

Figure 34.6: Stock Index Futures: Pricing and Arbitrage

<div style="border: 1px solid black; padding: 5px; text-align: center; margin: 0 auto; width: fit-content;"> <math>F^* = S (1+r-y)^t</math> </div>			
If $F > F^*$		If $F < F^*$	
Action	Cashflows	Action	Cashflows
1. Sell futures contract	0	1. Buy futures contract	0
2. Borrow spot price of index at riskfree r	$S$	2. Sell short stocks in the index	$S$
3. Buy stocks in index	$-S$	3. Lend money at riskfree rate	$-S$
1. Collect dividends on stocks	$S((1+y)^t-1)$	1. Collect on loan	$S(1+r)^t$
2. Delivery on futures contract	$F$	2. Take delivery of futures contract	$-F$
3. Pay back loan	$-S(1+r)^t$	3. Return borrowed stocks; Pay foregone dividends	$-S((1+y)^t-1)$
<b><math>F - S(1+r-y)^t &gt; 0</math></b>		<b><math>S(1+r-y)^t - F &gt; 0</math></b>	

**Notations:**

Theoretical futures price

 $r$  = Riskless rate of interest (annualized)

Actual futures price

 $t$  = Time to expiration on the futures contract

Current level of index

 $y$  = Dividend yield over lifetime of futures contract as % of current index level**Assumptions**

Investor can lend and borrow at the riskless rate.

There are no transactions costs associated with buying or selling short stocks.

Dividends are known with certainty.

Figure 34.7: Stock Index Futures: Pricing and Arbitrage with modified assumptions

**Assumptions**

Investor can borrow at  $r_b$  ( $r_b > r$ ) and lend at  $r_a$  ( $r_a < r$ ).

Transactions cost associated with selling short is  $t_s$  (where  $t_s$  is the dollar transactions cost) and the transactions cost associated with buying the stock is  $t_c$ .

		$F_h^* = (S + t_c) (1 + r_b - y)^t$ $F_l^* = (S - t_s) (1 + r_a - y)^t$	
If $F > F_h^*$		If $F < F_l^*$	
Action	Cashflows	Action	Cashflows
1. Sell futures contract	0	1. Buy futures contract	0
2. Borrow spot price at $r_b$	$S + t_c$	2. Sell short stocks in the index	$S - t_s$
3. Buy stocks in the index	$-S - t_c$	3. Lend money at $r_a$	$-(S - t_s)$
1. Collect dividends on stocks	$S((1+y)^t - 1)$	1. Collect on loan	$(S - t_s)(1 + r_a)^t$
2. Delivery on futures contract	$F$	2. Take delivery of futures contract	$-F$
3. Pay back loan	$-(S + t_c)(1 + r_b)^t$	3. Return borrowed stocks; Pay foregone dividends	$-S((1+y)^t - 1)$
<b><math>F - (S + t_c) (1 + r_b - y)^t &gt; 0</math></b>		<b><math>(S - t_s) (1 + r_a - y)^t - F &gt; 0</math></b>	
Upper limit for arbitrage bound on futures prices		$F_l$ = Lower limit for arbitrage bound on futures prices	