that market capitalization and price to book ratios are correlated with returns; stocks with small market capitalization and low price to book ratios have historically earned higher returns than large market stocks with higher price to book ratios. Using the historical data, we can then estimate the expected return for a company, based on its market capitalization and price to book ratio.

$$\text{Expected Return} = a + b(\text{Market Capitalization}) + c (\text{Price to Book Ratio})$$

Since we are no longer working within the confines of an economic model, it is not surprising that researchers keep finding new variables (trading volume, price momentum) that improve the predictive power of these models. The open question, though, is whether these variables are truly proxies for risk or indicators of market inefficiency. In effect, we may be explaining away the misvaluation of classes of stock by the market by using proxy models for risk.

Estimation Issues

With the CAPM and multi-factor models, the inputs that we need for the expected return are straightforward. We need to come up with a risk free rate and an equity risk

premium (or premiums in the multi-factor models) to use across all investments. Once we have these market-wide estimates, we then have to measure the risk (beta or betas) in individual investments. In this section, we will lay out the broad principles that will govern these estimates:

- The riskfree rate is the expected return on an investment with guaranteed returns; in effect, your expected return is also your actual return. Since the return is guaranteed, there are two conditions that an investment has to meet to be riskfree. The first is that the entity making the guarantee has to have no default risk; this is why we use government securities to derive riskfree rates, a necessary though not always a sufficient condition. There is default risk in many government securities that is priced into the expected return. The second is that the time horizon matters. A six-month treasury bill is not riskfree, if you are looking at a five-year time horizon, since we are exposed to reinvestment risk. In fact, even a 5-year treasury bond may not be riskless, since the coupons received every six months have to be reinvested. Clearly, getting a riskfree rate is not as simple as it looks at the outset.
- The equity risk premium is the premium that investors demand for investing in risky assets (or equities) as a class, relative to the riskfree rate. It will be a function not only of how much risk investors perceive in equities, as a class, but the risk aversion that they bring to the market. It also follows that the equity risk premium can change over time, as market risk and risk aversion both change. The conventional practice for estimating equity risk premiums is to use the historical risk premium, i.e., the premium investors have earned over long periods (say 75 years) investing in equities instead of riskfree (or close to riskfree) investments. An alternative approach is to estimate the equity risk premium, implied by current stock prices and expected future cash flows. This approach will yield more dynamic premiums, but they will also be more volatile.
- To estimate the beta in the CAPM and betas in multi-factor models, we draw on statistical techniques and historical data. The standard approach for estimating the CAPM beta is to run a regression of returns on a stock against returns on a broad equity market index, with the slope capturing how much the stock moves, for any given market move. To estimate betas in the arbitrage pricing model, we use

historical return data on stocks and factor analysis to extract both the number of factors in the models, as well as factor betas for individual companies. As a consequence, the beta estimates that we obtain will always be backward looking (since they are derived from past data) and noisy (they are statistical estimates, with standard errors). In addition, these approaches clearly will not work for investments that do not have a trading history (young companies, divisions of publicly traded companies). One solution is to replace the regression beta with a *bottom-up beta*, i.e., a beta that is based upon industry averages for the businesses that the firm is in, adjusted for differences in financial leverage.⁵ Since industry averages are more precise than individual regression betas, and the weights on the businesses can reflect the current mix of a firm, bottom up betas generally offer better estimates for the future.

The Cost of Debt

While equity investors receive residual cash flows and bear the bulk of the operating risk in most firms, lenders to the firm also face the risk that they will not receive their promised payments – interest expenses and principal repayments. It is to cover this default risk that lenders add a “default spread” to the riskless rate when they lend money to firms; the greater the perceived risk of default, the greater the default spread and the cost of debt. The other dimension on which debt and equity can vary is in their treatment for tax purposes, with cashflows to equity investors (dividends and stock buybacks) coming from after-tax cash flows, whereas interest payments are tax deductible. In effect, the tax law provides a benefit to debt and lowers the cost of borrowing to businesses.

⁵ The simplest and most widely used equation relating betas to debt to equity ratios is based on the assumption that debt provides a tax advantage and that the beta of debt is zero.

$$\text{Beta for equity} = \text{Beta of business} * (1 + (1 - \text{tax rate}) (\text{Debt} / \text{Equity}))$$

The beta for equity is a levered beta, whereas the beta of the business is titled an unlevered beta. Regression betas are equity betas and are thus levered – the debt to equity ratio over the regression period is embedded in the beta.

To estimate the cost of debt for a firm, we need three components. The first is the riskfree rate, an input to the cost of equity as well. As a general rule, the riskfree rate used to estimate the cost of equity should be used to compute the cost of debt as well; if the cost of equity is based upon a long-term riskfree rate, as it often is, the cost of debt should be based upon the same rate. The second is the default spread and there are three approaches that are used, depending upon the firm being analyzed.

- If the firm has traded bonds outstanding, the current market interest rate on the bond (yield to maturity) is used as the cost of debt. This is appropriate only if the bond is liquid and is representative of the overall debt of the firm; even risky firms can issue safe bonds, backed up by the most secure assets of the firms.
- If the firm has a bond rating from an established ratings agency such as S&P or Moody's, we can estimate a default spread based upon the rating. In September 2008, for instance, the default spread for BBB rated bonds was 2% and would have been used as the spread for any BBB rated company.
- If the firm is unrated and has debt outstanding (bank loans), we can estimate a "synthetic" rating for the firm, based upon its financial ratios. A simple, albeit effective approach for estimating the synthetic ratio is to base it entirely on the interest coverage ratio (EBIT/ Interest expense) of a firm; higher interest coverage ratios will yield higher ratings and lower interest coverage ratios.

The final input needed to estimate the cost of debt is the tax rate. Since interest expenses save you taxes at the margin, the tax rate that is relevant for this calculation is not the effective tax rate but the marginal tax rate. In the United States, where the federal corporate tax rate is 35% and state and local taxes add to this, the marginal tax rate for corporations in 2008 was close to 40%, much higher than the average effective tax rate, across companies, of 28%. The after-tax cost of debt for a firm is therefore:

After-tax cost of debt = (Riskfree Rate + Default Spread) (1- Marginal tax rate)

The after-tax cost of debt for most firms will be significantly lower than the cost of equity for two reasons. First, debt in a firm is generally less risky than its equity, leading to lower expected returns. Second, there is a tax saving associated with debt that does not exist with equity.

Debt Ratios and the Cost of Capital

Once we have estimated the costs of debt and equity, we still have to assign weights for the two ingredients. To come up with this value, we could start with the mix of debt and equity that the firm uses right now. In making this estimate, the values that we should use are market values, rather than book values. For publicly traded firms, estimating the market value of equity is usually a trivial exercise, where we multiply the share price by the number of shares outstanding. Estimating the market value of debt is usually a more difficult exercise, since most firms have some debt that is not traded. Though many practitioners fall back on book value of debt as a proxy of market value, estimating the market value of debt is still a better practice.

Once we have the current market value weights for debt and equity for use in the cost of capital, we have a follow up judgment to make in terms of whether these weights will change or remain stable. If we assume that they will change, we have to specify both what the right or target mix for the firm will be and how soon the change will occur. In an acquisition, for instance, we can assume that the acquirer can replace the existing mix with the target mix instantaneously. As passive investors in publicly traded firms, we have to be more cautious, since we do not control how a firm funds its operations. In this case, we may adjust the debt ratio from the current mix to the target over time, with concurrent changes in the costs of debt, equity and capital. In fact, the last point about debt ratios and costs of capital changing over time is worth reemphasizing. As companies change over time, we should expect the cost of capital to change as well.

Growth Rates

There is no other ingredient in discounted cash flow valuation that evokes as much angst as estimating future growth. Unlike cash flows and discount rates, where we often have the security of historical data, growth rates require us to grapple with the future. In this section, we will examine two of the standard approaches for estimating growth (by looking at the past and using analyst estimates) and close with a discussion of the fundamentals that determine growth.

Historical and Forecasted Growth Rates

When confronted with the task of estimating growth, it is not surprising that analysts turn to the past. In effect, they use growth in revenues or earnings in the recent past as a predictor of growth in the future. Before we put this practice under the microscope, we should add that the historical growth rates for the same company can yield different estimates for the following reasons:

1. Earnings measure: The growth rates in earnings per share, net income, operating income and revenues can be very different for the same firm over a specified time period.
2. Period of analysis: For firms that have been in existence for long periods, the growth rates can be very different if we look at ten years of history as opposed to five years.
3. Averaging approach: Even if we agree on an earnings measure and time period for the analysis, the growth rates we derive can be different, depending upon how we compute the values. We could, for instance, compute the growth rate in each period and average the growth rates over time, yielding an arithmetic average. Alternatively, we could use just the starting and ending values for the measure and compute a geometric average. For firms with volatile earnings, the latter can generate a very different (and lower) value for growth than the former.

A debate how best to estimate historical growth makes sense only if it is a good predictor of future growth. Unfortunately, studies that have looked at the relationship have generally concluded that (a) the relationship between past and future growth is a very weak one, (b) scaling matters, with growth dropping off significantly as companies grow and (c) firms and sectors grow through growth cycles, with high growth in one period followed by low growth in the next.

If historical growth is not a useful predictor of future growth, there is another source that we can use for future growth. We can draw on those who know the firm better than we do – equity research analysts who have tracked the firm for years or the managers in the firm – and use their estimates of growth. On the plus side, these forecasts should be based upon better information than we have available to us. After all, managers should have a clearer sense of how much they will reinvest in their own businesses and what the potential returns on investments are when they do, and equity research analysts

have sector experience and informed sources that they can draw on for better information. On the minus side, neither managers nor equity research analysts are objective about the future; managers are likely to over estimate their capacity to generate growth and analysts have their own biases. In addition, both analysts and managers can get caught up in the mood of the moment, over estimating growth in buoyant times and under estimating growth in down times. As with historical growth, studies indicate that neither analyst estimates nor management forecasts are good predictors of future growth.

Fundamental Growth Rates

If we cannot draw on history or trust managers and analysts, how then do we estimate growth? The answer lies in the fundamentals within a firm that ultimately determine its growth rate. In this section, we will consider the two sources for growth – new investments that expand the business and improved efficiency on existing investments.

Growth from new investments

In the long term, growth in earnings comes from how much a firm reinvests back into the business and the return it generates on these investments. When looking at operating earnings, the focus is on the investment in capital and the return is the return on capital. In our earlier discussion of FCFE, we measured the magnitude of reinvestment with the reinvestment rate (RIR), defined as the portion of after-tax operating income put back into the business in long term and short term investments. Staying consistent with the focus on overall capital, we define the returns that we generate on investments with a return on invested capital (ROIC)

$$\text{Return on Capital (ROIC)} = \frac{\text{Operating Income}_t (1 - \text{tax rate})}{\text{Book Value of Invested Capital}_{t-1}}$$

Book value of Invested Capital = Book value of Debt + Book value of equity - Cash

Note that this is perhaps the only place in valuation where we use book values, with the rationale being that we want to measure what has actually been invested in projects, rather than their current market value.

Bringing together the reinvestment rate and the return on capital, we obtain the measure for organic or fundamental growth:

Expected Growth in operating income = Reinvestment Rate * ROIC

There are two issues that arise in this estimate. The first is that uaccounting measures (earnings and book value) can be skewed by accounting choices on restructuring charges, amortization and capitalization all making a difference in the final numbers.⁶ The other issue that we have to consider is the difference between marginal and average returns. Note that the return on investment that we use to compute the growth from new investments should be the return earned on those investments alone, i.e, a marginal return. The return on existing assets is an average return on a portfolio of investments already made. While we often use the same value for both numbers in valuation, they can be different, in fact, very different in practice.

Efficiency Growth

For many mature firms with limited investment opportunities, the potential for growth from new investments is limited. These firms cannot maintain a high reinvestment rate and deliver a high return on capital with that reinvestment. However, they can still grow at healthy rates if they can improve the returns that they earn on existing assets or margins on revenues. Conversely, declines in returns on existing assets can translate into drops in earnings growth rates. The simplest way to present efficiency growth is in terms of improving operating margins over time. If we state operating earnings in terms of revenues and margins, we get the following:

EBIT in period t = Revenues in period t * Expected operating margin in period t

Thus, if you expect operating margins to change over time, you should not use the fundamental growth equation developed in the last section as your base growth rate. Instead, you should forecast revenues and margins over time, and estimate the operating income as the product of the two.

When valuing companies, efficiency growth is pure gravy in terms of value created, since the growth comes with no concurrent cost. Unlike growth from new investments, where the positive effects of growth have to be offset against the negative

⁶ To get a sense of the problems with using accounting numbers, and how best to correct for them, see: Damodaran, A., 2007, Return on capital, Return on Invested Capital and Return on Equity: Measurement and Implications, Working Paper, SSRN.

effect of more investment, improving the return on capital on existing assets increases the growth rate without adversely affecting the cash flows. It should as come as no surprise, then, that analysts who want to increase the value of a company draw on the efficiency argument to justify much higher growth rates than those estimated using fundamentals.

While the potential for efficiency growth is always there, you can draw on increased efficiency to justify growth only for finite periods. After all, a firm cannot be infinitely inefficient. In discounted cash flow valuation, this has a practical consequence: you can draw on both efficiency and new investments to justify growth during the high growth period, but only on new investments to justify growth forever (in the terminal value computation).

Terminal Value

Publicly traded firms do not have finite lives. Given that we cannot estimate cash flows forever, we generally impose closure in valuation models by stopping our estimation of cash flows sometime in the future and then computing a terminal value that reflects all cash flows beyond that point. There are three approaches generally used to estimate the terminal value. The most common approach, which is to apply a multiple to earnings in the terminal year to arrive at the terminal value, is inconsistent with intrinsic valuation. Since these multiples are usually obtained by looking at what comparable firms are trading at in the market today, this is a relative valuation, rather than a discounted cash flow valuation. The two more legitimate ways of estimating terminal value are to estimate a liquidation value for the assets of the firm, assuming that the assets are sold in the terminal year, and the other is to estimate a going concern or a terminal value.

1. Liquidation Value

If we assume that the business will be ended in the terminal year and that its assets will be liquidated at that time, we can estimate the proceeds from the liquidation. This liquidation value still has to be estimated, using a combination of market-based numbers (for assets that have ready markets) and cashflow-based estimates. For firms that have finite lives and marketable assets (like real estate), this represents a fairly

conservative way of estimating terminal value. For other firms, estimating liquidation value becomes more difficult to do, either because the assets are not separable (brand name value in a consumer product company) or because there is no market for the individual assets. One approach is to use the estimated book value of the assets as a starting point, and to estimate the liquidation value, based upon the book value.

2. Going Concern or Terminal value

If we treat the firm as a going concern at the end of the estimation period, we can estimate the value of that concern by assuming that cash flows will grow at a constant rate forever afterwards. This perpetual growth model draws on a simple present value equation to arrive at terminal value:

$$\text{Terminal Value}_n = \frac{\text{Cashflow in year } n+1}{(\text{Discount rate} - \text{Perpetual growth rate})}$$

Our definitions of cash flow and growth rate have to be consistent with whether we are valuing dividends, cash flows to equity or cash flows to the firm; the discount rate will be the cost of equity for the first two and the cost of capital for the last. The perpetual growth model is a powerful one, but it can be easily misused. In fact, analysts often use it as a piggy bank that they go to whenever they feel that the value that they have derived for an asset is too low or high. Small changes in the inputs can alter the terminal value dramatically. Consequently, there are three key constraints that should be imposed on its estimation:

a. Cap the growth rate: Small changes in the stable growth rate can change the terminal value significantly and the effect gets larger as the growth rate approaches the discount rate used in the estimation. The fact that a stable growth rate is constant forever, however, puts strong constraints on how high it can be. Since no firm can grow forever at a rate higher than the growth rate of the economy in which it operates, the constant growth rate cannot be greater than the overall growth rate of the economy. So, what is the maximum stable growth rate that you can use in a valuation? The answer will depend on whether the valuation is being done in real or nominal terms, and if the latter, the currency used to estimate cash flows. With the former, you would use the real growth rate in the economy as your constraint, whereas with the latter, you would add expected inflation in the

currency to the real growth. Setting the stable growth rate to be less than or equal to the growth rate of the economy is not only the consistent thing to do but it also ensures that the growth rate will be less than the discount rate. This is because of the relationship between the riskless rate that goes into the discount rate and the growth rate of the economy. Note that the riskless rate can be written as:

Nominal riskless rate = Real riskless rate + Expected inflation rate

In the long term, the real riskless rate will converge on the real growth rate of the economy and the nominal riskless rate will approach the nominal growth rate of the economy. In fact, a simple rule of thumb on the stable growth rate is that it should not exceed the riskless rate used in the valuation.

b. Use mature company risk characteristics: As firms move from high growth to stable growth, we need to give them the characteristics of stable growth firms. A firm in stable growth is different from that same firm in high growth on a number of dimensions. In general, you would expect stable growth firms to be less risky and use more debt. In practice, we should move betas for even high risk firms towards one in stable growth and give them debt ratios, more consistent with larger, more stable cashflows.

c. Reinvestment and Excess Return Assumptions: Stable growth firms tend to reinvest less than high growth firms and it is critical that we both capture the effects of lower growth on reinvestment and that we ensure that the firm reinvests enough to sustain its stable growth rate in the terminal phase. Given the relationship between growth, reinvestment rate and returns that we established in the section on expected growth rates, we can estimate the reinvestment rate in terminal value:

Reinvestment rate in terminal year = $\frac{\text{Stable growth rate}}{\text{ROIC in stable phase}}$

Linking the reinvestment rate and retention ratio to the stable growth rate also makes the valuation less sensitive to assumptions about stable growth. While increasing the stable growth rate, holding all else constant, can dramatically increase value, changing the reinvestment rate as the growth rate changes will create an offsetting effect.

Terminal Value = $\frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment Rate})}{\text{Cost of Capital}_n - \text{Stable Growth Rate}}$

The gains from increasing the growth rate will be partially or completely offset by the loss in cash flows because of the higher reinvestment rate. Whether value increases or decreases as the stable growth increases will entirely depend upon what you assume about excess returns. If the return on capital is higher than the cost of capital in the stable growth period, increasing the stable growth rate will increase value. If the return on capital is equal to the stable period cost of capital, increasing the stable growth rate will have no effect on value. Substituting in the stable growth rate as a function of the reinvestment rate, from above, you get:

$$\text{Terminal Value} = \frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment Rate})}{\text{Cost of Capital}_n - (\text{Reinvestment Rate} * \text{Return on Capital})}$$

Setting the return on capital equal to the cost of capital, you arrive at:

$$\text{Terminal Value}_{\text{ROC=WACC}} = \frac{\text{EBIT}_{n+1}(1-t)}{\text{Cost of Capital}_n}$$

In closing, the key assumption in the terminal value computation is not what growth rate you use in the valuation, but what excess returns accompany that growth rate. If you assume no excess returns, the growth rate becomes irrelevant. There are some valuation experts who believe that this is the only sustainable assumption, since no firm can maintain competitive advantages forever. In practice, though, there may be some wiggle room, insofar as the firm may become a stable growth firm before its excess returns go to zero. If that is the case and the competitive advantages of the firm are strong and sustainable (even if they do not last forever), we may be able to give the firm some excess returns in perpetuity. As a simple rule of thumb again, these excess returns forever should be modest (<4-5%) and will affect the terminal value.

Tying up loose ends

We have covered the four inputs that go into discounted cash flow valuation models – cash flows, discount rates, growth rates and the terminal value. The present value we arrive at, when we discount the cash flows at the risk-adjusted rates should yield an estimate of value, but getting from that number to what we would be willing to pay per share for equity does require use to consider a few other factors.

- a. Cash and Marketable Securities: Most companies have cash balances that are not insignificant in magnitude. Is this cash balance already incorporated into the present value? Since the FCFF is based on operating income, and interest income from cash is not part of operating income we have not valued cash yet and it should be added on to the present value.
- b. Cross Holdings in other companies: Companies sometimes invest in other firms, and these cross holdings can generally be categorized as either minority or majority holdings. With the former, the holdings are usually less than 50%, and the income from the holdings are reported in the income statement below the operating income line. If we use free cash flow to the firm to value the operating assets, we have not valued these minority holdings yet, and they have to be valued explicitly and added to present value. With majority holdings, which generally exceed 50%, firms usually consolidate the entire subsidiary in their financials, and report 100% of the operating income and assets of the subsidiary. To reflect the portion of the subsidiary that does not belong to them, they report the book value of that portion as minority interest in a balance sheet. If we compute cash flows from consolidated financial statements, we have to subtract out the estimated market value of the minority interest.
- c. Debt: Discounting cash flows at the cost of capital yields the value of operating assets. To get to the value of equity, we should subtract out the debt outstanding at the current time. Note that any debt that may be raised in the future is captured in the debt ratio used in the cost of capital computation and should not be subtracted out to get to today's equity value. If there are other liabilities that are a potential drain on value – unfunded pension or health care obligations or potential losses from lawsuits, they should also be netted out to arrive at equity value
- d. Employee Options: Having arrived at the value of equity in the firm, there is one final estimate that we have to make, especially if the firm has made it a practice to grant options to managers. Since many of these options will be still outstanding, we have to consider them as another (and different) claim on equity. While analysts often use short cuts (such as adjusting the number of shares for dilution) to deal with these options, the right approach is to value the options (using an option pricing model),

reduce the value of equity by the option value and then divide by the actual number of shares outstanding.

In summary, to get to value of equity per share from the present value of the cash flows (value of operating assets):

$$\text{Value of equity/share} = \frac{\text{Value of operating assets} + \text{Cash} - \text{Debt} - \text{Employee options}}{\text{Primary number of shares outstanding}}$$

Conclusion

The intrinsic value of a company reflects its fundamentals. The primary tool for estimating intrinsic value is the discounted cash flow model. We started by looking at the contrast between valuing the equity in a business and valuing the entire business, and then moved on to the four inputs that we need for the model. The cash flows to equity investors can be defined strictly as dividends, more expansively as dividends augmented with stock buybacks and most generally as free cash flows to equity (potential dividends). The cash flow to the firm is the cumulative cash flow to both equity investors and lenders, and thus is a pre-debt cash flow. The discount rates we apply have to be consistent with the cash flow definition, with the cost of equity used to discount cash flows to equity and the cost of capital to discount cash flows to the firm. When estimating growth, we noted the limitations of historical growth numbers and outside estimates, and the importance of linking growth to fundamentals. Finally, we applied closure to the models by assuming that cash flows will settle into stable growth, sometime in the future, but imposed constraints on what this growth rate can be and the characteristics of stable growth companies.

CHAPTER 4

 INTRINSIC VALUATION: THE APV MODEL

In the *adjusted present value (APV) approach*, we separate the effects on value of debt financing from the value of the assets of a business. In contrast to the cost of capital approach, where the effects of debt financing are captured in the discount rate, the APV approach attempts to estimate the expected dollar value of debt benefits and costs separately from the value of the operating assets. In this chapter, we start by looking at the basis for the APV approach and then move on to ways of putting it into practice.

Basis for APV Approach

In the APV approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. In general, using debt to fund a firm's operations creates tax benefits (because interest expenses are tax deductible) on the plus side and increases bankruptcy risk (and expected bankruptcy costs) on the minus side. The value of a firm can be written as follows:

$$\text{Value of business} = \text{Value of business with 100\% equity financing} + \text{Present value of Expected Tax Benefits of Debt} - \text{Expected Bankruptcy Costs}$$

The first attempt to isolate the effect of tax benefits from borrowing was in Miller and Modigliani (1963), where they valued the present value of the tax savings in debt as a perpetuity using the cost of debt as the discount rate. The adjusted present value approach, in its current form, was first presented in Myers (1974) in the context of examining the interrelationship between investment and financing decisions.⁷

Implicitly, the adjusted present value approach is built on the presumption that it is easier and more precise to compute the valuation impact of debt in absolute terms rather than in proportional terms. Firms, it is argued, do not state target debt as a ratio of market value (as implied by the cost of capital approach) but in dollar value terms.

⁷ Modigliani, F. and M. Miller, 1958, The Cost of Capital, Corporation Finance and the Theory of Investment, American Economic Review, v48, 261-297; Myers, S., 1974, Interactions in Corporate Financing and Investment Decisions—Implications for Capital Budgeting, Journal of Finance, v29, March, 1-25.

Measuring Adjusted Present Value

In the adjusted present value approach, we estimate the value of the firm in three steps. We begin by estimating the value of the firm with no leverage. We then consider the present value of the interest tax savings generated by borrowing a given amount of money. Finally, we evaluate the effect of borrowing the amount on the probability that the firm will go bankrupt, and the expected cost of bankruptcy.

a. Unlevered (all-equity) value

The first step in the APV approach is the estimation of the value of the unlevered firm, i.e., the value of the firm as an all-equity funded entity. This can be accomplished by discounting the expected free cash flow to the firm at the cost of equity that the firm would have had, if it had no debt. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

$$\text{Value of Unlevered Firm} = \frac{\text{FCFF}_0(1+g)}{\rho_u - g}$$

where FCFF_0 is the current after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity and g is the expected growth rate. In the more general case, we can value the firm using any set of growth assumptions we believe are reasonable for the firm.

The inputs needed for this valuation are the expected cashflows, growth rates and the unlevered cost of equity. Since the free cash flows to the firm that we estimated for the cost of capital approach were unlevered or operating cash flows, we use precisely the same cash flows in the APV approach. Using the same principles in measuring after-tax operating income and reinvestment that we developed in chapter 3, we obtain the following:

$$\text{FCFF} = \text{EBIT} (1-t) - (\text{Capital Expenditure} - \text{Depreciation}) - \text{Change in non-cash working capital}$$

The growth rate in these cash flows is also estimated exactly the same way as it was in the cost of capital approach, with the same caveats applying: don't trust historical growth rates or analyst forecasts and tie growth to fundamentals:

$$\text{Expected growth rate} = \text{Reinvestment Rate} * \text{ROIC}$$

The only input where the two approaches diverge is on the discount rate. In the cost of capital approach, we used a weighted average of the cost of equity and the after-tax cost of debt, with the intent of capturing the effect of debt in the discount rate. Since the APV approach separately evaluates the effects of debt separately, the discount rate used to compute the unlevered firm value should be the cost of equity that the firm would have had, as an all-equity funded entity. There are two ways in which this unlevered cost of equity can be obtained:

- a. Bottom up approach: If you can obtain the unlevered or pure play betas of different businesses, you can estimate the unlevered beta for a company by taking a weighted average of the betas of the businesses in which it operates, with the weights being based upon the estimated values of the businesses. Thus, if you are working with a company that generates 60% of its value from steel and 40% from chemicals, its unlevered beta will be the weighted average of the betas of the two businesses.
- b. Regression beta unlevered: In the second approach, we start with a levered beta, obtained from a regression or alternate technique and try to back out an unlevered beta for the firm. There are three variants that are used to make this adjustment.
 - In the Hamada approach, we assume that debt creates tax benefits (which are manifested as an asset) and that the beta of debt is zero. Note that this is not equivalent to assuming that debt is riskless, but essentially assumes (perhaps unrealistically) that default risk is not related to overall market risk. With these assumptions, the unlevered beta can be written as:

$$\text{Unlevered beta} = \frac{\text{Levered Beta}}{(1 + (1 - \text{tax rate}) (\text{Debt/Equity}))}$$

The tax rate used is the marginal tax rate and the debt to equity ratio is the market debt to equity ratio.

- In a slight variant of this approach, we assume that debt has no effect on firm value (with the tax benefits being offset by expected bankruptcy costs) and estimate the levered beta, purely as a function of the debt to equity ratio.

$$\text{Unlevered beta} = \frac{\text{Levered Beta}}{(1 + \text{Debt/Equity})}$$

This approach will yield lower unlevered betas than the Hamada approach.

- In the third approach, we relax the assumption that the beta of debt is zero by explicitly estimate a debt beta. We can then modify either the Hamada equation or its no-tax counterpart to estimate unlevered betas for the firm.

$$\text{Unlevered } \beta_{\text{Hamada}} = \frac{\text{Levered Beta} + \text{Beta of Debt} (1 - \text{tax rate}) (\text{Debt/Equity})}{(1 + (1 - \text{tax rate}) (\text{Debt/Equity}))}$$

$$\text{Unlevered } \beta_{\text{No tax asset}} = \frac{\text{Levered Beta} + \text{Beta of Debt} (\text{Debt/Equity})}{(1 + \text{Debt/Equity})}$$

This approach will yield higher unlevered betas than the other two approaches, because it brings in the additional market risk embedded in the debt.

So, which approach yields the best answers? The debt beta approach is the most realistic but it will yield answers that are very different only if the firm is exposed to a significant amount of default risk in the first place. One very simple way to compute a debt beta is to compare the default spread on the company's debt to the overall equity risk premium. Thus, a AA rated firm that has a default spread of 0.5% would have a debt beta of 0.1, if the equity risk premium of is 5%.

$$\text{Beta of debt} = \frac{\text{Default Spread of Debt}}{\text{Equity Risk Premium}} = \frac{0.5\%}{5.0\%} = 0.10$$

While this approach is simple, it does assume that all of the default risk is market risk.

Once the unlevered beta is computed, it can be used to compute the cost of equity for the firm, which will also be its cost of capital as an all-equity funded firm.

Tax Benefits of Debt

The second step in this approach is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate of the firm and is discounted to reflect the riskiness of this cash flow.

$$\text{Value of Tax Benefits} = \sum_{t=1}^{t=\infty} \frac{\text{Tax Rate}_t * \text{Interest Rate}_t * \text{Debt}_t}{(1+r)^t}$$

There are three estimation questions that we have to address here. The first is what tax rate to use in computing the tax benefit and whether that rate can change over time. The second is the dollar debt to use in computing the tax savings and whether that amount can vary across time. The final issue relates to what discount rate to use to compute the present value of the tax benefits. In the early iterations of APV, the tax rate and dollar debt were viewed as constants (resulting in tax savings as a perpetuity) and the pre-tax cost of debt was used as the discount rate leading to a simplification of the tax benefit value:

$$\begin{aligned} &= \frac{(\text{Tax Rate})(\text{Cost of Debt})(\text{Debt})}{\text{Cost of Debt}} \\ \text{Value of Tax Benefits} &= (\text{Tax Rate})(\text{Debt}) \\ &= t_c D \end{aligned}$$

Subsequent adaptations of the approach allowed for variations in both the tax rate and the dollar debt level, and raised questions about whether it was appropriate to use the cost of debt as the discount rate.

- a. The tax rate question: Since interest expenses save you taxes on your last dollar (or marginal dollar), there is a strong case to be made that the marginal or statutory tax rate should be used in computing the tax benefits from debt. While this may be straight forward for a company with purely domestic operations that generates all of its income from one country, it can get complicated for multinationals. Notwithstanding this nuance, at least for US companies, it can be argued that the marginal tax rate that is used should be the one for the US. In 2010, KPMG estimated this tax rate, including state and local taxes, to be about 40%.⁸
- b. Dollar debt and interest expense: While the early versions of the APV locked in debt and interest expenses at current levels and treated them as perpetuities, there

⁸ The argument is that even if a significant portion of a multinational's income comes from lower tax locales, it can strategically raise debt in the locations that provide the greatest tax benefit. Since the US marginal tax rate is the second highest in the world, after Japan, that would be the US.

is no reason why the model cannot be expanded to cover debt levels and interest expenses that change over time. This does put the onus on the analyst to estimate how much the firm will have in dollar debt each period into the future and the interest rate to attach to the debt. One simple variant is to keep the debt ratio fixed and to allow the dollar debt to increase proportionately with firm value, while keeping the pre-tax cost of debt fixed. Thus, if you have \$ 100 million in debt currently and expect the firm to grow 3% a year, your dollar debt will grow 3% a year and interest expenses (and tax benefits) will keep pace. In some cases, though, you will have access to explicit debt repayment schedules and interest rate estimates. This is often true in leveraged transactions, where a firm is acquired with a disproportionately large debt burden, with plans in place for the debt to be repaid over time. The dollar debt level can therefore be entered each period.

- c. Discount rate for tax benefits: There are two choices for discount rate to use in computing the present value of tax benefits. The original version of the APV argued for the use of the pre-tax cost of debt, on the assumption that the tax benefit of debt has the same risk profile as the debt, since it is based upon interest expenses. The alternative view is that the real risk that the firms face on the tax benefit issue stem not from the interest expenses but from the taxable income side. In other words, if the firm does not make enough taxable income, it cannot get the tax benefit from interest expenses. This make the case for the use of the cost of equity, and in this case the unlevered cost of equity, as the discount rate for tax benefits.

Expected Bankruptcy Costs

The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. In theory, at least, this requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

$$\begin{aligned} \text{PV of Expected Bankruptcy cost} &= (\text{Probability of Bankruptcy})(\text{PV of Bankruptcy Cost}) \\ &= \pi_a BC \end{aligned}$$

This step of the adjusted present value approach poses the most significant estimation problem, since neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly.

Probability of Bankruptcy

There are two basic ways in which the probability of bankruptcy can be estimated indirectly. One is to estimate a bond rating, as we did in the cost of capital approach, at each level of debt and use the empirical estimates of default probabilities for each rating. For instance, Table 8.19, extracted from an annually updated study by Altman, summarizes the probability of default over ten years by bond rating class.⁹

Table 8.19 Default Rates by Bond Rating Classes

<i>Rating</i>	<i>Likelihood of Default</i>
AAA	0.07%
AA	0.51%
A+	0.60%
A	0.66%
A-	2.50%
BBB	7.54%
BB	16.63%
B+	25.00%
B	36.80%
B-	45.00%
CCC	59.01%
CC	70.00%
C	85.00%
D	100.00%

Source: Altman (2008).

Thus, a firm that has a BBB rating (actual or synthetic) can be assumed to have a probability of default in the long term of 7.54%.

The other is to use a statistical approach, such as a probit to estimate the probability of default, based on the firm's observable characteristics, at each level of

⁹ Altman, E.I., 2008, *The Default Experience of U.S. Bonds*, Working Paper, Salomon Center, New York University. This study estimated default rates over ten years only for some of the ratings classes. We extrapolated the rest of the ratings.

debt. The other is to use a statistical approach to estimate the probability of default, based upon the firm's observable characteristics, at each level of debt.

The Cost of Bankruptcy

The cost of going bankrupt is neither obvious nor easily quantified. It is true that bankruptcy is a disaster for all involved in the firm—lenders often get a fraction of what they are owed, and equity investors get nothing—but the overall cost of bankruptcy includes the indirect costs on operations of being perceived as having high default risk.

a. Direct Costs

The direct, or deadweight, cost of bankruptcy is that which is incurred in terms of cash outflows at the time of bankruptcy. These costs include the legal and administrative costs of a bankruptcy, as well as the present value effects of delays in paying out the cash flows. In a widely quoted study of railroad bankruptcies in the 1970s, Warner estimated the legal and administrative costs of eleven railroads to be on average 5.3 percent of the value of the assets at the time of the bankruptcy. He also estimated that it took, on average, thirteen years before the railroads were reorganized and released from the bankruptcy costs.¹⁰ These costs, although certainly not negligible, are not overwhelming, especially in light of two additional factors. First, the direct cost as a percentage of the value of the assets decreases to 1.4 percent if the asset value is computed five years before the bankruptcy. Second, railroads in general are likely to have higher bankruptcy costs than other companies because of the nature of their assets (real estate and fixed equipment).

b. Indirect Costs

If the only costs of bankruptcy were the direct costs, the low leverage maintained by many firms would be puzzling. There are, however, much larger costs associated with taking on debt and increasing default risk, which arise prior to the bankruptcy, largely as a consequence of the perception that a firm is in financial trouble. The first is the perception on the part of the customers that the firm is in trouble. When this happens, customers may stop buying the product or service because of the fear that the company

¹⁰ Warner, J.N., 1977, *Bankruptcy Costs: Some Evidence*, *Journal of Finance*, v32, 337-347.

will go out of business. In 1980, for example, when car buyers believed that Chrysler was on the verge of bankruptcy, they chose to buy from Ford and GM, largely because they were concerned about receiving service and parts for their cars after their purchases. Similarly, in the late 1980s, when Continental Airlines found itself in financial trouble, business travelers switched to other airlines because they were unsure about whether they would be able to accumulate and use their frequent-flier miles on the airline. The second indirect cost is the stricter terms suppliers start demanding to protect themselves against the possibility of default, leading to an increase in working capital and a decrease in cash flows. The third cost is the difficulty the firm may experience trying to raise fresh capital for its projects—both debt and equity investors are reluctant to take the risk, leading to capital rationing constraints and the rejection of good projects.

Given this reasoning, the indirect costs of bankruptcy are likely to be higher for the following types of firms:¹¹

- *Firms that sell durable products with long lives that require replacement parts and service:* Thus, a personal computer manufacturer would have higher indirect costs associated with bankruptcy than would a grocery store.
- *Firms that provide goods or services for which quality is an important attribute but is difficult to determine in advance:* Because the quality cannot be determined easily in advance, the reputation of the firm plays a significant role in whether the customer will buy the product in the first place. For instance, the perception that an airline is in financial trouble may scare away customers who worry that the planes belonging to the airline will not be maintained.
- *Firms producing products whose value to customers depends on the services and complementary products supplied by independent companies:* Returning to the example of personal computers, a computer system is valuable only insofar as there is software available to run on it. If the firm manufacturing the computers is perceived to be in trouble, it is entirely possible that the independent suppliers that produce the software might stop providing it. Thus, if Apple Computers gets into financial

¹¹ See Shapiro, A., 1989, *Modern Corporate Finance*, Macmillan, New York; Titman, S., 1984, *The Effect of Capital Structure on a Firm's Liquidation Decision*, *Journal of Financial Economics*, v13, 1371-51.

trouble, many software manufacturers might stop producing software for its machines, leading to an erosion in its potential market.

- *Firms that sell products that require continuous service and support from the manufacturer:* A manufacturer of copying machines, for which constant service seems to be a necessary operating characteristic, would be affected more adversely by the perception of default risk than would a furniture manufacturer, for example.

The bankruptcy cost can be estimated, albeit with considerable error, from studies that have looked at the magnitude of this cost in actual bankruptcies. Research that has looked at the direct cost of bankruptcy concludes that they are small¹², relative to firm value and the variation across firms is not dramatic. However, indirect bankruptcy costs can be catastrophic for many firms and essentially make the perception of distress into a reality. The magnitude of these costs is not only much larger than the direct bankruptcy costs, but also vary much more widely across firms, with estimates ranging from 10-40% of firm value.¹³

Variants on APV

While the original version of the adjusted present value model was fairly rigid in its treatment of the tax benefits of debt and expected bankruptcy costs, subsequent variations allow for more flexibility in the treatment of both. Some of these changes can be attributed to pragmatic considerations, primarily because of the absence of information, whereas others represented theoretical corrections.

One adaptation of the model was suggested by Luehrman (1997), where he presents an example where the dollar debt level, rather than remain fixed as it does in conventional APV, changes over time as a fraction of book value.¹⁴ The interest tax

¹² Warner, J.N., 1977, Bankruptcy Costs: Some Evidence, *Journal of Finance*, v32, 337-347. He studies railroad bankruptcies, and concludes that the direct cost of bankruptcy was only 5% on the day before bankruptcy. In fact, it is even lower when assessed five years ahead of the bankruptcy.

¹³ For an examination of the theory behind indirect bankruptcy costs, see Opler, T. and S. Titman, 1994. For an estimate on how large these indirect bankruptcy costs are in the real world, see Andrade, G. and S. Kaplan, 1998. They look at highly levered transactions that subsequently became distressed and conclude that the magnitude of these costs ranges from 10% to 23% of firm value.

¹⁴ Luehrman, T. A., 1997, Using APV: A Better Tool for Valuing Operations, *Harvard Business Review*, May-June, 145-154.

savings reflect the changing debt but the present value of the tax savings is still computed using the cost of debt.

Another variation on adjusted present value was presented by Kaplan and Ruback (1995) in a paper where they compared the discounted cash flow valuations of companies to the prices paid in leveraged transactions.¹⁵ They first estimated what they termed capital cash flows which they defined to be cash flows to both debt and equity investors and thus inclusive of the tax benefits from interest payments on debt. This is in contrast with the conventional unlevered firm valuation, which uses only operating cash flows and does not include interest tax savings. These capital cash flows are discounted back at the unlevered cost of equity to arrive at firm value. In effect, the compressed adjusted present value approach differs from the conventional adjusted present value approach on two dimensions. First, the tax savings from debt are discounted back at the unlevered cost of equity rather than the cost of debt. Second, the expected bankruptcy costs are effectively ignored in the computation. Kaplan and Ruback argue that this approach is simpler to use than the conventional cost of capital approach in levered transactions because the leverage changes over time, which will result in time-varying costs of capital. In effect, they are arguing that it is easier to reflect the effects of changing leverage in the cash flows than it is in debt ratios. Gilson, Hotchkiss and Ruback (2000) use the compressed APV approach to value bankrupt firms that are reorganized and conclude that while the approach yields unbiased estimates of value, the valuation errors remain large.¹⁶ The key limitation of the compressed APV approach, notwithstanding its simplicity, is that it ignores expected bankruptcy costs. In fact, using the compressed adjusted present value approach will lead to the conclusion that a firm is always worth more with a higher debt ratio than with a lower one. Kaplan and Ruback justify their approach by noting that the values that they arrive at are very similar to the values obtained using comparable firms, but this cannot be viewed as vindication.

¹⁵ Kaplan, S.N. and R.S. Ruback, 1995, The Valuation of Cash Flow Forecasts, *Journal of Finance*, v50, 1059-1093.

¹⁶ Gilson, S.C., E. S. Hotchkiss and R. Ruback, 1998, Valuation of Bankrupt Firms, *Review of Financial Studies*, v13, 43-74. The one modification they introduce is that the tax savings from net operating loss carryforwards are discounted back at the cost of debt.

Ruback (2000) provides a more extensive justification of the capital cash flow approach to valuation.¹⁷ He notes that the conventional APV's assumption that interest tax savings have the same risk as the debt (and thus get discounted back at the cost of debt) may be justifiable for a fixed dollar debt but that it is more reasonable to assume that interest tax savings share the same risk as the operating assets, when dollar debt is expected to change over time. He also notes that the capital cash flow approach assumes that debt grows with firm value and is thus closer to the cost of capital approach, where free cash flows to the firm are discounted back at a cost of capital. In fact, he shows that when the dollar debt raised each year is such that the debt ratio stays constant, the cost of capital approach and the capital cash flows approach yield identical results.

Cost of Capital versus APV Valuation

To understand when the cost of capital approach, the adjusted present value approach and the modified adjusted present value approach (with capital cash flows) yield similar and different results, we consider the mechanics of each approach in table 1:

Table 1: Cost of Capital, APV and Compressed APV

	<i>Cost of Capital</i>	<i>Conventional APV</i>	<i>Compressed APV</i>
Cash flow discounted	Free Cash Flow to Firm (prior to all debt payments)	Free Cash Flow to Firm (prior to debt payments)	Free Cash Flow to Firm + Tax Savings from Interest Payments
Discount Rate used	Weighted average of cost of equity and after-tax cost of debt = Cost of capital	Unlevered cost of equity	Weighted average of cost of equity and pre-tax cost of debt = Unlevered cost of equity
Tax Savings from Debt	Shows up through the discount rate	Added on separately as present value of tax savings (using cost of debt as discount rate)	Shows up through cash flow
Dollar debt levels	Determined by debt ratios used in cost of capital. If debt ratio	Fixed dollar debt	Dollar debt can change over time – increase or decrease.

¹⁷ Ruback, R.S., 2000, Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows, Working Paper, Harvard Business School.

	stays fixed, dollar debt increases with firm value		
Discount rate for tax benefits from interest expenses	Discounted at unlevered cost of equity	Discounted at pre-tax cost of debt	Discounted at unlevered cost of equity
Bankruptcy Costs	Reflected as higher costs of equity and debt, as default risk increases.	Can be computed separately, based upon likelihood of distress and the cost of such distress. (In practice, often ignored)	Can be computed separately, based upon likelihood of distress and the cost of such distress. (In practice, often ignored)

In an APV valuation, the value of a levered firm is obtained by adding the net effect of debt to the unlevered firm value.

$$\text{Value of Levered Firm} = \frac{\text{FCFF}_0(1+g)}{\rho_u - g} + t_c D - \pi_a BC$$

The tax savings from debt are discounted back at the cost of debt. In the cost of capital approach, the effects of leverage show up in the cost of capital, with the tax benefit incorporated in the after-tax cost of debt and the bankruptcy costs in both the levered beta and the pre-tax cost of debt. Inselbag and Kaufold (1997) provide examples where they get identical values using the APV and Cost of Capital approaches, but only because they infer the costs of equity to use in the latter.¹⁸

Will the approaches yield the same value? Not necessarily. The first reason for the differences is that the models consider bankruptcy costs very differently, with the adjusted present value approach providing more flexibility in allowing you to consider indirect bankruptcy costs. To the extent that these costs do not show up or show up inadequately in the pre-tax cost of debt, the APV approach will yield a more conservative estimate of value. The second reason is that the conventional APV approach considers the tax benefit from a fixed dollar debt value, usually based upon existing debt. The cost of capital and compressed APV approaches estimate the tax benefit from a debt ratio that

¹⁸ Inselbag, I. and H. Kaufold, 1997, Two DCF approaches for valuing companies under alternative financing strategies and how to choose between them, *Journal of Applied Corporate Finance*, v10, 114-122.

may require the firm to borrow increasing amounts in the future. For instance, assuming a market debt to capital ratio of 30% in perpetuity for a growing firm will require it to borrow more in the future and the tax benefit from expected future borrowings is incorporated into value today. Finally, the discount rate used to compute the present value of tax benefits is the pre-tax cost of debt in the conventional APV approach and the unlevered cost of equity in the compressed APV and the cost of capital approaches. As we noted earlier, the compressed APV approach yields equivalent values to the cost of capital approach, if we allow dollar debt to reflect changing firm value (and debt ratio assumptions) and ignore the effect of indirect bankruptcy costs. The conventional APV approach yields a higher value than either of the other two approaches because it views the tax savings from debt as less risky and assigns a higher value to it.

Which approach will yield more reasonable estimates of value? The dollar debt assumption in the APV approach is a more conservative one but the fundamental flaw with the APV model lies in the difficulties associated with estimating expected bankruptcy costs. As long as that cost cannot be estimated, the APV approach will continue to be used in half-baked form where the present value of tax benefits will be added to the unlevered firm value to arrive at total firm value.

Conclusion

In the adjusted present value approach, we separate debt from the operating assets of the firm, and value its effects independently of the firm. The APV model is particularly useful for levered transactions where the initial high debt level used to fund assets is expected to decrease over time and you have access to the repayment and interest expense schedule. It allows the analyst to first value the operating assets, without the distraction of the heavy debt load, and to then assess the value added (or not) by the additional debt in the transaction. Note, though, that to obtain realistic values for equity, you have to incorporate the expected bankruptcy costs into your APV model.

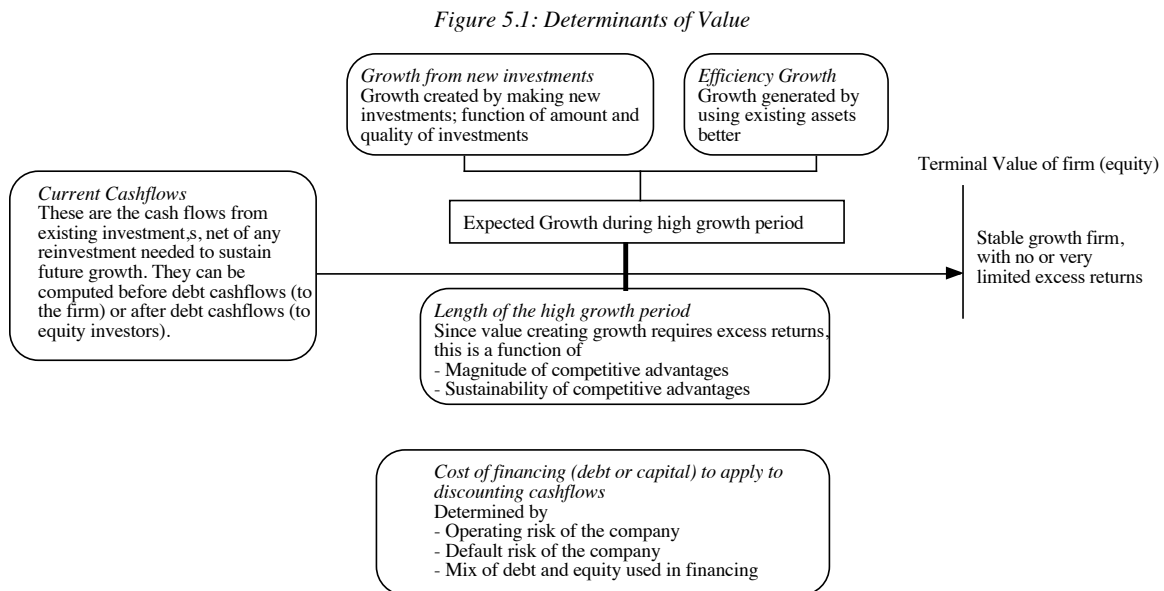
CHAPTER 5

INTRINSIC VALUATION: EQUITY VALUATION MODELS

In chapters 3 and 4, we looked at two approaches that can be used to value an entire business or firm – discounting free cash flows to the firm at the cost of capital and the APV approach. In this chapter, we turn our attention to an alternate way of thinking about valuation, where we focus on valuing the equity in the firm directly, by discounting cash flows to equity investors at the rate of return that these investors demand for investing in the equity – the cost of equity.

Inputs to a Equity DCF Valuation

In chapter 3, we outlined the key determinants to value and we reproduce the figure below in Figure 5.1.



From the perspective of equity valuation, the first input is the cashflow from existing assets, defined as post-debt (and to equity) earnings, net of reinvestment to generate future growth. In fact, we can use an even stricter definition of cash flow and consider only dividends paid. The second input is growth, with growth in equity income (net income or earnings per share) becoming the focus when valuing equity. The third input is the discount rate, defined as the cost of equity, since we are valuing equity. The final input, allowing for closure, is the terminal value, defined as the estimated value of equity at the end of the forecast period in equity valuation.

For the rest of this section, we will focus on estimating the inputs into discounted cash flow models, starting with the cashflows, moving on to risk (and discount rates) and then closing with a discussion of how best to estimate the growth rate for the high growth period and the value at the end of that period.

Cash Flows

In this section, we will begin with the strictest measure of cash flow to equity, i.e. the dividends received by investors, and then progressively move to more expansive measures of cash flows, which generally require more information.

Dividends

When an investor buys stock, he generally expects to get two types of cash flows - dividends during the holding period and an expected price at the end of the holding period. Since this expected price is itself determined by future dividends, the value of a stock is the present value of just expected dividends. If we accept this premise, the only cash flow to equity that we should be considering in valuation is the dividend paid, and estimating that dividend for the last period should be a simple exercise. Since many firms do not pay dividends, this number can be zero, but it should never be negative.

Augmented Dividends

One of the limitations of focusing on dividends is that many companies, especially in the United States but increasingly around the world, have shifted from dividends to stock buybacks as their mechanism for returning cash to stockholders. While only those stockholders who sell their stock back receive cash, it still represents cash returned to equity investors. In 2007, for instance, firms in the United States returned twice as much cash in the form of stock buybacks than they did in dividends, and focusing only on dividends will result in the under valuation of equity. One simple way of adjusting for this is to *augment the dividend* with stock buybacks and look at the cumulative cash returned to stockholders.

$$\text{Augmented Dividends} = \text{Dividends} + \text{Stock Buybacks}$$

One problem, though, is that unlike dividends that are smoothed out over time, stock buybacks can spike in some years and be followed by years of inaction. We therefore will

have to normalize buybacks by using average buybacks over a period of time (say, 5 years) to arrive at more reasonable annualized numbers.

Potential Dividends (Free Cash flow to Equity)

With both dividends and augmented dividends, we are trusting managers at publicly traded firms to return to pay out to stockholders any excess cash left over after meeting operating and reinvestment needs. However, we do know that managers do not always follow this practice, as evidenced by the large cash balances that you see at most publicly traded firms. To estimate what managers could have returned to equity investors, we develop a measure of potential dividends that we term the *free cash flow to equity*. Intuitively, the free cash flow to equity measures the cash left over after taxes, reinvestment needs and debt cash flows have been met. It is measured as follows:

$$\begin{aligned} \text{FCFE} &= \text{Net Income} - \text{Reinvestment Needs} - \text{Debt Cash flows} \\ &= \text{Net Income} - (\text{Capital Expenditures} - \text{Depreciation} + \text{Change in non-cash} \\ &\quad \text{working Capital} - \text{Principal}) - (\text{Repayments} - \text{New Debt Issues}) \end{aligned}$$

Consider the equation in pieces. We begin with net income, since that is the earnings generated for equity investors; it is after interest expenses and taxes. We compute what the firm has to reinvest in long term assets as net capital expenditures and what it has to invest in short term assets as the change in non-cash working capital, thus sticking with the definitions we attached to each term in the cost of capital model. The final input into the process are the negative cash flows associated with the repayment of old debt and the positive cash flows to equity investors from raising new debt. If old debt is replaced with new debt of exactly the same magnitude, this term will be zero, but it will generate positive (negative) cash flows when debt issues exceed (are less than) debt repayments.

Focusing on just debt cash flows allows us to zero in on a way to simplify this computation. In the special case where the capital expenditures and the working capital are expected to be financed at a fixed debt ratio δ , and principal repayments are made from new debt issues, the FCFE is measured as follows:

$$\text{FCFE} = \text{Net Income} - (1-\delta) (\text{Capital Expenditures} - \text{Depreciation}) - (1-\delta) \Delta \text{Working Capital}$$

In effect, we are assuming that a firm with a 30% debt ratio that is growing through reinvestment will choose to fund 30% of its reinvestment needs with new debt and replace old debt that comes due with new debt.

There is one more way in which we can present the free cash flow to equity. If we define the portion of the net income that equity investors reinvest back into the firm as the equity reinvestment rate, we can state the FCFE as a function of this rate.

$$\begin{aligned} & \text{Equity Reinvestment Rate} \\ & = \frac{(\text{Capital Expenditures} - \text{Depreciation} + \Delta \text{ Working Capital}) (1 - \delta)}{\text{Net Income}} \end{aligned}$$

$$\text{FCFE} = \text{Net Income} (1 - \text{Equity Reinvestment Rate})$$

A final note on the contrast between the first two measures of cash flows to equity (dividends and augmented dividends) and this measure. Unlike those measures, which can never be less than zero, the free cash flow to equity can be negative for a number of reasons. The first is that the net income could be negative, a not uncommon phenomenon even for mature firms. The second is that reinvestment needs can overwhelm net income, which is often the case for growth companies, especially early in the life cycle. The third is that large debt repayments coming due that have to be funded with equity cash flows can cause negative FCFE; highly levered firms that are trying to bring their debt ratios down can go through years of negative FCFE. The fourth is that the quirks of the reinvestment process, where firms invest large amounts in long-lived and short-lived assets in some years and nothing in others, can cause the FCFE to be negative in the big reinvestment years and positive in others. As with buybacks, we have to consider normalizing reinvestment numbers across time when estimating cash flows to equity. If the FCFE is negative, it is indicative of the firm needing to raise fresh equity.

Risk and Discount Rates

In chapter 3, we laid out the core issues in estimating the cost of equity. We noted first the difficulty in estimating an implicit cost (cost of equity is implicit whereas the interest rate on a loan is explicit) and argued that we should estimate the cost of equity through the eyes of the marginal investor in the company. In fact, if that marginal investor is diversified, the risk that we measure is only the risk that cannot be diversified

away. We also looked at the different models for estimating cost of equity (the CAPM, the Arbitrage Pricing Model and Multi-factor Models) and the challenges in estimating the inputs to these models.

We would like to add a couple of intuitive points to that discussion. The first is that the cost of equity for a firm can and often should change over time, as its growth rate changes and it approaches maturity. Using beta as a measure for growth, for instance, you would expect to see the beta for a young, growth firm go from 1.80 today (when it is young, high growth and risky) to a number closer to one as the firm approaches maturity. The second is to reemphasize a point we made in the earlier chapter on estimating betas. It is bad practice to trust a single regression (or a service that reports that regression) for the beta for your company. You will be better served using industry averages for the businesses in which your firm operates and correcting for financial and operating leverage differences.

Growth Rates

To estimate growth rates in equity earnings, we can draw on some of the same approaches we used for operating earnings, looking at historical growth or trusting in analyst forecasts, but they are just as flawed in this context as they were in the operating income context. Consequently, we will fall back on the same approach that we focused on for operating income, which is to link expected growth in earnings to how much a firm reinvests and how well it does so.

While investment and return on investment are generic terms, the way in which we define them will depend upon whether we are looking at equity earnings or operating income. When looking at equity earnings, our focus is on the investment in equity and the return is the return on equity. The change in investment is computed as the reinvestment, with the measurement of the reinvestment again varying depending upon the cash flow being discounted. In dividend discount models, reinvestment is defined as retained earnings (i.e., any income not paid out as dividends). In free cash flow to equity (firm) models, reinvestment is defined in terms of the equity reinvestment rate (reinvestment rate).

Central to any estimate of fundamental growth is the estimate of return on capital or equity. Table 2.4 summarizes the inputs for each measure depending on the measure of cash flow that we are focused on:

Table 2.4: Measuring Investment and Return on Investment

	<i>Change in Investment</i>	<i>Return on Investment</i>
Net Income (Non-cash)	Equity Reinvestment Rate = $\frac{(\text{Cap Ex} - \text{Deprec'n} + \Delta\text{WC} - \Delta\text{Debt})}{\text{Net Income}}$	Non-cash Return on Equity (NCROE)
Earnings per share	Retention Ratio = $1 - \frac{\text{Dividends}}{\text{Net Income}}$	Return on Equity (ROE)

It is conventional practice to use accounting measures of investment and return on investment. Thus, the book values of equity and invested capital and accounting earnings are used to compute returns on equity and capital:

$$\text{Non-cash Return on Equity (NCROE)} = \frac{\text{Net Income}_t - \text{Interest Income from Cash}_t (1 - \text{tax rate})}{\text{Book Value of Equity}_{t-1} - \text{Cash}_{t-1}}$$

$$\text{Return on Equity (ROE)} = \frac{\text{Net Income}_t}{\text{Book Value of Equity}_{t-1}}$$

It is worth noting that the book value of equity can be skewed by restructuring charges and stock buybacks, resulting in high returns on equity for firms with poor investments.

Just as there was a second component to growth, from running existing investments more efficiently, with growth in operating income, there can efficiency driven growth in net income. This can either manifest itself as an improved net profit margin over time or as the change in the return on equity from period to period.

Table 2.5: Measuring Investment and Return on Investment

	<i>Measure of return on existing assets</i>	<i>Efficiency growth</i>
Net Income (Non-cash)	Non-cash Return on Equity	$\frac{(\text{NCROE}_t - \text{NCROE}_{t-1})}{\text{NCROE}_{t-1}}$
Earnings per share	Return on Equity	$\frac{(\text{ROE}_t - \text{ROE}_{t-1})}{\text{ROE}_{t-1}}$

Terminal Value

When discussing terminal value in the context of the cost of capital approach, we noted three key constraints: (a) Make sure that the growth rate you estimate in perpetuity is less than the growth rate of the economy. In fact, we suggested using the riskfree rate as a proxy for long term growth in the economy and setting the growth rate at or below the riskfree rate (b) Give the company the characteristics of a mature company in stable growth. In the context of equity models, this requires us to bring the beta of the company closer to one, the beta for the market and (c) Make sure you reinvest to sustain the stable growth rate.

Stable growth firms tend to reinvest less than high growth firms and it is critical that we both capture the effects of lower growth on reinvestment and that we ensure that the firm reinvests enough to sustain its stable growth rate in the terminal phase. Given the relationship between growth, reinvestment rate and returns that we established in the section on expected growth rates, we can estimate the reinvestment rate that is consistent with expected growth in table 2.7:

Table 2.7: Reinvestment in Stable Growth

Model	Reinvestment Rate in stable growth
Dividend	$\frac{\text{Stable Growth rate}}{\text{Return on Equity in stable growth}}$
FCFE	$\frac{\text{Stable Growth rate}}{\text{Non - cash Return on Equity in stable growth}}$

Linking the equity reinvestment rate and retention ratio to the stable growth rate also makes the valuation less sensitive to assumptions about stable growth. While increasing the stable growth rate, holding all else constant, can dramatically increase value, changing the reinvestment rate as the growth rate changes will create an offsetting effect.

$$\text{Terminal Value of Equity} = \frac{\text{Net Income}_{n+1} \left(1 - \frac{g_n}{\text{ROE}_n} \right)}{(\text{Cost of Equity}_n - g_n)}$$

In the special case where the cost of equity = ROE, the growth rate becomes irrelevant and the terminal value of equity can be written as:

$$\text{Terminal Value}_{\text{ROE=Cost of Equity}} = \frac{\text{Net Income}_{n+1}}{\text{Cost of Equity}_n}$$

Tying up loose ends

When using equity valuation models, there are fewer loose ends to tie up, partly because net income already includes the income from cash and cross holdings but that I always not a hard and fast rule. Thus, if the cash flows are based non-cash net income, we have not valued cash yet and it should be added on to the present value. If, on the other hand, we estimate cash flows from the cumulative net income or use the dividend discount model, cash already has been implicitly valued; the income from cash is part of the final cash flow and the discount rate presumably has been adjusted to reflect the presence of cash.

There is no need to net debt out, since we have estimated the equity value in the business, though we may still net out underfunded pension/health care obligations or the costs of a potential lawsuit from the equity value. We should still subtract out the estimated value of employee options, since that represents a drain on equity value to common shareholders. Table 5.3 summarizes how we deal with loose ends in different discounted cash flow models.

Table 2.9: Dealing with loose ends in valuation

<i>Loose End</i>	<i>Dividend Discount Model</i>	<i>FCFE Model</i>	<i>FCFF Model</i>
Cash and Marketable Securities	Ignore, since net income includes interest income from cash.	Ignore, if FCFE is computed using total net income. Add, if FCFE is computed using non-cash net income	Add. Operating income does not include income from cash.
Cross Holdings	Ignore, since net income includes income from cross holdings.	Ignore, since net income includes income from cross holdings.	Add market value of minority holdings and subtract market value of minority interests.
Other Liabilities	Ignore. The	Subtract out expected	Subtract out under

	assumption is that the firm is considering costs when setting dividends.	litigation costs.	funded pension obligations, health care obligations and expected litigation costs.
Employee options	Ignore.	Subtract out value of equity options outstanding	Subtract out value of equity options outstanding

FCFE versus Dividend Discount Model Valuation

The FCFE model can be viewed as an alternative to the dividend discount model. Since the two approaches sometimes provide different estimates of value for equity, it is worth examining when they provide similar estimates of value, when they provide different estimates of value and what the difference tells us about the firm.

a. When they are similar

There are two conditions under which the value from using the FCFE in discounted cashflow valuation will be the same as the value obtained from using the dividend discount model. The first is the obvious one, where the dividends are equal to the FCFE. There are firms that maintain a policy of paying out excess cash as dividends either because they have pre-committed to doing so or because they have investors who expect this policy of them.

The second condition is more subtle, where the FCFE is greater than dividends, but the excess cash (FCFE - Dividends) is invested in fairly priced assets (i.e. assets that earn a fair rate of return and thus have zero net present value). For instance, investing in financial assets that are fairly priced should yield a net present value of zero. To get equivalent values from the two approaches, though, we have to keep track of accumulating cash in the dividend discount model and add it to the value of equity (as shown in illustration 5.10 at the end of this section).

b. When they are different

There are several cases where the two models will provide different estimates of value. First, when the FCFE is greater than the dividend and the excess cash either earns

below-market interest rates or is invested in negative net present value assets, the value from the FCFE model will be greater than the value from the dividend discount model. There is reason to believe that this is not as unusual as it would seem at the outset. There are numerous case studies of firms, which having accumulated large cash balances by paying out low dividends relative to FCFE, have chosen to use this cash to finance unwise takeovers (where the price paid is greater than the value received from the takeover). Second, the payment of dividends less than FCFE lowers debt-equity ratios and may lead the firm to become under levered, causing a loss in value.

In the cases where dividends are greater than FCFE, the firm will have to issue either new stock or debt to pay these dividends or cut back on its investments, leading to at least one of three negative consequences for value. If the firm issues new equity to fund dividends, it will face substantial issuance costs that decrease value. If the firm borrows the money to pay the dividends, the firm may become over levered (relative to the optimal) leading to a loss in value. Finally, if paying too much in dividends leads to capital rationing constraints where good projects are rejected, there will be a loss of value (captured by the net present value of the rejected projects).

There is a third possibility and it reflects different assumptions about reinvestment and growth in the two models. If the same growth rate used in the dividend discount and FCFE models, the FCFE model will give a higher value than the dividend discount model whenever FCFE are higher than dividends and a lower value when dividends exceed FCFE. In reality, the growth rate in FCFE should be different from the growth rate in dividends, because the free cash flow to equity is assumed to be paid out to stockholders. This will affect the equity reinvestment rate of the firm. In addition, the return on equity used in the FCFE model should reflect the return on equity on non-cash investments, whereas the return on equity used in the dividend discount model should be the overall return on equity. In general, when firms pay out much less in dividends than they have available in FCFE, the expected growth rate and terminal value will be higher in the dividend discount model, but the year-to-year cash flows will be higher in the FCFE model.

3. What does it mean when they are different?

When the value using the FCFE model is different from the value using the dividend discount model, with consistent growth assumptions, there are two questions that need to be addressed - What does the difference between the two models tell us? Which of the two models is the appropriate one to use in evaluating the market price?

The more common occurrence is for the value from the FCFE model to exceed the value from the dividend discount model. The difference between the value from the FCFE model and the value using the dividend discount model can be considered one component of the value of controlling a firm - it measures the value of controlling dividend policy. In a hostile takeover, the bidder can expect to control the firm and change the dividend policy (to reflect FCFE), thus capturing the higher FCFE value.

As for which of the two values is the more appropriate one for use in evaluating the market price, the answer lies in the openness of the market for corporate control. If there is a sizable probability that a firm can be taken over or its management changed, the market price will reflect that likelihood and the appropriate benchmark to use is the value from the FCFE model. As changes in corporate control become more difficult, either because of a firm's size and/or legal or market restrictions on takeovers, the value from the dividend discount model will provide the appropriate benchmark for comparison.

Per Share versus Aggregate Valuation

In this chapter, some of the valuations that we did use per share values for earnings and cash flows and arrived at a per share estimate of value for equity. Other valuations used aggregate net income and cash flows and arrived at the aggregate value for equity. Why use one approach over the other and what are the pros and cons?

The per share approach tends to be a little simpler and information is usually more accessible. Most data services report earnings per share and analyst estimates of growth in earnings per share. There are two reasons, though, for sticking with aggregate valuation. The first is that it is easier to keep operating assets separate from cash, if we begin with net income rather than earnings per share, and break it down into net income from operating assets and cash income. The second is that the number of shares to use to compute per share values can be subject to debate when there are options, warrants and

convertible bonds outstanding. These equity options issued by the firm can be converted into shares, thus altering the number of shares outstanding. Analysts do try to factor in these options by computing the partially diluted (where options in the money are counted as shares outstanding) or fully diluted (where all options are counted) per share values. However, options do not lend themselves easily to this characterization. A much more robust way of dealing with options is to value them as options and to subtract this value from the aggregate value of equity estimated for a firm to arrive at an equity value for common stock. Dividing this value by the actual number of shares outstanding should yield the correct value for equity per share. We will deal with this question much more extensively later in this book, when we look at employee stock options and their effects on value.

Firm valuation versus equity valuation

The equity valuation models, whether they use dividends or free cash flows to equity, yield a value for the equity in a company. The value of equity, however, can be extracted from the value of the firm by subtracting out the market value of outstanding debt. Since these are alternative ways of valuing equity, two questions arise - Why value the firm rather than equity? Will the values for equity obtained from the firm valuation approach be consistent with the values obtained from the equity valuation approaches?

The advantage of using the firm valuation approach is that cashflows relating to debt do not have to be considered explicitly, since the FCFF is a pre-debt cashflow, while they have to be taken into account in estimating FCFE. In cases where the leverage is expected to change significantly over time, this is a significant saving, since estimating new debt issues and debt repayments when leverage is changing can become increasingly messy the further into the future you go. The firm valuation approach does, however, require information about debt ratios and interest rates to estimate the weighted average cost of capital.

The value for equity obtained from the firm valuation and equity valuation approaches will be the same if you make consistent assumptions about financial leverage. Getting them to converge in practice is much more difficult. Let us begin with the simplest case – a no-growth, perpetual firm. Assume that the firm has \$166.67 million in

earnings before interest and taxes and a tax rate of 40%. Assume that the firm has equity with a market value of \$600 million, with a cost of equity of 13.87% debt of \$400 million and with a pre-tax cost of debt of 7%. The firm's cost of capital can be estimated.

$$\text{Cost of capital} = \left(13.87\% \left(\frac{600}{1000}\right)\right) + \left(7\% \right)(1 - 0.4) \left(\frac{400}{1000}\right) = 10\%$$

$$\text{Value of the firm} = \frac{\text{EBIT}(1 - t)}{\text{Cost of capital}} = \frac{166.67(1 - 0.4)}{0.10} = \$1,000$$

Note that the firm has no reinvestment and no growth. We can value equity in this firm by subtracting out the value of debt.

$$\text{Value of equity} = \text{Value of firm} - \text{Value of debt} = \$1,000 - \$400 = \$600 \text{ million}$$

Now let us value the equity directly by estimating the net income:

$$\text{Net Income} = (\text{EBIT} - \text{Pre-tax cost of debt} * \text{Debt}) (1 - t) = (166.67 - 0.07 * 400) (1 - 0.4) = 83.202 \text{ million}$$

The value of equity can be obtained by discounting this net income at the cost of equity:

$$\text{Value of equity} = \frac{\text{Net Income}}{\text{Cost of equity}} = \frac{83.202}{0.1387} = \$600 \text{ million}$$

Even this simple example works because of the following assumptions that we made implicitly or explicitly during the valuation.

1. The values for debt and equity used to compute the cost of capital were equal to the values that we obtained in the valuation. Notwithstanding the circularity in reasoning – you need the cost of capital to obtain the values in the first place – it indicates that a cost of capital based upon market value weights will not yield the same value for equity as an equity valuation model, if the firm is not fairly priced in the first place.
2. There are no extraordinary or non-operating items that affect net income but not operating income. Thus, to get from operating to net income, all we do is subtract out interest expenses and taxes.
3. The interest expenses are equal to the pre-tax cost of debt multiplied by the market value of debt. If a firm has old debt on its books, with interest expenses that are different from this value, the two approaches will diverge.

If there is expected growth, the potential for inconsistency multiplies. You have to ensure that you borrow enough money to fund new investments to keep your debt ratio at a level consistent with what you are assuming when you compute the cost of capital.

Conclusion

The intrinsic value of a company reflects its fundamentals. The primary tool for estimating intrinsic value is the discounted cash flow model. We started by looking at the contrast between valuing the equity in a business and valuing the entire business, and then moved on to the four inputs that we need for the model. The cash flows to equity investors can be defined strictly as dividends, more expansively as dividends augmented with stock buybacks and most generally as free cash flows to equity (potential dividends). The cash flow to the firm is the cumulative cash flow to both equity investors and lenders, and thus is a pre-debt cash flow. The discount rates we apply have to be consistent with the cash flow definition, with the cost of equity used to discount cash flows to equity and the cost of capital to discount cash flows to the firm. When estimating growth, we noted the limitations of historical growth numbers and outside estimates, and the importance of linking growth to fundamentals. Finally, we applied closure to the models by assuming that cash flows will settle into stable growth, sometime in the future, but imposed constraints on what this growth rate can be and the characteristics of stable growth companies.

We closed the chapter by looking at three variations on the discounted cash flow model. In the certainty equivalent approach, we adjusted the cash flows for risk and discounted back at the riskfree rate. In the adjusted present value approach, we separated debt from the operating assets of the firm, and valued its effects independently of the firm. In the excess return model, we zeroed in on the fact that it is not growth per se that creates value but growth with excess returns. However, we noted that the models agree at the core, though there are minor differences in assumptions.

CHAPTER 6

RELATIVE VALUATION

In discounted cash flow valuation, the objective is to find the value of an asset, given its cash flow, growth and risk characteristics. In relative valuation, the objective is to value an asset, based upon how similar assets are currently priced by the market. Consequently, there are two components to relative valuation. The first is that to value assets on a relative basis, prices have to be standardized, usually by converting prices into multiples of some common variable. While this common variable will vary across assets, it usually takes the form of earnings, book value or revenues for publicly traded stocks.. The second is to find similar assets, which is difficult to do since no two assets are exactly identical. With real assets like antiques and baseball cards, the differences may be small and easily controlled for when pricing the assets. In the context of valuing equity in firms, the problems are compounded since firms in the same business can still differ on risk, growth potential and cash flows. The question of how to control for these differences, when comparing a multiple across several firms, becomes a key one.

While relative valuation is easy to use and intuitive, it is also easy to misuse. In this chapter, we will develop a four-step process for doing relative valuation. In the process, we will also develop a series of tests that can be used to ensure that multiples are correctly used.

What is relative valuation?

In relative valuation, we value an asset based upon how similar assets are priced in the market. A prospective house buyer decides how much to pay for a house by looking at the prices paid for similar houses in the neighborhood. A baseball card collector makes a judgment on how much to pay for a Mickey Mantle rookie card by checking transactions prices on other Mickey Mantle rookie cards. In the same vein, a potential investor in a stock tries to estimate its value by looking at the market pricing of “similar” stocks.

Embedded in this description are the three essential steps in relative valuation. The first step is finding comparable assets that are priced by the market, a task that is

easier to accomplish with real assets like baseball cards and houses than it is with stocks. All too often, analysts use other companies in the same sector as comparable, comparing a software firm to other software firms or a utility to other utilities. The second step is scaling the market prices to a common variable to generate standardized prices that are comparable. While this may not be necessary when comparing identical assets (Mickey Mantle rookie cards), it is necessary when comparing assets that vary in size or units. Other things remaining equal, a smaller house or apartment should trade at a lower price than a larger residence. In the context of stocks, this equalization usually requires converting the market value of equity or the firm into multiples of earnings, book value or revenues. The third and last step in the process is adjusting for differences across assets when comparing their standardized values. Again, using the example of a house, a newer house with more updated amenities should be priced higher than a similar sized older house that needs renovation. With stocks, differences in pricing across stocks can be attributed to all of the fundamentals that we talked about in discounted cash flow valuation. Higher growth companies, for instance, should trade at higher multiples than lower growth companies in the same sector. Many analysts adjust for these differences qualitatively, making every relative valuation a story telling experience; analysts with better and more believable stories are given credit for better valuations.

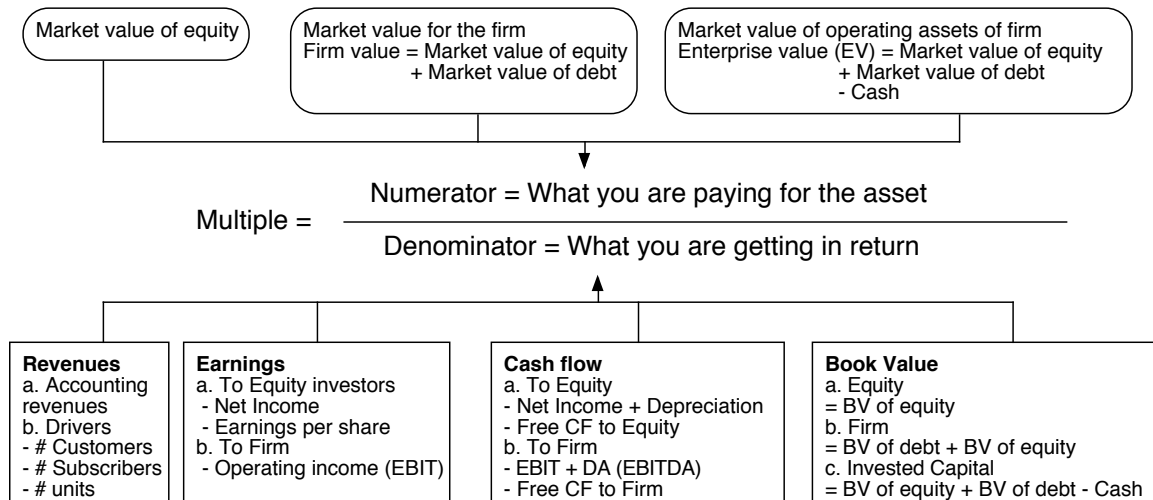
There is a significant philosophical difference between discounted cash flow and relative valuation. In discounted cash flow valuation, we are attempting to estimate the intrinsic value of an asset based upon its capacity to generate cash flows in the future. In relative valuation, we are making a judgment on how much an asset is worth by looking at what the market is paying for similar assets. If the market is correct, on average, in the way it prices assets, discounted cash flow and relative valuations may converge. If, however, the market is systematically over pricing or under pricing a group of assets or an entire sector, discounted cash flow valuations can deviate from relative valuations.

Standardized Values and Multiples

Comparing assets that are not exactly similar can be a challenge. If we have to compare the prices of two buildings of different sizes in the same location, the smaller building will look cheaper unless we control for the size difference by computing the

price per square foot. When comparing publicly traded stocks across companies, the price per share of a stock is a function both of the value of the equity in a company and the number of shares outstanding in the firm. To compare the values of “similar” firms in the market, we need to standardize the values in some way by scaling them to a common variable. In general, values can be standardized relative to the earnings firms generate, to the books values, to the revenues that firms generate or to measures that are specific to firms in a sector. Figure 6.1 lists the alternatives:

Figure 6.1: Choices in Standardization



To provide a couple of illustrations, we can divide the market value of equity by the net income to estimate the PE ratio (measuring how much equity investors are paying per dollar of earnings), or the enterprise value to EBITDA to get a sense of the market value of operating assets, relative to operating cash flows. We can even estimate what we are paying as a market value per subscriber in a cable company or what the market value is relative to the accounting estimate of value (book value). The central reason for standardizing, though, does not change. We want to compare these numbers across companies.

The Four Basic Steps to Using Multiples

Multiples are easy to use and easy to misuse. There are four basic steps to using multiples wisely and for detecting misuse in the hands of others. The first step is to ensure that the multiple is defined consistently and that it is measured uniformly across the firms being compared. The second step is to be aware of the cross sectional

distribution of the multiple, not only across firms in the sector being analyzed but also across the entire market. The third step is to analyze the multiple and understand not only what fundamentals determine the multiple but also how changes in these fundamentals translate into changes in the multiple. The final step is finding the right firms to use for comparison and controlling for differences that may persist across these firms.

1. Definitional Tests

Even the simplest multiples are defined differently by different analysts. Consider, for instance, the price earnings ratio (PE), the most widely used valuation multiple in valuation. Analysts define it to be the market price divided by the earnings per share but that is where the consensus ends. There are a number of variants on the PE ratio. While the current price is conventionally used in the numerator, there are some analysts who use the average price over the last six months or a year. The earnings per share in the denominator can be the earnings per share from the most recent financial year (yielding the current PE), the last four quarters of earnings (yielding the trailing PE) and expected earnings per share in the next financial year (resulting in a forward PE). In addition, earnings per share can be computed based upon primary shares outstanding or fully diluted shares and can include or exclude extraordinary items.

Not only can these variants on earnings yield vastly different values for the price earnings ratio but the one that gets used by analysts depends upon their biases. For instance, in periods of rising earnings, the forward PE will yield consistently lower values than the trailing PE, which, in turn, will be lower than the current PE. A bullish analyst will tend to use the forward PE to make the case that the stock is trading at a low multiple of earnings, while a bearish analyst will focus on the current PE to make the case that the multiple is too high. The first step when discussing a valuation based upon a multiple is to ensure that everyone in the discussion is using the same definition for that multiple.

Consistency

Every multiple has a numerator and a denominator. The numerator can be either an equity value (such as market price or value of equity) or a firm value (such as enterprise value, which is the sum of the values of debt and equity, net of cash). The denominator can be an equity measure (such as earnings per share, net income or book

value of equity) or a firm measure (such as operating income, EBITDA or book value of capital).

One of the key tests to run on a multiple is to examine whether the numerator and denominator are defined consistently. *If the numerator for a multiple is an equity value, then the denominator should be an equity value as well. If the numerator is a firm value, then the denominator should be a firm value as well.* To illustrate, the price earnings ratio is a consistently defined multiple, since the numerator is the price per share (which is an equity value) and the denominator is earnings per share (which is also an equity value). So is the Enterprise value to EBITDA multiple, since the numerator and denominator are both firm value measures; the enterprise value measures the market value of the operating assets of a company and the EBITDA is the cashflow generated by the operating assets, prior to taxes and reinvestment needs.

Are there any multiples in use that are inconsistently defined? Consider the price to EBITDA multiple, a multiple that has acquired adherents in the last few years among analysts. The numerator in this multiple is an equity value and the denominator is a measure of earnings to the firm. The analysts who use this multiple will probably argue that the inconsistency does not matter since the multiple is computed the same way for all of the comparable firms; but they would be wrong. If some firms on the list have no debt and others carry significant amounts of debt, the latter will look cheap on a price to EBITDA basis, when in fact they might be over or correctly priced.

Uniformity

In relative valuation, the multiple is computed for all of the firms in a group and then compared across these firms to make judgments on which firms are over priced and which are under priced. For this comparison to have any merit, the multiple has to be defined uniformly across all of the firms in the group. Thus, if the trailing PE is used for one firm, it has to be used for all of the others as well. In fact, one of the problems with using the current PE to compare firms in a group is that different firms can have different fiscal-year ends. This can lead to some firms having their prices divided by earnings from July 2007 to June 2008, with other firms having their prices divided by earnings from January 2008 to December 2008. While the differences can be minor in mature sectors,

where earnings do not make quantum jumps over six months, they can be large in high-growth sectors.

With both earnings and book value measures, there is another component to be concerned about and that is the accounting standards used to estimate earnings and book values. Differences in accounting standards can result in very different earnings and book value numbers for similar firms. This makes comparisons of multiples across firms in different markets, with different accounting standards, very difficult. Even with the same accounting standards governing companies, there can be differences in firms that arise because of discretionary accounting choices. There is also the additional problem posed by the fact that some firms use different accounting rules (on depreciation and expensing) for reporting purposes and tax purposes and others do not.¹⁹ In summary, companies that use aggressive assumptions in measuring earnings will look cheaper on earnings multiples than firms that adopt conservative accounting practices.

2. Descriptive Tests

When using a multiple, it is always useful to have a sense of what a high value, a low value or a typical value for that multiple is in the market. In other words, knowing the distributional characteristics of a multiple is a key part of using that multiple to identify under or over valued firms. In addition, we need to understand the effects of outliers on averages and unearth any biases in these estimates, introduced in the process of estimating multiples. In the final part of this section, we will look at how the distributions of multiples shift over time.

Distributional Characteristics

When using multiples to value companies, we often compare the values across companies in a sector. What is often lacking, however, is a sense of how the multiple is distributed across the entire market. The standard statistics – the average and standard deviation – are where we should start, but they represent the beginning of the exploration.

¹⁹ Firms that adopt different rules for reporting and tax purposes generally report higher earnings to their stockholders than they do to the tax authorities. When they are compared on a price earnings basis to firms that do not maintain different reporting and tax books, they will look cheaper (lower PE).

Table 6.1 summarizes key statistics for three widely used multiples -price earnings ratios, price to book value ratios and enterprise value to EBITDA multiple – in January 2010 in the United States

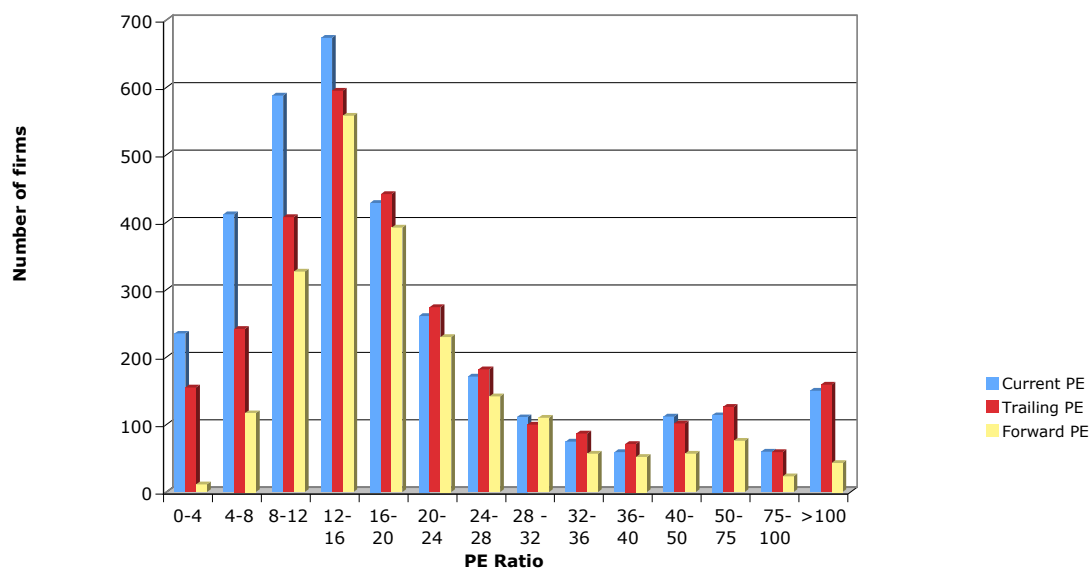
Table 6.1: Summary Statistics on Multiples – January 2010

	<i>Current PE</i>	Price to Book Equity	EV/EBITDA	EV/Sales
Mean	29.57	3.81	36.27	13.35
Standard Error	1.34	0.30	17.04	5.70
Median	14.92	1.51	5.86	1.13
Skewness	12.12	41.64	64.64	68.99
Maximum	1570.00	1294.00	5116.05	28846.00

Since the lowest value for any of these multiples is zero and the highest can be huge, the distributions for these multiples are skewed towards the positive values as evidenced by the distribution of PE ratios of US companies in January 2010 in figure 6.2:

Figure 6.2: PE Ratio Distribution: US stocks in January 2010

PE Ratios for US: January 2010



The consequences of skewed distributions for investors and analysts are significant:

- Average versus Median values: As a result of the positively skewed distributions, the average values for multiples will be higher than median values²⁰. For instance, the

²⁰ With the median, half of all firms in the group fall below this value and half lie above.

median PE ratio in January 2010 was about 15, well below the average PE reported in table 4.1 and this is true for all multiples. The median value is much more representative of the typical firm in the group and any comparisons should be made to medians. The standard sales pitch of a stock being cheap because it trades at a multiple less than the average for the sector should be retired in favor of one which compares the stock's pricing to the median for the sector.

- b. Probabilistic statements: As a result of the focus on normal distributions in most statistics classes, we begin attributing its properties to all distributions. For instance, it is true that the probability of values in a normal distribution falling more than two standard deviations away from the mean is very small. In the case of the PE ratio, this rule would suggest that few companies should have PE ratios that fall below 27.89 (which is the average of 29.57 minus two standard errors) or above 32.25 (the average plus two standard errors). The reality is that there are thousands of firms that fall outside this range because the distributions are not normal.
- c. Outliers and averages: Since multiples can take on extreme values, data reporting services that compute and report average values for multiples either throw out these outliers when computing the averages or constrain the multiples to be less than or equal to a fixed number. For instance, any firm that has a price earnings ratio greater than 500 will be assumed to have a price earnings ratio of 500. The consequence is that the averages reported by two services for the same sector or market will almost never match up because they deal with outliers differently.

Analytical Tests

In discussing why analysts were so fond of using multiples, we argued that relative valuations require fewer assumptions than discounted cash flow valuations. While this is technically true, it is only so on the surface. In reality, we make just as many assumptions when we do a relative valuation as we do in a discounted cash flow valuation. The difference is that the assumptions in a relative valuation are implicit and unstated, whereas those in discounted cash flow valuation are explicit and stated. The two primary questions that we need to answer before using a multiple are: What are the fundamentals

that determine at what multiple a firm should trade? How do changes in the fundamentals affect the multiple?

Determinants

In the introduction to discounted cash flow valuation, we observed that the value of a firm is a function of three variables – its capacity to generate cash flows, its expected growth in these cash flows and the uncertainty associated with these cash flows. Every multiple, whether it is of earnings, revenues or book value, is a function of the same three variables – risk, growth and cash flow generating potential. Intuitively, then, firms with higher growth rates, less risk and greater cash flow generating potential should trade at higher multiples than firms with lower growth, higher risk and less cash flow potential.

The specific measures of growth, risk and cash flow generating potential that are used will vary from multiple to multiple. To look under the hood, so to speak, of equity and firm value multiples, we can go back to fairly simple discounted cash flow models for equity and firm value and use them to derive the multiples.

In the simplest discounted cash flow model for equity, which is a stable growth dividend discount model, the value of equity is:

$$\text{Value of Equity} = P_0 = \frac{\text{DPS}_1}{k_e - g_n}$$

where DPS_1 is the expected dividend in the next year, k_e is the cost of equity and g_n is the expected stable growth rate. Dividing both sides by the earnings, we obtain the discounted cash flow equation specifying the PE ratio for a stable growth firm.

$$\frac{P_0}{\text{EPS}_0} = \text{PE} = \frac{\text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

The key determinants of the PE ratio are the expected growth rate in earnings per share, the cost of equity and the payout ratio. Other things remaining equal, we would expect higher growth, lower risk and higher payout ratio firms to trade at higher multiples of earnings than firms without these characteristics.

Dividing both sides by the book value of equity, we can estimate the price/book value ratio for a stable growth firm.

$$\frac{P_0}{BV_0} = \text{PBV} = \frac{\text{ROE} * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

where ROE is the return on equity and is the only variable in addition to the three that determine PE ratios (growth rate, cost of equity and payout) that affects price to book equity.

Dividing by the Sales per share, the price/sales ratio for a stable growth firm can be estimated as a function of its profit margin, payout ratio, risk and expected growth.

$$\frac{P_0}{\text{Sales}_0} = \text{PS} = \frac{\text{Profit Margin} * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

The net margin is the new variable that is added to the process. While all of these computations are based upon a stable growth dividend discount model, we will show that the conclusions hold even when we look at companies with high growth potential and with other equity valuation models.

We can do a similar analysis to derive the firm value multiples. The value of a firm in stable growth can be written as:

$$\text{Value of Firm} = V_0 = \frac{\text{FCFF}_1}{k_c - g_n}$$

Dividing both sides by the expected free cash flow to the firm yields the Value/FCFF multiple for a stable growth firm.

$$\frac{V_0}{\text{FCFF}_1} = \frac{1}{k_c - g_n}$$

The multiple of FCFF that a firm commands will depend upon two variables – its cost of capital and its expected stable growth rate. Since the free cash flow the firm is the after-tax operating income netted against the net capital expenditures and working capital needs of the firm, the multiples of revenues, EBIT and after-tax EBIT can also be estimated similarly.

$$\frac{V_0}{\text{EBIT}_1(1-t)} = \frac{(1 - \text{Reinvestment Rate})}{k_c - g_n}$$

$$\frac{V_0}{EBIT_1} = \frac{(1 - \text{Reinvestment Rate})(1 - t)}{k_c - g_n}$$

$$\frac{V_0}{\text{Sales}} = \frac{\text{After - tax Operating Margin} (1 - \text{Reinvestment Rate})}{k_c - g_n}$$

Table 6.2 summarizes the multiples and the key variables that determine each multiple, with the sign of the relationship in brackets next to each variable: $\hat{\uparrow}$ indicates that an increase in this variable will increase the multiple, whereas \downarrow indicates that an increase in this variable will decrease the multiple, holding all else constant.

Table 6.2: Fundamentals Determining Multiples

<i>Multiple</i>	<i>Fundamental Determinants</i>
Price Earnings Ratio	Expected Growth($\hat{\uparrow}$), Payout($\hat{\uparrow}$), Risk(\downarrow)
Price to Book Equity Ratio	Expected Growth($\hat{\uparrow}$), Payout($\hat{\uparrow}$), Risk(\downarrow), ROE($\hat{\uparrow}$)
Price to Sales Ratio	Expected Growth($\hat{\uparrow}$), Payout($\hat{\uparrow}$), Risk(\downarrow), Net Margin($\hat{\uparrow}$)
EV to FCFF	Cost of capital(\downarrow), Growth Rate($\hat{\uparrow}$)
EV to EBITDA	Expected Growth($\hat{\uparrow}$), Reinvestment Rate(\downarrow), Risk(\downarrow), ROC($\hat{\uparrow}$), Tax rate(\downarrow)
EV to Capital Ratio	Expected Growth($\hat{\uparrow}$), Reinvestment Rate(\downarrow), Risk(\downarrow), ROC($\hat{\uparrow}$)
EV to Sales	Expected Growth($\hat{\uparrow}$), Reinvestment Rate(\downarrow), Risk(\downarrow), Operating Margin($\hat{\uparrow}$)

The point of this analysis is not to suggest that we go back to using discounted cash flow valuation, but to understand the variables that may cause these multiples to vary across firms in the same sector. If we ignore these variables, we might conclude that a stock with a PE of 8 is cheaper than one with a PE of 12 when the true reason may be that the latter has higher expected growth or we might decide that a stock with a P/BV ratio of 0.7 is cheaper than one with a P/BV ratio of 1.5 when the true reason may be that the latter has a much higher return on equity.

Companion Variable

While the variables that determine a multiple can be extracted from a discounted cash flow model and the relationship between each variable and the multiple can be developed by holding all else constant and asking what-if questions, there is a single variable that dominates when it comes to explaining each multiple (and it is not the same variable for every multiple). This variable, which is called the companion variable, is critical to using multiples wisely in making valuation judgments and can be identified by looking for the variable that best explain differences across firms using a particular multiple.

So, what are the companion variables for the most widely used multiples? To arrive at this judgment, we looked at which of the variables listed in table 6.2 was most useful in explaining differences across firms with each multiple and came up with the list in table 6.3:

Table 6.3: Companion Variables for Multiples

<i>Multiple</i>	<i>Companion variable</i>	<i>Valuation mismatch</i>
PE Ratio	Expected Growth	Low PE stock with high expected growth rate in earnings per share
PBV Ratio	ROE	Low PBV stock with high ROE
PS Ratio	Net Margin	Low PS stock with high net profit margin.
EV/EBITDA	Reinvestment Rate	Low EV/EBITDA stock with low reinvestment needs
EV/Capital	Return on Capital	Low EV/Capital stock with high return on capital.
EV/Sales	After-tax operating margin	Low EV/Sales ratio with a high after-tax operating margin.

Relationship

Knowing the fundamentals that determine a multiple is a useful first step, but understanding how the multiple changes as the fundamentals change is just as critical to using the multiple. To illustrate, knowing that higher growth firms have higher PE ratios is not a sufficient insight if we are called upon to analyze whether a firm with a growth

rate that is twice as high as the average growth rate for the sector should have a PE ratio that is 1.5 times or 1.8 times or 2 times the average price earnings ratio for the sector. To make this judgment, we need to know how the PE ratio changes as the growth rate changes.

A surprisingly large number of valuation analyses are based upon the assumption that there is a linear relationship between multiples and fundamentals. For instance, the PEG ratio, which is the ratio of the PE to the expected growth rate of a firm and widely used to analyze high growth firms, implicitly assumes that PE ratios and expected growth rates are linearly related.

One of the advantages of deriving the multiples from a discounted cash flow model, as was done in the last section, is that we can analyze the relationship between each fundamental variable and the multiple by keeping everything else constant and changing the value of that variable. When we do this, we will find that there are very few linear relationships in valuation.

3. Application Tests

When multiples are used, they tend to be used in conjunction with comparable firms to determine the value of a firm or its equity. But what is a comparable firm? While the conventional practice is to look at firms within the same industry or business, this is not necessarily always the correct or the best way of identifying these firms. In addition, no matter how carefully we choose comparable firms, differences will remain between the firm we are valuing and the comparable firms. Figuring out how to control for these differences is a significant part of relative valuation.

What is a comparable firm?

A comparable firm is one with cash flows, growth potential, and risk similar to the firm being valued. It would be ideal if we could value a firm by looking at how an exactly identical firm - in terms of risk, growth and cash flows - is priced. Nowhere in this definition is there a component that relates to the industry or sector to which a firm belongs. Thus, a telecommunications firm can be compared to a software firm, if the two are identical in terms of cash flows, growth and risk. In most analyses, however, analysts define comparable firms to be other firms in the firm's business or businesses. If there are

enough firms in the industry to allow for it, this list is pruned further using other criteria; for instance, only firms of similar size may be considered. The implicit assumption being made here is that firms in the same sector have similar risk, growth, and cash flow profiles and therefore can be compared with much more legitimacy.

This approach becomes more difficult to apply when there are relatively few firms in a sector. In most markets outside the United States, the number of publicly traded firms in a particular sector, especially if it is defined narrowly, is small. It is also difficult to define firms in the same sector as comparable firms if differences in risk, growth and cash flow profiles across firms within a sector are large. Thus, there are hundreds of computer software companies listed in the United States, but the differences across these firms are also large. The tradeoff is therefore a simple one. Defining an industry more broadly increases the number of comparable firms, but it also results in a more diverse group of companies.

There are alternatives to the conventional practice of defining comparable firms. One is to look for firms that are similar in terms of valuation fundamentals. For instance, to estimate the value of a firm with a beta of 1.2, an expected growth rate in earnings per share of 20% and a return on equity of 40%²¹, we would find other firms across the entire market with similar characteristics.²² The other is consider all firms in the market as comparable firms and to control for differences on the fundamentals across these firms, using statistical techniques.

Controlling for Differences across Firms

No matter how carefully we construct our list of comparable firms, we will end up with firms that are different from the firm we are valuing. The differences may be small on some variables and large on others and we will have to control for these differences in a relative valuation. There are three ways of controlling for these differences:

²¹ The return on equity of 40% becomes a proxy for cash flow potential. With a 20% growth rate and a 40% return on equity, this firm will be able to return half of its earnings to its stockholders in the form of dividends or stock buybacks.

²² Finding these firms manually may be tedious when your universe includes 10000 stocks. You could draw on statistical techniques such as cluster analysis to find similar firms.

1. Subjective Adjustments

Relative valuation begins with two choices - the multiple used in the analysis and the group of firms that comprises the comparable firms. In many relative valuation, the multiple is calculated for each of the comparable firms and the average is computed. To evaluate an individual firm, the analyst then compare the multiple it trades at to the average computed; if it is significantly different, the analyst can make a subjective judgment about whether the firm's individual characteristics (growth, risk or cash flows) may explain the difference. Thus, a firm may have a PE ratio of 22 in a sector where the average PE is only 15, but the analyst may conclude that this difference can be justified because the firm has higher growth potential than the average firm in the industry. If, in the judgment of the analyst, the difference on the multiple cannot be explained by the fundamentals, the firm will be viewed as over valued (if its multiple is higher than the average) or undervalued (if its multiple is lower than the average).

The weakness in this approach is not that analysts are called upon to make subjective judgments, but that the judgments are often based upon little more than guesswork. All too often, these judgments confirm their biases about companies.

2. Modified Multiples

In this approach, we modify the multiple to take into account the most important variable determining it – the companion variable. To provide an illustration, analysts who compare PE ratios across companies with very different growth rates often divide the PE ratio by the expected growth rate in EPS to determine a growth-adjusted PE ratio or the PEG ratio. This ratio is then compared across companies with different growth rates to find under and over valued companies.

There are two implicit assumptions that we make when using these modified multiples. The first is that these firms are comparable on all the other measures of value, other than the one being controlled for. In other words, when comparing PEG ratios across companies, we are assuming that they are all of equivalent risk. The other assumption generally made is that that the relationship between the multiples and fundamentals is linear. Again, using PEG ratios to illustrate the point, we are assuming that as growth doubles, the PE ratio will double; if this assumption does not hold up and

PE ratios do not increase proportional to growth, companies with high growth rates will look cheap on a PEG ratio basis.

3. Statistical Techniques

In a regression, we attempt to explain a dependent variable by using independent variables that we believe influence the dependent variable. This mirrors what we are attempting to do in relative valuation, where we try to explain differences across firms on a multiple (PE ratio, EV/EBITDA) using fundamental variables (such as risk, growth and cash flows). Regressions offer three advantages over the subjective approach:

- a. The output from the regression gives us a measure of how strong the relationship is between the multiple and the variable being used. Thus, if we are contending that higher growth companies have higher PE ratios, the regression should yield clues to both how growth and PE ratios are related (through the coefficient on growth as an independent variable) and how strong the relationship is (through the t statistics and R squared).
- b. If the relationship between a multiple and the fundamental we are using to explain it is non-linear, the regression can be modified to allow for the relationship.
- c. Unlike the modified multiple approach, where we were able to control for differences on only one variable, a regression can be extended to allow for more than one variable and even for cross effects across these variables.

In general, regressions seem particularly suited to our task in relative valuation, which is to make sense of voluminous and sometimes contradictory data. There are two key questions that we face when running sector regressions:


- The first relates to how we define the sector. If we define sectors too narrowly, we run the risk of having small sample sizes, which undercut the usefulness of the regression. Defining sectors broadly entails fewer risks. While there may be large differences across firms when we do this, we can control for those differences in the regression.
- The second involves the independent variables that we use in the regression. While the focus in statistics classes is increasing the explanatory power of the regression (through the R-squared) and including any variables that accomplish this, the focus of

regressions in relative valuations is narrower. Since our objective is not to explain away all differences in pricing across firms but only those differences that are explained by fundamentals, we will use only those variables that are related to those fundamentals. The last section where we analyzed multiples using DCF models should yield valuable clues. As an example, consider the PE ratio. Since it is determined by the payout ratio, expected growth and risk, we will include only those variables in the regression. We will not add other variables to this regression, even if doing so increases the explanatory power, if there is no fundamental reason why these variables should be related to PE ratios.

Summary

In relative valuation, we estimate the value of an asset by looking at how similar assets are priced. To make this comparison, we begin by converting prices into multiples – standardizing prices – and then comparing these multiples across firms that we define as comparable. Prices can be standardized based upon earnings, book value, revenue or sector-specific variables.

While the allure of multiples remains their simplicity, there are four steps in using them soundly. First, we have to define the multiple consistently and measure it uniformly across the firms being compared. Second, we need to have a sense of how the multiple varies across firms in the market. In other words, we need to know what a high value, a low value and a typical value are for the multiple in question. Third, we need to identify the fundamental variables that determine each multiple and how changes in these fundamentals affect the value of the multiple. Finally, we need to find truly comparable firms and adjust for differences between the firms on fundamental characteristics.



CHAPTER 7

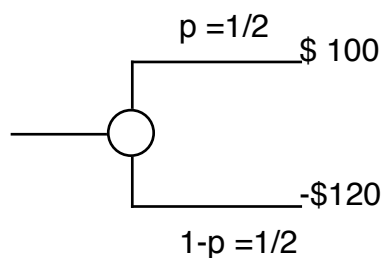
REAL OPTION VALUATION

The real options approach has its foundations in two elements – the capacity of individuals or entities to learn from what is happening around them and their willingness and the ability to modify behavior based upon that learning. In this chapter, we examine possible uses for real options in valuation, with caveats on the values.

The Essence of Real Options

To understand the basis of the real options argument and the reasons for its allure, it is easiest to think in terms of decision trees. Consider a very simple example of a decision tree in figure 7.1:

Figure 7.1: Simple Decision Tree

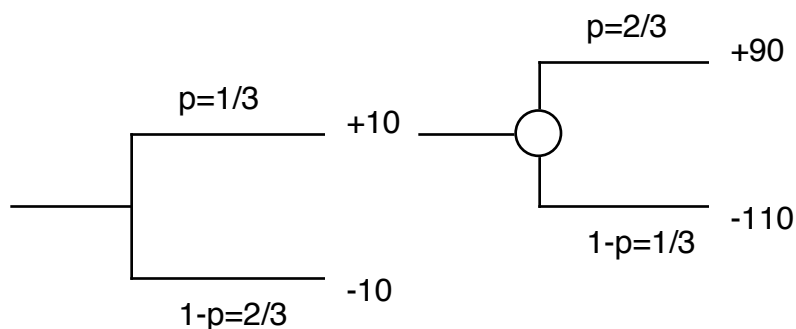


Given the equal probabilities of up and down movements, and the larger potential loss, the expected value for this investment is negative.

$$\text{Expected Value} = 0.50 (100) + 0.5 (-120) = -\$10$$

Now contrast this with the slightly more complicated two-phase decision tree in figure 7.2:

Figure 7.2: Two-phase Decision Tree



Note that the total potential profits and losses over the two phases in the tree are identical to the profit and loss of the simple tree in figure 8.1; your total gain is \$ 100 and your total loss is \$120. Note also that the cumulative probabilities of success and failure remain at the 50% that we used in the simple tree. When we compute the expected value of this tree, though, the outcome changes:

$$\text{Expected Value} = (2/3) (-10) + 1/3 [10+(2/3)(90) + (1/3)(-110)] = \$4.44$$

What is it about the second decision tree that makes a potentially bad investment (in the first tree) into a good investment (in the second)? We would attribute the change to two factors. First, by allowing for an initial phase where you get to observe the cashflows on a first and relatively small try at the investment, we allow for learning. Thus, getting a bad outcome in the first phase (-10 instead of +10) is an indicator that the overall investment is more likely to be money losing than money making. Second, you act on the learning by abandoning the investment, if the outcome from the first phase is negative; we will call this adaptive behavior.

In essence, the value of real options stems from the fact that when investing in risky assets, we can learn from observing what happens in the real world and adapting our behavior to increase our potential upside from the investment and to decrease the possible downside. In the real options framework, we use updated knowledge or information to expand opportunities while reducing danger. In the context of a risky investment, there are three potential actions that can be taken based upon this updated knowledge. The first is that you build on good fortune to increase your possible profits; this is the option to expand. For instance, a market test that suggests that consumers are far more receptive to a new product than you expected them to be could be used as a basis for expanding the scale of the project and speeding its delivery to the market. The second is to scale down or even abandon an investment when the information you receive contains bad news; this is the option to abandon and can allow you to cut your losses. The third is to hold off on making further investments, if the information you receive suggests ambivalence about future prospects; this is the option to delay or wait. You are, in a sense, buying time for the investment, hoping that product and market developments will make it attractive in the future.

We would add one final piece to the mix that is often forgotten but is just as important as the learning and adaptive behavior components in terms of contributing to the real options arguments. The value of learning is greatest, when you and only you have access to that learning and can act on it. After all, the expected value of knowledge that is public, where anyone can act on that knowledge, will be close to zero. We will term this third condition “exclusivity” and use it to scrutinize when real options have the most value.

Real Option Examples

As we noted in the introductory section, there are three types of options embedded in investments – the option to expand, delay and abandon an investment. In this section, we will consider each of these options and how they made add value to an investment, as well as potential implications for valuation and risk management.

The Option to Delay an Investment

Investments are typically analyzed based upon their expected cash flows and discount rates at the time of the analysis; the net present value computed on that basis is a measure of its value and acceptability at that time. The rule that emerges is a simple one: negative net present value investments destroy value and should not be accepted. Expected cash flows and discount rates change over time, however, and so does the net present value. Thus, a project that has a negative net present value now may have a positive net present value in the future. In a competitive environment, in which individual firms have no special advantages over their competitors in taking projects, this may not seem significant. In an environment in which a project can be taken by only one firm (because of legal restrictions or other barriers to entry to competitors), however, the changes in the project’s value over time give it the characteristics of a call option.

Basic Setup

In the abstract, assume that a project requires an initial up-front investment of X , and that the present value of expected cash inflows computed right now is V . The net present value of this project is the difference between the two:

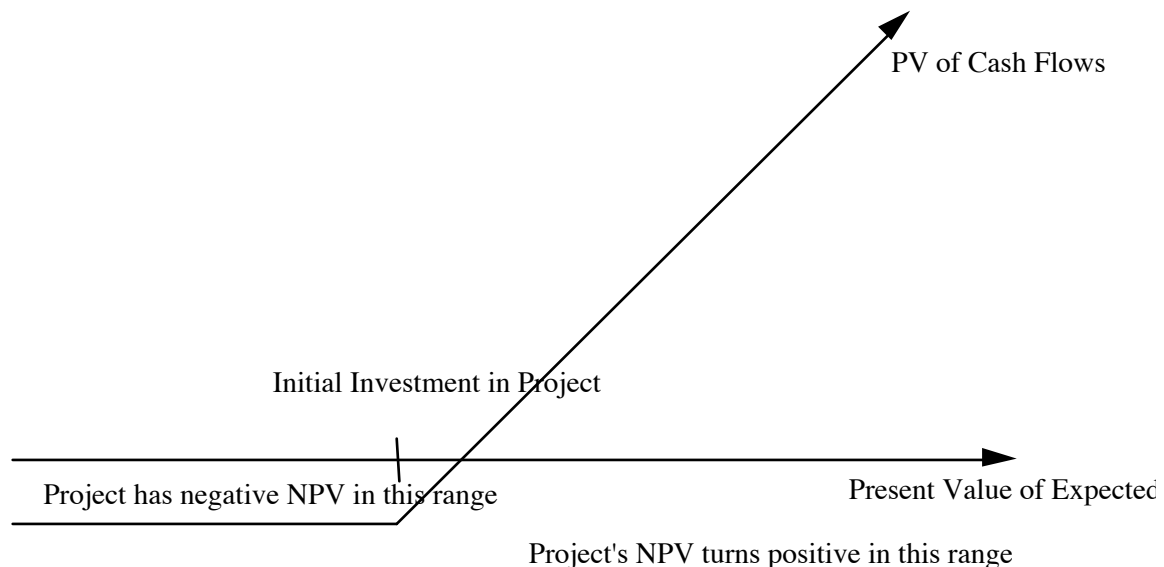
$$NPV = V - X$$

Now assume that the firm has exclusive rights to this project for the next n years, and that the present value of the cash inflows may change over that time, because of changes in either the cash flows or the discount rate. Thus, the project may have a negative net present value right now, but it may still be a good project if the firm waits. Defining V again as the present value of the cash flows, the firm's decision rule on this project can be summarized as follows:

If $V > X$ Take the project: Project has positive net present value
 $V < X$ Do not take the project: Project has negative net present value

If the firm does not invest in the project, it incurs no additional cash flows, though it will lose what it originally invested in the project. This relationship can be presented in a payoff diagram of cash flows on this project, as shown in Figure 7.3, assuming that the firm holds out until the end of the period for which it has exclusive rights to the project:²³

Figure 7.3: The Option to Delay



Note that this payoff diagram is that of a call option — the underlying asset is the investment, the strike price of the option is the initial outlay needed to initiate the investment; and the life of the option is the period for which the firm has rights to the

²³ McDonald, R. and D. Siegel, 2002, The Value of Waiting to Invest, Quarterly Journal of Economics, v101, 707-728.

investment. The present value of the cash flows on this project and the expected variance in this present value represent the value and variance of the underlying asset.

Valuing an Option to Delay

On the surface, the inputs needed to apply option pricing theory to valuing the option to delay are the same as those needed for any option. We need the value of the underlying asset, the variance in that value, the time to expiration on the option, the strike price, the riskless rate and the equivalent of the dividend yield (cost of delay). Actually estimating these inputs for product patent valuation can be difficult, however.

a. Value Of The Underlying Asset: In this case, the underlying asset is the investment itself. The current value of this asset is the present value of expected cash flows from initiating the project now, not including the up-front investment, which can be obtained by doing a standard capital budgeting analysis.

b. Variance in the value of the asset: The present value of the expected cashflows that measures the value of the asset will change over time, partly because the potential market size for the product may be unknown, and partly because technological shifts can change the cost structure and profitability of the product. The variance in the present value of cash flows from the project can be estimated in one of three ways.

- If similar projects have been introduced in the past, the variance in the cash flows from those projects can be used as an estimate. This may be the way that a consumer product company like Gillette might estimate the variance associated with introducing a new blade for its razors.
- Probabilities can be assigned to various market scenarios, cash flows estimated under each scenario and the variance estimated across present values. Alternatively, the probability distributions can be estimated for each of the inputs into the project analysis - the size of the market, the market share and the profit margin, for instance - and simulations used to estimate the variance in the present values that emerge.
- The variance in the market value of publicly traded firms involved in the same business (as the project being considered) can be used as an estimate of the variance. Thus, the average variance in firm value of firms involved in the software business can be used as the variance in present value of a software project.

The value of the option is largely derived from the variance in cash flows - the higher the variance, the higher the value of the project delay option. Thus, the value of an option to delay a project in a stable business will be less than the value of a similar option in an environment where technology, competition and markets are all changing rapidly.

c. Exercise Price On Option: A project delay option is exercised when the firm owning the rights to the project decides to invest in it. The cost of making this investment is the exercise price of the option. The underlying assumption is that this cost remains constant (in present value dollars) and that any uncertainty associated with the product is reflected in the present value of cash flows on the product.

d. Expiration Of The Option And The Riskless Rate The project delay option expires when the rights to the project lapse; investments made after the project rights expire are assumed to deliver a net present value of zero as competition drives returns down to the required rate. The riskless rate to use in pricing the option should be the rate that corresponds to the expiration of the option. While this input can be estimated easily when firms have the explicit right to a project (through a license or a patent, for instance), it becomes far more difficult to obtain when firms only have a competitive advantage to take a project.

d. Cost of Delay (Dividend Yield): There is a cost to delaying taking a project, once the net present value turns positive. Since the project rights expire after a fixed period, and excess profits (which are the source of positive present value) are assumed to disappear after that time as new competitors emerge, each year of delay translates into one less year of value-creating cash flows.²⁴ If the cash flows are evenly distributed over time, and the life of the patent is n years, the cost of delay can be written as:

$$\text{Annual cost of delay} = \frac{1}{n}$$

Thus, if the project rights are for 20 years, the annual cost of delay works out to 5% a year. Note, though, that this cost of delay rises each year, to 1/19 in year 2, 1/18 in year 3 and so on, making the cost of delaying exercise larger over time.

²⁴ A value-creating cashflow is one that adds to the net present value because it is in excess of the required return for investments of equivalent risk.

Practical Considerations

While it is quite clear that the option to delay is embedded in many investments, there are several problems associated with the use of option pricing models to value these options. First, the *underlying asset in this option, which is the project, is not traded*, making it difficult to estimate its value and variance. We would argue that the value can be estimated from the expected cash flows and the discount rate for the project, albeit with error. The variance is more difficult to estimate, however, since we are attempting the estimate a variance in project value over time.

Second, the *behavior of prices over time may not conform to the price path assumed by the option pricing models*. In particular, the assumption that prices move in small increments continuously (an assumption of the Black-Scholes model), and that the variance in value remains unchanged over time, may be difficult to justify in the context of a real investment. For instance, a sudden technological change may dramatically change the value of a project, either positively or negatively.

Third, *there may be no specific period for which the firm has rights to the project*. For instance, a firm may have significant advantages over its competitors, which may, in turn, provide it with the virtually exclusive rights to a project for a period of time. The rights are not legal restrictions, however, and could erode faster than expected. In such cases, the expected life of the project itself is uncertain and only an estimate. Ironically, uncertainty about the expected life of the option can increase the variance in present value, and through it, the expected value of the rights to the project.

Applications of Option to Delay

The option to delay provides interesting perspectives on three common investment problems. The first is in the valuation of patents, especially those that are not viable today but could be viable in the future; by extension, this will also allow us to look at whether R&D expenses are delivering value. The second is in the analysis of natural resource assets – vacant land, undeveloped oil reserves etc.

Patents

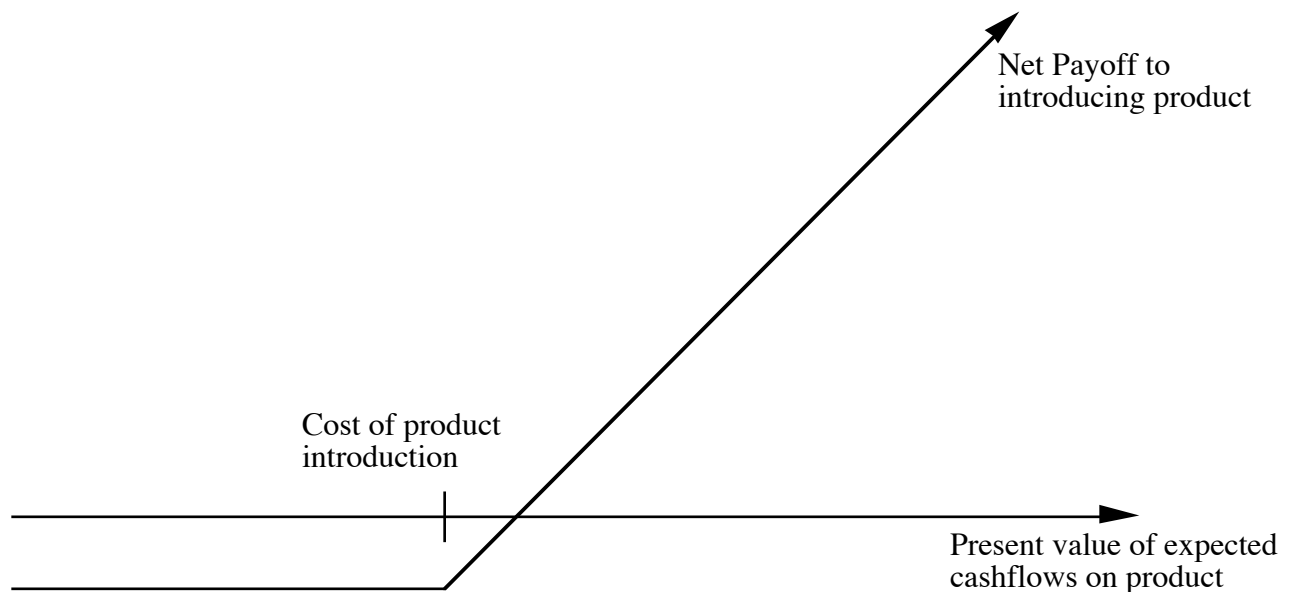
A product patent provides a firm with the right to develop and market a product. The firm will do so only if the present value of the expected cash flows from the product

sales exceed the cost of development, however, as shown in Figure 7.4. If this does not occur, the firm can shelve the patent and not incur any further costs. If I is the present value of the costs of developing the product, and V is the present value of the expected cash flows from development, the payoffs from owning a product patent can be written as:

$$\begin{aligned} \text{Payoff from owning a product patent} &= V - I && \text{if } V > I \\ &= 0 && \text{if } V \leq I \end{aligned}$$

Thus, a product patent can be viewed as a call option, where the product itself is the underlying asset.²⁵

Figure 7.4: Payoff to Introducing Product



We will illustrate the use of option pricing to value Avonex, a drug to treat multiple sclerosis, right after it had received FDA approval in 1997, but before its parent company, Biogen, had decided whether to commercialize the drug or not. We arrived at the following estimates for use in the option pricing model:

- An internal analysis of the drug at the time, based upon the potential market and the price that the firm can expect to charge, yielded a present value of expected cash flows of \$ 3.422 billion, prior to considering the initial development cost.

²⁵ Schwartz, E., 2002, Patents and R&D as Real Options, Working Paper, Anderson School at UCLA.

- The initial cost of developing the drug for commercial use was estimated to be \$2.875 billion, if the drug was introduced immediately.
- The firm had the patent on the drug for the next 17 years, and the 17-year Treasury bond rate was 6.7%.
- The average historical variance in market value for publicly traded bio-technology firms was 0.224.
- It was assumed that the potential for excess returns exists only during the patent life, and that competition will wipe out excess returns beyond that period. Thus, any delay in introducing the drug, once it is viable, will cost the firm one year of patent-protected excess returns. (For the initial analysis, the cost of delay will be 1/17, the following year, it will be 1/16, the year after, 1/15 and so on.)

Based on these assumptions, we obtained the following inputs to the option pricing model.

Present Value of Cash Flows from Introducing Drug Now = $S = \$ 3.422$ billion

Initial Cost of Developing Drug for commercial use = $K = \$ 2.875$ billion

Patent life = $t = 17$ years Riskless Rate = $r = 6.7\%$ (17-year T.Bond rate)

Variance in Expected Present Values = $\sigma^2 = 0.224$

Expected Cost of Delay = $y = 1/17 = 5.89\%$

Using these inputs in an option pricing model, we derived a value of \$907 million for the option,²⁶ and this can be considered to be the real options value attached to the patent on Avonex. To provide a contrast, the net present value of this patent is only \$ 547 million:

$$\text{NPV} = \$3,422 \text{ million} - \$ 2,875 \text{ million} = \$ 547 \text{ million}$$

The time premium of \$ 360 million (\$907 million -\$547 million) on this option suggests that the firm will be better off waiting rather than developing the drug immediately, the cost of delay notwithstanding. However, the cost of delay will increase over time, and make exercise (development) more likely. Note also that we are assuming that the firm is protected from all competition for the life of the patent. In reality, there are other pharmaceutical firms working on their own drugs to treat multiple sclerosis and that can

²⁶ This value was derived from using a Black Scholes model with these inputs. With a binomial model, the estimated value increases slightly to \$915 million.

affect both the option value and the firm's behavior. In particular, if we assume that Upjohn or Pfizer has a competing drug working through the FDA pipeline and that the drug is expected to reach the market in 6 years, the cost of delay will increase to 16.67% (1/6) and the option value will dissipate.

The implications of viewing patents as options can be significant. First, it implies that non-viable patents will continue to have value, especially in businesses where there is substantial volatility. Second, it indicates that firms may hold off on developing viable patents, if they feel that they gain more from waiting than they lose in terms of cash flows; this behavior will be more common if there is no significant competition on the horizon. Third, the value of patents will be higher in risky businesses than in safe businesses, since option value increases with volatility. If we consider R&D to be the expense associated with acquiring these patents, this would imply that research should have its biggest payoff when directed to areas where less is known and there is more uncertainty. Consequently, we should expect pharmaceutical firms to spend more of their R&D budgets on gene therapy than on flu vaccines.²⁷

Natural Resource Options

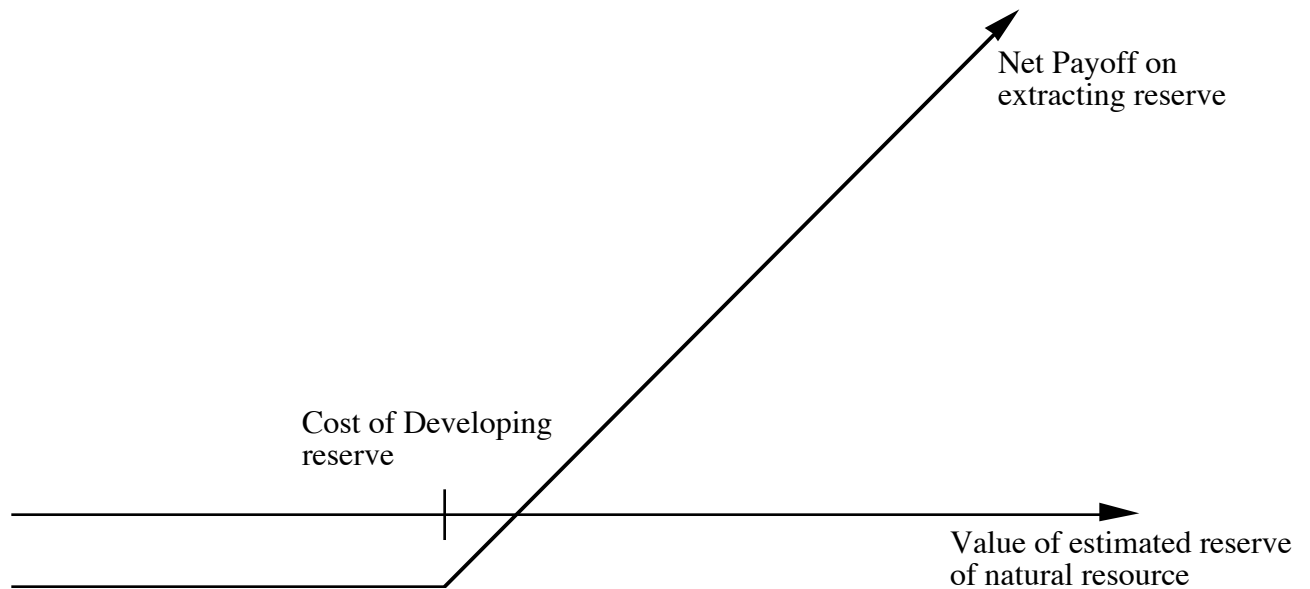
In a natural resource investment, the underlying asset is the natural resource and the value of the asset is based upon two variables - (1) the estimated quantity, and (2) the price of the resource. Thus, in a gold mine, for example, the value of the underlying asset is the value of the estimated gold reserves in the mine, based upon the current price of gold. In most such investments, there is an initial cost associated with developing the resource; the difference between the value of the asset extracted and the cost of the development is the profit to the owner of the resource (see Figure 7.5). Defining the cost of development as X , and the estimated value of the developed resource as V , the potential payoffs on a natural resource option can be written as follows:

²⁷ Pakes, A., 1986, Patents as Options: Some Estimates of the Value of Holding European Patent Stocks, *Econometrica*, v54, 755-784. While this paper does not explicitly value patents as options, it examines the returns investors would have earned investing in companies that derive their value from patents. The return distribution resembles that of a portfolio of options, with most investments losing money but the winners providing disproportionate gains.

$$\begin{aligned} \text{Payoff on natural resource investment} &= V - X && \text{if } V > X \\ &= 0 && \text{if } V \leq X \end{aligned}$$

Thus, the investment in a natural resource option has a payoff function similar to a call option.²⁸

Figure 7.5: Payoff from Developing Natural Resource Reserves



To value a natural resource investment as an option, we need to make assumptions about a number of variables:

- 1. Available reserves of the resource:* Since this is not known with certainty at the outset, it has to be estimated. In an oil tract, for instance, geologists can provide reasonably accurate estimates of the quantity of oil available in the tract.
- 2. Estimated cost of developing the resource:* The estimated development cost is the exercise price of the option. Again, a combination of knowledge about past costs and the specifics of the investment have to be used to come up with a reasonable measure of development cost.
- 3. Time to expiration of the option:* The life of a natural resource option can be defined in one of two ways. First, if the ownership of the investment has to be relinquished at the end of a fixed period of time, that period will be the life of the option. In many offshore

²⁸ Brennan, M. and E. Schwartz, 1985, Evaluating Natural Resource Investments, *The Journal of Business*, v58, 135-157.

oil leases, for instance, the oil tracts are leased to the oil company for several years. The second approach is based upon the inventory of the resource and the capacity output rate, as well as estimates of the number of years it would take to exhaust the inventory. Thus, a gold mine with a mine inventory of 3 million ounces and a capacity output rate of 150,000 ounces a year will be exhausted in 20 years, which is defined as the life of the natural resource option.

4. Variance in value of the underlying asset: The variance in the value of the underlying asset is determined by two factors – (1) variability in the price of the resource, and (2) variability in the estimate of available reserves. In the special case where the quantity of the reserve is known with certainty, the variance in the underlying asset's value will depend entirely upon the variance in the price of the natural resource. In the more realistic case where the quantity of the reserve and the oil price can change over time, the option becomes more difficult to value; here, the firm may have to invest in stages to exploit the reserves.

5. Cost of Delay: The net production revenue as a percentage of the market value of the reserve is the equivalent of the dividend yield and is treated the same way in calculating option values. An alternative way of thinking about this cost is in terms of a cost of delay. Once a natural resource option is in-the-money (Value of the reserves > Cost of developing these reserves), the firm, by not exercising the option, is costing itself the production revenue it could have generated by developing the reserve.

An important issue in using option pricing models to value natural resource options is the effect of development lags on the value of these options. Since the resources cannot be extracted instantaneously, a time lag has to be allowed between the decision to extract the resources and the actual extraction. A simple adjustment for this lag is to reduce the value of the developed reserve to reflect the loss of cash flows during the development period. Thus, if there is a one-year lag in development, the current value of the developed reserve will be discounted back one year at the net production revenue/asset value ratio²⁹ (which we also called the dividend yield above).³⁰

²⁹ Intuitively, it may seem like the discounting should occur at the riskfree rate. The simplest way of explaining why we discount at the dividend yield is to consider the analogy with a listed option on a stock.

To illustrate the use of option pricing to value natural reserves, consider an offshore oil property with an estimated reserve of 50 million barrels of oil; the cost of developing the reserve is expected to be \$ 600 million, and the development lag is two years. The firm has the rights to exploit this reserve for the next 20 years, and the marginal value per barrel of oil is \$12 currently³¹ (price per barrel - marginal cost per barrel). Once developed, the net production revenue each year will be 5% of the value of the reserves. The riskless rate is 8%, and the variance in $\ln(\text{oil prices})$ is 0.03. Given this information, the inputs to the option pricing model can be estimated as follows:

Current Value of the asset = $S = \text{Value of the developed reserve discounted back the length of the development lag at the dividend yield} = \$12 * 50 / (1.05)^2 = \$544.22$

If development is started today, the oil will not be available for sale until two years from now. The estimated opportunity cost of this delay is the lost production revenue over the delay period; hence, the discounting of the reserve back at the dividend yield.

Exercise Price = Cost of developing reserve = \$ 600 million (assumed to be both known and fixed over time)

Time to expiration on the option = 20 years

In this example, we assume that the only uncertainty is in the price of oil, and the variance therefore becomes the variance in oil prices.

Variance in the value of the underlying asset (oil) = 0.03

Riskless rate = 8%

Dividend Yield = Net production revenue / Value of reserve = 5%

Assume that on exercising a listed option on a stock, you had to wait six months for the stock to be delivered to you. What you lose is the dividends you would have received over the six-month period by holding the stock. Hence, the discounting is at the dividend yield.

³⁰ Brennan, M.J., and E.S. Schwartz, 1985, Evaluating Natural Resource Investments, *Journal of Business* 58, pp. 135-157.

³¹ For simplicity, we will assume that while this marginal value per barrel of oil will grow over time, the present value of the marginal value will remain unchanged at \$ 12 per barrel. If we do not make this assumption, we will have to estimate the present value of the oil that will be extracted over the extraction period.

Based upon these inputs, the option pricing model yields an estimate of value of \$97.08 million.³² This oil reserve, though not viable at current prices, is still a valuable property because of its potential to create value if oil prices go up.³³

The same type of analysis can be extended to any other commodity company (gold and copper reserves, for instance) and even to vacant land or real estate properties. The owner of vacant land in Manhattan can choose whether and when to develop the land and will make that decision based upon real estate values.³⁴

What are the implications of viewing natural resource reserves as options? The first is that the value of a natural resource company can be written as a sum of two values: the conventional risk adjusted value of expected cash flows from developed reserves and the option value of undeveloped reserves. While both will increase in value as the price of the natural resource increases, the latter will respond positively to increases in price volatility. Thus, the values of oil companies should increase if oil price volatility increases, even if oil prices themselves do not go up. The second is that conventional discounted cash flow valuation will understate the value of natural resource companies, even if the expected cash flows are unbiased and reasonable because it will miss the option premium inherent in their undeveloped reserves. The third is that development of natural resource reserves will slow down as the volatility in prices increases; the time premium on the options will increase, making exercise of the options (development of the reserves) less likely.

Mining and commodity companies have been at the forefront in using real options in decision making and their usage of the technology predates the current boom in real options. One reason is that natural resource options come closest to meeting the prerequisites for the use of option pricing models. Firms can learn a great deal by observing

³² This is the estimate from a Black-Scholes model, with a dividend yield adjustment. Using a binomial model yields an estimate of value of \$ 101 million.

³³ Paddock, J.L. & D. R. Siegel & J.L. Smith (1988): "Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases", *Quarterly Journal of Economics*, August 1988, pp.479-508. This paper provides a detailed examination of the application of real options to value oil reserves. They applied the model to examine the prices paid for offshore oil leases in the US in 1980 and concluded that companies over paid (relative to the option value).

³⁴ Quigg, L, 1993] Empirical Testing of Real Option-Pricing Models », *Journal of Finance*, vol.48, 621-640. The author examined transaction data on 2700 undeveloped and 3200 developed real estate properties between 1976-79 and found evidence of a premium arising from the option to wait in the former.

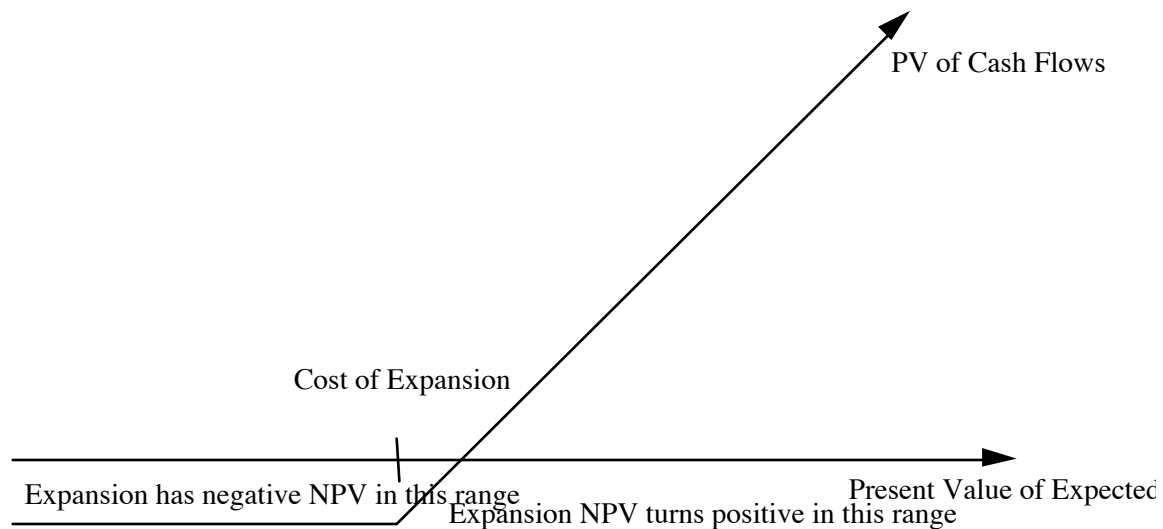
commodity prices and can adjust their behavior (in terms of development and exploration) quickly. In addition, if we consider exclusivity to be a pre-requisite for real options to have value, that exclusivity for natural resource options derives from their natural scarcity; there is, after all, only a finite amount of oil and gold under the ground and vacant land in Manhattan. Finally, natural resource reserves come closest to meeting the arbitrage/replication requirements that option pricing models are built upon; both the underlying asset (the natural resource) and the option can often be bought and sold.

The Option to Expand

In some cases, a firm will take an investment because doing so allows it either to make other investments or to enter other markets in the future. In such cases, it can be argued that the initial investment provides the firm with an option to expand, and the firm should therefore be willing to pay a price for such an option. Consequently, a firm may be willing to lose money on the first investment because it perceives the option to expand as having a large enough value to compensate for the initial loss.

To examine this option, assume that the present value of the expected cash flows from entering the new market or taking the new project is V , and the total investment needed to enter this market or take this project is X . Further, assume that the firm has a fixed time horizon, at the end of which it has to make the final decision on whether or not to take advantage of this opportunity. Finally, assume that the firm cannot move forward on this opportunity if it does not take the initial investment. This scenario implies the option payoffs shown in Figure 7.6.

Figure 7.6: The Option to Expand



As you can see, at the expiration of the fixed time horizon, the firm will enter the new market or take the new investment if the present value of the expected cash flows at that point in time exceeds the cost of entering the market.

Consider a simple example of an option to expand. Disney is considering starting a Spanish version of the Disney Channel in Mexico and estimates the net present value of this investment to be -\$150 million. While the negative net present value would normally suggest that rejecting the investment is the best course, assume that if the Mexican venture does better than expected, Disney plans to expand the network to the rest of South America at a cost of \$ 500 million. Based on its current assessment of this market, Disney believes that the present value of the expected cash flows on this investment is only \$ 400 million (making it a negative net present value investment as well). The saving grace is that the latter present value is an estimate and Disney does not have a firm grasp of the market; a Monte Carlo simulation of the investments yields a standard deviation of 50% in value. Finally, assume that Disney will have to make this expansion decision within 5 years of the Mexican investment, and that the five-year riskfree rate is 4%. The value of the expansion option can now be computed using the inputs:

$$S = \text{Present value of expansion cash flows} = \$ 400 \text{ million}$$

K = Cost of expansion = \$ 500 million

σ = Standard deviation in value (from simulation) = 50%

t = 5 years

r = 4%

The resulting option value is \$167 million.³⁵

The practical considerations associated with estimating the value of the option to expand are similar to those associated with valuing the option to delay. In most cases, firms with options to expand have no specific time horizon by which they have to make an expansion decision, making these open-ended options, or, at best, options with arbitrary lives. Even in those cases where a life can be estimated for the option, neither the size nor the potential market for the product may be known, and estimating either can be problematic. To illustrate, consider the Disney example discussed above. While we adopted a period of five years, at the end of which the Disney has to decide one way or another on its future expansion into South America, it is entirely possible that this time frame is not specified at the time the store is opened. Furthermore, we have assumed that both the cost and the present value of expansion are known initially. In reality, the firm may not have good estimates for either before making the first investment, since it does not have much information on the underlying market.

Implications

The option to expand is implicitly used by firms to rationalize taking investments that have negative net present value, but provide significant opportunities to tap into new markets or sell new products. While the option pricing approach adds rigor to this argument by estimating the value of this option, it also provides insight into those occasions when it is most valuable. In general, the option to expand is clearly more valuable for more volatile businesses with higher returns on projects (such as biotechnology or computer software), than in stable businesses with lower returns (such as housing, utilities or automobile production). Specifically, the option to expand is at the basis of arguments that an investment should be made because of strategic considerations or that large investments should be broken up into smaller phases. It can also be

³⁵ This value was computed using the Black-Scholes model. A binomial model yields a similar value.

considered a rationale for why firms may accumulate cash or hold back on borrowing, thus preserving financial flexibility.

Strategic Considerations

In many acquisitions or investments, the acquiring firm believes that the transaction will give it competitive advantages in the future. These competitive advantages range the gamut, and include:

- Entrée into a Growing or Large Market: An investment or acquisition may allow the firm to enter a large or potentially large market much sooner than it otherwise would have been able to do so. A good example of this would be the acquisition of a Mexican retail firm by a US firm, with the intent of expanding into the Mexican market.
- Technological Expertise: In some cases, the acquisition is motivated by the desire to acquire a proprietary technology, that will allow the acquirer to expand either its existing market or into a new market.
- Brand Name: Firms sometime pay large premiums over market price to acquire firms with valuable brand names, because they believe that these brand names can be used for expansion into new markets in the future.

While all of these potential advantages may be used to justify initial investments that do not meet financial benchmarks, not all of them create valuable options. The value of the option is derived from the degree to which these competitive advantages, assuming that they do exist, translate into sustainable excess returns. As a consequence, these advantages can be used to justify premiums only in cases where the acquiring firm believes that it has some degree of exclusivity in the targeted market or technology. Two examples can help illustrate this point. A telecommunications firm should be willing to pay a premium for Chinese telecomm firm, if the latter has exclusive rights to service a large segment of the Chinese market; the option to expand in the Chinese market could be worth a significant amount.³⁶ On the other hand, a developed market retailer should be

³⁶ A note of caution needs to be added here. If the exclusive rights to a market come with no pricing power – in other words, the Government will set the price you charge your customers – it may very well translate into zero excess returns (and no option value).

wary about paying a real option premium for an Indian retail firm, even though it may believe that the Indian market could grow to be a lucrative one. The option to expand into this lucrative market is open to all entrants and not just to existing retailers and thus may not translate into sustainable excess returns.

Multi-Stage Projects/ Investments

When entering new businesses or making new investments, firms sometimes have the option to enter the business in stages. While doing so may reduce potential upside, it also protects the firm against downside risk, by allowing it, at each stage, to gauge demand and decide whether to go on to the next stage. In other words, a standard project can be recast as a series of options to expand, with each option being dependent on the previous one. There are two propositions that follow:

- Some projects that do not look good on a full investment basis may be value creating if the firm can invest in stages.
- Some projects that look attractive on a full investment basis may become even more attractive if taken in stages.

The gain in value from the options created by multi-stage investments has to be weighed off against the cost. Taking investments in stages may allow competitors who decide to enter the market on a full scale to capture the market. It may also lead to higher costs at each stage, since the firm is not taking full advantage of economies of scale.

There are several implications that emerge from viewing this choice between multi-stage and one-time investments in an option framework. The projects where the gains will be largest from making the investment in multiple stages include:

- (1) Projects where there are significant barriers to entry from competitors entering the market, and taking advantage of delays in full-scale production. Thus, a firm with a patent on a product or other legal protection against competition pays a much smaller price for starting small and expanding as it learns more about the product
- (2) Projects where there is significant uncertainty about the size of the market and the eventual success of the project. Here, starting small and expanding allows the firm to reduce its losses if the product does not sell as well as anticipated, and to learn more about the market at each stage. This information can then be useful in subsequent

stages in both product design and marketing. Hsu argues that venture capitalists invest in young companies in stages, partly to capture the value of option of waiting/learning at each stage and partly to reduce the likelihood that the entrepreneur will be too conservative in pursuing risky (but good) opportunities.³⁷

- (3) Projects where there is a substantial investment needed in infrastructure (large fixed costs) and high operating leverage. Since the savings from doing a project in multiple stages can be traced to investments needed at each stage, they are likely to be greater in firms where those costs are large. Capital intensive projects as well as projects that require large initial marketing expenses (a new brand name product for a consumer product company) will gain more from the options created by taking the project in multiple stages.

Growth Companies

In the stock market boom in the 1990s, we witnessed the phenomenon of young, start-up, internet companies with large market capitalizations but little to show in terms of earnings, cash flows or even revenues. Conventional valuation models suggested that it would be difficult, if not impossible, to justify these market valuations with expected cash flows. In an interesting twist on the option to expand argument, there were some who argued that investors in these companies were buying options to expand and be part of a potentially huge e-commerce market, rather than conventional stock.³⁸

While the argument is alluring and serves to pacify investors in growth companies who may feel that they are paying too much, there are clearly dangers in making this stretch. The biggest one is that the “exclusivity” component that is necessary for real options to have value is being given short shrift. Consider investing in an internet stock in 1999 and assume that you are paying a premium to be part of a potentially large online market in 2008. Assume further that this market comes to fruition. Could you partake in this market without paying that upfront premium a dot-com company? We don’t see why

³⁷ Hsu, Y., 2002, Staging of Venture Capital Investment: A Real Options Analysis, Working paper, University of Cambridge.

³⁸ Schwartz, E.S. and M. Moon, 2001, Rational Pricing of Internet Companies Revisited, *The Financial Review* 36, pp. 7-26. A simpler version of the same argument was made in Mauboussin, M., 1998, *Get Real: Using Real Options in Security Analysis*, CSFB Publication, June 23, 1999.

not. After all, GE and Nokia are just as capable of being part of this online market, as are any number of new entrants into the market.³⁹

The Option to Abandon an Investment

The final option to consider here is the option to abandon a project when its cash flows do not measure up to expectations. One way to reflect this value is through decision trees. The decision tree has limited applicability in most real world investment analyses; it typically works only for multi-stage projects, and it requires inputs on probabilities at each stage of the project. The option pricing approach provides a more general way of estimating and building in the value of abandonment into investment analysis. To illustrate, assume that V is the remaining value on a project if it continues to the end of its life, and L is the liquidation or abandonment value for the same project at the same point in time. If the project has a life of n years, the value of continuing the project can be compared to the liquidation (abandonment) value. If the value from continuing is higher, the project should be continued; if the value of abandonment is higher, the holder of the abandonment option could consider abandoning the project .

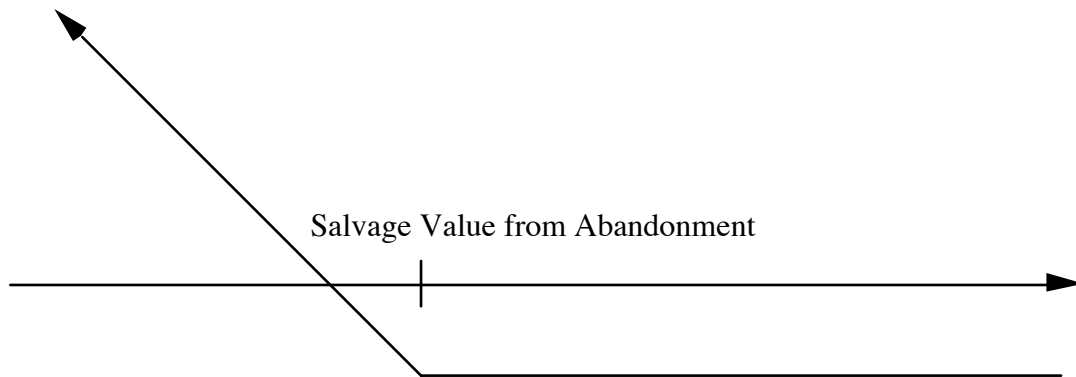
$$\begin{aligned} \text{Payoff from owning an abandonment option} &= 0 && \text{if } V > L \\ &= L - V && \text{if } V \leq L \end{aligned}$$

These payoffs are graphed in Figure 7.7, as a function of the expected value from continuing the investment.

³⁹ This argument is fleshed out in my book, "The Dark Side of Valuation", published by Prentice-Hall.

Figure 7.7: The Option to Abandon

PV of Cash Flows from project



Unlike the prior two cases, the option to abandon takes on the characteristics of a put option.

Consider a simple example. Assume that a firm is considering taking a 10-year project that requires an initial investment of \$ 100 million in a real estate partnership, where the present value of expected cash flows is \$ 110 million. While the net present value of \$ 10 million is small, assume that the firm has the option to abandon this project anytime in the next 10 years, by selling its share of the ownership to the other partners in the venture for \$ 50 million. Assume that the variance in the present value of the expected cash flows from being in the partnership is 0.09.

The value of the abandonment option can be estimated by determining the characteristics of the put option:

Value of the Underlying Asset (S) = PV of Cash Flows from Project
= \$ 110 million

Strike Price (K) = Salvage Value from Abandonment = \$ 50 million

Variance in Underlying Asset's Value = 0.09

Time to expiration = Period for which the firm has abandonment option = 10 years

The project has a 25-year life and is expected to lose value each year; for simplicity, we will assume that the loss is linear (4% a year).

Loss in value each year = $1/n = 1/25 = 4\%$

Assume that the ten-year riskless rate is 6%. The value of the put option can be estimated as follows:

$$\text{Call Value} = 110 \exp^{(-.04)(10)} (0.9737) - 50 (\exp^{(-0.06)(10)} (0.8387)) = \$ 84.09 \text{ million}$$

$$\text{Put Value} = \$ 84.09 - 110 + 50 \exp^{(-0.06)(10)} = \$ 1.53 \text{ million}$$

The value of this abandonment option has to be added on to the net present value of the project of \$ 10 million, yielding a total net present value with the abandonment option of \$ 11.53 million. Note though that abandonment becomes a more and more attractive option as the remaining project life decreases, since the present value of the remaining cash flows will decrease.

In the above analysis, we assumed, rather unrealistically, that the abandonment value was clearly specified up front and that it did not change during the life of the project. This may be true in some very specific cases, in which an abandonment option is built into the contract. More often, however, the firm has the option to abandon, and the salvage value from doing so has to be estimated (with error) up front. Further, the abandonment value may change over the life of the project, making it difficult to apply traditional option pricing techniques. Finally, it is entirely possible that abandoning a project may not bring in a liquidation value, but may create costs instead; a manufacturing firm may have to pay severance to its workers, for instance. In such cases, it would not make sense to abandon, unless the present value of the expected cash flows from continuing with the investment are even more negative.

Implications

The fact that the option to abandon has value provides a rationale for firms to build in operating flexibility to scale back or terminate projects if they do not measure up to expectations. It also indicates that firms that focus on generating more revenues by offering their customers the option to walk away from commitments may be giving up more than they gain, in the process.

1. Escape Clauses

When a firm enters into a long term risky investment that requires a large up front investment, it should do so with the clear understanding that it may regret making this

investment fairly early in its life. Being able to get out of such long-term commitments that threaten to drain more resources in the future is at the heart of the option to abandon. It is true that some of this flexibility is determined by the business that you are in; getting out of bad investments is easier to do in service businesses than in heavy infrastructure businesses. However, it is also true that there are actions that firms can take at the time of making these investments that give them more choices, if things do not go according to plan.

The first and most direct way is to build operating flexibility contractually with those parties that are involved in the investment. Thus, contracts with suppliers may be written on an annual basis, rather than long term, and employees may be hired on a temporary basis, rather than permanently. The physical plant used for a project may be leased on a short-term basis, rather than bought, and the financial investment may be made in stages rather than as an initial lump sum. While there is a cost to building in this flexibility, the gains may be much larger, especially in volatile businesses. The initial capital investment can be shared with another investor, presumably with deeper pockets and a greater willingness to stay with the investment, even if it turns sour. This provides a rationale for joint venture investing, especially for small firms that have limited resources; finding a cash-rich, larger company to share the risk may well be worth the cost.

None of these actions are costless. Entering into short term agreements with suppliers and leasing the physical plant may be more expensive than committing for the life of the investment, but that additional cost has to be weighed off against the benefit of maintaining the abandonment option.

2. Customer Incentives

Firms that are intent on increasing revenues sometimes offer abandonment options to customers to induce them to buy their products and services. As an example, consider a firm that sells its products on multi-year contracts and offers customers the option to cancel their contracts at any time, with no cost. While this may sweeten the deal and increase sales, there is likely to be a substantial cost. In the event of a recession, customers that are unable to meet their obligations are likely to cancel their contracts. In effect, the firm has made its good times better and its bad times worse; the cost of this

increased volatility in earnings and revenues has to be measured against the potential gain in revenue growth to see if the net effect is positive.

This discussion should also act as a cautionary note for those firms that are run with marketing objectives such as maximizing market share or posting high revenue growth. Those objectives can often be accomplished by giving valuable options to customers – sales people will want to meet their sales targets and are not particularly concerned about the long term costs they may create with their commitments to customers – and the firm may be worse off as a consequence.

3. Switching Options

While the abandonment option considers the value of shutting an investment down entirely, there is an intermediate alternative that is worth examining. Firms can sometimes alter production levels in response to demand and being able to do so can make an investment more valuable. Consider, for instance, a power company that is considering a new plant to generate electricity and assume that the company can run the plant at full capacity and produce 1 million kilowatt hours of power or at half capacity (and substantially less cost) and produce 500,000 kilowatt hours of power. In this case, the company can observe both the demand for power and the revenues per kilowatt-hour and decide whether it makes sense to run at full or half capacity. The value of this switching option can then be compared to the cost of building in this flexibility in the first place.

The airline business provides an interesting case study in how different companies manage their cost structure and the payoffs to their strategies. One reason that Southwest Airlines has been able to maintain its profitability in a deeply troubled sector is that the company has made cost flexibility a central component in its decision process. From its choice of using only one type of aircraft for its entire fleet⁴⁰ to its refusal, for the most part, to fly into large urban airports (with high gate costs), the company's operations have created the most flexible cost structure in the business. Thus, when revenues dip (as they

⁴⁰ From its inception until recently, Southwest used the Boeing 737 as its workhorse, thus reducing its need to maintain different maintenance crews at each airport it flies into.

inevitably do at some point in time when the economy weakens), Southwest is able to trim its costs and stay profitable while other airlines teeter on the brink of bankruptcy.

Caveats on Real Options

The discussion on the potential applications of real options should provide a window into why they are so alluring to practitioners and businesses. In essence, we are ignoring that the time honored rules of capital budgeting, which include rejecting investments that have negative net present value, when real options are present. Not only does the real options approach encourage you to make investments that do not meet conventional financial criteria, it also makes it more likely that you will do so, the less you know about the investment. Ignorance, rather than being a weakness, becomes a virtue because it pushes up the uncertainty in the estimated value and the resulting option value. To prevent the real options process from being hijacked by managers who want to rationalize bad (and risky) decisions, we have to impose some reasonable constraints on when it can be used and when it is used, how to estimate its value.

First, not all investments have options embedded in them, and not all options, even if they do exist, have value. To assess whether an investment creates valuable options that need to be analyzed and valued, three key questions need to be answered affirmatively.

- Is the first investment a pre-requisite for the later investment/expansion? If not, how necessary is the first investment for the later investment/expansion? Consider our earlier analysis of the value of a patent or the value of an undeveloped oil reserve as options. A firm cannot generate patents without investing in research or paying another firm for the patents, and it cannot get rights to an undeveloped oil reserve without bidding on it at a government auction or buying it from another oil company. Clearly, the initial investment here (spending on R&D, bidding at the auction) is required for the firm to have the second option. Now consider the Disney expansion into Mexico. The initial investment in a Spanish channel provides Disney with information about market potential, without which presumably it is unwilling to expand into the larger South American market. Unlike the patent and undeveloped reserves illustrations, the initial investment is not a pre-requisite for the second, though management might view it as such. The connection gets even weaker when

we look at one firm acquiring another to have the option to be able to enter a large market. Acquiring an internet service provider to have a foothold in the internet retailing market or buying a Brazilian brewery to preserve the option to enter the Brazilian beer market would be examples of such transactions.

- Does the firm have an exclusive right to the later investment/expansion? If not, does the initial investment provide the firm with significant competitive advantages on subsequent investments? The value of the option ultimately derives not from the cash flows generated by then second and subsequent investments, but from the excess returns generated by these cash flows. The greater the potential for excess returns on the second investment, the greater the value of the option in the first investment. The potential for excess returns is closely tied to how much of a competitive advantage the first investment provides the firm when it takes subsequent investments. At one extreme, again, consider investing in research and development to acquire a patent. The patent gives the firm that owns it the exclusive rights to produce that product, and if the market potential is large, the right to the excess returns from the project. At the other extreme, the firm might get no competitive advantages on subsequent investments, in which case, it is questionable as to whether there can be any excess returns on these investments. In reality, most investments will fall in the continuum between these two extremes, with greater competitive advantages being associated with higher excess returns and larger option values.
- How sustainable are the competitive advantages? In a competitive market place, excess returns attract competitors, and competition drives out excess returns. The more sustainable the competitive advantages possessed by a firm, the greater will be the value of the options embedded in the initial investment. The sustainability of competitive advantages is a function of two forces. The first is the nature of the competition; other things remaining equal, competitive advantages fade much more quickly in sectors where there are aggressive competitors and new entry into the business is easy. The second is the nature of the competitive advantage. If the resource controlled by the firm is finite and scarce (as is the case with natural resource reserves and vacant land), the competitive advantage is likely to be sustainable for longer periods. Alternatively, if the competitive advantage comes from

being the first mover in a market or technological expertise, it will come under assault far sooner. The most direct way of reflecting this in the value of the option is in its life; the life of the option can be set to the period of competitive advantage and only the excess returns earned over this period counts towards the value of the option.

Second, when real options are used to justify a decision, the justification has to be in more than qualitative terms. In other words, managers who argue for taking a project with poor returns or paying a premium on an acquisition on the basis of real options, should be required to value these real options and show, in fact, that the economic benefits exceed the costs. There will be two arguments made against this requirement. The first is that real options cannot be easily valued, since the inputs are difficult to obtain and often noisy. The second is that the inputs to option pricing models can be easily manipulated to back up whatever the conclusion might be. While both arguments have some basis, an estimate with error is better than no estimate at all, and the process of quantitatively trying to estimate the value of a real option is, in fact, the first step to understanding what drives its value.

There is one final note of caution that we should add about the use of option pricing models to assess the value of real options. Option pricing models, be they of the binomial or Black Scholes variety, are based on two fundamental precepts – replication and arbitrage. For either to be feasible, you have to be able to trade on the underlying asset and on the option. This is easy to accomplish with a listed option on a traded stock; you can trade on both the stock and the listed option. It is much more difficult to pull off when valuing a patent or an investment expansion opportunity; neither the underlying asset (the product that emerges from the patent) nor the option itself are traded. This does not mean that you cannot estimate the value of a patent as an option but it does indicate that monetizing this value will be much more difficult to do. In the Avonex example from earlier in the chapter, the option value for the patent was \$907 million whereas the conventional risk adjusted value was only \$547 million. Much as you may believe in the former as the right estimate of value, it is unlikely that any potential buyer of the patent will come close to paying that amount.

Conclusion

In this chapter, we considered three potential real options and applications of each. The first is the option to delay, where a firm with exclusive rights to an investment has the option of deciding when to take that investment and to delay taking it, if necessary. The second is the option to expand, where a firm may be willing to lose money on an initial investment, in the hope of expanding into other investments or markets further down the road. The third is the option to abandon an investment, if it looks like a money loser, early in the process.

While it is clearly appropriate to attach value to real options in some cases – patents, reserves of natural resources or exclusive licenses – the argument for an option premium gets progressively weaker as we move away from the exclusivity inherent in each of these cases. In particular, a firm that invests into an emerging market in a money-losing enterprise, using the argument that that market is a large and potentially profitable one, could be making a serious mistake. After all, the firm could be right in its assessment of the market, but absent barriers to entry, it may not be able to earn excess returns in that market or keep the competition out. Not all opportunities are options and not all options have significant economic value.

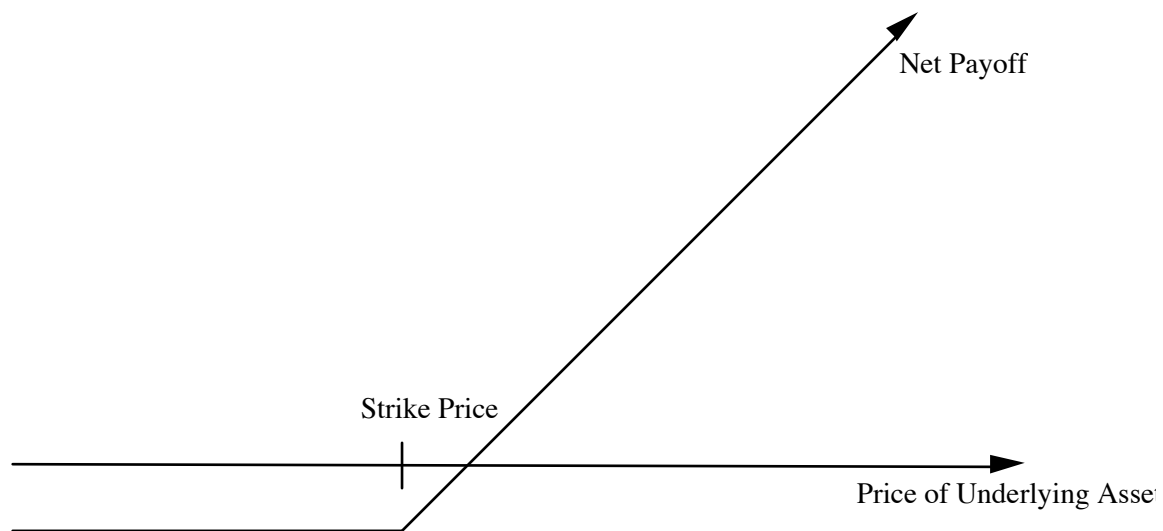
Appendix: Basics of Options and Option Pricing

An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to expire. There are two types of options - call options and put options.

Option Payoffs

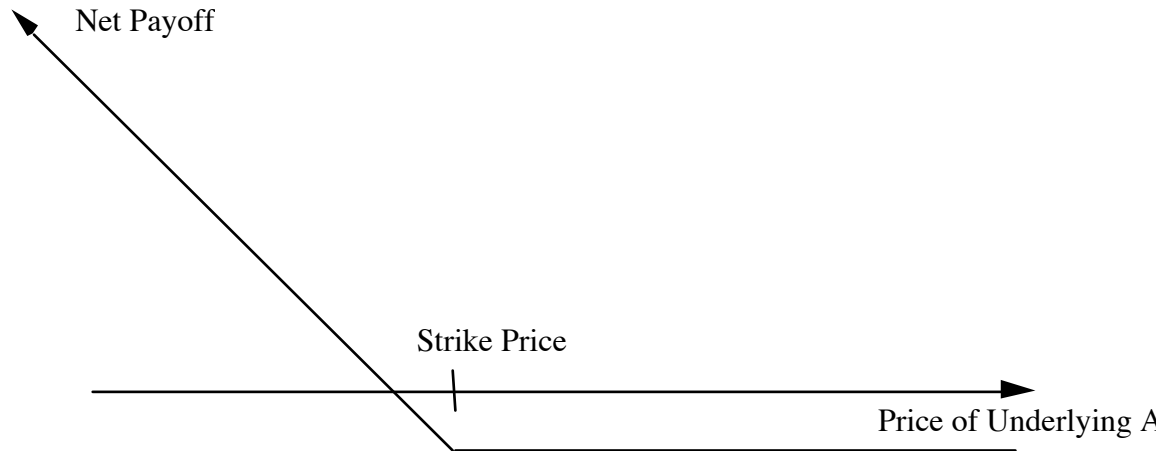
A call option gives the buyer of the option the right to buy the underlying asset at a fixed price, called the strike or the exercise price, at any time prior to the expiration date of the option: the buyer pays a price for this right. If at expiration, the value of the asset is less than the strike price, the option is not exercised and expires worthless. If, on the other hand, the value of the asset is greater than the strike price, the option is exercised - the buyer of the option buys the stock at the exercise price and the difference between the asset value and the exercise price comprises the gross profit on the investment. The net profit on the investment is the difference between the gross profit and the price paid for the call initially. A payoff diagram illustrates the cash payoff on an option at expiration. For a call, the net payoff is negative (and equal to the price paid for the call) if the value of the underlying asset is less than the strike price. If the price of the underlying asset exceeds the strike price, the gross payoff is the difference between the value of the underlying asset and the strike price, and the net payoff is the difference between the gross payoff and the price of the call. This is illustrated in the figure 7A.1:

Figure 7A.1: Call Option – Payoff Diagram



A put option gives the buyer of the option the right to sell the underlying asset at a fixed price, again called the strike or exercise price, at any time prior to the expiration date of the option. The buyer pays a price for this right. If the price of the underlying asset is greater than the strike price, the option will not be exercised and will expire worthless. If on the other hand, the price of the underlying asset is less than the strike price, the owner of the put option will exercise the option and sell the stock at the strike price, claiming the difference between the strike price and the market value of the asset as the gross profit. Again, netting out the initial cost paid for the put yields the net profit from the transaction. A put has a negative net payoff if the value of the underlying asset exceeds the strike price, and has a gross payoff equal to the difference between the strike price and the value of the underlying asset if the asset value is less than the strike price. This is summarized in figure 7A.2.

Figure 7A.2: Put Option – Payoff Diagram



There is one final distinction that needs to be made. Options are usually categorized as American or European options. A primary distinction between two is that American options can be exercised at any time prior to its expiration, while European options can be exercised only at expiration. The possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. There is one compensating factor that enables the former to be valued using models designed for the latter. In most cases, the time premium associated with the remaining life of an option and transactions costs makes early exercise sub-optimal. In other words, the holders of in-the-money options will generally get much more by selling the option to someone else than by exercising the options.⁴¹

⁴¹ While early exercise is not optimal generally, there are at least two exceptions to this rule. One is a case where the underlying asset pays large dividends, thus reducing the value of the asset, and any call options on that asset. In this case, call options may be exercised just before an ex-dividend date, if the time premium on the options is less than the expected decline in asset value as a consequence of the dividend payment. The other exception arises when an investor holds both the underlying asset and deep in-the-money puts on that asset at a time when interest rates are high. In this case, the time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

Determinants of Option Value

The value of an option is determined by a number of variables relating to the underlying asset and financial markets.

1. Current Value of the Underlying Asset : Options are assets that derive value from an underlying asset. Consequently, changes in the value of the underlying asset affect the value of the options on that asset. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increase.

2. Variance in Value of the Underlying Asset: The buyer of an option acquires the right to buy or sell the underlying asset at a fixed price. The higher the variance in the value of the underlying asset, the greater the value of the option. This is true for both calls and puts. While it may seem counter-intuitive that an increase in a risk measure (variance) should increase value, options are different from other securities since buyers of options can never lose more than the price they pay for them; in fact, they have the potential to earn significant returns from large price movements.

3. Dividends Paid on the Underlying Asset: The value of the underlying asset can be expected to decrease if dividend payments are made on the asset during the life of the option. Consequently, the value of a call on the asset is a decreasing function of the size of expected dividend payments, and the value of a put is an increasing function of expected dividend payments. A more intuitive way of thinking about dividend payments, for call options, is as a cost of delaying exercise on in-the-money options. To see why, consider a option on a traded stock. Once a call option is in the money, i.e, the holder of the option will make a gross payoff by exercising the option, exercising the call option will provide the holder with the stock, and entitle him or her to the dividends on the stock in subsequent periods. Failing to exercise the option will mean that these dividends are foregone.

4. Strike Price of Option: A key characteristic used to describe an option is the strike price. In the case of calls, where the holder acquires the right to buy at a fixed price, the value of the call will decline as the strike price increases. In the case of puts, where the holder has the right to sell at a fixed price, the value will increase as the strike price increases.

5. Time To Expiration On Option: Both calls and puts become more valuable as the time to expiration increases. This is because the longer time to expiration provides more time for the value of the underlying asset to move, increasing the value of both types of options. Additionally, in the case of a call, where the buyer has to pay a fixed price at expiration, the present value of this fixed price decreases as the life of the option increases, increasing the value of the call.

6. Riskless Interest Rate Corresponding To Life Of Option: Since the buyer of an option pays the price of the option up front, an opportunity cost is involved. This cost will depend upon the level of interest rates and the time to expiration on the option. The riskless interest rate also enters into the valuation of options when the present value of the exercise price is calculated, since the exercise price does not have to be paid (received) until expiration on calls (puts). Increases in the interest rate will increase the value of calls and reduce the value of puts.

Table 7A.1 below summarizes the variables and their predicted effects on call and put prices.

Table 7A.1: Summary of Variables Affecting Call and Put Prices

<i>Factor</i>	<i>Effect on</i>	
	<i>Call Value</i>	<i>Put Value</i>
Increase in underlying asset's value	Increases	Decreases
Increase in Strike Price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interest rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

Option Pricing Models

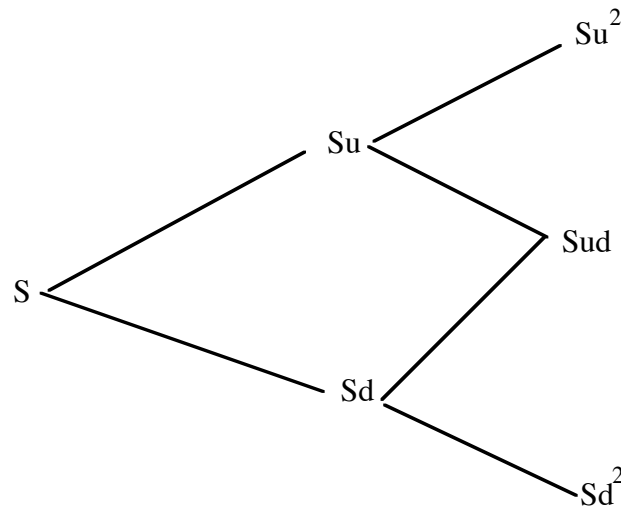
Option pricing theory has made vast strides since 1972, when Black and Scholes published their path-breaking paper providing a model for valuing dividend-protected European options. Black and Scholes used a “replicating portfolio” — a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued— to come up with their final formulation. While their derivation

is mathematically complicated, there is a simpler binomial model for valuing options that draws on the same logic.

The Binomial Model

The binomial option pricing model is based upon a simple formulation for the asset price process, in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial is shown in figure 7A.3.

Figure 7A.3: General Formulation for Binomial Price Path



In this figure, S is the current stock price; the price moves up to S_u with probability p and down to S_d with probability $1-p$ in any time period.

The objective in creating a replicating portfolio is to use a combination of risk-free borrowing/lending and the underlying asset to create the same cash flows as the option being valued. The principles of arbitrage apply here, and the value of the option must be equal to the value of the replicating portfolio. In the case of the general formulation above, where stock prices can either move up to S_u or down to S_d in any time period, the replicating portfolio for a call with strike price K will involve borrowing $\$B$ and acquiring Δ of the underlying asset, where:

$$\Delta = \text{Number of units of the underlying asset bought} = (C_u - C_d)/(S_u - S_d)$$

where,

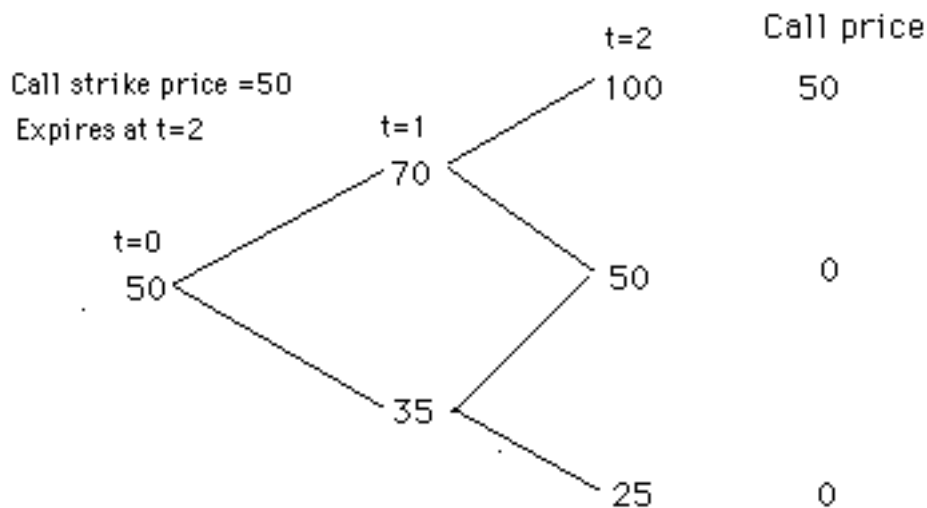
C_u = Value of the call if the stock price is S_u

C_d = Value of the call if the stock price is S_d

In a multi-period binomial process, the valuation has to proceed iteratively; i.e., starting with the last time period and moving backwards in time until the current point in time. The portfolios replicating the option are created at each step and valued, providing the values for the option in that time period. The final output from the binomial option pricing model is a statement of the value of the option in terms of the replicating portfolio, composed of Δ shares (option delta) of the underlying asset and risk-free borrowing/lending.

Value of the call = Current value of underlying asset * Option Delta - Borrowing needed to replicate the option

Consider a simple example. Assume that the objective is to value a call with a strike price of 50, which is expected to expire in two time periods, on an underlying asset whose price currently is 50 and is expected to follow a binomial process:



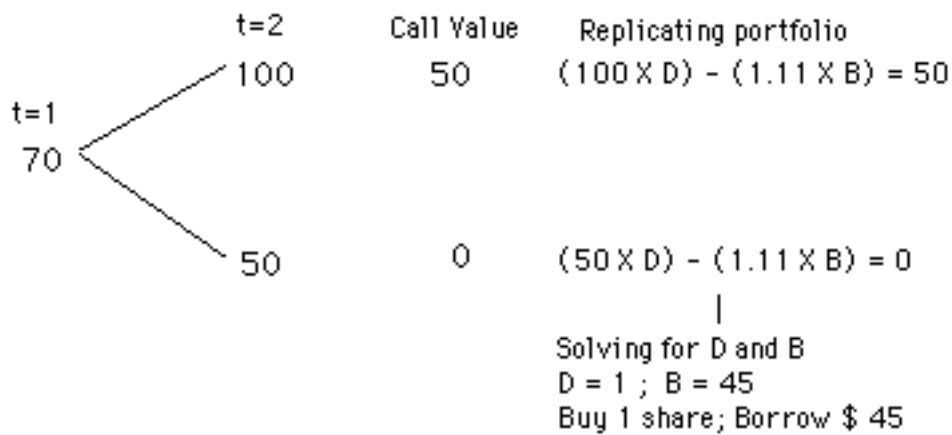
Now assume that the interest rate is 11%. In addition, define

Δ = Number of shares in the replicating portfolio

B = Dollars of borrowing in replicating portfolio

The objective is to combine Δ shares of stock and B dollars of borrowing to replicate the cash flows from the call with a strike price of \$ 50. This can be done iteratively, starting with the last period and working back through the binomial tree.

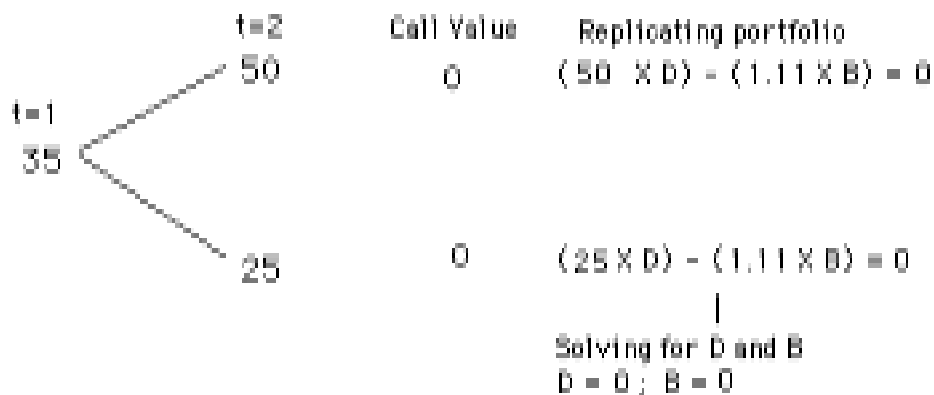
Step 1: Start with the end nodes and work backwards:



Thus, if the stock price is \$70 at t=1, borrowing \$45 and buying one share of the stock will give the same cash flows as buying the call. The value of the call at t=1, if the stock price is \$70, is therefore:

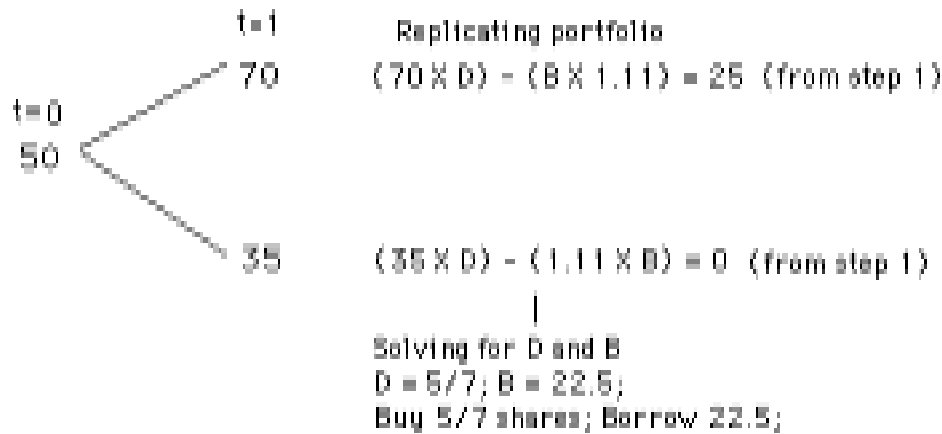
$$\text{Value of Call} = \text{Value of Replicating Position} = 70 \Delta - B = 70 - 45 = 25$$

Considering the other leg of the binomial tree at t=1,



If the stock price is 35 at t=1, then the call is worth nothing.

Step 2: Move backwards to the earlier time period and create a replicating portfolio that will provide the cash flows the option will provide.



In other words, borrowing \$22.5 and buying 5/7 of a share will provide the same cash flows as a call with a strike price of \$50. The value of the call therefore has to be the same as the value of this position.

$$\text{Value of Call} = \text{Value of replicating position} = 5/7 \times \text{Current stock price} - \$ 22.5 = \$ 13.20$$

The binomial model provides insight into the determinants of option value. The value of an option is not determined by the expected price of the asset but by its current price, which, of course, reflects expectations about the future. This is a direct consequence of arbitrage. If the option value deviates from the value of the replicating portfolio, investors can create an arbitrage position, i.e., one that requires no investment, involves no risk, and delivers positive returns. To illustrate, if the portfolio that replicates the call costs more than the call does in the market, an investor could buy the call, sell the replicating portfolio and be guaranteed the difference as a profit. The cash flows on the two positions will offset each other, leading to no cash flows in subsequent periods. The option value also increases as the time to expiration is extended, as the price movements (u and d) increase, and with increases in the interest rate.

The Black-Scholes Model

The binomial model is a discrete-time model for asset price movements, including a time interval (t) between price movements. As the time interval is shortened, the limiting distribution, as t approaches 0, can take one of two forms. If as t approaches 0, price changes become smaller, the limiting distribution is the normal distribution and the price process is a continuous one. If as t approaches 0, price changes remain large, the

limiting distribution is the Poisson distribution, i.e., a distribution that allows for price jumps. The Black-Scholes model applies when the limiting distribution is the normal distribution,⁴² and it explicitly assumes that the price process is continuous.

The Model

The original Black and Scholes model was designed to value European options, which were dividend-protected. Thus, neither the possibility of early exercise nor the payment of dividends affects the value of options in this model. The value of a call option in the Black-Scholes model can be written as a function of the following variables:

S = Current value of the underlying asset

K = Strike price of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the $\ln(\text{value})$ of the underlying asset

The model itself can be written as:

$$\text{Value of call} = S N(d_1) - K e^{-rt} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

The process of valuation of options using the Black-Scholes model involves the following steps:

Step 1: The inputs to the Black-Scholes are used to estimate d_1 and d_2 .

Step 2: The cumulative normal distribution functions, $N(d_1)$ and $N(d_2)$, corresponding to these standardized normal variables are estimated.

⁴² Stock prices cannot drop below zero, because of the limited liability of stockholders in publicly listed firms. Hence, stock prices, by themselves, cannot be normally distributed, since a normal distribution requires some probability of infinitely negative values. The distribution of the natural logs of stock prices is assumed to be log-normal in the Black-Scholes model. This is why the variance used in this model is the variance in the log of stock prices.

Step 3: The present value of the exercise price is estimated, using the continuous time version of the present value formulation:

$$\text{Present value of exercise price} = K e^{-rt}$$

Step 4: The value of the call is estimated from the Black-Scholes model.

The determinants of value in the Black-Scholes are the same as those in the binomial - the current value of the stock price, the variability in stock prices, the time to expiration on the option, the strike price, and the riskless interest rate. The principle of replicating portfolios that is used in binomial valuation also underlies the Black-Scholes model. In fact, embedded in the Black-Scholes model is the replicating portfolio.

$$\text{Value of call} = \underbrace{S N(d_1)}_{\text{Buy } N(d_1) \text{ shares}} - \underbrace{K e^{-rt} N(d_2)}_{\text{Borrow this amount}}$$

$N(d_1)$, which is the number of shares that are needed to create the replicating portfolio is called the option delta. This replicating portfolio is self-financing and has the same value as the call at every stage of the option's life.

Model Limitations and Fixes

The version of the Black-Scholes model presented above does not take into account the possibility of early exercise or the payment of dividends, both of which impact the value of options. Adjustments exist, which while not perfect, provide partial corrections to value.

1. Dividends

The payment of dividends reduces the stock price. Consequently, call options will become less valuable and put options more valuable as dividend payments increase. One approach to dealing with dividends to estimate the present value of expected dividends paid by the underlying asset during the option life and subtract it from the current value of the asset to use as “S” in the model. Since this becomes impractical as the option life becomes longer, we would suggest an alternate approach. If the dividend yield ($y = \text{dividends} / \text{current value of the asset}$) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S e^{-yt} N(d_1) - K e^{-rt} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

From an intuitive standpoint, the adjustments have two effects. First, the value of the asset is discounted back to the present at the dividend yield to take into account the expected drop in value from dividend payments. Second, the interest rate is offset by the dividend yield to reflect the lower carrying cost from holding the stock (in the replicating portfolio). The net effect will be a reduction in the value of calls, with the adjustment, and an increase in the value of puts.

2. *Early Exercise*

The Black-Scholes model is designed to value European options, whereas most options that we consider are American options, which can be exercised anytime before expiration. Without working through the mechanics of valuation models, an American option should always be worth at least as much and generally more than a European option because of the early exercise option. There are three basic approaches for dealing with the possibility of early exercise. The first is to continue to use the unadjusted Black-Scholes, and regard the resulting value as a floor or conservative estimate of the true value. The second approach is to value the option to each potential exercise date. With options on stocks, this basically requires that we value options to each ex-dividend day and chooses the maximum of the estimated call values. The third approach is to use a modified version of the binomial model to consider the possibility of early exercise.

While it is difficult to estimate the prices for each node of a binomial, there is a way in which variances estimated from historical data can be used to compute the expected up and down movements in the binomial. To illustrate, if σ^2 is the variance in $\ln(\text{stock prices})$, the up and down movements in the binomial can be estimated as follows:

$$u = \text{Exp} \left[(r - \sigma^2/2)(T/m) + \sqrt{(\sigma^2 T/m)} \right]$$

$$d = \text{Exp} [(r - \sigma^2/2)(T/m) - \sqrt{(\sigma^2 T/m)}]$$

where u and d are the up and down movements per unit time for the binomial, T is the life of the option and m is the number of periods within that lifetime. Multiplying the stock price at each stage by u and d will yield the up and the down prices. These can then be used to value the asset.

3. *The Impact Of Exercise On The Value Of The Underlying Asset*

The derivation of the Black-Scholes model is based upon the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for some types of options. For instance, the exercise of warrants increases the number of shares outstanding and brings fresh cash into the firm, both of which will affect the stock price.⁴³ The expected negative impact (dilution) of exercise will decrease the value of warrants compared to otherwise similar call options. The adjustment for dilution in the Black-Scholes to the stock price is fairly simple. The stock price is adjusted for the expected dilution from the exercise of the options. In the case of warrants, for instance:

$$\text{Dilution-adjusted } S = (S n_s + W n_w) / (n_s + n_w)$$

where

S = Current value of the stock
 n_w = Number of warrants outstanding

W = Market value of warrants outstanding
 n_s = Number of shares outstanding

When the warrants are exercised, the number of shares outstanding will increase, reducing the stock price. The numerator reflects the market value of equity, including both stocks and warrants outstanding. The reduction in S will reduce the value of the call option.

There is an element of circularity in this analysis, since the value of the warrant is needed to estimate the dilution-adjusted S and the dilution-adjusted S is needed to estimate the value of the warrant. This problem can be resolved by starting the process

⁴³ Warrants are call options issued by firms, either as part of management compensation contracts or to raise equity.

off with an estimated value of the warrant (say, the exercise value), and then iterating with the new estimated value for the warrant until there is convergence.

Valuing Puts

The value of a put can be derived from the value of a call with the same strike price and the same expiration date through an arbitrage relationship that specifies that:

$$C - P = S - K e^{-rt}$$

where C is the value of the call and P is the value of the put (with the same life and exercise price).

This arbitrage relationship can be derived fairly easily and is called put-call parity. To see why put-call parity holds, consider creating the following portfolio:

- (a) Sell a call and buy a put with exercise price K and the same expiration date "t"
- (b) Buy the stock at current stock price S

The payoff from this position is riskless and always yields K at expiration (t). To see this, assume that the stock price at expiration is S*:

<i>Position</i>	<i>Payoffs at t if S* > K</i>	<i>Payoffs at t if S* < K</i>
Sell call	-(S*-K)	0
Buy put	0	K-S*
Buy stock	S*	S*
<i>Total</i>	<i>K</i>	<i>K</i>

Since this position yields K with certainty, its value must be equal to the present value of K at the riskless rate ($K e^{-rt}$).

$$S+P-C = K e^{-rt}$$

$$C - P = S - K e^{-rt}$$

This relationship can be used to value puts. Substituting the Black-Scholes formulation for the value of an equivalent call,

$$\text{Value of put} = S e^{-yt} (N(d_1) - 1) - K e^{-rt} (N(d_2) - 1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

CHAPTER 8

MODEL CHOICES AND A PHILOSOPHICAL DISCOURSE

The problem in valuation is not that there are not enough models to value an asset, it is that there are too many. Choosing the right model to use in valuation is as critical to arriving at a reasonable value as understanding how to use the model. This chapter attempts to provide an overview of the valuation models introduced in this book and a general framework that can be used to pick the right model for any task. It closes with some thoughts about valuation in general that are worth keeping in mind, as you start valuing assets or companies.

Choices in valuation models

In the broadest possible terms, firms or assets can be valued in one of four ways – asset based valuation approaches where you estimate what the assets owned by a firm are worth currently, discounted cashflow valuation approaches that discount cashflows to arrive at a value of equity or the firm, relative valuation approaches that base value upon multiples and option pricing approaches that use contingent claim valuation. Within each of these approaches, there are further choices that help determine the final value.

There are at least two ways in which you can value a firm using *asset based valuation techniques*. One is liquidation value, where you consider what the market will be willing to pay for assets, if the assets were liquidated today. The other is replacement cost, where you evaluate how much it would cost you to replicate or replace the assets that a firm has in place today.

In the context of *discounted cashflow valuation*, cashflows to equity can be discounted at the cost of equity to arrive at a value of equity or cashflows to the firm can be discounted at the cost of capital to arrive at the value for the firm. The cashflows to equity themselves can be defined in the strictest sense as dividends or in a more expansive sense as free cashflows to equity. These models can be further categorized on the basis of assumptions about growth into stable growth, two-stage and three-stage models. Finally, the measurement of earnings and cashflows may be modified to match the special characteristics of the firm/asset - current earnings for firms/assets which have

normal earnings or normalized earnings for firms/assets whose current earnings may be distorted either by temporary factors or cyclical effects.

In the context of *multiples*, you can use either equity or firm value as your measure of value and relate it to a number of firm-specific variables – earnings, book value and sales. The multiples themselves can be estimated by using comparable firms in the same business or from cross-sectional regressions that use the broader universe. For other assets, such as real estate, the price can similarly be expressed as a function of gross income or per square foot of space. Here, the comparables would be other properties in the same locale with similar characteristics.

Contingent claim models can also be used in a variety of scenarios. When you consider the option that a firm has to delay making investment decisions, you can value a patent or an undeveloped natural resource reserve as an option. The option to expand may make young firms with potentially large markets trade at a premium on their discounted cashflow values. Finally, equity investors may derive value from the option to liquidate troubled firms with substantial debt.

Which approach should you use?

The values that you obtain from the four approaches described above can be very different and deciding which one to use can be a critical step. This judgment, however, will depend upon several factors, some of which relate to the business being valued but many of which relate to you, as the analyst.

Asset or Business Characteristics

The approach that you use to value a business will depend upon how marketable its assets are, whether it generates cash flows and how unique it is in terms of its operations.

Marketability of Assets

Liquidation valuation and replacement cost valuation are easiest to do for firms that have assets that are separable and marketable. For instance, you can estimate the liquidation value for a real estate company because its properties can be sold individually and you can estimate the value of each property easily. The same cannot be said of

companies like Coca Cola or Disney, which have valuable assets that are linked to each other or to a brand name. You can also use this same analysis to see why the liquidation or replacement cost value of a high growth business will bear little resemblance to true value. Unlike assets in place, growth assets cannot be easily identified or sold.

Cash Flow Generating Capacity

You can categorize assets into three groups based upon their capacity to generate cash flows – assets that are either generating cash flows currently or are expected to do so in the near future, assets that are not generating cash flows currently but could in the future in the event of a contingency and assets that will never generate cash flows.

- The first group includes most publicly traded companies and these firms can be valued using discounted cash flow models. Note that we do not draw a distinction between negative and positive cash flows and young, start-up companies that generate negative cash flow can still be valued using discounted cash flow models.
- The second group includes assets such as drug patents, promising (but not viable) technology, undeveloped oil or mining reserves and undeveloped land. These assets may generate no cash flows currently and could generate large cash flows in the future but only under certain conditions – if the FDA approves the drug patent, if the technology becomes commercially viable, if oil prices and commercial property values go up. While you could estimate expected values using discounted cash flow models by assigning probabilities to these events, you will understate the value of the assets if you do so. You should value these assets using option pricing models.
- Assets that are never expected to generate cash flows include your primary residence, a baseball card collection or fine art. These assets can only be valued using relative valuation models.

Uniqueness (or presence of comparables)

In a market where thousands of stocks are traded and tens of thousands of assets are bought and sold every day, it may be difficult to visualize an asset or business that is so unique that you cannot find comparable assets. On a continuum, though, some assets

and businesses are part of a large group of similar assets, with no or very small differences across the assets. These assets are tailor-made for relative valuation, since assembling comparable assets (businesses) and controlling for differences is simple. The further you move from this ideal, the less reliable is relative valuation. For businesses that are truly unique, discounted cash flow valuation will yield much better estimates of value.

Analyst Characteristics and Beliefs

The valuation approach that you choose to use will depend upon your time horizon, the reason that you are doing the valuation in the first place and what you think about markets – whether they are efficient and if they are not, what form the inefficiency takes.

Time Horizon

At one extreme, in discounted cash flow valuation you consider a firm as a going concern that may last into perpetuity. At the other extreme, with liquidation valuation, you are estimating value on the assumption that the firm will cease operations today. With relative valuation and contingent claim valuation, you take an intermediate position between the two. Not surprisingly, then, you should be using discounted cash flow valuation, if you have a long time horizon, and relative valuation, if you have a shorter time horizon. This may explain why discounted cash flow valuation is more prevalent in valuing a firm for an acquisition and relative valuation is more common in equity research and portfolio management.

Reason for doing the valuation

Analysts value businesses for a number of reasons and the valuation approach used will vary depending upon the reason. If you are an equity research analyst following steel companies, your job description is simple. You are asked to find the most under and over valued companies in the sector and not to take a stand on whether the sector overall is under or over valued. You can see why multiples would be your weapon of choice when valuing companies. This effect is likely to be exaggerated if the way you are judged

and rewarded is on a relative basis, i.e., your recommendations are compared to those made by other steel company analysts. If you are an individual investor setting money aside for retirement or a private businessperson valuing a business for purchase, on the other hand, you want to estimate intrinsic value. Consequently, discounted cash flow valuation is likely to be more appropriate for your needs.

Beliefs about Markets

Embedded in each approach are assumptions about markets and how they work or fail to work. With discounted cash flow valuation, you are assuming that market prices deviate from intrinsic value but that they correct themselves over long periods. With relative valuation, you are assuming that markets are on average right and that while individual firms in a sector or market may be mispriced, the sector or overall market is fairly priced. With asset-based valuation models, you are assuming that the markets for real and financial assets can deviate and that you can take advantage of these differences. Finally, with option pricing models, you are assuming that markets are not very efficient at assessing the value of flexibility that firms have and that option pricing models will therefore give you an advantage. In each and every one of these cases, though, you are assuming that markets will eventually recognize their mistakes and correct them.

Choosing the right DCF Model

The model used in valuation should be tailored to match the characteristics of the asset being valued. The unfortunate truth is that the reverse is often true. Time and resources are wasted trying to make assets fit a pre-specified valuation model, either because it is considered to be the 'best' model or because not enough thought goes into the process of model choice. There is no one 'best' model. The appropriate model to use in a particular setting will depend upon a number of the characteristics of the asset or firm being valued.

Firm versus Equity Valuation

With consistent assumptions about growth and leverage, you should get the same value for your equity using the firm approach (where you value the firm and subtract outstanding debt) and the equity approach (where you value equity directly). If this is the

case, you might wonder why you would pick one approach over the other. The answer is purely pragmatic and tilts us towards firm valuation, over equity valuation, for most firms:

(1) For most publicly traded firms, it is not only easier to estimate cash flows prior to debt payments and to adjust the discount rate for risk (or compute the tax benefits of debt separately) but it also preserves more flexibility. In other words, you can go back and change your assumptions about debt usage in the future easily in a firm valuation, but it is far more difficult to do in equity valuation

(2) By separating operating assets from non-operating assets (cash, cross holdings), firm valuation allows us to focus on the risk and cash flow characteristics of the business decisions made by a firm. While you do have to take care to bring in the value of cash and cross holdings into the final value, you can value these investments using much simpler techniques than you use for the operating assets.

Are there dangers? Sure. By focusing on cash flows prior to debt payments, you may be missing signals that the firm may be headed towards default. However, there is no reason why you cannot value a firm using firm valuation approaches and estimate the cash flow for equity separately.

So, when should you use equity valuation? For financial service firms, where debt is both difficult to define and determine and where it is more raw material than a source of capital, the focus should stay firmly on equity valuation. For real estate or private equity investments, where the focus is on highly levered equity investors and their returns, it may also be useful to focus on the equity invested and the cash flows to these equity investors.

Cost of Capital versus APV valuation

In the chapter on APV valuation, we noted that the cost of capital approaches and APV will yield the same value for a business, with consistent assumptions on growth and risk. The key difference between the approaches is that the tax benefits of debt are incorporated into the discount rate (cost of capital) in the cost of capital approach and are counted separately, net of bankruptcy costs, in the APV approach. Again, the choice has to be a pragmatic one:

- a. For companies where the debt ratio is expected to change over time, it is easier to adjust the cost of capital for time varying debt ratios than it is to estimate the net value added by debt.
- b. In transactions where the focus is on dollar debt and changes in that value over time, the APV approach is easier to use. Estimating debt ratios in future periods can be problematic, with dollar debt values, because estimating equity values in future periods puts you at risk of circularity in the valuation.

Even in the latter case, a key input into the APV model is the expected bankruptcy cost. If it is difficult to estimate bankruptcy costs, analysts often ignore these costs and the values obtained from APV models have to be taken with a grain of salt, since they will be overstated.

Dividends or Free Cash flow to Equity

In valuing equity, you can discount dividends or free cashflows to equity. In the chapter on equity valuation models, we examined why the models may give you different answers, and what you would need to assume for the models to reconcile. If you can estimate the free cash flow to equity (the potential dividend), you will get a more realistic estimate of value using it to compute value rather than the dividends. In effect, for a firm that pays too little in dividends (relative to its free cash flow to equity), you are incorporating the effect of the cash build up in the firm into the value of equity. For a firm that pays out too much in dividends, you are reducing the value of equity to reflect your expectation that the dividends are unsustainable.

In some cases, though, especially with financial service companies, it may be difficult, if not impossible, to estimate the free cash to equity, because capital expenditures and working capital are not identifiable easily. In those cases, you have no choice but to focus on the dividend discount model.

Choosing the Right Relative Valuation Model

Many analysts choose to value assets using relative valuation models. In making this choice, two basic questions have to be answered -- Which multiple will be used in the valuation? Will this multiple be arrived at using the sector or the entire market?

Which multiple should I use?

In the chapters on multiples, we presented a variety of multiples. Some were based upon earnings, some on book value and some on revenues. For some multiples, we used current values and for others, we used forward or forecast values. Since the values you obtain are likely to be different using different multiples, deciding which multiple to use can make a big difference to your estimate of value. There are three ways you can answer this question –the first is to adopt the cynical view that you should use the multiple that reflects your biases, the second is to value your firm with different multiples and try to use all of the values that you obtain and the third is to pick the best multiple and base your valuation on it.

The most sensible choice is to pick a multiple that resonates in the sector that your company operates in. One way to find this multiple is to look at the variables that managers in the sector focus and work backwards to the multiple that matches that variable. In retailing, for instance, the focus on margins and same store sales would lead us to use revenue multiples. In banking, the emphasis on return on equity would suggest that price to book ratios be used to value banks. The other is to find the variable that is most highly correlated with firm value in the sector. For instance, current earnings and value are much more highly correlated in consumer product companies than in technology companies. Using price earnings ratios makes more sense for the former than for the latter.

How should you define comparable companies?

There are three choices. One is to define comparable narrowly and to look for companies that resemble the company that you are valuing on multiple dimensions – size, growth potential and risk. The net result will be a small sample of firms that are “similar” to the firm you are valuing. The second is to define comparable firms more broadly, and to bring in firms that operate in the same sector, even if they differ from your firm on key dimensions. This will allow for much larger samples of firms, albeit some with different characteristic than the firm you are valuing.

If you want to compare the pricing multiple (PE, EV/EBITDA) that your firm is trading at to an average or median value, not correcting for differences across companies,

you should go with the former: a small sample of similar companies. The risk, though, is that you may be finding a small cluster of mispriced companies. If you plan to control for differences across firms – by modifying the multiple or with a regression- you will be better served with a larger sample of less comparable firms.

Intrinsic versus Relative Value

The two approaches to valuation – discounted cash flow valuation and relative valuation – will generally yield different estimates of value for the same firm at the same point in time. It is even possible for one approach to generate the result that the stock is under valued while the other concludes that it is over valued. Furthermore, even within relative valuation, we can arrive at different estimates of value depending upon which multiple we use and what firms we based the relative valuation on.

The differences in value between discounted cash flow valuation and relative valuation come from different views of market efficiency, or put more precisely, market inefficiency. In discounted cash flow valuation, we assume that markets make mistakes, that they correct these mistakes over time, and that these mistakes can often occur across entire sectors or even the entire market. In relative valuation, we assume that while markets make mistakes on individual stocks, they are correct on average. In other words, when we value a new software company relative to other small software companies, we are assuming that the market has priced these companies correctly, on average, even though it might have made mistakes in the pricing of each of them individually. Thus, a stock may be over valued on a discounted cash flow basis but under valued on a relative basis, if the firms used for comparison in the relative valuation are all overpriced by the market. The reverse would occur, if an entire sector or market were underpriced.

When should you use the option pricing models?

In the chapters on applying option pricing models to valuation, we presented a number of scenarios where option pricing may yield a premium on traditional discounted cash flow valuation. We do not intend to revisit those scenarios, but offer the following general propositions that you should keep in mind when using option pricing models.

- *Use Options sparingly*: Restrict your use of options to where they make the biggest difference in valuation. In general, options will affect value the most at smaller firms that derive the bulk of their value from assets that resemble options. Therefore, valuing patents as options to estimate firm value makes more sense for a small biotechnology firm than it does for a drug giant like Merck. While Merck may have dozens of patents, it derives much of its value from a portfolio of developed drugs and the cash flows they generate.
- *Opportunities are not always options*: You should be careful not to mistake opportunities for options. Analysts often see a firm with growth potential and assume that there must be valuable options embedded in the firm. For opportunities to become valuable options, you need some degree of exclusivity for the firm in question – this can come from legal restrictions on competition or a significant competitive edge.
- *Do not double count options*: All too often, analysts incorporate the effect of options on fundamentals in the company value and then proceed to add on premiums to reflect the same options. Consider, for instance, the undeveloped oil reserves owned by an oil company. While it is legitimate to value these reserves as options, you should not add this value to a discounted cashflow valuation of the company, if your expected growth rate in the valuation is set higher because of the firm's undeveloped reserves.

Some fundamental (philosophical) propositions about valuation

When confronted with uncertainty or missing information, we are tempted to adopt loosely backed rules of thumb and make inconsistent assumptions about growth, risk, and cash flows. The following sections outline a few propositions that can guide us in making better judgments and estimates, and result in better valuations.

Proposition 1: Be steadfast on principles, open to new tools and flexible on estimates.

A principle is a core idea, a tool is what you use to put that idea into practice, and an estimate is what emerges from that tool. There are a few basic principles in valuation that we should never compromise on, no matter what the counterarguments are. They include the following (a) that an asset derives its value from its capacity to generate cash flows in

the future, (b) that risk affects value, (c) that growth has to be earned (not endowed) and (d) that the laws of demand and supply cannot be repealed. An analyst who argues that firms can grow forever without reinvesting is violating a first principle. Lest this be seen as a sign of rigidity, we would hasten to add that we should always be willing to compromise on, and accept, better tools, and be open to alternative estimates for inputs into value.

Proposition 2: Pay Heed to Markets, But Don't Let Them Determine Your Valuations

Many of the companies that we are called on to value are traded in financial markets and have market prices. Without making any judgments about markets or their efficiency, we believe that there is valuable information in how the market is pricing assets. When valuing a company, we should pay attention to the market price for two reasons:

- The information we extract from markets on implied growth rates, risk, and cash flows can be used to improve our valuations.
- Ultimately, we make money not from our estimates of intrinsic values, but from the market prices moving to those intrinsic values. Thus, we need to understand why market prices deviate from value and how they will adjust over time.

At the same time, paying too much attention to markets (and prices) can lead to paralysis, since there is always an alternative set of assumptions under which the market price is justified. In other words, if we start by assuming that the market is always right, we will end up confirming that assumption in our valuations.

Proposition 3: Risk Matters

It is true that considerable resources have been invested in both academia and practice in coming up with models to assess risk and derive expected returns. It is also true that these models often make assumptions about the real world that do not withstand close scrutiny. In the capital asset pricing model, for instance, we assume no transaction costs and private information to derive the relationship between expected returns and betas. Analysts, looking at these models, find themselves disagreeing with the assumptions. That is perfectly understandable, but jumping from that disagreement to a conclusion that risk does not matter in value is not.

Proposition 4: Growth Is Not Free and Is Not Always Value-Adding

If there is a theme to our discussions of growth across the chapters, it is that growth is not free. Ultimately, the expected growth in a company's earnings and cash flows must come from either new investments or improved efficiency. The latter is finite growth and there is a limit to how efficient you can become as a firm, whereas new investment growth can be for the long term (and potentially forever). It is also critical to remember that growth by itself is not always a plus for a company, since the value added by growth is a function of the quality of the investments that generated that growth.

Proposition 5: All Good Things Come to an End

In addition to looking at the value added or destroyed by growth, we often have to make estimates of future growth rates for companies. In making this assessment, we must pay attention to two key variables. The first is the *scaling effect*. As companies get bigger, it becomes more and more difficult for them to keep delivering the excess returns and growth rates they did in the past. Thus, it is almost a certainty that a firm that has grown 100% a year for the last three years will grow at a slower rate in the next three years. The second factor is *competition*. When firms are successful, they attract attention, which in turn leads to imitation and increased competition. While firms may be successful at keeping this competition at bay for extended periods (using legal and other tools), the new competitors will inevitably chip away at profitability and growth.

Proposition 6: Watch Out for Truncation Risk

In conventional discounted cash flow valuation, we value firms as going concerns, with cash flows continuing into perpetuity. While this may not be an unreasonable assumption for some firms, the reality is that most firms do not make it to this stable-growth phase. Many young firms run out of cash and have to shut down, some mature firms become the targets of acquirers, and most distressed firms end up defaulting on debt and going out of business. The optimism that underlies discounted cash flow valuation can lead us to overestimate the values of firms, where the risk of not making it (truncation risk) is high.

Proposition 7: Look at the Past, But Think About the Future

One of the conundrums we face in valuation is that although almost all the data we have available to us is about the past, reflecting the company's history (past financial statements, betas), the sector (industry average margins and returns on capital), and macroeconomic variables (interest rates, exchange rates, stock returns), all the forecasts

we have to make are for the future. While we cannot manufacture data for the future, we can follow some simple rules to minimize the damage. First, use historical data, but do not be bound by it. Second, trust in mean reversion, but watch out for structural breaks and challenges. Third, use forward estimates as alternatives or checks to historical data.

Proposition 8: Draw on the Law of Large Numbers

In many cases, the numbers that we use in valuation come from looking at samples of past data. The historical equity risk premium, for instance, is estimated by averaging the premium earned by stocks over treasuries over long time periods. Even if these estimates are constantly updated and are from services that have impeccable reputations, these numbers cannot be treated as facts. They are estimates, based on samples, and they come with substantial standard errors. While making estimates based on data will always be subject to noise, we can do some things to make these estimates more precise. The first is to use larger samples. With historical risk premiums, for instance, an equity risk premium estimated over 100 years of data will have a standard error of only 2% associated with it. With betas, replacing a single regression beta with a sector average can yield far more precision in your estimate. The second is to use statistical tools, like multiple regressions, to reduce the estimation error. We should make a final point about the law of large numbers. Since the values we obtain for individual companies are estimates, the likelihood that we will be wrong about any one company in any one period is high. However, if we assess value well, our chances of being right will improve if we extend to multiple periods (longer time horizons) and across multiple stocks (portfolios).

Proposition 9: Accept Uncertainty, and Deal with It

Accept uncertainty as a given, understanding that no matter how carefully we construct models, we cannot make uncertainty go away. Thus, adding more detail to models or making models more complicated, both of which are tactics that analysts use to reduce uncertainty, often does little to alleviate the problem and may actually make it worse. Uncertainty and risk are part of life and investing. When valuing companies, we can demand a premium for taking this risk in the discount rate and arrive at a risk-adjusted value, but it will change as circumstances change.

Proposition 10: Convert Stories to Numbers

One of the critiques of discounted cash flow valuation is that it is so focused on numbers that it misses the qualitative variables. Included in the list of qualitative variables are items such as customer loyalty, brand name, and good management. Every number in a discounted cash flow valuation should have an economic rationale[md]a good economic story behind it. Thus, when we set a firm's return on capital at 15%, well above its cost of capital, it behooves us to think about this firm's competitive advantages (most of which are qualitative) and how they translate into the return on capital. On the flip side, every solid economic story should find a place in the numbers. Brand name affects value, but it does so by increasing margins and excess returns, not as a premium at the end of the process.

Conclusion

The analyst faced with the task of valuing a firm/asset or its equity has to choose among three different approaches -- discounted cashflow valuation, relative valuation and option pricing models; and within each approach, they must also choose among different models. These choices will be driven largely by the characteristics of the firm/asset being valued - the level of its earnings, its growth potential, the sources of earnings growth, the stability of its leverage and its dividend policy. Matching the valuation model to the asset or firm being valued is as important a part of valuation as understanding the models and having the right inputs.

