

CHAPTER 15

FIRM VALUATION: COST OF CAPITAL AND APV APPROACHES

In the last two chapters, we examined two approaches to valuing the equity in the firm -- the dividend discount model and the FCFE valuation model. This chapter develops another approach to valuation where the entire firm is valued, by either discounting the cumulated cashflows to all claim holders in the firm by the weighted average cost of capital (the cost of capital approach) or by adding the marginal impact of debt on value to the unlevered firm value (adjusted present value approach).

In the process of looking at firm valuation, we also look at how leverage may or may not affect firm value. We note that in the presence of default risk, taxes and agency costs, increasing leverage can sometimes increase firm value and sometimes decrease it. In fact, we argue that the optimal financing mix for a firm is the one that maximizes firm value.

The Free Cashflow to the Firm

The free cashflow to the firm is the sum of the cashflows to all claim holders in the firm, including stockholders, bondholders and preferred stockholders. There are two ways of measuring the free cashflow to the firm (FCFF).

One is to add up the cashflows to the claim holders, which would include cash flows to equity (defined either as free cash flow to equity or dividends), cashflows to lenders (which would include principal payments, interest expenses and new debt issues) and cash flows to preferred stockholders (usually preferred dividends).

FCFF = Free Cashflow to Equity

$$+ \text{Interest Expense (1 - tax rate)} + \text{Principal Repayments} - \text{New Debt Issues} \\ + \text{Preferred Dividends}$$

Note, however, that we are reversing the process that we used to get to free cash flow to equity, where we subtracted out payments to lenders and preferred stockholders to estimate the cash flow left for stockholders. A simpler way of getting to free cash flow to the firm is to estimate the cash flows prior to any of these claims. Thus, we could begin

with the earnings before interest and taxes, net out taxes and reinvestment needs and arrive at an estimate of the free cash flow to the firm.

$$\text{FCFF} = \text{EBIT} (1 - \text{tax rate}) + \text{Depreciation} - \text{Capital Expenditure} - \text{Working Capital}$$

Since this cash flow is prior to debt payments, it is often referred to as an unlevered cash flow. Note that this free cash flow to the firm does not incorporate any of the tax benefits due to interest payments. This is by design, because the use of the after-tax cost of debt in the cost of capital already considers this benefit and including it in the cash flows would double count it.

FCFF and other cashflow measures

The differences between FCFF and FCFE arise primarily from cashflows associated with debt -- interest payments, principal repayments, new debt issues and other non-equity claims such as preferred dividends. For firms at their desired debt level, which finance their capital expenditures and working capital needs with this mix of debt and equity. As for the use of debt issues to finance principal repayments, the free cashflow to the firm will exceed the free cashflow to equity.

One measure that is widely used in valuation is the earnings before interest, taxes, depreciation and amortization (EBITDA). The free cashflow to the firm is a closely related concept but it takes into account the potential tax liability from the earnings as well as capital expenditures and working capital requirements.

Three measures of earnings are also often used to derive cash flows. The earnings before interest and taxes (EBIT) or operating income comes directly from a firm's income statements. Adjustments to EBIT yield the net operating profit or loss after taxes (NOPLAT) or the net operating income (NOI). The net operating income is defined to be the income from operations, prior to taxes and non-operating expenses.

Each of these measures is used in valuation models and each can be related to the free cashflow to the firm. Each, however, makes some assumptions about the relationship between depreciation and capital expenditures that are made explicit in the Table 15.1.

Table 15.1: Free Cash Flows to the Firm: Comparison to other measures

<i>Cashflow used</i>	<i>Definition</i>	<i>Use in valuation</i>
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FCFF	Free Cashflow to firm	Discounting free cash flow to the firm at the cost of capital will yield the value of the operating assets of the firm. To this, you would add on the value of non-operating assets to arrive at firm value.
FCFE	FCFF - Interest (1-t) – Principal repaid + New Debt Issued – Preferred Dividend	Discounting free cash flows to equity at the cost of equity will yield the value of equity in a business.
EBITDA	FCFF + EBIT(t) + Capital Expenditures + Change in working capital	If you discount EBITDA at the cost of capital to value an asset, you are assuming that there are no taxes and that the firm will actively disinvest over time. It would be inconsistent to assume a growth rate or an infinite life for this firm.
EBIT (1-t) (NOPLAT is a slightly modified version of this estimate and it removes any non-operating items that might affect the reported EBIT.)	FCFF + Capital Expenditures – Depreciation + Change in working capital	If you discount after-tax operating income at the cost of capital to value a firm, you are assuming no reinvestment. The depreciation is reinvested back into the firm to maintain existing assets. You can assume an infinite

		life but no growth.
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Growth in FCFE versus Growth in FCFF

Will equity cashflows and firm cashflows grow at the same rate? Consider the starting point for the two cash flows. Equity cash flows are based upon net income or earnings per share – measures of equity income. Firm cash flows are based upon operating income – i.e. income prior to debt payments. As a general rule, you would expect growth in operating income to be lower than growth in net income, because financial leverage can augment the latter. To see why, let us go back to the fundamental growth equations we laid out in Chapter 11.

Expected growth in net income = Equity Reinvestment rate * Return on Equity

Expected growth in operating income = Reinvestment Rate * Return on Capital

We also defined the return on equity in terms of the return on capital:

$$\text{Return on Equity} = \text{Return on Capital} + \frac{\text{Debt}}{\text{Equity}} (\text{Return on capital} - \text{After-tax cost of debt})$$

When a firm borrows money and invests in projects that earn more than the after-tax cost of debt, the return on equity will be higher than the return on capital. This, in turn, will translate into a higher growth rate in equity income at least in the short term.

In stable growth, though, the growth rates in equity income and operating income have to converge. To see why, assume that you have a firm whose revenues and operating income are growing at 5% a year forever. If you assume that the same firm’s net income grows at 6% a year forever, the net income will catch up with operating income at some point in time in the future and exceed revenues at a later point in time. In stable growth, therefore, even if return on equity exceeds the return on capital, the expected growth will be the same in all measures of income.¹

Firm Valuation: The Cost of Capital Approach

¹ The equity reinvestment rate and firm reinvestment rate will adjust to ensure that this happens. The equity reinvestment rate will be a lower number than the firm reinvestment rate in stable growth for any levered firm.

The value of the firm is obtained by discounting the free cashflow to the firm at the weighted average cost of capital. Embedded in this value are the tax benefits of debt (in the use of the after-tax cost of debt in the cost of capital) and expected additional risk associated with debt (in the form of higher costs of equity and debt at higher debt ratios). Just as with the dividend discount model and the FCFE model, the version of the model used will depend upon assumptions made about future growth.

Stable Growth Firm

As with the dividend discount and FCFE models, a firm that is growing at a rate that it can sustain in perpetuity – a stable growth rate – can be valued using a stable growth model.

The Model

A firm with free cashflows to the firm growing at a stable growth rate can be valued using the following equation:

$$\text{Value of firm} = \frac{\text{FCFF}_1}{\text{WACC} - g_n}$$

where,

FCFF₁ = Expected FCFF next year

WACC = Weighted average cost of capital

g_n = Growth rate in the FCFF (forever)

The Caveats

There are two conditions that need to be met in using this model. First, the growth rate used in the model has to be less than or equal to the growth rate in the economy – nominal growth if the cost of capital is in nominal terms, or real growth if the cost of capital is a real cost of capital. Second, the characteristics of the firm have to be consistent with assumptions of stable growth. In particular, the reinvestment rate used to estimate free cash flows to the firm should be consistent with the stable growth rate. The best way of enforcing this consistency is to derive the reinvestment rate from the stable growth rate.

$$\text{Reinvestment rate in stable growth} = \frac{\text{Growth rate}}{\text{Return on capital}}$$

If reinvestment is estimated from net capital expenditures and change in working capital, the net capital expenditures should be similar to those other firms in the industry (perhaps by setting the ratio of capital expenditures to depreciation at industry averages) and the change in working capital should generally not be negative. A negative change in working capital creates a cash inflow and while this may, in fact, be viable for a firm in the short term, it is dangerous to assume it in perpetuity.² The cost of capital should also be reflective of a stable growth firm. In particular, the beta should be close to one – the rule of thumb presented in the earlier chapters that the beta should be between 0.8 and 1.2 still holds. While stable growth firms tend to use more debt, this is not a pre-requisite for the model, since debt policy is subject to managerial discretion.

Limitations

Like all stable growth models, this one is sensitive to assumptions about the expected growth rate. This is accentuated, however, by the fact that the discount rate used in valuation is the WACC, which is significantly lower than the cost of equity for most firms. Furthermore, the model is sensitive to assumptions made about capital expenditures relative to depreciation. If the inputs for reinvestment are not a function of expected growth, the free cashflow to the firm can be inflated (deflated) by reducing (increasing) capital expenditures relative to depreciation. If the reinvestment rate is estimated from the return on capital, changes in the return on capital can have significant effects on firm value.

Illustration 15.1: Valuing a firm with a stable growth FCFF Model: Tube Investments of India (TI)

Tube Investments of India is a diversified manufacturing firm, with its headquarters in South India. In 1999, the firm reported operating income of Rs. 632.2 million and paid faced a tax rate of 30% on income. The firm had a book value of equity of

² Carried to its logical extreme, this will push net working capital to a very large (potentially infinite) negative number.

Rs 3432.1 million rupees and book value of debt of Rs. 1377.2 million at the end of 1998.

The firm's return on capital can be estimated as follows:

$$\begin{aligned} \text{Return on capital} &= \frac{\text{EBIT}(1-t)}{\text{Book value of debt} + \text{Book value of Equity}} \\ &= \frac{632.2(1-0.30)}{3432.1 + 1377.2} = 9.20\% \end{aligned}$$

The firm is in stable businesses and expects to grow only 5% a year.³ Assuming that it maintains its current return on capital, the reinvestment rate for the firm will be:

$$\text{Reinvestment rate} = \frac{g}{\text{ROC}} = \frac{5\%}{9.20\%} = 54.34\%$$

The firm's expected free cash flow to the firm next year can be estimated as follows:

$$\begin{aligned} \text{Expected EBIT (1-t) next year} &= 632.2 (1-0.30) (1.05) &&= 464.7 \\ \text{- Expected Reinvestment next year} &= \text{EBIT}(1-t) (\text{Reinvestment rate}) \\ &= 464.7 (0.5435) &&= 252.5 \\ \text{Expected Free Cash flow to the firm} &&&= 212.2 \end{aligned}$$

To estimate the cost of capital, we use a bottom-up beta (adjusted to 1.17 to reflect TI's additional leverage), a nominal rupee riskfree rate of 10.50% and a risk premium of 9.23% (4% for the mature market premium and 5.23% for country risk in India). The cost of equity can then be estimated as follows:

$$\text{Cost of Equity} = 10.5\% + 1.17 (9.23\%) = 21.30\%$$

The cost of debt for Tube Investments is 12%, which in conjunction with their market debt to capital ratio of 44.19% - the market value of equity at the time of the valuation was Rs.2282 million and the market value of debt was Rs. 1807.3 million - yields a cost of capital of 15.60%:

$$\begin{aligned} \text{Cost of capital} &= (\text{Cost of Equity}) \frac{E}{D+E} + (\text{After-tax Cost of Debt}) \frac{D}{D+E} \\ &= (21.30\%)(0.5581) + (12\%)(1-0.3)(0.4419) = 15.60\% \end{aligned}$$

With the perpetual growth of 5%, the expected free cash flow to the firm shown above (Rs 212.2 million) and the cost of capital of 15.60%, we obtain a value for the firm of:

³ Note that while this resembles growth rates we have used for other firms, it is a low growth rate given that this valuation is in Indian rupees. As a simple check, note that the riskfree rate that we use is 10.50%.

$$\text{Value of the operating assets of firm} = \frac{212.2}{0.156 - 0.05} = \text{Rs } 2002 \text{ million}$$

Adding back cash and marketable securities with a value of Rs 1365.3 million and subtracting out the debt outstanding of Rs 1807.3 million yields a value for the equity of Rs 1560 million and a value per share of Rs. 63.36 (based upon the 24.62 million shares outstanding). The stock was trading at Rs 92.70 at the time of this valuation.

An interesting aspect of this valuation is that the return on capital used to compute the reinvestment rate is significantly lower than the cost of capital. In other words, we are locking in this firm into investing in negative excess return projects forever. If we assume that the firm will find a way to earn its cost of capital of 15.6% on investments, the reinvestment rate would be much lower.

$$\text{Reinvestment rate}_{\text{ROC}=\text{Cost of capital}} = \frac{g}{\text{ROC}} = \frac{0.05}{0.156} = 32.05\%$$

$$\text{Value of operating assets} = (464.7) \frac{1 - 0.3205}{0.1560 - 0.05} = \text{Rs. } 2979 \text{ million}$$

$$+ \text{ Value of cash and marketable securities} = \text{Rs } 1365 \text{ million}$$

$$- \text{ Debt} = \text{Rs } 1807 \text{ million}$$

$$\text{Value of equity} = \text{Rs } 2537 \text{ million}$$

$$\text{Value per share} = \frac{2537}{24.62} = \text{Rs } 103.04 \text{ per share}$$

Market Value Weights, Cost of Capital and Circular Reasoning

To value a firm, you first need to estimate a cost of capital. Every textbook is categorical that the weights in the cost of capital calculation be market value weights. The problem, however, is that the cost of capital is then used to estimate new values for debt and equity that might not match the values used in the original calculation. One defense that can be offered for this inconsistency is that if you went out and bought all of the debt and equity in a publicly traded firm, you would pay current market value and not your estimated value and your cost of capital reflects this.

To those who are bothered by this inconsistency, there is a way out. You could do a conventional valuation using market value weights for debt and equity, but then use the estimated values of debt and equity from the valuation to re-estimate the cost of

capital. This, of course, will change the values again, but you could feed the new values back and estimate cost of capital again. Each time you do this, the differences between the values you use for the weights and the values you estimate will narrow, and the values will converge sooner rather than later.

How much of a difference will it make in your ultimate value? The greater the difference between market value and your estimates of value, the greater the difference this iterative process will make. In the valuation of Tube Investments above, we began with a market price of Rs 92.70 per share and estimated a value of Rs 63.36. If we substituted back this estimated value and iterated to a solution, we would arrive at an estimate of value of Rs 70.66 per share.⁴

The General Version of the FCFF Model

Rather than break the free cash flow model into two-stage and three-stage models and risk repeating what was said in the last chapter, we present the general version of the model in this section. We follow up by examining a range of companies – a traditional manufacturing firm, a firm with operating leases and a firm with substantial R&D investments – to illustrate the differences and similarities between this approach and the FCFE approach.

The Model

The value of the firm, in the most general case, can be written as the present value of expected free cashflows to the firm.

$$\text{Value of Firm} = \sum_{t=1}^{t=\infty} \frac{\text{FCFF}_t}{(1 + \text{WACC})^t}$$

where,

FCFF_t = Free Cashflow to firm in year t

WACC = Weighted average cost of capital

⁴ In Microsoft Excel, it is easy to set this process up. You should first go into calculation options and put a check in iteration box. You can then make the cost of capital a function of your estimated values for debt and equity.

If the firm reaches steady state after n years and starts growing at a stable growth rate g_n after that, the value of the firm can be written as:

$$\text{Value of Firm} = \sum_{t=1}^{t=n} \frac{\text{FCFF}_t}{(1 + \text{WACC})^t} + \frac{[\text{FCFF}_{n+1} / (\text{WACC} - g_n)]}{(1 + \text{WACC})^n}$$

Best suited for:

Firms that have very high leverage or are in the process of changing their leverage are best valued using the FCFE approach. The calculation of FCFE is much more difficult in these cases because of the volatility induced by debt payments (or new issues) and the value of equity, which is a small slice of the total value of the firm, is more sensitive to assumptions about growth and risk. It is worth noting, though, that in theory, the two approaches should yield the same value for the equity. Getting them to agree in practice is an entirely different challenge and we will return to examine it later in this chapter.

Best suited for:

There are three problems that we see with the free cash flow to the firm model. The first is that the free cash flows to equity are a much more intuitive measure of cash flows than cash flows to the firm. When asked to estimate cash flows, most of us look at cash flows after debt payments (free cash flows to equity), because we tend to think like business owners and consider interest payments and the repayment of debt as cash outflows. Furthermore, the free cash flow to equity is a real cash flow that can be traced and analyzed in a firm. The free cash flow to the firm is the answer to a hypothetical question: What would this firm's cash flow be, if it had no debt (and associated payments)?

The second is that its focus on pre-debt cash flows can sometimes blind us to real problems with survival. To illustrate, assume that a firm has free cash flows to the firm of \$100 million but because of its large debt load makes the free cash flows to equity equal to -\$50 million. This firm will have to raise \$50 million in new equity to survive and, if it cannot, all cash flows beyond this point are put in jeopardy. Using free cash flows to equity would have alerted you to this problem, but free cash flows to the firm are unlikely to reflect this.

The final problem is that the use of a debt ratio in the cost of capital to incorporate the effect of leverage requires us to make implicit assumptions that might not be feasible or reasonable. For instance, assuming that the market value debt ratio is 30% will require a growing firm to issue large amounts of debt in future years to reach that ratio. In the process, the book debt ratio might reach stratospheric proportions and trigger covenants or other negative consequences. In fact, we count the expected tax benefits from future debt issues implicitly into the value of equity today.

Illustration 15.2: Valuing The Gap: Dealing with Operating Leases

The Gap is one of the largest specialty retailers in the world and sells its products at Gap, GapKids, babyGap, Banana Republic and Old Navy stores. While it has operations around the world, it gets the bulk of its revenues from the United States.

Rationale for using Model

- *Why two-stage?* While the Gap is one of the largest and most successful specialty retailers in the world, its dependence on the mature U.S. market for growth restricts its capacity to maintain high growth in the future. We will assume a high growth period of 5 years and then put the firm into stable growth.
- *Why FCF?* The Gap has a significant operating lease commitments and the firm has increased its leverage aggressively over the last few years.

Background Information

In 2000, the Gap reported operating income \$1,445 million on revenues of \$13,673 million. The firm also reported capital expenditures of \$1,859 million and depreciation of \$590 million for the year, and its non-cash working capital increased by \$323 million during the year. The operating lease expenses for the year were \$705.8 million and Table 15.2 reports the lease commitments for future years.

Table 15.2: Lease Commitments for future years: The Gap

<i>Year</i>	<i>Commitment</i>
1	\$774.60
2	\$749.30
3	\$696.50
4	\$635.10

5	\$529.70
6 and beyond	\$5,457.90

To convert these operating lease expenses into debt, we first compute a pre-tax cost of debt for the firm based upon its rating of A. The default spread for A rated firms is 1.80%, which when added to the riskfree rate of 5.4%, yields a pre-tax cost of debt of 7.2%. Treating the commitment in year 6 and beyond as an annuity of \$682.24 million for 8 years, we estimate a debt value for the operating leases in Table 15.3.

Table 15.3: Present value of lease commitments

<i>Year</i>	<i>Commitment</i>	<i>Present Value</i>
1	\$ 774.60	\$722.57
2	\$ 749.30	\$652.03
3	\$ 696.50	\$565.38
4	\$ 635.10	\$480.91
5	\$ 529.70	\$374.16
6 and beyond	\$ 682.24	\$2,855.43
Debt Value of leases =		\$5,650.48

This amount is added on to the debt outstanding on the balance sheet of \$1,809.90 million to arrive at a total value for debt of \$7,460.38 million. The Gap's market value of equity at the time of this valuation was \$28,795 million, yielding a market debt to capital ratio of:

$$\begin{aligned} \text{Market Debt to Capital} &= \frac{\text{Debt}}{\text{Debt} + \text{Market value of equity}} \\ &= \frac{7,460}{7,460 + 28,795} = 20.58\% \end{aligned}$$

The operating income is also adjusted to reflect this shift by adding the imputed interest expense on the debt value of operating leases:

Adjusted Operating Income

$$= \text{Operating Income} + \text{Debt value of operating leases} * \text{Pre-tax cost of debt}$$

$$= 1445 + 5650 * 0.072 = \$1,851 \text{ million}$$

Multiplying by (1- tax rate), using a marginal tax rate of 35%, we get an after-tax operating income of \$1,203 million.

Adjusted after-tax operating income
 = Adjusted Operating Income (1- tax rate)
 = 1851 (1-0.35) = \$1,203 million

Dividing this value by the book value of debt (including capitalized operating leases) and the book value of equity at the end of the previous year yields an adjusted return on capital of 13.61% in 2000 for the firm.

$$\begin{aligned} \text{ROC}_{2000} &= \frac{\text{EBIT}_{2000} (1-t)}{\text{BV of Debt}_{1999} + \text{BV of Equity}_{1999}} \\ &= \frac{1203}{6604 + 2233} = 13.61\% \end{aligned}$$

We will assume that the firm will be able to maintain this return on capital in perpetuity.

Valuation

We will begin with a cost of equity estimate for the Gap, using a bottom-up beta of 1.20 (based upon the betas of specialty retailers) for the high growth period, a riskfree rate of 5.4% and a mature market premium of 4%. In stable growth, we will lower the beta to 1.00, keeping the riskfree rate and risk premium unchanged.

Cost of equity_{High Growth} = 5.4% + 1.2 (4%) = 10.2%

Cost of equity_{Stable Growth} = 5.4% + 1.0 (4%) = 9.4%

To estimate the cost of capital during the high growth and stable growth phases, we will assume that the pre-tax cost of debt will remain at 7.2% in perpetuity and that the current market debt ratio of 20.58% will remain the debt ratio.

Cost of capital_{High Growth} = 10.2%(0.7942)+ 7.2% (1-0.35)(0.2058) = 9.06%

Cost of capital_{Stable Growth} = 9.4%(0.7942)+ 7.2% (1-0.35)(0.2058) = 8.43%

To estimate the expected growth in operating earnings during the high growth period, we will assume that the firm will continue to earn 13.61% as its return on capital and that its reinvestment rate will equal its average reinvestment rate over the last 4 years.⁵

Average reinvestment rate over last 4 years = 93.53%

Expected Growth rate = Reinvestment rate * Return on Capital = 0.9353*0.1361 = 12.73%

⁵ The Gap has had volatile capital expenditures and working capital changes. This is our attempt to average out this volatility.

Table 15.4 summarizes the expected cash flows for the high growth period.

Table 15.4: Estimated FCFF: The Gap

Year	EBIT(1-t)	Reinvestment rate	Reinvestment	FCFF	Present Value
Current	\$1,203				
1	\$1,356	93.53%	\$1,269	\$88	\$80
2	\$1,529	93.53%	\$1,430	\$99	\$83
3	\$1,732	93.53%	\$1,620	\$112	\$86
4	\$1,952	93.53%	\$1,826	\$126	\$89
5	\$2,190	93.53%	\$2,049	\$142	\$92
Sum of present values of cash flows =					\$430

Note that the cash flows during the high growth period are discounted back at 9.06%. To estimate the terminal value at the end of year 5, we assume that this cash flow will grow forever at 5%. The reinvestment rate can then be estimated and used to measure the free cash flow to the firm in year 6:

Expected growth rate = 5%

$$\text{Reinvestment rate in stable growth} = \frac{g}{\text{Stable period ROC}} = \frac{5\%}{13.61\%} = 36.73\%$$

$$\begin{aligned} \text{FCFF}_6 &= \text{EBIT}_5 (1-t) (1 + g_{\text{Stable Period}}) (1 - \text{Reinvestment Rate}) \\ &= 2190 (1.05) (1 - 0.3673) = 1455 \end{aligned}$$

The terminal value is:

$$\begin{aligned} \text{Terminal value} &= \frac{\text{FCFF}_{11}}{\text{Cost of capital in stable growth} - \text{Growth rate}} \\ &= \frac{1455}{0.0843 - 0.05} = \$42,441 \text{ million} \end{aligned}$$

Discounting the terminal value to the present and adding it to the present value of the cash flows over the high growth period yields a value for the operating assets of the firm.

Value of Operating assets = PV of cash flows during high growth + PV of terminal value

$$= \$430 + \frac{\$42,441}{1.0906^5} = \$27,933 \text{ million}$$

Adding back the firm's cash and marketable securities (estimated to be \$409 million at the end of 2000) and subtracting out the value of the debt yields a value for the equity in the firm:

Value of the equity

= Value of the operating assets + Cash and Marketable securities – net debt

= 27,933 + 409 – 7460 = \$20,882 million

Note that the debt subtracted out includes the present value of operating leases. At its prevailing market value of equity of \$27,615 million, the Gap is overvalued.

Illustration 15.3: Valuing Amgen: Effects of R&D

As a leading biotechnology firm, Amgen has substantial research and development expenses and we capitalized these expenses earlier in this book. In this valuation, we will consider the implications of this capitalization for firm and equity values.

Rationale for using Model

- *Why three-stage?* Amgen, in spite of being one of the largest biotechnology firms has significant potential for future growth, both because of drugs that it has in commercial production and other drugs in the pipeline. We will assume that the firm will continue to grow for 10 years, five at a high growth rate followed by five year in transition to stable growth.
- *Why FCFF?* The firm has little debt on its books currently but will come under increasing pressure to increase its leverage as its cash flows become larger and more stable.

Background Information

In 2000, Amgen reported operating income \$1,549 million on revenues of \$3,629 million. The firm also reported capital expenditures of \$437 million and depreciation of \$212 million for the year, and its non-cash working capital increased by \$146 million during the year. Recapping the analysis of Amgen's R&D from Chapter 9, we will use a 10-year amortizable life to estimate the value of the research asset in Table 15.5.

Table 15.5: Capitalizing the Research Asset

Year	R&D Expense	Unamortized portion	Amortization this year

Current	845.00	1.00	845.00	
-1	822.80	0.90	740.52	\$82.28
-2	663.30	0.80	530.64	\$66.33
-3	630.80	0.70	441.56	\$63.08
-4	528.30	0.60	316.98	\$52.83
-5	451.70	0.50	225.85	\$45.17
-6	323.63	0.40	129.45	\$32.36
-7	255.32	0.30	76.60	\$25.53
-8	182.30	0.20	36.46	\$18.23
-9	120.94	0.10	12.09	\$12.09
-10	0.00	0.00	0.00	\$0.00
Value of Research Asset =			\$3,355.15	\$397.91

The operating income is adjusted by adding back the current year's R& D expense and subtracting out the amortization of the research asset.

Adjusted operating income

= Operating income + Current year's R&D – Amortization of Research asset

= \$1,549+ \$845 - \$398 = \$1996 million

To get to the after-tax operating income, we also consider the tax benefits from expensing R&D (as opposed to just the amortization of the research asset).

Adjusted after-tax operating income

= Adjusted Operating Income (1- tax rate) + (Current year R&D – Amortization) Tax rate

= 1996 (1-0.35) + (845-398) (0.35) = \$1,454 million

The current year's R&D expense is added to the capital expenditures for the year, and the amortization to the depreciation. In conjunction with an increase in working capital of \$146 million, we estimate an adjusted reinvestment rate for the firm of 56.27%.

Adjusted Capital expenditures = 437+ 845 = \$1,282 million

Adjusted Depreciation = 212 + 398 = \$610 million

Adjusted Reinvestment rate

$$= \frac{\text{Capital Expenditures} - \text{Depreciation} + \text{WC}}{\text{Adjusted EBIT}(1 - t)}$$

$$= \frac{1282 - 610 + 146}{1454} = 56.27\%$$

To estimate the return on capital, we estimated the value of the research asset at the end of the previous year and added it to the book value of equity. The resultant return on capital for the firm is shown.

Return on capital

$$= \frac{\text{Adjusted EBIT}(1 - t)}{\text{Adjusted book value of equity (includes research asset)} + \text{Book value of debt}}$$

$$= \frac{1454}{5932 + 323} = 23.24\%$$

Valuation

To value Amgen, we will begin with the estimates for the 5-year high growth period. We use a bottom-up beta estimate of 1.35, a riskfree rate of 5.4% and a risk premium of 4% to estimate the cost of equity:

$$\text{Cost of equity} = 5.4\% + 1.35(4\%) = 10.80\%$$

We estimate a synthetic rating of AAA for the firm, and use it to come up with a pre-tax cost of borrowing of 6.15% by adding a default spread of 0.75% to the treasury bond rate of 5.4%. With a marginal tax rate of 35% and a debt ratio of 0.55%, the firm's cost of capital closely tracks its cost of equity.

$$\text{Cost of capital} = 13.08\% (0.9945) + 6.15\%(1-0.35)(0.0055) = 10.76\%$$

To estimate the expected growth rate during the high growth period, we will assume that the firm can maintain its current return on capital and reinvestment rate estimated in the section above.

$$\text{Expected Growth rate} = \text{Reinvestment rate} * \text{Return on capital} = 0.5627 * 0.2324 = 13.08\%$$

Before we consider the transition period, we estimate the inputs for the stable growth period. First, we assume that the beta for Amgen will drop to 1, and that the firm will raise its debt ratio to 10%. Keeping the cost of debt unchanged, we estimate a cost of capital of

$$\text{Cost of equity} = 5.4\% + 1(4\%) = 9.4\%$$

$$\text{Cost of capital} = 9.4\% (0.9) + 6.15\% (1-0.35) (0.1) = 8.86\%$$

We assume that the stable growth rate will be 5% and that the firm will have a return on capital of 20% in stable growth. This allows us to estimate the reinvestment rate in stable growth.

$$\text{Reinvestment rate in stable growth} = \frac{g}{\text{ROC}} = \frac{5\%}{20\%} = 25\%$$

During the transition period, we adjust growth, reinvestment rate and the cost of capital from high growth levels to stable growth levels in linear increments. Table 15.6 summarizes the inputs and cash flows for both the high growth and transition period.

Table 15.6: Free Cashflows to Firm: Amgen

<i>Year</i>	<i>Expected Growth</i>	<i>EBIT(1-t)</i>	<i>Reinvestment Rate</i>	<i>FCFF</i>	<i>Cost of Capital</i>	<i>Present Value</i>
Current		\$1,454				
1	13.08%	\$1,644	56.27%	\$719	10.76%	\$649
2	13.08%	\$1,859	56.27%	\$813	10.76%	\$663
3	13.08%	\$2,102	56.27%	\$919	10.76%	\$677
4	13.08%	\$2,377	56.27%	\$1,040	10.76%	\$691
5	13.08%	\$2,688	56.27%	\$1,176	10.76%	\$705
6	11.46%	\$2,996	50.01%	\$1,498	10.38%	\$814
7	9.85%	\$3,291	43.76%	\$1,851	10.00%	\$914
8	8.23%	\$3,562	37.51%	\$2,226	9.62%	\$1,003
9	6.62%	\$3,798	31.25%	\$2,611	9.24%	\$1,077
10	5.00%	\$3,988	25.00%	\$2,991	8.86%	\$1,133
Sum of the present value of the FCFF during high growth =						\$8,327 m

Finally, we estimate the terminal value, based upon the growth rate, cost of capital and reinvestment rate estimated above.

$$\text{FCFF}_{11} = \text{EBIT}_{11} (1-t)(1 - \text{Reinvestment rate}) = 3988(1.05)(1 - 0.25) = \$3,140 \text{ million}$$

$$\begin{aligned} \text{Terminal value}_{10} &= \frac{\text{FCFF}_{11}}{\text{Cost of capital in stable growth} - \text{Growth rate}} \\ &= \frac{3140}{0.0886 - 0.05} = \$81,364 \text{ million} \end{aligned}$$

Adding the present value of the terminal value to the present value of the free cash flows to the firm in the first 10 years, we get:

Value of the operating assets of the firm

$$= \$8,327 \text{ million} + \frac{\$81,364}{(1.1076^5)(1.1038)(1.10)(1.0962)(1.0924)(1.0886)}$$

$$= \$39,161 \text{ million}$$

Adding the value of cash and marketable securities (\$2,029 million) and subtracting out debt (\$323 million) yields a value for the equity of \$40,867 million. At the time of this valuation in May 2001, the equity was trading at a market value of \$58,000 million.

Illustration 15.4: Valuing Embraer: Dealing with Country Risk

Embraer is a Brazilian aerospace firm that manufactures and sells both commercial and military aircraft. In this valuation, we will consider the implications of valuing the firm in the context of country risk and uncertainty about expected inflation.

Rationale for using Model

- *Why two-stage?* Embraer has done exceptionally well in the last few years though it operates in a mature business with strong competition from giants such as Boeing and Airbus. We believe that it can sustain growth for a long period (10 years) and that there will be a transition to stable growth in the second half of this growth period.
- *Why FCF?* The firm's debt ratio has been volatile. While it does not use much debt to fund its operations currently, it does have the capacity to raise more debt now, especially in the United States.
- *Why real cash flows?* We had two choices when it came to valuation – to work with U.S. dollars or work in real cash flows. We avoided working with nominal real, largely because of the difficulties associated with getting a riskfree rate in that currency.

Background Information

In 2000, Embraer reported operating income of 810.32 million BR on revenues of 4560 million BR, and faced a marginal tax rate of 33% on its income. At the end of 2000, the firm had net debt (debt minus cash) of 215.5 million BR on which their net interest expenses for 2000 were 28.20 million BR. The firm's non-cash working capital at the end

of 2000 amounted to 915 million BR, an increase of 609.7 million BR over the previous year's amount.

The firm's capital expenditures were 233.5 million BR and depreciation was 127.5 million for the year, yielding a reinvestment rate of 131.83% for the year.

$$\text{Reinvestment Rate}_{2000} = \frac{233.5 - 127.5 + 609.7}{810.32(1 - 0.33)} = 131.83\%$$

Normalizing the non-cash working capital component⁶ yields a change in non-cash working capital of 239.59 million BR and a normalized reinvestment rate.

$$\text{Normalized Reinvestment Rate}_{2000} = \frac{233.5 - 127.5 + 239.59}{810.32(1 - 0.33)} = 63.65\%$$

Based upon the capital invested of 1,470 million BR in the firm at the beginning of 2000, the return on capital at Embraer in 2000 was 36.94%.

$$\text{Return on capital} = \frac{810.32(1 - 0.33)}{1470} = 36.94\%$$

Valuation

We first have to estimate a country risk premium for Brazil. Drawing on the approach developed in Chapter 7, we estimate a country risk premium for Brazil of 10.24%.

Country rating for Brazil = B1

Default spread on Brazilian Government C-bond (U.S. dollar denominated) = 5.37%

To estimate the country equity risk premium, we estimated the standard deviation in weekly returns over the last 2 years in both the Bovespa (the Brazilian equity index) and the C-Bond.

Standard deviation in the Bovespa = 32.6%

Standard deviation in the C-Bond = 17.1%

$$\begin{aligned} \text{Country risk premium} &= (\text{Default Spread}) \frac{\text{Standard Deviation}_{\text{Equity}}}{\text{Standard Deviation}_{\text{C-Bond}}} \\ &= (5.37\%) \frac{32.6\%}{17.1\%} = 10.24\% \end{aligned}$$

⁶ The normalized change in non-cash working capital was computed as follows:
 Normalized change = (Non-cash WC₂₀₀₀/Revenues₂₀₀₀)*(Revenues₂₀₀₀-Revenues₁₉₉₉)

To make an estimate of Embraer's beta, we used a bottom-up unlevered beta of 0.87 and Embraer's market net debt to equity ratio (to stay consistent with our use of net debt in the valuation) of 2.45%.

$$\text{Levered beta} = 0.87 (1 + (1-0.33) (0.0245)) = 0.88$$

Finally, to estimate the cost of equity, we used a real riskless rate of 4.5% and a mature market risk premium of 4% (in addition to the country risk premium of 10.24%):

$$\text{Cost of equity} = 4.5\% + 0.88 (4\%+10.24\%) = 17.03\%$$

We estimate a synthetic rating of AAA for Embraer and use it to come up with a pre-tax cost of borrowing of 10.62% by adding a default spread of 0.75% to the real riskless rate of 4.5%, and then adding the country default spread of 5.37%.⁷

$$\text{Pre-tax cost of debt} = \text{Real riskfree rate} + \text{Country default spread} + \text{Company default spread} = 4.5\% + 5.37\% + 0.75\% = 10.62\%$$

With a marginal tax rate of 33% and a net debt to capital ratio of 2.40%, the firm's cost of capital is shown below:

$$\text{Cost of capital} = 17.03\% (0.976) + 0.1062(1-0.33)(0.024) = 16.79\%$$

To estimate the expected growth rate during the high growth period, we will assume that the firm can maintain its current return on capital and use the normalized reinvestment rate.

$$\begin{aligned} \text{Expected Growth rate} &= \text{Normalized Reinvestment rate} * \text{Return on capital} \\ &= 0.6365 * 0.3694 = 23.51\% \end{aligned}$$

In stable growth, we assume that the beta for Embrarer will rise slightly to 0.90, that its net debt ratio will remain unchanged at 2.40% and that the country risk premium will drop to 5.37%. We also assume that the pre-tax cost of debt will decline to 7.50%

$$\text{Cost of equity} = 4.5\% + 0.9(4\%+ 5.37\%) = 12.93\%$$

$$\text{Cost of capital} = 12.93\% (0.976) + 7.5\% (1-0.33) (0.024) = 12.74\%$$

We assume that the stable real growth rate will be 3% and that the firm will have a return on capital of 15% in stable growth. This is a significant drop from its current return on

⁷ This is a conservative estimate. It is entirely possible that the market will not assess Embraer with all of the country risk and may view Embraer as safer than the Brazilian government.

capital but reflect the returns of more mature firms in the business. This allows us to estimate the reinvestment rate in stable growth

$$\text{Reinvestment rate in stable growth} = \frac{g}{\text{ROC}} = \frac{3\%}{15\%} = 20\%$$

During the transition period, we adjust growth, reinvestment rate and the cost of capital from high growth levels to stable growth levels in linear increments. Table 15.6 summarizes the inputs and cash flows for both the high growth and transition period.

Table 15.7: Free Cashflows to Firm: Embraer

<i>Year</i>	<i>Expected Growth</i>	<i>EBIT(1-t)</i>	<i>Reinvestment Rate</i>	<i>FCFF</i>	<i>Cost of Capital</i>	<i>Present Value</i>
Current		BR 543				
1	23.51%	BR 671	63.65%	BR 244	16.79%	BR 209
2	23.51%	828	63.65%	301	16.79%	221
3	23.51%	1,023	63.65%	372	16.79%	233
4	23.51%	1,264	63.65%	459	16.79%	247
5	23.51%	1,561	63.65%	567	16.79%	261
6	19.41%	1,864	54.92%	840	15.98%	333
7	15.31%	2,149	46.19%	1,156	15.17%	398
8	11.21%	2,390	37.46%	1,495	14.36%	450
9	7.10%	2,559	28.73%	1,824	13.55%	484
10	3.00%	2,636	20.00%	2,109	12.74%	496
Sum of the present value of the FCFF during high growth =						BR 3,333 m

Finally, we estimate the terminal value, based upon the growth rate, cost of capital and reinvestment rate estimated.

$$\text{FCFF}_{11} = \text{EBIT}_{11} (1 - t) (1 - \text{Reinvestment rate}) = 2636 (1.03) (1 - 0.2) = 2172 \text{ million BR}$$

$$\begin{aligned} \text{Terminal value}_{10} &= \frac{\text{FCFF}_{11}}{\text{Cost of capital in stable growth} - \text{Growth rate}} \\ &= \frac{2172}{0.1274 - 0.03} = 22,295 \text{ million BR} \end{aligned}$$

Adding the present value of the terminal value to the present value of the free cash flows to the firm in the first 10 years, we get:

Value of the operating assets of the firm

$$= 3,333 \text{ million BR} + \frac{22,295}{(1.1679)^5 (1.1598)(1.1517)(1.1436)(1.1355)(1.1274)}$$

$$= 8,578 \text{ million BR}$$

We do not add back cash and marketable securities, because we are using net debt (and the cash has therefore already been netted out against debt). Adding the value of non-operating assets (\$510 million) and subtracting out net debt (\$223 million) yields a value for the equity of 8,865 million BR and a per-share value of 14.88 BR. At the time of this valuation in March 2001, the equity was trading at a market price of 15.2 BR per share.

Doing a valuation is only the first part of the process. Presenting it to others is the second and perhaps just as important. Valuations can be complicated and it is easy to lose your audience (and yourself) in the details. Presenting a big picture of the valuation often helps. In Figure 15.1, for instance, the valuation of Embraer is presented in a picture. The valuation contains all of the details presented in the Amgen and Gap valuations but they are presented both in a more concise format and the connections between the various inputs are much more visible.

 *fcffginzu.xls*: This spreadsheet allows you to estimate the value of a firm using the FCFF approach.

Net Debt versus Gross Debt

In valuing Embrarer, we used net debt where cash was netted out against debt. In all of the earlier valuations, we used gross debt. What is the difference between the two approaches and will the valuations from the two approaches agree?

A comparison of the Embraer and the earlier valuations reveals the differences in the way we approach the calculation of key inputs to the valuation. We summarize these.

	<i>Gross Debt</i>	<i>Net Debt</i>
Levered Beta	Unlevered beta is levered using gross debt to market equity ratio.	Unlevered beta is levered using net debt to market equity ratio.
Cost of capital	Debt to capital ratio used is based upon net debt.	Debt to capital ratio used is based upon gross debt.
Treatment of cash & debt	Cash is added to value of operating assets and net debt is subtracted out to get to equity value.	Cash is not added back to operating assets and gross debt is subtracted out to get to equity value.

While working with net debt in valuation is not difficult to do, the more interesting question is whether the value that emerges will be different from the value that would have been estimated using gross debt. In general, the answer is no and the reason usually lies in the cost of debt used in the net debt valuation. Intuitively, what you are doing when you use net debt is break the firm into two parts – a cash business, which is funded 100% with riskless debt, and an operating business, funded partly with risky debt. Carrying this to its logical conclusion, the cost of debt you would have for the operating business would be significantly higher than the firm's current cost of debt. This is because the current lenders to the firm will factor in the firm's cash holdings when setting the cost of debt.

To illustrate, assume that you have a firm with an overall value of \$1 billion - \$200 million in cash and \$800 million in operating assets – with \$400 million in debt and \$600 million in equity. The firm's cost of debt is 7%, a 2% default spread over the riskfree rate of 5%; note that this cost of debt is set based upon the firm's substantial cash holdings. If you net debt against cash, the firm would have \$200 million in net debt and \$600 million in equity. If you use the 7% cost of debt to value the firm now, you will overstate its value. Instead, the cost of debt you should use in the valuation is 9%.

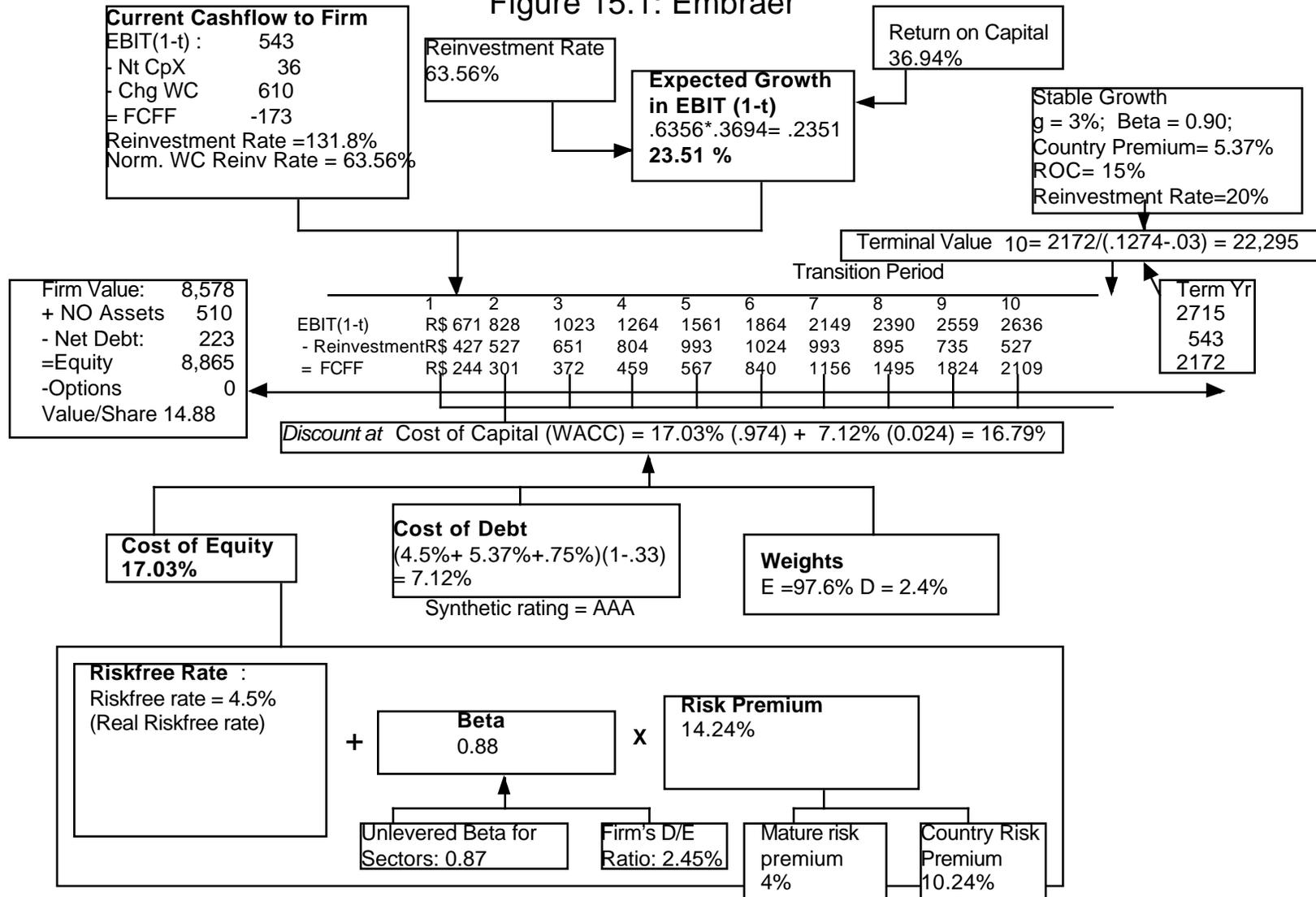
Cost of debt on net debt

$$= \frac{(\text{Pre - tax cost of debt}_{\text{Gross Debt}})(\text{Gross Debt}) - (\text{Pre - tax cost of debt}_{\text{Net Debt}})(\text{Net Debt})}{\text{Gross Debt} - \text{Net Debt}}$$

$$= \frac{(0.07)(400) - (0.05)(200)}{400 - 200} = 0.09$$

In general, we would recommend using gross debt rather than net debt for two other reasons. First, the net debt can be a negative number, if cash exceeds the gross debt. If this occurs, you should set the net debt to zero and consider the excess cash just as you would cash in a gross debt valuation. Second, maintaining a stable net debt ratio in a growing firm will require that cash balances increase as the firm value increases.

Figure 15.1: Embraer



Will equity value be the same under firm and equity valuation?

This model, unlike the dividend discount model or the FCFE model, values the firm rather than equity. The value of equity, however, can be extracted from the value of the firm by subtracting out the market value of outstanding debt. Since this model can be viewed as an alternative way of valuing equity, two questions arise - Why value the firm rather than equity? Will the values for equity obtained from the firm valuation approach be consistent with the values obtained from the equity valuation approaches described in the previous chapter?

The advantage of using the firm valuation approach is that cashflows relating to debt do not have to be considered explicitly, since the FCFE is a pre-debt cashflow, while they have to be taken into account in estimating FCFE. In cases where the leverage is expected to change significantly over time, this is a significant saving, since estimating new debt issues and debt repayments when leverage is changing can become increasingly messy the further into the future you go. The firm valuation approach does, however, require information about debt ratios and interest rates to estimate the weighted average cost of capital.

The value for equity obtained from the firm valuation and equity valuation approaches will be the same if you make consistent assumptions about financial leverage. Getting them to converge in practice is much more difficult. Let us begin with the simplest case – a no-growth, perpetual firm. Assume that the firm has \$166.67 million in earnings before interest and taxes and a tax rate of 40%. Assume that the firm has equity with a market value of \$600 million, with a cost of equity of 13.87% debt of \$400 million and with a pre-tax cost of debt of 7%. The firm's cost of capital can be estimated.

$$\text{Cost of capital} = (13.87\%) \frac{600}{1000} + (7\%)(1 - 0.4) \frac{400}{1000} = 10\%$$

$$\text{Value of the firm} = \frac{\text{EBIT}(1 - t)}{\text{Cost of capital}} = \frac{166.67(1 - 0.4)}{0.10} = \$1,000$$

Note that the firm has no reinvestment and no growth. We can value equity in this firm by subtracting out the value of debt.

$$\text{Value of equity} = \text{Value of firm} - \text{Value of debt} = \$1,000 - \$400 = \$600 \text{ million}$$

Now let us value the equity directly by estimating the net income:

$$\text{Net Income} = (\text{EBIT} - \text{Pre-tax cost of debt} * \text{Debt}) (1-t) = (166.67 - 0.07*400) (1-0.4) = 83.202 \text{ million}$$

The value of equity can be obtained by discounting this net income at the cost of equity:

$$\text{Value of equity} = \frac{\text{Net Income}}{\text{Cost of equity}} = \frac{83.202}{0.1387} = \$ 600 \text{ million}$$

Even this simple example works because of the following assumptions that we made implicitly or explicitly during the valuation.

1. The values for debt and equity used to compute the cost of capital were equal to the values that we obtained in the valuation. Notwithstanding the circularity in reasoning – you need the cost of capital to obtain the values in the first place – it indicates that a cost of capital based upon market value weights will not yield the same value for equity as an equity valuation model, if the firm is not fairly priced in the first place.
2. There are no extraordinary or non-operating items that affect net income but not operating income. Thus, to get from operating to net income, all we do is subtract out interest expenses and taxes.
3. The interest expenses are equal to the pre-tax cost of debt multiplied by the market value of debt. If a firm has old debt on its books, with interest expenses that are different from this value, the two approaches will diverge.

If there is expected growth, the potential for inconsistency multiplies. You have to ensure that you borrow enough money to fund new investments to keep your debt ratio at a level consistent with what you are assuming when you compute the cost of capital.

 *fcaffvsfcfe.xls*: This spreadsheet allows you to compare the equity values obtained using FCFF and FCFE models.

Firm Valuation: The APV approach

In the adjusted present value (APV) approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. To do this, we assume that the

primary benefit of borrowing is a tax benefit and that the most significant cost of borrowing is the added risk of bankruptcy.

The Mechanics of APV Valuation

We estimate the value of the firm in three steps. We begin by estimating the value of the firm with no leverage. We then consider the present value of the interest tax savings generated by borrowing a given amount of money. Finally, we evaluate the effect of borrowing the amount on the probability that the firm will go bankrupt, and the expected cost of bankruptcy.

Value of Unlevered Firm

The first step in this approach is the estimation of the value of the unlevered firm. This can be accomplished by valuing the firm as if it had no debt, i.e., by discounting the expected free cash flow to the firm at the unlevered cost of equity. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

$$\text{Value of Unlevered Firm} = \frac{\text{FCFF}_0 (1 + g)}{\rho_u - g}$$

where FCFF_0 is the current after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity and g is the expected growth rate. In the more general case, you can value the firm using any set of growth assumptions you believe are reasonable for the firm.

The inputs needed for this valuation are the expected cashflows, growth rates and the unlevered cost of equity. To estimate the latter, we can draw on our earlier analysis and compute the unlevered beta of the firm.

$$\beta_{\text{unlevered}} = \frac{\beta_{\text{current}}}{1 + (1 - t) \frac{D}{E}}$$

where

unlevered = Unlevered beta of the firm

current = Current equity beta of the firm

t = Tax rate for the firm

D/E = Current debt/equity ratio

This unlevered beta can then be used to arrive at the unlevered cost of equity.

Expected Tax Benefit from Borrowing

The second step in this approach is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

$$\begin{aligned} \text{Value of Tax Benefits} &= \frac{(\text{Tax Rate})(\text{Cost of Debt})(\text{Debt})}{\text{Cost of Debt}} \\ &= (\text{Tax Rate})(\text{Debt}) \\ &= t_c D \end{aligned}$$

The tax rate used here is the firm's marginal tax rate and it is assumed to stay constant over time. If we anticipate the tax rate changing over time, we can still compute the present value of tax benefits over time, but we cannot use the perpetual growth equation cited above.

Estimating Expected Bankruptcy Costs and Net Effect

The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. In theory, at least, this requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

$$\begin{aligned} \text{PV of Expected Bankruptcy cost} &= (\text{Probability of Bankruptcy})(\text{PV of Bankruptcy Cost}) \\ &= \pi_a BC \end{aligned}$$

This step of the adjusted present value approach poses the most significant estimation problem, since neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly.

There are two basic ways in which the probability of bankruptcy can be estimated indirectly. One is to estimate a bond rating, as we did in the cost of capital approach, at each level of debt and use the empirical estimates of default probabilities for each rating.

For instance, Table 15.8, extracted from a study by Altman and Kishore, summarizes the probability of default over ten years by bond rating class in 1998.⁸

Table 15.8: Default Rates by Bond Rating Classes

<i>Bond Rating</i>	<i>Default Rate</i>
D	100.00%
C	80.00%
CC	65.00%
CCC	46.61%
B-	32.50%
B	26.36%
B+	19.28%
BB	12.20%
BBB	2.30%
A-	1.41%
A	0.53%
A+	0.40%
AA	0.28%
AAA	0.01%

Source: Altman and Kishore (1998)

The other is to use a statistical approach, such as a **probit** to estimate the probability of default, based upon the firm's observable characteristics, at each level of debt.

The bankruptcy cost can be estimated, albeit with considerable error, from studies that have looked at the magnitude of this cost in actual bankruptcies. Research that has looked at the direct cost of bankruptcy concludes that they are small⁹, relative to firm value. The indirect costs of bankruptcy can be substantial, but the costs vary widely

⁸ This study estimated default rates over ten years only for some of the ratings classes. We extrapolated the rest of the ratings.

⁹ In Warner's study of railroad bankruptcies, the direct cost of bankruptcy seems to be about 5%.

across firms. Shapiro and Titman speculate that the indirect costs could be as large as 25% to 30% of firm value but provide no direct evidence of the costs.

Illustration 15.5: Valuing a firm with the APV approach: Tube Investments

In Illustration 15.1, we valued Tube Investments, using a cost of capital approach. Here, we re-estimate the value of the firm using an adjusted present value approach in three steps.

Step 1: Unlevered firm value

To estimate the unlevered firm value, we first compute the unlevered beta. Tube Investment's beta is 1.17, its current market debt to equity ratio is 79% and the firm's tax rate is 30%.

$$\text{Unlevered beta} = \frac{1.17}{1 + (1 - 0.3)(0.79)} = 0.75$$

Using the rupee riskfree rate of 10.5% and the risk premium of 9.23% for India, we estimate an unlevered cost of equity.

$$\text{Unlevered cost of equity} = 10.5\% + 0.75(9.23\%) = 17.45\%$$

Using the free cash flow to the firm that we estimated in Illustration 15.1 of Rs 212.2 million and the stable growth rate of 5%, we estimate the unlevered firm value:

$$\text{Unlevered firm value} = \frac{212.2}{0.1745 - 0.05} = \$1704.6 \text{ million}$$

Step 2: Tax benefits from debt

The tax benefits from debt are computed based upon Tube Investment's existing dollar debt of Rs. 1807.3 million and the tax rate of 30%:

$$\text{Expected tax benefits in perpetuity} = \text{Tax rate (Debt)} = 0.30 (1807.3) = \text{Rs } 542.2 \text{ million}$$

Step 3: Expected bankruptcy costs

To estimate this, we made two assumptions. First, based upon its existing rating, the probability of default at the existing debt level is 10%. Second, the cost of bankruptcy is 40% of unlevered firm value.

$$\text{Expected bankruptcy cost} = \text{Probability of bankruptcy} * \text{Cost of bankruptcy} * \text{Unlevered firm value} = 0.10 * 0.40 * 1704.6 = \text{Rs } 68.2 \text{ million}$$

The value of the operating assets of the firm can now be estimated.

Value of the operating assets

= Unlevered firm value + PV of tax benefits – Expected Bankruptcy Costs

= 1704.6 + 542.2 – 68.2 = Rs 2178.6 million

Adding to this the value of cash and marketable securities of Rs. 1365.3 million, we obtain a value for the firm of Rs 3543.9 million. In contrast, we valued the firm at Rs. 3367.3 million with the cost of capital approach.

Cost of Capital versus APV Valuation

In an APV valuation, the value of a levered firm is obtained by adding the net effect of debt to the unlevered firm value.

$$\text{Value of Levered Firm} = \frac{\text{FCFF}_0 (1 + g)}{\rho_u - g} + t_c D - \pi_a BC$$

In the cost of capital approach, the effects of leverage show up in the cost of capital, with the tax benefit incorporated in the after-tax cost of debt and the bankruptcy costs in both the levered beta and the pre-tax cost of debt. Will the two approaches yield the same value? Not necessarily. The first reason for the differences is that the models consider bankruptcy costs very differently, with the adjusted present value approach providing more flexibility in allowing you to consider indirect bankruptcy costs. To the extent that these costs do not show up or show up inadequately in the pre-tax cost of debt, the APV approach will yield a more conservative estimate of value. The second reason is that the APV approach considers the tax benefit from a dollar debt value, usually based upon existing debt. The cost of capital approach estimates the tax benefit from a debt ratio that may require the firm to borrow increasing amounts in the future. For instance, assuming a market debt to capital ratio of 30% in perpetuity for a growing firm will require it to borrow more in the future and the tax benefit from expected future borrowings is incorporated into value today.

APV, without bankruptcy costs

There are many who believe that adjusted present value is a more flexible way of approaching valuation than traditional discounted cash flow models. This may be true in a generic sense, but APV valuation in practice has significant flaws. The first and most important is that most practitioners who use the adjusted present value model ignore

expected bankruptcy costs. Adding the tax benefits to unlevered firm value to get to the levered firm value makes debt seem like an unmixed blessing. Firm value will be overstated, especially at very high debt ratios, where the cost of bankruptcy is clearly not zero and, in some instances, the cost of bankruptcy is higher than the tax benefit of debt.

The Effect of Leverage in Firm Value

Both the cost of capital approach and the APV approach make the value of a firm a function of its leverage. It follows directly, then, that there is some mix of debt and equity at which firm value is maximized. In the rest of this chapter, we consider how best to make this link.

Cost of Capital and Optimal Leverage

In order to understand the relationship between the cost of capital and optimal capital structure, we rely on the relationship between firm value and the cost of capital. In the earlier section, we noted that the value of the entire firm can be estimated by discounting the expected cash flows to the firm at the firm's cost of capital. The cash flows to the firm can be estimated as cash flows after operating expenses, taxes and any capital investments needed to create future growth in both fixed assets and working capital, but before financing expenses.

$$\text{Cash Flow to Firm} = \text{EBIT} (1-t) - (\text{Capital Expenditures} - \text{Depreciation}) - \text{Change in Working Capital}$$

The value of the firm can then be written as:

$$\text{Value of Firm} = \sum_{t=1}^{t=n} \frac{\text{CF to Firm}_t}{(1+\text{WACC})^t}$$

and is a function of the firm's cash flows and its cost of capital. If we assume that the cash flows to the firm are unaffected by the choice of financing mix and the cost of capital is reduced as a consequence of changing the financing mix, the value of the firm will increase. If the objective in choosing the financing mix for the firm is the maximization of firm value, we can accomplish it, in this case, by *minimizing the cost of capital*. In the

more general case where the cash flows to the firm are a function of the debt-equity mix, the optimal financing mix is the mix *that maximizes firm value*.¹⁰

Illustration 15.6: WACC, Firm Value, and Leverage

Assume that you are given the costs of equity and debt at different debt levels for Strunks Inc., a leading manufacturer of chocolates and other candies, and that the cash flows to this firm are currently \$200 million. Strunks is in a relatively stable market. The cash flows are expected to grow at 6% forever and are unaffected by the debt ratio of the firm. The cost of capital schedule is provided in Table 15.9, along with the value of the firm at each level of debt.

Table 15.9: Cost of Capital, Firm Value and Debt Ratios

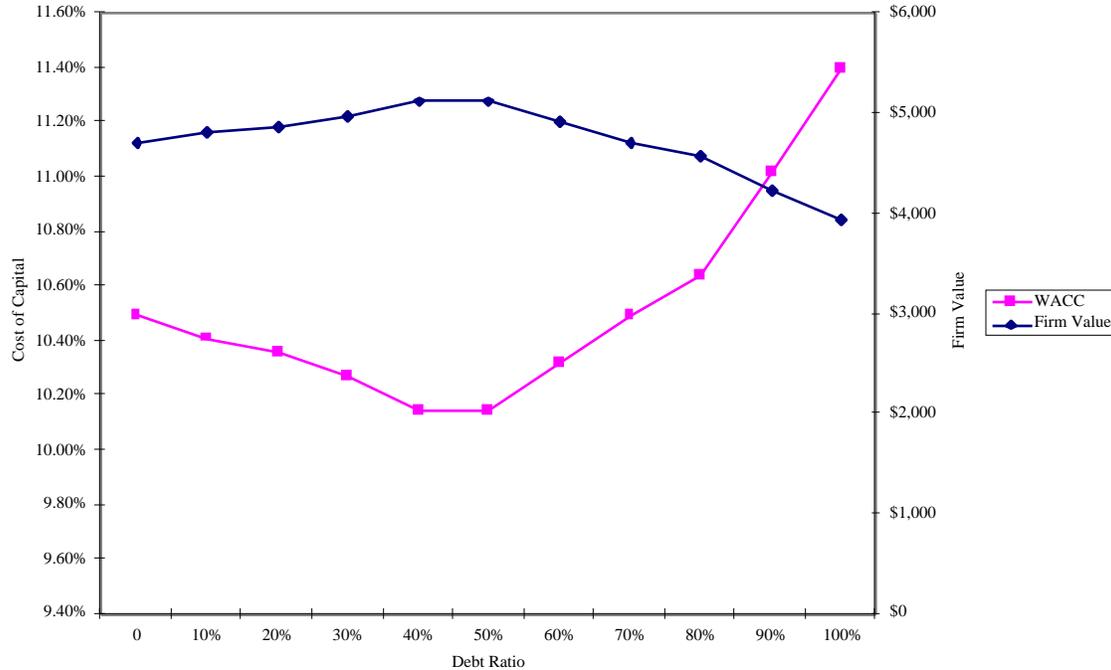
$D/(D+E)$	<i>Cost of Equity</i>	<i>Cost of Debt</i>	<i>WACC</i>	<i>Firm Value</i>
0	10.50%	4.80%	10.50%	\$4,711
10%	11.00%	5.10%	10.41%	\$4,807
20%	11.60%	5.40%	10.36%	\$4,862
30%	12.30%	5.52%	10.27%	\$4,970
40%	13.10%	5.70%	10.14%	\$5,121
50%	14.00%	6.30%	10.15%	\$5,108
60%	15.00%	7.20%	10.32%	\$4,907
70%	16.10%	8.10%	10.50%	\$4,711
80%	17.20%	9.00%	10.64%	\$4,569
90%	18.40%	10.20%	11.02%	\$4,223
100%	19.70%	11.40%	11.40%	\$3,926

$$\begin{aligned} \text{Note that the value of the firm} &= \frac{\text{Cash flows to firm}(1 + g)}{\text{Cost of Capital} - g} \\ &= \frac{200(1.06)}{\text{Cost of Capital} - 0.06} \end{aligned}$$

¹⁰ In other words, the value of the firm might not be maximized at the point that cost of capital is minimized, if firm cash flows are much lower at that level.

The value of the firm increases as the cost of capital decreases, and decreases as the cost of capital increases. This is illustrated in Figure 15.2.

Figure 15.2: Cost of Capital and Firm Value



While this illustration makes the choice of an optimal financing mix seem easy, it obscures problems that may arise in its practice. First, we typically do not have the benefit of having the entire schedule of costs of financing prior to an analysis. In most cases, the only level of debt at which we have information on the cost of debt and equity financing is the current level. Second, the analysis assumes implicitly that the level of operating income of the firm is unaffected by the financing mix of the firm and, consequently, by the default risk (or bond rating) for the firm. While this may be reasonable in some cases, it might not be in others. Firms that borrow too much might find that there are indirect bankruptcy costs that affect revenues and operating income.

Steps in Cost of Capital Approach

We need three basic inputs to compute the cost of capital – the cost of equity, the after-tax cost of debt and the weights on debt and equity. The costs of equity and debt

change as the debt ratio changes, and the primary challenge of this approach is in estimating each of these inputs.

Let us begin with the cost of equity. We argued that the beta of equity will change as the debt ratio changes. In fact, we estimated the levered beta as a function of the market debt to equity ratio of a firm, the unlevered beta and the firm's marginal tax rate:

$$\beta_{levered} = \beta_{unlevered} \left(1 + (1 - t) \frac{D}{E} \right)$$

Thus, if we can estimate the unlevered beta for a firm, we can use it to estimate the levered beta of the firm at every debt ratio. This levered beta can then be used to compute the cost of equity at each debt ratio.

$$\text{Cost of Equity} = \text{Riskfree rate} + \beta_{levered} (\text{Risk Premium})$$

The cost of debt for a firm is a function of the firm's default risk. As firms borrow more, their default risk will increase and so will the cost of debt. If we use bond ratings as our measure of default risk, we can estimate the cost of debt in three steps. First, we estimate a firm's dollar debt and interest expenses at each debt ratio; as firms increase their debt ratio, both dollar debt and interest expenses will rise. Second, at each debt level, we compute a financial ratio or ratios that measures default risk and use the ratio(s) to estimate a rating for the firm; again, as firms borrow more, this rating will decline. Third, a default spread, based upon the estimated rating, is added on to the riskfree rate to arrive at the pre-tax cost of debt. Applying the marginal tax rate to this pre-tax cost yields an after-tax cost of debt.

Once we estimate the costs of equity and debt at each debt level, we weight them based upon the proportions used of each to estimate the cost of capital. While we have not explicitly allowed for a preferred stock component in this process, we can have preferred stock as a part of capital. However, we have to keep the preferred stock portion fixed, while changing the weights on debt and equity. The debt ratio at which the cost of capital is minimized is the optimal debt ratio.

In this approach, the effect on firm value of changing the capital structure is isolated by keeping the operating income fixed and varying only the cost of capital. In practical terms, this requires us to make two assumptions. First, the debt ratio is decreased by raising new equity and/or retiring debt; conversely, the debt ratio is

increased by borrowing money and buying back stock. This process is called *recapitalization*. Second, the pre-tax operating income is assumed to be unaffected by the firm's financing mix and, by extension, its bond rating. If the operating income changes with a firm's default risk, the basic analysis will not change, but minimizing the cost of capital may not be the optimal course of action, since the value of the firm is determined by both the cashflows and the cost of capital. The value of the firm will have to be computed at each debt level and the optimal debt ratio will be that which maximizes firm value.

Illustration 15.7: Analyzing the Capital Structure for Boeing – March 1999

The cost of capital approach can be used to find the optimal capital structure for a firm, as we will for Boeing in March 1999. Boeing had \$6,972 million in debt on its books at that time, with an estimated market value¹¹, inclusive of operating leases, of \$8,194 million. The market value of equity at the same time was \$32,595 million; the market price per share was \$32.25 and there were 1010.7 million shares outstanding. Proportionally, 20.09% of the overall financing mix was debt and the remaining 79.91% was equity.

The beta for Boeing's stock in March 1999 was 1.01. The treasury bond rate at that time was 5%. Using an estimated market risk premium of 5.5%, we estimated the cost of equity for Boeing to be 10.58%.

$$\begin{aligned}\text{Cost of Equity} &= \text{Riskfree rate} + \text{Beta} * (\text{Market Premium}) \\ &= 5.00\% + 1.01 (5.5\%) = 10.58\%\end{aligned}$$

Boeing's senior debt was rated AA. Based upon this rating, the estimated pre-tax cost of debt for Boeing is 5.50%. The tax rate used for the analysis is 35%.

$$\begin{aligned}\text{Value of Firm} &= 8,194 + 32,595 = \$ 40,789 \text{ million} \\ \text{After-tax Cost of debt} &= \text{Pre-tax interest rate} (1 - \text{tax rate}) \\ &= 5.50\% (1 - 0.35) = 3.58\%\end{aligned}$$

The cost of capital was calculated using these costs and the weights based upon market value:

¹¹ The details of this calculation are in Chapter 7.

$$\begin{aligned}
 \text{WACC} &= (\text{Cost of Equity}) \frac{\text{Equity}}{\text{Equity} + \text{Debt}} + (\text{After-tax Cost of Debt}) \frac{\text{Debt}}{\text{Equity} + \text{Debt}} \\
 &= (10.58\%) \frac{32,595}{40,789} + (3.58\%) \frac{8,194}{40,789} = 9.17\%
 \end{aligned}$$

I. Boeing's Cost of Equity and Leverage

The cost of equity for Boeing at different debt ratios can be computed using the unlevered beta of the firm and the debt equity ratio at each level of debt. We use the levered betas that emerge to estimate the cost of equity. The first step in this process is to compute the firm's current unlevered beta, using the current market debt to equity ratio and a tax rate of 35%.

$$\begin{aligned}
 \text{Unlevered Beta} &= \frac{\text{Current Beta}}{1 + (1 - t) \frac{D}{E}} \\
 &= \frac{1.014}{1 + (1 - 0.35) \frac{8,194}{32,595}} \\
 &= 0.87
 \end{aligned}$$

The recomputed betas are reported in Table 15.10. We use the treasury bond rate of 5% and the market premium of 5.5% to compute the cost of equity. Note that the tax rate above a 50% debt ratio is adjusted to reflect the fact that Boeing does not have enough operating income to cover its interest expenses.

Table 15.10: Leverage, Betas And The Cost Of Equity

<i>Debt Ratio</i>	<i>Beta</i>	<i>Cost of Equity</i>
0%	0.87	9.79%
10%	0.93	10.14%
20%	1.01	10.57%
30%	1.11	11.13%
40%	1.25	11.87%
50%	1.51	13.28%
60%	1.92	15.54%
70%	2.56	19.06%
80%	3.83	26.09%

90%	7.67	47.18%
-----	------	--------

In calculating the levered beta in this table, we assumed that all market risk is borne by the equity investors; this may be unrealistic especially at higher levels of debt. We could also consider an alternative estimate of levered betas that apportions some of the market risk to the debt:

$$\beta_{\text{levered}} = \beta_u \left[1 + (1 - t) \frac{D}{E} \right] - \beta_{\text{debt}} (1 - t) \frac{D}{E}$$

The beta of debt is based upon the rating of the bond and is estimated by regressing past returns on bonds in each rating class against returns on a market index. The levered betas estimated using this approach will generally be lower than those estimated with the conventional model.

II. Boeing's Cost of Debt and Leverage

We assume that bond ratings are determined solely by the interest coverage ratio, which is defined as:

$$\text{Interest Coverage Ratio} = \frac{\text{Earnings before interest \& taxes}}{\text{Interest Expense}}$$

We chose the interest coverage ratio for three reasons. First, it is a ratio¹² used by both Standard and Poor's and Moody's to determine ratings. Second, there is significant correlation not only between the interest coverage ratio and bond ratings, but also between the interest coverage ratio and other ratios used in analysis, such as the debt coverage ratio and the funds flow ratios. Third, the interest coverage ratio changes as a firm changes its financing mix and decreases as the debt ratio increases. The ratings agencies would argue, however, that subjective factors, such as the perceived quality of management, are part of the ratings process. One way to build these factors into the analysis would be to modify the ratings obtained from the financial ratio analysis across the board to reflect the ratings agencies' subjective concerns¹³.

¹² S&P lists interest coverage ratio first among the nine ratios that it reports for different ratings classes on its web site.

¹³ For instance, assume that a firm's current rating is AA, but that its financial ratios would result in an A rating. It can then be argued that the ratings agencies are, for subjective reasons, rating the company one

The data in Table 15.11 were obtained based upon an analysis of the interest coverage ratios of large manufacturing firms in different ratings classes.

Table 15.11: Bond Ratings and Interest Coverage Ratios

<i>Interest Coverage Ratio</i>	<i>Rating</i>
> 8.5	AAA
6.50 - 8.50	AA
5.50 - 6.50	A+
4.25 - 5.50	A
3.00 - 4.25	A-
2.50 - 3.00	BBB
2.00 - 2.50	BB
1.75 - 2.00	B+
1.50 - 1.75	B
1.25 - 1.50	B-
0.80 - 1.25	CCC
0.65 - 0.80	CC
0.20 - 0.65	C
< 0.65	D

Source: Compustat

Using this table as a guideline, a firm with an interest coverage ratio of 1.65 would have a rating of B for its bonds.

The relationship between bond ratings and interest rates in February 1999 was obtained by looking at the typical default spreads¹⁴ for bonds in different ratings classes. Table 15.12 summarizes the interest rates/rating relationship and reports the spread for these bonds over treasury bonds and the resulting interest rates, using the treasury bond rate of 5%.

notch higher than the rating obtained from a purely financial analysis. The ratings obtained for each debt level can then be increased by one notch across the board to reflect these subjective considerations.

¹⁴ These default spreads were estimated from bondsonline.com, a service that provides, among other data on fixed income securities, updated default spreads for each ratings class.

Table 15.12: Bond Ratings And Market Interest Rates, February 1999

<i>Rating</i>	<i>Spread</i>	<i>Interest Rate on Debt</i>
AAA	0.20%	5.20%
AA	0.50%	5.50%
A+	0.80%	5.80%
A	1.00%	6.00%
A-	1.25%	6.25%
BBB	1.50%	6.50%
BB	2.00%	7.00%
B+	2.50%	7.50%
B	3.25%	8.25%
B-	4.25%	9.25%
CCC	5.00%	10.00%
CC	6.00%	11.00%
C	7.50%	12.50%
D	10.00%	15.00%

Source: bondsonline.com

Table 15.13 summarizes Boeing's projected operating income statement for the financial year 1998. It shows that Boeing had earnings before interest, taxes, and depreciation (EBITDA) of \$3,237 million and paid interest expenses of \$453 million.

Table 15.13: Boeing's Income Statement for 1998

Sales & Other Operating Revenues	\$56,154.00
- Operating Costs & Expenses	\$52,917.00
EBITDA	\$3,237.00
- Depreciation	\$1,517.00
EBIT	\$1,720.00
+ Extraordinary Income	\$130.00
EBIT with extraordinary income	\$1,850.00

- Interest Expenses	\$453.00
Earnings before Taxes	\$1,397.00
- Income Taxes	\$277.00
Net Earnings (Loss)	\$1,120.00

Based upon the current earnings before interest and taxes (EBIT) of \$1,720 million and interest expenses of \$453 million, Boeing has an interest coverage ratio of 3.80 and should command a rating of A-. Boeing's earnings before interest, taxes and depreciation (EBITDA) for the year was \$3,237 million. The actual rating of the firm which is AA reflects the ratings agency view that Boeing had sub-par years in both 1997 and 1998, and is capable of earning more on a regular basis. In our analysis, we adjust the EBIT and EBITDA for the imputed interest expenses on Boeing's operating leases¹⁵; this results in an increase of \$31 million in both numbers – to \$1,751 million in EBIT and \$3,268 million in EBITDA.

Finally, to compute Boeing's ratings at different debt levels, we redo the operating income statement at each level of debt, compute the interest coverage ratio at that level of debt and find the rating that corresponds to that level of debt. For example, Table 15.14 estimates the interest expenses, interest coverage ratios and bond ratings for Boeing at 0% and 10% debt ratios, at the existing level of operating income.

Table 15.14: Effect of Moving to Higher Debt Ratios, Boeing

D/(D+E)	0.00%	10.00%
D/E	0.00%	11.11%
\$ Debt	\$0	\$4,079
EBITDA	\$3,268	\$3,268
Depreciation	\$1,517	\$1,517
EBIT	\$1,751	\$1,751
Interest Expense	\$0	\$227
Pre-tax Int. cov		7.80

¹⁵ The details of this adjustment are provided in Chapter 4.

Likely Rating	AAA	AA
Interest Rate	5.20%	5.50%
Eff. Tax Rate	35.00%	35.00%

The dollar debt is computed to be 10% of the current value of the firm, which we compute by adding the market values of debt and equity.

Dollar Debt at 10% debt ratio = (Debt Ratio)(Market Value of Equity + Market Value of Debt) = 0.10 (32,595 + 8,194) = \$4,079 million

There is circular reasoning involved in estimating the interest expense. The interest rate is needed to calculate the interest coverage ratio and the coverage ratio is necessary to compute the interest rate. To get around the problem, we began our analysis by assuming that you could borrow \$4,079 million at the AAA rate of 5.20%; we computed an interest expense and interest coverage ratio using that rate; we estimated a new rating of AA for Boeing. We recomputed the interest expense using the AA rate¹⁶ of 5.50% as our cost of debt. This process is repeated for each level of debt from 10% to 90%, and the after-tax costs of debt are obtained at each level of debt in Table 15.15.

Table 15.15: Boeing: Cost of Debt and Debt Ratios

<i>Debt Ratio</i>	<i>\$ Debt</i>	<i>Interest Expense</i>	<i>Interest Coverage Ratio</i>	<i>Bond Rating</i>	<i>Pre-tax Cost of Debt</i>	<i>Tax Rate</i>	<i>After-tax Cost of Debt</i>
0.00%	\$0	\$0		AAA	5.20%	35.00%	3.38%
10.00%	\$4,079	\$224	7.80	AA	5.50%	35.00%	3.58%
20.00%	\$8,158	\$510	3.43	A-	6.25%	35.00%	4.06%
30.00%	\$12,237	\$857	2.04	BB	7.00%	35.00%	4.55%
40.00%	\$16,316	\$1,632	1.07	CCC	10.00%	35.00%	6.50%
50.00%	\$20,394	\$2,039	0.86	CCC	10.00%	30.05%	7.00%
60.00%	\$24,473	\$2,692	0.65	CC	11.00%	22.76%	8.50%
70.00%	\$28,552	\$3,569	0.49	C	12.50%	17.17%	10.35%

¹⁶ Since the interest expense rises, it is possible that for the rating to drop again. Thus, a third iteration might be necessary in some cases.

80.00%	\$32,631	\$4,079	0.43	C	12.50%	15.02%	10.62%
90.00%	\$36,710	\$4,589	0.38	C	12.50%	13.36%	10.83%

There are two points to make about this computation. We assume that at every debt level, all existing debt will be refinanced at the new interest rate that will prevail after the capital structure change. For instance, Boeing's existing debt, which has an AA rating, is assumed to be refinanced at the interest rate corresponding to a BB rating when Boeing moves to a 30% debt ratio. This is done for two reasons. The first is that existing debt-holders might have protective puts that enable them to put their bonds back to the firm and receive face value.¹⁷ The second is that the refinancing eliminates “wealth expropriation” effects — the effects of stockholders expropriating wealth from bondholders when debt is increased and vice versa when debt is reduced. If firms can retain old debt at lower rates, while borrowing more and becoming riskier, the lenders of the old debt will lose wealth. If we lock in current rates on existing bonds and recalculate the optimal debt ratio, we will allow for this wealth transfer.¹⁸

While it is conventional to leave the marginal tax rate unchanged as the debt ratio is increased, we adjust the tax rate to reflect the potential loss of the tax benefits of debt at higher debt ratios, where the interest expenses exceed the earnings before interest and taxes. To illustrate this point, note that the earnings before interest and taxes at Boeing is \$1,751 million. As long as interest expenses are less than \$1,751 million, interest expenses remain fully tax deductible and earn the 35% tax benefit. For instance, at a 40% debt ratio, the interest expenses are \$1,632 million and the tax benefit is therefore 35% of this amount. At a 50% debt ratio, however, the interest expenses balloon to \$2,039 million, which is greater than the earnings before interest and taxes of \$1,751 million. We consider the tax benefit on the interest expenses up to this amount.

$$\text{Tax Benefit} = \$1,751 \text{ million} * 0.35 = \$612.85 \text{ million}$$

As a proportion of the total interest expenses, the tax benefit is now less than 35%.

¹⁷ If they do not have protective puts, it is in the best interests of the stockholders not to refinance the debt (as in the leveraged buyout of RJR Nabisco) if debt ratios are increased.

¹⁸ This will have the effect of reducing interest cost, when debt is increased, and thus interest coverage ratios. This will lead to higher ratings, at least in the short term, and a higher optimal debt ratio.

$$\begin{aligned} \text{Effective Tax Rate} &= \frac{\text{EBIT}}{\text{interest expense}} t \\ &= \frac{\$1,751}{\$2,039} 0.35 = 30.05\% \end{aligned}$$

This, in turn, raises the after-tax cost of debt. This is a conservative approach, since losses can be carried forward. Given that this is a permanent shift in leverage, it does make sense to be conservative.

III. Leverage and Cost of Capital

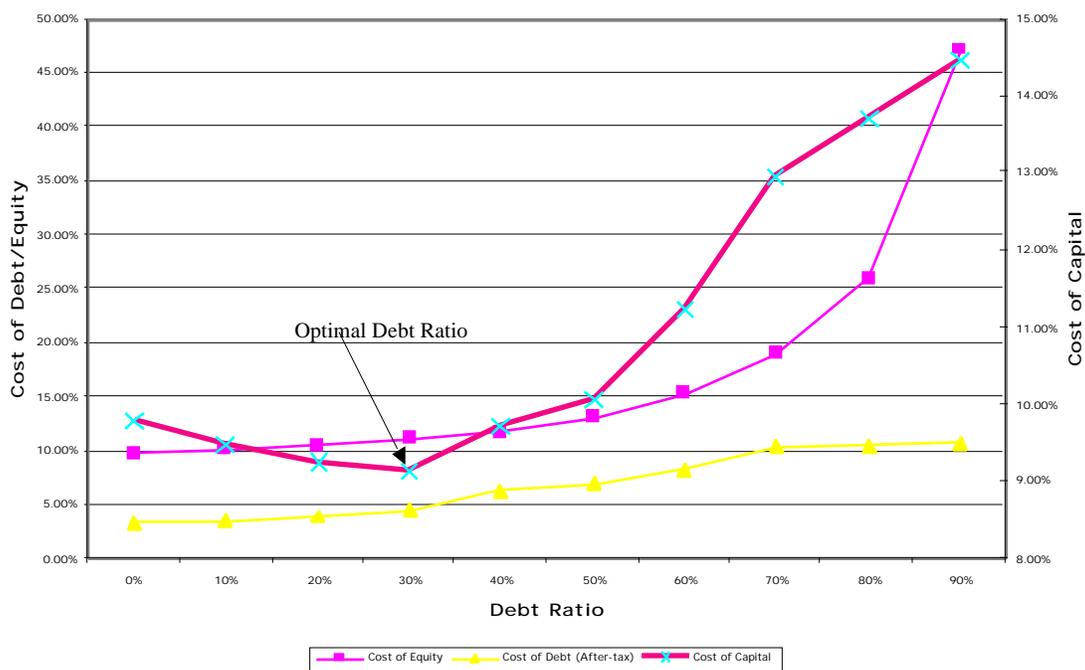
Now that we have estimated the cost of equity and the cost of debt at each debt level, we can compute Boeing's cost of capital. This is done for each debt level in Table 15.16. The cost of capital, which is 9.79%, when the firm is unlevered, decreases as the firm initially adds debt, reaches a minimum of 9.16% at 30% debt and then starts to increase again.

Table 15.16: Cost of Equity, Debt and Capital, Boeing

Debt Ratio	Beta	Cost of Equity	Cost of Debt (After-tax)	Cost of Capital
0%	0.87	9.79%	3.38%	9.79%
10%	0.93	10.14%	3.58%	9.48%
20%	1.01	10.57%	4.06%	9.27%
30%	1.11	11.13%	4.55%	9.16%
40%	1.25	11.87%	6.50%	9.72%
50%	1.48	13.15%	7.00%	10.07%
60%	1.88	15.35%	8.50%	11.24%
70%	2.56	19.06%	10.35%	12.97%
80%	3.83	26.09%	10.62%	13.72%
90%	7.67	47.18%	10.83%	14.47%

The optimal debt ratio is shown graphically in Figure 15.3.

Figure 15.3: Costs of Equity, Debt and Capital: Boeing



To illustrate the robustness of this solution to alternative measures of levered betas, we re-estimate the costs of debt, equity and capital under the assumption that debt bears some market risk. The results are summarized in Table 15.17.

Table 15.17: Costs of Equity, Debt and Capital with Debt carrying Market Risk, Boeing

Debt Ratio	Beta	Cost of Equity	Beta of Debt	Bond Rating	Interest rate on debt	Tax Rate	Cost of Debt (after-tax)	Cost of Capital
0%	0.89	9.92%	0.02	AAA	5.20%	35.00%	3.38%	9.92%
10%	0.96	10.26%	0.05	AA	5.50%	35.00%	3.58%	9.59%
20%	1.02	10.62%	0.11	A-	6.25%	35.00%	4.06%	9.31%
30%	1.10	11.04%	0.18	BB	7.00%	35.00%	4.55%	9.09%
40%	1.11	11.08%	0.45	CCC	10.00%	35.00%	6.50%	9.25%
50%	1.24	11.80%	0.45	CCC	10.00%	29.81%	7.02%	9.41%
60%	1.24	11.80%	0.68	C	12.50%	19.87%	10.02%	10.73%

70%	1.44	12.94%	0.68	C	12.50%	17.03%	10.37%	11.14%
80%	1.86	15.24%	0.68	C	12.50%	14.91%	10.64%	11.56%
90%	3.11	22.13%	0.68	C	12.50%	13.25%	10.84%	11.97%

If the debt holders bear some market risk¹⁹, the cost of equity is lower at higher levels of debt and Boeing's optimal debt ratio is still 30%, which is unchanged from the optimal calculated under the conventional calculation of the levered beta.

IV. Firm Value and Cost of Capital

The reason for minimizing the cost of capital is that it maximizes the value of the firm. To illustrate the effects of moving to the optimal on Boeing's firm value, we use the model described earlier in the chapter designed to value a firm in stable growth.

$$\text{Firm Value} = \frac{\text{Expected FCF}_{\text{next year}}}{\text{WACC} - g}$$

where

g = Stable growth rate

We begin by computing Boeing's current free cash flow using its current earnings before interest and taxes of \$1,753 million, its tax rate of 35% and its reinvestments in 1998 in working capital and net fixed assets:

EBIT (1- tax rate) =	\$ 1,138
+ Depreciation & Amortization =	\$ 1,517
- Capital Expenditures =	\$ 1,584
- Change in Working Capital =	\$ (105)
Free Cash Flow to the Firm =	\$ 1,176

The market value of the firm at the time of this analysis was obtained by adding up the estimated market values of debt and equity:

¹⁹ To estimate the beta of debt, we used the default spread at each level of debt and assumed that half this risk is market risk. Thus, at a C rating, the default spread is 9%. Based upon the market risk premium of 5.5% and the riskfree rate of 5% that we used elsewhere, we estimated the beta at a C rating to be:

$$\text{Imputed Debt Beta at a C rating} = (9\%/5.5\%)*0.5 = 0.8182$$

Market Value of Equity =	\$ 32,595
+ Market Value of Debt =	\$ 8,194
= Value of the Firm	\$ 40,789

Based upon the current cost of capital of 9.17%, we solve for the implied growth rate.

$$\text{Growth rate} = \frac{(\text{Firm Value})(\text{Cost of Capital}) - \text{CF to Firm}}{\text{Firm Value} + \text{CF to Firm}}$$

$$= \frac{(40,789)(0.0917) - 1,176}{40,789 + 1,176} = 0.0611 = 6.11\%$$

Now assume that Boeing shifts to 30% Debt and a WACC of 9.16%. The firm can now be valued using the following parameters.

Cash flow to Firm = \$1,176 million

WACC = 9.16%

Growth rate in Cash flows to Firm = 6.11%

$$\text{Firm Value} = \frac{(1,176)(1.0611)}{0.0916 - 0.0611} = \$ 40,990 \text{ million}$$

The value of the firm²⁰ will increase from \$40,789 million to \$40,990 million if the firm moves to the optimal debt ratio.

Increase in firm value = \$ 40,990 mil - \$ 40,789 mil = \$201 million

With 1010.7 million shares outstanding, assuming that stockholders can evaluate the effect of this refinancing, we can calculate the increase in the stock price.

$$\text{Increase in stock price} = \frac{\text{Increase in Firm Value}}{\text{Number of shares outstanding}}$$

$$= \frac{201}{1010.7} = \$0.20$$

Since the current stock price is \$32.25, the stock price can be expected to increase to \$32.45, which translates into a 0.62% increase in the price. The change is negligible

²⁰ This approach works best for firms with growth rates close to or below the growth rate of the economy, since this is a model that assumes perpetual growth. When this is not the case, i.e., when implied growth is much higher than 6%, we would suggest a modified approach, in which the present value of savings in firm value each year from going to the lower cost of capital is computed using a stable growth rate capped at about 6%. In the case of Boeing, this calculation would have yielded the following:

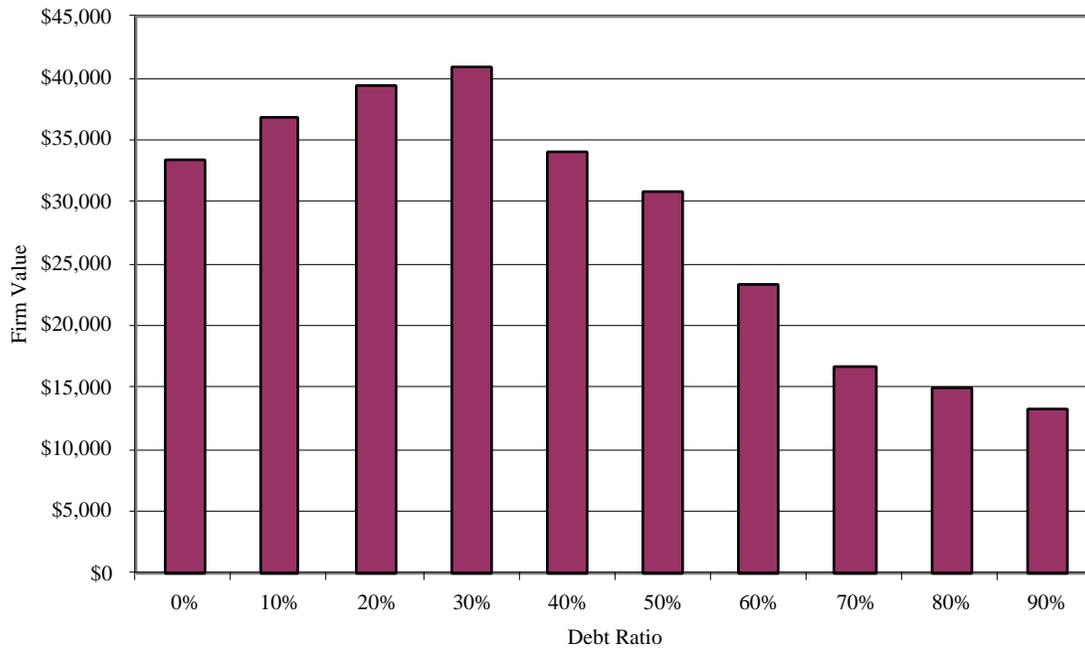
Savings each year = \$ 40,789 (0.0917 - 0.0916) = \$ 6.14 million

Present Value of Savings = \$ 6.14 / (0.0916 - 0.06) = \$ 206 million

Increase in value per share = \$206 million / 1010.7 = \$ 0.20 million

because the change in the cost of capital is small. The firm value and cost of capital at different debt ratios are summarized in Figure 15.4.

Figure 15.4: Debt Ratios and Firm Value



Since the asset side of the balance sheet is kept fixed and changes in capital structure are made by borrowing funds and repurchasing stock, this analysis implies that the stock price would increase to \$32.45 on the announcement of the repurchase. Implicit in this analysis is the assumption that the increase in firm value will be spread evenly across both stockholders who sell their stock back to the firm and those who do not. To the extent that stock can be bought back at the current price of \$32.25 or some value lower than \$32.45, the change in stock price will be larger. For instance, if Boeing could have bought stock back at the existing price of \$32.25, the increase²¹ in value per share would be \$0.23.

²¹ To compute this change in value per share, we first compute how many shares we would buy back with the additional debt of \$4.043 billion (Debt at 30% optimal – Current Debt) and the stock price of \$32.25. We then divide the increase in firm value of \$202 million by the remaining shares outstanding.



captstr.xls: This spreadsheet allows you to compute the optimal debt ratio firm value for any firm, using the same information used for Boeing. It has updated interest coverage ratios and default spreads built in.

Default Risk, Operating Income and Optimal Leverage

In the analysis we just completed on Boeing, we assumed that operating income would remain constant while the debt ratios changed. While this assumption simplifies our analysis substantially, it is not realistic. The operating income, for many firms, will drop as the default risk increases; this, in fact, is the cost we labeled as an indirect bankruptcy cost earlier in this chapter. The drop is likely to become more pronounced as the default risk falls below an acceptable level; for instance, a bond rating below investment grade may trigger significant losses in revenues and increases in expenses.

A general model for optimal capital structure would allow both operating income and cost of capital to change as the debt ratio changes. We have already described how we can estimate cost of capital at different debt ratios, but we could also attempt to do the same with operating income. For instance, we could estimate how the operating income for the Boeing would change as debt ratios and default risk changes by looking at the effects of rating downgrades on the operating income of other retailers.

If both operating income and cost of capital change, the optimal debt ratio may no longer be the point at which the cost of capital is minimized. Instead, the optimal has to be defined as that debt ratio at which the value of the firm is maximized.

APV and Financial Leverage

In the adjusted present value (APV) approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. The value of the levered firm can

$$\text{Change in stock price} = \frac{\$202 \text{ million}}{1010.7 - \frac{4043}{32.25}} = \$0.23 \text{ per share}$$

then be estimated at different levels of the debt and the debt level that maximizes firm value is the optimal debt ratio.

Steps in the Adjusted Present Value approach

The unlevered firm value is not a function of expected leverage and can be estimated as described in the earlier section – by discounting the free cash flows to the firm at the unlevered cost of equity. In fact, if you do not want to estimate this value and take the market value of the firm as correct, you could back out the unlevered firm value by subtracting out the tax benefits and adding back the expected bankruptcy cost from the existing debt.

$$\text{Current Firm Value} = \text{Value of Unlevered firm} + \text{PV of tax benefits} - \text{Expected Bankruptcy cost}$$

$$\text{Value of Unlevered firm} = \text{Current Firm Value} - \text{PV of tax benefits} + \text{Expected Bankruptcy costs}$$

The only components that change as a firm changes its leverage are the expected tax benefits and the expected bankruptcy costs. To obtain these values as you change leverage, you would go through the following steps.

1. Estimate the dollar debt outstanding at each debt ratio. This process mirrors what was done in the cost of capital approach. Keeping firm value fixed, we consider how much debt the firm will have at 20% debt, 30% debt and so on.
2. Estimate the tax benefits of debt by multiplying the dollar debt by the tax rate. This essentially assumes that the debt is permanent and that the tax benefits will continue in perpetuity.
3. Estimate the rating, interest rate and interest expense at each debt ratio. This process again replicates what was done in the cost of capital approach.
4. Use the rating to estimate a probability of default. Note that Table 15.8 provides these probabilities for each rating.
5. Estimate the expected bankruptcy cost by multiplying the probability of bankruptcy by the cost of bankruptcy, stated as a percent of unlevered firm value.

We compute the value of the levered firm at different levels of debt. The debt level that maximizes the value of the levered firm is the optimal debt ratio.

Illustration 15.8: Using the Adjusted Present Value Approach to calculate Optimal Debt Ratio for Boeing in 1999.

This approach can be applied to estimating the optimal capital structure for Boeing. The first step is to estimate the value of the unlevered firm. To do so, we start with the firm value of Boeing in 1999 and net the effect of the tax savings and bankruptcy costs arising from the existing debt.

Value of Boeing in 1999 = Value of Equity + Value of Debt = \$32,595 + \$8,194 = \$40,789

We compute the present value of the tax savings from the existing debt, assuming that the interest payments on the debt constitute a perpetuity.

$$\begin{aligned} \text{PV of Tax Savings from Existing Debt} &= \text{Existing Debt} * \text{Tax Rate} \\ &= \$8,194 * 0.35 = \$ 2,868 \text{ million} \end{aligned}$$

Based upon Boeing's current rating of AA, we estimate a probability of bankruptcy of 0.28% from Table 15.8. The bankruptcy cost is assumed to be 30% of the unlevered firm value.²² The cost is high because the perception of default risk is likely to be very damaging for a firm like Boeing, whose customers depend upon it for long-term service and support, and whose sales contracts are often spread out over a decade or more.

$$\begin{aligned} \text{PV of Expected Bankruptcy Cost} &= \text{Probability of Default} * \text{Bankruptcy cost} \\ &= 0.28\% * (30\% * (40,789 - 2,868)) = \$ 32 \end{aligned}$$

We then compute the value of Boeing as an unlevered firm.

$$\begin{aligned} \text{Value of Boeing as an Unlevered Firm} \\ &= \text{Current Market Value} - \text{PV of Tax Savings} + \text{Expected Bankruptcy Costs} \\ &= \$ 40,789 - \$ 2,868 + \$ 32 \\ &= \$ 37,953 \text{ million} \end{aligned}$$

The next step in the process is to estimate the tax savings at different levels of debt in Table 15.18. While we use the standard approach of assuming that the present value is calculated over a perpetuity, we reduce the tax rate used in the calculation if interest expenses exceed the earnings before interest and taxes. The adjustment to the tax rate was described more fully earlier in the cost of capital approach.

²² This estimate is based upon the Warner study, which estimates bankruptcy costs for large companies to be 10% of the value and upon the qualitative analysis of indirect bankruptcy costs in Shapiro and Cornell.

Table 15.18: Tax Savings From Debt ($t_c D$): Boeing

<i>Debt Ratio</i>	<i>\$ Debt</i>	<i>Tax Rate</i>	<i>Tax Benefits</i>
0%	\$0	35.00%	\$0
10%	\$4,079	35.00%	\$1,428
20%	\$8,158	35.00%	\$2,855
30%	\$12,237	35.00%	\$4,283
40%	\$16,316	35.00%	\$5,710
50%	\$20,394	30.05%	\$6,128
60%	\$24,473	22.76%	\$5,571
70%	\$28,552	17.17%	\$4,903
80%	\$32,631	15.02%	\$4,903
90%	\$36,710	13.36%	\$4,903

The final step in the process is to estimate the expected bankruptcy cost, based upon the bond ratings, the probabilities of default and the assumption that the bankruptcy cost is 30% of firm value. Table 15.19 summarizes these probabilities and the expected bankruptcy cost, computed based on the unlevered firm value.

Table 15.19: Expected Bankruptcy Cost, Boeing

<i>Debt Ratio</i>	<i>Bond Rating</i>	<i>Probability of Default</i>	<i>Expected Bankruptcy Cost</i>
0%	AA	0.28%	\$32
10%	AA	0.28%	\$32
20%	A-	1.41%	\$161
30%	BB	12.20%	\$1,389
40%	CCC	50.00%	\$5,693
50%	CCC	50.00%	\$5,693
60%	CC	65.00%	\$7,401
70%	C	80.00%	\$9,109
80%	C	80.00%	\$9,109
90%	C	80.00%	\$9,109

The value of the levered firm is estimated in Table 15.20 by aggregating the effects of the tax savings and the expected bankruptcy costs.

Table 15.20: Value of Boeing with Leverage

<i>Debt Ratio</i>	<i>Unlevered Firm Value</i>	<i>Tax Benefits</i>	<i>Expected Bankruptcy Cost</i>	<i>Value of Levered Firm</i>
0%	\$37,953	\$0	\$32	\$37,921
10%	\$37,953	\$1,428	\$32	\$39,349
20%	\$37,953	\$2,855	\$161	\$40,648
30%	\$37,953	\$4,283	\$1,389	\$40,847
40%	\$37,953	\$5,710	\$5,693	\$37,970
50%	\$37,953	\$6,128	\$5,693	\$38,388
60%	\$37,953	\$5,571	\$7,401	\$36,123
70%	\$37,953	\$4,903	\$9,109	\$33,747
80%	\$37,953	\$4,903	\$9,109	\$33,747
90%	\$37,953	\$4,903	\$9,109	\$33,747

The firm value is optimized at between 20% and 30% debt, which is consistent with the results of the other approaches. These results are, however, very sensitive to both the estimate of bankruptcy cost as a percent of firm value and the probabilities of default.

 *apv.xls*: This spreadsheet allows you to compute the value of a firm, with leverage, using the adjusted present value approach.

Benefits and Limitations of the Adjusted Present Value Approach

The advantage of this approach is that it separates the effects of debt into different components and allows the analyst to use different discount rates for each component. In this method, we do not assume that the debt ratio stays unchanged forever, which is an implicit assumption in the cost of capital approach. [NOTE: This is not true. In the CoC approach, I could adjust the debt ratio at any stage (year). The cumulative discount rate will be messy though.] Instead, we have the flexibility to keep the dollar value of debt fixed and to calculate the benefits and costs of the fixed dollar debt.

These advantages have to be weighed against the difficulty of estimating probabilities of default and the cost of bankruptcy. In fact, many analyses that use the adjusted present value approach ignore the expected bankruptcy costs leading them to the conclusion that firm value increases as firms borrow money. Not surprisingly, they conclude that the optimal debt ratio for a firm is 100% debt.

In general, with the same assumptions, the APV and the Cost of Capital conclusions give identical answers. However, the APV approach is more practical when firms are evaluating a dollar amount of debt, while the cost of capital approach is easier when firms are analyzing debt proportions.²³

Valuing the pieces rather than the whole

In the adjusted present value model, we value debt separately from the operating assets and firm value is the sum of the two components. In fact, one of the strongest benefits of discounted cash flow valuation is that breaking up cash flows into individual components and valuing them separately should not change the value. Thus, you could value a firm like General Electric by valuing each of its divisions separately and adding them up or Coca Cola by valuing its operations in each country separately and summing those up.

The advantage of piece-wise valuation is that you can estimate cash flows and discount rates separately for each piece and thus get more precise estimates of value. For example, you would use very different assumptions about operating margins, reinvestment needs and costs of capital when valuing the appliance and aircraft engine divisions of GE. Similarly, you could apply different country risk premiums for each country that Coca Cola operates in to value the firm. Since this is always the case, you might wonder why we do not do this for all firms. The problem is with the information. Many firms do not break down their earnings and cash flows in sufficient detail to allow for piece-wise valuation. Even firms that do, like GE, they often have large centralized expenses that get allocated, often arbitrarily, to individual divisions.

²³ See Inselbag and Kaufold (1997).

The benefits of breaking a firm down into pieces clearly increase as a firm becomes more diverse in its operations. These benefits have to be weighed off against the costs associated with more imprecise information and greater estimation problems.

Conclusion

This chapter develops an alternative approach to discounted cashflow valuation. The cashflows to the firm are discounted at the weighted average cost of capital to obtain the value of the firm, which when reduced by the market value of outstanding debt, yields the value of equity. Since the cashflow to the firm is a cashflow prior to debt payments, this approach is more straightforward to use when there is significant leverage or when leverage changes over time, though the weighted average cost of capital, used to discount free cashflows to the firm, has to be adjusted for changes in leverage. Finally, the costs of capital can be estimated at different debt ratios and used to estimate the optimal debt ratio for a firm.

The alternative approach to firm valuation is the APV approach, where we add the effect on value of debt (tax benefits – bankruptcy costs) to the unlevered firm value. This approach can also be used to estimate the optimal debt ratio for the firm.

Problems

1. Respond true or false to the following statements about the free cash flow to the firm.
 - A. The free cash flow to the firm is always higher than the free cash flow to equity.
 - B. The free cash flow to the firm is the cumulated cash flow to all investors in the firm, though the form of their claims may be different.
 - C. The free cash flow to the firm is a pre-debt, pre-tax cash flow.
 - D. The free cash flow to the firm is an after-debt, after-tax cash flow.
 - E. The free cash flow to the firm cannot be estimated without knowing interest and principal payments, for a firm with debt.

2. Union Pacific Railroad reported net income of \$770 million in 1993, after interest expenses of \$320 million. (The corporate tax rate was 36%.) It reported depreciation of \$960 million in that year, and capital spending was \$1.2 billion. The firm also had \$4 billion in debt outstanding on the books, rated AA (carrying a yield to maturity of 8%), trading at par (up from \$3.8 billion at the end of 1992). The beta of the stock is 1.05, and there were 200 million shares outstanding (trading at \$60 per share), with a book value of \$5 billion. Union Pacific paid 40% of its earnings as dividends and working capital requirements are negligible. (The treasury bond rate is 7%.)
 - a. Estimate the free cash flow to the firm in 1993.
 - b. Estimate the value of the firm at the end of 1993.
 - c. Estimate the value of equity at the end of 1993 and the value per share, using the FCFF approach.

3. Lockheed Corporation, one of the largest defense contractors in the US, reported EBITDA of \$1290 million in 1993, prior to interest expenses of \$215 million and depreciation charges of \$400 million. Capital Expenditures in 1993 amounted to \$450 million and working capital was 7% of revenues (which were \$13,500 million). The firm had debt outstanding of \$3.068 billion (in book value terms), trading at a market value of \$3.2 billion and yielding a pre-tax interest rate of 8%. There were 62 million shares outstanding trading at \$64 per share and the most recent beta is 1.10. The tax rate for the firm is 40%. (The treasury bond rate is 7%.)

The firm expects revenues, earnings, capital expenditures and depreciation to grow at 9.5% a year from 1994 to 1998, after which the growth rate is expected to drop to 4%. (Capital spending will offset depreciation in the steady state period.) The company also plans to lower its debt/equity ratio to 50% for the steady state (which will result in the pre-tax interest rate dropping to 7.5%).

 - a. Estimate the value of the firm.

b. Estimate the value of the equity in the firm, and the value per share.

4. In the face of disappointing earnings results and increasingly assertive institutional stockholders, Eastman Kodak was considering a major restructuring in 1993. As part of this restructuring, it was considering the sale of its health division, which earned \$560 million in earnings before interest and taxes in 1993, on revenues of \$5.285 billion. The expected growth in earnings was expected to moderate to 6% between 1994 and 1998, and to 4% after that. Capital expenditures in the health division amounted to \$420 million in 1993, while depreciation was \$350 million. Both are expected to grow 4% a year in the long term. Working capital requirements are negligible.

The average beta of firms competing with Eastman Kodak's health division is 1.15. While Eastman Kodak has a debt ratio ($D/(D+E)$) of 50%, the health division can sustain a debt ratio ($D/(D+E)$) of only 20%, which is similar to the average debt ratio of firms competing in the health sector. At this level of debt, the health division can expect to pay 7.5% on its debt, before taxes. (The tax rate is 40% and the treasury bond rate is 7%.)

a. Estimate the cost of capital for the division.

b. Estimate the value of the division.

c. Why might an acquirer pay more than this estimated value for the division?

5. You are analyzing a valuation done on a stable firm by a well-known analyst. Based upon the expected free cash flow to firm, next year, of \$30 million and an expected growth rate of 5%. The analyst has estimated a value of \$750 million. However, he has made the mistake of using the book values of debt and equity in his calculation. While you do not know the book value weights he used, you know that the firm has a cost of equity of 12% and an after-tax cost of debt of 6%. You also know that the market value of equity is three times the book value of equity, while the market value of debt is equal to the book value of debt. Estimate the correct value for the firm.

6. Santa Fe Pacific, a major rail operator with diversified operations, had earnings before interest, taxes and depreciation, of \$637 million in 1993, with depreciation amounting to \$235 million (offset by capital expenditure of an equivalent amount). The firm is in steady state and expected to grow 6% a year in perpetuity. Santa Fe Pacific had a beta of 1.25 in 1993 and debt outstanding of \$1.34 billion. The stock price was \$18.25 at the end of 1993, and there were 183.1 million shares outstanding. The expected ratings and the costs of debt at different levels of debt for Santa Fe are shown in the following table (The treasury bond rate is 7% and the firm faced a tax rate of 40%).

$D/(D+E)$	Rating	Cost of Debt (Pre-tax)
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0%	AAA	6.23%
10%	AAA	6.23%
20%	A+	6.93%
30%	A-	7.43%
40%	BB	8.43%
50%	B+	8.93%
60%	B-	10.93%
70%	CCC	11.93%
80%	CCC	11.93%
90%	CC	13.43%

The earnings before interest and taxes are expected to grow 3% a year in perpetuity with capital expenditures offset by depreciation. (The tax rate is 40%, the treasury bond rate is 7% and the market risk premium is 5.5%.)

- a. Estimate the cost of capital at the current debt ratio.
- b. Estimate the costs of capital at debt ratios ranging from 0% to 90%.
- c. Estimate the value of the firm at debt ratios ranging from 0% to 90%.