e. Currency Futures

In a currency futures contract, you enter into a contract to buy a foreign currency at a price fixed today. To see how spot and futures currency prices are related, note that holding the foreign currency enables the investor to earn the risk-free interest rate (R_f) prevailing in that country while the domestic currency earn the domestic riskfree rate (R_d). Since investors can buy currency at spot rates and assuming that there are no restrictions on investing at the riskfree rate, we can derive the relationship between the spot and futures prices. *Interest rate parity* relates the differential between futures and spot prices to interest rates in the domestic and foreign market.

$$\frac{\text{Futures Price}_{d,f}}{\text{Spot Price}_{d,f}} = \frac{(1+R_d)}{(1+R_f)}$$

where $\text{Futures Price}_{d,f}$ is the number of units of the domestic currency that will be received for a unit of the foreign currency in a forward contract and $\text{Spot Price}_{d,f}$ is the number of units of the domestic currency that will be received for a unit of the same foreign currency in a spot contract. For instance, assume that the one-year interest rate in the United States is 5% and the one-year interest rate in Germany is 4%. Furthermore, assume that the spot exchange rate is $0.65 per Deutsche Mark. The one-year futures price, based upon interest rate parity, should be as follows:

$$\frac{\text{Futures Price}_{d,f}}{\$\ 0.65} = \frac{(1.05)}{(1.04)}$$

resulting in a futures price of $0.65625 per Deutsche Mark.

Why does this have to be the futures price? If the futures price were greater than $0.65625, say $0.67, an investor could take advantage of the mispricing by selling the futures contract, completely hedging against risk and ending up with a return greater than the riskfree rate. When a riskless position yields a return that exceeds the riskfree rate, it is called an **arbitrage position**. The actions the investor would need to take are summarized in Table 34.3, with the cash flows associated with each action in brackets next to the action.

Table 34.3: Arbitrage when currency futures contracts are mispriced

Forward Rate Mispricing	*Actions to take today*	*Actions at expiration of* futures *contract*
If futures price > $0.65625 e.g. $0.67	1. Sell a futures contract at $0.67 per Deutsche Mark. ($0.00) 2. Borrow the spot price in the U.S. domestic markets @ 5%. (+$0.65) 3. Convert the dollars into Deutsche Marks at spot price. (-$0.65/+1 DM) 4. Invest Deustche Marks in the German market @ 4%. (-1 DM)	1. Collect on Deutsche Mark investment. (+1.04 DM) 2. Convert into dollars at futures price. (-1.04 DM/ +$0.6968) 3. Repay dollar borrowing with interest. (-$0.6825) **Profit = $0.6968 - $0.6825 = $ 0.0143**
If futures price < $0.65625 e.g. $0.64	1. Buy a futures price at $0.64 per Deutsche Mark. ($0.00) 2. Borrow the spot rate in the German market @4%. (+1 DM) 3. Convert the Deutsche Marks into Dollars at spot rate. (-1 DM/+$0.65) 4. Invest dollars in the U.S. market @ 5%. (-$0.65)	1. Collect on Dollar investment. (+$0.6825) 2. Convert into dollars at futures price. (-$0.6825/1.0664 DM) 3. Repay DM borrowing with interest. (1.04 DM) **Profit = 1.0664-1.04 = 0.0264 DM**

The first arbitrage of Table 34.3 results in a riskless profit of $0.0143, with no initial investment. The process of arbitrage will push down futures price towards the equilibrium price.

If the futures price were lower than $0.65625, the actions would be reversed, with the same final conclusion. Investors would be able to take no risk, invest no money and

still end up with a positive cash flow at expiration. In the second arbitrage of Table 34.3, we lay out the actions that would lead to a riskless profit of .0164 DM.

Effects of Special Features in Futures Contracts

The arbitrage relationship provides a measure of the determinants of futures prices on a wide range of assets. There are however some special features that affect futures prices. One is the fact that futures contracts require marking to the market, while forward contracts do not. Another is the existence of trading restrictions, such as price limits on futures contracts. The following section examines the pricing effects of each of these special features.

a. Futures versus Forward Contracts

As described earlier in this section, futures contracts require marking to market while forward contracts do not. If interest rates are constant and the same for all maturities, there should be no difference between the value of a futures contract and the value of an equivalent forward contract. When interest rates vary unpredictably, forward prices can be different from futures prices. This is because of the reinvestment assumptions that have to be made for intermediate profits and losses on a futures contract, and the borrowing and lending rates assumptions that have to be made for intermediate losses and profits, respectively. The effect of this interest rate induced volatility on futures prices will depend upon the relationship between spot prices and interest rates. If they move in opposite directions (as is the case with stock indices and treasury bonds), the interest rate risk will make futures prices greater than forward prices. If they move together (as is the case with some real assets), the interest rate risk can actually counter price risk and make futures prices less than forward prices. In most real world scenarios, and in empirical studies, the difference between futures and forward prices is fairly small and can be ignored.

There is another difference between futures and forward contracts that can cause their prices to deviate and it relates to credit risk. Since the futures exchange essentially guarantees traded futures contracts, there is relatively little credit risk. Essentially, the exchange has to default for buyers or sellers of contracts to not be paid. Forward contracts are between individual buyers and sellers. Consequently, there is potential for

significant default risk which has to be taken into account when valuing a forward contract.

b. Trading Restrictions

The existence of price limits and margin requirements on futures contract are generally ignored in the valuation and arbitrage conditions described in this chapter. It is however possible that these restrictions on trading, if onerous enough, could impact value. The existence of price limits, for instance, has two effects. One is that it might reduce the volatility in prices, by protecting against market overreaction to information and thus make futures contracts more valuable. The other is that it makes futures contracts less liquid and this may make them less valuable. The net effect could be positive, negative or neutral.

Conclusion

The value of a futures contract is derived from the value of the underlying asset. The opportunity for arbitrage will create a strong linkage between the futures and spot prices; and the actual relationship will depend upon the level of interest rates, the cost of storing the underlying asset and any yield that can be made by holding the asset. In addition the institutional characteristics of the futures markets, such as price limits and ‘marking to market’, as well as delivery options, can affect the futures price.

Problems

1. The following is an excerpt from the Wall Street Journal futures page. It includes the futures prices of gold. The current cash (spot) price of gold is $403.25. Make your best estimates of the implied interest rates (from the arbitrage relationship) in the futures prices. (You can assume zero carrying costs for gold.)

Contract expiring in	*Trading at*
1 month	$404.62
2 months	$406.11
3 months	$407.70
6 months	$412.51
12 months	$422.62

2. You are a portfolio manager who has just been exposed to the possibilities of stock index futures. Respond to the following situations.

(a) Assume that you have the resources to buy and hold the stocks in the S&P 500. You are given the following data. (Assume that today is January 1.)

Level of the S&P 500 index = 258.90

June S&P 500 futures contract = 260.15

Annualized Rate on T.Bill expiring June 26 (expiration date) = 6%

Annualized Dividend yield on S&P 500 stocks = 3%

Assume that dividends are paid out continuously over the year. Is there potential for arbitrage? How would you go about setting up the arbitrage?

(b) Assume now that you are known for your stock selection skills. You have 10,000 shares of Texaco in your portfolio (now selling for 38) and are extremely worried about the direction of the market until June. You would like to protect yourself against market risk by using the December S&P 500 futures contract (which is at 260.15). If Texaco's beta is 0.8, how would you go about creating this protection?

3. Assume that you are a mutual fund manager with a total portfolio value of $100 million. You estimate the beta of the fund to be 1.25. You would like to hedge against market movements by using stock index futures. You observe that the S&P 500 June futures are selling for 260.15 and that the index is at 258.90. Answer the following questions.

(a) How many stock index futures would you have to sell to protect against market risk?

(b) If the riskfree rate is 6% and the market risk premium is 8%, what return would you expect to make on the mutual fund? (Assuming you don't hedge.)

(c) How much would you expect to make if you hedge away all market risk?

4. Given the following information on gold futures prices, the spot price of gold, the riskless interest rate and the carrying cost of gold, construct an arbitrage position. (Assume that it is December 1987 now.)

December 1988 futures contract price = 515.60/troy oz

Spot price of gold = 481.40/troy oz

Interest rate (annualized) = 6%

Carrying cost (annualized) = 2%

a. What would you have to do right now to set up the arbitrage?

b. What would you have to do in December to unwind the position? How much arbitrage profit would you expect to make?

c. Assume now that you can borrow at 8%, but you can lend at only 6%. Establish a price band for the futures contract, within which arbitrage is not feasible.

5. The following is a set of prices for stock index futures on the S&P 500.

Maturity	Futures price
March	246.25
June	247.75

The current level of the index is 245.82 and the current annualized T.Bill rate is 6%. The annualized dividend yield is 3%. (Today is January 14. The March futures expire on March 18 and the June futures on June 17.)

(a) Estimate the theoretical basis and actual basis in each of these contracts.

(b) Using one of the two contracts, set up an arbitrage. Also show how the arbitrage will be resolved at expiration. [You can assume that you can lend or borrow at the riskfree rate and that you have no transaction costs or margins.]

(c) Assume that a good economic report comes out on the wire. The stock index goes up to 247.82 and the T.Bill rate drops to 5%. Assuming arbitrage relationships hold and that the dollar dividends paid do not change, how much will the March future go up by?

6. You are provided the following information.

Current price of wheat = $19,000 for 5000 bushels

Riskless rate = 10 % (annualized)

Cost of storage = $200 a year for 5000 bushels

One-year futures contract price = $20,400 (for a contract for 5000 bushels)

a. What is F* (the theoretical price)?

b. How would you arbitrage the difference between F and F*? (Specify what you do now and at expiration and what your arbitrage profits will be.)

c. If you can sell short (Cost $100 for 5000 bushels) and cannot claim any of the storage cost for yourself on short sales[2], at what rate would you have to be able to lend for this arbitrage to be feasible?

[2] In theory, we make the unrealistic assumption that a person who sells short (i.e. borrows somebody else's property and sells it now) will be able to collect the storage costs saved by the short sales from the other party to the transaction.