CHAPTER 28

The Option to Delay and Valuation Implications

In traditional investment analysis, a project or new investment should be accepted only if the returns on the project exceed the hurdle rate; in the context of cash flows and discount rates, this translates into investing in projects with positive net present values (NPVs). The limitation of this view of the world, which analyzes projects on the basis of expected cash flows and discount rates, is that it fails to consider fully the options that are usually associated with many investments.

This chapter will consider an option that is embedded in many projects, namely the option to wait and take the project in a later period. Why might a firm want to do this? If the present value of the cash flows on the project are volatile and can change over time, a project with a negative net present value today may have a positive net present value in the future. Furthermore, a firm may gain by waiting on a project even after a project has a positive net present value, because the option has a time premium that exceeds the cash flows that can be generated in the next period by accepting the project. This option is most valuable in projects where a firm has the exclusive right to invest in a project and becomes less valuable as the barriers to entry decline.

There are three cases where the option to delay can make a difference when valuing a firm. The first is undeveloped land in the hands of real estate investor or company. The choice of when to develop rests in the hands of the owner and presumably development will occur when real estate values increase. The second is a firm that owns a patent or patents. Since a patent provides a firm with the exclusive rights to produce the patented product or service, it can and should be valued as an option. The third is a natural resource company that has undeveloped reserves that it can choose to develop at a time of its choosing—presumably when the price of the resource is high.

THE OPTION TO DELAY A PROJECT

Projects are typically analyzed based on their expected cash flows and discount rates at the time of the analysis; the net present value computed on that basis is a measure of its value and acceptability at that time. Expected cash flows and discount rates change over time, however, and so does the net present value. Thus, a project that has a negative net present value now may have a positive net present value in the future. In a competitive environment, in which individual firms have no special advantages over their competitors in taking projects, the fact that net present values can be positive in the future may not be significant. In an environment in which a project can be taken by only one firm because of legal restrictions or other barriers to entry to competitors, however, the changes in the project's value over time give it the characteristics of a call option.

Payoff on the Option to Delay

Assume that a project requires an initial up-front investment of X, and that the present value of expected cash inflows from investing in the project, computed today, is V. The net present value of this project is the difference between the two:

$$NPV = V - X$$

Now assume that the firm has exclusive rights to this project for the next n years, and that the present value of the cash inflows may change over that time, because of changes in either the cash flows or the discount rate. Thus, the project may have a negative net present value right now, but it may still be a good project if the firm waits. Defining V again as the present value of the cash flows, the firm's decision rule on this project can be summarized as follows:

If $V > X$ Invest in the project: Project has positive net present value.
$V < X$ Do not invest in the project: Project has negative net present value.

If the firm does not invest in the project over its life, it incurs no additional cash flows, though it will lose what it invested to get exclusive rights to the project. This relationship can be presented in a payoff diagram of cash flows on this project, as shown in Figure 28.1, assuming that the firm holds out until the end of the period for which it has exclusive rights to the project.

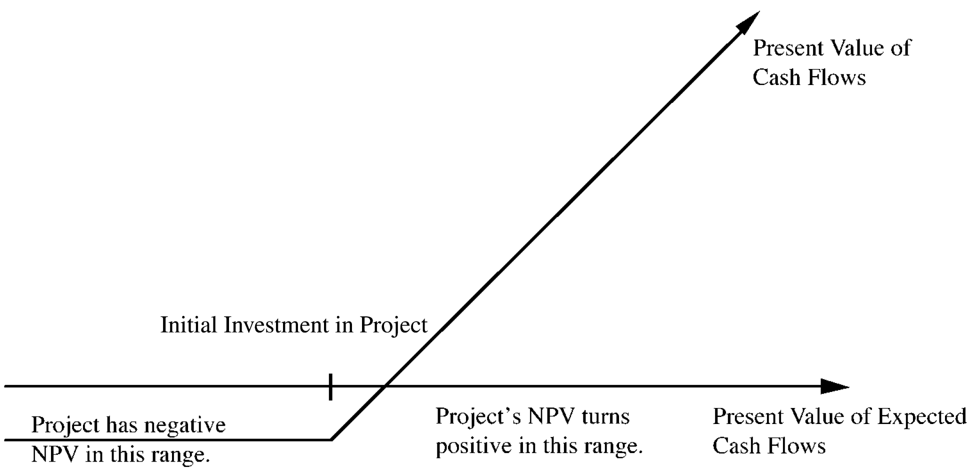

FIGURE 28.1 The Option to Delay a Project

Note that this payoff diagram is that of a call option—the underlying asset is the project, the strike price of the option is the investment needed to take the project, and the life of the option is the period for which the firm has rights to the project. The present value of the cash flows on this project and the expected variance in this present value represent the value and variance of the underlying asset.

Inputs for Valuing the Option to Delay

The inputs needed to apply option pricing theory to valuing the option to delay are the same as those needed for any option. We need the value of the underlying asset, the variance in that value, the time to expiration on the option, the strike price, the riskless rate, and the equivalent of the dividend yield.

Value of the Underlying Asset In the case of product options, the underlying asset is the project to which the firm has exclusive rights. The current value of this asset is the present value of expected cash flows from initiating the project now, not including the up-front investment. This present value can be obtained by doing a standard investment analysis. There is likely to be a substantial amount of error in the cash flow estimates and the present value, however. Rather than being viewed as a problem, this uncertainty should be viewed as the reason the project delay option has value. If the expected cash flows on the project were known with certainty and were not expected to change, there would be no need to adopt an option pricing framework, since there would be no value to the option.

Variance in the Value of the Asset As noted in the prior section, there is likely to be considerable uncertainty associated with the cash flow estimates and the present value that measures the value of the project now. This is partly because the potential market for the product may be unknown, and partly because technological shifts can change the cost structure and profitability of the product. The variance in the present value of cash flows from the project can be estimated in one of three ways.

1. If we have invested in similar projects in the past, the variance in the cash flows from those projects can be used as an estimate. This may be the way that a consumer product company like Gillette might estimate the variance associated with introducing a new blade for its razors.
2. We can assign probabilities to various market scenarios, estimate cash flows and a present value under each scenario, and then calculate the variance across present values. Alternatively, the probability distributions can be estimated for each of the inputs into the project analysis—the size of the market, the market share, and the profit margin, for instance—and simulations used to estimate the variance in the present values that emerge. This approach tends to work best when there are only one or two sources[1] of significant uncertainty about future cash flows.

[1]In practical terms, the probability distributions for inputs like market size and market share can often be obtained from market testing.

3. We can use the variance in the value of firms involved in the same business (as the project being considered) as an estimate of the variance. Thus, the average variance in the value of firms involved in the software business can be used as the variance in present value of a software project.

The value of the option is largely derived from the variance in cash flows; the higher the variance, the higher the value of the project delay option. Thus, the value of an option to invest in a project in a stable business will be less than the value of one in an environment where technology, competition, and markets are all changing rapidly.

Exercise Price on Option The option to delay a project is exercised when the firm owning the rights to the project decides to invest in it. The cost of making this initial investment is the exercise price of the option. The underlying assumption is that this cost remains constant (in present value dollars) and that any uncertainty associated with the investment is reflected in the present value of cash flows on the product.

Expiration of the Option and the Riskless Rate The project delay option expires when the rights to the project lapse. Investments made after the project rights expire are assumed to deliver a net present value of zero as competition drives returns down to the required rate. The riskless rate to use in pricing the option should be the rate that corresponds to the expiration of the option. While expiration dates can be estimated easily when firms have the explicit right to a project (through a license or a patent, for instance), they become far more difficult to obtain when the right is less clearly defined. If, for instance, a firm has a competitive advantage on a product or project, the option life can be defined as the expected period over which the advantage can be sustained.

Cost of Delay Chapter 5 noted that an American option generally will not be exercised prior to expiration. When you have the exclusive rights to a project, though, and the net present value turns positive, you would not expect the owner of the rights to wait until the rights expire to exercise the option (invest in the project). Note that there is a cost to delaying investing in a project, once the net present value turns positive. If you wait an additional period, you may gain if the variance pushes value higher but you also lose one period of protection against competition. You have to consider this cost when analyzing the option and there are two ways of estimating it:

1. Since the project rights expire after a fixed period, and excess profits (which are the source of positive present value) are assumed to disappear after that time as new competitors emerge, each year of delay translates into one less year of value-creating cash flows.[2] If the cash flows are evenly distributed over time, and the life of the patent is n years, the cost of delay can be written as:

$$\text{Annual cost of delay} = \frac{1}{n}$$

[2]A value-creating cash flow is one that adds to the net present value because it is in excess of the required return for investments of equivalent risk.

Thus, if the project rights are for 20 years, the annual cost of delay works out to $^1/_{20}$ or 5% a year. Note, though, that this cost of delay rises each year, to $^1/_{19}$ in year 2, $^1/_{18}$ in year 3, and so on, making the cost of delaying exercise larger over time.

2. If the cash flows are uneven, the cost of delay can be more generally defined in terms of the cash flow that can be expected to occur over the next period as a percent of the present value today:

$$\text{Cost of delay} = \frac{\text{Cash flow}_{\text{next period}}}{\text{Present value}_{\text{now}}}$$

In either case, the likelihood that a firm will delay investing in a project is higher early in the exclusive rights period rather than later and will increase as the loss in present value from waiting a period increases.

***optvar.xls:* This dataset on the Web summarizes standard deviations in firm value and equity value by industry group in the United States.**

ILLUSTRATION 28.1: Valuing the Option to Delay a Project

Assume that you are interested in acquiring the exclusive rights to market a new product that will make it easier for people to access their e-mail on the road. If you do acquire the rights to the product, you estimate that it will cost you $50 million up-front to set up the infrastructure needed to provide the service. Based on your current projections, you believe that the service will generate only $10 million in after-tax cash flows each year. In addition, you expect to operate without serious competition for the next five years.

From a static standpoint, the net present value of this project can be computed by taking the present value of the expected cash flows over the next five years. Assuming a discount rate of 15% (based on the riskiness of this project), we obtain the following net present value for the project:

NPV of project = – $50 million + $10 million(PV of annuity, 15%, 5 years)
= – $50 million + $33.5 million = –$16.5 million

This project has a negative net present value.

The biggest source of uncertainty about this project is the number of people who will be interested in the product. While current market tests indicate that you will capture a relatively small number of business travelers as your customers, they also indicate the possibility that the potential market could be much larger. In fact, a simulation of the project's cash flows yields a standard deviation of 42% in the present value of the cash flows, with an expected value of $33.5 million.

To value the exclusive rights to this project, we first define the inputs to the option pricing model:

Value of underlying asset (S) = PV of cash flows from product if introduced now = $33.5 million
Strike price (K) = Initial investment needed to introduce the product = $50 million
Variance in underlying asset's value = $0.42^2 = 0.1764$
Time to expiration = Period of exclusive rights to product = 5 years
Dividend yield = 1/Life of the patent = 1/5 = 0.20

Assume that the five-year riskless rate is 5%. The value of the option can be estimated as follows:

$$\text{Call value} = 33.5\ \exp^{(-0.2)(5)}(0.2250) - 50.0\ \exp^{(-0.05)(5)}(0.0451) = \$1.019\text{ million}$$

The rights to this product, which has a negative net present value if introduced today, is $1.019 million. Note, though, as measured by N(d1) and N(d2), the likelihood is low that this project will become viable before expiration.

delay.xls: **This spreadsheet allows you to estimate the value of an option to delay an investment.**

ARBITRAGE POSSIBILITIES AND OPTION PRICING MODELS

The discussion of option pricing models in Chapter 5 noted that they are based on two powerful constructs—the idea of replicating portfolios and arbitrage. Models such as the Black-Scholes and binomial assume that you can create a replicating portfolio, using the underlying asset and riskless borrowing or lending, that has cash flows identical to those on an option. Furthermore, these models assume that since investors can then create riskless positions by buying the option and selling the replicating portfolio, they have to sell for the same price. If they do not, investors should be able to create riskless positions and walk away with guaranteed profits—the essence of arbitrage. This is why the interest rate used in option pricing models is the riskless rate.

With listed options on traded stocks or assets, arbitrage is clearly feasible, at least for some investors. With options on nontraded assets, it is almost impossible to trade the replicating portfolio, although you can create it on paper. In Illustration 28.1, for instance, you would need to buy 0.225 units (the option delta) of the underlying project (a nontraded asset) to create a portfolio that replicates the call option.

There are some who argue that the impossibility of arbitrage makes it inappropriate to use option pricing models to value real options, whereas others try to adjust for this limitation by using an interest rate higher than the riskless rate in the option pricing model. We do not think that either of these responses is appropriate. Note that while you cannot trade on the replicating portfolios in many real options, you still can create them on paper (as we did in Illustration 28.1) and value the options. The difficulties in creating arbitrage positions may result in prices that deviate by a large amounts from this value, but that is an argument for using real option pricing models and not for avoiding them. Increasing the riskless rate to reflect the higher risk associated with real options may seem like an obvious fix, but doing this will only make call options (such as the one valued in Illustration 28.1) more valuable, not less.

If you want to be more conservative in your estimate of value for real options to reflect the difficulty of arbitrage, you have two choices. One is to use a higher discount rate in computing the present value of the cash flows that you would expect to make from investing in the project today, thus lowering the value of the underlying asset (S) in the model. In Illustration 28.1, using a 20 percent discount rate rather than a 15 percent rate would result in a present value of $29.1 million, which would replace the $33.5 million as S in the model. The other choice is to value the option and then apply an illiquidity discount to it (similar to the one we used in valuing private companies) because you cannot trade it easily.

Problems in Valuing the Option to Delay

While it is quite clear that the option to delay is embedded in many projects, several problems are associated with the use of option pricing models to value these options. First, the underlying asset in this option, which is the project, is not traded, making it difficult to estimate its value and variance. The value can be estimated from the expected cash flows and the discount rate for the project, albeit with error. The variance is more difficult to estimate, however, since we are attempting the estimate a variance in project value over time.

Second, the behavior of prices over time may not conform to the price path assumed by the option pricing models. In particular, the assumption that value follows a diffusion process, and that the variance in value remains unchanged over time, may be difficult to justify in the context of a project. For instance, a sudden technological change may dramatically change the value of a project, either positively or negatively.

Third, there may be no specific period for which the firm has rights to the project. Unlike the case of a patent, for instance, in which the firm has exclusive rights to produce the patented product for a specified period, the firm's rights often are less clearly defined, in terms of both exclusivity and time. For instance, a firm may have significant advantages over its competitors, which may, in turn, provide it with the virtually exclusive rights to a project for a period of time. An example would be a company with strong brand name recognition in retailing or consumer products. The rights are not legal restrictions, however, and will erode over time. In such cases, the expected life of the project itself is uncertain and only an estimate. In the valuation of the rights to the product in the previous section a life of five years for the option was used, but competitors could in fact enter sooner than anticipated. Alternatively, the barriers to entry may turn out to be greater than expected, and allow the firm to earn excess returns for longer than five years. Ironically, uncertainty about the expected life of the option can increase the variance in present value, and through it, the expected value of the rights to the project.

Implications and Extensions of Delay Options

Several interesting implications emerge from the analysis of the option to delay a project as an option. First, a project may have a negative net present value currently based on expected cash flows, but the rights to it may still be valuable because of the option characteristics.

Second, a project may have a positive net present value but still not be accepted right away. This can happen because the firm may gain by waiting and accepting the project in a future period, for the same reasons that investors do not always exercise an option that is in the money. A firm is more likely to wait if it has the rights to the project for a long time, and the variance in project inflows is high. To illustrate, assume a firm has the patent rights to produce a new type of disk drive for computer systems and building a new plant will yield a positive net present value today. If the technology for manufacturing the disk drive is in flux, however, the firm may delay investing in the project in the hopes that the improved technology will increase the expected cash flows and consequently the value of the project. It has to weigh this benefit against the cost of delaying the project, which will be the cash flows that will be forsaken by not investing in it.

Third, factors that can make a project less attractive in a static analysis can actually make the rights to the project more valuable. As an example, consider the effect of uncertainty about the size of the potential market and the magnitude of excess returns. In a static analysis, increasing this uncertainty increases the riskiness of the project and may make it less attractive. When the project is viewed as an option, an increase in the uncertainty may actually make the option more valuable, not less. The chapter will consider two cases, product patents and natural resource reserves, where the project delay option allows value to be estimated more precisely.

Option Pricing Models

Once you have identified the option to delay a project as a call option and identified the inputs needed to value the option, it may seem like a trivial task to actually value the option. There are, however, some serious estimation issues that we have to deal with in valuing these options. Chapter 5 noted that while the more general model for valuing options is the binomial model, many practitioners use the Black-Scholes model, which makes far more restrictive assumptions about price processes and early exercise to value options. With listed options on traded assets, you can do this at fairly low cost. With real options, there can be a substantial cost to this practice for the following reasons:

- Unlike listed options, real options tend to be exercised early, if they are in the money. While there are ways in which the Black-Scholes model can be adjusted to allow for this early exercise, the binomial model allows for much more flexibility.
- The binomal option pricing model allows for a much wider range of price processes for the underlying asset than the Black-Scholes model, which assumes that prices are not only continuous but log-normally distributed. With real options, where the present value of the cash flows is often equivalent to the price, the assumptions of nonnormality and continuous distributions may be difficult to sustain.

The biggest problem with the binomial model is that the prices at each node of the binomial tree have to be estimated. As the number of periods expands, this will become more and more difficult to do. You can, however, use the variance estimate in the Black-Scholes to come up with measures of the magnitude of the up and down movements, which can be used to obtain the binomial tree.

Having made a case for the binomial model, you may find it surprising that we use the Black-Scholes model to value any real options. We do so not only because the model is more compact and elegant to present, but because we believe that it will provide a lower bound on the value in most cases. To provide a frame of reference, we will present the values that we would have obtained using a binomial model in each case.

From Black-Scholes to Binomial It is a fairly simple exercise to convert the inputs to the Black-Scholes model into a binomial model. To make the adjustment, you have to assume a multiplicative binomial process, where the magnitude of the jumps, in percent terms, remains unchanged from period to period. If you assume symmetric probabilities, the up (u) and down (d) movements can be estimated as a function of the annualized variance in the price process and how many periods you decide to break each year into (t).

$$u = \exp^{\sigma\sqrt{dt}+\left(r-y-\frac{\sigma^2}{2}\right)dt}$$

$$d = \exp^{-\sigma\sqrt{dt}+\left(r-y-\frac{\sigma^2}{2}\right)dt}$$

where dt = 1/Number of periods each year

To illustrate, consider the project delay option valued in Illustration 28.1. The standard deviation in the value was assumed to be 42 percent, the risk-free rate was 5%, and the dividend yield was 20 percent. To convert the inputs into a binomial model, assume that each year is a time period and estimate the up and down movements as follows:

$$u = \exp^{.42\sqrt{1}+\left(.05-.20-\frac{.42^2}{2}\right)\sqrt{t}} = 1.1994$$

$$d = \exp^{-.42\sqrt{1}+\left(.05-.20-\frac{.42^2}{2}\right)1t} = 0.5178$$

The value today is \$33.5 million. To estimate the end values for the first branch:

Value with up movement = \$33.5(1.1994) = \$40.179 million

Value with down movement = \$33.5(0.5178) = \$17.345 million

You could use these values then to get the three potential values at the second branch. Note that the value of \$17.345 million growing at 19.94 percent is exactly equal to the value of \$40.179 million dropping by 48.22 percent. The binomial tree for the five periods is shown in Figure 28.2.

You could estimate the value of the option from this binomial tree to be \$1.02 million, slightly higher than the estimate obtained from the Black-Scholes model of \$1.019 million. The differences will narrow as the option becomes more in-the-money and you shorten the time periods you use in the binomial model.

VALUING A PATENT

A number of firms, especially in the technology and pharmaceutical sectors, can patent products or services. A product patent provides a firm with the right to develop and market a product, and thus can be viewed as an option.

Patents as Call Options

The firm will develop a patent only if the present value of the expected cash flows from the product sales exceed the cost of development, as shown in Figure 28.3. If

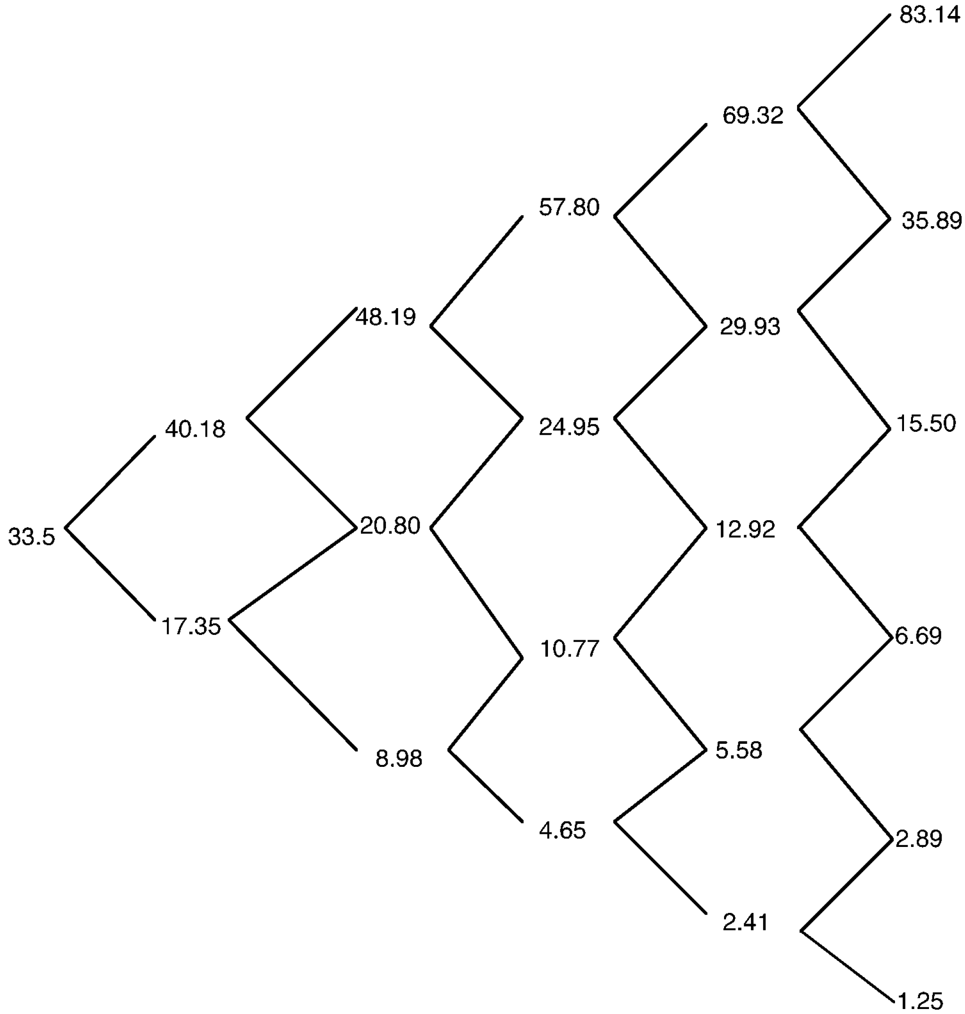

FIGURE 28.2 Binomial Tree for Delay Option

this does not occur, the firm can shelve the patent and not incur any further costs. If I is the present value of the costs of commercially developing the patent and V is the present value of the expected cash flows from development, then:

$$\text{Payoff from owning a product patent} = V - I \quad \text{if } V > I$$
$$= 0 \quad \text{if } V \leq I$$

Thus a product patent can be viewed as a call option, where the product is the underlying asset.

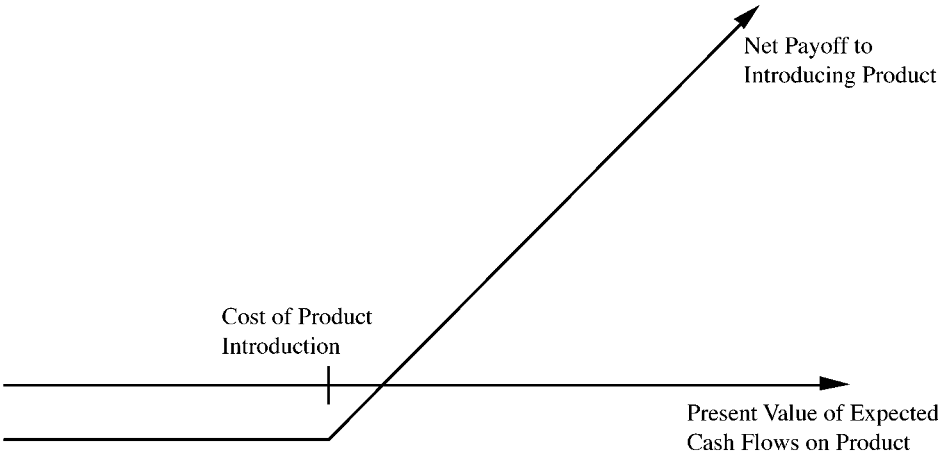

FIGURE 28.3 Payoff to Introducing Product

ILLUSTRATION 28.2: Valuing a Patent: Avonex in 1997

Biogen is a biotechnology firm with a patent on a drug called Avonex, which has received FDA approval for use in treating multiple sclerosis (MS). Assume you are trying to value the patent and that you have the following estimates for use in the option pricing model:

- An internal analysis of the financial viability of the drug today, based on the potential market and the price that the firm can expect to charge for the drug, yields a present value of cash flows of \$3.422 billion prior to considering the initial development cost.
- The initial cost of developing the drug for commercial use is estimated to be \$2.875 billion, if the drug is introduced today.
- The firm has the patent on the drug for the next 17 years, and the current long-term Treasury bond rate is 6.7%.
- The average variance in firm value for publicly traded biotechnology firms is 0.224.

We assume that the potential for excess returns exists only during the patent life, and that competition will eliminate excess returns beyond that period. Thus, any delay in introducing the drug, once it becomes viable, will cost the firm one year of patent-protected returns. (For the initial analysis, the cost of delay will be $\frac{1}{17}$, next year it will be $\frac{1}{16}$, the year after $\frac{1}{15}$, and so on.)

Based on these assumptions, we obtain the following inputs to the option pricing model.

Present value of cash flows from introducing the drug now = S = \$3.422 billion
Initial cost of developing drug for commercial use (today) = K = \$2.875 billion
Patent life = t = 17 years
Riskless rate = r = 6.7% (17-year Treasury bond rate)
Variance in expected present values = σ^2 = 0.224
Expected cost of delay = y = 1/17 = 5.89%

These yield the following estimates for d and N(d):

d1 = 1.1362 N(d1) = 0.8720
d2 = –0.8512 N(d2) = 0.2076

Plugging back into the dividend-adjusted Black-Scholes option pricing model,[3] we get:

Value of the patent = 3,422 exp(–0.0589)(17)(0.8720) – 2,875 exp(–0.067)(17)(0.2076) = $907 million

To provide a contrast, the net present value of this project is only $547 million:

NPV = $3,422 million – $2,875 million = $547 million

The time premium of $360 million on this option ($907 – $547) suggests that the firm will be better off waiting rather than developing the drug immediately, the cost of delay notwithstanding. However, the cost of delay will increase over time, and make exercise (development) more likely in future years.

To illustrate, we will value the call option, assuming that all of the inputs, other than the patent life, remain unchanged and changing the patent life. For instance, assume that there are 16 years left on the patent. Holding all else constant, the cost of delay increases as a result of the shorter patent life:

Cost of delay = 1/16

The decline in the present value of cash flows (which is S) and increase in the cost of delay (y) reduce the expected value of the patent. Figure 28.4 graphs the option value and the net present value of the project each year.

Based on this analysis, if nothing changes, you would expect Avonex to be worth more as a commercial product than as a patent if there were less than eight years left on the patent, which would also then be the optimal time to commercially develop the product.

product.xls: **This spreadsheet allows you to estimate the value of a patent.**

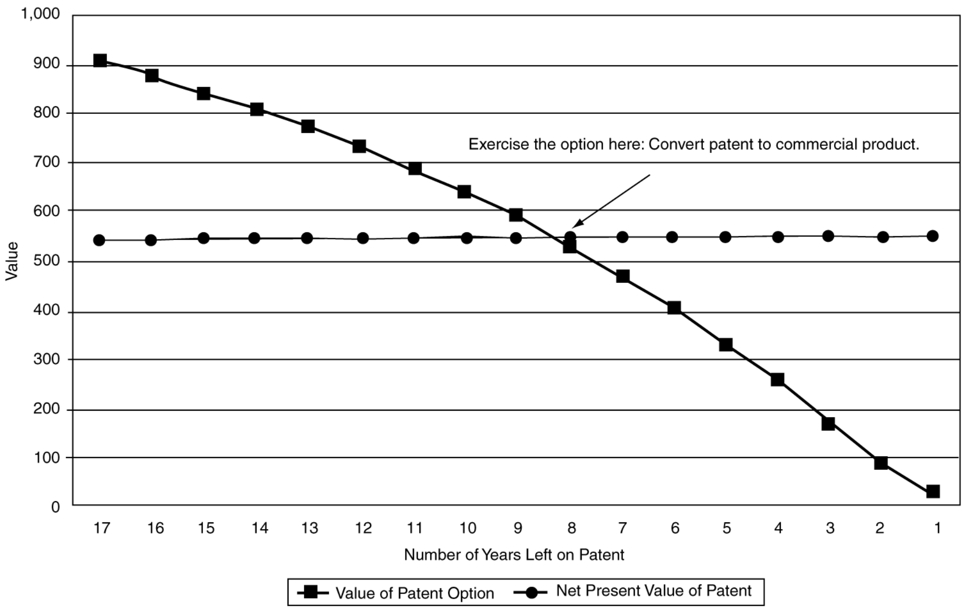

FIGURE 28.4 Patent Value versus Net Present Value

[3]With a binomial model, we estimate a value of $915 million for the same option.

COMPETITIVE PRESSURES AND OPTION VALUES

The preceding section has taken the view that a firm is protected from competition for the life of the patent. This is generally true only for the patented product or process, but the firm may still face competition from other firms that come up with their own products to serve the same market. More specifically, Biogen can patent Avonex, but Merck or Pfizer can come up with their own drugs to treat multiple sclerosis and compete with Biogen.

What are the implications for the value of the patent as an option? First, the life of the option will no longer be the life of the patent but the lead time that the firm has until a competing product is developed. For instance, if Biogen knows that another pharmaceutical firm is working on a drug to treat MS and where this drug is in the research pipeline (early research or stage in the FDA approval process), it can use its estimate of how long it will take before the drug is approved for use as the life of the option. This will reduce the value of the option and make it more likely that the drug will be commercially developed earlier rather than later.

The presence of these competitive pressures may explain why commercial development is much quicker with some drugs than with others, and why the value of patents is not always going to be greater than a discounted cash flow valuation. Generally speaking, the greater the number of competing products in the research pipeline, the less likely it is that the option pricing model will generate a value that is greater than the traditional discounted cash flow model.

Valuing a Firm with Patents

If the patents owned by a firm can be valued as options, how can this estimate be incorporated into firm value? The value of a firm that derives its value primarily from commercial products that emerge from its patents can be written as a function of three variables:

1. The cash flows it derives from patents that it has already converted into commercial products.
2. The value of the patents that it already possesses that have not been commercially developed.
3. The expected value of any patents that the firm can be expected to generate in future periods from new patents that it might obtain as a result of its research.

$$\begin{aligned}\text{Value of firm} = {} & \text{Value of commercial products} + \text{Value of existing patents} \\ & + (\text{Value of new patents that will be obtained in the future} \\ & - \text{Cost of obtaining these patents})\end{aligned}$$

The value of the first component can be estimated using traditional cash flow models. The expected cash flows from existing products can be estimated for their commercial lives and discounted back to the present at the appropriate cost of capital to arrive at the value of these products. The value of the second component can be obtained using the option pricing model described earlier to value each patent.

The value of the third component will be based on perceptions of a firm's research capabilities. In the special case where the expected cost of research and development in future periods is equal to the value of the patents that will be generated by this research, the third component will become zero. In the more general case, firms such as Merck and Pfizer that have a history of generating value from research will derive positive value from this component as well.

How would the estimate of value obtained using this approach contrast with the estimate obtained in a traditional discounted cash flow model? In traditional discounted cash flow valuation, the second and the third components of value are captured in the expected growth rate in cash flows. Firms such as Pfizer are allowed to grow at much higher rates for longer periods because of the technological edge they possess and their research prowess. In contrast, the approach described in this section looks at each patent separately and allows for the option component of value explicitly.

The biggest limitation of the option-based approach is the information that is needed to put it in practice. To value each patent separately, you need access to proprietary information that is usually available only to managers of the firm. In fact, some of the information, such as the expected variance to use in option pricing, may not even be available to insiders and will have to be estimated for each patent separately.

Given these limitations, the real option approach should be used to value small firms with one or two patents and little in terms of established assets. A good example would be Biogen in 1997, which was valued in the preceding section. For firms such as Merck and Pfizer that have significant assets in place and dozens of patents, discounted cash flow valuation is a more pragmatic choice. Viewing new technology as options provides insight into Cisco's successful growth strategy over the previous decade. Cisco has been successful at buying firms with nascent and promising technologies (options) and converting them into commercial success (exercising these options).

ILLUSTRATION 28.3: Valuing Biogen as a Firm

In illustration 28.2, the patent that Biogen owns on Avonex was valued as a call option and the estimated value was $907 million. To value Biogen as a firm two other components of value would have to be considered:

1. Biogen had two commercial products (a drug to treat hepatitis B and a drug called Intron) at the time of this valuation that it had licensed to other pharmaceutical firms. The license fees on these products were expected to generate $50 million in after-tax cash flows each year for the next 12 years. To value these cash flows, which were guaranteed contractually, the pretax cost of debt of the licensing firms (7%) was used:

$$\text{Present value of license fees} = \$50 \text{ million}\left[\frac{1-1.07^{-12}}{.07}\right] = \$397.13 \text{ million}$$

2. Biogen continued to fund research into new products, spending about $100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years and 5% thereafter. While it was difficult to forecast the specific patents that would emerge from this research, it was assumed that every dollar invested in research would create $1.25 in value in patents[4] (valued using the option pricing model described earlier) for the next 10 years, and break even after that (i.e., generate $1 in patent value for every $1 invested in R&D). There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.[5] The value of this component was then estimated as follows:

$$\text{Value of future research} = \sum_{t=1}^{t=\infty} \frac{(\text{Value of patents}_t - \text{R\&D}_t)}{(1 + r)^t}$$

The following table summarizes the value of patents generated each period and the R&D costs in that period. Note that there is no surplus value created after the tenth year:

Year	*Value of Patents Generated*	*R&D Cost*	*Excess Value*	*Present Value at 15%*
1	$150.00	$120.00	$ 30.00	$ 26.09
2	$180.00	$144.00	$ 36.00	$ 27.22
3	$216.00	$172.80	$ 43.20	$ 28.40
4	$259.20	$207.36	$ 51.84	$ 29.64
5	$311.04	$248.83	$ 62.21	$ 30.93
6	$373.25	$298.60	$ 74.65	$ 32.27
7	$447.90	$358.32	$ 89.58	$ 33.68
8	$537.48	$429.98	$107.50	$ 35.14
9	$644.97	$515.98	$128.99	$ 36.67
10	$773.97	$619.17	$154.79	$ 38.26
				$318.30

The total value created by new research is $318.3 million.

The value of Biogen as a firm is the sum of all three components—the present value of cash flows from existing products, the value of Avonex (as an option), and the value created by new research:

Value = CF: commercial products + Value: undeveloped patents + Value: future R&D
= $397.13 million + $907 million + $318.30 million = $1,622.43 million

Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.5 million) to arrive at a value per share:

Value per share = $1,622.43 million/35.5 = $45.70

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[4]To be honest, this is not an estimate based on any significant facts other than Biogen's history of success in coming up with new products. You can obtain an estimate of this number from the return and cost of capital. For instance, if you assume a return on capital of 15 percent and cost of capital of 10 percent in perpetuity, $1 invested would yield the following:

$$\text{Value of created} = 1 + \frac{(\text{ROC} - \text{Cost of capital})}{\text{Capital invested}} = 1 + \frac{(.15 - .10)}{.10} = \$1.50$$

[5]This discount rate was estimated by looking at the costs of equity of young publicly traded biotechnology firms with little or no revenue from commercial products.

IS THERE LIFE AFTER THE PATENT EXPIRES?

In these valuations it has been assumed that the excess returns are restricted to the patent life and that they disappear the instant the patent expires. In the pharmaceutical sector, the expiration of a patent does not necessarily mean the loss of excess returns. In fact, many firms continue to be able to charge a premium price for their products and earn excess returns even after the patent expires, largely as a consequence of the brand name image that they built up over the project life. A simple way of adjusting for this reality is to increase the present value of the cash flows on the project (S) and decrease the cost of delay (y) to reflect this reality. The net effect is a greater likelihood that firms will delay commercial development while they wait to collect more information and assess market demand.

The other thing that might increase the value of the patent is the capacity that drug companies have shown to lobby legislators to extend the patent life of profitable drugs. If we consider this as a possibility when we value a patent, it will increase the expected life of the patent and its value as an option.

NATURAL RESOURCE OPTIONS

Natural resource companies, such as oil and mining companies, generate cash flows from their existing reserves but also have undeveloped reserves that they can develop if they choose to do so. They will be much more likely to develop these reserves if the price of the resource (oil, gold, copper) increases and these undeveloped reserves can be viewed as call options. This section will begin by looking at the value of an undeveloped reserve and then consider how this can be extended to look at natural resource companies that have both developed and undeveloped reserves.

Undeveloped Reserves as Options

In a natural resource investment, the underlying asset is the natural resource and the value of the asset is based on the estimated quantity and the price of the resource. Thus, in a gold mine, the underlying asset is the value of the estimated gold reserves in the mine, based on the price of gold. In most such investments, there is an initial cost associated with developing the resource; the difference between the value of the estimated reserves and the cost of the development is the profit to the owner of the resource (see Figure 28.5). Defining the cost of development as X, and the estimated value of the resource as V makes the potential payoffs on a natural resource option the following:

$$\text{Payoff on natural resource investment} = \begin{cases} V - X & \text{if } V > X \\ 0 & \text{if } V \leq X \end{cases}$$

Thus the investment in a natural resource option has a payoff function that resembles a call option.

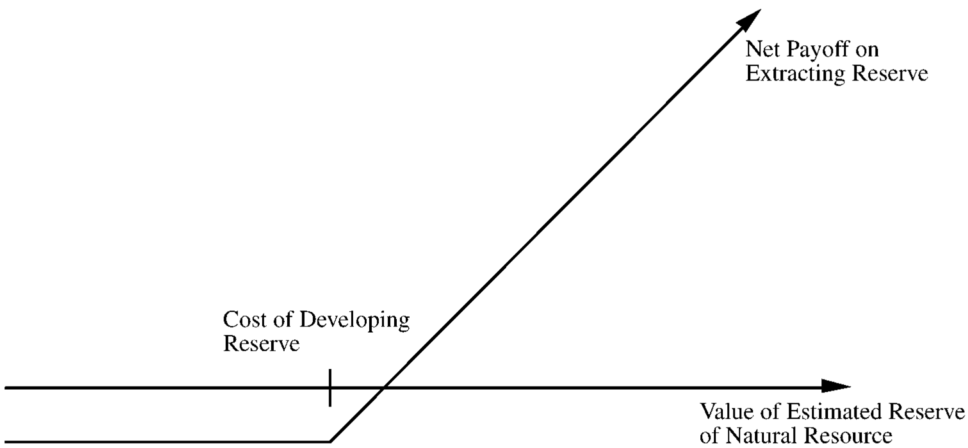

FIGURE 28.5 Payoff from Developing Natural Resource Reserves

Inputs for Valuing a Natural Resource Option To value a natural resource investment as an option, we need to make assumptions about a number of variables:

- *Available reserves of the resource and estimated value if extracted today.* Since the quantity of the reserve is not known with certainty at the outset, it has to be estimated. In an oil tract, for instance, geologists can provide reasonably accurate estimates of the quantity of oil available in the tract. The value of the reserves is then the product of the estimated reserves and the contribution (price of the resource minus variable cost of extraction) per unit of reserve.
- *Estimated cost of developing the resource.* The estimated cost of developing the resource reserve is the exercise price of the option. In an oil reserve, this would be the fixed cost of installing the rigs to extract oil from the reserve. With a mine, it would be the cost associated with making the mine operational. Since oil and mining companies have done this before in a variety of settings, they can use their experience to come up with a reasonable measure of development cost.
- *Time to expiration of the option.* The life of a natural resource option can be defined in one of two ways. First, if the ownership of the investment has to be relinquished at the end of a fixed period of time, that period will be the life of the option. In many offshore oil leases, for instance, the oil tracts are leased to the oil company for a fixed period. The second approach is based on the inventory of the resource and the capacity output rate, as well as estimates of the number of years it would take to exhaust the inventory. Thus, a gold mine with a mine inventory of 3 million ounces and a capacity output rate of 150,000 ounces a year will be exhausted in 20 years, which is defined as the life of the natural resource option.
- *Variance in value of the underlying asset.* The variance in the value of the underlying asset is determined by the variability in the price of the resource and the variability in the estimate of available reserves. In the special case where the quantity of the reserve is known with certainty, the variance in the underlying asset's value will depend entirely on the variance in the price of the natural resource.
- *Cost of delay.* The net production revenue is the annual cash flow that will be generated, once a resource reserve has been developed, as a percentage of the

market value of the reserve. This is the equivalent of the dividend yield and is treated the same way in calculating option values. An alternative way of thinking about this cost is in terms of a cost of delay. Once a natural resource option is in-the-money (value of the reserves is greater than the cost of developing these reserves), by not developing the reserve the firm is costing itself the net production revenue it could have generated by doing so.

An important issue in using option pricing models to value natural resource options is the effect of development lags on the value of these options. Since oil or gold or any other natural resource reserve cannot be developed instantaneously, a time lag has to be allowed between the decision to extract the resources and the actual extraction. A simple adjustment for this lag is to reduce the value of the developed reserve for the loss of cash flows during the development period. Thus, if there is a one-year lag in development, you can estimate the cash flow you would make over the year as a percent of your reserve value, and discount the current value of the developed reserve at that rate. This is the equivalent of removing the first year's cash flow from your investment analysis and lowering the present value of your cash flows.

ILLUSTRATION 28.4: Valuing an Oil Reserve[6]

Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil; the cost of developing the reserve is expected to be $600 million, and the development lag is two years. Exxon has the rights to exploit this reserve for the next 20 years, and the marginal value (price per barrel minus marginal cost per barrel) per barrel of oil is currently $12.[7] Once developed, the net production revenue each year will be 5% of the value of the reserves. The riskless rate is 8%, and the variance in oil prices is 0.03.

Given this information, the inputs to the Black-Scholes model can be estimated as follows:

Current value of the asset = S = Value of the developed reserve discounted back the length of the development lag at the dividend yield = $\$12 \times 50/(1.05)^2 = \$544.22$

Exercise price = Cost of developing reserve = $600 million

Time to expiration on the option = 20 years

Variance in the value of the underlying asset[8] = 0.03

Riskless rate = 8%

Dividend yield = Net production revenue/Value of reserve = 5%

Based on these inputs, the Black-Scholes model provides the following call value:

$$d1 = 1.0359 \qquad N(d1) = 0.8498$$

$$d2 = 0.2613 \qquad N(d2) = 0.6030$$

$$\text{Call value} = 544.22\ \exp^{(-0.05)(20)}(0.8498) - 600\ \exp^{(-0.08)(20)}(0.6030) = \$97.08 \text{ million}$$

This oil reserve, though not viable at current prices, is still valuable because of its potential to create value if oil prices go up.[9]

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[6]The following is a simplified version of the illustration provided by Siegel, Smith, and Paddock to value an offshore oil property.

[7]For simplicity, we will assume that while this marginal value per barrel of oil will grow over time, the present value of the marginal value will remain unchanged at $12 per barrel. If we do not make this assumption, we will have to estimate the present value of the oil that will be extracted over the extraction period.

[8]In this example, we assume that the only uncertainty is in the price of oil, and the variance therefore becomes the variance in ln(oil prices).

[9]With a binomial model, we arrive at an estimate of value of $99.15 million.

natres.xls: **This spreadsheet allows you to estimate the value of an undeveloped natural resource reserve.**

MULTIPLE SOURCES OF UNCERTAINTY

In the preceding example, we assumed that there was no uncertainty about the quantity of the reserve. Realistically, the oil company has an estimate of the reserve of 50 million barrels but does not know it with certainty. If we introduce uncertainty about the quantity of the reserve into the analysis, there will be two sources of variance and both can affect value. There are two ways we can address this problem:

1. *Combine the uncertainties into one value.* If we consider the value of the reserves to be the product of the price of oil and the oil reserves, the variance in the value should reflect the combined effect of the variances in each input.[10] This would be the variance we would use in the option pricing model to estimate a new value for the reserve.
2. *Keep the variances separate and value the option as a rainbow option.* A rainbow option allows explicitly for more than one source of variance and allows us to keep the variances separate and still value the option. While option pricing becomes more complicated, you may need to do this if you expect the two sources of uncertainty to evolve differently over time—the variance from one source (say, oil prices) may increase over time whereas the variance from the other source (say, oil reserves) may decrease over time.

Valuing a Firm with Undeveloped Reserves

The examples provided above illustrate the use of option pricing theory in valuing individual mines and oil tracts. Since the assets owned by a natural resource firm can be viewed primarily as options, the firm itself can be valued using option pricing models.

Individual Reserves versus Aggregate Reserves The preferred approach would be to consider each option separately, value it and cumulate the values of the options

[10]This is the variance of a product of two variables.

to get the value of the firm. Since this information is likely to be difficult to obtain for large natural resource firms, such as oil companies, which own hundreds of such assets, a variant of this approach is to value the entire firm's undeveloped reserves as one option. A purist would probably disagree, arguing that valuing an option on a portfolio of assets (as in this approach) will provide a lower value than valuing a portfolio of options (which is what the natural resource firm really owns). Nevertheless, the value obtained from the model still provides a reasonable estimate of the value of undeveloped reserves.

Inputs to Option Valuation If you decide to apply the option pricing approach to estimate the value of aggregate undeveloped reserves, you have to estimate the inputs to the model. In general terms, while the process resembles the one used to value an individual reserve, there are a few differences.

- *Value of underlying asset.* You should cumulate all of the undeveloped reserves owned by a company and estimate the value of these reserves, based on the price of the resource today and the average variable cost of extracting these reserves today. The variable costs are likely to be higher for some reserves and lower for others, and weighting the variable costs at each reserve by the quantity of the resource of that reserve should give you a reasonable approximation of this value. At least hypothetically, we are assuming that the company can decide to extract all of its undeveloped reserves at one time and not affect the price of the resource.
- *Exercise price.* For this input, you should consider what it would cost the company today to develop all of its undeveloped reserves. Again, the costs might be higher for some reserves than for others, and you can use a weighted average cost.
- *Life of the option.* A firm will probably have different lives for each of its reserves. As a consequence, you will have to use a weighted average of the lives of the different reserves.[11]
- *Variance in the value of the asset.* Here, there is a strong argument for looking at only the oil price as the source of variance, since a firm should have a much more precise estimate of its total reserves than it does of any one of its reserves.
- *Dividend yield (cost of delay).* As with an individual reserve, a firm with viable reserves will be giving up the cash flows it could receive in the next period from developing these reserves if it delays exercise. This cash flow, stated as a percent of the value of the reserves, becomes the equivalent of the dividend yield. The development lag reduces the value of this option just as it reduces the value of an individual reserve. The logical implication is that undeveloped reserves will be worth more at oil companies that can develop their reserves quicker than at less efficient companies.

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[11]If you own some reserves in perpetuity, you should cap the life of the reserve at a large value—say, 30 years—in making this estimate.

ILLUSTRATION 28.4: Valuing an Oil Company: Gulf Oil in 1984

Gulf Oil was the target of a takeover in early 1984 at $70 per share (It had 165.30 million shares outstanding and total debt of $9.9 billion). It had estimated reserves of 3,038 million barrels of oil and the total cost of developing these reserves at that time was estimated to be $30.38 billion dollars (the development lag is approximately two years). The average relinquishment life of the reserves is 12 years. The price of oil was $22.38 per barrel, and the production costs, taxes, and royalties were estimated at $7 per barrel. The bond rate at the time of the analysis was 9.00%. If Gulf were to choose to develop these reserves, it was expected to have cash flows next year of approximately 5% of the value of the developed reserves. The variance in oil prices is 0.03.

Value of underlying asset = Value of estimated reserves discounted back for period of development lag
= $3{,}038 \times (\$22.38 - \$7)/1.05^2 = \$42{,}380.44$

Note that you could have used forecasted oil prices and estimated cash flows over the production period and estimated the value of the underlying asset to be the present value of all of these cash flows. We have used a shortcut of assuming that the current contribution margin of $15.38 a barrel will remain unchanged in present value terms over the production period.

Exercise price = Estimated cost of developing reserves today = $30,380 million
Time to expiration = Average length of relinquishment option = 12 years
Variance in value of asset = Variance in oil prices = 0.03
Riskless interest rate = 9%
Dividend yield = Net production revenue/Value of developed reserves = 5%

Based on these inputs, the Black-Scholes model provides the following value for the call:[12]

$$d1 = 1.6548 \qquad N(d1) = 0.9510$$
$$d2 = 1.0548 \qquad N(d2) = 0.8542$$

$$\text{Call value} = 42{,}380.44\ \exp^{(-0.05)(12)}(0.9510) - 30{,}380\ \exp^{(-0.09)(12)}(0.8542) = \$13{,}306 \text{ million}$$

This stands in contrast to the discounted cash flow value of $12 billion that you obtain by taking the difference between the present value of the cash flows of developing the reserve today ($42.38 billion) and the cost of development ($30.38 billion). The difference can be attributed to the option possessed by Gulf to choose when to develop its reserves.

The option value ($13.3 billion) represents the value of the undeveloped reserves of oil owned by Gulf Oil. In addition, Gulf Oil had free cash flows to the firm from its oil and gas production of $915 million from already developed reserves and assume that these cashflows are likely to be constant and continue for 10 years (the remaining lifetime of developed reserves). The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:

$$\text{Value of already developed reserves} = 915(1 - 1.125^{-10})/.125 = \$5{,}065.83$$

Adding the value of the developed and undeveloped reserves of Gulf Oil provides the value of the firm.

Value of undeveloped reserves	$13,306 million
Value of production in place	$ 5,066 million
Total value of firm	$18,372 million
Less outstanding debt	$ 9,900 million
Value of equity	$ 8,472 million
Value per share	$ 8,472/165.3 = $51.25

This analysis would suggest that Gulf Oil was overvalued at $70 per share.

[12]With a binomial model, we estimate the value of the reserves to be $13.73 billion.

PRICE VOLATILITY AND NATURAL RESOURCE COMPANY VALUATION

An interesting implication of this analysis is that the value of a natural resource company depends not just on the price of the natural resource but also on the expected volatility in that price. Thus, if the price of oil goes from $25 a barrel to $40 a barrel, you would expect all oil companies to become more valuable. If the price drops back to $25, the values of oil companies may not decline to their old levels, since the perceived volatility in oil prices may have changed. If investors believe that the volatility in oil prices has increased, you would expect an increase in values but the increase will be greatest for companies that derive a higher proportion of their value from undeveloped reserves.

If you regard undeveloped reserves as options, discounted cash flow valuation will generally underestimate the value of natural resource companies, because the expected price of the commodity is used to estimate revenues and operating profits. As a consequence, you miss the option component of value. Again, the difference will be greatest for firms with significant undeveloped reserves and with commodities where price volatility is highest.

OTHER APPLICATIONS

While patents and undeveloped reserves of natural resource companies lend themselves best to applying option pricing, there are other assets referenced in earlier chapters that can also be valued as options.

- Chapter 26, in the context of real estate valuation, noted that vacant land could be viewed as an option on commercial development.
- Chapter 27 presented an argument that copyrights and licenses could be viewed as options, even if they are not commercially viable today.

Table 28.1 presents the inputs you would use to value each of these options in an option pricing model. Much of what we have said about the other option applications apply here as well. The value is derived from the exclusivity that you have to commercially develop the asset. That exclusivity is obtained by legal sanction in the case of licenses and copyrights, and from the scarcity of land in the case of undeveloped land.

CONCLUSION

In traditional investment analysis, we compute the net present value of a project's cash flows and conclude that firms should not invest in a project with a negative net present value. This is generally good advice, but it does not imply that the rights to this project are not valuable. Projects that have negative net present values today may have positive net present values in the future, and the likelihood of this occurring is directly a function of the volatility in the present value of the cash flows from the project.

TABLE 28.1 Inputs to Value Other Options to Delay

	Undeveloped Land	License/Copyright
Value of the underlying asset	Present value of the cash flows that would be obtained from commercial development of land today.	Present value of the cash flows that would be obtained from commercially utilizing the license or copyright today.
Variance in value of underlying asset	Variance in the values of commercial property in the area where the real estate is located.	Variance in the present values from commercial utilization of copyright or license (from a simulation).
Exercise price	Cost of commercially developing land today.	Up-front cost of commercially utilizing copyright or license today.
Life of the option	If land is under long-term lease, you could use the lease period. If not, you should set the option life equal to the period when the loan that you used to buy the land comes due.	Period for which you have rights to copyright or license.
Cost of delay	Interest payments you have to make on the loan each year.	Cash flow you could generate in next year as a percent of present value of the cash flows today.

This chapter valued the option to delay an investment and considered the implications of this option for three valuation scenarios—the value of a firm that derives all or a significant portion of value from patents that have not been commercially exploited yet, the value of a natural resource company with undeveloped reserves of the resource, and the value of a real estate firm with undeveloped land. In each case, we showed that using discounted cash flow valuation would result in an understatement of the values of these firms.

QUESTIONS AND SHORT PROBLEMS

1. A company is considering delaying a project with after-tax cash flows of $25 million but that costs $300 million to take on (the life of the project is 20 years, and the cost of capital is 16%). A simulation of the cash flows leads you to conclude that the standard deviation in the present value of cash inflows is 20%. If you can acquire the rights to the project for the next 10 years, what is the value of the rights? (The six-month T-bill rate is 8%, the 10-year bond rate is 12%, and the 20-year bond rate is 14%.)
2. You are examining the financial viability of investing in some abandoned copper mines in Chile, which still have significant copper deposits in them. A geologist survey suggests that there might be 10 million pounds of copper in the mines still, and that the cost of opening up the mines will be $3 million (in present value dollars). The capacity output rate is 400,000 pounds a year, and the price of copper is expected to increase 4% a year. The Chilean government is willing

to grant a 25-year lease on the mine. The average production cost is expected to be 40 cents a pound, and the current price per pound of copper is 85 cents. (The production cost is expected to grow 3% a year, once initiated.) The annualized standard deviation in copper prices is 25%, and the 25-year bond rate is 7%.

a. Estimate the value of the mine using traditional capital budgeting techniques.
b. Estimate the value of the mine based on an option pricing model.
c. How would you explain the difference between the two values?

3. You have been asked to analyze the value of an oil company with substantial oil reserves. The estimated reserves amount to 10 million barrels, and the estimated cost of developing these reserves today is $120 million. The current price of oil is $20 per barrel, and the average production cost is estimated to be $6 per barrel. The company has the rights to these reserves for the next 20 years, and the 20-year bond rate is 7%. The company also proposes to extract 4% of its reserves each year to meet cash flow needs. The annualized standard deviation in the price of the oil is 20%. What is the value of this oil company?
4. You are analyzing a capital budgeting project. The project is expected to have a PV of cash inflows of $250 million and will cost $200 million today to take on. You have done a simulation of the project cash flows, and the simulation yields a variance in present value of cash inflows of 0.04. You have the rights to this project for the next 20 years. The 20-year Treasury bond rate is 8%.
   a. What is the value of the project based on traditional NPV?
   b. What is the value of the project as an option?
   c. Why are the two values different? What factor or factors determine the magnitude of this difference?
5. Cyclops Inc., a high technology company specializing in state-of-the-art visual technology, is considering going public. While the company has no revenues or profits yet on its products, it has a 10-year patent to a product that will enable contact lens users to get no-maintenance lenses that will last for years. While the product is technically viable, it is exorbitantly expensive to manufacture, and the potential market for it will be relatively small currently. (A cash flow analysis of the project suggests that the present value of the cash inflows on the project, if adopted now, would be $250 million, while the cost of the project will be $500 million.) The technology is rapidly evolving, and a simulation of alternative scenarios yields a wide range of present values, with an annualized standard deviation of 60%. The 10-year bond rate is 6%.
   a. Estimate the value of this company.
   b. How sensitive is this value estimate to the variance in project cash flows? What broader lessons would you draw from this analysis?

CHAPTER 29

The Options to Expand and to Abandon: Valuation Implications

The preceding chapter noted that traditional discounted cash flow valuation does not consider the value of the option that many firms have to delay making an investment and consequently understates the value of these firms. This chapter considers two other options that are often embedded in investments (and consequently in the values of the firms that possess them). The first of these is the option to expand an investment not only in new markets but to new products, to take advantage of favorable conditions. We argue that this option may sometimes make young start-up firms significantly more valuable than the present value of their expected cash flows. The second option is the option to abandon or scale down investments, which can reduce the risk and downside from large investments and therefore make them more valuable.

THE OPTION TO EXPAND

Firms sometimes invest in projects because the investments allow them either to make further investments or to enter other markets in the future. In such cases, we can view the initial projects as yielding options allowing the firm to invest in other projects, and we should therefore be willing to pay a price for such options. Put another way, a firm may accept a negative net present value on the initial project because of the possibility of high positive net present values on future projects.

Payoff on the Option to Expand

The option to expand can be evaluated at the time the initial project is analyzed. Assume that this initial project will give the firm the right to expand and invest in a new project in the future. Assessed today, the expected present value of the cash flows from investing in the future project is V, and the total investment needed for this project is X. The firm has a fixed time horizon, at the end of which it has to make the final decision on whether or not to make the future investment. Finally, the firm cannot move forward on this future investment if it does not take the initial project. This scenario implies the option payoffs shown in Figure 29.1. As you can see, at the expiration of the fixed time horizon the firm will expand into the new project if the present value of the expected cash flows at that point in time exceeds the cost of expansion.

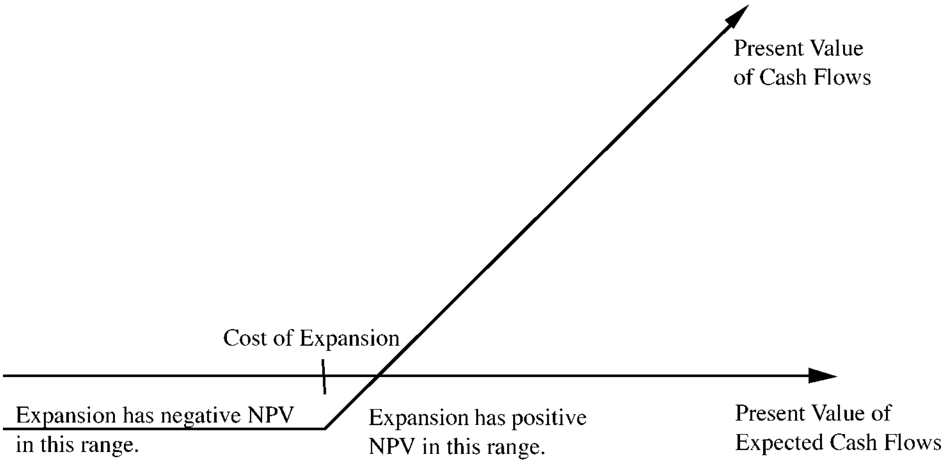

FIGURE 29.1 The Option to Expand a Project

Inputs to Value the Option to Expand To understand how to estimate the value of the option to expand, let us begin by recognizing that there are two projects usually that drive this option. The first project generally has a negative net present value and is recognized as a poor investment, even by the firm investing in it. The second project is the potential to expand that comes with the first project. It is the second project that represents the underlying asset for the option. The inputs have to be defined accordingly:

- The present value of the cash flows that you would generate if you were to invest in the second project today (the expansion option) is the value of the underlying asset—S in the option pricing model.
- If there is substantial uncertainty about the expansion potential, the present value is likely to be volatile and change over time as circumstances change. It is the variance in this present value that you would want to use to value the expansion option. Since projects are not traded, you have to either estimate this variance from simulations or use the variance in values of publicly traded firms in the business.
- The cost that you would incur up front, if you invest in the expansion today, is the equivalent of the strike price.
- The life of the option is fairly difficult to define, since there is usually no externally imposed exercise period. (This is in contrast to the patents valued in the preceding chapter, which have a legal life that can be used as the option life.) When valuing the option to expand, the life of the option will be an internal constraint imposed by the firm on itself. For instance, a firm that invests on a small scale in China might impose a constraint that it either will expand within five years or pull out of the market. Why might it do so? There may be considerable costs associated with maintaining the small presence or the firm may have scarce resources that have to be committed elsewhere.
- As with other real options, there may be a cost to waiting once the expansion option becomes viable. That cost may take the form of cash flows that will be lost on the expansion project if it is not taken or a cost imposed on the firm until it makes its final decision. For instance, the firm may have to pay a fee every year until it makes its final decision.

ILLUSTRATION 29.1: Valuing an Option to Expand: Ambev and Guarana

Guarana is a very popular caffeine-based soft drink in Brazil, and Ambev is the Brazilian beverage manufacturer that is the largest producer of Guarana in the world. Assume that Ambev is considering introducing the drink into the United States and that it has decided to do so in two steps:

1. Ambev will initially introduce Guarana in just the large metropolitan areas of the United States to gauge potential demand. The expected cost of this limited introduction is $500 million and the estimated present value of the expected cash flows is only $400 million. In other words, Ambev expects to have a negative net present value of $100 million on this first investment.
2. If the limited introduction turns out to be a success, Ambev expects to introduce Guarana to the rest of the U.S. market. At the moment, though, the firm is not optimistic about this expansion potential and believes that while the cost of the full-scale introduction will be $1 billion, the expected present value of the cash flows is only $750 million (making this a negative net present value investment as well).

At first sight, investing in a poor project to get a chance to invest in an even poorer project may seem like a bad deal, but the second investment does have a redeeming feature. It is an option and Ambev will not make the second investment (of $1 billion) if the expected present value of the cash flows stays below that number. Furthermore, there is considerable uncertainty about the size and potential for this market, and the firm may well find itself with a lucrative investment.

To estimate the value of the second investment as an option, we begin by first identifying the underlying asset—the expansion project—and using the current estimate of expected value ($750 million) as the value of the underlying asset. Since the investment needed for the investment of $1 billion is the exercise price, this option is an out-of-the-money option. The two most problematic assumptions relate to the variance in the value of the underlying asset and the life of the option:

- We estimated the average standard deviation of 35% in firm values of small, publicly traded beverage companies in the United States and assumed that this would be a good proxy for the standard deviation in the value of the expansion option.
- We assumed that Ambev would have a five-year window to make its decision. We admit that this is an arbitrary constraint but, in the real world, it may be driven by any of the following:
  - Financing constraints (loans will come due).
  - Strategic prerogatives (you have to choose where your resources will be invested).
  - Personnel decisions (management has to be hired and put in place).

Based on these inputs, we had the following inputs to the option pricing model:

S = Present value of cash flows from expansion option today = $750
K = Exercise price = $1,000
t = 5 years
Standard deviation in value = 35%

We used a riskless rate of 5% and derived the expected up and down movements from the standard deviation:

$$u = 1.4032$$

$$d = 0.6968$$

The binomial tree is presented in Figure 29.2.

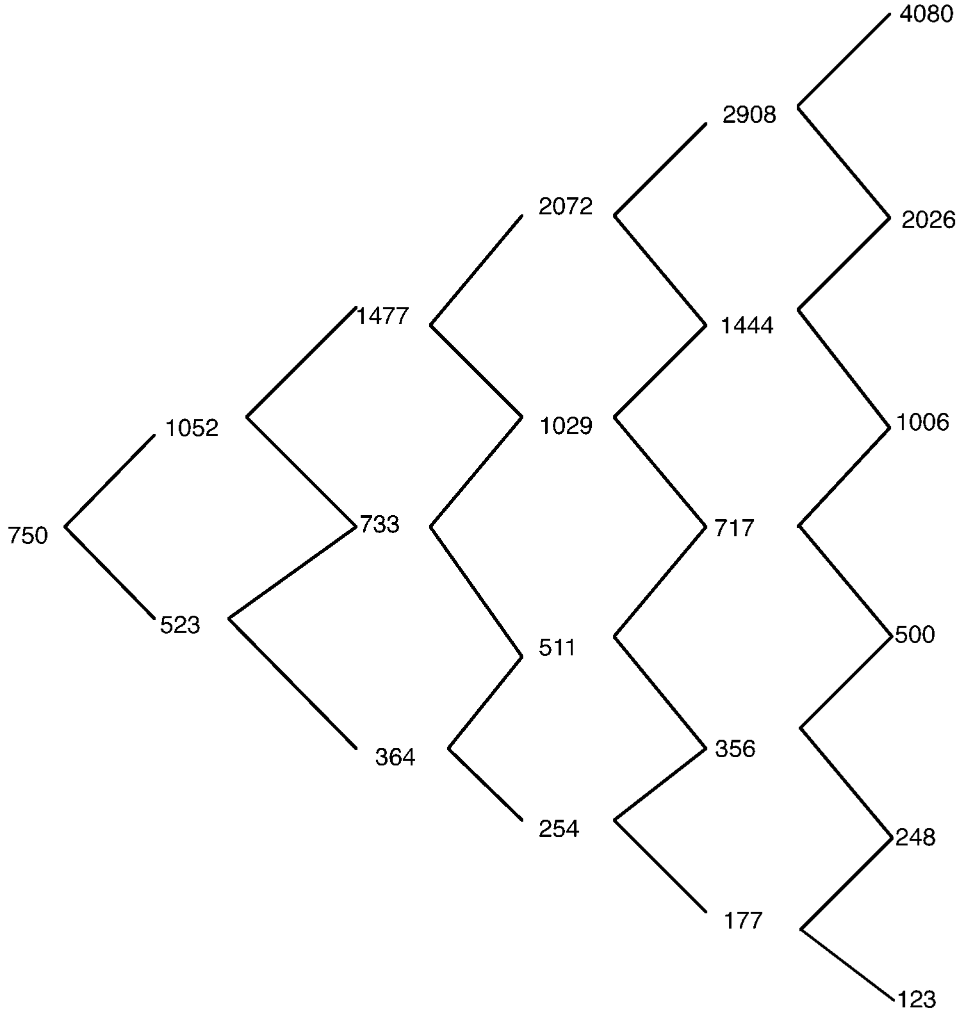

FIGURE 29.2 Binomial Tree—Ambev Expansion Option

Using the replicating portfolio framework described in Chapter 5, we estimate the value of the expansion option to be \$203 million. This value can be added on to the net present value of the original project under consideration.

NPV of limited introduction = –500 + 400 = – \$100 million

Value of option to expand = \$203 million

NPV with option to expand = –\$100 million + \$203 million = \$103 million

Ambev should go ahead with the limited introduction, even though it has a negative net present value, because it acquires an option of much greater value as a consequence.

ESTIMATING VARIANCES FROM MONTE CARLO SIMULATIONS

It has been suggested a couple of times in the last two chapters that the variances to be used in real option pricing models be derived from simulations. A Monte Carlo simulation requires the following three steps:

1. You define probability distributions for each of the key inputs that underlie the cash flows, and the parameters of the distributions—the average and the standard deviation, if it is a normal distribution, for instance.
2. In each simulation, you draw one outcome from each distribution and estimate the present value of the cash flows based on these draws.
3. After repeated simulations you should have a distribution of present values. The mean of this distribution should be the expected value of the project, and the standard deviation of the distribution can be used as the variance in the value to value options on the project.

While the process of running these simulations is straightforward and there are a number of software packages that exist that allow you to do this,[1] we would add the following notes of caution:

- The most difficult step is estimating the probability distributions and parameters for the key variables. It is easier to do when a firm has had experience with similar projects in the past—a retail store considering a new store, for instance—than for a new product or a new market. If the distributions that feed into a simulation are random, the output, impressive though it might look on paper, is meaningless.
- The standard deviation or variance that you want to use in option pricing models is a variance in value over time and not at a point in time. What is the difference, you might ask? Market testing, for instance, provides a distribution for the market potential today and reflects estimation uncertainty. The market itself will evolve over time, and it is the variance in that distribution that we would like to estimate.[2]
- You should estimate the standard deviation in the value of the project—the sum of the present value of the cash flows—rather than the standard deviation in annual income or annual cash flows.

expand.xls: **This spreadsheet allows you to estimate the value of the option to expand a project to cover new markets or new products, using the Black-Scholes model.**

[1]Crystal Ball and @Risk are both add-on packages to Excel that allow you to run simulations.
[2]You could, for instance, be fairly certain about the size of the market today—the variance would be low or even zero—but be uncertain about what the market will look like a year from now or three years from now. It is the latter variance that determines the value of the option.

Problems in Valuing the Option to Expand

The practical considerations associated with estimating the value of the option to expand are similar to those associated with valuing the option to delay. In most cases, firms with options to expand have no specific time horizon by which they have to make an expansion decision, making these open-ended options or, at best, options with arbitrary lives. Even in those cases where a life can be estimated for the option, neither the size nor the potential market for the product may be known, and estimating either can be problematic. To illustrate, consider the Ambev example discussed earlier. While we adopted a period of five years, at the end of which Ambev has to decide one way or another on its future expansion in United States, it is entirely possible that this time frame is not specified at the time the initial investment is made. Futhermore, we have assumed that both the cost and the present value of expansion are known at the time of the initial investment. In reality, the firm may not have good estimates for either input before opening the first store, since it does not have much information on the underlying market.

Extensions and Implications of Expansion Options

The option to expand can be used by firms to rationalize investing in projects that have negative net present values but provide significant opportunities to enter new markets or to sell new products. The option pricing approach adds rigor to this argument by estimating the value of this option, and it also provides insight into those occasions when it is most valuable. The option to expand is clearly more valuable for more volatile businesses with higher returns on projects (such as biotechnology or computer software), than it is for stable businesses with lower returns (such as automobile production). We will consider three cases where the expansion option may yield useful insights—strategic acquisitions, research and development expenses, and multistage projects.

Strategic Considerations in Acquisitions In many acquisitions or investments, the acquiring firm believes that the transaction will give it competitive advantages in the future. These competitive advantages include:

- *Entry into a large or growing market.* An investment or acquisition may allow the firm to enter a large or potentially large market much sooner than it otherwise would have been able to do so. A good example of this is the acquisition of a Mexican retail firm by a U.S. firm, with the intent of expanding into the Mexican market.
- *Technological expertise.* In some cases, the acquisition is motivated by the desire to acquire a proprietary technology that will allow the acquirer to either expand its existing market or enter a new market.
- *Brand name.* Firms sometime pay large premiums over market price to acquire firms with valuable brand names, because they believe that these brand names can be used for expansion into new markets in the future.

While all these potential advantages may be used to justify large acquisition premiums, not all of them create valuable options. Even if these advantages can be viewed as valuable expansion options, the value has to be greater than the acquisition premium for stockholders to gain.

Research, Development, and Test Market Expenses Firms that spend considerable amounts of money on research and development and test marketing are often stymied when they try to evaluate these expenses, since the payoffs are in terms of future projects. At the same time, there is the very real possibility that after the money has been spent the products or projects may turn out not to be viable; consequently, the expenditure must be treated as a sunk cost. In fact, R&D has the characteristics of a call option—the amount spent on the R&D is the cost of the call option, and the projects or products that might emerge from the research provide the payoffs on the options. If these products are viable (i.e., the present value of the cash inflows exceeds the needed investment), the payoff is the difference between the two.

Several logical implications emerge from this view of R&D. First, other things remaining equal, research expenditures should provide much higher value for firms that are in volatile businesses, since the variance in product or project cash flows is positively correlated with the value of the call option. Thus, Minnesota Mining and Manufacturing (3M), which expends a substantial amount on R&D on basic office products such as the Post-it pad, should receive less value for its research than does Amgen, whose research primarily concerns biotechnology products.[3] Second, the value of research and the optimal amount to be spent on research will change over time as businesses mature. The best example is the pharmaceutical industry: Pharmaceutical companies spent most of the 1980s investing substantial amounts in research and earning high returns on new products as health-care costs expanded. In the 1990s, however, as health-care costs started leveling off and the business matured, many of these companies found that they were not getting the same payoffs on research and started cutting back. Some companies moved research dollars from conventional drugs to biotechnology products, where uncertainty about future cash flows remains high.

Multistage Projects/Investments When entering new businesses or taking new investments, firms sometimes have the option to move in stages. While doing so may reduce potential upside, it also protects the firm against downside risk by allowing it at each stage to gauge demand and decide whether to go on to the next stage. In other words, a standard project can be recast as a series of options to expand, with each option being dependent on the previous one. There are two propositions that follow:

1. Some projects that are unattractive on a full-investment basis may be value-creating if the firm can invest in stages.
2. Some projects that look attractive on a full-investment basis may become even more attractive if taken in stages.

The gain in value from the options created by multistage investments has to be weighed against the cost. Taking investments in stages may allow competitors who decide to enter the market on a full scale to capture the market. It may also lead to

[3]This statement is based on the assumption that the quality of research is the same at both firms, though the research is in different businesses, and that the only difference is in the volatility of the underlying businesses.

higher costs at each stage, since the firm is not taking full advantage of economies of scale.

Several implications emerge from viewing this choice between multistage and one-time investments in an option framework. The projects where the gains will be largest from making the investment in multiple stages include:

- *Projects where there are significant barriers to entry to competitors entering the market and taking advantage of delays in full-scale production.* Thus a firm with a patent on a product or other legal protection against competition pays a much smaller price for starting small and expanding as it learns more about the market.
- *Projects where there is uncertainty about the size of the market and the eventual success of the project.* Here, starting small and expanding in stages allows the firm to reduce its losses if the product does not sell as well as anticipated, and to learn more about the market at each stage. This information can be useful in both product design and marketing in subsequent stages.
- *Projects where there is a substantial investment needed in infrastructure and high operating leverage (fixed costs).* Since the savings from doing a project in multiple stages can be traced to the investments needed at each stage, the benefit is likely to be greater in firms where those costs are large. Capital-intensive projects as well as projects that require large initial marketing expenses (a new brand name product for a consumer product company), for example, will gain more from the options created by investing in the projects in multiple stages.

SEQUENTIAL AND COMPOUND OPTIONS: SOME THOUGHTS

A compound option is an option on an option. A simple example would be a call option on a small company that has only one asset—a patent. Last chapter, we argued that a patent could be viewed as an option, and thus the call option on the company becomes a compound option. You can also have a sequence of options where the value of each option is dependent on whether the previous option is exercised. For instance, a five-stage project has sequential options. Whether you reach the fifth stage is obviously a function of whether you make it through the first four stages; the value of the fifth option in the sequence is determined by what happens to the first four options.

Needless to say, option pricing becomes more complicated when you have sequential and compound options. There are two choices. One is to value these options as simple options and accept the fact that the value that you obtain will be an approximation. The other is to modify the option pricing model to allow for the special characteristics of these options. While we do not consider these models in this book, you can modify both the Black-Scholes and binomial models to allow them to price compound and sequential options.

WHEN ARE EXPANSION OPTIONS VALUABLE?

While the argument that some or many investments have valuable strategic or expansion options embedded in them has great allure, there is a danger that this argument can be used to justify poor investments. In fact, acquirers have long justified huge premiums on acquisitions on synergistic and strategic grounds. We need to be more rigorous in our measurement of the value of real options and in our use of real options as justification for paying high prices or making poor investments.

Quantitative Estimation

When real options are used to justify a decision, the justification has to be in more than qualitative terms. In other words, managers who argue for investing in a project with poor returns or paying a premium on an acquisition on the basis of the real options generated by this investment should be required to value these real options and show that the economic benefits exceed the costs. There will be two arguments made against this requirement. The first is that real options cannot be easily valued, since the inputs are difficult to obtain and often noisy. The second is that the inputs to option pricing models can be easily manipulated to back up whatever the conclusion might be. While both arguments have some basis, an estimate is better than no estimate at all, and the process of trying to estimate the value of a real option is, in fact, the first step to understanding what drives its value.

Tests for Expansion Option to Have Value

Not all investments have options embedded in them, and not all options, even if they do exist, have significant value. To assess whether an investment creates valuable options that need to be analyzed and valued, we need to answer three key questions.

1. *Is the first investment a prerequisite for the later investment/expansion? If not, how necessary is the first investment for the later investment/expansion?* Consider our earlier analysis of the value of a patent or the value of an undeveloped oil reserve as options. A firm cannot generate patents without investing in research or paying another firm for the patents, and it cannot get rights to an undeveloped oil reserve without spending on exploration, bidding on it at a government auction, or buying it from another oil company. Clearly, the initial investment here (spending on R&D, bidding at the auction) is required for the firm to have the second investment. Now consider the Ambev investment in a limited introduction and the option to expand into the U.S. market later. The initial investment provides Ambev with information about market potential, without which presumably it is unwilling to expand into the larger market. Unlike the patent and undeveloped reserves examples, the initial investment is not a prerequisite for the second, though management might view it as such. The connection gets even weaker, and the option value lower, when we look at one firm acquiring another to have the option to be able to enter a large market. Acquiring an Internet service provider in order to have a foothold in the Internet retailing market or buying a Chinese brewery to preserve the option to enter the Chinese beer market would be examples of less valuable options.

2. *Does the firm have an exclusive right to the later investment/expansion? If not, does the initial investment provide the firm with significant competitive advantages on subsequent investments?* The value of the option ultimately derives not from the cash flows generated by the second and subsequent investments, but from the excess returns generated by these cash flows. The greater the potential for excess returns on the second investment, the greater the value of the expansion option in the first investment. The potential for excess returns is closely tied to how much of a competitive advantage the first investment provides the firm when it takes subsequent investments. At one extreme, again, consider investing in research and development to acquire a patent. The patent gives the firm that owns it the exclusive rights to produce that product, and if the market potential is large, the right to the excess returns from the project. At the other extreme, the firm might get no competitive advantages on subsequent investments, in which case, it is questionable as to whether there can be any excess returns on these investments. In reality, most investments will fall in the continuum between these two extremes, with greater competitive advantages being associated with higher excess returns and larger option values.

3. *Are the competitive advantages sustainable?* In a competitive marketplace, excess returns attract competitors, and competition drives out excess returns. The more sustainable the competitive advantages possessed by a firm, the greater will be the value of the options embedded in the initial investment. The sustainability of competitive advantages is a function of two forces. The first is the nature of the competition; other things remaining equal, competitive advantages fade much more quickly in sectors where there are aggressive competitors. The second is the nature of the competitive advantage. If the resource controlled by the firm is finite and scarce (as is the case with natural resource reserves and vacant land), the competitive advantage is likely to be sustainable for longer periods. Alternatively, if the competitive advantage comes from being the first mover in a market or from having technological expertise, it will come under assault far sooner. The most direct way of reflecting this competitive advantage in the value of the option is to estimate the period of competitive advantage, and only the excess returns earned over this period count toward the value of the option.

If the answer is yes to all three questions, then the option to expand can be valuable. Applying the last two tests to the Ambev expansion option, you can see the potential problems. While Ambev is the largest producer of Guarana in the world, it does not have a patent on the product. If the initial introduction proves successful, it is entirely possible that Coke and Pepsi could produce their own versions of Guarana for the national market. If this occurs, Ambev will have expended $100 million of its funds to provide market information to its competitors. Thus, if Ambev gets no competitive advantage in the expansion market because of its initial investment, the option to expand ceases to have value and cannot be used to justify the initial investment. Now consider two intermediate scenarios: If Ambev gets a lead time on the expansion investment because of its initial investment, you could build in higher cash flows for that lead time and a fading off to lower cash flows thereafter. This will lower the present value of the cash flows for the expansion and the value of the option. A simpler adjustment would be to cap the present value of the cash flows, the argument being that competition will restrict how large the net present value can become, and value the option with the cap. For instance, if you assume that the present value of the cash flows from the expansion option cannot exceed $2 billion, the value of the expansion option drops to $142 million.[4]

VALUING A FIRM WITH THE OPTION TO EXPAND

Is there an option to expand embedded in some firms that can lead to these firms to trade at a premium over their discounted cash flow values? At least in theory, there is a rationale for making this argument for a small, high-growth firm in a large and evolving market. The discounted cash flow valuation is based on expected cash flows and expected growth and these expectations should reflect the probability that the firm could be hugely successful (or a huge failure). What the expectations might fail to consider is that, in the event of success, the firm could invest more, add new products or expand into new markets and augment this success. This is the real option that is creating the additional value.

Relationship to Discounted Cash Flow Valuation

If the value of this option to expand is estimated, the value of a firm can be written as the sum of two components—a discounted cash flow value based on expected cash flows and a value associated with the option to expand:

$$\text{Value of firm} = \text{Discounted cash flow value} + \text{Option to expand}$$

The option pricing approach adds rigor to this argument by estimating the value of the option to expand, and it also provides insight into those occasions when it is most valuable. In general, the option to expand is clearly more valuable for more volatile businesses with higher returns on projects and greater barriers to competitive entry (such as biotechnology), than in stable businesses with lower returns (such as housing, utilities, or automobile production).

Again, though, you have to be careful not to double count the value of the option. If you use a higher growth rate than would be justified based on expectations because of the option to expand, you have already counted the value of the option in the discounted cash flow valuation. Adding an additional component to reflect the value of the option would be double counting.

Inputs for Valuing Expansion Option To value a firm with the option to expand, you have to begin by defining the market that the firm has the option to enter and specify the competitive advantages that you believe will give it some degree of exclusivity to make this entry. Once you are convinced that there is this exclusivity, you should then estimate the expected cash flows you would get if you entered the market today and the cost of entering that market. Presumably, the costs will exceed the expected cash flows, or you would have entered the market already. The cost of entering the market will become the exercise price of the option and the expected cash flows from entering the market today will become the value of the underlying asset.

[4]You can value the capped call by valuing the expansion option twice in the Black-Scholes model, once with a strike price of $1,000 (yielding the original expansion option value of $218 million) and once with the strike price of $2,000 (yield an option value of $76 million). The difference between the two is the value of the expansion option with a cap on the present value. You could also value it explicitly in the binomial by setting the value to $2,000 whenever it exceeds that number in the binomial tree.

To estimate the variance in the value, you can either run simulations on how the market will evolve over time or use the variances of publicly traded firms that service that market today, and assume that this variance is a good proxy for the volatility in the underlying market. You also have to specify a period by which you have to make the decision of whether to enter the market; this will become the life of the option. You may tie this assumption to the assumptions you made about competitive advantages. For instance, if you have the exclusive license to enter a market for the next 10 years, you would use 10 years as your option life.

ILLUSTRATION 29.2: Considering the Value of the Option to Expand

Rediff.com is an Internet portal serving the Indian subcontinent. In June 2000 the firm had only a few million in revenues, but tremendous growth potential as a portal and electronic marketplace. Using a discounted cash flow model, you could value Rediff.com at $474 million, based on its expected cash flows in the Internet portal business. Assume that in buying Rediff.com, you are in fact buying an option to expand in the online market in India. This market is a small one now, but could potentially be much larger in 5 or 10 years.

In more specific terms, assume that Rediff.com has the option to enter the Internet retailing business in India in the future. The cost of entering this business is expected to be $1 billion, and, based on current expectations, the present value of the cash flows that would be generated by entering this business today is only $500 million. Based on current expectations of the growth in the Indian e-commerce business, this investment clearly does not make sense.

There is substantial uncertainty about future growth in online retailing in India and the overall performance of the Indian economy. If the economy booms and the online market grows faster than expected over the next five years, Rediff.com might be able to create value from entering this market. If you leave the cost of entering the online retailing business at $1 billion, the present value of the cash flows would have to increase above this value for Rediff to enter this business and add value. The standard deviation in the present value of the expected cash flows (which is currently $500 million) is assumed to be 50%.

The value of the option to expand into Internet retailing can now be estimated using an option pricing model, with the following parameters:

S = Present value of the expected cash flows from entering market today = $500 million
K = Cost of entering the market today = $1 billion
σ^2 = Variance in the present value of expected cash flows = $0.5^2 = 0.25$
r = 5.8% (This is a five-year Treasury bond rate; the analysis is being done in U.S. dollar terms)
t = 5 years

The value of the option to expand can be estimated as follows:

$$\text{Option to expand} = 500(0.5786) - 1{,}000\ \exp^{-(0.058)(5)}(0.1789) = \$155.47 \text{ million}$$

Why does the option expire in five years? If the online retail market in India expands beyond this point in time, it is assumed that there will be other potential entrants into this market and that Rediff.com will have no competitive advantages and hence no good reason for entering this market. If the online retail market in India expands sooner than expected, it is assumed that Rediff.com, as one of the few recognized names in the market, will be able to parlay its brand name and the visitors to its portal to establish competitive advantages.

The value of Rediff.com as a firm can now be estimated as the sum of the discounted cash flow value of $474 million and the value of the option to expand into the retail market ($155 million). It is true that the discounted cash flow valuation is based on a high growth rate in revenues, but all of this growth is assumed to occur in the Internet portal business and not in online retailing.

In fact, the option to enter online retailing is only one of several options available to Rediff. Another path it might embark on is to become a development exchange for resources—software developers and programmers in India looking for programming work in the United States and other developed markets. The value of this option can also be estimated using an approach similar to the one just shown.

expand.xls: **This spreadsheet allows you to estimate the value of the option to expand an investment or project.**

VALUE OF FINANCIAL FLEXIBILITY

When making financial decisions, managers consider the effects of such decisions on their capacity to make new investments or meet unanticipated contingencies in future periods. Practically, this translates into firms maintaining excess debt capacity or larger cash balances than are warranted by current needs in order to meet unexpected future requirements. While maintaining this financing flexibility has value to firms, it also has a cost; the large cash balances might earn below-market returns, and excess debt capacity implies that the firm is giving up some value and has a higher cost of capital.

Determinants of the Value of Financial Flexibility

One reason that a firm maintains large cash balances and excess debt capacity is to have the future option to take unexpected projects with high returns. To value financial flexibility as an option, assume that a firm has expectations about how much it will need to reinvest in future periods, based on its own past history and current conditions in the industry. Assume also that a firm has expectations about how much it can raise from internal funds and its normal access to capital markets in future periods. There is uncertainty about future reinvestment needs; for simplicity, we will assume that the capacity to generate funds is known with certainty to the firm. The advantage (and value) of having excess debt capacity or large cash balances is that the firm can meet any reinvestment needs, in excess of funds available, using its debt capacity. The payoff from these projects, however, comes from the excess returns the firm expects to make on them. To value financial flexibility on an annualized basis, therefore, we will use the measures listed in Table 29.1.

TABLE 29.1 Inputs to Option Valuation: Financing Flexibility

Input to Model	Measure	Estimation Approach
S	Expected annual reinvestment needs as percent of firm value	Use historical average of (Net cap ex + Change in noncash working capital)/Market value of firm
K	Annual reinvestment needs as percent of firm value that can be raised without financing flexibility	If firm does not want to or cannot use external financing: (Net income – Dividend + Depreciation)/ Market value of firm If firm uses external capital (bank debt, bonds, or equity) regularly: (Net income + Depreciation + Net external financing)/Market value of firm
σ^s	Variance in reinvestment needs	Variance in the reinvestment as percent of firm value (using historical data)
t	1 year	To get an annual estimate of the value of flexibility

ILLUSTRATION 29.3: Valuing Financial Flexibility at the Home Depot

The Home Depot is a giant retail chain that sells home improvement products, primarily in the United States. This firm traditionally has not been a heavy user of debt and has also grown at an extraordinary rate over the past decade. To estimate the value of financial flexibility for the Home Depot, we began by estimating reinvestments as a percent of firm value from 1989 to 1998 in the following table:

Year	*Reinvestment Needs*	*Firm Value*	*Reinvestment Needs as Percent of Firm Value*	*ln (Reinvestment Needs)*
1989	$ 175	$ 2,758	6.35%	–2.7574751
1990	$ 374	$ 3,815	9.80%	–2.3224401
1991	$ 427	$ 5,137	8.31%	–2.4874405
1992	$ 456	$ 7,148	6.38%	–2.7520951
1993	$ 927	$ 9,239	10.03%	–2.2992354
1994	$1,176	$12,477	9.43%	–2.3617681
1995	$1,344	$15,470	8.69%	–2.4432524
1996	$1,086	$19,535	5.56%	–2.8897065
1997	$1,589	$24,156	6.58%	–2.7214279
1998	$1,817	$30,219	6.01%	–2.8112841

Average reinvestment needs as % of firm value = 7.71%

Standard deviation in ln(Reinvestment needs) = 22.36%

We followed up by estimating internal funds as a percent of firm value, using the sum of net income and depreciation as a measure of internal funds:

Year	*Net Income*	*Depreciation*	*Firm Value*	*Internal Funds/ Value*
1989	$ 112	$ 21	$ 2,758	4.82%
1990	$ 163	$ 34	$ 3,815	5.16%
1991	$ 249	$ 52	$ 5,137	5.86%
1992	$ 363	$ 70	$ 7,148	6.06%
1993	$ 457	$ 90	$ 9,239	5.92%
1994	$ 605	$130	$12,477	5.89%
1995	$ 732	$181	$15,470	5.90%
1996	$ 938	$232	$19,535	5.99%
1997	$1,160	$283	$24,156	5.97%
1998	$1,614	$373	$30,219	6.58%

Internal funds, on average, were 5.82% of firm value between 1989 and 1998. Since the firm uses almost no external debt, the firm made up the difference between its reinvestment needs (7.71%) and internal fund generation (5.82%) by issuing equity. We will assume, looking forward, that the Home Depot will no longer issue new equity.

The Home Depot's current debt ratio is 4.55%, and its current cost of capital is 9.51%. Using the cost of capital framework developed in Chapter 15, we estimated its optimal debt ratio to be 20%, and its cost of capital at that debt level is 9.17%. Finally, the Home Depot in 1998 earned a return on capital of 16.37%, and we will assume that this is the expected return on new projects as well.

S = Expected reinvestment needs as percent of firm value = 7.71%
K = Reinvestment needs that can be financed without flexibility = 5.82%
t = 1 year
σ^2 = Variance in ln(Net capital expenditures) = $(.2237)^2$ = .05

With a risk-free rate of 6%, the option value that we estimate using these inputs is .02277. We then convert this option value into a measure of value over time by multiplying the value by the annual excess return and then assuming that the firm forgoes this excess return forever:[5]

$$\begin{aligned}\text{Value of flexibility} &= .02277(\text{Return on capital} - \text{Cost of capital})/\text{Cost of capital}\\ &= .02277(.1637 - .0951)/.0951 = 1.6425\%\end{aligned}$$

On an annual basis, the flexibility generated by the excess debt capacity is worth 1.6425% of firm value at the Home Depot, which is well in excess of the savings (9.51% – 9.17% = 0.34%) in the cost of capital that would be accomplished, if it used up the excess debt capacity.

The one final consideration here is that this estimate does not consider the fact that the Home Depot does not have unlimited financial flexibility. In fact, assume that excess debt capacity of the Home Depot (which is 15.45%, the difference between the optimal debt ratio and the current debt ratio) is the upside limit on financial flexibility. We can value the effect of this limit, by valuing a call with the same parameters as the call described earlier, but with a strike price of 21.27% (15.45% + 5.82%). In this case, the effect of imposing this constraint on the value of flexibility is negligible.

finflex.xls: **This spreadsheet allows you to estimate the value of financial flexibility as an option.**

Implications of Financial Flexibility Option

Looking at financial flexibility as an option yields valuable insights on when financial flexibility is most valuable. Using the approach developed earlier, for instance, we would argue that:

- Other things remaining equal, firms operating in businesses where projects earn substantially higher returns than their hurdle rates should value flexibility more than those that operate in stable businesses where excess returns are small. This would imply that firms such as Microsoft and Dell, which earn large excess returns on their projects, can use the need for financial flexibility as justification for holding large cash balances and maintaining excess debt capacity.
- Since a firm's ability to fund these reinvestment needs is determined by its capacity to generate internal funds, other things remaining equal, financial flexibility should be worth less to firms with large and stable earnings as a percent of firm value. Firms that have small or negative earnings, and therefore much lower capacity to generate internal funds, will value flexibility more.
- Firms with limited internal funds can still get away with little or no financial flexibility if they can tap external markets for capital—bank debt, bonds, and new equity issues. Other things remaining equal, the greater the capacity (and the willingness) of a firm to raise funds from external capital markets, the less should be the value of flexibility. This may explain why private or small firms,

[5]We are assuming that the project that a firm is unable to take because it lacks financial flexibility is lost forever, and that the excess returns on this project would also have lasted forever. Both assumptions are strong and may result in overstatement of the lost value.

which have far less access to capital, will value financial flexibility more than larger firms. The existence of corporate bond markets can also make a difference in how much flexibility is valued. In markets where firms cannot issue bonds and have to depend entirely on banks for financing, there is less access to capital and a greater need to maintain financial flexibility. In the Home Depot example, a willingness to tap external funds—debt or equity—would reduce the value of flexibility substantially.

- The need for and the value of flexibility is a function of how uncertain a firm is about future reinvestment needs. Firms with predictable reinvestment needs should value flexibility less than firms in businesses where reinvestment needs are volatile on a period-to-period basis.

In our analysis of Home Depot, we considered the firm's gross debt ratio, which cannot be less than 0 percent. If we consider a firm's net debt ratio (gross debt minus cash), we see it is entirely possible for a firm to have a negative net debt ratio. Extending the financing flexibility argument, you could argue that in extreme circumstances—low or negative internal cash flows and no access to capital markets—firms not only will not use their debt capacity (thus driving the gross debt ratio to zero) but will accumulate cash. This may explain why many emerging market firms and young technology firms use no debt and accumulate large cash balances.

THE OPTION TO ABANDON

When investing in new projects, firms worry about the risk that the investment will not pay off, and that actual cash flows will not measure up to expectations. Having the option to abandon a project that does not pay off can be valuable, especially on projects with a significant potential for losses. This section examines the value of the option to abandon and its determinants.

Payoff on the Option to Abandon

The option pricing approach provides a general way of estimating and building in the value of abandonment. To illustrate, assume that V is the remaining value on a project if it continues to the end of its life, and L is the liquidation or abandonment value for the same project at the same point in time. If the project has a remaining life of n years, the value of continuing the project can be compared to the liquidation (abandonment) value. If the value from continuing is higher, the project should be continued; if the value of abandonment is higher, the holder of the abandonment option could consider abandoning the project. The payoffs can be written as:

$$\begin{aligned}\text{Payoff from owning an abandonment option} &= 0 \qquad \text{if } V > L\\ &= L - V \quad \text{if } V \le L\end{aligned}$$

These payoffs are graphed in Figure 29.3, as a function of the expected stock price. Unlike the prior two cases, the option to abandon takes on the characteristics of a put option.

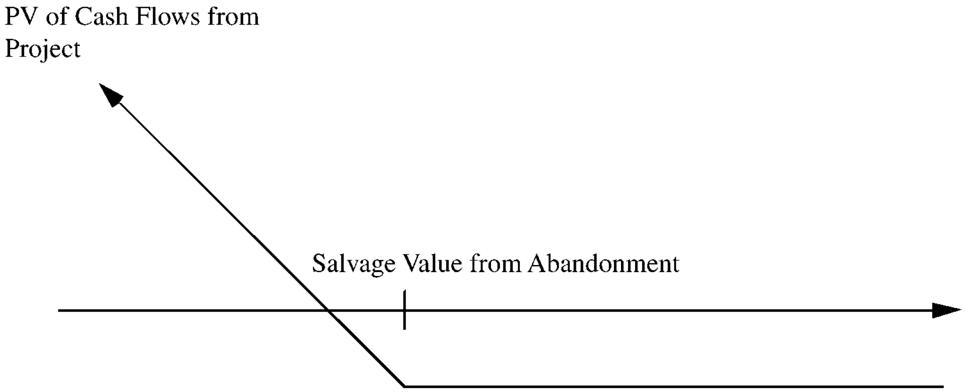

FIGURE 29.3 The Option to Abandon a Project

ILLUSTRATION 29.4: Valuing an Option to Abandon: Airbus and Lear Aircraft

Assume that Lear Aircraft is interested in building a small passenger plane and that it approaches Airbus with a proposal for a joint venture. Each firm will invest $500 million in the joint venture and produce the planes. The investment is expected to have a 30-year life. Airbus works through a traditional investment analysis and concludes that its share of the present value of the expected cash flows would be only $480 million. The net present value of the project would therefore be negative and Airbus would not want to be part of this joint venture.

On rejection of the joint venture, Lear approaches Airbus with a sweetener, offering to buy out Airbus's 50% share of the joint venture any time over the next five years for $400 million. This is less than what Airbus will invest initially but it puts a floor on its losses and thus gives Airbus an abandonment option. To value this option to Airbus, note that the inputs are as follows:

S = Present value of the share of cash flows from the investment today = $480 million
K = Abandonment value = $400 million
t = Period for which abandonment option holds = 5 years

To estimate the variance, assume that Airbus employs a Monte Carlo simulation on the project analysis and estimates a standard deviation in project value of 25%. Finally, note that since the project is a finite-life project, the present value will decline over time, because there will be fewer years of cash flows left. For simplicity, we will assume that this will be proportional to the time left on the project:

Dividend yield = 1/Remaining life of the project = 1/30 = 3.33%

Inputting these values into the Black-Scholes model and using a 5% riskless rate, we value the put option as follows:

$$\text{Value of abandonment option} = 400\ \exp^{(-0.05)(5)}(1 - 0.5776) - 480\ \exp^{(-0.033)(5)}(1 - 0.7748)$$
$$= \$40.09 \text{ million}$$

Since this is greater than the negative net present value of the investment, Airbus should enter into this joint venture. On the other hand, Lear needs to be able to generate a positive net present value of at least $40.09 million to compensate for giving up this option.[6]

[6]The binomial model yields a value of $46.44 million for this option.

abandon.xls: **This spreadsheet allows you to estimate the value of the option to abandon an investment.**

Problems in Valuing the Option to Abandon

Illustration 29.4 assumed, rather unrealistically, that the abandonment value was clearly specified and did not change during the life of the project. This may be true in some very specific cases, in which an abandonment option is built into the contract. More often, however, the firm has the option to abandon, and the salvage value from abandonment can only be estimated. Further, the abandonment value may change over the life of the project, making it difficult to apply traditional option pricing techniques. Finally, it is entirely possible that abandoning a project may not bring in a liquidation value but may create costs instead; a manufacturing firm may have to pay severance to its workers, for instance. In such cases, it would not make sense to abandon unless the cash flows on the project are even more negative.

Extensions and Implications of Abandonment Options

The fact that the option to abandon has value provides a rationale for firms to build the operating flexibility to scale back or terminate projects if they do not measure up to expectations. It also indicates that firms that try to generate more revenues by offering their customers the option to walk away from commitments will have to weigh the higher revenues against the cost of the options that have been granted to these customers.

Escape Clauses in Contracts The first and most direct way of creating an abandonment option is to build operating flexibility contractually with other parties that are involved in a project. Thus contracts with suppliers may be written on an annual basis rather than be long-term, and employees may be hired on a temporary basis rather than permanently. The physical plant used for a project may be leased on a short-term basis rather than bought, and the financial investment may be made in stages rather than as an initial lump sum. While there is a cost to building in this flexibility, the gains may be much larger, especially in volatile businesses.

Customer Incentives On the other side of the transaction, offering abandonment options to customers and partners in joint ventures can have a negative impact on value. As an example, assume that a firm that sells its products on multiyear contracts offers customers the option to cancel the contract at any time. While this may increase sales, there is likely to be a substantial cost. In the event of a recession, customers that are unable to meet their obligations are likely to cancel their contracts. Any benefits gained by the initial sale (obtained by offering the inducement of cancellation by the buyer) may be offset by the cost of the option provided to customers.

RECONCILING NET PRESENT VALUE AND REAL OPTION VALUATIONS

Why does an investment sometimes have higher value when you value it using real option approaches than with traditional discounted cash flow models? The answer lies in the flexibility that firms have to change the way they invest in and run a project, based on what they observe in the market. Thus, an oil company will not produce the same amount of oil or drill as many new wells if oil prices go to $15 a barrel as it would if oil prices go up to $35 a barrel.

In traditional net present value, we consider the expected actions and the cash flow consequences of those actions to estimate the value of an investment. If there is a potential for further investments, expansion, or abandonment down the road, all you can do is consider the probabilities of such actions and build them into your cash flows. Analysts often allow for flexibility by using decision trees and mapping out the optimal path, given each outcome. You can then estimate the value of a project today, using the probabilities of each branch and estimating the present value of the cash flows from each branch. For instance, you have a decision tree for a new investment for the Home Depot in Figure 29.4.

This decision tree does bear a significant resemblance to the binomial tree approach that we use to value real options, but there are two differences. The first is that the probabilities of the outcomes are not used directly to value the real option, and the second is that you have only two branches at each node in the binomial tree. Notwithstanding this, you might wonder why the two approaches will yield different values for the project. The answer is surprisingly simple. It lies in the discount rate assumptions we make to compute the value. In the real options approach, you use a replicating portfolio to compute value. In the decision tree, you used the cost of capital for the project as the discount rate all through the process. If the exposure to market risk, which is what determines the cost of capital, changes at each node, you can argue that using the same cost of capital all the way through is incorrect and that you should be modifying the discount rate as you move through time. If you do, you will obtain the same value with both approaches. The real options approach does allow

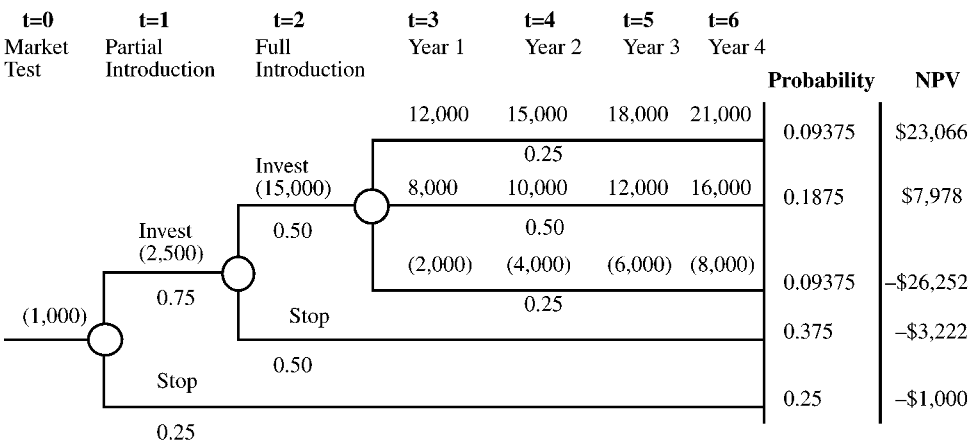

FIGURE 29.4 Decision Tree for the Home Depot Home Shopping

for far more complexity and is simpler to employ with continuous distributions (as opposed to the discrete outcomes that we assume in decision trees).

CONCLUSION

This chapter considers two options that are embedded in many investments—the option to expand an investment and the option to abandon it. When a firm has an option to expand an investment, the value of this expansion option may sometimes allow it to override the fact that the initial investment has a negative net present value. Extending this concept to firm valuation, you may sometimes add a premium to the value obtained from a discounted cash flow valuation for a firm that has the potential to enter new markets or create new products. This expansion option has maximum value when the firm has the exclusive right to make these investments, and the value decreases as the competitive advantages enjoyed by the firm decline.

The option to abandon refers to the right that firms often possess to walk away from poor investments. To the extent that this reduces the firm's exposure to the worst outcomes, it can make the difference between investing in a new project and not investing.

QUESTIONS AND SHORT PROBLEMS

1. NBC has the rights to televise the Winter Olympics in two years, and is trying to estimate the value of these rights for possible sale to another network. NBC expects it to cost $40 million (in present value terms) to televise the Olympics, and based on current assessments expects to have a Nielsen rating[7] of 15 for the games. Each rating point is expected to yield net revenue of $2 million to NBC (in present value terms). There is substantial variability in this estimate, and the standard deviation in the expected net revenues is 30%. The riskless rate is 5%.
   a. What is the net present value of these rights, based on current assessments?
   b. Estimate the value of these rights for sale to another network.
2. You are analyzing Skates Inc., a firm that manufactures skateboards. The firm is currently unlevered and has a cost of equity of 12%. You estimate that Skates would have a cost of capital of 11% at its optimal debt ratio of 40%. The management, however, insists that it will not borrow the money because of the value of maintaining financial flexibility and has provided you with the following information:
   - Over the past 10 years, reinvestment (net capital expenditures + working capital investments) has amounted to 10% of firm value, on an annual basis. The standard deviation in this reinvestment has been 0.30.
   - The firm has traditionally used only internal funding (net income + depreciation) to meet these needs, and these have amounted to 6% of firm value.
   - In the most recent year, the firm earned $180 million in net income on a book value of equity of $1 billion, and it expects to earn these excess returns on new investments in the future.
   - The riskless rate is 5%.

[7] There are 99.4 million households in the United States. Each rating point represents 1 percent of roughly 994,000 households.

a. Estimate the value of financial flexibility as a percent of firm value on an annual basis.
b. Based on part a, would you recommend that Skates use its excess debt capacity?

3. Disney is considering entering into a joint venture to build condominiums in Vail, Colorado, with a local real estate developer. The development is expected to cost $1 billion overall and, based on Disney's estimate of the cash flows, generate $900 million in present value cash flows over 25 years. Disney will have a 40% share of the joint venture (requiring it to put up $400 million of the initial investment and entitling it to 40% of the cash flows) but it will have the right to sell its share of the venture back to the developer for $300 million anytime over the next five years. (The project life is 25 years.)
   a. If the standard deviation in real estate values in Vail is 30% and the riskless rate is 5%, estimate the value of the abandonment option to Disney.
   b. Would you advise Disney to enter into the joint venture?
   c. If you were advising the developer, how much would he need to generate in present value cash flows from the investment to make this a good investment?
4. Quality Wireless is considering making an investment in China. While it knows that the investment will cost $1 billion and generate only $800 million in cash flows (in present value terms), the proponents of expansion are arguing that the potential market is huge and that Quality should go ahead with its investment.
   a. Under what conditions will the expansion potential have option value?
   b. Assume now that there is an option value to expansion that exactly offsets the negative net present value on the initial investment. If the cost of the subsequent expansion in five years is $2.5 billion, what is your current estimate of the present value of the cash flows from expansion? (You can assume that the standard deviation in the present value of the cash flows is 25% and that the riskless rate is 6%.)
5. Reliable Machinery Inc. is considering expanding its operations in Thailand. The initial analysis of the project yields the following results:
   - The project is expected to generate $85 million in after-tax cash flows every year for the next 10 years.
   - The initial investment in the project is expected to be $750 million.
   - The cost of capital for the project is 12%.

   If the project generates much higher cash flows than anticipated, you will have the exclusive right for the next 10 years (from a manufacturing license) to expand operations into the rest of Southeast Asia. A current analysis suggests the following about the expansion opportunity:
   - The expansion will cost $2 billion (in current dollars).
   - The expansion is expected to generate $150 million in after-tax cash flows each year for 15 years. There is substantial uncertainty about these cash flows, and the standard deviation in the present value is 40%.
   - The cost of capital for this investment is expected to be 12% as well. The risk-free rate is 6.5%.

   a. Estimate the net present value of the initial investment.
   b. Estimate the value of the expansion option.