

### ***b. Storable Commodities***

The distinction between storable and perishable goods is that storable goods can be acquired at the spot price and stored till the expiration of the futures contract, which is the practical equivalent of buying a futures contract and taking delivery at expiration. Since the two approaches provide the same result, in terms of having possession of the commodity at expiration, the futures contract, if priced right, should cost the same as a strategy of buying and storing the commodity. The two additional costs of the latter strategy are as follows.

(a) Since the commodity has to be acquired now, rather than at expiration, there is an added financing cost associated with borrowing the funds needed for the acquisition now.

$$\text{Added Interest Cost} = (\text{Spot price}) \left( (1 + \text{Interest Rate})^{\text{Life of Futures contract}} - 1 \right)$$

(b) If there is a storage cost associated with storing the commodity until the expiration of the futures contract, this cost has to be reflected in the strategy as well. In addition, there may be a benefit to having physical ownership of the commodity. This benefit is called the convenience yield and will reduce the futures price. The net storage cost is defined to be the difference between the total storage cost and the convenience yield.

If  $F$  is the futures contract price,  $S$  is the spot price,  $r$  is the annualized interest rate,  $t$  is the life of the futures contract and  $k$  is the net annual storage costs (as a percentage of the spot price) for the commodity, the two equivalent strategies and their costs can be written as follows.

Strategy 1: Buy the futures contract. Take delivery at expiration. Pay  $\$F$ .

Strategy 2: Borrow the spot price ( $S$ ) of the commodity and buy the commodity. Pay the additional costs.

$$(a) \text{ Interest cost} = S \left( (1 + r)^t - 1 \right)$$

$$(b) \text{ Cost of storage, net of convenience yield} = S k t$$

If the two strategies have the same costs,

$$\begin{aligned} F^* &= S \left( (1 + r)^t - 1 \right) + S k t \\ &= S \left( (1 + r)^t + k t \right) \end{aligned}$$

This is the basic arbitrage relationship between futures and spot prices. Any deviation from this arbitrage relationship should provide an opportunity for arbitrage, i.e., a strategy with no risk and no initial investment, and for positive profits. These arbitrage opportunities are described in Figure 34.4.

This arbitrage is based upon several assumptions. First, investors are assumed to borrow and lend at the same rate, which is the riskless rate. Second, when the futures contract is over priced, it is assumed that the seller of the futures contract (the arbitrageur) can sell short on the commodity and that he can recover, from the owner of the commodity, the storage costs that are saved as a consequence. To the extent that these assumptions are unrealistic, the bounds on prices within which arbitrage is not feasible expand. Assume, for instance, that the rate of borrowing is  $r_b$  and the rate of lending is  $r_a$ , and that short seller cannot recover any of the saved storage costs and has to pay a transactions cost of  $t_s$ . The futures price will then fall within a bound.

$$(S - t_s)(1 + r_a)^t < F^* < S(1 + r_b)^t + kt$$

If the futures price falls outside this bound, there is a possibility of arbitrage and this is illustrated in Figure 34.5.