

Figure 34.4: Storable Commodity Futures: Pricing and Arbitrage

$$F^* = S((1+r)^t + kt)$$

If $F > F^*$			If $F < F^*$	
Time	Action	Cashflows	Action	Cashflows
Now:	1. Sell futures contract	0	1. Buy futures contract	0
	2. Borrow spot price at riskfree r	S	2. Sell short on commodity	S
	3. Buy spot commodity	-S	3. Lend money at riskfree rate	-S
At t:	1. Collect commodity; Pay storage cost.	-Skt	1. Collect on loan	$S(1+r)^t$
	2. Deliver on futures contract	F	2. Take delivery of futures contract	-F
	3. Pay back loan	$-S(1+r)^t$	3. Return borrowed commodity; Collect storage costs	+Skt
NCF=		$F - S((1+r)^t + kt) > 0$	$S((1+r)^t + kt) - F > 0$	

Key inputs:

F* = Theoretical futures price

r = Riskless rate of interest (annualized)

F = Actual futures price

t = Time to expiration on the futures contract

S = Spot price of commodity

k = Annualized carrying cost, net of convenience yield (as % of spot price)

Key assumptions

1. The investor can lend and borrow at the riskless rate.
2. There are no transactions costs associated with buying or selling short the commodity.
3. The short seller can collect all storage costs saved because of the short selling.

Figure 34.5: storable commodity futures: pricing and arbitrage with modified assumptions

Modified Assumptions

1. Investor can borrow at r_b ($r_b > r$) and lend at r_a ($r_a < r$).
2. The transactions cost associated with selling short is t_s (where t_s is the dollar transactions cost).
3. The short seller does not collect any of the storage costs saved by the short selling.

$$F_h^* = S ((1+r_b)^t + k t)$$

$$F_l^* = (S-t_s) (1+r_a)^t$$

If $F > F_h^*$

If $F < F_l^*$

Time	Action	Cashflows	Action	Cashflows
Now:	1. Sell futures contract	0	1. Buy futures contract	0
	2. Borrow spot price at r_b	S	2. Sell short on commodity	S - t_s
	3. Buy spot commodity	-S	3. Lend money at r_a	-(S - t_s)
At t:	1. Collect commodity from storage	-Skt	1. Collect on loan	(S- t_s)(1+ r_a) ^t
	2. Delivery on futures contract	F	2. Take delivery of futures contract	-F
	3. Pay back loan	-S(1+ r_b) ^t	3. Return borrowed commodity; Collect storage costs	0
NCF=		F-S((1+r_b)^t - kt)> 0		(S-t_s) (1+r_a)^t - F > 0

F_h = Upper limit for arbitrage bound on futures prices

F_l = Lower limit for arbitrage bound on futures prices