

**CHAPTER 33****Probabilistic Approaches in  
Valuation: Scenario Analysis,  
Decision Trees, and Simulations**

Through much of this book, we have focused on discounted cash flow and relative valuation approaches to valuation. Notwithstanding their popularity, these approaches share a common theme. The riskiness of an asset is encapsulated in one number—a higher discount rate, lower cash flows, or a discount—and the computation almost requires us to make assumptions (often unrealistic) about the nature of risk.

In this chapter, we consider a different and potentially more informative way of assessing the value of an investment. Rather than compute an expected value for an asset that tries to capture in one number all of the different possible outcomes, we could provide information on what the value of the asset will be under each outcome or at least a subset of outcomes. We will begin this section by looking at the simplest version, which is an analysis of an asset's value under three scenarios—a best case, most likely case, and worse case—and then extend the discussion to look at scenario analysis more generally. We will move on to examine the use of decision trees, a more complete approach to dealing with discrete risk. We will close the chapter by evaluating Monte Carlo simulations, the most complete approach to assessing risk across the spectrum.

**SCENARIO ANALYSIS**

The expected cash flows that we use to value risky assets can be estimated in one of two ways. They can represent a probability-weighted average of cash flows under all possible scenarios, or they can be the cash flows under the most likely scenario. While the former is the more precise measure, it is seldom used simply because it requires far more information to compile. In both cases, there are other scenarios where the cash flows will be different from expectations: higher than expected in some and lower than expected in others. In scenario analysis, we estimate expected cash flows and asset values under various scenarios, with the intent of getting a better sense of the effect of risk on value. In this section, we first consider an extreme version of scenario analysis where we consider the value in the best and the worst-case scenarios and then a more generalized version of scenario analysis.

### Best Case / Worst Case

With risky assets, the actual cash flows can be very different from expectations. At the **minimum**, we can estimate the cash flows if everything works to perfection—a best-case scenario—and if nothing does—a worst-case scenario. In practice, there are two ways in which this analysis can be structured. In the first, each input into asset value is set to its best (or worst) possible outcome and the cash flows are estimated with those values. Thus, when valuing a firm, you may set the revenue growth rate and operating margin at the highest possible level while setting the discount rate at its lowest level, and compute the value as the best-case scenario. The problem with this approach is that it may not be feasible; after all, to get the high revenue growth, the firm may have to lower prices and accept lower margins. In the second, the best possible scenario is defined in terms of what is feasible while allowing for the relationship between the inputs. Thus, instead of assuming that revenue growth and margins will both be maximized, we will choose that combination of growth and margin that is feasible and yields the maximum value. While this approach is more realistic, it does require more work to put into practice.

How useful is best-case/worse-case analysis? There are two ways in which the results from this analysis can be useful to decision makers. First, the difference between the best-case and worst-case value can be used as a measure of risk on an asset; the range in value (scaled to size) should be higher for riskier investments. Second, investors that are concerned about the potential spill-over effects on their portfolio of an investment going bad may be able to gauge the effects by looking at the worst-case outcome.

In general, though, best-case/worse-case analyses are not very informative. After all, there should be no surprise in knowing that an asset will be worth a lot in the best case and not very much in the worst case. Thus, an equity research analyst who uses this approach to value a stock, priced at \$50, may arrive at values of \$80 for the best case and \$10 for the worst case; with a range that large, it will be difficult to make a judgment on whether the stock is a good investment or not.

### Multiple-Scenario Analysis

Scenario analysis does not have to be restricted to the best and worst cases. In its most general form, the value of a risky asset can be computed under a number of different scenarios, varying the assumptions about both macro-economic and asset-specific variables.

### Steps in Scenario Analysis

While the concept of sensitivity analysis is a simple one, it has four critical components:

1. The first is the determination of which factors the scenarios will be built around. These factors can range from the state of the economy for an automobile firm considering a new plant, to the response of competitors for a consumer product firm introducing a new product, to the behavior of regulatory authorities for a regulated company considering a new product or service. In general, **analysts** should focus on the two or three most critical factors that will determine the value of the asset and build scenarios around these factors.

2. The second component is determining the number of scenarios to analyze for each factor. While more scenarios may be more realistic than fewer, it becomes more difficult to collect information and differentiate between the scenarios in terms of asset cash flows. Thus, estimating cash flows under each scenario will be easier if ~~the firm lays~~ out 5 scenarios, for instance, than if ~~it lays~~ out 15 scenarios. The question of how many scenarios to consider will depend then upon how different the scenarios are, and how well ~~the analyst~~ can forecast cash flows under each scenario.
3. The third component is the estimation of asset cash flows under each scenario. It is to ease the estimation at this step that ~~we~~ focus on only two or three critical factors and build relatively few scenarios for each factor.
4. The final component is the assignment of probabilities to each scenario. For some scenarios, involving macro-economic factors such as exchange rates, interest rates, and overall economic growth, ~~we~~ can draw on the expertise of services that forecast these variables. For other scenarios, involving either the sector or competitors, ~~we~~ have to draw on ~~our~~ knowledge about the industry. Note, though, that this makes sense only if the scenarios cover the full spectrum of possibilities. If the scenarios represent only a subset of the possible outcomes on an investment, the probabilities will not add up to one.

The output from a scenario analysis can be presented as values under each scenario and as an expected value across scenarios (if the probabilities can be estimated in the fourth step). If the scenarios are incomplete, the expected value cannot be computed.

This quantitative view of scenario analysis will be challenged by strategists, who have traditionally viewed scenario analysis as a qualitative exercise, whose primary benefit is to broaden the thinking of decision makers. As one strategist put it, scenario analysis is about devising “plausible future narratives” rather than probable outcomes; in other words, there are benefits to considering scenarios that have a very low probability of occurring. The benefit of the exercise is that it forces ~~decision makers~~ to consider views of what may unfold ~~than~~ differ from the official view.

### Use in Valuation and Decision Making

How useful is scenario analysis in value assessment and decision making? The answer, as with all tools, depends upon how it is used. The most ~~useful~~ information from a scenario analysis is the range of values across different scenarios, which provides a snapshot of the riskiness of the asset; riskier assets will have values that vary more across scenarios, and safer assets will have manifest more ~~value~~ stability. In addition, scenario analysis can be useful in determining the inputs into an analysis that have the most effect on value. There is another advantage to doing scenario analysis. To the extent that value is much lower under some scenarios than others, investors can try to find ways to hedge against those scenarios’ occurring.

If nothing else, the process of thinking through scenarios is a useful exercise in examining how the competition will react under different macro-economic environments and what can be done to minimize the effect of downside risk and maximize the effect of potential upside on the value of a risky asset.

## Issues

Multiple-scenario analysis provides more information than a best-case/worst-case analysis by providing asset values under each of the specified scenarios. It does, however, have its own set of problems:

- *Garbage in, garbage out:* ~~It goes without saying that the~~ key to doing scenario analysis well is the setting up of the scenarios and the estimation of cash flows under each one. Not only do the outlined scenarios have to be realistic, but they also have to try to cover the spectrum of possibilities. Once the scenarios have been laid out, the cash flows have to be estimated under each one; this trade-off has to be considered when determining how many scenarios will be run.
- *Continuous Risk:* Scenario analysis is best suited for dealing with risk that takes the form of discrete outcomes. When the outcomes can take on any of a very large number of potential values or the risk is continuous, it becomes more difficult to set up scenarios.
- *Double counting of risk:* As with the best-case/worst-case analysis, there is the danger that decision makers will double-count risk when they do scenario analysis. Thus, an analyst, may decide to reject the investment, even though it looks undervalued, because it looks significantly overvalued under at least one scenario. Since the expected value is already risk-adjusted, this would represent a double counting of potentially the same risk or risk that should not be a factor in the decision in the first place (because it is diversifiable).

### ILLUSTRATION 33.1: Valuing a Company Facing the Threat of Nationalization

While the global threat of nationalization has diminished over the last few decades, there remain parts of the world where investors remain wary that ~~their~~ firms might be expropriated or nationalized by a government. Analysts who attempt to incorporate the risk of nationalization into the expected cash flows or discount rates find very quickly that it is difficult to do so. Discount rates, in particular, are blunt instruments that do not lend themselves easily to the consideration of discrete risks (such as nationalization, distress, or regulatory change).

As an alternative, consider a very simple scenario analysis where you value the firm under two scenarios: as a going concern where the owners of the business keep the cash flows, and under nationalization, where the proceeds to the owners are not commensurate with fair value. The expected value for the firm will be a weighted average of the two estimates.

To move from abstractions, assume that you are valuing a Venezuelan company that expects to generate \$10 million in after-tax operating income next year and grow 3 percent a year, in U.S. dollar terms, in perpetuity. Assume also that this firm generates a 20 percent return on capital (the book value of capital invested is \$50 million) and faces a 12 percent cost of capital, with the latter including a country risk component for the macroeconomic (but not nationalization) risk. To value the firm as a going concern, we use the expected cash flows and cost of capital to arrive at a value of \$94.44 million for the operating assets of the firm:

$$\text{Reinvestment rate} = g/\text{ROC} = 3\%/20\% = 15\%$$

$$\text{Value of operating assets}_{\text{Going concern}} = \frac{\text{EBIT}(1-t)\left(1 - \frac{g}{\text{ROC}}\right)}{(\text{Cost of capital} - g)} = \frac{\$10(1 - .15)}{(.12 - .03)} = \$94.44 \text{ million}$$

Now assume that you believe that there is a likelihood that the firm will be nationalized and that the government will pay only the book value of operating assets to the owners of nationalized companies. The proceeds from nationalization can be estimated as follows:

$$\text{Value of operating assets}_{\text{Nationalization}} = \text{Book value} = \$50 \text{ million}$$

Bringing in a probability of nationalization (25%) allows us to estimate the expected value for the operating assets:

$$\begin{aligned} \text{Expected value} &= \text{Value of operating assets}_{\text{Going concern}} (1 - \text{Probability of nationalization}) \\ &\quad + \text{Value of operating assets}_{\text{Nationalization}} (\text{Probability of nationalization}) \\ &= \$94.44(.75) + \$50(.25) \\ &= \$83.33 \text{ million} \end{aligned}$$

Note that expected value will decrease as the likelihood of nationalization increases and the proceeds from nationalization decreases.

### ILLUSTRATION 33.2: Valuing a Regulated Company with Shifting Regulatory Risk

Scenario analysis becomes more complicated and potentially more useful as the number of scenarios increases and uncertainty magnifies. To illustrate, assume that you were valuing Wells Fargo, one of the largest commercial banks in the United States, in early 2009. The banking crisis of 2008 had not only wreaked havoc on the profitability of banks but had also increased the likelihood that regulatory authorities would tighten regulatory capital rules, requiring banks to set aside more capital to cover potential losses from their operations.

To see the impact of the crisis, the table following lists key financial variables for Wells Fargo from 2001 to 2008:

Year	2008	2007	2006	2005	2004	2003	2002	2001	Average: 01–07
Dividends	<b>\$ 5,751</b>	\$ 3,955	\$ 3,641	\$ 3,375	\$ 3,150	\$ 2,527	\$ 1,873	\$ 1,710	
Net Income	<b>\$ 2,842</b>	\$ 8,057	\$ 8,482	\$ 7,671	\$ 7,014	\$ 6,202	\$ 5,434	\$ 3,423	
Book Equity	<b>\$47,628</b>	\$45,876	\$40,660	\$37,866	\$34,469	\$30,319	\$27,214	\$26,488	
Growth Rate	<b>−64.73%</b>	−5.01%	10.57%	9.37%	13.09%	14.13%	58.75%	−14.98%	12.28%
Payout ratio	<b>202.36%</b>	49.09%	42.93%	44.00%	44.91%	40.74%	34.47%	49.96%	43.73%
ROE	<b>5.97%</b>	17.56%	20.86%	20.26%	20.35%	20.46%	19.97%	12.92%	18.91%

Note that while dividends increased in 2008, net income and return on equity dropped precipitously. In early 2009, here were some key questions that an analyst would have faced in valuing Wells Fargo:

- What should you use as your base year values for earnings, dividends, and return on investment (ROE)? In other words, could you assume that 2008 is an aberration and go back to normalized values that reflect the average from 2001 to 2007?
- Historically banks have had a beta close to one, which would have given Wells Fargo a cost of equity of about 9 percent in February 2009 (T. Bond rate = 3%; ERP = 6%). Given that the crisis has shed new light on the hidden risks in banks, would you continue to use this beta in the valuation?

Rather than try to come up with a composite value for Wells Fargo across all possible scenarios, we developed three scenarios:

1. *Quick bounce back to normalcy (10 percent probability):* This was the most optimistic scenario, in which we assumed that the crisis would pass quickly, that regulatory capital ratios would not

be changed and that the return on equity and beta would revert back quickly to pre-crisis levels (beta = 1, ROE = 18.91%)

2. *Slow bounce back to normalcy (60 percent probability)*: This is a more pessimistic (and more realistic) scenario, where the crisis does fade slowly but is accompanied by increased regulatory capital requirements (which will lower return on equity to 15 percent) and higher volatility for banks (which will raise the cost of equity to 10 percent).
3. *New World Order (30 percent probability)*: This is the most pessimistic of the scenarios, where the crisis drags on and regulatory capital changes are draconian, with return on equity dropping to 12 percent and the cost of equity rising to 11 percent.

Using the return on equity and cost of equity as the levers in a dividend discount model, we were able to derive the value of equity:

$$\begin{aligned} \text{Value of Equity} &= \frac{\text{Expected Dividends}_{\text{Next year}}}{(\text{Cost of Equity} - \text{Expected growth rate})} \\ &= \frac{\text{Book Equity}_{\text{Base}} \times \text{ROE} \times \left(1 - \frac{g}{\text{ROE}}\right)}{\text{Cost of Equity} - \text{Expected growth rate}} \end{aligned}$$

Assuming that Wells Fargo is in stable growth, growing 3 percent a year, we can estimate the value of equity as a function of the expected ROE and cost of equity. The table below summarizes the estimates for the aggregate value of equity (using \$47,628 million as the base book equity) in Wells Fargo under all three scenarios:

	Probability	Expected Net Income	ROE	Cost of Equity	Value of Equity (in millions)
Quick bounce back to normalcy	10%	\$9,006.45	18.91%	9%	\$126,294
Slow bounce back to normalcy	60%	\$7,144.20	15.00%	10%	\$81,648
New World Order	30%	\$5,715.36	12.00%	11%	\$53,582
<b>Expected value</b>	$0.30 \times (\$126,294) + 0.60(\$81,648) + 0.10(\$53,582)$				<b>\$77,693</b>

At its market capitalization of \$66.643 billion in early 2009, Wells Fargo looks undervalued, but the valuation is sensitive to the probabilities attached to the three scenarios.

## DECISION TREES

In some cases, risk is not only discrete but is sequential. In other words, for the asset to have value, it has to pass through a series of tests, with failure at any point potentially translating into a complete loss of value. This is the case, for instance, with a pharmaceutical drug that is just being tested ~~on human beings~~. The three-stage FDA approval process lays out the hurdles that have to be passed for this drug to be commercially sold, and failure at any of the three stages dooms the drug's chances. When valuing a large pharmaceutical company with a portfolio of drugs, the risk that a drug will not make it through the process will be averaged out across the portfolio, thus allowing us to use conventional discounted cash flow

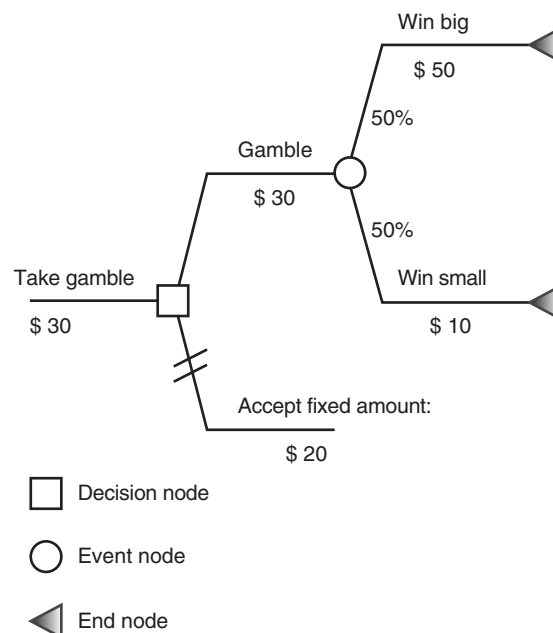
models. In contrast, if you are valuing a small, biotechnology company with only one patent making its way through the approval process, the value rests entirely on the sequential risk process. Decision trees allow us to not only consider the risk in stages but also to devise the right response to outcomes at each stage.

### Steps in Decision Tree Analysis

The first step in understanding decision trees is to distinguish between root nodes, decision nodes, event nodes, and end nodes.

- The root node represents the start of the decision tree, where a decision maker can be faced with a decision choice or an uncertain outcome. The objective of the exercise is to evaluate what a risky investment is worth at this node.
- Event nodes represent the possible outcomes on a risky gamble; whether a drug passes the first stage of the FDA approval process or not is a good example. We have to figure out the possible outcomes and the probabilities of the outcomes occurring, based upon the information we have available today.
- Decision nodes represent choices that can be made by the decision maker—to expand from a test market to a national market, after a test market's outcome is known.
- End nodes usually represent the final outcomes of earlier risky outcomes and decisions made in response.

Consider a very simple example. You are offered a choice where you can take a certain amount of \$20 or partake in a gamble, where you can win \$50 with probability 50 percent and \$10 with probability 50 percent. The decision tree for this offered gamble is shown in Figure 33.1.



**FIGURE 33.1** Simple Decision Tree

Note the key elements in the decision tree. First, only the event nodes represent uncertain outcomes and have probabilities attached to them. Second, the decision node represents a choice. On a pure expected value basis, the gamble is better (with an expected value of \$30) than the guaranteed amount of \$20; the double slash on the latter branch indicates that it would not be selected. While this example may be simplistic, the elements of building a decision tree are in it.

*Step 1: Divide analysis into risk phases:* The key to developing a decision tree is outlining the phases of risk that you will be exposed to in the future. In some cases, such as the FDA approval process, this will be easy to do since there are only two outcomes—the drug gets approved to move on to the next phase, or it does not. In other cases, it will be more difficult. For instance, a test market of a new consumer product can yield hundreds of potential outcomes; here, you will have to create discrete categories for the success of the test market.

*Step 2: In each phase, estimate the probabilities of the outcomes:* Once the phases of risk have been put down and the outcomes at each phase are defined, the probabilities of the outcomes have to be computed. In addition to the obvious requirement that the probabilities across outcomes have to sum up to one, the analyst will also have to consider whether the probabilities of outcomes in one phase can be affected by outcomes in earlier phases. For example, how does the probability of a successful national product introduction change when the test market outcome is only average?

*Step 3: Define decision points:* Embedded in the decision tree will be decision points where you will get to determine what your best course of action will be, based upon observing the outcomes at earlier stages, and expectations of what will occur in the future. With the test market example, for instance, you will get to determine, at the end of the test market, whether you want to conduct a second test market, abandon the product or move directly to a national product introduction.

*Step 4: Compute cash flows/value at end nodes:* The next step in the decision tree process is estimating what the final cash flow and value outcomes will be at each end node. In some cases, such as abandonment of a test market product, this will be easy to do and will represent the money spent on the test marketing of the product. In other cases, such as a national launch of the same product, this will be more difficult to do since you will have to estimate expected cash flows over the life of the product and discount these cash flows to arrive at value.

*Step 5: Folding back the tree:* The last step in a decision tree analysis is termed *folding back* the tree, where the expected values are computed working backwards through the tree. If the node is a chance node, the expected value is computed as the probability-weighted average of all of the possible outcomes. If it is a decision node, the expected value is computed for each branch, and the highest value is chosen (as the optimal decision). The process culminates in an expected value for the asset or investment today.<sup>1</sup>

There are two key pieces of output that emerge from a decision tree. The first is the expected value today of going through the entire decision tree. This expected value will incorporate the potential upside and downside from risk and the actions

<sup>1</sup>There is a significant body of literature examining the assumptions that have to hold for this folding back process to yield consistent values. In particular, if a decision tree is used to portray concurrent risks, the risks should be independent of each other. See Sarin, R. and P. Wakker (1994).



that you will take along the way in response to this risk. In effect, this is analogous to the risk-adjusted value that we talked about in the last chapter. The second is the range of values at the end nodes, which should encapsulate the potential risk in the investment.

### ILLUSTRATION 33.3: Decision Tree Valuation—Valuing a Young Pharmaceutical Company

To illustrate the steps involved in developing a decision tree, let us value a small biotechnology company, with only one product: a pharmaceutical drug for treating Type 1 diabetes, that has gone through preclinical testing and is about to enter phase 1 of the FDA approval process.<sup>2</sup> Assume that you are provided with the additional information on each of the three phases:

1. Phase 1 is expected to cost \$50 million and will involve 100 volunteers to determine safety and dosage; it is expected to last 1 year. There is a 70 percent chance that the drug will successfully complete the first phase.
2. In phase 2, the drug will be tested on 250 volunteers for effectiveness in treating diabetes over a two-year period. This phase will cost \$100 million, and the drug will have to show a statistically significant impact on the disease to move on to the next phase. There is a 30 percent chance that the drug will prove successful in treating Type 1 diabetes but there is a 10 percent chance that it will be successful in treating both Type 1 and Type 2 diabetes and a 10 percent chance that it will succeed only in treating Type 2 diabetes.
3. In phase 3, the testing will expand to 4,000 volunteers to determine the long-term consequences of taking the drug. If the drug is tested on only Type 1 or Type 2 diabetes patients, this phase will last 4 years and cost \$250 million; there is an 80 percent chance of success. If it is tested on both types, the phase will last 4 years and cost \$300 million; there is a 75 percent chance of success.

If the drug passes through all three phases, the costs of developing the drug and the annual cash flows are provided as follows:

<i>Disease Treatment</i>	<i>Cost of Development</i>	<i>Annual Cash Flow</i>
Type 1 diabetes only	\$500 million	\$300 million for 15 years
Type 2 diabetes only	\$500 million	\$125 million for 15 years
Type 1 and 2 diabetes	\$600 million	\$400 million for 15 years

Assume that the cost of capital for the firm is 10 percent.

We now have the information to draw the decision tree for this drug. We will first draw the tree in Figure 33.2, specifying the phases, the cash flows at each phase, and the probabilities.

The decision tree shows the probabilities of success at each phase and the additional cash flow or marginal cash flow associated with each step. Since it takes time to go through the phases, there is a time value effect that has to be built into the expected cash flows for each path. We introduce the time value effect and compute the cumulative present value (today) of cash flows from each path, using the 10 percent cost of capital as the discount rate, in Figure 33.3.

Note that the present value of the cash flows from development after the third phase get discounted back an additional seven years (to reflect the time it takes to get through three phases). In the last step in the process, we compute the expected values by working backwards through the tree and estimating the optimal action in each decision phase in Figure 33.4.

<sup>2</sup>In Type 1 diabetes, the pancreas do not produce insulin. The patients are often young children and the disease is unrelated to diet and activity; they have to receive insulin to survive. In Type 2 diabetes, the pancreas produce insufficient insulin. The disease manifests itself in older people and can be sometimes controlled by changing lifestyle and diet.

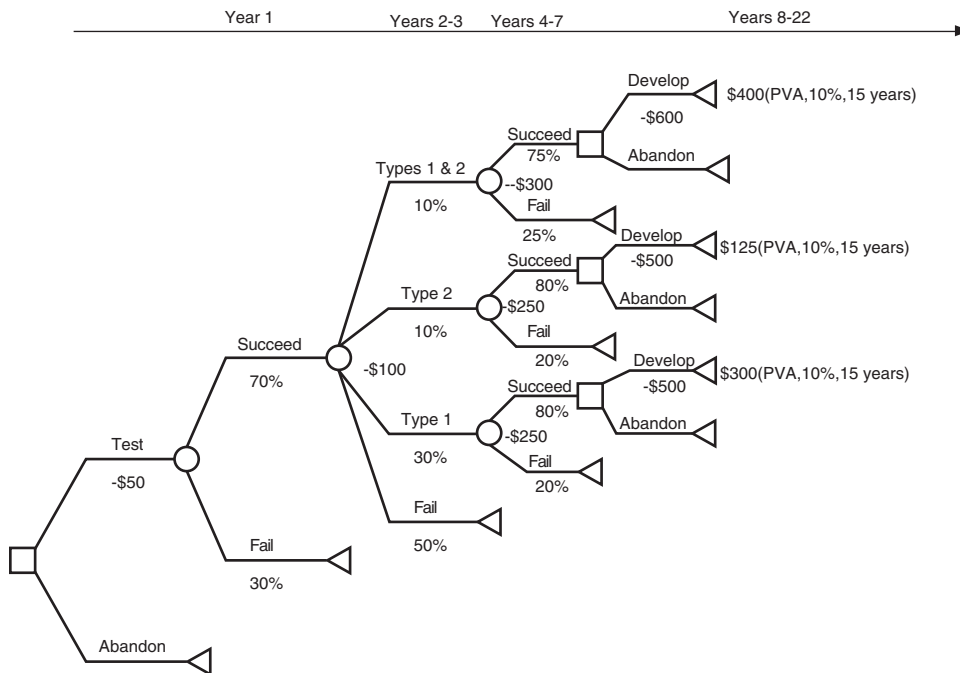



FIGURE 33.2 Decision Tree for Drug Development

AU: The equations in 33.3——do they need some parens to indicate the order of operations? They seem confusing as is.

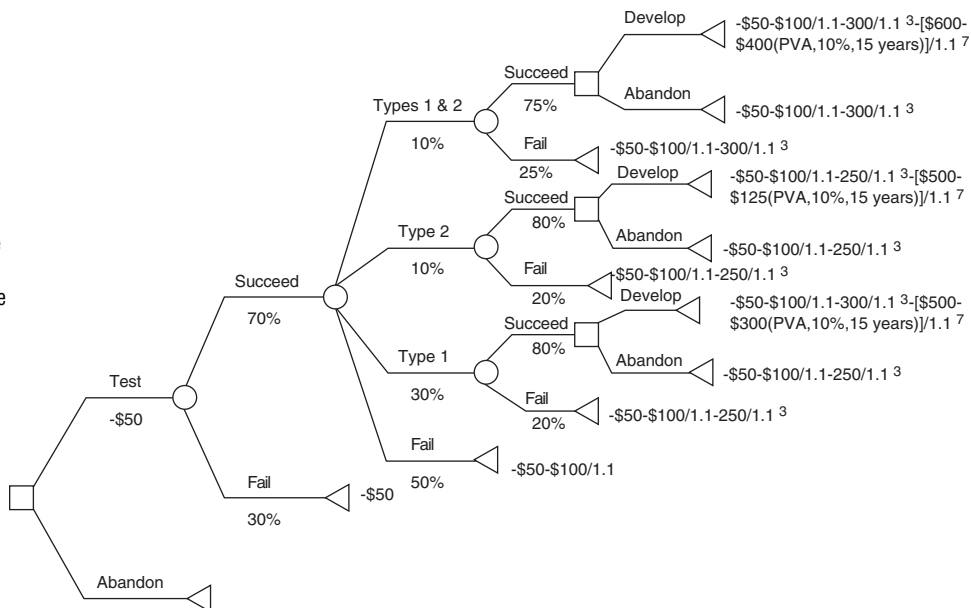
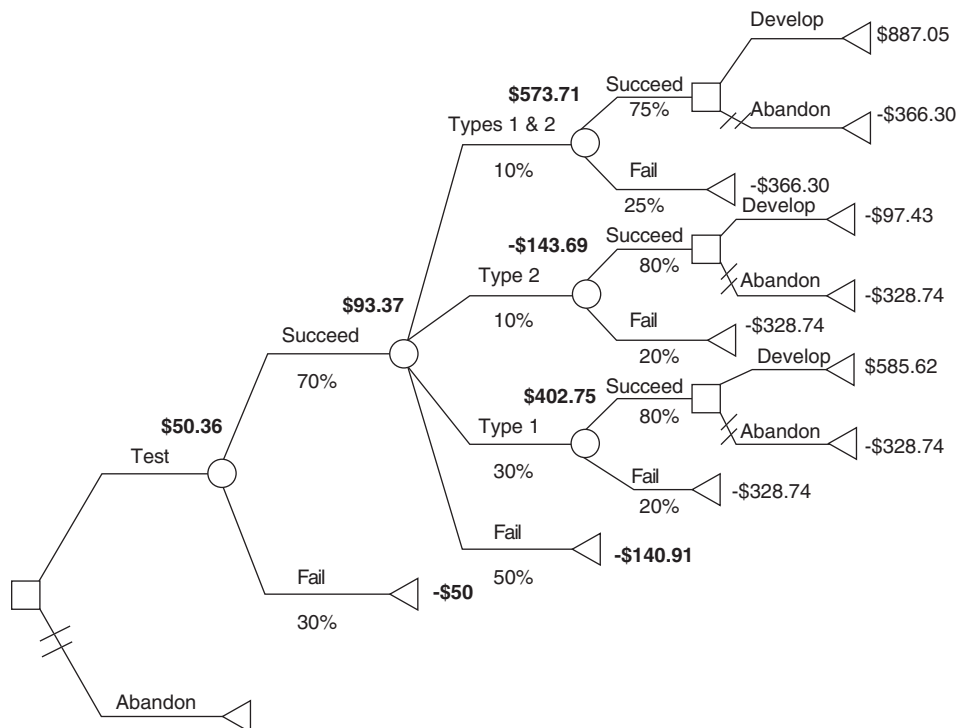


FIGURE 33.3 Present Value of Cash Flows at End Nodes: Drug Development Tree



**FIGURE 33.4** Drug Decision Tree Folded Back

The expected value of the drug today, given the uncertainty over its success, is \$50.36 million. Since it is the only product for the biotechnology company, it is also the value for the company. This value reflects all of the possibilities that can unfold over time and shows the choices at each decision branch that are suboptimal and thus should be rejected. The decision tree also provides a range of outcomes, with the worst outcome being failure in phase 3 of the drug as a treatment for both phase 1 and 2 diabetes (–\$366.30 million in today's dollars) to the best-case outcome of approval and development of the drug as treatment for both types of diabetes (\$887.05 million in today's dollars).

There may one element in the last set of branches that may seem puzzling. Note that the present value of developing the drug as a treatment for just Type 2 diabetes is negative (–\$97.43 million). Why would the company still develop the drug? Because the alternative of abandoning the drug at the late stage in the process has an even more negative net present value (–\$328.74 million). Another way to see this is to look at the marginal effect of developing the drug just for Type 2 diabetes. Once the firm has expended the resources to take the firm through all three phases of testing, the testing cost becomes a sunk cost and is not a factor in the decision.<sup>3</sup> The marginal cash flows from developing the drug after phase 3 yield a positive net present value of \$451 million (in year 7 cash flows):

$$\begin{aligned} &\text{Present value of developing drug to treat Type 2 diabetes in year 7} \\ &= -500 + 125(\text{PV of annuity, 10 percent, 15 years}) = \$451 \text{ million} \end{aligned}$$

Rolling back the decision tree allows you to see what the value of the drug/company is at each phase in the process.

<sup>3</sup>It would be more accurate to consider only the costs of the first two phases as sunk, since by the end of phase 2, the firm knows that the drug is effective only against Type 2 diabetes. Even if we consider only the costs of the first 2 phases as sunk, it still makes sense on an expected value basis to continue to phase 3.

## Use in Decision Making

There are several benefits that accrue from using decision trees, and it is surprising that they are not used more often in analysis.

- *Dynamic response to risk:* By linking actions and choices to outcomes of uncertain events, decision trees encourage firms to consider how they should act under different circumstances. As a consequence, firms will be prepared for whatever outcome may arise rather than be surprised. In the example in the last section, for instance, the firm will be ready with a plan of action, no matter what the outcome of phase 3 happens to be.
- *Value of information:* Decision trees provide a useful perspective on the value of information in decision making. While it is not as obvious in the drug development example, it can be seen clearly when a firm considers whether to test market a product before commercially developing it. By test marketing a product, you acquire more information on the chances of eventual success. We can measure the expected value of this improved information in a decision tree and compare it to the test marketing cost.
- *Risk management:* Since decision trees provide a picture of how cash flows unfold over time, they are useful in deciding what risks should be protected against and the benefits of doing so. Consider a decision tree on an asset, where the worst-case scenario unfolds if the dollar is weak against the euro. Since we can hedge against this risk, the cost of hedging the risk can be compared to the loss in cash flows in the worst-case scenario.

In summary, decision trees provide a flexible and powerful approach for dealing with risk that occurs in phases, with decisions in each phase depending upon outcomes in the previous one. In addition to providing us with measures of risk exposure, they also force us to think through how we will react to both adverse and positive outcomes that may occur at each phase.

## Issues

There are some types of risk that decision trees are capable of handling and others that they are not. In particular, decision trees are best suited for risk that is sequential; the FDA process where approval occurs in phases is a good example. Risks that affect an asset concurrently cannot be easily modeled in a decision tree.<sup>4</sup>

As with scenario analysis, decision trees generally look at risk in terms of discrete outcomes. Again, this is not a problem with the FDA approval process where there are only two outcomes—success or failure. There is a much wider range of outcomes with most other risks, and we have to create discrete categories for the outcomes to stay within the decision-tree framework. For instance, when looking at a market test, we may conclude that selling more than 100,000 units in a test market qualifies as a great success, between 60,000 and 100,000 units as an average outcome, and below 60,000 as a failure.

<sup>4</sup>If we choose to model such risks in a decision tree, they have to be independent of each other. In other words, the sequencing should not matter.

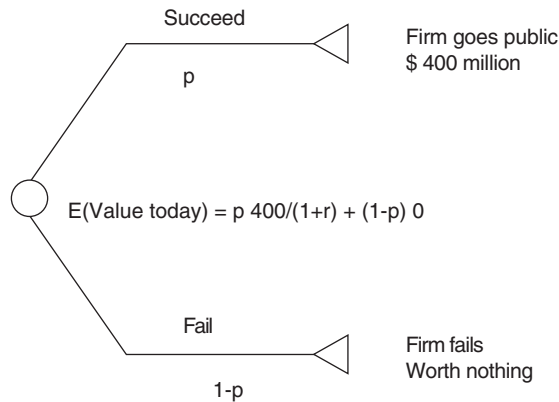
Assuming risk is sequential and can be categorized into discrete boxes, we are faced with estimation questions to which there may be no easy answers. In particular, we have to estimate the cash flow under each outcome and the associated probability. With the drug development example, we had to estimate the cost and the probability of success of each phase. The advantage that we have when it comes to these estimates is that we can draw on empirical data on how frequently drugs that enter each phase make it to the next one and historical costs associated with drug testing. To the extent that there may be wide differences across different phase 1 drugs in terms of success—some may be longer shots than others—there can still be errors that creep into decision trees.

The expected value of a decision tree is heavily dependent upon the assumption that we will stay disciplined at the decision points in the tree. In other words, if the optimal decision is to abandon if a test market fails and the expected value is computed, based on this assumption, the integrity of the process and the expected value will quickly fall apart, if managers decide to overlook the market testing failure and go with a full launch of the product anyway.

### Risk-Adjusted Value and Decision Trees

Are decision trees an alternative or an addendum to discounted cash flow valuation? The question is an interesting one because there are some analysts who believe that decision trees, by factoring in the possibility of good and bad outcomes, are already risk adjusted. In fact, they go on to make the claim that the right discount rate to use estimating present value in decision trees is the riskfree rate; using a risk-adjusted discount rate, they argue, would be double counting the risk. Barring a few exceptional circumstance, they are incorrect in their reasoning.

- *Expected values are not risk adjusted:* Consider decision trees, where we estimate expected cash flows by looking at the possible outcomes and their probabilities of occurrence. The probability-weighted expected value that we obtain is not risk adjusted. The only rationale that can be offered for using a risk-free rate is that the risk embedded in the uncertain outcomes is asset-specific and will be diversified away, in which case the risk-adjusted discount rate would be the riskfree rate. In the FDA drug development example, for instance, this may be offered as the rationale for why we would use the risk-free rate to discount cash flows for the first seven years, when the only risk we face is drug approval risk. After year 7, though, the risk is likely to contain a market element and the risk-adjusted rate will be higher than the risk-free rate.
- *Double counting of risk:* We do have to be careful about making sure that we don't double count for risk in decision trees by using risk-adjusted discount rates that are set high to reflect the possibility of failure at the earlier phases. One common example of this phenomenon is in venture capital valuation. In Chapter 24, we noted that venture capitalists often value young start-up companies by estimating an exit value, based on projected earnings and a multiple of those earnings in the future, and then discounting the exit value at a target rate of return. Using this approach, for instance, the value today for a firm that is losing money currently but is expected to make

**FIGURE 33.5** Decision Tree for Start-Up Firm

\$10 million in 5 years (when the earnings multiple at which it will be taken public is estimated to be 40) can be computed as follows (if the target rate is 35 percent):

Value of the firm in 5 years = Earnings in year 5  $\times$  PE =  $10 \times 40 = \$400$  million

Value of firm today =  $\$400/1.35^5 = \$89.20$  million

Note, however, that the target rate is set at a high level (35 percent) because of the probability that this young firm will not survive. In fact, we could frame this as a simple decision tree in Figure 33.5.

Assume that  $r$  is the correct discount rate, based upon the business risk that the venture capitalist faces on this venture. Going back to the numeric example, assume that this discount rate would have been 15 percent for this venture. We can solve for the implied probability of failure, embedded in the venture capitalist's estimate of value of \$89.20 million:

$$\text{Estimated Value} = \$89.20 = \frac{\$400}{1.15^5} (p)$$

Solving for  $p$ , we estimate the probability of success at 44.85 percent. With this estimate of probability in the decision tree, we would have arrived at the same value as the venture capitalist, assuming that we use the right discount rate. Using the target rate of 35 percent as the discount rate in a decision tree would lead to a drastically lower value, because risk would have been counted twice. Using the same reasoning, we can see why using a high discount rate in assessing the value of a biotechnology drug in a decision tree will undervalue the drug, especially if the discount rate already reflects the probability that the drug will not make it to commercial production. If the risk of the approval process is drug-specific and thus diversifiable, this would suggest that discount rates should be moderate in decision tree analysis, even for products with very high likelihoods of not making it through the early stages in the process.

- *The right discount rate:* If the right discount rate to use in a decision tree should reflect the business risk looking forward, it is not only possible but

likely that discount rates were different at different points in the tree. For instance, extraordinary success at the test market stage may yield more predictable cash flows than an average test market outcome; this would lead us to use a lower discount rate to value the former and a higher discount rate to value the latter. In the drug development example, it is possible that the expected cash flows, if the drug works for both types of diabetes, will be more stable than if it is a treatment for only one type. It would follow that a discount rate of 8 percent may be the right one for the first set of cash flows, whereas a 12 percent discount rate may be more appropriate for the second.

Reviewing the discussion, decision trees are not alternatives to risk-adjusted valuation. Instead, they can be viewed as a different way of adjusting for discrete risk that may be difficult to bring into expected cash flows or into risk-adjusted discount rates.

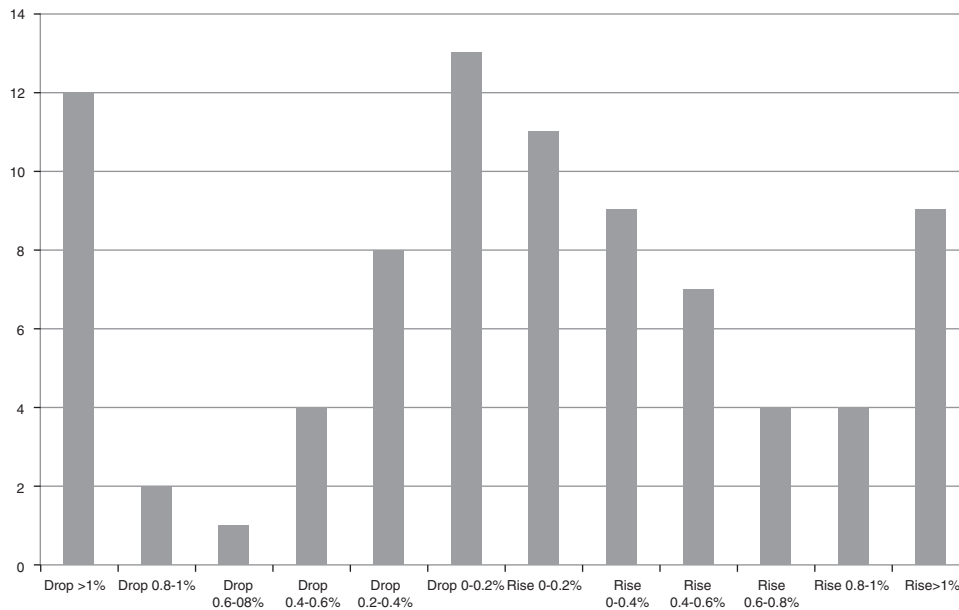
## SIMULATIONS

If scenario analysis and decision trees are techniques that help us to assess the effects of discrete risk, simulations provide a way of examining the consequences of continuous risk. To the extent that most risks that we face in the real world can generate hundreds of possible outcomes, a simulation will give us a fuller picture of the risk in an asset or investment.

### Steps in Simulation

Unlike scenario analysis, where we look at the values under discrete scenarios, simulations allow for more flexibility in how we deal with uncertainty. In its classic form, distributions of values are estimated for each parameter in the valuation (growth, market share, operating margin, beta, etc.). In each simulation, we draw one outcome from each distribution to generate a unique set of cash flows and value. Across a large number of simulations, we can derive a distribution for the value of investment or an asset that will reflect the underlying uncertainty we face in estimating the inputs to the valuation. The steps associated with running a simulation are as follows:

1. *Determine probabilistic variables:* In any analysis, there are potentially dozens of inputs, some of which are predictable and some of which are not. Unlike scenario analysis and decision trees, where the number of variables that are changed and the potential outcomes have to be few in number, there is no constraint on how many variables can be allowed to vary in a simulation. At least in theory, we can define probability distributions for each and every input in a valuation. The reality, though, is that this will be time consuming and may not provide much of a payoff, especially for inputs that have only marginal impact on value. Consequently, it makes sense to focus attention on a few variables that have a significant impact on value.
2. *Define probability distributions for these variables:* This is a key and the most difficult step in the analysis. Generically, there are three ways in which you can go about defining probability distributions:
  - i. *Historical data:* For variables that have a long history and reliable data over that history, it is possible to use the historical data to develop distributions. Assume, for instance, that you are trying to develop a distribution of



**FIGURE 33.6** One-Year Change in U.S. 10-Year Treasury Rate

expected changes in the long-term Treasury bond rate (to use as an input in investment analysis). You could use the histogram in Figure 33.6, based upon the annual changes in Treasury bond rates every year from 1928 to 2010, as the distribution for future changes.

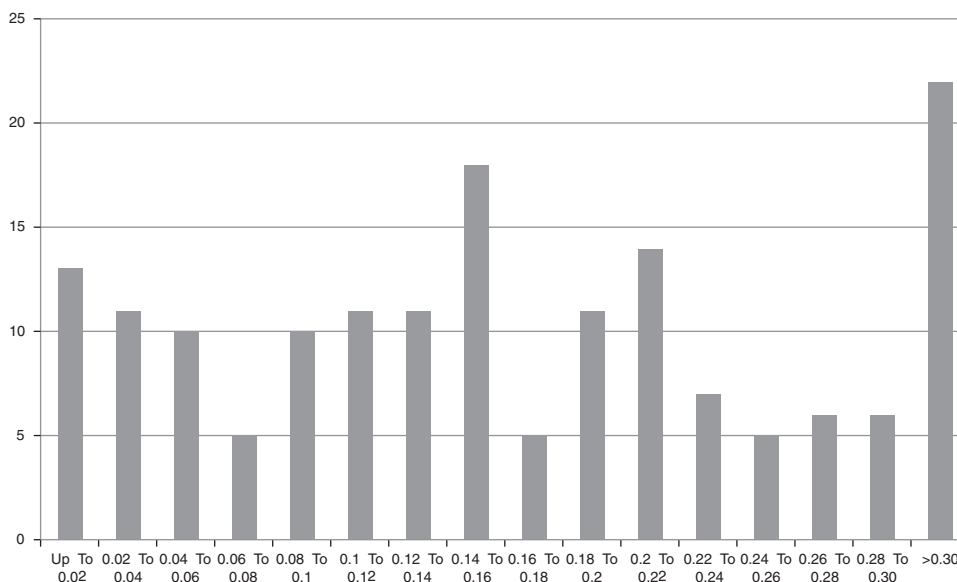
Implicit in this approach is the assumption that there have been no structural shifts in the market that will render the historical data unreliable.

- ii. *Cross-sectional data:* In some cases, you may be able to substitute data on differences in a specific variable across existing investments that are similar to the investment being analyzed. ~~Consider two examples.~~ Assume that you are valuing a software firm and are concerned about the volatility in operating margins. Figure 33.7 provides a distribution of pretax operating margins across software companies in 2011.

In a simulation, you could either use this distribution directly or find a standardized statistical distribution that is close in terms of characteristics. Note, though, that if you use this distribution, you are in effect assuming that the underlying distribution of margins is the same across software firms.

- iii. *Statistical distribution and parameters:* For most variables that you are trying to forecast, the historical and cross-sectional data will be insufficient or unreliable. In these cases, you have to pick a statistical distribution that best captures the variability in the input and estimate the parameters for that distribution. Thus, you may conclude that operating margins for software companies will be distributed uniformly, with a minimum of 0 percent and a maximum of 35 percent and that revenue growth is normally distributed with an expected value of 15 percent and a standard deviation of 10 percent. Many simulation packages available for personal computers now provide a rich array of distributions, but picking the right distribution and the parameters for the distribution remains difficult for two reasons.





**FIGURE 33.7** Pre-Tax Operating Margins: U.S. Software Companies in January 2011

The first is that few inputs that we see in practice meet the stringent requirements that statistical distributions demand; revenue growth, for instance, cannot really be normally distributed because the lowest value it can take on is  $-100$  percent. Consequently, you have to settle for statistical distributions that are close enough to the real distribution that the resulting errors will not wreak havoc on your conclusion. The second is that the parameters still need to be estimated, once the distribution is picked. For this, you can draw on historical or cross-sectional data; for the revenue growth input, you can look at revenue growth in prior years or revenue growth rate differences across peer group companies. The caveats about structural shifts that make historical data unreliable and peer group companies not being truly comparable continue to apply.

In summary, the probability distributions will be discrete for some inputs and continuous for others, be based upon historical data for some and statistical distributions for others.

3. *Check for correlation across variables:* While it is tempting to jump to running simulations right after the distributions have been specified, it is important that we check for correlations across variables. Assume, for instance, that you are developing probability distributions for both interest rates and inflation. While both inputs may be critical in determining value, they are likely to be correlated with each other; high inflation is usually accompanied by high interest rates. When there is strong correlation, positive or negative, across inputs, you have two choices. One is to pick only one of the two inputs to vary; it makes sense to focus on the input that has the bigger impact on value. The other is to build the correlation explicitly into the simulation; this requires more sophisticated simulation packages and adds more detail to the estimation process.
4. *Run the simulation:* For the first simulation, you draw one outcome from each distribution and compute the value based upon those outcomes. This process

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can be repeated as many times as desired, though the marginal contribution of each simulation drops off as the number of simulations increases. The number of simulations you run should be determined by the following:

- *Number of probabilistic inputs:* The larger the number of inputs that have probability distributions attached to them, the greater will be the required number of simulations.
- *Characteristics of probability distributions:* The greater the diversity of distributions in an analysis, the larger will be the number of required simulations. Thus, the number of required simulations will be smaller in a simulation where all of the inputs have normal distributions than in one where some have normal distributions, some are based upon historical data distributions, and some are discrete.
- *Range of outcomes:* The greater the potential range of outcomes on each input, the greater will be the number of simulations.

Most simulation packages allow users to run thousands of simulations, with little or no cost attached to increasing that number. Given that reality, it is better to err on the side of too many simulations rather than too few.

There have generally been two impediments to good simulations. The first is informational: estimating distributions of values for each input into a valuation is difficult to do. In other words, it is far easier to estimate an expected growth rate of 8 percent in revenues for the next 5 years than it is to specify the distribution of expected growth rates—the type of distribution, parameters of that distribution—for revenues. The second is computational; until the advent of personal computers, simulations tended to be too time and resource intensive for the typical analyst. Both these constraints have eased in recent years and simulations have become more feasible.

#### ILLUSTRATION 33.4: Valuing Exxon Mobil—Monte Carlo Simulation

~~Exxon Mobil is the largest of the global oil companies, with diversified operations in multiple locations, but it is as dependent upon oil prices as the rest of the companies in its sector. In Figure 33.8, we graph Exxon's operating income as a function of the average oil price each year from 1985 to 2008.~~

~~The operating income clearly increases (decreases) as the oil price increases (decreases). We regressed the operating income against the oil price per barrel over the period and obtained the following:~~

$$\text{Operating Income} = -6,395 + 911.32 (\text{Average Oil Price}) \quad R^2 = 90.2\%$$

(2.95) (14.59)

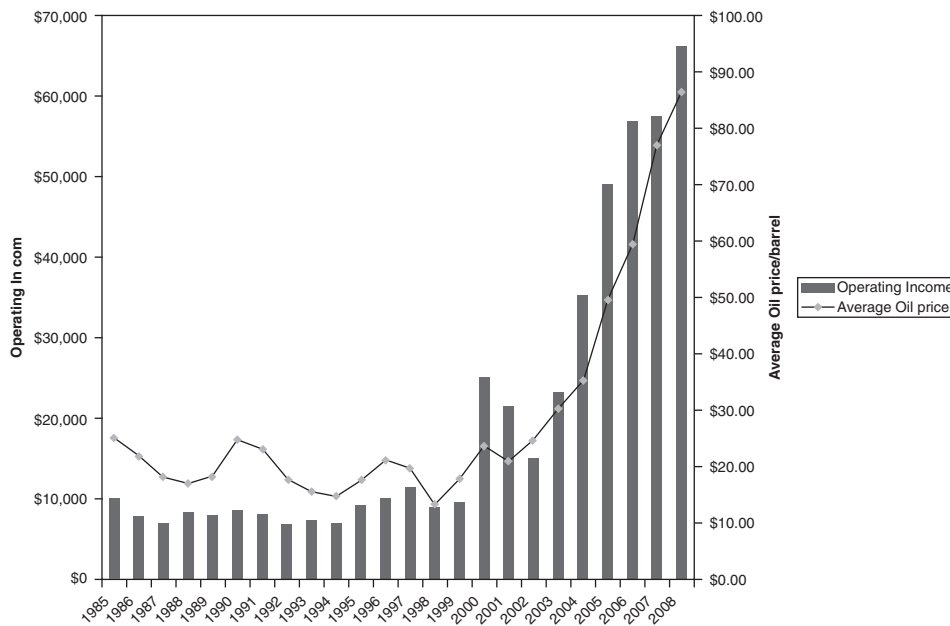
~~Put another way, Exxon Mobil's operating income increases about \$9.11 billion for every \$10 increase in the price per barrel of oil, and 90 percent of the variation in Exxon's earnings over time comes from movements in oil prices.<sup>5</sup>~~

~~To get from operating income to equity value at Exxon, we made the following assumptions:~~

- We estimated a bottom-up beta of 0.90 for Exxon Mobil, and then used the Treasury bond rate of 2.5 percent and an equity risk premium of 6.5 percent to estimate a cost of equity.

$$\text{Cost of equity} = 2.5\% + 0.90 (6.5\%) = 8.35\%$$

<sup>5</sup>The relationship is very strong at Exxon because it has been a large and stable firm for decades. It is likely that the relationship between earnings and oil prices will be weaker at smaller, evolving oil companies.



**FIGURE 33.8** Operating Income versus Oil Prices for Exxon Mobil: 1985–2008

Exxon has \$9.4 billion of debt outstanding and a market capitalization of \$320.4 billion (4941.63 million shares, trading at \$64.83/share), resulting in a debt ratio of 2.85 percent. As an AAA-rated company, its cost of debt is expected to be 3.75 percent, reflecting a default spread of 1.25 percent over the riskfree rate. Using a marginal tax rate of 38 percent (rather than the effective tax rate) and Exxon's debt to capital ratio of 2.85%, we estimated a cost of capital of 8.18 percent for the firm.

$$\text{Cost of capital} = 8.35\%(.9715) + 3.75\%(1 - .38)(.0285) = 8.18\%$$

- Exxon Mobil is in stable growth with the operating income growing at 2 percent a year in perpetuity. New investments are expected to generate a return on capital that reflects the normalized operating income and current capital invested; this return on capital is used to compute a reinvestment rate.

Exxon reported pretax operating income in excess of \$60 billion in 2008, but that reflects the fact that the average oil price during the year was \$86.55. By March 2009, the price per barrel of oil had dropped to \$45, and the operating income for the coming year can be expected to be lower. Using the regression results, the expected operating income at this oil price is \$34,614 billion.

$$\text{Normalized Operating Income} = -6,395 + 911.32(\$45) = \$34,614$$

This operating income translates into a return on capital of approximately 21 percent and a reinvestment rate of 9.52 percent, based upon a 2 percent growth rate.<sup>6</sup>

<sup>6</sup>To compute the return on capital, we aggregated the book value of equity (\$126,044 million), the book value of debt (\$9,566 million) and netted out cash (\$33,981 million) from the end of 2007, to arrive at an invested capital value of \$101,629 million. The return on capital is computed as follows:

$$\text{Return on capital} = \text{Operating Income} (1 - \text{tax rate}) / \text{Invested Capital} = 34614 (1 - .38) / 101629 = 21.1\%$$

$$\text{Reinvestment Rate} = g/\text{ROC} = 2/21\% = 9.52\%$$

$$\begin{aligned} \text{Value of Operating Assets} &= \frac{\text{Operating Income}(1 + g)(1 - \text{tax rate})\left(1 - \frac{g}{\text{ROC}}\right)}{(\text{Cost of capital} - g)} \\ &= \frac{34614(1.02)(1 - .38)\left(1 - \frac{2\%}{21\%}\right)}{(.0818 - .02)} = \$320,472 \text{ million} \end{aligned}$$

Adding the current cash balance (\$32,007 million), subtracting out debt (\$9,400 million) and dividing by the number of shares (4,941.63 million) yields the value per share.

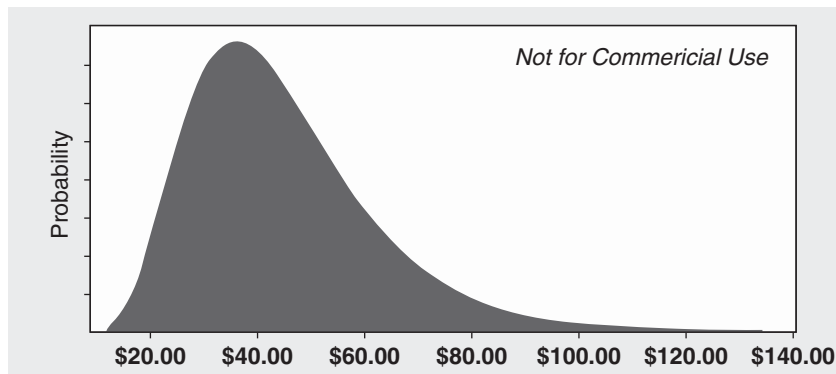
$$\begin{aligned} \text{Value per share} &= \frac{\text{Operating Assets} + \text{Cash} - \text{Debt}}{\text{Number of shares}} \\ &= \frac{320472 + 32007 - 9400}{4941.63} = \$69.43/\text{share} \end{aligned}$$

At its current stock price of \$64.83, the stock looks slightly undervalued. However, that reflects the assumption that the current oil price (of \$45) is the normalized price.

Since the value per share is so dependent on the oil price, it would make more sense to allow the oil price to vary and value the company as a function of this price and simulations can help flesh out the numbers:

**Step 1: Determine the probability distribution for the oil prices:** We used historical data on oil prices, adjusted for inflation, to both define the distribution and estimate its parameters. Figure 33.9 summarizes the distribution.

Note that oil prices can vary from about \$8 a barrel at the minimum to more than \$120 a barrel. While we have used the current price of \$45 as the mean of the distribution, we could have inserted a price view into the distribution by choosing a higher or lower mean value.<sup>7</sup>



**FIGURE 33.9** Oil Price Distribution—Historical data

<sup>7</sup>We used thirty years of historical data on oil prices, adjusted for inflation, to create an empirical distribution. We then chose the statistical distribution that seemed to provide the closest fit (lognormal) and chose parameter values that yielded numbers closest to the historical data.

Step 2: *Link the operating results to commodity price:* To link the operating income to commodity prices, we used the regression results from earlier in the illustration:

$$\text{Operating Income} = -6,395 + 911.32 (\text{Average Oil Price}) \quad R^2 = 90.2\%$$

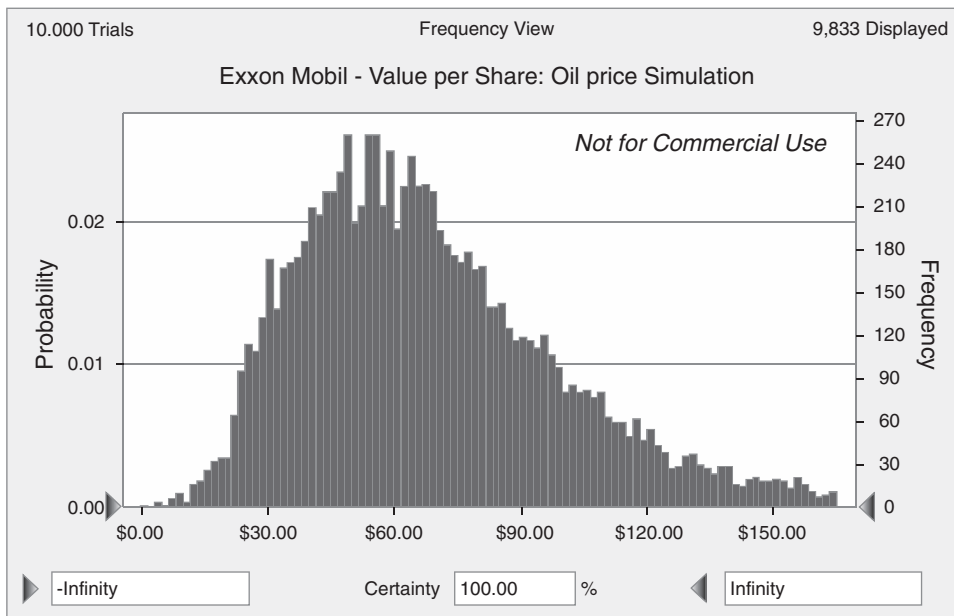
(2.95) (14.59)

As we noted in the earlier section, the regression approach works well for Exxon but may not for smaller, more volatile commodity companies.

Step 3: *Estimate the value as a function of the operating results:* As the operating income changes, there are two levels at which the value of the firm is affected. The first is that lower operating income, other things remaining equal, lowers the ~~base~~-free cash flow, and reduces value. The second is that the return on capital is recomputed, holding the capital invested fixed, as the operating income changes. As operating income declines, the return on capital drops, and the firm will have to reinvest more to sustain the stable growth rate of 2 percent. While we could also have allowed the cost of capital and the growth rate to vary, we feel comfortable with both numbers and have left them fixed.

Step 4: *Develop a distribution for the value:* We ran 10,000 simulations, letting the oil price vary and valuing the firm and equity value per share in each simulation. The results are summarized in Figure 33.10.

The average value per share across the simulations was \$69.59, with a minimum value of \$2.25 and a maximum value of \$324.42; there is, however, a greater than 50 percent chance that the value per share will be less than \$64.83 (the current stock price). Put differently, this approach yields not only the value per share for the company but also the likelihood that the firm is really under valued (less than 50%). That information can feed into the investment decision.



**FIGURE 33.9** Oil Price Distribution—Historical data

## Use in Decision Making

A well-done simulation provides us with more than just an expected value for an asset or investment.

- *Better input estimation:* In an ideal simulation, analysts will examine both the historical and cross-sectional data on each input variable before making a judgment on what distribution to use and the parameters of the distribution. In the process, they may be able to avoid the sloppiness that is associated with the use of point estimates; many discounted cash flow valuations are based upon expected growth rates that are obtained from managers or analysts, often with no basis in ~~facts~~.
- *It yields a distribution for expected value rather than a point estimate:* Consider the valuation of Exxon Mobil that we completed in the last illustration. In addition to reporting an expected value of \$69.59/share, we also estimated a standard deviation in that value and a breakdown of the values, by percentile. The distribution reinforces the obvious but important point that valuation models yield estimates of value for risky assets that are imprecise and explains why different analysts valuing the same asset may arrive at different estimates of value.

### MARGIN OF SAFETY AND SIMULATION

The margin of safety (MOS) is a measure used widely by value investors to control risk in investing. Put simply, here is how it works. A conservative value investor will require that a stock be priced more than X percent (10, 15, or 20 percent) below the value before she buys it; the margin of safety is the X percent. While intuitive, the margin of safety is not an alternative to traditional risk measures, since you need to estimate the value of a stock first, before you can use the margin of safety. It is, however, a way in which you can bring in your uncertainty and risk aversion into the decision process, with the margin of safety increasing as both increase. In practice, though, the margin of safety is often arbitrarily set and can vary widely across investors and investments.

A well-run simulation can be used to set margins of safety, since it provides information on the estimation error in value. For instance, assume that your margin of safety is 20 percent and that you are looking at Exxon Mobil in early 2009. In the Exxon Mobil simulation, for instance, where the expected value is \$69.59/share, adding this requirement would require that the stock be trading at less than \$45/share (~~the 20th percentile of the simulated values~~), for it to be a stock that you would buy. Obviously, ~~tightening (loosening) the constraint~~ would give you a lower (higher) price threshold. Furthermore, the margin of safety will be greater for firms where you are more uncertain about the future than for firms where you feel more confident in your inputs.

Note that there are two claims about simulations that we are unwilling to make. The first is that simulations yield better estimates of expected value than conventional risk-adjusted value models. In fact, the expected values from simulations should be fairly close to the expected value that we would obtain using the expected values for each of the inputs (rather than the entire distribution). The second is that simulations, by providing estimates of the expected value and the distribution in that value, lead to better decisions. This may not always be the case since the benefits that decision makers get by getting a fuller picture of the uncertainty in value in a risky asset may be more than offset by misuse of that risk measure. As we will argue later in this chapter, it is all too common for risk to be double-counted in simulations and for decisions to be based upon the wrong type of risk.

### Simulations with Constraints

There is a second use for simulations, and that is when you have a constraint, which, if violated, creates very large costs for the firm and perhaps even causes its demise. We can examine the likelihood that the constraint will be violated, given the firm's current characteristics, and the implications for value. In this section, we will look at some of these constraints.

**Book Value Constraints** The book value of equity is an accounting construct and, by itself, means little. Firms like Microsoft and Intel trade at market values that are several times their book values. At the other extreme, there are firms that trade at well below book value. In fact, there are several hundred firms in the United States, some with significant market values that have negative book values for equity. There are two types of restrictions on book value of equity that may affect value.

1. *Regulatory capital restrictions:* Financial service firms such as banks and insurance companies are required to maintain book equity as a fraction of loans or other assets at or above a floor ratio specified by the authorities. Firms that violate these capital constraints can be taken over by the regulatory authorities with the equity investors losing everything if that occurs. Not surprisingly, financial service firms not only keep a close eye on their book value of equity (and the related ratios) but they are also conscious of the possibility that the risk in their investments or positions can manifest itself as a drop in book equity. In fact, value at risk or VAR, a risk measurement device used by many financial service firms, represents the efforts to understand the potential risks in their investments and to be ready for the possibility of a catastrophic outcome, though the probability of it occurring might be very small. By simulating the values of their investments under a variety of scenarios, investors can identify not only the possibility of a bank falling below the regulatory ratios but also the valuation consequences, which can range from loss of all equity value, as a worst case, or dilution from new equity capital issues, as a best case.
2. *Negative book value for equity:* As noted, there are hundreds of firms in the United States with negative book values of equity that survive its occurrence

and have high market values for equity. There are some countries where a negative book value of equity can create substantial costs for the firm and its investors. For instance, companies with negative book values of equity in parts of Europe are required to raise fresh equity capital to bring their book values above zero. In some countries in Asia, companies that have negative book values of equity are barred from paying dividends. Even in the United States, lenders to firms can have loan covenants that allow them to gain at least partial control of a firm if its book value of equity turns negative. As with regulatory capital restrictions, investors can use simulations to assess the probability of a negative book value for equity and to capture the consequences for value.

**Market Value Constraints** In discounted cash flow valuation, the value of the firm is computed as a going concern, by discounting expected cash flows at a risk-adjusted discount rate. Deducting debt from this estimate yields equity value. The possibility and potential costs of not being able to meet debt payments is considered only peripherally in the discount rate. In reality, the costs of not meeting contractual obligations can be substantial. In fact, these costs are generally categorized as indirect bankruptcy costs and could include the loss of customers, tighter supplier credit, and higher employee turnover. The perception that a firm is in trouble can lead to further trouble. By allowing investors to compare the value of a business to its outstanding claims in all possible scenarios (rather than just the most likely one), simulations allow them to not only quantify the likelihood of distress but also build in the cost of indirect bankruptcy costs into valuation. In effect, analysts can explicitly model the effect of distress on expected cash flows and discount rates.

## Issues

The use of simulations in investment analysis was first suggested in an article by David Hertz in the Harvard Business Review.<sup>8</sup> He argued that using probability distributions for input variables, rather than single best estimates, would yield more informative output. In the example that he provided in the paper, he used simulations to compare the distributions of returns of two investments; the investment with the higher expected return also had a higher chance of losing money (which was viewed as an indicator of its riskiness). In the aftermath, there were several analysts who jumped on the simulation bandwagon, with mixed results. In recent years, there has been a resurgence in interest in simulations as a tool for risk assessment, especially in the context of derivatives. There are several key issues, though, that we have to deal with in the context of using simulations in risk assessment:

- *Garbage in, garbage out:* For simulations to have value, the distributions chosen for the inputs should be based upon analysis and data, rather than guesswork. It is worth noting that simulations yield great-looking output, even when the inputs are random. Unsuspecting decision makers may therefore be getting meaningless pictures of the risk in an investment. It is also worth noting that

<sup>8</sup>Hertz (1964) has a classic paper on the use of probabilistic approaches in decision making.



simulations require more than a passing knowledge of statistical distributions and their characteristics; analysts who ~~cannot assess~~ the difference between normal and ~~log-normal~~ distributions should not be doing simulations.

- *Real data may not fit distributions:* The problem with the real world is that the data seldom fits the stringent requirements of statistical distributions. Using a probability distribution that bears little resemblance to the true distribution underlying an input variable will yield misleading results.
- *Nonstationary distributions:* Even when the data fits a statistical distribution or where historical data distributions are available, shifts in the market structure can lead to shifts in the distribution as well. In some cases, this can change the form of the distribution and in others, it can change the parameters of the distribution. Thus, the mean and variance estimated from historical data for an input that is normally distributed may change for the next period. What we would really like to use in simulations, but seldom can assess, are forward-looking probability distributions.
- *Changing correlation across inputs:* Earlier in this chapter, we noted that correlation across input variables can be modeled into simulations. However, this works only if the correlations remain stable and predictable. To the extent that correlations between input variables change over time, it becomes far more difficult to model them.

### Risk Adjusted Value and Simulations

In our discussion of decision trees, we referred to the common misconception that decision trees are risk adjusted because they consider the likelihood of adverse events. The same misconception is prevalent in simulations, where the argument is that the cash flows from simulations are somehow risk adjusted because of the use of probability distributions and that the riskfree rate should be used in discounting these cash flows. With one exception, this argument does not make sense. Looking across simulations, the cash flows that we obtain are expected cash flows and are not risk adjusted. Consequently, we should be discounting these cash flows at a risk-adjusted rate.

The exception occurs when you use the standard deviation in values from a simulation as a measure of investment or asset risk and make decisions based upon that. In this case, using a risk-adjusted discount rate will result in a double-counting of risk. Consider a simple example. Assume that you are trying to choose between two assets, both of which you have valued using simulations and risk-adjusted discount rates. The table below summarizes your findings:

Asset	Risk-adjusted Discount Rate	Simulation: Expected Value	Simulation: Std deviation
A	12%	\$100	15%
B	15%	\$100	21%

Note that you view asset B to be riskier and have used a higher discount rate to compute value. If you now proceed to reject asset B, because the standard deviation is higher across the simulated values, you would be penalizing it twice. You can redo the simulations using the riskfree rate as the discount rate for both assets, but a note

of caution needs to be introduced. If we then base our choice between these assets on the standard deviation in simulated values, we are assuming that all risk matters in investment choice, rather than only the risk that cannot be diversified away. Put another way, we may end up rejecting an asset because it has a high standard deviation in simulated values, even though adding that asset to a portfolio may result in little additional risk (because much of its risk can be diversified away).

This is not to suggest that simulations are not useful to us in understanding risk. Looking at the variance of the simulated values around the expected value provides a visual reminder that we are estimating value in an uncertain environment. It is also conceivable that we can use it as a decision tool in portfolio management in choosing between two stocks that are equally undervalued but have different value distributions. The stock with the less-volatile value distribution may be considered a better investment than another stock with a more volatile value distribution.

## **AN OVERALL ASSESSMENT OF PROBABILISTIC RISK-ASSESSMENT APPROACHES**

Now that we have looked at scenario analysis, decision trees, and simulations, we can consider not only when each one is appropriate but also how these approaches complement or replace risk-adjusted value approaches.

### **Comparing the Approaches**

Assuming that we decide to use a probabilistic approach to assess risk and could choose between scenario analysis, decision trees, and simulations, which one should we pick? The answer will depend upon how you plan to use the output and what types of risk you are facing:

- *Selective versus full risk analysis:* In the best-case/worst-case scenario analysis, we look at only three scenarios (the best case, the most likely case, and the worst case) and ignore all other scenarios. Even when we consider multiple scenarios, we will not have a complete assessment of all possible outcomes from risky investments or assets. With decision trees and simulations, we attempt to consider all possible outcomes. In decision trees, we try to accomplish this by converting continuous risk into a manageable set of possible outcomes. With simulations, we can use distributions to capture all possible outcomes. Put in terms of probability, the sum of the probabilities of the scenarios we examine in scenario analysis can be less than one, whereas the sum of the probabilities of outcomes in decision trees and simulations has to equal one. As a consequence, we can compute expected values across outcomes in the latter, using the probabilities as weights, and these expected values are comparable to the single-estimate risk-adjusted values that we talked about in the last chapter.
- *Discrete versus continuous risk:* As noted earlier, scenario analysis and decision trees are generally built around discrete outcomes in risky events whereas simulations are better suited for continuous risks. Focusing on just scenario analysis and decision trees, the latter are better suited for sequential risks, since risk is considered in phases, whereas the former is easier to use when risks occur concurrently.
- *Correlation across risks:* If the various risks that an investment is exposed to are correlated, simulations allow for explicitly modeling these correlations (assuming that you can estimate and forecast them). In scenario analysis, we can

**TABLE 33.1** Risk Type and Probabilistic Approaches

Discrete/Continuous	Correlated/Independent	Sequential/Concurrent	Risk Approach
Discrete	Independent	Sequential	Decision Tree
Discrete	Correlated	Concurrent	Scenario Analysis
Continuous	Either	Either	Simulations

deal with correlations subjectively by creating scenarios that allow for them; the high (low) interest rate scenario will also include slower (higher) economic growth. Correlated risks are difficult to model in decision trees.

Table 33.1 summarizes the relationship between risk type and the probabilistic approach used.

Finally, the quality of the information will be a factor in your choice of approach. Since simulations are heavily dependent upon being able to assess probability distributions and parameters, they work best in cases where there is substantial historical and cross-sectional data available that can be used to make these assessments. With decision trees, you need estimates of the probabilities of the outcomes at each chance node, making them best suited for risks where these risks can be assessed, either using past data or population characteristics. Thus, it should come as no surprise that when confronted with new and unpredictable risks, analysts continue to fall back on scenario analysis, notwithstanding its slapdash and subjective ways of dealing with risk.

### Complement or Replacement for Risk-Adjusted Value

As we noted in our discussion of both decision trees and simulations, these approaches can be used as either complements to or substitutes for risk-adjusted value. Scenario analysis, on the other hand, will always be a complement to risk-adjusted value, since it does not look at the full spectrum of possible outcomes.

When any of these approaches are used as complements to risk-adjusted value, the caveats that we offered earlier in the chapter continue to apply and bear repeating. All of these approaches use expected rather than risk-adjusted cash flows and *the discount rate that is used should be a risk-adjusted discount rate*; the riskfree rate cannot be used to discount expected cash flows. In all three approaches, though, we still preserve the flexibility to change the risk-adjusted discount rate for different outcomes. Since all of these approaches will also provide a range for estimated value and a measure of variability (in terms of value at the end nodes in a decision tree or as a standard deviation in value in a simulation), it is important that we do not double-count for risk. In other words, it is patently unfair to risky investments to discount their cash flows back at a risk-adjusted rate (in simulations and decision trees) and to then reject them because the variability in value is high.

Both simulations and decision trees can be used as alternatives to risk-adjusted valuation, but there are constraints on the process. The first is that the cash flows will be discounted back at a riskfree rate to arrive at value. The second is that we now use the measure of variability in values that we obtain in both these approaches as a measure of risk in the investment. Comparing two assets with the same expected value (obtained with riskless rates as discount rates) from a simulation, we

will pick the one with the lower variability in simulated values as the better investment. If we do this, we are assuming that all of the risks that we have built into the simulation are relevant for the investment decision. In effect, we are ignoring the line drawn between risks that could have been diversified away in a portfolio and asset-specific risk on which much of modern finance is built. For investors considering investing all of their wealth in one asset, this should be reasonable. For a portfolio managers comparing two risky stocks that they are considering adding to a diversified portfolio, it can yield misleading results; the rejected stock with the higher variance in simulated values may be uncorrelated with the other investments in the portfolio and thus have little marginal risk.

The use of probabilistic approaches has become more common with the surge in data availability and computing power. It is not uncommon now to see a capital budgeting analysis, with a 20 to 30 ~~additional~~ scenarios, or a Monte Carlo simulation attached to an equity valuation.

## CONCLUSION

Estimating the risk-adjusted value for a risky asset or investment may seem like an exercise in futility. After all, the value is a function of the assumptions that we make about how the risk will unfold in the future. With probabilistic approaches to valuation, we estimate not only an expected value but also get a sense of the range of possible outcomes for value, across good and bad scenarios.

- In the most extreme form of scenario analysis, you look at the value in the best-case and worst-case scenarios and contrast them with the expected value. In its more general form, you estimate the value under a small number of likely scenarios, ranging from optimistic to pessimistic.
- Decision trees are designed for sequential and discrete risks, where the risk in an investment is considered into phases and the risk in each phase is captured in the possible outcomes and the probabilities that they will occur. A decision tree provides a complete assessment of risk and can be used to determine the optimal courses of action at each phase and an expected value for an asset today.
- Simulations provide the most complete assessments of risk since they are based upon probability distributions for each input (rather than just discrete outcomes). The output from a simulation takes the form of an expected value across simulations and a distribution for the simulated values.

With all three approaches, the keys are to avoid double counting risk (by using a risk-adjusted discount rate and considering the variability in estimated value as a risk measure) or making decisions based upon the wrong types of risk.

## QUESTIONS AND SHORT PROBLEMS

*In the problems following, use an equity risk premium of 5.5 percent if none is specified.*

1. You have estimated the value per share for Littlefield Inc., a transportation company, under three scenarios: \$5/share under the worst-case scenario, \$30/share under the best-case scenario, and \$18/share under the most likely scenario. If the stock is trading at \$15, would you buy the stock? Why or why not?

2. You are analyzing Delta Enterprises, a small publicly traded company with \$50 million of debt outstanding, and 25 million shares, trading at \$10/share. The firm generated \$40 million in after-tax cash flows last year, and you estimate that these cash flows will grow 2 percent a year in perpetuity and that the cost of capital for the firm is 12 percent.
  - a. Estimate a value per share for the firm, assuming it has no cash balance.
  - b. Now assume that you find out that a significant portion of the firm's revenues comes from one customer and that there is a 20 percent chance that this contract will be lost next year. Assuming that a lost contract will result in a 50 percent drop in the after-tax cash flows, estimate the value per share today. (You can assume that the growth rate and cost of capital will be unaffected).
3. You are valuing Signet Bank during a period of substantial uncertainty about future regulatory rules. Signet Bank generated \$100 million in net income last year on a book value of equity of \$1 billion and paid out \$70 million in dividends. While you expect Signet to be a stable growth company, you envision three regulatory scenarios for the future:
  - a. Status quo—No changes in regulatory rules (40 percent probability): Signet will continue to generate its current return on equity in perpetuity and maintain its existing dividend payout ratio.
  - b. Regulatory easing—Lower regulatory capital ratios (25 percent probability): Signet will be able to generate a return on equity of 12 percent on future investments, while maintaining its stable growth rate (at status quo levels).
  - c. Regulatory tightening—Higher regulatory capital ratios (35 percent probability): Signet will see its return on equity drop to 9 percent, while maintaining its stable growth rate (at status quo levels).
  - d. Estimate the value of equity in Signet Bank under each scenario. (You can leave current net income unchanged under all scenarios.)
  - e. Given the probabilities of each scenario unfolding, estimate the value of equity in Signet Bank today.
4. Sigma Energy is an alternative energy company that produces solar energy panels. The company is expected to generate \$50 million in after-tax operating income in the next year, but its prospects for future growth are dependent upon the level of oil prices and access to low-cost (and subsidized) government financing. You have outlined the following scenarios (with consequences for growth (g), return on invested capital (ROC) and cost of capital (r)):

	<i>Government Subsidies Continue</i>	<i>Government Subsidies End</i>
Oil prices > \$100/barrel (30% probability)	g = 4%, ROC = 12%, r = 8%	g = 4%, ROC = 12%, r = 10%
Oil prices between \$60 and \$100/barrel (50% probability)	g = 3%, ROC = 10%, r = 8%	g = 3%, ROC = 10%, r = 10%
Oil prices < \$60/barrel (20% probability)	g = 2%, ROC = 8%, r = 8%	g = 2%, ROC = 8%, r = 10%

- a. Estimate the value of the firm under each scenario. (You can leave the after-tax operating income for next year unchanged at \$50 million under all the scenarios.)

- b. If the probability that the government will end subsidies is 40 percent, estimate the value of the firm across the scenarios
- 5. Chavez Enterprises is a small, Venezuelan company. The firm is profitable and is expected to generate 120 million bolivars in after-tax operating income next year on capital invested (book value) of 600 million bolivars. You have estimated a cost of capital of 12 percent for the firm and expect it to grow 4 percent a year in perpetuity.
  - a. Estimate the value of the firm today.
  - b. Now assume that you are concerned that the firm may be nationalized. If nationalization occurs, you will be paid book value for the assets. Assuming a 30 percent probability for nationalization, estimate the value of the firm today.
- 6. Loral Drugs is a biotechnology firm that is working on a new drug to treat insomnia. The drug is working its way through the FDA approval process and is expected to generate \$150 million in after-tax cash flows each year for 15 years, once it is approved. There are two more hurdles that the drug has to cross before approval:
  - a. A small sample test on laboratory animals that will take a year to complete and is expected to cost \$100 million (to be spent today); there is an 80 percent chance that it will succeed.
  - b. If the lab animal test succeeds, it will be followed by a study on human subjects that will take two additional years to complete and cost \$250 million (to be spent at the end of year 1); there is a 60 percent chance that this study will yield favorable results.

The cost of capital for biotechnology firms is 10 percent.

- a. Outline the decision tree for the insomnia drug.
  - b. Estimate the expected value of the drug to the company today.
- 7. You are a venture capitalist who is interested in investing \$50 million in Friends Online, a social media company. The business will take three years to become operational (during which period it will generate no positive cash flows), and once operational, it will generate \$27 million in after-tax cash flows, growing 3 percent in perpetuity. The cost of capital for established social media companies is 12 percent, but there is a 60 percent chance that the business will not survive (in which case it will have no assets of value to liquidate).
  - a. Estimate the value of Friends Online, once it is operational.
  - b. Given the likelihood of failure, estimate the proportion of equity in Friends Online you would demand today, in return for your \$50 million investment. (You can assume that Friends Online has no debt or cash.)
  - c. Estimate the target return you would demand over the next three years, with the risk of failure incorporated into the return.

*Simulations: To solve the next two problems you will need access to a simulation program like Crystal Ball or @Risk.*

- 8. You are valuing Stedman Inc., a chemical firm in stable growth (growing 3 percent a year) that is expected to generate \$100 million in after-tax operating income next year. You have estimated the following:
  - a. The return on invested capital on new investments is normally distributed, with an expected value of 15 percent, with a standard deviation of 3 percent.
  - b. The cost of capital for the firm is 10 percent, uniformly distributed, with a minimum value of 8 percent and a maximum value of 12 percent.

The firm has \$500 million in debt outstanding and a cash balance of \$200 million. Develop a distribution of simulated values for Stedman Inc.

9. Simon Gold is a mature gold mining firm that has mines that are expected to generate 100,000 ounces of gold every year for the next 25 years; at the end of that period, the mines will be exhausted and will be worth nothing. The company has fixed costs of \$100 million that are expected to remain unchanged over the next 25 years and no variable costs. The cost of capital for gold mining companies is 8 percent.
  - a. If the current gold price is \$1500 an ounce and is expected to remain unchanged for the next 25 years, estimate the value of Simon Gold.
  - b. Now assume that you believe that gold prices will be normally distributed with an expected value of \$1500 an ounce and a standard deviation of \$200 an ounce. Develop a distribution of simulated values for Simon Gold.
10. You are a portfolio manager and have analysts who have used Monte Carlo simulations to find 10 undervalued companies, listed here:

<i>Company</i>	<i>Price</i>	<i>Expected Value</i>	<i>Standard Deviation</i>	<i>% Chance That the Firm Is Undervalued</i>	<i>Lowest Value</i>	<i>Highest Value</i>
A	\$ 8.00	\$ 10.00	\$ 1.00	80%	\$ 7.00	\$ 13.00
B	\$ 12.00	\$ 13.50	\$ 0.50	75%	\$10.00	\$ 16.00
C	\$ 15.00	\$ 20.00	\$ 5	50%	\$ 4.00	\$ 50.00
D	\$ 9.00	\$ 10.00	\$ 0.20	85%	\$ 8.50	\$ 13.00
E	\$ 50.00	\$ 80.00	\$10	80%	\$40.00	\$150.00
F	\$ 22.00	\$ 25.00	\$ 1	88%	\$18.00	\$ 28.00
G	\$ 3.00	\$ 5.00	\$ 0.50	70%	\$ 2.50	\$ 6.00
H	\$150.00	\$200.00	\$30	60%	\$40.00	\$500.00
I	\$ 35.00	\$ 70.00	\$20	65%	\$ 0.00	\$200.00
J	\$ 80.00	\$100.00	\$ 5	90%	\$70.00	\$115.00

- a. Purely on an expected value basis, rank these companies (from best to worst investments).
- b. If you wanted to incorporate the uncertainty in the estimates into your ranking, how would the rankings change?
- c. If you were worried about downside risk (because you are highly levered), how would you incorporate that risk into your rankings?
- d. Under what conditions might you incorporate the “highest value” into your ranking process?