

### c. Stock Index Futures

Futures on stock indices have become an important and growing part of most financial markets. Today, you can buy or sell futures on the Dow Jones, the S&P 500, the NASDAQ and the Value Line indices.

An index future entitles the buyer to any appreciation in the index over and above the index futures price and the seller to any depreciation in the index from the same benchmark. To evaluate the arbitrage pricing of an index future, consider the following strategies.

*Strategy 1:* Sell short on the stocks in the index for the duration of the index futures contract. Invest the proceeds at the riskless rate. (This strategy requires that the owners of the index be compensated for the dividends they would have received on the stocks.)

*Strategy 2:* Sell the index futures contract.

Both strategies require the same initial investment, have the same risk and should provide the same proceeds. Again, if  $S$  is the spot price of the index,  $F$  is the futures prices,  $y$  is the annualized dividend yield on the stock and  $r$  is the riskless rate, the cash flows from the two contracts at expiration can be written.

$$F^* = S (1 + r - y)^t$$

If the futures price deviates from this arbitrage price, there should be an opportunity from arbitrage. This is illustrated in Figure 34.6.

This arbitrage is conditioned on several assumptions. First, it, like the commodity futures arbitrage, assumes that investors can lend and borrow at the riskless rate. Second, it ignores transactions costs on both buying stock and selling short on stocks. Third, it assumes that the dividends paid on the stocks in the index are known with certainty at the start of the period. If these assumptions are unrealistic, the index futures arbitrage will be feasible only if prices fall outside a band, the size of which will depend upon the seriousness of the violations in the assumptions.

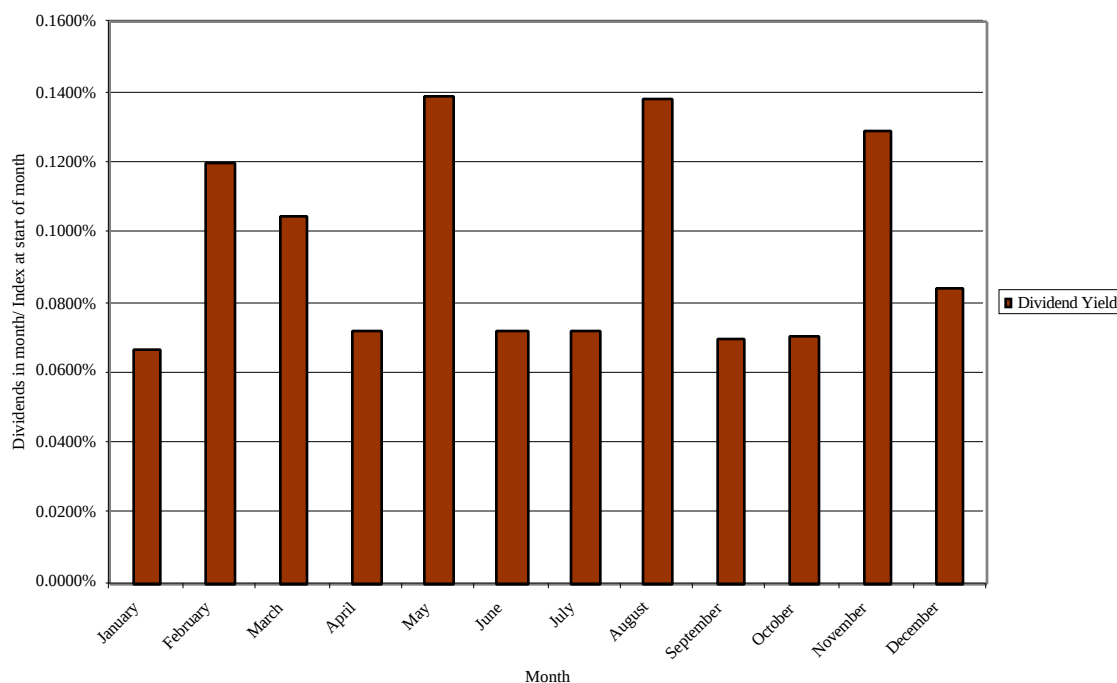
Assume that investors can borrow money at  $r_b$  and lend money at  $r_a$  and that the transactions costs of buying stock is  $t_c$  and selling short is  $t_s$ . The band within which the futures price must stay can be written as:

$$(S - t_s)(1 + r_a - y)^t < F^* < (S + t_c)(1 + r_b - y)^t$$

The arbitrage that is possible if the futures price strays outside this band is illustrated in Figure 34.7.

In practice, one of the issues that you have to factor in is the seasonality of dividends since the dividends paid by stocks tend to be higher in some months than others. Figure 34.8 graphs out dividends paid as a percent of the S&P 500 index on U.S. stocks in 2000 by month of the year.

Figure 34.8: Dividend Yields by Month of Year- 2000



Thus, dividend yields seem to peak in February, May, August and November.