CHAPTER 15

Firm Valuation: Cost of Capital and Adjusted Present Value Approaches

The preceding two chapters examined two approaches to valuing the equity in the firm—the dividend discount model and the free cash flow to equity (FCFE) valuation model. This chapter develops another approach to valuation where the entire firm is valued, by either discounting the cumulated cash flows to all claim holders in the firm by the weighted average cost of capital (the cost of capital approach) or by adding the marginal impact of debt on value to the unlevered firm value—the adjusted present value (APV) approach).

In the process of looking at firm valuation, we also look at how leverage may or may not affect firm value. We note that in the presence of default risk, taxes, and agency costs, increasing leverage can sometimes increase firm value and sometimes decrease it. In fact, we argue that the optimal financing mix for a firm is the one that maximizes firm value.

FREE CASH FLOW TO THE FIRM

The free cash flow to the firm (FCFF) is the sum of the cash flows to all claim holders in the firm, including stockholders, bondholders, and preferred stockholders. There are two ways of measuring the free cash flow to the firm.

One is to add up the cash flows to the claim holders, which would include cash flows to equity (defined either as free cash flow to equity or dividends), cash flows to lenders (which would include principal payments, interest expenses, and new debt issues), and cash flows to preferred stockholders (usually preferred dividends):

$$\begin{aligned}\text{FCFF} = {} & \text{Free cash flow to equity} + \text{Interest expense}(1 - \text{Tax rate}) \\ & + \text{Principal repayments} - \text{New debt issues} + \text{Preferred dividends}\end{aligned}$$

Note, however, that we are reversing the process that we used to get to free cash flow to equity, where we subtracted out payments to lenders and preferred stockholders to estimate the cash flow left for stockholders. A simpler way of getting to free cash flow to the firm is to estimate the cash flows prior to any of these claims. Thus we could begin with the earnings before interest and taxes, net out

taxes and reinvestment needs, and arrive at an estimate of the free cash flow to the firm:

$$\text{FCFF} = \text{EBIT}(1 - \text{Tax rate}) + \text{Depreciation} - \text{Capital expenditure} - \Delta \text{ Working capital}$$

Since this cash flow is prior to debt payments, it is often referred to as an unlevered cash flow. Note that this free cash flow to the firm does not incorporate any of the tax benefits due to interest payments. This is by design, because the use of the after-tax cost of debt in the cost of capital already considers this benefit, and including it in the cash flows would double count it.

FCFF and Other Cash Flow Measures

The differences between FCFF and FCFE arise primarily from cash flows associated with debt—interest payments, principal repayments, and new debt issues—and other nonequity claims, such as preferred dividends. For firms at their desired debt level, which finance their capital expenditures and working capital needs with this mix of debt and equity and use debt issues to finance principal repayments, the free cash flow to the firm will exceed the free cash flow to equity.

One measure that is widely used in valuation is the earnings before interest, taxes, depreciation, and amortization (EBITDA). The free cash flow to the firm is a closely related concept but it takes into account the potential tax liability from the earnings as well as capital expenditures and working capital requirements.

Three measures of earnings are also often used to derive cash flows. The amount of earnings before interest and taxes (EBIT) or operating income comes directly from a firm's income statements. Adjustments to EBIT yield the net operating profit or loss after taxes (NOPLAT) or the net operating income (NOI). The net operating income is defined to be the income from operations prior to taxes and non-operating expenses.

Each of these measures is used in valuation models, and each can be related to the free cash flow to the firm. Each, however, makes some assumptions about the relationship between depreciation and capital expenditures that are made explicit in Table 15.1.

Growth in FCFE versus Growth in FCFF

Will equity cash flows and firm cash flows grow at the same rate? Consider the starting point for the two cash flows. Equity cash flows are based on net income or earnings per share—measures of equity income. Firm cash flows are based on operating income (i.e., income prior to debt payments). As a general rule, you would expect growth in operating income to be lower than growth in net income, because financial leverage can augment the latter. To see why, let us go back to the fundamental growth equations laid out in Chapter 11:

$$\text{Expected growth in net income} = \text{Equity reinvestment rate} \times \text{Return on equity}$$

$$\text{Expected growth in operating income} = \text{Reinvestment rate} \times \text{Return on capital}$$

TABLE 15.1 Free Cash Flows to the Firm: Comparison to Other Measures

Cash Flow Used	Definition	Use in Valuation
FCFF	Free cash flow to firm	Discounting free cash flow to the firm at the cost of capital will yield the value of the operating assets of the firm. To this, you would add on the value of nonoperating assets to arrive at firm value.
FCFE	FCFF – Interest (1 – t) – Principal repaid + New debt issued – Preferred dividend	Discounting free cash flows to equity at the cost of equity will yield the value of equity in a business.
EBITDA	FCFF + EBIT(t) + Capital expenditures + Change in working capital	If you discount EBITDA at the cost of capital to value an asset, you are assuming that there are no taxes and that the firm will actively disinvest over time. It would be inconsistent to assume a growth rate or an infinite life for this firm.
EBIT (1 – t) (NOPLAT is a slightly modified version of this estimate and it removes any non-operating items that might affect the reported EBIT.)	FCFF + Capital expenditures – Depreciation + Change in working capital	If you discount after-tax operating income at the cost of capital to value a firm, you are assuming no reinvestment. The depreciation is reinvested back into the firm to maintain existing assets. You can assume an infinite life but no growth.

We also defined the return on equity in terms of the return on capital:

$$\text{Return on equity} = \text{Return on capital} + \frac{\text{Debt}}{\text{Equity}} \times (\text{Return on capital} - \text{After-tax cost of debt})$$

When a firm borrows money and invests in projects that earn more than the after-tax cost of debt, the return on equity will be higher than the return on capital. This, in turn, will translate into a higher growth rate in equity income at least in the short term.

In stable growth, though, the growth rates in equity income and operating income have to converge. To see why, assume that you have a firm whose revenues and operating income and growing at 5 percent a year forever. If you assume that the same firm's net income grows at 6 percent a year forever, the net income will catch up with operating income at some point in time in the future and exceed revenues at a later point in time. In stable growth, therefore, even if return on equity

exceeds the return on capital, the expected growth will be the same in all measures of income.[1]

FIRM VALUATION: THE COST OF CAPITAL APPROACH

The value of the firm is obtained by discounting the free cash flow to the firm at the weighted average cost of capital. Embedded in this value are the tax benefits of debt (in the use of the after-tax cost of debt in the cost of capital) and expected additional risk associated with debt (in the form of higher costs of equity and debt at higher debt ratios). Just as with the dividend discount model and the FCFE model, the version of the model used will depend on assumptions made about future growth.

Stable Growth Firm

As with the dividend discount and FCFE models, a firm that is growing at a rate that it can sustain in perpetuity—a stable growth rate—can be valued using a stable growth model.

The Model A firm with free cash flows to the firm growing at a stable growth rate can be valued using the following equation:

$$\text{Value of firm} = \frac{FCFF_1}{(WACC - g_n)}$$

where $FCFF_1$ = Expected FCFF next year
WACC = Weighted average cost of capital
g_n = Growth rate in the FCFF forever

The Caveats There are two conditions that need to be met in using this model. First, the growth rate used in the model has to be less than or equal to the growth rate in the economy—nominal growth, if the cost of capital is in nominal terms, or real growth, if the cost of capital is a real cost of capital. Second, the characteristics of the firm have to be consistent with assumptions of stable growth. In particular, the reinvestment rate used to estimate free cash flows to the firm should be consistent with the stable growth rate. The best way of enforcing this consistency is to derive the reinvestment rate from the stable growth rate:

$$\text{Reinvestment rate in stable growth} = \frac{\text{Growth rate}}{\text{Return on capital}}$$

If reinvestment is estimated from net capital expenditures and change in working capital, the net capital expenditures should be similar to those other firms in the

[1]The equity reinvestment rate and firm reinvestment rate will adjust to ensure that this happens. The equity reinvestment rate will be a lower number than the firm reinvestment rate in stable growth for any levered firm.

industry (perhaps by setting the ratio of capital expenditures to depreciation at industry averages) and the change in working capital should generally not be negative. A negative change in working capital creates a cash inflow, and while this may, in fact, be viable for a firm in the short term, it is dangerous to assume it in perpetuity.[2] The cost of capital should also be reflective of a stable growth firm. In particular, the beta should be close to 1—the rule of thumb presented in the earlier chapters that the beta should be between 0.8 and 1.2 still holds. While stable growth firms tend to use more debt, this is not a prerequisite for the model, since debt policy is subject to managerial discretion.

Limitations Like all stable growth models, this one is sensitive to assumptions about the expected growth rate. This is accentuated, however, by the fact that the discount rate used in valuation is the WACC, which is significantly lower than the cost of equity for most firms. Furthermore, the model is sensitive to assumptions made about capital expenditures relative to depreciation. If the inputs for reinvestment are not a function of expected growth the free cash flow to the firm can be inflated (deflated) by reducing (increasing) capital expenditures relative to depreciation. If the reinvestment rate is estimated from the return on capital, changes in the return on capital can have significant effects on firm value.

ILLUSTRATION 15.1: Valuing a Firm with a Stable Growth FCFF Model: Tube Investments of India

Tube Investments of India (TI) is a diversified manufacturing firm, with its headquarters in South India. In 1999, the firm reported operating income of Rs 632.2 million and faced a tax rate of 30% on income. The firm had a book value of equity of Rs 3,432.1 million and book value of debt of Rs 1,377.2 million at the end of 1998. The firm's return on capital can be estimated as follows:

$$\text{Return on capital} = \text{EBIT}(1-t)/(\text{Book value of debt} + \text{Book value of equity})$$
$$= 632.2(1-.3)/(3,432.1 + 1,377.2) = 9.20\%$$

The firm is in stable businesses and expects to grow only 5% a year.[3] Assuming that it maintains its current return on capital, the reinvestment rate for the firm will be:

$$\text{Reinvestment rate} = g/\text{ROC} = 5\%/9.20\% = 54.35\%$$

The firm's expected free cash flow to the firm next year can be estimated as follows:

Expected EBIT(1 – t) next year = 632.2(1 – .30)(1.05)	464.7
– Expected reinvestment next year = EBIT(1 – t)(Reinvestment rate) = 464.7(.5435)	252.5
Expected free cash flow to the firm	212.2

To estimate the cost of capital, we use a bottom-up beta (adjusted to 1.17 to reflect TI's additional leverage), a nominal rupee risk-free rate of 10.50%, and a risk premium of 9.23% (4% for the

[2]Carried to its logical extreme, this will push net working capital to a very large (potentially infinite) negative number.

[3]Note that while this resembles growth rates we have used for other firms, it is a low growth rate given that this valuation is in Indian rupees. As a simple check, note that the risk-free rate used is 10.50 percent.

mature market premium and 5.23% for country risk in India). The cost of equity can then be estimated as follows:

$$\text{Cost of equity} = 10.5\% + 1.17(9.23\%) = 21.30\%$$

The pretax cost of debt for Tube Investments is 12%, which in conjunction with its market debt-to-capital ratio of 44.19% yields a cost of capital of 15.60%:

$$\text{Cost of capital} = \text{Cost of equity}[E/(D+E)] + \text{After-tax cost of debt}[D/E+E)]$$
$$= 21.30\%(.5581) + 12\%(1-.3)(.4419) = 15.60\%$$

With the perpetual growth of 5%, the expected free cash flow to the firm shown (Rs 212.2 million) and the cost of capital of 15.60%, we obtain a value for the firm of:

$$\text{Value of the operating assets of firm} = 212.2/(.156-.05) = \text{Rs } 2{,}002 \text{ million}$$

Adding back cash and marketable securities with a value of Rs 1,365.3 million and subtracting out the debt outstanding of Rs 1,807.3 million yields a value for the equity of Rs 1,560 million and a value per share of Rs 63.36 (based on the 24.62 million shares outstanding). The stock was trading at Rs 92.70 at the time of this valuation.

An interesting aspect of this valuation is that the return on capital used to compute the reinvestment rate is significantly lower than the cost of capital. In other words, we are locking in this firm into investing in negative excess return projects forever. If we assume that the firm will find a way to earn its cost of capital of 15.6% on investments, the reinvestment rate would be much lower:

$$\text{Reinvestment rate}_{\text{ROC=cost of capital}} = g/\text{ROC} = .05/.156 = 32.05\%$$

Value of operating assets = 464.7 (1 – .3205)/(.156 – .05)	= Rs 2,979 million
+ Value of cash and marketable securities	= Rs 1,365 million
– Debt	= Rs 1,807 million
Value of equity	= Rs 2,537 million
Value per share	= 2,537/24.62 = Rs 103.04 per share

General Version of the FCFF Model

Rather than break the free cash flow model into two-stage and three-stage models and risk repeating what was said in the preceding chapter, we present the general version of the model in this section. We follow up by examining a range of companies—a traditional manufacturing firm, a firm with operating leases, and a firm with substantial R&D investments—to illustrate the differences and similarities between this approach and the FCFE approach.

The Model The value of the firm, in the most general case, can be written as the present value of expected free cash flows to the firm:

$$\text{Value of firm} = \sum_{t=1}^{t=\infty} \frac{\text{FCFF}_t}{(1+\text{WACC})^t}$$

where FCFF_t = Free cash flow to firm in year t
WACC = Weighted average cost of capital

MARKET VALUE WEIGHTS, COST OF CAPITAL, AND CIRCULAR REASONING

To value a firm, you first need to estimate a cost of capital. Every textbook is categorical that the weights in the cost of capital calculation be market value weights. The problem, however, is that the cost of capital is then used to estimate new values for debt and equity that might not match the values used in the original calculation. One defense that can be offered for this inconsistency is that if you bought all of the debt and equity in a publicly traded firm, you would pay current market value and not your estimated value, and your cost of capital reflects this.

For those who are bothered by this inconsistency, there is a way out. You could do a conventional valuation using market value weights for debt and equity, but then use the estimated values of debt and equity from the valuation to reestimate the cost of capital. This, of course, will change the values again, but you could feed the new values back and estimate cost of capital again. Each time you do this, the differences between the values you use for the weights and the values you estimate will narrow, and the values will converge sooner rather than later.

How much of a difference will it make in your ultimate value? The greater the difference between market value and your estimates of value, the greater the difference this iterative process will make. In the valuation of Tube Investments, we began with a market price of Rs 92.70 per share and estimated a value of Rs 63.36. If we substituted back this estimated value and iterated to a solution, we would arrive at an estimate of value of $70.66 per share.[4]

If the firm reaches steady state after n years and starts growing at a stable growth rate g_n after that, the value of the firm can be written as:

$$\text{Value of firm} = \sum_{t=1}^{t=n} \frac{\text{FCFF}_t}{(1+\text{WACC}_{hg})^t} + \frac{\left[\text{FCFF}_{n+1}/(\text{WACC}_{st} - g_n)\right]}{(1+\text{WACC}_{hg})^n}$$

where WACC = Cost of capital (hg: high growth; st: stable growth)

Firms Model Best Suited For Firms that either have very high leverage or are in the process of changing their leverage are best valued using the FCFF approach. The calculation of FCFE is much more difficult in these cases because of the volatility induced by debt payments (or new issues), and the value of equity, which is a small slice of the total value of the firm, is more sensitive to assumptions about growth and risk. It is worth noting, though, that in theory the two approaches should yield the same value for the equity. Getting them to agree in practice is an entirely different challenge and we will return to examine it later in this chapter.

[4]In Microsoft Excel, it is easy to set this process up. You should first go into calculation options and put a check in iteration box. You can then make the cost of capital a function of your estimated values for debt and equity.

Problems There are three problems that we see with the free cash flow to the firm model. The first is that the free cash flows to equity are a much more intuitive measure of cash flows than cash flows to the firm. When asked to estimate cash flows, most of us look at cash flows after debt payments (free cash flows to equity), because we tend to think like business owners and consider interest payments and the repayment of debt as cash outflows. Furthermore, the free cash flow to equity is a real cash flow that can be traced and analyzed in a firm. The free cash flow to the firm is the answer to a hypothetical question: What would this firm's cash flow be if it had no debt (and associated payments)?

The second is that its focus on predebt cash flows can sometimes blind us to real problems with survival. To illustrate, assume that a firm has free cash flows to the firm of $100 million but that its large debt load makes its free cash flows to equity equal to –$50 million. This firm will have to raise $50 million in new equity to survive, and if it cannot, all cash flows beyond this point are put in jeopardy. Using free cash flows to equity would have alerted you to this problem, but free cash flows to the firm are unlikely to reflect this.

The final problem is that the use of a debt ratio in the cost of capital to incorporate the effect of leverage requires us to make implicit assumptions that might not be feasible or reasonable. For instance, assuming that the market value debt ratio is 30 percent will require a growing firm to issue large amounts of debt in future years to reach that ratio. In the process, the book debt ratio might reach stratospheric proportions and trigger covenants or other negative consequences. In fact, we count the expected tax benefits from future debt issues implicitly in the value of equity today.

ILLUSTRATION 15.2: Valuing the Gap (July 2001): Dealing with Operating Leases

The Gap is one of the largest specialty retailers in the world and sells its products at Gap, GapKids, babyGap, Banana Republic, and Old Navy stores. While it has operations around the world, it gets the bulk of its revenues from the United States.

Rationale for Using the Model

- *Why two-stage?* While the Gap is one of the largest and most successful specialty retailers in the world, its dependence on the mature U.S. market for growth restricts its capacity to maintain high growth in the future. We will assume a high-growth period of five years and then put the firm into stable growth.
- *Why FCFF?* The Gap has a significant operating lease commitments, and the firm has increased its leverage aggressively over the past few years.

Background Information

In 2000, the Gap reported operating income $1,445 million on revenues of $13,673 million. The firm also reported capital expenditures of $1,859 million and depreciation of $590 million for the year, and its noncash working capital increased by $323 million during the year. The operating lease expenses for the year were $705.8 million, and the following table reports the lease commitments for future years (in $millions):

Year	*Commitment*
1	$ 774.60
2	$ 749.30
3	$ 696.50
4	$ 635.10
5	$ 529.70
6 and beyond	$5,457.90

To convert these operating lease expenses into debt, we first compute a pretax cost of debt for the firm based on its rating of A. The default spread for A-rated firms is 1.80%, which when added to the risk-free rate of 5.4%, yields a pretax cost of debt of 7.2%. Treating the commitment in year 6 and beyond as an annuity of $682.24 million for eight years, we estimate a debt value for the operating leases:

Year	*Commitment*	*Present Value*
1	$774.60	$ 722.57
2	$749.30	$ 652.03
3	$696.50	$ 565.38
4	$635.10	$ 480.91
5	$529.70	$ 374.16
6 and beyond	$682.24	$2,855.43
Debt value of leases		$5,650.48

This amount is added on to the debt outstanding on the balance sheet of $1,809.90 million to arrive at a total value for debt of $7,460.38 million. The Gap's market value of equity at the time of this valuation was $28,795 million, yielding a market debt to capital ratio of:

Market debt to capital = Debt/(Debt + Market value of equity) = $7,460/($7,460 + $28,795) = 20.58%

The operating income is also adjusted to reflect this shift by adding the imputed interest expense on the debt value of operating leases:

$$\text{Adjusted operating income} = \text{Operating income} + \text{Debt value of operating leases} \times \text{Pretax cost of debt}$$
$$= 1{,}445 + 5{,}650 \times .072 = \$1{,}851 \text{ million}$$

Multiplying by (1 – Tax rate), using a marginal tax rate of 35%, we get an after-tax operating income of $1,203 million:

$$\text{Adjusted after-tax operating income} = \text{Adjusted operating income}(1 - \text{Tax rate})$$
$$= 1{,}851(1 - .35) = \$1{,}203 \text{ million}$$

Dividing this value by the book value (BV) of debt (including capitalized operating leases) and the book value of equity at the end of the previous year yields an adjusted return on capital of 13.61% in 2000 for the firm:

$$ROC_{2000} = EBIT_{2000}(1 - t)/(\text{BV of debt}_{1999} + \text{BV of equity}_{1999})$$
$$= 1{,}203/(6{,}604 + 2{,}233) = 13.61\%$$

We will assume that the firm will be able to maintain this return on capital in perpetuity.

VALUATION

We will begin with a cost of equity estimate for the Gap, using a bottom-up beta of 1.20 (based on the betas of specialty retailers) for the high-growth period, a risk-free rate of 5.4%, and a mature market premium of 4%. In stable growth, we will lower the beta to 1.00, keeping the risk-free rate and risk premium unchanged.

$$\text{Cost of equity}_{\text{high growth}} = 5.4\% + 1.2(4\%) = 10.2\%$$

$$\text{Cost of equity}_{\text{stable growth}} = 5.4\% + 1.0(4\%) = 9.4\%$$

To estimate the cost of capital during the high-growth and stable growth phases, we will assume that the pretax cost of debt will remain at 7.2% in perpetuity and that the current market debt ratio of 20.58% will remain the debt ratio:

$$\text{Cost of capital}_{\text{high growth}} = 10.2\%(.7942) + 7.2\%\ (1 - .35)(.2058) = 9.06\%$$

$$\text{Cost of capital}_{\text{stable growth}} = 9.4\%(.7942) + 7.2\%\ (1 - .35)(.2058) = 8.43\%$$

To estimate the expected growth in operating earnings during the high-growth period, we will assume that the firm will continue to earn 13.61% as its return on capital and that its reinvestment rate will equal its average reinvestment rate over the past four years:[5]

$$\text{Average reinvestment rate over past four years} = 93.53\%$$

$$\text{Expected growth rate} = \text{Reinvestment rate} \times \text{Return on capital} = .9353 \times 1{,}361 = 12.73\%$$

The following table summarizes the expected cash flows for the high-growth period:

Year	*EBIT(1 – t)*	*Reinvestment Rate*	*Reinvestment*	*FCFF*	*Present Value*
Current	$1,203				
1	$1,356	93.53%	$1,269	$ 88	$ 80
2	$1,529	93.53%	$1,430	$ 99	$ 83
3	$1,732	93.53%	$1,620	$112	$ 86
4	$1,952	93.53%	$1,826	$126	$ 89
5	$2,190	93.53%	$2,049	$142	$ 92
Sum of present values of cash flows					$430

Note that the cash flows during the high-growth period are discounted back at 9.06%. To estimate the terminal value at the end of year 5, we assume that this cash flow will grow forever at 5%. The reinvestment rate can then be estimated and used to measure the free cash flow to the firm in year 6:

$$\text{Expected growth rate} = 5\%$$

$$\text{Reinvestment rate in stable growth} = g/\text{Stable period ROC} = 5\%/13.61\% = 36.73\%$$

$$\begin{aligned}\text{FCFF}_6 &= \text{EBIT}_5(1 - t)(1 + \text{Stable period g})(1 - \text{Reinvestment rate}) \\ &= 2{,}190(1.05)(1 - .3673) = 1{,}455\end{aligned}$$

The terminal value is:

$$\begin{aligned}\text{Terminal value} &= \text{FCFF}_6/(\text{Stable period cost of capital} - \text{Stable growth rate}) \\ &= 1{,}455/(.0843 - .05) = \$42{,}441 \text{ million}\end{aligned}$$

Discounting the terminal value to the present and adding it to the present value (PV) of the cash flows over the high-growth period yields a value for the operating assets of the firm:

$$\begin{aligned}\text{Value of operating assets} &= \text{PV of cash flows during high growth} + \text{PV of terminal value} \\ &= \$430 + \$42{,}441/1.0906^5 = \$27{,}933 \text{ million}\end{aligned}$$

Adding back the firm's cash and marketable securities (estimated to be $409 million at the end of 2000) and subtracting out the value of the debt yields a value for the equity in the firm:

$$\begin{aligned}\text{Value of the equity} &= \text{Value of the operating assets} + \text{Cash and marketable securities} - \text{Debt} \\ &= 27{,}933 + 409 - 7{,}460 = \$20{,}882 \text{ million}\end{aligned}$$

Note that the debt subtracted includes the present value of operating leases. At its prevailing market value of equity of $27,615 million, the Gap is overvalued.

[5]The Gap has had volatile capital expenditures and working capital changes. This is our attempt to average out this volatility.

ILLUSTRATION 15.3: Valuing Amgen: Effects of R&D

As a leading biotechnology firm, Amgen has substantial research and development expenses that were capitalized earlier in this book. In this valuation, we will consider the implications of this capitalization for firm and equity values.

Rationale for Using Model

- *Why three-stage?* Amgen, in spite of being one of the largest biotechnology firms in the world, has significant potential for future growth because of drugs that it has in commercial production and other drugs in the pipeline. We will assume that the firm will continue to grow for 10 years, five at a high-growth rate followed by five years in transition to stable growth.
- *Why FCFF?* The firm has little debt on its books currently but will come under increasing pressure to increase its leverage as its cash flows become larger and more stable.

Background Information

In 2000, Amgen reported operating income $1,549 million on revenues of $3,629 million. The firm also reported capital expenditures of $437 million and depreciation of $212 million for the year, and its noncash working capital (WC) increased by $146 million during the year. Recapping the analysis of Amgen's R&D from Chapter 9, we will use a 10-year amortizable life to estimate the value of the research asset:

Year	*R&D Expense*	*Unamortized Portion*		*Amortization This Year*
Current	$845.00	1.00	$ 845.00	
–1	$822.80	0.90	$ 740.52	$ 82.28
–2	$663.30	0.80	$ 530.64	$ 66.33
–3	$630.80	0.70	$ 441.56	$ 63.08
–4	$528.30	0.60	$ 316.98	$ 52.83
–5	$451.70	0.50	$ 225.85	$ 45.17
–6	$323.63	0.40	$ 129.45	$ 32.36
–7	$255.32	0.30	$ 76.60	$ 25.53
–8	$182.30	0.20	$ 36.46	$ 18.23
–9	$120.94	0.10	$ 12.09	$ 12.09
–10	$ 0.00	0.00	$ 0.00	$ 0.00
Value of research asset			$3,355.15	$397.91

The operating income is adjusted by adding back the current year's R&D expense and subtracting out the amortization of the research asset.

$$\text{Adjusted operating income} = \text{Operating income} + \text{Current year's R\&D} - \text{Amortization of research asset}$$
$$= \$1{,}549 + \$845 - \$398 = \$1{,}996 \text{ million}$$

To get to the after-tax operating income, we also consider the tax benefits from expensing R&D (as opposed to just the amortization of the research asset):

$$\text{Adjusted after-tax operating income} = \text{Adjusted operating income}(1 - \text{Tax rate}) + (\text{Current year R\&D} - \text{Amortization})\text{Tax rate}$$
$$= 1{,}996(1 - .35) + (845 - 398)(.35) = \$1{,}454 \text{ million}$$

The current year's R&D expense is added to the capital expenditures for the year, and the amortization to the depreciation. In conjunction with an increase in working capital of $146 million, we estimate an adjusted reinvestment rate for the firm of 56.27%.

Adjusted capital expenditures = 437 + 845 = $1,282 million
Adjusted depreciation = 212 + 398 = $610 million
Adjusted reinvestment rate = (Capital expenditures – Depreciation + Change in working capital)
/Adjusted EBIT(1 – t) = (1,282 – 610 + 146)/1,454 = 56.27%

To estimate the return on capital, we estimated the value of the research asset at the end of the previous year and added it to the book value of equity. The resultant return on capital for the firm is:

Return on capital = Adjusted EBIT(1 – t)/(Adjusted book value of equity, including research asset) + Book value of debt) = 1,454/(5,932 + 323) = 23.24%

VALUATION

To value Amgen, we will begin with the estimates for the five-year high growth period. We use a bottom-up beta estimate of 1.35, a risk-free rate of 5.4%, and a risk premium of 4% to estimate the cost of equity:

Cost of equity = 5.4% + 1.35(4%) = 10.80%

We estimate a synthetic rating of AAA for the firm, and use it to come up with a pretax cost of borrowing of 6.15% by adding a default spread of 0.75% to the Treasury bond rate of 5.4%. With a marginal tax rate of 35% and a debt ratio of 0.55%, the firm's cost of capital closely tracks its cost of equity:

Cost of capital = 10.80%(.9945) + .0615(1 – .35)(.0055) = 10.76%

To estimate the expected growth rate during the high growth period, we will assume that the firm can maintain its current return on capital and reinvestment rate estimated in the preceding section:

Expected growth rate = Reinvestment rate × Return on capital = .5627 × .2324 = 13.08%

Before we consider the transition period, we estimate the inputs for the stable growth period. First, we assume that the beta for Amgen will drop to 1, and that the firm will raise its debt ratio to 10%. Keeping the cost of debt unchanged, we estimate a cost of capital of:

Cost of equity = 5.4% + 1(4%) = 9.4%

Cost of capital = 9.4%(.9) + 6.15%(1 – .35)(.1) = 8.86%

We assume that the stable growth rate will be 5% and that the firm will have a return on capital of 20% in stable growth. This allows us to estimate the reinvestment rate in stable growth:

Reinvestment rate in stable growth = g/ROC = 5%/20% = 25%

During the transition period, we adjust growth, the reinvestment rate, and the cost of capital from high-growth levels to stable growth levels in linear increments. The following table summarizes the inputs and cash flows for both the high-growth and transition periods (in $millions):

Year	*Expected Growth*	*EBIT(1 – t)*	*Reinvestment Rate*	*FCFF*	*Cost of Capital*	*Present Value*
Current	$1,454					
1	13.08%	$1,644	56.27%	$ 719	10.76%	$ 649
2	13.08%	$1,859	56.27%	$ 813	10.76%	$ 663
3	13.08%	$2,102	56.27%	$ 919	10.76%	$ 677
4	13.08%	$2,377	56.27%	$1,040	10.76%	$ 691
5	13.08%	$2,688	56.27%	$1,176	10.76%	$ 705
6	11.46%	$2,996	50.01%	$1,498	10.38%	$ 814
7	9.85%	$3,291	43.76%	$1,851	10.00%	$ 914
8	8.23%	$3,562	37.51%	$2,226	9.62%	$1,003
9	6.62%	$3,798	31.25%	$2,611	9.24%	$1,077
10	5.00%	$3,988	25.00%	$2,991	8.86%	$1,133
		Sum of the present value of the FCFF during high growth =				$8,327

Finally, we estimate the terminal value, based on the estimated growth rate, cost of capital, and reinvestment rate:

$FCFF_{11}$ = $EBIT_{11}$(1 – t)(1 – Reinvestment rate) = 3,988 (1.05)(1 – .25) = $3,140 million

Terminal $value_{10}$ = $FCFF_{11}$/(Cost of capital in stable growth – Growth rate)
= 3,140/(.0886 –.05) = $81,364 million

Adding the present value of the terminal value to the present value of the free cash flows to the firm in the first 10 years, we get:

$$\begin{aligned}\text{Value of the operating assets of the firm} &= \$8{,}327 \text{ million} + \$81{,}364/(1.1076^5 \times 1.1038 \times 1.10 \\ &\quad \times 1.0962 \times 1.0924 \times 1.0886) \\ &= \$39{,}161 \text{ million}\end{aligned}$$

Adding the value of cash and marketable securities ($2.029 million) and subtracting debt ($323 million) yields a value for the equity of $40,867 million. At the time of this valuation in May 2001, the equity was trading at a market value of $58,000 million.

ILLUSTRATION 15.4: Valuing Embraer: Dealing with Country Risk

Embraer is a Brazilian aerospace firm that manufactures and sells both commercial and military aircraft. In this valuation, we will consider the implications of valuing the firm in the context of country risk and uncertainty about expected inflation.

Rationale for Using Model

- *Why two-stage?* Embraer has done exceptionally well in the past few years despite the fact that it operates in a mature business with strong competition from giants such as Boeing and Airbus. We believe that it can sustain growth for a long period (10 years) and that there will be a transition to stable growth in the second half of this growth period.
- *Why FCFF?* The firm's debt ratio has been volatile. While it does not use much debt to fund its operations currently, it does have the capacity to raise more debt now, especially in the United States.
- *Why real cash flows?* We had two choices when it came to valuation—to work with U.S. dollars or to work in real cash flows. We avoided working with nominal BR, largely because of the difficulties associated with getting a risk-free rate in that currency.

Background Information

In 2000, Embraer reported operating income of 810.32 million BR on revenues of 4,560 million BR, and faced a marginal tax rate of 33% on its income. At the end of 2000, the firm had net debt (debt minus cash) of 215.5 million BR on which its net interest expenses for 2000 were 28.20 million BR. The firm's noncash working capital at the end of 2000 amounted to 915 million BR, an increase of 609.7 million BR over the previous year's amount.

The firm's capital expenditures were 233.5 million BR, and depreciation was 127.5 million for the year, yielding a reinvestment rate of 131.83% for the year:

$$\text{Reinvestment rate}_{2000} = (233.5 - 127.5 + 609.7)/[810.32 \times (1 - .33)] = 131.83\%$$

Normalizing the noncash working capital component[6] yields a change in noncash working capital of 239.59 million BR and a normalized reinvestment rate of:

$$\text{Normalized reinvestment rate}_{2000} = (233.5 - 127.5 + 239.59)/[810.32 \times (1 - .33)] = 63.65\%$$

[6]The normalized change in noncash working capital was computed as follows:

$$\text{Normalized change} = (\text{Noncash WC}_{2000}/\text{Revenues}_{2000}) \times (\text{Revenues}_{2000} - \text{Revenues}_{1999})$$

Based on the capital invested of 1,470 million BR in the firm at the beginning of 2000, the return on capital at Embraer in 2000 was 36.94%:

$$\text{Return on capital} = 810.32(1 - .33)/1{,}470 = 36.94\%$$

VALUATION

We first have to estimate a country risk premium for Brazil. Drawing on the approach developed in Chapter 7, we estimate a country risk premium for Brazil of 10.24%:

Country rating for Brazil = B1
Default spread on Brazilian government C-bond (U.S. dollar–denominated) = 5.37%

To estimate the country equity risk premium, we estimated the standard deviation in weekly returns over the last two years in both the Bovespa (the Brazilian equity index) and the C-bond:

Standard deviation in the Bovespa = 32.6%
Standard deviation in the C-bond = 17.1%
Country risk premium = Default spread(Standard deviation$_{\text{equity}}$/Standard deviation$_{\text{C-bond}}$)
= 5.37%(32.6/17.1) = 10.24%

To make an estimate of Embraer's beta, we used a bottom-up unlevered beta of 0.87 and Embraer's market net debt-to-equity ratio (to stay consistent with use of net debt in the valuation) of 2.45%:

$$\text{Levered beta} = 0.87[1 + (1 - .33)(.0245)] = 0.88$$

Finally, to estimate the cost of equity, we used a real riskless rate of 4.5% and a mature market risk premium of 4% (in addition to the country risk premium of 10.24%):

$$\text{Cost of equity} = 4.5\% + 0.88(4\% + 10.24\%) = 17.03\%$$

We estimate a synthetic rating of AAA for Embraer, and use it to come up with a pretax cost of borrowing of 10.62% by adding a default spread of 0.75% to the real riskless rate of 4.5%, and then adding the country default spread of 5.37%:[7]

Pretax cost of debt = Real risk-free rate + Country default spread + Company default spread
= 4.5% + 5.37% + 0.75% = 10.62%

With a marginal tax rate of 33% and a net debt to capital ratio of 2.40%, the firm's cost of capital is:

$$\text{Cost of capital} = 17.03\%(.976) + .1062(1 - .35)(.024) = 16.79\%$$

To estimate the expected growth rate during the high-growth period, we will assume that the firm can maintain its current return on capital and use the normalized reinvestment rate:

Expected growth rate = Normalized reinvestment rate $\times$ Return on capital
= .6365 $\times$ .3694 = 23.51%

[7]This is a conservative estimate. It is entirely possible that the market will not assess Embraer with all of the country risk, and may view Embraer as safer than the Brazilian government.

In stable growth, we assume that the beta for Embraer will rise slightly to 0.90, that its net debt ratio will remain unchanged at 2.40% and that the country risk premium will drop to 5.37% (which is the bond default spread). We also assume that the pretax cost of debt will decline to 7.50%.

$$\text{Cost of equity} = 4.5\% + 0.9(4\% + 5.37\%) = 12.93\%$$

$$\text{Cost of capital} = 12.93\%(.976) + 7.5\%(1 - .33)(.024) = 12.74\%$$

We assume that the stable real growth rate will be 3% and that the firm will have a return on capital of 15% in stable growth. This is a significant drop from its current return on capital but reflect the returns of more mature firms in the business. This allows us to estimate the reinvestment rate in stable growth:

$$\text{Reinvestment rate in stable growth} = g/ROC = 3\%/15\% = 20\%$$

During the transition period, we adjust growth, reinvestment rate, and the cost of capital from high-growth levels to stable-growth levels in linear increments. The following table summarizes the inputs and cash flows for both the high-growth and transition periods:

Year	*Expected Growth*	*EBIT(1 – t)*	*Reinvestment Rate*	*FCFF*	*Cost of Capital*	*Present Value*
Current		BR 543				
1	23.51%	BR 671	63.65%	BR 244	16.79%	BR 209
2	23.51%	828	63.65%	301	16.79%	221
3	23.51%	1,023	63.65%	372	16.79%	233
4	23.51%	1,264	63.65%	459	16.79%	247
5	23.51%	1,561	63.65%	567	16.79%	261
6	19.41%	1,864	54.92%	840	15.98%	333
7	15.31%	2,149	46.19%	1,156	15.17%	398
8	11.21%	2,390	37.46%	1,495	14.36%	450
9	7.10%	2,559	28.73%	1,824	13.55%	484
10	3.00%	2,636	20.00%	2,109	12.74%	496
		Sum of the present value of the FCFF during high growth				BR 3,333

Finally, we estimate the terminal value, based on the growth rate, cost of capital, and reinvestment rate estimated previously:

$$FCFF_{11} = EBIT_{11}(1 - t)(1 - \text{Reinvestment rate}) = 2{,}636(1.03)(1 - .2) = 2{,}172 \text{ million BR}$$

$$\text{Terminal value}_{10} = FCFF_{11}/(\text{Cost of capital in stable growth} - \text{Growth rate})$$
$$= 2{,}172/(.1274 - .03) = 22{,}295 \text{ million BR}$$

Adding the present value of the terminal value to the present value of the free cash flows to the firm in the first 10 years, we get:

$$\text{Value of the operating assets of the firm} = 3{,}333 \text{ million BR} + 22{,}295/(1.1679^5 \times 1.1598 \times 1.1517 \times 1.1436 \times 1.1355 \times 1.1274)$$
$$= 8{,}578 \text{ million BR}$$

We do not add back cash and marketable securities, because we are using net debt (and the cash has therefore already been netted out against debt). Adding the value of nonoperating assets ($510 million) and subtracting out net debt ($223 million) yields a value for the equity of 8,865 million BR and a per-share value of 14.88 BR. At the time of this valuation in March 2001, the equity was trading at a market price of 15.2 BR per share.

Doing a valuation is only the first part of the process. Presenting it to others is the second part and perhaps just as important. Valuations can be complicated, and it is easy to lose your audience (and yourself) in the details. Presenting a big picture of the valuation often helps. In Figure 15.1, for instance, the valuation of a Embraer is presented in a picture. The valuation contains all of the details presented in the Amgen and Gap valuations, but they are presented in a more concise format and the connections between the various inputs are much more visible.

fcffginzu.xls: **This spreadsheet allows you to estimate the value of a firm using the FCFF approach.**

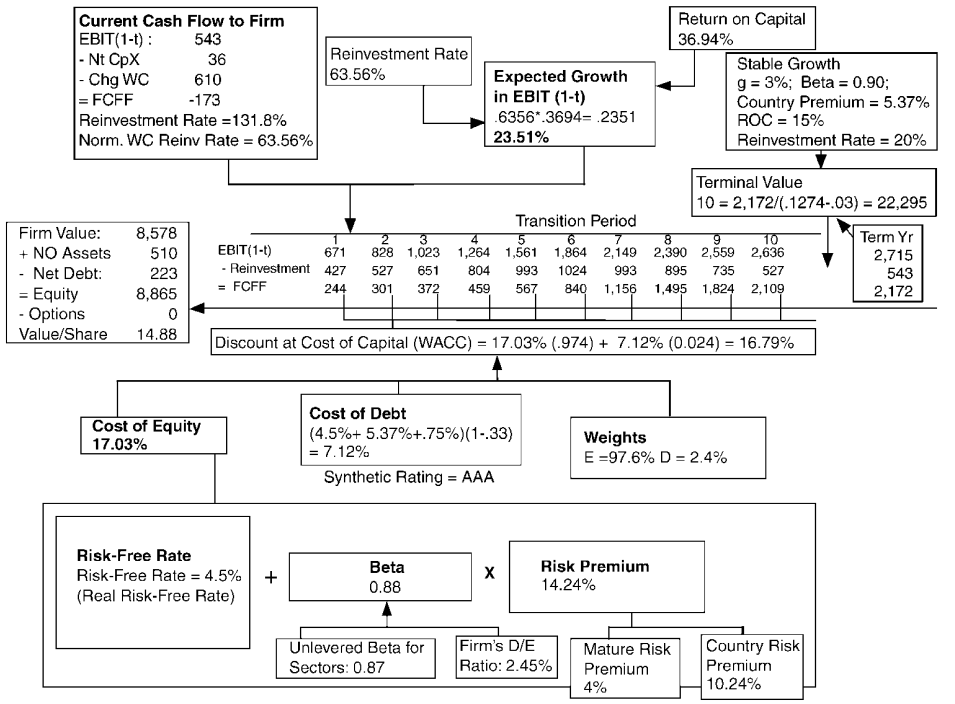

FIGURE 15.1 Embraer

NET DEBT VERSUS GROSS DEBT

In valuing Embraer, we used net debt where cash was netted out against debt. In all of the earlier valuations, we used gross debt. What is the difference between the two approaches, and will the valuations from the two approaches agree?

A comparison of the Embraer and the earlier valuations reveals the differences in the way we approach the calculation of key inputs to the valuation, summarized as follows:

	Gross Debt	*Net Debt*
Levered beta	Unlevered beta is levered using gross debt to market equity ratio.	Unlevered beta is levered using net debt to market equity ratio.
Cost of capital	Debt-to-capital ratio used is based on gross debt.	Debt-to-capital ratio used is based on net debt.
Treatment of cash and debt	Cash is added to value of operating assets and gross debt is subtracted to get to equity value.	Cash is not added back to operating assets and net debt is subtracted to get to equity value.

While working with net debt in valuation is not difficult to do, the more interesting question is whether the value that emerges will be the same as the value that would have been estimated using gross debt. In general the answer is no, and the reason usually lies in the cost of debt used in the net debt valuation. Intuitively, what you are doing when you use net debt is break the firm into two parts—a cash business, which is funded 100 percent with riskless debt, and an operating business funded partly with risky debt. Carrying this to its logical conclusion, the cost of debt you would have for the operating business would be significantly higher than the firm's current cost of debt. This is because the current lenders to the firm will factor in the firm's cash holdings when setting the cost of debt.

To illustrate, assume that you have a firm with an overall value of $1 billion—$200 million in cash and $800 million in operating assets—with $400 million in debt and $600 million in equity. The firm's cost of debt is 7 percent, a 2 percent default spread over the risk-free rate of 5 percent; note that this cost of debt is set based on the firm's substantial cash holdings. If you net debt against cash, the firm would have $200 million in net debt and $600 million in equity. If you use the 7 percent cost of debt to value the firm now, you will overstate its value. Instead, the cost of debt you should use in the valuation is 9 percent:

$$\begin{aligned}\text{Cost of debt on net debt} &= (\text{Pretax cost of debt}_{\text{gross debt}} \times \text{Gross debt} \\ &\quad - \text{Risk rate}_{\text{net debt}} \times \text{Cash})/(\text{Gross debt} - \text{Cash}) \\ &= (.07 \times 400 - .05 \times 200)/(400 - 200) = .09\end{aligned}$$

In general, we would recommend using gross debt rather than net debt for two other reasons. First, the net debt can be a negative number if cash exceeds the gross debt. If this occurs, you should set the net debt to zero and consider the excess cash just as you would cash in a gross debt valuation. Second, maintaining a stable net debt ratio in a growing firm will require that cash balances increase as the firm value increases.

Will Equity Value Be the Same under Firm and Equity Valuation?

This model, unlike the dividend discount model or the FCFE model, values the firm rather than equity. The value of equity, however, can be extracted from the value of the firm by subtracting the market value of outstanding debt. Since this model can be viewed as an alternative way of valuing equity, two questions arise: Why value the firm rather than equity? Will the values for equity obtained from the firm valuation approach be consistent with the values obtained from the equity valuation approaches described in the previous chapter?

The advantage of using the firm valuation approach is that cash flows relating to debt do not have to be considered explicitly since the FCFF is a predebt cash flow, while they have to be taken into account in estimating FCFE. In cases where the leverage is expected to change significantly over time, this is a significant savings, since estimating new debt issues and debt repayments when leverage is changing can become increasingly messy the further into the future you go. The firm valuation approach does, however, require information about debt ratios and interest rates to estimate the weighted average cost of capital.

The value for equity obtained from the firm valuation and equity valuation approaches will be the same if you make consistent assumptions about financial leverage. Getting them to converge in practice is much more difficult. Let us begin with the simplest case—a no-growth, perpetual firm. Assume that the firm has $166.67 million in earnings before interest and taxes and a tax rate of 40 percent. Assume that the firm has equity with a market value of $600 million, with a cost of equity of 13.87 percent, and debt of $400 million, with a pretax cost of debt of 7 percent. The firm's cost of capital can be estimated as follows:

$$\text{Cost of capital} = 13.87\%(700/1{,}000) + 7\%(1 - .4)(300/1{,}000) = 10\%$$

$$\begin{aligned}\text{Value of the firm} &= \text{Earnings before interest and taxes}(1 - t)/\text{Cost of capital} \\ &= 166.67(1 - .4)/.10 = \$1{,}000\end{aligned}$$

Note that the firm has no reinvestment and no growth. We can value equity in this firm by subtracting the value of debt:

$$\text{Value of equity} = \text{Value of firm} - \text{Value of debt} = \$1{,}000 - \$400 = \$600 \text{ million}$$

Now let us value the equity directly by estimating the net income:

$$\begin{aligned}\text{Net income} &= (\text{EBIT} - \text{Pretax cost of debt} \times \text{Debt})(1 - t) \\ &= (166.67 - .07 \times 400)(1 - .4) = \$83.202 \text{ million}\end{aligned}$$

The value of equity can be obtained by discounting this net income at the cost of equity:

$$\text{Value of equity} = \text{Net income}/\text{Cost of equity} = 83.202/.1387 = \$600 \text{ million}$$

Even this simple example works because of the following three assumptions made implicitly or explicitly during the valuation:

1. The values for debt and equity used to compute the cost of capital were equal to the values obtained in the valuation. Notwithstanding the circularity in reasoning—you need the cost of capital to obtain the values in the first place—it indicates that a cost of capital based on market value weights will not yield the

same value for equity as an equity valuation model if the firm is not fairly priced in the first place.
2. There are no extraordinary or nonoperating items that affect net income but not operating income. Thus, to get from operating to net income all we do is subtract interest expenses and taxes.
3. The interest expenses are equal to the pretax cost of debt multiplied by the market value of debt. If a firm has old debt on its books, with interest expenses that are different from this value, the two approaches will diverge.

If there is expected growth, the potential for inconsistency multiplies. You have to ensure that you borrow enough money to fund new investments to keep your debt ratio at a level consistent with what you are assuming when you compute the cost of capital.

fcffvsfcfe.xls: **This spreadsheet allows you to compare the equity values obtained using FCFF and FCFE models.**

FIRM VALUATION: THE ADJUSTED PRESENT VALUE APPROACH

The adjusted present value (APV) approach begins with the value of the firm without debt. As debt is added to the firm, the net effect on value is examined by considering both the benefits and the costs of borrowing. To do this, it is assumed that the primary benefit of borrowing is a tax benefit, and that the most significant cost of borrowing is the added risk of bankruptcy.

Mechanics of APV Valuation

We estimate the value of the firm in three steps:

1. Estimate the value of the firm with no leverage.
2. Consider the present value of the interest tax savings generated by borrowing a given amount of money.
3. Evaluate the effect of borrowing the amount on the probability that the firm will go bankrupt, and the expected cost of bankruptcy.

Value of Unlevered Firm The first step in this approach is the estimation of the value of the unlevered firm. This can be accomplished by valuing the firm as if it had no debt (i.e., by discounting the expected free cash flow to the firm at the unlevered cost of equity). In the special case where cash flows grow at a constant rate in perpetuity,

$$\text{Value of unlevered firm} = E(FCFF_1)/(\rho_u - g)$$

where $FCFF_1$ is the expected after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity, and g is the expected growth rate. In the more general case, you can value the firm using any set of growth assumptions you believe are reasonable for the firm.

The inputs needed for this valuation are the expected cash flows, growth rates, and the unlevered cost of equity. To estimate the unlevered cost of equity, we can draw on our earlier analysis and compute the unlevered beta of the firm:

$$\beta_{unlevered} = \beta_{current}/[1 + (1 - t)D/E]$$

where $\beta_{unlevered}$ = Unlevered beta of the firm
$\beta_{current}$ = Current equity beta of the firm
t = Tax rate for the firm
D/E = Current debt/equity ratio

This unlevered beta can then be used to arrive at the unlevered cost of equity.

Expected Tax Benefit from Borrowing The second step in this approach is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate and interest payments of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

$$\begin{aligned}\text{Value of tax benefits} &= (\text{Tax rate} \times \text{Cost of debt} \times \text{Debt})/\text{Cost of debt} \\ &= \text{Tax rate} \times \text{Debt} = t_c D\end{aligned}$$

The tax rate used here is the firm's marginal tax rate, and it is assumed to stay constant over time. If we anticipate the tax rate changing over time, we can still compute the present value of tax benefits over time, but we cannot use the perpetual growth equation cited earlier. In addition, you would have to modify this equation if the current interest expenses do not reflect the current cost of debt.

Estimating Expected Bankruptcy Costs and Net Effect The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. In theory, at least, this requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value (PV) of expected bankruptcy cost can be estimated:

$$\begin{aligned}\text{PV of expected bankruptcy cost} &= \text{Probability of bankruptcy} \times \text{PV of bankruptcy cost} \\ &= \pi_a BC\end{aligned}$$

This step of the adjusted present value approach poses the most significant estimation problems, since neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly.

There are two basic ways in which the probability of bankruptcy can be estimated indirectly. One is to estimate a bond rating and use the empirical estimates of default probabilities for the rating. For instance, Table 15.2, extracted from a study by Altman and Kishore, summarizes the probability of default over 10 years by bond rating class in 1998.[8]

[8]This study estimated default rates over 10 years only for some of the ratings classes. We extrapolated the rest of the ratings.

TABLE 15.2 Default Rates by Bond Rating Classes

Bond Rating	Default Rate
D	100.00%
C	80.00%
CC	65.00%
CCC	46.61%
B–	32.50%
B	26.36%
B+	19.28%
BB	12.20%
BBB	2.30%
A–	1.41%
A	0.53%
A+	0.40%
AA	0.28%
AAA	0.01%

Source: Altman and Kishore (1998).

The other way is to use a statistical approach such as a probit to estimate the probability of default, based on the firm's observable characteristics, at each level of debt.

The bankruptcy cost can be estimated, albeit with considerable error, from studies that have looked at the magnitude of this cost in actual bankruptcies. Research that has looked at the direct cost of bankruptcy concludes that they are small[9] relative to firm value. The indirect costs of bankruptcy can be substantial, but the costs vary widely across firms. Shapiro and Titman speculate that the indirect costs could be as large as 25 to 30 percent of firm value but provide no direct evidence of the costs.

ILLUSTRATION 15.5: Valuing a Firm with the APV Approach: Tube Investments

Illustration 15.1 valued Tube Investments using a cost of capital approach. Here, we reestimate the value of the firm using an adjusted present value approach in three steps:

Step 1: Unlevered Firm Value

To estimate the unlevered firm value, we first compute the unlevered beta. Tube Investments' beta is 1.17, its current market debt to equity ratio is 79%, and the firm's tax rate is 30%:

$$\text{Unlevered beta} = 1.17/[1 + (1 - .3)(.79)] = 0.75$$

[9]In Warner's study of railroad bankruptcies, the direct cost of bankruptcy seems to be about 5 percent.

Using the rupee risk-free rate of 10.5% and the risk premium of 9.23% for India, we estimate an unlevered cost of equity:

Unlevered cost of equity = 10.5% + 0.75(9.23%) = 17.45%

Using the free cash flow to the firm estimated in Illustration 15.1 of Rs 212.2 million and the stable growth rate of 5%, we estimate the unlevered firm value:

Unlevered firm value = 212.2/(.1745 – .05) = $1,704.6 million

STEP 2: TAX BENEFITS FROM DEBT

The tax benefits from debt are computed based un Tube Investments' existing dollar debt of Rs 1,807.3 million and the tax rate of 30%:

Expected tax benefits in perpetuity = Tax rate(Debt) = .30(1,807.3) = Rs 542.2 million

STEP 3: EXPECTED BANKRUPTCY COSTS

To estimate this, we made two assumptions. One, based on the firm's existing synthetic rating, is that the probability of default at its existing debt level is 10%. The other is that the cost of bankruptcy is 40% of unlevered firm value.

Expected bankruptcy cost = Probability of bankruptcy × Cost of bankruptcy × Unlevered firm value
= .10 × .40 × 1,704.6 = Rs 68.2 million

The value of the operating assets of the firm can now be estimated:

Value of the operating assets = Unlevered firm value + PV of tax benefits – Expected bankruptcy costs
= 1,704.6 + 542.2 – 68.2 = Rs 2,178.6 million

Adding to this the value of cash and marketable securities of Rs 1,365.3 million, we obtain a value for the firm of Rs 3,543.9 million. In contrast, we valued the firm at Rs 3,367.3 million with the cost of capital approach.

Cost of Capital versus APV Valuation

In an APV valuation, the value of a levered firm is obtained by adding the net effect of debt to the unlevered firm value.

$$\text{Value of levered firm} = FCFF_o(1 + g)/(\rho_u - g) + t_c D - \pi_a BC$$

In the cost of capital approach, the effects of leverage show up in the cost of capital, with the tax benefit incorporated in the after-tax cost of debt and the bankruptcy costs in both the levered beta and the pretax cost of debt. Will the two approaches yield the same value? Not necessarily. The first reason for differences is that the models consider bankruptcy costs very differently, with the adjusted present value approach providing more flexibility in allowing you to consider indirect bankruptcy costs. To the extent that these costs do not show up or show up inadequately in the pretax cost of debt, the APV approach will yield a more conservative estimate of value. The second reason is that the APV approach considers the tax

APV WITHOUT BANKRUPTCY COSTS

There are many who believe that adjusted present value is a more flexible way of approaching valuation than traditional discounted cash flow models. This may be true in a generic sense, but APV valuation in practice has significant flaws. The first and most important is that most practitioners who use the adjusted present value model ignore expected bankruptcy costs. Adding the tax benefits to unlevered firm value to get to levered firm value makes debt seem like an unmixed blessing. Firm value will be overstated, especially at very high debt ratios, where the cost of bankruptcy is clearly not zero.

benefit from a dollar debt value, usually based on existing debt. The cost of capital approach estimates the tax benefit from a debt ratio that may require the firm to borrow increasing amounts in the future. For instance, assuming a market debt to capital ratio of 30 percent in perpetuity for a growing firm will require it to borrow more in the future, and the tax benefit from expected future borrowings is incorporated into value today.

EFFECT OF LEVERAGE ON FIRM VALUE

Both the cost of capital approach and the APV approach make the value of a firm a function of its leverage. It follows directly, then, that there is some mix of debt and equity at which firm value is maximized. The rest of this chapter considers how best to make this link.

Cost of Capital and Optimal Leverage

In order to understand the relationship between the cost of capital and optimal capital structure, we rely on the relationship between firm value and the cost of capital. The earlier section noted that the value of the entire firm can be estimated by discounting the expected cash flows to the firm at the firm's cost of capital.

The firm value can then be written as follows:

$$\text{Value of firm} = \sum_{t=1}^{t=n} \frac{\text{CF to firm}_t}{(1+\text{WACC})^t}$$

and is a function of the firm's cash flows and its cost of capital. If we assume that the cash flows to the firm are unaffected by the choice of financing mix, and the cost of capital is reduced as a consequence of changing the financing mix, the value of the firm will increase. If the objective in choosing the financing mix for the firm is the maximization of firm value, we can accomplish it, in this case, by minimizing the cost of capital. In the more general case where the cash flows to the firm are a function of the debt-equity mix, the optimal financing mix is the mix that maximizes firm value.[10]

[10]In other words, the value of the firm might not be maximized at the point that cost of capital is minimized, if firm cash flows are much lower at that level.

ILLUSTRATION 15.6: WACC, Firm Value, and Leverage

Assume that you are given the costs of equity and debt at different debt levels for Strunks Inc., a leading manufacturer of chocolates and other candies, and that the cash flows to this firm are currently $200 million. Strunks is in a relatively stable market, and these cash flows are expected to grow at 6% forever and to be unaffected by the debt ratio of the firm. The cost of capital schedule is provided in the following table, along with the value of the firm at each level of debt.

D/(D + E)	*Cost of Equity*	*Cost of Debt*	*WACC*	*Firm Value*
0%	10.50%	4.80%	10.50%	$4,711
10%	11.00%	5.10%	10.41%	$4,807
20%	11.60%	5.40%	10.36%	$4,862
30%	12.30%	5.52%	10.27%	$4,970
40%	13.10%	5.70%	10.14%	$5,121
50%	14.00%	6.30%	10.15%	$5,108
60%	15.00%	7.20%	10.32%	$4,907
70%	16.10%	8.10%	10.50%	$4,711
80%	17.20%	9.00%	10.64%	$4,569
90%	18.40%	10.20%	11.02%	$4,223
100%	19.70%	11.40%	11.40%	$3,926

Note that:

$$\text{Value of firm} = \text{Cash flows to firm} \times (1 + g)/(\text{Cost of capital} - g) = \$200 \times 1.06/(\text{Cost of capital} - .06)$$

The value of the firm increases as the cost of capital decreases, and decreases as the cost of capital increases. This is illustrated in Figure 15.2. While this illustration makes the choice of an optimal financing mix seem easy, it obscures problems that may arise in its practice. First, we typically do not have the benefit of having the entire schedule of costs of financing prior to an analysis. In most cases, the only level of debt at which we have information on the cost of debt and equity financing is the current level. Second, the analysis assumes implicitly that the level of operating income of the firm is unaffected by the financing mix of the firm and, consequently, by the default risk (or bond rating) for the firm. While this may be reasonable in some cases, it might not be in others. Firms that borrow too much might find that there are indirect bankruptcy costs that affect revenues and operating income.

Steps in Cost of Capital Approach We need three basic inputs to compute the cost of capital—the cost of equity, the after-tax cost of debt, and the weights on debt and equity. The costs of equity and debt change as the debt ratio changes, and the primary challenge of this approach is in estimating each of these inputs.

Let us begin with the cost of equity. We argued that the beta of equity will change as the debt ratio changes. In fact, we estimated the levered beta as a function of the market debt to equity ratio of a firm, the unlevered beta and the firm's marginal tax rate:

$$\beta_{\text{levered}} = \beta_{\text{unlevered}}[1 + (1 - t)\text{Debt/Equity}]$$

Thus, if we can estimate the unlevered beta for a firm, we can use it to estimate the levered beta of the firm at every debt ratio. This levered beta can then be used to compute the cost of equity at each debt ratio.

$$\text{Cost of equity} = \text{Risk-free rate} + \beta_{\text{levered}}(\text{Risk premium})$$

The cost of debt for a firm is a function of the firm's default risk. As firms borrow more, their default risk will increase and so will the cost of debt. If we use bond ratings as our measure of default risk, we can estimate the cost of debt in three steps. First, estimate a firm's dollar debt and interest expenses at each debt ratio; as firms increase their debt ratio, both dollar debt and interest expenses will rise. Second, at each debt level, compute a financial ratio or ratios that measures default risk and use the ratio(s) to estimate a rating for the firm; again, as firms borrow more, this rating will decline. Third, a default spread, based on the estimated rating, is added to the risk-free rate to arrive at the pretax cost of debt. Applying the marginal tax rate to this pretax cost yields an after-tax cost of debt.

Once we estimate the costs of equity and debt at each debt level, we weight them based on the proportions used of each to estimate the cost of capital. While we have not explicitly allowed for a preferred stock component in this process, we can have preferred stock as a part of capital. However, we have to keep the preferred stock portion fixed, while changing the weights on debt and equity. The debt ratio at which the cost of capital is minimized is the optimal debt ratio.

In this approach, the effect on firm value of changing the capital structure is isolated by keeping the operating income fixed and varying only the cost of capital. In practical terms, this requires us to make two assumptions. First, the debt ratio is decreased by raising new equity and retiring debt; conversely, the debt ratio is increased by borrowing money and buying back stock. This process is called recapitalization. Second, the pretax operating income is assumed to be unaffected by the firm's financing mix and, by extension, its bond rating. If the operating income

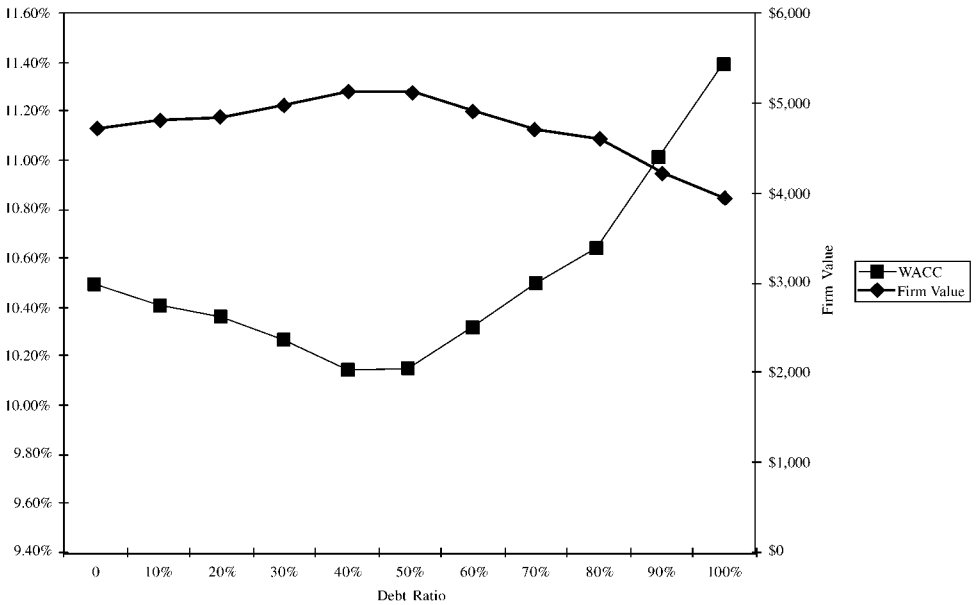

FIGURE 15.2 Cost of Capital and Firm Value

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changes with a firm's default risk, the basic analysis will not change, but minimizing the cost of capital may not be the optimal course of action, since the value of the firm is determined by both the cash flows and the cost of capital. The value of the firm will have to be computed at each debt level and the optimal debt ratio will be that which maximizes firm value.

ILLUSTRATION 15.7: Analyzing the Capital Structure for Boeing—March 1999

The cost of capital approach can be used to find the optimal capital structure for a firm, as in this case for Boeing in March 1999. Boeing had $6,972 million in debt on its books at that time, with an estimated market value, inclusive of operating leases, of $8,194 million.[11] The market value of equity at the same time was $32,595 million; the market price per share was $32.25, and there were 1,010.7 million shares outstanding. Proportionally, 20.09% of the overall financing mix was debt, and the remaining 79.91% was equity.

The beta for Boeing's stock in March 1999 was 1.01. The Treasury bond rate at that time was 5%. Using an estimated market risk premium of 5.5%, we estimated the cost of equity for Boeing to be 10.58%:

$$\begin{aligned}\text{Cost of equity} &= \text{Risk-free rate} + \text{Beta} \times (\text{Market premium}) \\ &= 5.00\% + 1.01(5.5\%) = 10.58\%\end{aligned}$$

Boeing's senior debt was rated AA. Based on this rating, the estimated pretax cost of debt for Boeing is 5.50%. The tax rate used for the analysis is 35%.

$$\text{Value of firm} = 8{,}194 + 32{,}595 = \$40{,}789 \text{ million}$$

$$\text{After-tax cost of debt} = \text{Pretax interest rate}(1 - \text{Tax rate}) = 5.50\%(1 - 0.35) = 3.58\%$$

The cost of capital was calculated using these costs and the weights based on market value:

$$\begin{aligned}\text{WACC} &= \text{Cost of equity}[\text{Equity}/(\text{Equity} + \text{Debt})] + \text{After-tax cost of debt}[\text{Debt}/(\text{Debt} + \text{Equity})] \\ &= 10.58\% \times [32{,}595/40{,}789] + 3.58\% \times [8{,}194/40{,}789] = 9.17\%\end{aligned}$$

BOEING'S COST OF EQUITY AND LEVERAGE

The cost of equity for Boeing at different debt ratios can be computed using the unlevered beta of the firm, and the debt-equity ratio at each level of debt. We use the levered betas that emerge to estimate the cost of equity. The first step in this process is to compute the firm's current unlevered beta, using the current market debt to equity ratio and a tax rate of 35%.

$$\begin{aligned}\text{Unlevered beta} &= \text{Current beta}/[1 + (1 - t)\text{Debt/Equity}] \\ &= 1.014/[1 + (1 - 0.35)(8{,}194/32{,}595)] = 0.87\end{aligned}$$

The recomputed betas are reported in the following table. We use the Treasury bond rate of 5% and the market premium of 5.5% to compute the cost of equity.

[11]The details of this calculation are in Chapter 7.

Debt Ratio	*Beta*	*Cost of Equity*
0%	0.87	9.79%
10%	0.93	10.14%
20%	1.01	10.57%
30%	1.11	11.13%
40%	1.25	11.87%
50%	1.51	13.28%
60%	1.92	15.54%
70%	2.56	19.06%
80%	3.83	26.09%
90%	7.67	47.18%

In calculating the levered betas in this table, we assumed that all market risk is borne by the equity investors; this may be unrealistic especially at higher levels of debt. We also adjusted the tax rate beyond a debt ratio of 50% to reflect the loss of tax benefits. We could also consider an alternative estimate of levered betas that apportions some of the market risk to the debt:

$$\beta_{levered} = \beta_u[1 + (1 - t)D/E] - \beta_{debt}(1 - t)D/E$$

The beta of debt is based on the rating of the bond and is estimated by regressing past returns on bonds in each rating class against returns on a market index. The levered betas estimated using this approach will generally be lower than those estimated with the conventional model.

Boeing's Cost of Debt and Leverage

We assume that bond ratings are determined solely by the interest coverage ratio, which is defined as:

Interest coverage ratio = Earnings before interest and taxes/Interest expense

We chose the interest coverage ratio for three reasons. First, it is a ratio used by both Standard & Poor's and Moody's to determine ratings.[12] Second, there is significant correlation not only between the interest coverage ratio and bond ratings, but also between the interest coverage ratio and other ratios used in analysis, such as the debt coverage ratio and the funds flow ratios. Third, the interest coverage ratio changes as a firm changes is financing mix and decreases as the debt ratio increases. The ratings agencies would argue, however, that subjective factors, such as the perceived quality of management, are part of the ratings process. One way to build these factors into the analysis would be to modify the ratings obtained from the financial ratio analysis across the board to reflect the ratings agencies' subjective concerns.[13]

The data in the following table were obtained based on an analysis of the interest coverage ratios of large manufacturing firms in different ratings classes.

[12]S&P lists interest coverage ratio first among the nine ratios that it reports for different ratings classes on its web site.

[13]For instance, assume that a firm's current rating is AA, but that its financial ratios would result in an A rating. It can then be argued that the ratings agencies are, for subjective reasons, rating the company one notch higher than the rating obtained from a purely financial analysis. The ratings obtained for each debt level can then be increased by one notch across the board to reflect these subjective considerations.

Interest Coverage Ratio	*Rating*
> 8.5	AAA
6.50–8.50	AA
5.50–6.50	A+
4.25–5.50	A
3.00–4.25	A–
2.50–3.00	BBB
2.00–2.50	BB
1.75–2.00	B+
1.50–1.75	B
1.25–1.50	B–
0.80–1.25	CCC
0.65–0.80	CC
0.20–0.65	C
< 0.65	D

Source: Compustat.

Using this table as a guideline, a firm with an interest coverage ratio of 1.65 would have a rating of B for its bonds.

The relationship between bond ratings and interest rates in February 1999 was obtained by looking at the typical default spreads for bonds in different ratings classes.[14] The following table summarizes the interest rates/rating relationship and reports the spreads for these bonds over Treasury bonds and the resulting interest rates, using the Treasury bond rate of 5%.

Rating	*Spread*	*Interest Rate on Debt*
AAA	0.20%	5.20%
AA	0.50%	5.50%
A+	0.80%	5.80%
A	1.00%	6.00%
A–	1.25%	6.25%
BBB	1.50%	6.50%
BB	2.00%	7.00%
B+	2.50%	7.50%
B	3.25%	8.25%
B–	4.25%	9.25%
CCC	5.00%	10.00%
CC	6.00%	11.00%
C	7.50%	12.50%
D	10.00%	15.00%

Source: bondsonline.com.

The following table summarizes Boeing's income statement for the financial year 1998. It shows that Boeing had earnings before interest, taxes, depreciation, and amortization (EBITDA) of $3,237 million, and paid interest expenses of $453 million.

[14]These default spreads were estimated from bondsonline.com, a service that provides, among other data on fixed income securities, updated default spreads for each ratings class.

Sales and other operating revenues	$56,154.00
– Operating costs and expenses	$52,917.00
EBITDA	$ 3,237.00
– Depreciation	$ 1,517.00
EBIT	$ 1,720.00
+ Extraordinary income	$ 130.00
EBIT with extraordinary income	$ 1,850.00
– Interest expenses	$ 453.00
Earnings before taxes	$ 1,397.00
– Income Taxes	$ 277.00
Net earnings (loss)	$ 1,120.00

Based on the earnings before interest and taxes (EBIT) of $1,720 million and interest expenses of $453 million, Boeing has an interest coverage ratio of 3.80 and should command a rating of A–. Boeing's earnings before interest, taxes, and depreciation for the year were $3,237 million. The actual rating of the firm, which is AA, reflects the ratings agency view that Boeing had subpar years in both 1997 and 1998, and is capable of earning more on a regular basis. In our analysis, we adjust the EBIT and EBITDA for the imputed interest expenses on Boeing's operating leases;[15] this results in an increase of $31 million in both numbers—to $1,751 million in EBIT and $3,268 million in EBITDA.

Finally, to compute Boeing's ratings at different debt levels, we redo the operating income statement at each level of debt, compute the interest coverage ratio at that level of debt, and find the rating that corresponds to that level of debt. For example, the following table estimates the interest expenses, interest coverage ratios, and bond ratings for Boeing at 0% and 10% debt ratios, at the existing level of operating income.

Debt/(Debt + Equity)	0.00%	10.00%
Debt/Equity	0.00%	11.11%
$ Debt	$0	$4,079
EBITDA	$3,268	$3,268
Depreciation	$1,517	$1,517
EBIT	$1,751	$1,751
Interest expense	$0	$227
Pretax int. coverage	∞	7.80
Likely rating	AAA	AA
Interest rate	5.20%	5.50%
Effective tax rate	35.00%	35.00%

The dollar debt is computed to be 10% of the current value of the firm by adding the market values of debt and equity:

$$\text{Dollar debt at 10\% debt ratio} = \text{Debt ratio}(\text{Market value of equity} + \text{Market value of debt})$$
$$= .10(32{,}595 + 8{,}194) = \$4{,}079 \text{ million}$$

There is circular reasoning involved in estimating the interest expense. The interest rate is needed to calculate the interest coverage ratio, and the coverage ratio is necessary to compute the interest rate. To get around the problem, we began our analysis by assuming that you could borrow $4.079 billion at the AAA rate of 5.20%; we then computed an interest expense and interest coverage ratio using that rate, and estimated a new rating of AA for Boeing. We recomputed the interest expense using the AA rate of 5.50% as our cost of debt.[16] This process is repeated for each level of debt from 10% to 90%, and the after-tax costs of debt are obtained at each level of debt in the following table:

[15]The details of this adjustment are provided in Chapter 9.

[16]Since the interest expense rises, it is possible for the rating to drop again. Thus a third iteration might be necessary in some cases.

Debt Ratio	*$ Debt*	*Interest Expense*	*Interest Coverage Ratio*	*Bond Rating*	*Pretax Cost of Debt*	*Tax Rate*	*After-Tax Cost of Debt*
0.00%	$ 0	$0	∞	AAA	5.20%	35.00%	3.38%
10.00%	$ 4,079	$224	7.80	AA	5.50%	35.00%	3.58%
20.00%	$ 8,158	$510	3.43	A–	6.25%	35.00%	4.06%
30.00%	$12,237	$857	2.04	BB	7.00%	35.00%	4.55%
40.00%	$16,316	$1,632	1.07	CCC	10.00%	35.00%	6.50%
50.00%	$20,394	$2,039	0.86	CCC	10.00%	30.05%	7.00%
60.00%	$24,473	$2,692	0.65	CC	11.00%	22.76%	8.50%
70.00%	$28,552	$3,569	0.49	C	12.50%	17.17%	10.35%
80.00%	$32,631	$4,079	0.43	C	12.50%	15.02%	10.62%
90.00%	$36,710	$4,589	0.38	C	12.50%	13.36%	10.83%

There are two points to make about this computation. We assume that at every debt level, all existing debt will be refinanced at the new interest rate that will prevail after the capital structure change. For instance, Boeing's existing debt, which has a AA rating, is assumed to be refinanced at the interest rate corresponding to a BB rating when Boeing moves to a 30% debt ratio. This is done for two reasons. The first is that existing debt holders might have protective puts that enable them to put their bonds back to the firm and receive face value.[17] The second is that the refinancing eliminates "wealth expropriation" effects—the effects of stockholders expropriating wealth from bondholders when debt is increased, and vice versa when debt is reduced. If firms can retain old debt at lower rates while borrowing more and becoming riskier, the lenders of the old debt will lose wealth. Locking in current rates on existing bonds and recalculating the optimal debt ratio will allow for this wealth transfer.[18]

While it is conventional to leave the marginal tax rate unchanged as the debt ratio is increased, we adjust the tax rate to reflect the potential loss of the tax benefits of debt at higher debt ratios, where the interest expenses exceed the earnings before interest and taxes. To illustrate this point, note that the amount of earnings before interest and taxes at Boeing is $1,751 million. As long as interest expenses are less than $1,751 million, interest expenses remain fully tax deductible and earn the 35% tax benefit. For instance, at a 40% debt ratio, the interest expenses are $1,632 million and the tax benefit is therefore 35% of this amount. At a 50% debt ratio, however, the interest expenses balloon to $2,039 million, which is greater than the earnings before interest and taxes of $1,751 million. Considering the tax benefit on the interest expenses up to this amount:

$$\text{Tax benefit} = \$1{,}751 \text{ million} \times .35 = \$612.85 \text{ million}$$

As a proportion of the total interest expenses, the tax benefit is now less than 35%:

$$\text{Effective tax rate} = \$613/\$1{,}751 = 30.05\%$$

This, in turn, raises the after-tax cost of debt. This is a conservative approach, since losses can be carried forward. Given that this is a permanent shift in leverage, it does make sense to be conservative.

[17]If they do not have protective puts, it is in the best interests of the stockholders not to refinance the debt (as in the leveraged buyout of RJR Nabisco) if debt ratios are increased.

[18]This will have the effect of reducing interest cost when debt is increased, and thus interest coverage ratios. This will lead to higher ratings, at least in the short term, and a higher optimal debt ratio.

Leverage and Cost of Capital

Now that we have estimated the cost of equity and the cost of debt at each debt level, we can compute Boeing's cost of capital. This is done for each debt level in the following table. The cost of capital, which is 9.79% when the firm is unlevered, decreases as the firm initially adds debt, reaches a minimum of 9.16% at 30% debt, and then starts to increase again.

Debt Ratio	*Beta*	*Cost of Equity*	*Cost of Debt (After-Tax)*	*Cost of Capital*
0%	0.87	9.79%	3.38%	9.79%
10%	0.93	10.14%	3.58%	9.48%
20%	1.01	10.57%	4.06%	9.27%
30%	1.11	11.13%	4.55%	9.16%
40%	1.25	11.87%	6.50%	9.72%
50%	1.48	13.15%	7.00%	10.07%
60%	1.88	15.35%	8.50%	11.24%
70%	2.56	19.06%	10.35%	12.97%
80%	3.83	26.09%	10.62%	13.72%
90%	7.67	47.18%	10.83%	14.47%

The optimal debt ratio is shown graphically in Figure 15.3.

To illustrate the robustness of this solution to alternative measures of levered betas, we reestimate the costs of debt, equity, and capital under the assumption that debt bears some market risk, and the results are summarized in the following table.

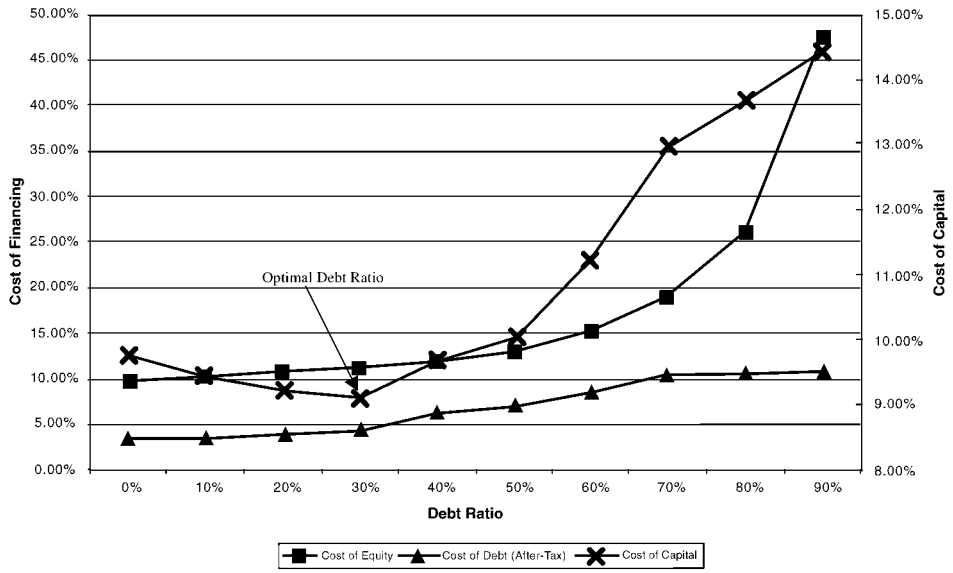

FIGURE 15.3 Costs of Equity, Debt, and Capital: Boeing

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Debt Ratio	*Beta*	*Cost of Equity*	*Beta of Debt*	*Bond Rating*	*Interest Rate on Debt*	*Tax Rate*	*Cost of Debt (After-Tax)*	*Cost of Capital*
0%	0.89	9.92%	0.02	AAA	5.20%	35.00%	3.38%	9.92%
10%	0.96	10.26%	0.05	AA	5.50%	35.00%	3.58%	9.59%
20%	1.02	10.62%	0.11	A–	6.25%	35.00%	4.06%	9.31%
30%	1.10	11.04%	0.18	BB	7.00%	35.00%	4.55%	9.09%
40%	1.11	11.08%	0.45	CCC	10.00%	35.00%	6.50%	9.25%
50%	1.24	11.80%	0.45	CCC	10.00%	29.81%	7.02%	9.41%
60%	1.24	11.80%	0.68	C	12.50%	19.87%	10.02%	10.73%
70%	1.44	12.94%	0.68	C	12.50%	17.03%	10.37%	11.14%
80%	1.86	15.24%	0.68	C	12.50%	14.91%	10.64%	11.56%
90%	3.11	22.13%	0.68	C	12.50%	13.25%	10.84%	11.97%

If the debt holders bear some market risk,[19] the cost of equity is lower at higher levels of debt and Boeing's optimal debt ratio is still 30%, which is unchanged from the optimal calculated under the conventional calculation of the levered beta.

Firm Value and Cost of Capital

The reason for minimizing the cost of capital is that it maximizes the value of the firm. To illustrate the effects of moving to the optimal on Boeing's firm value, we use the model described earlier in the chapter designed to value a firm in stable growth:

$$\text{Firm value} = \text{Expected FCFF}_{\text{next year}}/(\text{WACC} - g)$$

where g is the stable growth rate.

We begin by computing Boeing's current free cash flow using its current earnings before interest and taxes of $1,753 million, its tax rate of 35%, and its reinvestments in 1998 in working capital and net fixed assets:

EBIT(1 – Tax rate)	$1,138
+ Depreciation and amortization	$1,517
– Capital expenditures	$1,584
– Change in working capital	$ (105)
Free cash flow to the firm	$1,176

The market value of the firm at the time of this analysis was obtained by adding up the estimated market values of debt and equity:

Market value of equity	$32,595
+ Market value of debt	$ 8,194
= Value of the firm	$40,789

[19]To estimate the beta of debt, we used the default spread at each level of debt, and assumed that half this risk is market risk. Thus, at a C rating, the default spread is 9 percent. Based on the market risk premium of 5.5 percent and the risk-free rate of 5 percent that we used elsewhere, we estimated the beta at a C rating to be:

$$\text{Imputed debt beta at a C rating} = (9\%/5.5\%) \times 0.5 = 0.68$$

Based on the current cost of capital of 9.17%, we solve for the implied growth rate:

$$\text{Growth rate} = (\text{Firm value} \times \text{Cost of capital} - \text{CF to firm})/(\text{Firm value} + \text{CF to firm})$$
$$= (40{,}789 \times .0917 - 1{,}176)/(40{,}789 + 1{,}176) = .0611 \text{ or } 6.11\%$$

Now assume that Boeing shifts to 30% debt and a WACC of 9.16%. The firm can now be valued using the following parameters:

Cash flow to firm = $1,176 million
WACC = 9.16%
Growth rate in cash flows to firm = 6.11%
Firm value = $(1{,}176 \times 1.0611)/(.0916 - .0611)$ = $40,990 million

The value of the firm[20] will increase from $40,789 million to $40,990 million if the firm moves to the optimal debt ratio:

Increase in firm value = $40,990 million – $ 40,789 million = $201 million

With 1,010.7 million shares outstanding, assuming that stockholders can evaluate the effect of this refinancing, we can calculate the increase in the stock price:

$$\text{Increase in stock price} = \text{Increase in firm value}/\text{Number of shares outstanding}$$
$$= \$201/1{,}010.7 = \$0.20$$

Since the current stock price is $32.25, the stock price can be expected to increase to $32.45, which translates into a 0.62% increase in the price. The change is negligible because the change in the cost of capital is small. The firm value and cost of capital at different debt ratios are summarized in Figure 15.4.

Since the asset side of the balance sheet is kept fixed and changes in capital structure are made by borrowing funds and repurchasing stock, this analysis implies that the stock price would increase to $32.45 on the announcement of the repurchase. Implicit in this analysis is the assumption that the increase in firm value will be spread evenly across both stockholders who sell their stock back to the firm and those who do not. To the extent that stock can be bought back at the current price of $32.25 or some value lower than $32.45, the change in stock price will be larger. For instance, if Boeing could have bought stock back at the existing price of $32.25, the increase in value per share would be $0.23.[21]

captstr.xls: **This spreadsheet allows you to compute the optimal debt ratio firm value for any firm, using the same information used for Boeing. It has updated interest coverage ratios and default spreads built in.**

[20]This approach works best for firms with growth rates close to or below the growth rate of the economy, since this is a model that assumes perpetual growth. When this is not the case (i.e., when implied growth is much higher than 6 percent, we would suggest a modified approach, in which the present value of savings in firm value each year from going to the lower cost of capital is computed using a stable growth rate capped at about 6 percent. In the case of Boeing, this calculation would have yielded the following:

Savings each year = $40,789(.0917 – .0916) = $6.14 million
Present value of savings = $6.14/(.0916 – .06) = $206 million
Increase in value per share = $206 million/1,010.7 = $0.20

[21]To compute this change in value per share, we first compute how many shares we would buy back with the additional debt taken on of $4.043 billion (debt at 30 percent optimal minus current debt) and the stock price of $32.25. We then divide the increase in firm value of $202 million by the remaining shares outstanding:

Change in stock price = $202 million/[1,010.7 – (4,043/32.25)] = $0.23 per share

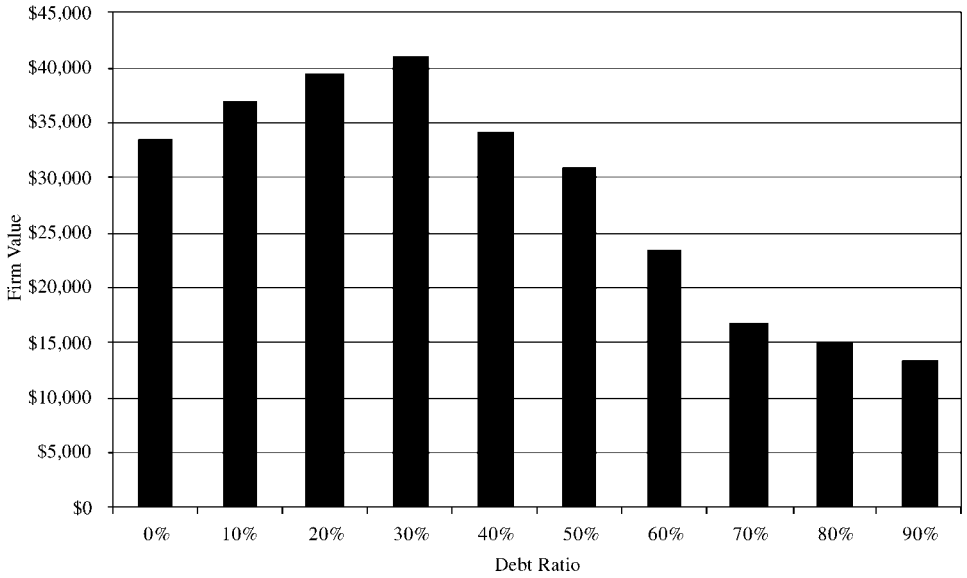

FIGURE 15.4 Debt Ratios and Firm Value

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DEFAULT RISK, OPERATING INCOME, AND OPTIMAL LEVERAGE

The Boeing analysis just completed assumed that operating income would remain constant while the debt ratios changed. While this assumption simplifies the analysis substantially, it is not realistic. The operating income, for many firms, will drop as the default risk increases; this, in fact, is the cost we label as an indirect bankruptcy cost. The drop is likely to become more pronounced as the default risk falls below an acceptable level; for instance, a bond rating below investment grade may trigger significant losses in revenues and increases in expenses.

A general model for optimal capital structure would allow both operating income and cost of capital to change as the debt ratio changes. We have already described how we can estimate cost of capital at different debt ratios, but we could also attempt to do the same with operating income. For instance, we could estimate how the operating income for the Boeing would change as debt ratios and default risk changes by looking at the effects of rating downgrades on the operating income of other retailers.

If both operating income and cost of capital change, the optimal debt ratio may no longer be the point at which the cost of capital is minimized. Instead, the optimal has to be defined as that debt ratio at which the value of the firm is maximized.

ADJUSTED PRESENT VALUE AND FINANCIAL LEVERAGE

In the adjusted present value (APV) approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. The value of the levered firm can then be estimated at different levels of the debt, and the debt level that maximizes firm value is the optimal debt ratio.

Steps in the Adjusted Present Value Approach

The unlevered firm value is not a function of expected leverage and can be estimated as described in the earlier section—by discounting the free cash flows to the firm at the unlevered cost of equity. In fact, if you do not want to estimate this value and take the market value of the firm as correct, you could back out the unlevered firm value by subtracting out the tax benefits and adding back the expected bankruptcy cost from the existing debt.

$$\text{Current firm value} = \text{Value of unlevered firm} + \text{Present value of tax benefits} - \text{Expected bankruptcy cost}$$

$$\text{Value of unlevered firm} = \text{Current firm value} - \text{Present value of tax benefits} + \text{Expected bankruptcy cost}$$

The only components that change as a firm changes its leverage are the expected tax benefits and the expected bankruptcy costs. To obtain these values as you change leverage, you would go through the following five steps:

1. *Estimate the dollar debt outstanding at each debt ratio.* This process mirrors what was done in the cost of capital approach. Keeping firm value fixed, consider how much debt the firm will have at 20 percent debt, 30 percent debt, and so on.
2. *Estimate the tax benefits of debt by multiplying the dollar debt by the tax rate.* This essentially assumes that the debt is permanent and that the tax benefits will continue in perpetuity.
3. *Estimate the rating, interest rate, and interest expense at each debt ratio.* This process again replicates what was done in the cost of capital approach.
4. *Use the rating to estimate a probability of default.* Note that Table 15.2 provides these probabilities for each rating.
5. *Estimate the expected bankruptcy cost* by multiplying the probability of bankruptcy by the bankruptcy cost, stated as a percent of unlevered firm value.

We compute the value of the levered firm at different levels of debt. The debt level that maximizes the value of the levered firm is the optimal debt ratio.

ILLUSTRATION 15.8: Using the Adjusted Present Value Approach to Calculate Optimal Debt Ratio for Boeing in 1999

This approach can be applied to estimating the optimal capital structure for Boeing. The first step is to estimate the value of the unlevered firm. To do so, we start with the firm value of Boeing in 1999 and net the effect of the tax savings and bankruptcy costs arising from the existing debt.

$$\begin{aligned}\text{Value of Boeing in 1999} &= \text{Value of equity} + \text{Value of debt}\\ &= \$32{,}595 + \$8{,}194 = \$40{,}789\end{aligned}$$

We compute the present value of the tax savings from the existing debt, assuming that the interest payments on the debt constitute a perpetuity.

$$\begin{aligned}\text{PV of tax savings from existing debt} &= \text{Existing debt} \times \text{Tax rate}\\ &= \$8{,}194 \times 0.35 = \$2{,}868 \text{ million}\end{aligned}$$

Based on Boeing's current rating of AA, we estimate a probability of bankruptcy of 0.28% from Table 15.2. The bankruptcy cost is assumed to be 30% of the unlevered firm value.[22] The cost is high because the perception of default risk is likely to be very damaging for a firm like Boeing, whose customers depend on it for long-term service and support, and whose sales contracts are often spread out over a decade or more.

$$\begin{aligned}\text{Present value of expected bankruptcy cost} &= \text{Probability of default} \times \text{Bankruptcy cost}\\ &= 0.28\% \times [0.30 \times (40{,}789 - 2{,}868)] = \$32\end{aligned}$$

We then compute the value of Boeing as an unlevered firm.

$$\begin{aligned}\text{Value of Boeing as unlevered firm} &= \text{Current market value} - \text{PV of tax savings}\\ &\quad + \text{Expected bankruptcy cost}\\ &= \$40{,}789 - \$2{,}868 + \$32 = \$37{,}953 \text{ million}\end{aligned}$$

The next step in the process is to estimate the tax savings at different levels of debt in the following table. While we use the standard approach of assuming that the present value is calculated over a perpetuity, we reduce the tax rate used in the calculation, if interest expenses exceed the earnings before interest and taxes. The adjustment to the tax rate was described more fully earlier in the cost of capital approach.

Debt Ratio	*$ Debt*	*Tax Rate*	*Tax Benefits*
0%	$ 0	35.00%	$ 0
10%	$ 4,079	35.00%	$1,428
20%	$ 8,158	35.00%	$2,855
30%	$12,237	35.00%	$4,283
40%	$16,316	35.00%	$5,710
50%	$20,394	30.05%	$6,128
60%	$24,473	22.76%	$5,571
70%	$28,552	17.17%	$4,903
80%	$32,631	15.02%	$4,903
90%	$36,710	13.36%	$4,903

[22]This estimate is based on the Warner study, which estimates bankruptcy costs for large companies to be 15 percent of the value, and on the qualitative analysis of indirect bankruptcy costs in Shapiro and Cornell.

The final step in the process is to estimate the expected bankruptcy cost, based on the bond ratings, the probabilities of default, and the assumption that the bankruptcy cost is 30% of unlevered firm value. The following table summarizes these probabilities and the expected bankruptcy cost, computed based on the unlevered firm value.

Debt Ratio	*Bond Rating*	*Probability of Default*	*Expected Bankruptcy Cost*
0%	AA	0.28%	$ 32
10%	AA	0.28%	$ 32
20%	A–	1.41%	$ 161
30%	BB	12.20%	$1,389
40%	CCC	50.00%	$5,693
50%	CCC	50.00%	$5,693
60%	CC	65.00%	$7,401
70%	C	80.00%	$9,109
80%	C	80.00%	$9,109
90%	C	80.00%	$9,109

The value of the levered firm is estimated in the following table by aggregating the effects of the tax savings and the expected bankruptcy costs:

Debt Ratio	*Unlevered Firm Value*	*Tax Benefits*	*Expected Bankruptcy Cost*	*Value of Levered Firm*
0%	$37,953	$ 0	$ 32	$37,921
10%	$37,953	$1,428	$ 32	$39,349
20%	$37,953	$2,855	$ 161	$40,648
30%	$37,953	$4,283	$1,389	$40,847
40%	$37,953	$5,710	$5,693	$37,970
50%	$37,953	$6,128	$5,693	$38,388
60%	$37,953	$5,571	$7,401	$36,123
70%	$37,953	$4,903	$9,109	$33,747
80%	$37,953	$4,903	$9,109	$33,747
90%	$37,953	$4,903	$9,109	$33,747

The firm value is optimized at between 20% and 30% debt, which is consistent with the results of the cost of capital approach. These results are, however, very sensitive to both the estimate of bankruptcy cost as a percent of firm value and the probabilities of default.

apv.xls: **This spreadsheet allows you to compute the value of a firm, with leverage, using the adjusted present value approach.**

Benefits and Limitations of the Adjusted Present Value Approach

The advantage of the APV approach is that it separates the effects of debt into different components and allows the analyst to use different discount rates for each component. In addition, we do not assume that the debt ratio stays unchanged forever, which is an implicit assumption in the cost of capital approach. Instead, we have the flexibility to keep the dollar value of debt fixed and to calculate the benefits and costs of the fixed dollar debt.

VALUING THE PIECES RATHER THAN THE WHOLE

The adjusted present value model values debt separately from the operating assets, and firm value is the sum of the two components. In fact, one of the biggest benefits of discounted cash flow valuation is that breaking up cash flows into individual components and valuing them separately should not change the value. Thus, you could value a firm like General Electric (GE) by valuing each of its divisions separately and adding them up, or Coca-Cola by valuing its operations in each country separately and summing those up.

The advantage of piecewise valuation is that you can estimate cash flows and discount rates separately for each piece and thus get more precise estimates of value. For example, you would use very different assumptions about operating margins, reinvestment needs, and costs of capital when valuing the appliance and aircraft engine divisions of GE. Similarly, you could apply different country risk premiums for each country that Coca-Cola operates in to value the firm. Since this is always the case, you might ask why we do not do this for all firms. The problem is with the information. Many firms do not break down their earnings and cash flows in sufficient detail to allow for piecewise valuation. Even firms that do, like GE, often have large centralized expenses that get allocated, often arbitrarily, to individual divisions.

The benefits of breaking a firm down into pieces clearly increase as a firm becomes more diverse in its operations. These benefits have to be weighed against the costs associated with more imprecise information and greater estimation problems.

These advantages have to be weighed against the difficulty of estimating probabilities of default and the cost of bankruptcy. In fact, many analyses that use the adjusted present value approach ignore the expected bankruptcy costs, leading them to the conclusion that firm value increases as firms borrow money. Not surprisingly, this will yield the conclusion that the optimal debt ratio for a firm is 100 percent debt.

In general, with the same assumptions, the APV and the cost of capital conclusions give identical answers. However, the APV approach is more practical when firms are evaluating a dollar amount of debt, while the cost of capital approach is easier when firms are analyzing debt proportions.[23]

CONCLUSION

This chapter develops an alternative approach to discounted cash flow valuation. The cash flows to the firm are discounted at the weighted average cost of capital to obtain the value of the firm, which when reduced by the market value

[23]See Inselbag and Kaufold (1997).

of outstanding debt yields the value of equity. Since the cash flow to the firm is a cash flow prior to debt payments, this approach is more straightforward to use when there is significant leverage or when leverage changes over time, though the weighted average cost of capital, used to discount free cash flows to the firm, has to be adjusted for changes in leverage. Finally, the costs of capital can be estimated at different debt ratios and used to estimate the optimal debt ratio for a firm.

The alternative approach to firm valuation is the APV approach, where the effect on value of debt (tax benefits minus bankruptcy costs) is added to the unlevered firm value. This approach can also be used to estimate the optimal debt ratio for the firm.

QUESTIONS AND SHORT PROBLEMS

1. Respond true or false to the following statements about the free cash flow to the firm:
   a. The free cash flow to the firm is always higher than the free cash flow to equity.
      True ____ False ____
   b. The free cash flow to the firm is the cumulated cash flow to all investors in the firm, though the form of their claims may be different.
      True ____ False ____
   c. The free cash flow to the firm is a predebt, pretax cash flow.
      True ____ False ____
   d. The free cash flow to the firm is an after-debt, after-tax cash flow.
      True ____ False ____
   e. The free cash flow to the firm cannot be estimated for a firm with debt without knowing interest and principal payments.
      True ____ False ____
2. Union Pacific Railroad reported net income of $770 million in 1993 after interest expenses of $320 million. (The corporate tax rate was 36%.) It reported depreciation of $960 million in that year, and capital spending was $1.2 billion. The firm also had $4 billion in debt outstanding on the books, rated AA (carrying a yield to maturity of 8%) and trading at par (up from $3.8 billion at the end of 1992). The beta of the stock was 1.05, and there were 200 million shares outstanding (trading at $60 per share), with a book value of $5 billion. Union Pacific's working capital requirements were negligible. (The Treasury bond rate was 7%, and the risk premium was 5.5%.)
   a. Estimate the free cash flow to the firm in 1993.
   b. Estimate the value of the firm at the end of 1993.
   c. Estimate the value of equity at the end of 1993, and the value per share, using the FCFF approach.
3. Lockheed Corporation, one of the largest defense contractors in the United States, reported EBITDA of $1,290 million in 1993, prior to interest expenses of $215 million and depreciation charges of $400 million. Capital expenditures in 1993 amounted to $450 million, and working capital was 7% of revenues (which were $13,500 million). The firm had debt outstanding of $3.068 billion

(in book value terms), trading at a market value of $3.2 billion and yielding a pretax interest rate of 8%. There were 62 million shares outstanding, trading at $64 per share, and the most recent beta was 1.10. The tax rate for the firm was 40%. (The Treasury bond rate was 7%, and the risk premium was 5.5%.)

The firm expected revenues, earnings, capital expenditures and depreciation to grow at 9.5% a year from 1994 to 1998, after which the growth rate was expected to drop to 4%. (Capital spending will be 120% of depreciation in the steady state period.) The company also planned to lower its debt/equity ratio to 50% for the steady state (which will result in the pretax interest rate dropping to 7.5%).

a. Estimate the value of the firm.
b. Estimate the value of the equity in the firm, and the value per share.

4. In the face of disappointing earnings results and increasingly assertive institutional stockholders, Eastman Kodak was considering a major restructuring in 1993. As part of this restructuring, it was considering the sale of its health division, which earned $560 million in earnings before interest and taxes in 1993, on revenues of $5.285 billion. The expected growth in earnings was expected to moderate to 6% between 1994 and 1998, and to 4% after that. Capital expenditures in the health division amounted to $420 million in 1993, while depreciation was $350 million. Both were expected to grow 4% a year in the long term. Working capital requirements were negligible.

   The average beta of firms competing with Eastman Kodak's health division was 1.15. While Eastman Kodak had a debt ratio [D/(D + E)] of 50%, the health division could sustain a debt ratio [D/(D + E)] of only 20%, which was similar to the average debt ratio of firms competing in the health sector. At this level of debt, the health division could expect to pay 7.5% on its debt, before taxes. (The tax rate is 40%, the Treasury bond rate is 7%, and the risk premium is 5.5%.)

   a. Estimate the cost of capital for the division.
   b. Estimate the value of the division.
   c. Why might an acquirer pay more than this estimated value for the division?

5. You are analyzing a valuation done on a stable firm by a well-known analyst. Based on the expected free cash flow to firm next year of $30 million and an expected growth rate of 5%, the analyst has estimated a value of $750 million. However, he has made the mistake of using the book values of debt and equity in his calculation. While you do not know the book value weights he used, you know that the firm has a cost of equity of 12% and an after-tax cost of debt of 6%. You also know that the market value of equity is three times the book value of equity, while the market value of debt is equal to the book value of debt. Estimate the correct value for the firm.

6. Santa Fe Pacific, a major rail operator with diversified operations, had earnings before interest, taxes, and depreciation of $637 million in 1993, with depreciation amounting to $235 million (offset by capital expenditure of an equivalent amount). The firm was in steady state and expected to grow 6% a year in perpetuity. Santa Fe Pacific had a beta of 1.25 in 1993, and debt outstanding of $1.34 billion. The stock price was $18.25 at the end of 1993, and there were 183.1 million shares outstanding. The expected ratings and the costs of debt at different levels of debt for Santa Fe are shown in the following table:

D/(D + E)	*Rating*	*Cost of Debt (Pretax)*
0%	AAA	6.23%
10%	AAA	6.23%
20%	A+	6.93%
30%	A–	7.43%
40%	BB	8.43%
50%	B+	8.93%
60%	B–	10.93%
70%	CCC	11.93%
80%	CCC	11.93%
90%	CC	13.43%

The earnings before interest and taxes were expected to grow 3% a year in perpetuity, with capital expenditures offset by depreciation. (The tax rate is 40%, and the Treasury bond rate is 7% and the market risk premium is 5.5%.)

a. Estimate the cost of capital at the current debt ratio.
b. Estimate the costs of capital at debt ratios ranging from 0% to 90%.
c. Estimate the value of the firm at debt ratios ranging from 0% to 90%.

7. You have been asked to estimate the value of Cavanaugh Motels, a motel chain. The firm reported earnings of $200 million before interest and taxes in the most recent year and paid 40% of its taxable income in taxes. The book value of capital at the firm is $1.2 billion, and the firm expects to grow 4% a year in perpetuity. The firm has a beta of 1.2, a pretax cost of debt of 6%, equity with a market value of $1 billion, and debt with a market value of $500 million. (The risk-free rate is 5%, and the market risk premium is 5.5%.)
   a. Estimate the value of the firm, using the cost of capital approach.
   b. If you were told the probability of default at this firm at its current debt level is 10% and that the cost of bankruptcy is 25% of unlevered firm value, estimate the value of the firm using the adjusted present value approach.
   c. How would you reconcile the two estimates of value?
8. Bethlehem Steel, one of the oldest and largest steel companies in the United States, is considering the question of whether it has any excess debt capacity. The firm has $527 million in market value of debt outstanding and $1.76 billion in market value of equity. The firm has earnings before interest and taxes of $131 million, and faces a corporate tax rate of 36%. The company's bonds are rated BBB, and the cost of debt is 8%. At this rating, the firm has a probability of default of 2.3%, and the cost of bankruptcy is expected to be 30% of firm value.
   a. Estimate the unlevered value of the firm from the current market value of the firm.
   b. Estimate the levered value of the firm, using the adjusted present value approach, at a debt ratio of 50%. At that debt ratio, the firm's bond rating will be CCC, and the probability of default will increase to 46.61% of unlevered firm value.

CHAPTER 16

Estimating Equity Value per Share

Chapter 15 considered how best to estimate the value of the operating assets of the firm. To get from that value to the firm value, you have to consider the value of cash, marketable securities, and other nonoperating assets held by a firm. In particular, you have to value holdings in other firms and deal with a variety of accounting techniques used to record such holdings. To get from firm value to equity value, you have to determine what should be subtracted from firm value (i.e., the value of the nonequity claims in the firm).

Once you have valued the equity in a firm, it may appear to be a relatively simple exercise to estimate the value per share. It seems that all you need to do is divide the value of the equity by the number of shares outstanding. But, in the case of some firms, even this simple exercise can become complicated by the presence of management and employee options. This chapter discusses the magnitude of this option overhang on valuation and then consider ways of incorporating the effect into the value per share.

VALUE OF NONOPERATING ASSETS

Firms have a number of assets on their books that can be categorized as nonoperating assets. The first and most obvious one is cash and near-cash investments—investments in riskless or very low-risk investments that most companies with large cash balances make. The second is investments in equities and bonds of other firms, sometimes for investment reasons and sometimes for strategic ones. The third is holdings in other firms, private and public, which are categorized in a variety of ways by accountants. Finally, there are assets that firms own that do not generate cash flows but nevertheless could have value—say, undeveloped land in New York City or Tokyo.

Cash and Near-Cash Investments

Investments in short-term government securities or commercial paper, which can be converted into cash quickly and with very low cost, are considered near-cash investments. This section considers how best to deal with these investments in valuation.

Operating Cash Requirements If a firm needs cash for its operations—an operating cash balance—and this cash does not earn a fair market return you should consider such cash part of working capital requirements rather than as a source of additional value. Any cash and near-cash investments that exceed the operating cash

requirements can be then added to the value of operating assets. How much cash does a firm need for its operations? The answer depends on both the firm and the economy in which the firm operates. A small retail firm in an emerging market, where cash transactions are more common than credit card transactions, may require an operating cash balance that is substantial. In contrast, a manufacturing firm in a developed market may not need any operating cash. If the cash held by a firm is interest-bearing and the interest earned on the cash reflects a fair rate of return,[1] you would not consider that cash to be part of working capital. Instead, you would add it to the value of operating assets to value the firm.

Dealing with Nonoperating Cash Holdings There are two ways in which we can deal with cash and marketable securities in valuation. One is to lump them in with the operating assets and value the firm (or equity) as a whole. The other is to value the operating assets and the cash and marketable securities separately.

Consolidated Valuation Is it possible to consider cash as part of the total assets of the firm, and to value it on a consolidated basis? The answer is yes, and it is, in a sense, what we do when we forecast the total net income for a firm and estimate dividends and free cash flows to equity from those forecasts. The net income will then include income from investments in government securities, corporate bonds, and equity investments. While this approach has the advantage of simplicity and can be used when financial investments comprise a small percent of the total assets, it becomes much more difficult to use when financial investments represent a larger proportion of total assets for two reasons:

First, the cost of equity or capital used to discount the cash flows has to be adjusted on an ongoing basis for the cash. In specific terms, you would need to use an unlevered beta that represents a weighted average of the unlevered beta for the operating assets of the firm and the unlevered beta for the cash and marketable securities. For instance, the unlevered beta for a steel company where cash represents 10 percent of the value would be a weighted average of the unlevered beta for steel companies and the beta of cash (which is usually zero). If the 10 percent were invested in riskier securities, you would need to adjust the beta accordingly. While this can be done if you use bottom-up betas, you can see that it would be much more difficult to do if you obtain a beta from a regression.[2]

Second, as the firm grows, the proportion of income that is derived from operating assets is likely to change. When this occurs, you have to adjust the inputs to the valuation model—cash flows, growth rates, and discount rates—to maintain consistency.

What will happen if you do not make these adjustments? You will tend to misvalue the financial assets. To see why, assume that you were valuing the aforementioned steel company with 10 percent of its value coming from cash. This cash is invested in government securities and earns an appropriate rate—say 5 percent. If

[1]Note that if the cash is invested in riskless assets such as Treasury bills, the riskless rate is a fair rate of return.

[2]The unlevered beta that you can back out of a regression beta reflects the average cash balance (as a percent of firm value) over the period of the regression. Thus, if a firm maintains this ratio at a constant level, you might be able to arrive at the correct unlevered beta.

this income is added on to the other income of the firm and discounted back at a cost of equity appropriate for a steel company—say 11 percent—the value of the cash will be discounted. A billion dollars in cash will be valued at $800 million, for instance, because the discount rate used is incorrect.

Separate Valuation It is safer to separate cash and marketable securities from operating assets and to value them individually. We do this almost always when we use the firm valuation approaches described in the preceding chapter. This is because we use operating income to estimate free cash flows to the firm, and operating income generally does not include income from financial assets. If, however, this is not the case and some of the investment income has found its way into the operating income, you would need to back it out before you did the valuation. Once you value the operating assets, you can add the value of the cash and marketable securities to it to arrive at firm value.

Can this be done with the FCFE valuation models described in Chapter 14? While net income includes income from financial assets, we can still separate cash and marketable securities from operating assets if we wanted to. To do this, we would first back out the portion of the net income that represents the income from financial investments (interest on bonds, dividends on stock) and use this adjusted net income to estimate free cash flows to equity. These free cash flows to equity would be discounted back using a cost of equity that would be estimated using a beta that reflected only the operating assets. Once the equity in the operating assets has been valued, you could add the value of cash and marketable securities to it to estimate the total value of equity. In fact, we used this approach to value Coca-Cola in Chapter 14.

ILLUSTRATION 16.1: Consolidated versus Separate Valuation

To examine the effects of a cash balance on firm value, consider a firm with investments of $1,200 million in noncash assets and $200 million in cash. For simplicity, let us assume the following:

- The noncash assets have a beta of 1, and are expected to earn $120 million in net income each year in perpetuity, and there are no reinvestment needs.
- The cash is invested at the riskless rate, which we assume to be 4.5%.
- The market risk premium is assumed to be 5.5%.

Under these conditions, we can value the equity using both the consolidated and separate approaches.

Let us first consider the consolidated approach. Here, we will estimate a cost of equity for all of the assets (including cash) by computing a weighted average beta of the noncash and cash assets:

$$\text{Beta of the firm} = \text{Beta}_{\text{noncash assets}} \times \text{Weight}_{\text{noncash assets}} + \text{Beta}_{\text{cash assets}} \times \text{Weight}_{\text{cash assets}}$$
$$= 1.00 \times (1{,}200/1{,}400) + 0 \times (200/1{,}400) = 0.8571$$

$$\text{Cost of equity for the firm} = 4.5\% + 0.8571(5.5\%) = 9.21\%$$

$$\text{Expected earnings for the firm} = \text{Net income from operating assets} + \text{Interest income from cash}$$
$$= (120 + .045 \times 200) = \$129 \text{ million (which is also the FCFE since there are no reinvestment needs)}$$

$$\text{Value of the equity} = \text{FCFE/Cost of equity} = 129/.0921 = \$1{,}400 \text{ million}$$

The equity is worth $1,400 million.

Now, let us try to value them separately, beginning with the noncash investments:

Cost of equity for noncash investments = Riskless rate + Beta × Risk premium
= 4.5% + 1.00 × 5.5% = 10%

Expected earnings from operating assets = $120 million (which is the FCFE from these assets)

Value of noncash assets = Expected earnings/Cost of equity for noncash assets
= 120/.10 = $1,200 million

To this we can add the value of the cash, which is $200 million, to get a value for the equity of $1,400 million.

To see the potential for problems with the consolidated approach, note that if you had discounted the total FCFE of $129 million at the cost of equity of 10% (which reflects only the operating assets) you would valued the firm at $1,290 million. The loss in value of $110 million can be traced to the mishandling of cash:

Interest income from cash = 4.5% × 200 = $9 million

If you discount the cash at 10%, you would value the cash at $90 million instead of the correct value of $200 million—hence the loss in value of $110 million.

Should You Ever Discount Cash? In Illustration 16.1, cash was reduced in value for the wrong reason—a riskless cash flow was discounted at a discount rate that reflects risky investments. However, there are two conditions under which you might legitimately apply a discount to a cash balance:

1. The cash held by a firm is invested at a rate that is lower than the market rate, given the riskiness of the investment.
2. The management is not trusted with the large cash balance because of its past track record on investments.

Cash Invested at Below-Market Rates The first and most obvious condition occurs when much or all the cash balance does not earn a market interest rate. If this is the case, holding too much cash will clearly reduce the firm's value. While most firms in the United States can invest in government bills and bonds with ease today, the options are much more limited for small businesses in the United States and for firms in many emerging markets. When this is the case, a large cash balance earning less than a fair return can destroy value over time.

ILLUSTRATION 16.2: Cash Invested at Below-Market Rates

Illustration 16.1 assumed that cash was invested at the riskless rate. Assume, instead, that the firm was able to earn only 3% on its cash balance, while the riskless rate is 4.5%. The estimated value of the cash kept in the firm would then be:

Estimated value of cash invested at 3% = (.03 × 200)/.045 = 133.33

The firm would have been worth only $1,333 million instead of $1,400 million. The cash returned to stockholders would have a value of $200 million. In this scenario, returning the cash to stockholders would yield them a surplus value of $66.67 million. In fact, liquidating any asset that has a return less than the required return would yield the same result, as long as the entire investment can be recovered on liquidation.[3]

[3]While this assumption is straightforward with cash, it is less so with real assets, where the liquidation value may reflect the poor earning power of the asset. Thus, the potential surplus from liquidation may not be as easily claimed.

Distrust of Management While making a large investment in low-risk or no-risk marketable securities by itself is value neutral, the burgeoning cash balance can tempt managers to accept large investments or make acquisitions even if these investments earn substandard returns. In some cases, managers may take these action to prevent the firm from becoming a takeover target.[4] To the extent that stockholders anticipate such substandard investments, the current value of the firm will reflect the cash at a discounted level. The discount is likely to be largest at firms with few investment opportunities and poor management, and there will be no discount in firms with significant investment opportunities and good management.

ILLUSTRATION 16.3: Discount for Poor Investments in the Future

Return now to the firm described in Illustration 16.1, where the cash is invested at the riskless rate of 4.5%. Normally, we would expect this firm to trade at a total value of $1,400 million. Assume, however, that the managers of this firm have a history of poor acquisitions and that the presence of a large cash balance increases the probability from 0% to 30% that they will try to acquire another firm. Further, assume that the market anticipates that they will overpay by $50 million on this acquisition. The cash will then be valued at $185 million, with the discount estimated as follows:

$$\begin{aligned}\text{Estimated discount on cash balance} &= \Delta\,\text{Probability}_{\text{acquisition}} \times \text{Expected overpayment}_{\text{acquisition}} \\ &= 0.30 \times \$50 \text{ million} = \$15 \text{ million} \\ \text{Value of cash} &= \text{Cash balance} - \text{Estimated discount} \\ &= \$200 \text{ million} - \$15 \text{ million} = \$185 \text{ million}\end{aligned}$$

The firm will therefore be valued at $1,385 million instead of $1,400 million. The two factors that determine this discount—the incremental likelihood of a poor investment and the expected net present value of the investment—are likely to be based on investors' assessments of management quality.

Investments in Risky Securities

So far this chapter has looked at how to value cash and near-cash investments. In some cases, firms invest in risky securities, which can range from investment-grade bonds to high-yield bonds to publicly traded equity in other firms. This section examines the motivation, consequences, and accounting for such investments.

Reasons for Holding Risky Securities Why do firms invest in risky securities? Some firms do so for the allure of the higher returns they can expect to make investing in stocks and corporate bonds, relative to Treasury bills. In recent years, there has also been a trend for firms to take equity positions in other firms to further their strategic interests. Still other firms take equity positions in firms they view as undervalued by the market; and finally, investing in risky securities is part of doing business for banks, insurance companies, and other financial service companies.

[4]Firms with large cash balances are attractive targets, since the cash balance reduces the cost of making the acquisition.

To Make a Higher Return Near-cash investments such as Treasury bills and commercial paper are liquid and have little or no risk, but they also earn low returns. When firms have substantial amounts invested in marketable securities, they can expect to earn considerably higher returns by investing in riskier securities. For instance, investing in corporate bonds will yield a higher interest rate than investing in Treasury bonds, and the rate will increase with the riskiness of the investment. Investing in stocks will provide an even higher expected return, though not necessarily a higher actual return, than investing in corporate bonds. Figure 16.1 summarizes returns on risky investments—corporate bonds, high-yield bonds, and equities—and compares them to the returns on near-cash investments between 1990 and 2000.

However, while investing in riskier investments may earn a higher return for the firm, it does not make the firm more valuable. In fact, using the same reasoning that we used to analyze near-cash investments, we can conclude that investing in riskier investments and earning a fair market return (which would reward the risk) has to be value neutral.

To Invest in Undervalued Securities A good investment is one that earns a return greater than its required return. That principle, developed in the context of investments in projects and assets, applies just as strongly to financial investments. A firm that invests in undervalued stocks is accepting positive net present value investments, since the return it will make on these equity investments will exceed the cost of equity on these investments. Similarly, a firm that invests in underpriced corporate bonds will also earn an excess return and a positive net present value.

How likely is it that a firm will find undervalued stocks and bonds to invest in? It depends on how efficient markets are and how good the managers of the firm are at finding undervalued securities. In unique cases, a firm may be more adept at find-

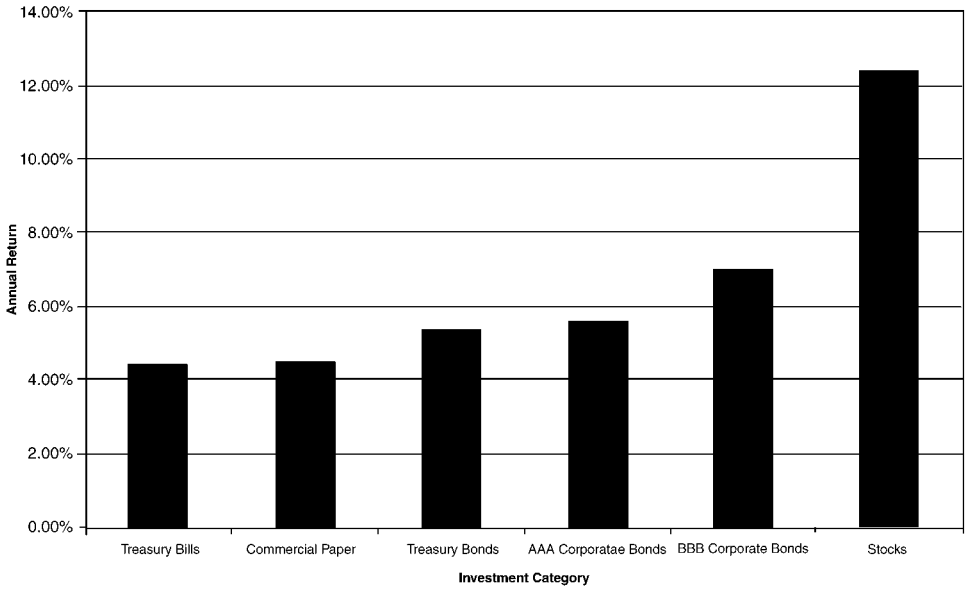

FIGURE 16.1 Returns on Investments—1990–2000

ing good investments in financial markets than it is at competing in product markets. Consider the case of Berkshire Hathaway, a firm that has been a vehicle for Warren Buffett's investing acumen over the past few decades. At the end of the second quarter of 1999, Berkshire Hathaway had $69 billion invested in securities of other firms. Among its holdings were investments of $12.4 billion in Coca-Cola, $6.6 billion in American Express, and $3.9 billion in Gillette. While Berkshire Hathaway also has real business interests, including ownership of a well-regarded insurance company (GEICO), investors in the firm get a significant portion of their value from the firm's passive equity investments.

Notwithstanding Berkshire Hathaway's success, most firms in the United States steer away from looking for bargains among financial investments. Part of the reason for this is their realization that it is difficult to find undervalued securities in financial markets. Part of the reluctance on the part of firms to make equity investments in other firms can be traced to a recognition that investors in firms like Procter & Gamble and Coca-Cola invest in these firms because of their competitive advantages in product markets (brand name, marketing skills, etc.) and not for their perceived skill at picking stocks.

Strategic Investments During the 1990s, Microsoft accumulated a huge cash balance in excess of $20 billion. It used this cash to make a series of investments in the equity of software, entertainment, and Internet-related firms. It did so for several reasons.[5] First, doing so gave Microsoft a say in the products and services these firms were developing and preempted competitors from forming partnerships with the firms. Second, it allowed Microsoft to work on joint products with these firms. In 1998 alone, Microsoft announced investments in 14 firms, including ShareWave, General Magic, RoadRunner, and Qwest Communications. In an earlier investment in 1995, Microsoft invested in NBC to create the MSNBC network in order to give it a foothold in the television and entertainment business.

Can strategic investments be value enhancing? As with all investments, it depends on how much is invested and what the firm receives as benefits in return. If the side benefits and synergies that are touted in these investments exist, investing in the equity of other firms can earn much higher returns than the hurdle rate and can create value. It is clearly a much cheaper option than acquiring the entire equity of the firm.

Business Investments Some firms hold marketable securities not as discretionary investments, but because it is the nature of their business. For instance, insurance companies and banks often invest in marketable securities in the course of their business, the former to cover expected liabilities on insurance claims and the latter in the course of trading. While these financial service firms have financial assets of substantial value on their balance sheets, these holdings are not comparable to those of the firms described so far. In fact, they are more akin to the raw material used by manufacturing firms than to discretionary financial investments.

[5]One of Microsoft's oddest investments was in one of its primary competitors, Apple Computer, early in 1998. The investment may have been intended to fight the antitrust suit brought against Microsoft by the Justice Department.

Dealing with Marketable Securities in Valuation Marketable securities can include corporate bonds, with default risk embedded in them, and traded equities, which have even more risk associated with them. As the marketable securities held by a firm become more risky, the choices on how to deal with them become more complex. You have three ways of accounting for marketable securities:

1. The simplest and most direct approach is to estimate the current market value of these marketable securities and add the value to the value of operating assets. For firms valued on a going-concern basis, with a large number of holdings of marketable securities, this may be the only practical option.
2. The second approach is to estimate the current market value of the marketable securities and net out the effect of capital gains taxes that may be due if those securities were sold today. This capital gains tax bite depends on how much was paid for these assets at the time of the purchase and the value today. This is the best way of estimating value when valuing a firm on a liquidation basis or when the firm has provided a clear indication that it plans to sell its holdings.
3. The third and most difficult way of incorporating the value of marketable securities into firm value is to value the firms (using a discounted cash flow approach) that issued these securities and estimate the value of these securities. This approach tends to work best for firms that have relatively few, but large, holdings in other publicly traded firms.

ILLUSTRATION 16.4: Microsoft's Cash and Marketable Securities

Over the past decade, Microsoft has accumulated a huge cash balance, largely as a consequence of holding back on free cash flows to equity that could have been paid to stockholders. In 1999 and 2000, for instance, the firm reported the following holdings of near-cash investments (in $millions):

	1999	*2000*
Cash and equivalents:		
Cash	$ 635	$ 849
Commercial paper	$ 3,805	$ 1,986
Certificates of deposit	$ 522	$ 1,017
U.S. government and agency securities	$ 0	$ 729
Corporate notes and bonds	$ 0	$ 265
Money market preferreds	$ 13	$ 0
Total cash and equivalents	$ 4,975	$ 4,846
Short-term investments:		
Commercial paper	$ 1,026	$ 612
U.S. government and agency securities	$ 3,592	$ 7,104
Corporate notes and bonds	$ 6,996	$ 9,473
Municipal securities	$ 247	$ 1,113
Certificates of deposit	$ 400	$ 650
Total short-term investments	$12,261	$18,952
Cash and short-term investments	*$17,236*	*$23,798*

When valuing Microsoft in 2000, we should clearly consider the $23.798 billion investment as part of the firm's value. The interesting question is whether there should be a discount reflecting investor's fears about poor investments in the future. Over its life, Microsoft has not been punished for holding on to cash, largely as a consequence of its impeccable track record in delivering ever-increasing profits on the one hand and high stock returns on the other. While 1999 and 2000 were not good years for the

firm, investors will probably give the firm the benefit of the doubt at least for the near future. We would add the cash balance at face value to the value of Microsoft's operating assets.

The more interesting component is the $17.7 billion that Microsoft shows as investments in riskier securities in 2000. Microsoft reports the following information about these investments (in $millions):

	Cost Basis	*Unrealized Gains*	*Unrealized Losses*	*Recorded Basis*
Debt securities recorded at market:				
Within one year	$ 498	$ 27	$ 0	$ 525
Between 2 and 10 years	$ 388	$ 11	–$ 3	$ 396
Between 10 and 15 years	$ 774	$ 14	–$ 93	$ 695
Beyond 15 years	$ 4,745		–$ 933	$ 3,812
Total debt securities recorded at market	$ 6,406	$ 52	–$1,029	$ 5,429
Equities:				
Common stock and warrants	$ 5,815	$5,655	–$1,697	$ 9,773
Preferred stock	$ 2,319			$ 2,319
Other investments	$ 205		$ 205	
Total equities and other investments	$14,745	$5,707	–$2,726	$17,726

Microsoft has generated a paper profit of almost $3 billion on its original cost of $14.745 billion, and reports a current value of $17.726 billion. Most of these investments are traded in the market and are recorded at market value. The easiest way to deal with these investments is to add the market value to the value of the operating assets of the firm to arrive at firm value. The most volatile item is the investment in common stock of other firms. The value of these holdings has almost doubled, as reflected in the recorded basis of $9,773 million. Should we reflect this at current market value when we value Microsoft? The answer is generally yes. However, if these investments are overvalued, you risk building in this overvaluation into your valuation. The alternative is to value each of the equities that the firm has invested in, but this will become increasingly cumbersome as the number of equity holdings increases.

In summary, then, you would add the values of both the near-cash investments of $23.798 billion and the equity investments of $17.726 billion to the value of the operating assets of Microsoft.

Premiums or Discounts on Marketable Securities? As a general rule, you should not attach a premium or discount for marketable securities. Thus, you would add the entire value of $17,726 million to the value of Microsoft. There is an exception to this rule, though, and it relates to firms that make it their business to buy and sell financial assets. These are the closed-end mutual funds, of which there are several hundred listed on the U.S. stock exchanges, and investment companies, such as Fidelity and T. Rowe Price. Closed-end mutual funds sell shares to investors and use the funds to invest in financial assets. The number of shares in a closed-end fund remains fixed, and the share price changes. Since the investments of a closed-end fund are in publicly traded securities, this sometimes creates a phenomenon where the market value of the shares in a closed-end fund is greater than or less than the market value of the securities owned by the fund. For these firms, it is appropriate to attach a discount or premium to the marketable securities to reflect their capacity to generate excess returns on these investments.

A closed-end mutual fund that consistently finds undervalued assets and delivers much higher returns than expected (given the risk) should be valued at a premium on the value of its marketable securities. The amount of the premium will

depend on how large the excess return is and how long you would expect the firm to continue to make these excess returns. Conversely, a closed-end fund that delivers returns that are much lower than expected should trade at a discount on the value of the marketable securities held by the fund. The stockholders in this fund would clearly be better off if it were liquidated, but that may not be a viable option.

ILLUSTRATION 16.5: Valuing a Closed-End Fund

The Pierce Regan Asia fund is a closed-end fund with investments in traded Asian stocks, valued at $4 billion at today's market prices. The fund has earned a return of 13% over the past 10 years, but based on the riskiness of its investments and the performance of the Asian market over the period, it should have earned 15%. Looking forward, your expected return for the Asian market for the future is 12%, but you anticipate that the Pierce Regan fund will continue to underperform the market by 2%.

To estimate the discount you would expect to see on the fund, let us begin by assuming that the fund will continue in perpetuity earning 2% less than the return on the market index. The discount would then be:

$$\text{Estimated discount} = \text{Excess return} \times \text{Fund value/Expected return on the market}$$
$$= (.10 - .12)(4{,}000)/.12 = \$667 \text{ million}$$

On a percent basis, the discount represents 16.67% of the market value of the investments.

If you assume that the fund will either be liquidated or begin earning the expected return at a point in time in the future—say 10 years from now—the expected discount will become smaller.

Holdings in Other Firms

In this category, we consider a broader category of nonoperating assets, where we look at holdings in other companies, public as well as private. We begin by looking at the differences in accounting treatment of different holdings and how this treatment can affect the way they are reported in financial statements.

Accounting Treatment The way in which these assets are valued depends on the way the investment is categorized and the motive behind the investment. In general, an investment in the securities of another firm can be categorized as a minority passive investment. a minority active investment, or a majority active investment, and the accounting rules vary depending on the categorization.

Minority Passive Investments If the securities or assets owned in another firm represent less than 20 percent of the overall ownership of that firm, an investment is treated as a minority passive investment. These investments have an acquisition value, which represents what the firm originally paid for the securities, and often a market value. Accounting principles require that these assets be subcategorized into one of three groups—investments that will be held to maturity, investments that are available for sale, and trading investments. The valuation principles vary for each.

- For investments that will be held to maturity, the valuation is at historical cost or book value, and interest or dividends from this investment are shown in the income statement.

- For investments that are available for sale, the valuation is at market value, but the unrealized gains or losses are shown as part of the equity in the balance sheet and not in the income statement. Thus, unrealized losses reduce the book value of the equity in the firm, and unrealized gains increase the book value of equity.
- For trading investments, the valuation is at market value, and the unrealized gains and losses are shown in the income statement.

Firms are allowed an element of discretion in the way they classify investments and through this choice in the way they value these assets. This classification ensures that firms such as investment banks, whose assets are primarily securities held in other firms for purposes of trading, revalue the bulk of these assets at market levels each period. This is called marking to market, and provides one of the few instances in which market value trumps book value in accounting statements.

Minority Active Investments If the securities or assets owned in another firm represent between 20 percent and 50 percent of the overall ownership of that firm, an investment is treated as a minority active investment. While these investments have an initial acquisition value, a proportional share (based on ownership proportion) of the net income and losses made by the firm in which the investment was made, is used to adjust the acquisition cost. In addition, the dividends received from the investment reduce the acquisition cost. This approach to valuing investments is called the equity approach.

The market value of these investments is not considered until the investment is liquidated, at which point the gain or loss from the sale relative to the adjusted acquisition cost is shown as part of the earnings in that period.

Majority Active Investments If the securities or assets owned in another firm represent more than 50 percent of the overall ownership of that firm, an investment is treated as a majority active investment.[6] In this case, the investment is no longer shown as a financial investment but is instead replaced by the assets and liabilities of the firm in which the investment was made. This approach leads to a consolidation of the balance sheets of the two firms, where the assets and liabilities of the two firms are merged and presented as one balance sheet. The share of the firm that is owned by other investors is shown as a minority interest on the liability side of the balance sheet. A similar consolidation occurs in the other financial statements of the firm as well, with the statement of cash flows reflecting the cumulated cash inflows and outflows of the combined firm. This is in contrast to the equity approach, used for minority active investments, in which only the dividends received on the investment are shown as a cash inflow in the cash flow statement.

Here again, the market value of this investment is not considered until the ownership stake is liquidated. At that point, the difference between the market price and the net value of the equity stake in the firm is treated as a gain or loss for the period.

[6]Firms have evaded the requirements of consolidation by keeping their share of ownership in other firms below 50 percent.

Valuing Cross Holdings in Other Firms Given that the holdings in other firms can be accounted for in three different ways, how do you deal with each in valuation? The best way to deal with each of them is exactly the same. You would value the equity in each holding separately, and estimate the value of the proportional holding. This value would then be added to the value of the equity of the parent company. Thus, to value a firm with minority holdings in three other firms, you would value the equity in each of these firms, take the percent share of the equity in each, and add it to the value of equity in the parent company.

When income statements are consolidated, you would first need to strip the income, assets, and debt of the subsidiary from the parent company's financials before you do any of the above. If you do not do so, you will double count the value of the subsidiary.

Why, you might ask, do we not value the consolidated firm? You could, and in some cases, because of the absence of information, you might have to. The reason we would suggest separate valuations is because the parent and the subsidiaries may have very different characteristics—costs of capital, growth rates, and reinvestment rates. Valuing the combined firm under these circumstances may yield misleading results. There is another reason: Once you have valued the consolidated firm, you will have to subtract the portion of the equity in the subsidiary that the parent company does not own. If you have not valued the subsidiary separately, it is not clear how you would do this. Note that the conventional practice of netting out the minority interest does not accomplish this, because minority interest reflects book rather than market value.

As a firm's holdings become more numerous, estimating the values of the holdings will become more onerous. If the holdings are publicly traded, substituting the market values of the holdings for estimated value is an alternative worth exploring. While you risk building into your valuation any mistakes the market might be making in valuing these holdings, this approach is more time efficient.

ESTIMATING THE VALUE OF HOLDINGS IN PRIVATE COMPANIES

When a publicly traded firm has a cross holding in a private company, it is often difficult to obtain information on the private company and to value it. Consequently, you might have to make your best estimate of how much this holding is worth based on the limited information that you have available. One way to do this is to estimate the multiple of book value at which firms in the same business (as the private business in which you have holdings) typically trade at and apply this multiple to the book value of the holding in the private business. Assume, for instance, that you are trying to estimate the value of the holdings of a pharmaceutical firm in five privately held biotechnology firms, and that these holdings collectively have a book value of $50 million. If biotechnology firms typically trade at 10 times book value, the estimated market value of these holdings would be $500 million.

In fact, this approach can be generalized to estimate the value of complex holdings where you lack the information to estimate the value for each holding or there are too many such holdings. For example, you could be valuing a Japanese firm with dozens of cross holdings. You could estimate a value for the cross holdings by applying a multiple of book value to their cumulative book value.

ILLUSTRATION 16.6: Valuing Holdings in Other Companies

Segovia Entertainment operates in a wide range of entertainment businesses. The firm reported $300 million in operating income (EBIT) on capital invested of $1,500 million in the current year; the total debt outstanding is $500 million. A portion of the operating income ($100 million), capital invested ($400 million), and debt outstanding ($150 million) represent Segovia's holdings in Seville Television, a television station owner. Segovia owns only 51% of Seville, but Seville's financials are consolidated with those of Segovia.[7] In addition, Segovia owns 15% of LatinWorks, a record and CD company. These holdings have been categorized as minority passive investments, and the dividends from the investments are shown as part of Segovia's net income but not as part of its operating income. LatinWorks reported operating income of $80 million on capital invested of $250 million in the current year; the firm has $100 million in debt outstanding. We will assume the following:

- The cost of capital for Segovia Entertainment, without considering its holdings in either Seville or LatinWorks, is 10%. The firm is in stable growth, with operating income (again not counting the holdings) growing 5% a year in perpetuity.
- Seville Television has a cost of capital of 9% and is in stable growth, with operating income growing 5% a year in perpetuity.
- LatinWorks has a cost of capital of 12% and is in stable growth, with operating income growing 4.5% a year in perpetuity.
- None of the firms has a significant balance of cash and marketable securities.
- The tax rate for all of these firms is 40%.

We can value Segovia Entertainment in three steps:

STEP 1: Value the equity in the operating assets of Segovia without counting any of the holdings. To do this, we first have to cleanse the operating income of the consolidation:

Operating income from Segovia's operating assets = Consolidated income – Income from Seville = $300 – $100 = $200 million

Capital invested in Segovia's operating assets = Consolidated capital – Capital from Seville = $1,500 – $400 = $1,100 million

Debt in Segovia's operating assets = Consolidated debt – Debt from Seville = $500 – $150 = $350 million

Return on capital invested in Segovia's operating assets = 200(1 – .4)/1,100 = 10.91%

Reinvestment rate = g/ROC = 5%/10.91% = 45.83%

Value of Segovia's operating assets = EBIT(1 – t)(1 – Reinvestment rate)(1 + g)/(Cost of capital – g) = 200(1 – .4)(1 – .4583)(1.05)/(.10 – .05) = $1,365 million

Value of equity in Segovia's operating assets = Value of operating assets – Value of Segovia's debt = 1,365 – 350 = $1,015 million

STEP 2: Value the 51% of equity in Seville Enterprises:

Operating income from Seville's operating assets = $100 million

Capital invested in Seville's operating assets = $400 million

Debt invested in Seville = $150 million

Return on capital invested in Seville's operating assets = 100(1 – .4)/400 = 15%

Reinvestment rate = g/ROC = 5%/15% = 33.33%

Value of Seville's operating assets = EBIT(1 – t)(1 – Reinvestment rate)(1 + g)/(Cost of capital – g) = 100(1 – .4)(1 – .3333)(1.05)/(.09 – .05) = $1,050 million

[7]Consolidation in the U.S. requires that you consider 100 percent of the subsidiary, even if you own less. There are other markets in the world where consolidation requires only that you consider the portion of the firm that you own.

Value of Seville's equity = Value of operating assets – Debt = 1,050 – 150 = $900 million

Value of Segovia's equity stake in Seville = .51(900) = $459 million

STEP 3: Value the 15% stake in LatinWorks:

Operating income from LatinWorks' operating assets = $75 million

Capital invested in LatinWorks' operating assets = $250 million

Return on capital invested in LatinWorks' operating assets = 75(1 – .4)/250 = 18%

Reinvestment rate = g/ROC = 4.5%/18% = 25%

Value of LatinWorks' operating assets = EBIT(1 – t)(1 – Reinvestment rate)(1 + g)/(Cost of capital – g) = 75(1 – .4)(1 – .25)(1.045)/(.12 – .045) = $470.25 million

Value of LatinWorks' equity = Value of operating assets – Debt = 470.25 – 100 = $370.25 million

Value of Segovia's equity stake in LatinWorks = .15(370.25) = $55 million

The value of Segovia as a firm can now be computed (assuming that it has no cash balance):

$$\begin{aligned}\text{Value of equity in Segovia} &= \text{Value of equity in Segovia} + 51\% \text{ of equity in Seville} \\ &\quad + 15\% \text{ of equity in LatinWorks} \\ &= \$1{,}015 + \$459 + \$55 = \$1{,}529 \text{ million}\end{aligned}$$

To provide a contrast, consider what would have happened if we had used the consolidated income statement and Segovia's cost of capital to do this valuation. We would have valued Segovia and Seville together as follows:

Operating income from Segovia's consolidated assets = $300 million

Capital invested in Segovia's consolidated assets = $1,500 million

Consolidated debt = $500 million

Return on capital invested in Segovia's operating assets = 300(1 – .4)/1,500 = 12%

Reinvestment rate = g/ROC = 5%/12% = 41.67%

Value of Segovia's operating assets = EBIT(1 – t)(1 – Reinvestment rate)(1 + g)/(Cost of capital – g) = 300(1 – .4)(1 – .4167)(1.05)/(.10 – .05) = $2,205 million

Value of equity in Segovia = Value of operating assets – Consolidated debt – Minority interests in Seville + Minority interest in LatinWorks = 2,205 – 500 – 122.5 + 22.5 = $1,605 million

Note that the minority interests in Seville are computed as 49% of the book value of equity at Seville.

$$\begin{aligned}\text{Book value of equity in Seville} &= \text{Capital invested in Seville} – \text{Seville's debt} \\ &= 400 – 150 = 250 \text{ million}\end{aligned}$$

$$\begin{aligned}\text{Minority interest} &= (1 – \text{Parent company holding})\text{Book value of equity} \\ &= (1 – .51)250 = \$122.5 \text{ million}\end{aligned}$$

The minority interests in LatinWorks are computed as 15% of the book value of equity in LatinWorks, which is $250 million (capital invested minus debt outstanding). It would be pure chance if this value were equal to the true value of equity, as first estimated, of $1,529 million.

You can see from the discussion that you need a substantial amount of information to value holdings correctly. This information may be difficult to come by when the holdings are in private companies.

VALUE OF TRANSPARENCY

The difficulty we often face in identifying and valuing holdings in other companies highlights a cost faced by firms that have complicated cross-holding structures and that make little or no effort to explain what they own to investors. In fact, many companies seem to adopt a strategy of making it difficult for their own stockholders to see what they own lest they be questioned about the wisdom of their choices. Not surprisingly, the market values of these firms often understate the value of these hidden holdings.

Many firms outside the United States use, as an excuse, the argument that the disclosure laws are not as strict in their countries as they are in the United States, but disclosure laws provide a floor for information that has to be revealed to markets and not a ceiling. For instance, InfoSys, an Indian software company, has one of the most informative financial reports of any company anywhere in the world. In fact, the firm has reaped substantial financial rewards because of its openness, as investors are better able to gauge how the firm is doing and tend to be much more willing to listen to management views.

So, what can undervalued firms with cross holdings do to improve their value? First, they can break down complicated holdings structures that impede understanding and valuation. Second, they can adopt a strategy of revealing as much as they can to investors about their holdings—private as well as public. Third, they need to stick with this strategy when they have bad news to report. A firm that is generous with positive information and stingy with negative information will rapidly lose credibility as an information source. Finally, if all else fails, they can consider divesting or spinning off their holdings.

Other Nonoperating Assets

Firms can have other nonoperating assets, but they are likely to be of less importance than those listed in the previous section. In particular, firms can have unutilized assets that do not generate cash flows and have book values that bear little resemblance to market values. An example would be prime real estate holdings that have appreciated significantly in value since the firm acquired them but produce little if any cash flows. An open question also remains about overfunded pension plans. Do the excess funds belong to stockholders, and, if so, how do you incorporate the effect into value?

Unutilized Assets The strength of discounted cash flow models is that they estimate the value of assets based on expected cash flows that these assets generate. In some cases, however, this can lead to assets of substantial value being ignored in the final valuation. For instance, assume that a firm owns a plot of land that has not been developed, and that the book value of the land reflects its original acquisition price. The land obviously has significant market value but does not generate any cash flow for the firm yet. If a conscious effort is not made to bring the expected cash flows from developing the land into the valuation, the value of the land will be left out of the final estimate.

How do you reflect the value of such assets in firm value? An inventory of all

such assets (or at least the most valuable ones) is a first step, followed up by estimates of market value for each of the assets. These estimates can be obtained by looking at what the assets would fetch in the market today or by projecting the cash flows that could be generated if the assets were developed and discounting the cash flows at the appropriate discount rate.

The problem with incorporating unutilized assets into firm value is an informational one. Firms do not reveal their unutilized assets as part of their financial statements. While it may sometimes be possible for investors and analysts to find out about such assets, it is far more likely that they will be uncovered only when you have access to information about what the firm owns and uses.

Pension Fund Assets Firms with defined pension liabilities sometimes accumulate pension fund assets in excess of these liabilities. While the excess does belong to stockholders, they usually face a tax liability if they claim it. The conservative rule in dealing with overfunded pension plans would be to assume that the social and tax costs of reclaiming the excess funds are so large that few firms would ever even attempt to do it. The more realistic approach would be to add the after-tax portion of the excess funds into the valuation.

As an illustration, consider a firm that reports pension fund assets that exceed its liabilities by $1 billion. Since a firm that withdraws excess assets from a pension fund is taxed at 50% on these withdrawals (in the United States), you would add $500 million to the estimated value of the operating assets of the firm. This would reflect the 50% of the excess assets that the firm will be left with after paying the taxes.

cash.xls: **This dataset on the Web summarizes the value of cash and marketable securities by industry group in the United States for the most recent quarter.**

FIRM VALUE AND EQUITY VALUE

Once you have estimates of the values of the operating assets, cash and marketable securities, and the other nonoperating assets owned by a firm, you can estimate the value of the firm as the sum of the three components. To get to the value of the equity from the firm value, you subtract out the nonequity claims on the firm. Nonequity claims would include debt and preferred stock, though the latter are often treated as equity in financial statements.

What Nonequity Claims Should Be Subtracted?

The general rule that you should use is that the debt you subtract from the value of the firm should be at least equal to the debt that you use to compute the cost of capital. Thus, if you decide to capitalize operating leases as debt, as we did with the Gap in the preceding chapter, to compute the cost of capital, you should subtract the debt value of operating leases from the value of operating assets to estimate the value of equity. If the firm you are valuing has preferred stock, you would use the

market value of the stock (if it is traded) or estimate a market value (if it is not)[8] and deduct it from firm value to get to the value of common equity.

There may be other claims on the firm that do not show up in debt that you should subtract from firm value.

- *Expected liabilities on lawsuits.* You could be analyzing a firm that is the defendant in a lawsuit, where it potentially could have to pay tens of millions of dollars in damages. You should estimate the probability that this will occur, and use this probability to estimate the expected liability. Thus, if there is a 10 percent chance that you could lose a case that you are defending, and the expected damage award is $1 billion, you would reduce the value of the firm by $100 million (probability × expected damages). If the expected liability is not anticipated until several years from now, you would compute the present value of the payment.
- *Unfunded pension and health care obligations.* If a firm has significantly underfunded a pension or a health plan, it will need to set aside cash in future years to meet these obligations. While it would not be considered debt for cost of capital purposes, it should be subtracted from firm value to arrive at equity value.
- *Deferred tax liability.* The deferred tax liability that shows up on the financial statements of many firms reflects the fact that firms often use tax-deferral strategies that reduce their taxes in the current year while increasing their taxes in future years. Of the three items listed here, this one is the least clearly defined, since it is not clear when or even whether the obligation will come due. Ignoring it may be foolhardy, though, since the firm could find itself making these tax payments in the future. The most sensible way of dealing with this item is to consider it an obligation, but one that will come due only when the firm's growth rate moderates. Thus, if you expect your firm to be in stable growth in 10 years, you would discount the deferred tax liability back 10 years and deduct this amount from firm value to get to equity value.

What about Future Claims?

As you forecast earnings growth for your firm, you generally also assume that the firm will increase its debt as it grows. A question that arises then is whether you should be subtracting the value of these future debt issues when estimating equity value today. The answer is no, since the value of the equity is a current value and these future claims do not exist today. To illustrate, assume that you have a firm with no debt today and that you assume that it will have a 30 percent debt ratio in stable growth. Assume further that your estimate of the terminal value for this firm is $10 billion in five years. You are implicitly assuming that your firm will borrow

[8]Estimating market value for preferred stock is relatively simple. Preferred stock generally is perpetual, and the estimated market value of the preferred stock is therefore:

Value of preferred stock = Preferred dividend/Cost of preferred stock

The cost of preferred stock should be higher than the pretax cost of debt, since debt has a prior claim on the cash flows and assets of the firm.

$3 billion in five years to raise its debt ratio to 30 percent. This higher debt ratio may affect your firm value today, but the value of equity today is the firm value less the current debt (which is zero).

MANAGEMENT AND EMPLOYEE OPTIONS

Firms use options to reward managers as well as other employees. There are two effects that these options have on value per share. One is created by options that have already been granted. These options, most of which have exercise prices well below the stock price, reduce the value of equity per share, since a portion of the existing equity in the firm has to be set aside to meet these eventual option exercises. The other is the likelihood that these firms will use options on a continuing basis to reward employees or to compensate them. These expected option grants reduce the portion of the expected future cash flows that accrue to existing stockholders.

Magnitude of the Option Overhang

The use of options in management compensation packages is not new to firms. Many firms in the 1970s and 1980s initiated option-based compensation packages to induce top managers to think like stockholders in their decision making. In most cases, though, the drain on value created by these options was small enough that it could be ignored without affecting the value per share substantially. In the past decade, however, the surge in both the number and the value of technology firms has highlighted the importance of dealing with these options in valuation.

What is different about technology firms? One is that management contracts at these firms are much more heavily weighted toward options than are those at other firms. The second is that the paucity of cash at these firms has meant that options are granted not just to top managers but to employees all through the organization, making the total option grants much larger. The third is that some of the smaller firms have used options to meet operating expenses and pay for supplies.

Figure 16.2 summarizes the number of options outstanding as a percent of outstanding stock at technology firms and compares them to options outstanding at nontechnology firms. As Figure 16.2 makes clear, the overhang is larger for younger new-technology firms.

Characteristics of Option Grants Firms that use employee options usually restrict when and whether these options can be exercised. It is standard, for instance, that the options granted to an employee cannot be exercised until they are vested. For this to occur, the employee usually has to remain for a period that is specified with the contract. While firms do this to keep employee turnover low, it also has implications for the value of these options. Firms that issue options do not face any tax consequences in the year in which they make the issue. When the options are exercised, however, they are allowed to treat the difference between the stock price and the exercise price as an employee expense. This tax deductibility also has implications for option value.

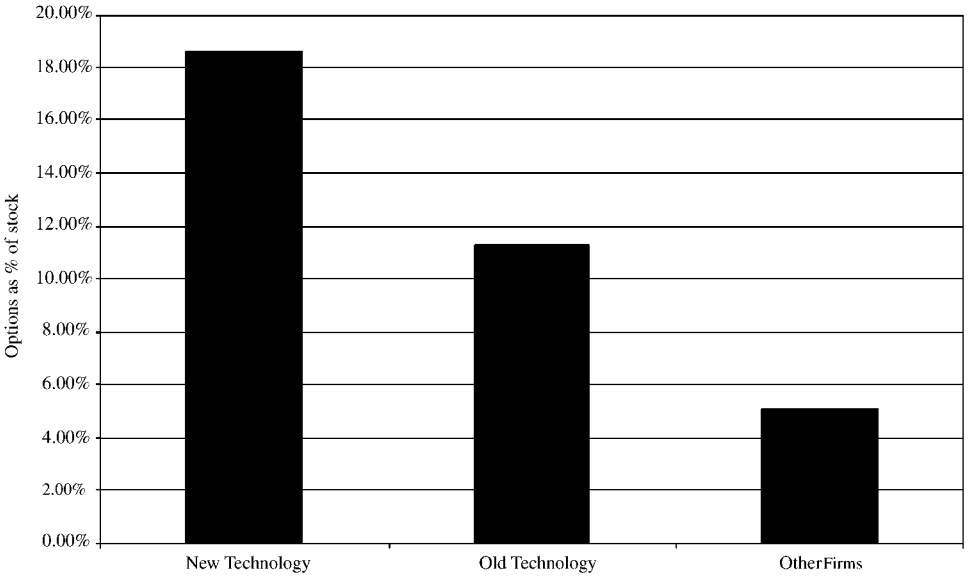

FIGURE 16.2 Options as Percent of Outstanding Stock
Source: Securities and Exchange Commission.

Options in Existence

Given the large number of options outstanding at many firms, our first task is to consider ways in which we can incorporate their effect into value per share. The section begins by presenting the argument for why these outstanding options matter when computing value per share, and then considers four ways in which we can incorporate their effect on value.

Why Options Affect Value per Share Why do existing options affect value per share? Note that not all options do. In fact, options issued and listed by the options exchanges have no effect on the value per share of the firms on which they are issued. The options issued by firms themselves do have an effect on value per share, since there is a chance that they will be exercised in the near or far future. Given that these options offer the right to individuals to buy stock at a fixed price, they will be exercised only if the stock price rises above that exercise price. When they are exercised, the firm has two choices, both of which have negative consequences for existing stockholders. It can issue additional shares to cover the option exercise. But this increases the number of shares outstanding and reduces the value per share to existing stockholders.[9] Alternatively, it can use cash flows from operations to

[9]This would be dilution in the true sense of the word, rather than the term that is used to describe any increase in the number of shares outstanding. The reason there is dilution is because the additional shares are issued only to the option holders at a price below the current price. In contrast, the dilution that occurs in a rights issue where every stockholder gets the right to buy additional shares at a lower price is value neutral. The shares will trade at a lower price but everyone will have more shares outstanding.

buy back shares in the open market and use these shares to meet the option exercise. This reduces the cash flows available to current equity investors in future periods, and makes their equity less valuable today.

Ways of Incorporating Existing Options into Value There are four approaches that are used to incorporate that effect of options that are already outstanding into the value per share. However, the first three approaches can lead to misleading estimates of value.

Use Fully Diluted Number of Shares to Estimate Per-Share Value The simplest way to incorporate the effect of outstanding options on value per share is to divide the value of equity by the number of shares that will be outstanding if all options are exercised today—the fully diluted number of shares. While this approach has the virtue of simplicity, it will lead to too low an estimate of value per share for two reasons:

1. It considers all options outstanding, not just ones that are in-the-money and vested. To be fair, there are variants of this approach where the shares outstanding are adjusted to reflect only in-the-money and vested options.
2. It does not incorporate the expected proceeds from exercise, which will comprise a cash inflow to the firm.

Finally, this approach does not build in the time premium on the options into the valuation.

Estimate Expected Option Exercises in the Future and Build In Expected Dilution In this approach, you forecast when in the future options will be exercised and build in the expected cash outflows associated with the exercise by assuming that the firm will go out and buy back stock to cover the exercise. The biggest limitation of this approach is that it requires estimates of what the stock price will be in the future and when options will be exercised on the stock. Given that your objective is to examine whether the price today is correct, forecasting future prices to estimate the current value per share seems circular. In general, this approach is neither practical nor particularly useful in coming up with reasonable estimates of value.

Use Treasury Stock Approach This approach is a variant of the fully diluted approach. Here the number of shares is adjusted to reflect options that are outstanding, but the expected proceeds from the exercise (exercise price times number of options) are added to the value of equity. The limitations of this approach are that, like the fully diluted approach, it does not consider the time premium on the options and there is no effective way of dealing with vesting. Generally this approach, by underestimating the value of options granted, will overestimate the value of equity per share.

ILLUSTRATION 16.7: Fully Diluted Approach to Estimating Value per Share: Commerce One

Commerce One, as a young and fast-growing B2B business, used options liberally in the period 1998 to 2000 to compensate employees. The following table summarizes the options granted, exercised, and canceled each year and also provides information on the total number of options outstanding at the firm at the end of each of these years:

Commerce One Options (in '000s)

	Granted	*Exercised*	*Canceled*	*Outstanding*
1998	7,336	462	1,338	11,334
1999	26,288	7,431	2,995	17,195
2000	29,023	8,033	2,275	45,911

At the end of 2000, Commerce One had options on 45.911 million shares outstanding, with a wide range of exercise prices and expiration dates. The following table summarizes the details of these options:

Exercise Price Range	*Number of Options*	*Remaining Life*	*Average Exercise Price*	*Exercisable and Vested*	*Average Exercise Price*
$0.00–$0.40	4,771,451	7.26	$ 0.19	1,889,590	$ 0.13
$0.67–$3.50	7,414,524	8.38	$ 2.33	1,672,662	$ 2.32
$4.71–$24.61	5,498,253	8.75	$ 15.42	1,036,632	$ 14.07
$25.31–$28.81	2,746,602	9.73	$ 27.88	274,724	$ 27.56
$30.00–$33.00	4,851,300	9.29	$ 32.70	1,053,513	$ 32.80
$34.17–$54.69	5,032,969	9.38	$ 42.75	631,181	$ 42.48
$54.88–$62.81	7,926,752	9.39	$ 59.75	919,951	$ 56.86
$64.19–$75.07	5,000,268	9.36	$ 72.12	837,853	$ 73.15
$78.50–$101.81	2,103,829	9.2	$ 86.94	387,099	$ 89.94
$104.44	565,275	9.16	$104.44	117,755	$104.44
Total or average	45,911,223	8.92	$ 35.49	8,820,960	$ 28.16

To apply the fully diluted approach to estimate the per share value, we first estimated the total value of equity for Commerce One using a discounted cash flow model. The value obtained was $4,941 million.[10] At the end of 2000, Commerce One had 228.32 million shares outstanding. To estimate the value of equity per share, we used the total number of shares that would be outstanding if all options were exercised:

$$\text{Value of equity per share} = \text{Value of equity}/(\text{Shares outstanding} + \text{Shares in options})$$
$$= 4{,}941/(228.32 + 45.911) = \$18.02$$

Note, though, that some of these options are not vested or exercisable. If only exercisable options were considered, we would estimate a value of equity per share that is higher:

$$\text{Value of equity per share} = \text{Value of equity}/(\text{Shares outstanding} + \text{Exercisable options})$$
$$= 4{,}941/(228.32 + 8.82) = \$20.84$$

[10]The details of this valuation are in Chapter 23.

The biggest advantage of this approach is that it does not require a value per share (or stock price) to incorporate the option value into per-share value. As you will see with the last (and recommended) approach, there is a circularity that is created when the stock price is an input when estimating value per share.

ILLUSTRATION 16.8: Treasury Stock Approach: Commerce One

To estimate the value per share with the treasury stock approach for Commerce One, we consider the expected proceeds for the exercise of the options today. To simplify calculations, we use the total number of options outstanding and the weighted average exercise price from the tables in Illustration 16.7:

$$\text{Expected proceeds from option exercise} = \text{Number of options} \times \text{Weighted exercise price}$$
$$= 45.911 \times 35.49 = \$1{,}629 \text{ million}$$

We add the expected proceeds from option exercise to the value of equity that we estimated for Commerce One, and then divide by the total number of shares outstanding to estimate the value of equity per share:

$$\text{Value per share} = (\text{Value of equity} + \text{Expected proceeds})/(\text{Shares outstanding} + \text{Shares underlying options})$$
$$= (4{,}941 + 1{,}629)/(228.32 + 45.911) = \$23.96$$

Here again, we could have used the modified approach of looking only at in-the-money options, which would have given us the following:

$$\text{Expected proceeds from option exercise} = \text{Number of exercisable options} \times \text{Weighted exercise price}$$
$$= 8.82 \times \$28.16 = \$248 \text{ million}$$

$$\text{Value per share} = (\text{Value of equity} + \text{Expected proceeds from in-the-money options})/(\text{Shares outstanding} + \text{Exercisable options})$$
$$= (4{,}941 + 248)/(228.32 + 8.82) = \$21.88$$

Note that the value per share using this approach is higher than the value per share using the fully diluted approach. The difference is greatest when options have a higher exercise price, relative to the current stock price. The estimated value per share still ignores the time premium of the options.

Value Options Using Option Pricing Model The correct approach to dealing with options is to estimate the value of the options today, given today's value per share and the time premium on the option. Once this value has been estimated, it is subtracted from the equity value, and then divided by the number of shares outstanding to arrive at value per share.

$$\text{Value of equity per share} = (\text{Value of equity} - \text{Value of options outstanding})/\text{Primary number of shares outstanding}$$

In valuing these options, however, there are four measurement issues that you have to confront. One relates to the fact that not all of the options outstanding are vested and some of the nonvested options might never be vested. The second relates to the stock price to use in valuing these options. As the description in the preced-

ing paragraph makes clear, the value per share is an input to the process as well as the output. The third issue is taxation. Since firms are allowed to deduct a portion of the expense associated with option exercises, there may be a potential tax saving when the options are exercised. The final issue relates to private firms or firms on the verge of a public offering. Key inputs to the option pricing model, including the stock price and the variance, cannot be obtained for these firms, but the options have to be valued nevertheless.

Dealing with Vesting As noted earlier in the chapter, firms granting employee options usually require that the employee receiving the options stay with the firm for a specified period, for the option to be vested. Consequently, when you examine the options outstanding at a firm, you are looking at a mix of vested and nonvested options. The nonvested options should be worth less than the vested options, but the probability of vesting will depend upon how in-the-money the options are and the period left for an employee to vest. While there have been attempts to develop option pricing models that allow for the possibility that employees may leave a firm before vesting and forfeit the value of their options,[11] the likelihood of such an occurrence when a manager's holdings are substantial should be small. Carpenter (1998) developed a simple extension of the standard option pricing model to allow for early exercise and forfeiture, and used it to value executive options.

Which Stock Price? The answer to this question may seem obvious. Since the stock is traded, and you can obtain a stock price, it would seem that you should be using the current stock price to value options. However, you are valuing these options to arrive at a value per share that you will then compare to the market price to decide whether a stock is under- or overvalued. Thus, using the current market price to arrive at the value of the options and then using this option value to estimate an entirely different value per share seems inconsistent.

There is a solution. You can value the options using the estimated value per share. This creates circular reasoning in your valuation. In other words, you need the option value to estimate value per share and value per share to estimate the option value. We would recommend that the value per share be initially estimated using the treasury stock approach, and that you then converge on the proper value per share by iterating.[12]

There is another related issue. When options are exercised, they increase the number of shares outstanding, and by doing so, there can have an effect on the stock price. In conventional option pricing models, the exercise of the option does not affect the stock price. These models have to be adapted to allow for the dilutive effect of option exercise. This can be done fairly simply by adjusting the current stock price for the expected effects of dilution (as we did with warrants in Chapter 5).

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[11]Cuny and Jorion (1995) examine the valuation of options when there is the possibility of forfeiture.

[12]The value per share, obtained using the treasury stock approach, will become the stock price in the option pricing model. The option value that results from using this price is used to compute a new value per share, which is fed back into the option pricing model, and so on.

Taxation When options are exercised, the firm can deduct for tax purposes the difference between the stock price at the time and the exercise price as an employee expense. This potential tax benefit reduces the drain on value created by having options outstanding. One way in which you could estimate the tax benefit is to multiply the difference between the stock price today and the exercise price by the tax rate; clearly, this would make sense only if the options are in-the-money. While this does not allow for the expected price appreciation over time, it has the benefit of simplicity. An alternative way of estimating the tax benefit is to compute the after-tax value of the options:

$$\text{After-tax value of options} = \text{Value from option pricing model}(1 - \text{Tax rate})$$

This approach is also straightforward and allows you to consider the tax benefits from option exercise in valuation. One of the advantages of this approach is that it can be used to consider the potential tax benefit even when options are out-of-the-money.

Nontraded Firms A couple of key inputs to the option pricing model—the current price per share and the variance in stock prices—cannot be obtained if a firm is not publicly traded. There are two choices in this scenario. One is to revert to the treasury stock approach to estimate the value of the options outstanding and abandon the option pricing models. The other is to stay with the option pricing models and to estimate the value per share, from the discounted cash flow model. The variance of similar firms that are publicly traded can be used to estimate the value of the options.

WHAT ABOUT OTHER OPTIONS?

While we have considered the effects of management options specifically in this section, everything that has been said here about management and employee options applies to other equity options issued by the firm as well. In particular, warrants issued to raise equity capital and conversion options in convertible securities (bonds and preferred stock) also dilute the value of the common stock in a firm. Consequently, you would need to reduce the value of equity by the value of these options as well. Generally speaking, though, warrants and conversions tend to be easier to value than management options because they are traded. The market values of the warrants and the conversion options can be used as measures of their estimated values.

ILLUSTRATION 16.9: Option Value Approach: Commerce One

We use an option pricing model and adjust for dilution to value all outstanding options at Commerce One. To estimate the value of the options, we first estimate the standard deviation in stock prices[13] over the previous two years. Weekly returns are used to make this estimate, and this estimate is annualized.[14] All options, vested as well as nonvested, are valued, and there is no adjustment for nonvesting.

Inputs to the Black-Scholes Model: Commerce One Options

Current stock price	\$8.28
Weighted average exercise price per option	\$35.49
Weighted average maturity of options	8.92 years
Standard deviation in stock price	135%
Riskless rate	5.40%
Number of options outstanding	45.911
Number of shares outstanding	228.32
Value of options outstanding	\$349
After-tax value of options outstanding	349(1 – .35) = \$227 million

In estimating the after-tax value of the options at Commerce One, we have used their prospective marginal tax rate of 35%. If the options are exercised prior to these firms reaching their marginal tax rates, the tax benefit is lower since the expenses are carried forward and offset against income in future periods.

The value per share can now be computed by subtracting the value of the options outstanding from the value of equity and dividing by the primary number of shares outstanding. Again, using Commerce One, we estimate a value for equity per share:

$$\begin{aligned}\text{Value of equity per share} &= (\text{Value of equity} - \text{Value of options outstanding}) \\ &\quad /\text{Number of shares outstanding} \\ &= (4{,}941 - 227)/228.32 = \$20.65 \text{ per share}\end{aligned}$$

The inconsistency averred to earlier is clear when you compare the value per share that is estimated here (\$20.65) to the price per share (\$8.28) used to estimate the value of the options. (Commerce One's value per share is \$20.65, whereas the price per share used in the option valuation is \$8.28.) If you choose to iterate, you would revalue the options using the estimated value of \$20.65, which would increase the value of the options and lower the value per share, leading to a second iteration and a third one and so on. The values converge to yield a consistent estimate.

$$\text{Estimated value of options with estimated value per share} = \$835 \text{ million}$$

$$\begin{aligned}\text{Value per share} &= (\text{Value of equity} - \text{Value of options outstanding})/\text{Number of shares outstanding} \\ &= [4{,}941 - 835 \times (1 - .35)]/228.32 = \$19.26 \text{ per share}\end{aligned}$$

The options are also valued using the same value per share.

[13]The variance estimate is actually on the natural log of the stock prices. This allows you to cling to at least the possibility of a normal distribution. Neither stock prices nor stock returns can be normally distributed since prices cannot fall below zero and returns cannot be lower than –100 percent.

[14]All of the inputs to the Black-Scholes model have to be in annual terms. To annualize a weekly variance, you multiply by 52.

THE REPRICING OF OPTIONS: EFFECTS ON VALUE

In recent years, firms that have seen their stock price drop have often reset their exercise prices on options closer to the market price to make them more attractive to management. This practice is obviously hazardous to stockholders since it increases the value of the option overhang. In fact, if this practice is flagrant at a firm, you should value the options with an exercise price of zero, which would make them each worth as much as a regular share. In effect, the fully diluted estimate of value per share will be the value you get even if you used the option pricing model.

Future Option Grants

While incorporating options that are already outstanding is fairly straightforward, incorporating the effects of future option grants is much more complicated. In this section, the argument for why these option issues affect value is presented, as well as how to incorporate these effects into value.

Why Future Options Issues Affect Value Just as options outstanding currently represent potential dilution or cash outflows to existing equity investors, expected option grants in the future will affect value per share by increasing the number of shares outstanding in future periods. The simplest way of thinking about this expected dilution is to consider the terminal value in the discounted cash flow model. As constructed in the last chapter, the terminal value is discounted to the present and divided by the shares outstanding today to arrive at the value per share. However, expected option issues in the future will increase the number of shares outstanding in the terminal year, and therefore reduce the portion of the terminal value that belongs to existing equity investors.

Ways of Incorporating Effect into Value per Share It is much more difficult to incorporate the effect of expected option issues into value than it is to consider existing options. This is because you have to forecast not only how many options will be issued by a firm in future periods, but also what the terms of these options will be. While this may be possible for a couple of periods with proprietary information (the firm lets you know how much it plans to issue and at what terms), it will become more difficult beyond that point. We will consider an approach in which you can obtain an estimate of the option value, and look at two ways of dealing with this estimate, once obtained.

Estimate Option Value as an Operating or a Capital Expense You can estimate the value of options that will be granted in future periods as a percentage of revenues or operating income. By doing so, you can avoid having to estimate the number and terms of future option issues. Estimation will also become easier since you can draw on the firm's own history (by looking at the value of option grants in previous years as a proportion of firm value) and the experiences of more mature firms in the sector. Generally, as firms become larger, the value of options granted as a percent of revenues should become smaller.

Having estimated the value of expected future option issues, you are left with another choice. You can consider this value each period as an operating expense

and compute the operating income after the expense. You are assuming, then, that option issues form part of annual compensation. Alternatively, you can treat it as a capital expense and amortize it over multiple periods. While the cash flow in the current period is unaffected by this distinction, it has consequences for the return on capital and reinvestment rates that you measure for a firm.

It is important that you do not double count future option issues. The current operating expenses of the firm already include the expenses associated with option exercises in the current period. The operating margins and returns on capital that you might derive by looking at industry averages reflect the effects of option exercise in the current period for the firms in the industry. If the effect on operating income of option exercise in the current period is less than the expected value of new option issues, you have to allow for an additional expense associated with option issues. Conversely, if a disproportionately large number of options were exercised in the last period, you have to reduce the operating expenses to allow for the fact that the expected effect of option issues in future periods will be smaller.

VALUE PER SHARE WHEN VOTING RIGHTS VARY

When you divide the value of the equity by the number of shares outstanding, you assume that the shares all have the same voting rights. If different classes of shares have different voting rights, the value of equity per share has to reflect these differences, with the shares with more voting rights having higher value. Note, though, that the total value of equity is still unchanged. To illustrate, assume that the value of equity in a firm is $500 million and that there are 50 million shares outstanding; 25 million of these shares have voting rights and 25 million do not. Furthermore, assume that the voting shares will have a value 10 percent higher than the nonvoting shares. To estimate the value per share:

$$\text{Value per nonvoting share} = \$500 \text{ million}/(25 \text{ million} \times 1.10 + 25 \text{ million}) = \$500 \text{ million}/52.5 \text{ million} = \$9.52$$

$$\text{Value per voting share} = \$9.52(1.10) = \$10.48$$

The key issue that you face in valuation then is in coming up with the discount to apply for nonvoting shares or, alternatively, the premium to attach to voting shares.

Voting Shares versus Nonvoting Shares

What premium should be assigned to the voting shares? You have two choices. One is to look at studies that empirically examine the size of the premium for voting rights and to assign this premium to all voting shares. Lease, McConnell, and Mikkelson (1983) examined 26 firms that had two classes of common stock outstanding, and they concluded that the voting shares traded at a premium relative to nonvoting shares.[15] The premium, on average, amounted to 5.44 percent, and the voting shares sold at a higher price in 88 percent of the months for which data were available. In four firms that also had voting preferred stock, however, the voting common stock traded at a discount of about 1.17 percent relative to nonvoting shares.

The other option is to be more discriminating and vary the premium depending on the firm. Voting rights have value because they give shareholders a say in the

[15]The two classes of stock received the same dividend.

ESTIMATING THE PREMIUM FOR VOTING RIGHTS

If one class of shares has significantly more voting rights than another, you would expect it to trade at a higher price. Estimating the premium for voting rights can be fairly complicated. While many analysts prefer to use ad hoc approaches, you can estimate a more precise estimate of the relative value of voting shares by valuing the firm twice—once under incumbent management and once with a new (and better) management. For instance, assume that you value a firm at $800 million with existing management and $1,200 million with new management. The value of control at this firm is $400 million. If you assume that this firm has 10 million voting shares and 10 million nonvoting shares, you could estimate the voting share premium by estimating two per-share values:

Value per share for nonvoting shares = Status quo value /(Voting + Nonvoting shares)
= 800/(10 + 10) = $40

Value per share for voting shares = Value per share without voting rights + (Value of firm with superior management – Value of firm status quo)/ Number of voting shares
= $40 + (1,200 – 800)/10 = $80 per share

The voting share premium will decrease as the difference between optimal and current value decreases and also if the likelihood of a hostile takeover lessens.

warrants.xls: **This spreadsheet allows you to value the options outstanding in a firm, allowing for the dilution effect.**

management of the firm. To the extent that voting shares can make a difference—by removing incumbent management, forcing management to change policy, or selling to a hostile bidder in a takeover—their price will reflect the possibility of a change in the way the firm is run.[16] Nonvoting shareholders do not participate in these decisions.

CONCLUSION

Incorporating the value of nonoperating assets into firm value can be very simple to do in some cases—cash and near-cash investments—and very complicated in other cases—holdings in private companies. The principle, though, should remain the same. You want to estimate a fair value for these nonoperating assets and bring them into value. As noted, it is often better to value nonoperating assets separately from operating assets, but the absence of information may impede this process.

[16]In some cases, the rights of nonvoting stockholders are protected in the specific instance of a takeover by forcing the bidder to buy the nonvoting shares as well.

The existence of options and the possibility of future option grants makes getting from equity value to value per share a complicated exercise. To deal with options outstanding at the time of the valuation, there are four approaches. The simplest is to estimate the value per share by dividing the value of equity by the fully diluted number of shares outstanding. This approach ignores both the expected proceeds from exercising the options and the time value of the options. The second approach of forecasting expected option exercises in the future and estimating the effect on value per share is not only tedious but unlikely to work. In the treasury stock approach, you add the expected proceeds from option exercise to the value of equity and then divide by the fully diluted number of shares outstanding. While this approach does consider the expected proceeds from exercise, it still ignores the option time premium.

In the final and preferred approach, the options are valued using an option pricing model, and the value is subtracted from the value of equity. The resulting estimate is divided by the primary shares outstanding to arrive at the value of equity per share. While the current price of the stock is usually used in option pricing models, the value per share estimated from the discounted cash flow valuation can be substituted to arrive at a more consistent estimate.

To deal with expected option grants in the future, the current operating income has to be dissected to consider how much of an effect option exercises in the current period had on operating expenses. If the options granted during the period had more value than the option expense resulting from exercise of options granted in prior periods, the current operating income has to be adjusted down to reflect the difference. Industry average margins and returns on capital will also have to be adjusted for the same reason.

Once the value per share of equity has been estimated, that value may need to be adjusted for differences in voting rights. Shares with disproportionately high voting rights will sell at a premium relative to shares with low or no voting rights. The difference will be larger for firms that are badly managed and smaller for well-managed firms.

QUESTIONS AND SHORT PROBLEMS

1. ABV Inc. has earnings before interest and taxes of $250 million, expected to grow 5% a year forever; the tax rate is 40%. Its cost of capital is 10%, its reinvestment rate is 33.33%, and it has 200 million shares outstanding. If the firm has $500 million in cash and marketable securities and $750 million in debt outstanding, estimate the value of equity per share.
2. How would your answer to the previous problem change if you were told that ABV had options outstanding for 50 million shares and that each option had a value of $5.
3. If you were told that the average exercise price of the 50 million options in the previous problem was $6, estimate the value per share for ABV using the treasury stock approach.
4. LSI Logic has 1 billion shares outstanding, trading at $25 per share. The firm also has $5 billion in debt outstanding. The cost of equity is 12.5% and the cost of debt, after taxes, is 5%. If the firm has $3 billion in cash outstanding and is fairly valued, estimate how much the firm earned in operating income in the current year. (The return on capital is 15%, the tax rate is 30% and earnings are growing 6% a year in perpetuity.)

5. Lava Lamps Inc. had $800 million in earnings before interest and taxes last year. It has just acquired a 50% stake in General Lamps Inc., which had $400 million in earnings before interest and taxes last year. Because Lava Lamps has a majority active stake, it has been asked to consolidate last year's income statements for the two firms. What earnings before interest and taxes would you see in the consolidated statement?
   a. If both firms have a 5% stable growth rate, a 10% cost of capital, a 40% tax rate, and a return on capital of 11%, estimate the value of equity in Lava Lamps.
   b. How would your answer change if you were told that General Lamps has a 9% cost of capital and a 15% return on capital?
6. Genome Sciences is a biotechnology firm that had after-tax operating income of $300 million last year; these earnings are expected to grow 6% a year forever, the reinvestment rate is 40% and the firm has a cost of capital of 12%. Genome also owns 10% of the stock of Gene Therapies Inc., another publicly traded firm. Gene Therapies has 100 million shares outstanding, trading at $50 per share. If Genome has $800 million in debt outstanding, estimate the value of equity per share in Genome Sciences. (Genome has 50 million shares outstanding.)
7. Fedders Asia Closed End fund is a closed-end equity fund that holds Asian securities with a market value of $1 billion. Over the past 10 years, the fund has earned a return of 9% a year, 3% less than the return earned by index funds investing in Asia. You expect annual returns in the future to be similar to those earned in the past, both for your fund and for index funds in general.
   a. Assuming no growth in the fund and investment in perpetuity, estimate the discount at which you would expect the fund to trade.
   b. How would your answer change if you expect the fund to be liquidated in 10 years?
8. You have been asked to review another analyst's valuation of System Logic Inc., a technology firm. The analyst estimated a value per share of $11 while the stock was trading at $12.50 per share. In making this estimate, however, she divided the value of equity by the fully diluted 1.4 million shares outstanding. Reviewing this number, you discover that the firm has only 1 million shares outstanding and that the remaining 400,000 shares represent options with an average maturity of three years and an average exercise price of $5.
   a. Estimate the correct value per share, using the treasury stock approach.
   b. If the standard deviation in the stock price is 80%, estimate the value of the options using an option pricing model (and the current stock price) and the correct value per share.
   c. Will your value per share increase or decrease if you reestimate the value of the options using your estimated value per share?