CHAPTER 7

Riskless Rates and Risk Premiums

All models of risk and return in finance are built around a rate that investors can make on riskless investments and the risk premium or premiums that investors should charge for investing in the average-risk investment. In the capital asset pricing model (CAPM), where there is only one source of market risk captured in the market portfolio, this risk premium becomes the premium that investors would demand when investing in that portfolio. In multifactor models, there are multiple risk premiums, each one measuring the premium demanded by investors for exposure to a specific market risk factor. This chapter examines how best to measure a riskless rate and to estimate a risk premium or premiums for use in these models.

As noted in Chapter 4, risk is measured in terms of default risk for bonds, and this default risk is captured in a default spread that firms have to pay over and above the riskless rate. This chapter closes by considering how best to estimate these default spreads and factors that may cause these spreads to change over time.

THE RISK-FREE RATE

Most risk and return models in finance start off with an asset that is defined as risk free, and use the expected return on that asset as the risk-free rate. The expected returns on risky investments are then measured relative to the risk-free rate, with the risk creating an expected risk premium that is added to the risk-free rate. But what makes an asset risk free? And what do we do when we cannot find such an asset? These are the questions that will be dealt with in this section.

Requirements for an Asset to Be Risk Free

Chapter 4 considered some of the requirements for an asset to be risk free. In particular, an asset is risk free if we know the expected returns on it with certainty (i.e., the actual return is always equal to the expected return). Under what conditions will the actual returns on an investment be equal to the expected returns? There are two basic conditions that have to be met. The first is that *there can be no default risk*. Essentially, this rules out any security issued by a private firm, since even the largest and safest firms have some measure of default risk. The only securities that have a chance of being risk free are government securities, not because governments are better run than corporations, but because they control the printing of currency. At least in nominal terms, they should be able to fulfill their promises.

Even this assumption, straightforward though it might seem, does not always hold up, especially when governments refuse to honor claims made by previous regimes and when they borrow in currencies other than their own.

There is a second condition that riskless securities need to fulfill that is often forgotten. For an investment to have an actual return equal to its expected return, *there can be no reinvestment risk*. To illustrate this point, assume that you are trying to estimate the expected return over a five-year period and that you want a risk-free rate. A six-month Treasury bill rate, while default free, will not be risk free, because there is the reinvestment risk of not knowing what the Treasury bill rate will be in six months. Even a five-year Treasury bond is not risk free, since the coupons on the bond will be reinvested at rates that cannot be predicted today. The risk-free rate for a five-year time horizon has to be the expected return on a default-free (government) five-year zero coupon bond. This clearly has painful implications for anyone doing corporate finance or valuation, where expected returns often have to be estimated for periods ranging from 1 to 10 years. A purist's view of risk-free rates would then require different risk-free rates for each period, and different expected returns.

As a practical compromise, however, it is worth noting that the present value effect of using year-specific risk-free rates tends to be small for most well-behaved term structures.[1] In these cases, we could use a duration matching strategy, where the duration of the default-free security used as the risk-free asset is matched up to the duration[2] of the cash flows in the analysis. If, however, there are very large differences, in either direction, between short-term and long-term rates, it does pay to stick with year-specific risk-free rates in computing expected returns.

Practical Implications When a Default-Free Entity Exists

In most developed markets, where the government can be viewed as a default-free entity, at least when it comes to borrowing in the local currency, the implications are simple. When doing investment analysis on longer-term projects or valuations, the risk-free rate should be the long-term government bond rate. If the analysis is shorter-term, the short-term government security rate can be used as the risk-free rate. The choice of a risk-free rate also has implications for how risk premiums are estimated. If, as is often the case, historical risk premiums are used, where the excess return earned by stocks over and above a government security rate over a past period is used as the risk premium, the government security chosen has to be same one as that used for the risk-free rate. Thus, the historical risk premium used in the United States should be the excess return earned by stocks over Treasury bonds, and not Treasury bills, for purposes of long-term analysis.

[1]Well-behaved term structures would include a normal upward-sloping yield curve, where long-term rates are at most 2 to 3 percent higher than short-term rates.

[2]In investment analysis, where we look at projects, these durations are usually between 3 and 10 years. In valuation, the durations tend to be much longer, since firms are assumed to have infinite lives. The durations in these cases are often well in excess of 10 years and increase with the expected growth potential of the firm.

Cash Flows and Risk-Free Rates: The Consistency Principle

The risk-free rate used to come up with expected returns should be measured consistently with how the cash flows are measured. Thus, if cash flows are estimated in nominal U.S. dollar terms, the risk-free rate will be the U.S. Treasury bond rate. This also implies that it is not where a firm is domiciled that determines the choice of a risk-free rate, but the currency in which the cash flows on the firm are estimated. Thus, Nestlé can be valued using cash flows estimated in Swiss francs, discounted back at an expected return estimated using a Swiss long-term government bond rate, or it can be valued in British pounds, with both the cash flows and the risk-free rate being in British pounds. Given that the same firm can be valued in different currencies, will the final results always be consistent? If we assume purchasing power parity, then differences in interest rates reflect differences in expected inflation. Both the cash flows and the discount rate are affected by expected inflation; thus, a low discount rate arising from a low risk-free rate will be exactly offset by a decline in expected nominal growth rates for cash flows, and the value will remain unchanged.

If the difference in interest rates across two currencies does not adequately reflect the difference in expected inflation in these currencies, the values obtained using the different currencies can be different. In particular, firms will be valued more highly when the currency used is the one with low interest rates relative to inflation. The risk, however, is that the interest rates will have to rise at some point to correct for this divergence, at which point the values will also converge.

Real versus Nominal Risk-Free Rates

Under conditions of high and unstable inflation, valuation is often done in real terms. Effectively, this means that cash flows are estimated using real growth rates and without allowing for the growth that comes from price inflation. To be consistent, the discount rates used in these cases have to be real discount rates. To get a real expected rate of return, we need to start with a real risk-free rate. While government bills and bonds offer returns that are risk free in nominal terms, they are not risk free in real terms, since expected inflation can be volatile. The standard approach of subtracting an expected inflation rate from the nominal interest rate to arrive at a real risk-free rate provides at best an estimate of the real risk-free rate.

Until recently, there were few traded default-free securities that could be used to estimate real risk-free rates, but the introduction of inflation-indexed Treasuries has filled this void. An inflation-indexed Treasury security does not offer a guaranteed nominal return to buyers, but instead provides a guaranteed real return. Thus, an inflation-indexed Treasury that offers a 3 percent real return will yield approximately 7 percent in nominal terms if inflation is 4 percent and only 5 percent in nominal terms if inflation is only 2 percent.

The only problem is that real valuations are seldom called for or done in the United States, which has stable and low expected inflation. The markets where we would most need to do real valuations, unfortunately, are markets without inflation-indexed default-free securities. The real risk-free rates in these markets can be estimated by using one of two arguments:

1. The first argument is that as long as capital can flow freely to those economies with the highest real returns, there can be no differences in real risk-free rates across markets. Using this argument, the real risk-free rate for the United States, estimated from the inflation-indexed Treasury, can be used as the real risk-free rate in any market.
2. The second argument applies if there are frictions and constraints in capital flowing across markets. In that case, the expected real return on a economy, in the long term, should be equal to the expected real growth rate, again in the long term, of that economy, for equilibrium. Thus, the real risk-free rate for a mature economy like Germany should be much lower than the real risk-free rate for a economy with greater growth potential, such as Hungary.

Risk-Free Rates When There Is No Default-Free Entity

Our discussion, hitherto, has been predicated on the assumption that governments do not default, at least on local borrowing. There are many emerging market economies where this assumption might not be viewed as reasonable. Governments in these markets are perceived as capable of defaulting even on local borrowing. When this is coupled with the fact that many governments do not borrow long-term locally, there are scenarios where obtaining a local risk-free rate, especially for the long term, becomes difficult. Under these cases, there are compromises that give us reasonable estimates of the risk-free rate:

- Look at the largest and safest firms in that market, and use the rate that they pay on their long-term borrowings in the local currency as a base. Given that these firms, in spite of their size and stability, still have default risk, you would use a rate that is marginally lower than the corporate borrowing rate.[3]
- If there are long-term dollar-denominated forward contracts on the currency, you can use interest rate parity and the Treasury bond rate (or riskless rate in any other base currency) to arrive at an estimate of the local borrowing rate.

$$\text{Forward rate}^t_{FC,\$} = \text{Spot rate}_{FC,\$}\frac{(1+\text{Interest rate}_{FC})^t}{(1+\text{Interest rate}_{\$})^t}$$

where $\text{Forward rate}^t_{FC,\$}$ = Forward rate for foreign currency units/$ in period t
$\text{Spot rate}_{FC,\$}$ = Spot rate for foreign currency units/$
$\text{Interest rate}_{FC}$ = Interest rate in foreign currency
$\text{Interest rate}_{\$}$ = Interest rate in U.S. dollars

For instance, if the current spot rate is 38.10 Thai baht per U.S. dollar, the 10-year forward rate is 61.36 baht per dollar, and the current 10-year U.S. Treasury bond rate is 5 percent, the 10-year Thai risk-free rate (in nominal baht) can be estimated as follows:

$$61.36 = 38.10(1 + \text{Interest rate}_{\text{Thai baht}})^{10}/1.05^{10}$$

[3]I would use 1 percent less than the corporate borrowing rate as my risk-free rate. This is roughly an AA default spread in the United States.

Solving for the Thai interest rate yields a 10-year risk-free rate of 10.12 percent. The biggest limitation of this approach, however, is that long-term forward rates are difficult to come by for periods beyond a year for many of the emerging markets, where we would be most interested in using them.[4]

- You could adjust the local currency government borrowing rate by the estimated default spread on the bond to arrive at a riskless local rate. The default spread on the government bond can be estimated using the local currency ratings[5] that are available for many countries. For instance, assume that the Indian government bond rate is 12 percent and that the rating assigned to the Indian government is A. If the default spread for A-rated bonds is 2 percent, the riskless Indian rupee rate would be 10 percent.

$$\begin{aligned}\text{Riskless rupee rate} &= \text{Indian government bond rate} - \text{Default spread}\\ &= 12\% - 2\% = 10\%\end{aligned}$$

EQUITY RISK PREMIUM

The notion that risk matters, and that riskier investments should have a higher expected return than safer investments, to be considered good investments, is intuitive. Thus, the expected return on any investment can be written as the sum of the risk-free rate and an extra return to compensate for the risk. The disagreement, in both theoretical and practical terms, remains on how to measure this risk, and how to convert the risk measure into an expected return that compensates for risk. This section looks at the estimation of an appropriate risk premium to use in risk and return models, in general, and in the capital asset pricing model, in particular.

Competing Views on Risk Premiums

In Chapter 4, we considered several competing models of risk ranging from the capital asset pricing model to multifactor models. Notwithstanding their different conclusions, they all share some common views about risk. First, they all define risk in terms of variance in actual returns around an expected return; thus, an investment is riskless when actual returns are always equal to the expected return. Second, they all argue that risk has to be measured from the perspective of the marginal investor in an asset, and that this marginal investor is well diversified. Therefore, the argument goes, it is only the risk that an investment adds on to a diversified portfolio that should be measured and compensated. In fact, it is this view

[4]In cases where only a one-year forward rate exists, an approximation for the long-term rate can be obtained by first backing out the one-year local currency borrowing rate, taking the spread over the one-year Treasury bill rate, and then adding this spread to the long-term Treasury bond rate. For instance, with a one-year forward rate of 39.95 on the Thai bond, we obtain a one-year Thai baht riskless rate of 9.04 percent (given a one-year T-bill rate of 4 percent). Adding the spread of 5.04 percent to the 10-year Treasury bond rate of 5 percent provides a 10-year Thai baht rate of 10.04 percent.

[5]Ratings agencies generally assign different ratings for local currency borrowings and dollar borrowing, with higher ratings for the former and lower ratings for the latter.

of risk that leads models of risk to break the risk in any investment into two components. There is a firm-specific component that measures risk that relates only to that investment or to a few investments like it, and a market component that contains risk that affects a large subset or all investments. It is the latter risk that is not diversifiable and should be rewarded.

While all risk and return models agree on this fairly crucial distinction, they part ways when it comes to how measure this market risk. Table 7.1 summarizes four models and the way each model attempts to measure risk.

In the first three models, the expected return on any investment can be written as:

$$\text{Expected return} = \text{Risk-free rate} + \sum_{j=1}^{j=k} \beta_j(\text{Risk premium}_j)$$

where β_j = Beta of investment relative to factor j
Risk premium_j = Risk premium for factor j

Note that in the special case of a single-factor model, like the CAPM, each investment's expected return will be determined by its beta relative to the single factor.

Assuming that the risk-free rate is known, these models all require two inputs. The first is the beta or betas of the investment being analyzed, and the second is the appropriate risk premium(s) for the factor or factors in the model. While the issue of beta estimation will be examined in the next chapter, this section will concentrate on the measurement of the risk premium.

TABLE 7.1 Comparing Risk and Return Models

Model	Assumptions	Measure of Market Risk
Capital asset pricing model (CAPM)	There are no transaction costs or private information. Therefore, the diversified portfolio includes all traded investments, held in proportion to their market value.	Beta measured against this market portfolio
Arbitrage pricing model (APM)	Investments with the same exposure to market risk have to trade at the same price (no arbitrage).	Betas measured against multiple (unspecified) market risk factors
Multifactor model	There is the same no-arbitrage assumption as with the APM.	Betas measured against multiple specified macroeconomic factors
Proxy model	Over very long periods, higher returns on investments must be compensation for higher market risk.	Proxies for market risk, for example, include market capitalization and price–book value ratios.

What We Would Like to Measure As far as the risk premium is concerned, we would like to know for each factor, what investors, on average, require as a premium over the risk-free rate for an investment with average risk.

Without any loss of generality, let us consider the estimation of the beta and the risk premium in the capital asset pricing model. Here, the risk premium should measure what investors, on average, demand as extra return for investing in the market portfolio relative to the risk-free asset.

Historical Risk Premiums

In practice, we usually estimate the risk premium by looking at the historical premium earned by stocks over default-free securities over long time periods. The historical premium approach is simple. The actual returns earned on stocks over a long time period are estimated, and compared to the actual returns earned on a default-free (usually government) security. The difference, on an annual basis, between the two returns is computed and represents the historical risk premium. This approach might yield reasonable estimates in markets like the United States, with a large and diversified stock market and a long history of returns on both stocks and government securities. However, they yield meaningless estimates for the risk premium in other countries, where the equity markets represent a small proportion of the overall economy, and the historical returns are available only for short periods.

While users of risk and return models may have developed a consensus that historical premium is, in fact, the best estimate of the risk premium looking forward, there are surprisingly large differences in the actual premiums we observe being used in practice. For instance, the risk premium estimated in the U.S. markets by different investment banks, consultants, and corporations range from 4 percent at the lower end to 12 percent at the upper end. Given that we almost all use the same database of historical returns, provided by Ibbotson Associates,[6] summarizing data from 1926, these differences may seem surprising. There are, however, three reasons for the divergence in risk premiums:

1. *Time period used.* While there are many who use all the data going back to 1926, there are almost as many using data over shorter time periods, such as 50, 20 or even 10 years to come up with historical risk premiums. The rationale presented by those who use shorter periods is that the risk aversion of the average investor is likely to change over time, and that using a shorter time period provides a more updated estimate. This has to be offset against a cost associated with using shorter time periods, which is the greater noise in the risk premium estimate. In fact, given the annual standard deviation in stock prices[7] between 1926 and 2000 of 20 per-

[6]See "Stocks, Bonds, Bills and Inflation," an annual edition that reports on annual returns on stocks, Treasury bonds, and Treasury bills, as well as inflation rates from 1926 to the present (www.ibbotson.com).

[7]For the historical data on stock returns, bond returns, and bill returns check under "Updated Data" in www.stern.nyu.edu/~adamodar.

cent, the standard error[8] associated with the risk premium estimate can be estimated as follows for different estimation periods in Table 7.2.

Note that to get reasonable standard errors, we need very long time periods of historical returns. Conversely, the standard errors from 10-year and 20-year estimates are likely to almost as large or larger than the actual risk premium estimated. This cost of using shorter time periods seems, in our view, to overwhelm any advantages associated with getting a more updated premium.

2. *Choice of risk-free security.* The Ibbotson database reports returns on both Treasury bills (T-bills) and Treasury bonds (T-bonds), and the risk premium for stocks can be estimated relative to each. Given that the yield curve in the United States has been upward-sloping for most of the past seven decades, the risk premium is larger when estimated relative to shorter-term government securities (such as Treasury bills). *The risk-free rate chosen in computing the premium has to be consistent with the risk-free rate used to compute expected returns.* Thus, if the Treasury bill rate is used as the risk-free rate, the premium has to be the premium earned by stocks over that rate. If the Treasury bond rate is used as the risk-free rate, the premium has to be estimated relative to that rate. For the most part, in corporate finance and valuation, the risk-free rate will be a long-term default-free (government) bond rate and not a Treasury bill rate. Thus, the risk premium used should be the premium earned by stocks over Treasury bonds.

3. *Arithmetic and geometric averages.* The final sticking point when it comes to estimating historical premiums relates to how the average returns on stocks, Treasury bonds, and Treasury bills are computed. The arithmetic average return measures the simple mean of the series of annual returns, whereas the geometric average looks at the compounded return.[9] Conventional wisdom argues for the use of the arithmetic average. In fact, if annual returns are uncorrelated over time, and our objective were to estimate the risk premium for the next year, the arithmetic average is the best unbiased estimate of the premium. In reality, however, there are strong arguments that can be made for the use of geometric averages. First, empiri-

TABLE 7.2 Standard Errors in Risk Premium Estimates

Estimation Period	Standard Error of Risk Premium Estimate
5 years	$20\%\sqrt{5} = 8.94\%$
10 years	$20\%\sqrt{10} = 6.32\%$
25 years	$20\%\sqrt{25} = 4.00\%$
50 years	$20\%\sqrt{50} = 2.83\%$

[8]These estimates of the standard error are probably understated, because they are based on the assumption that annual returns are uncorrelated over time. There is substantial empirical evidence that returns are correlated over time, which would make this standard error estimate much larger.

[9]The compounded return is computed by taking the value of the investment at the start of the period (Value_0) and the value at the end (Value_N), and then computing the following:

$$\text{Geometric average} = \left(\frac{\text{Value}_N}{\text{Value}_0}\right)^{1/N} - 1$$

cal studies seem to indicate that returns on stocks are negatively correlated over time.[10] Consequently, the arithmetic average return is likely to overstate the premium. Second, while asset pricing models may be single-period models, the use of these models to get expected returns over long periods (such as 5 or 10 years) suggests that the single period may be much longer than a year. In this context, the argument for geometric average premiums becomes even stronger.

In summary, the risk premium estimates vary across users because of differences in time periods used, the choice of Treasury bills or bonds as the risk-free rate and the use of arithmetic as opposed to geometric averages. The effect of these choices is summarized in Table 7.3, which uses returns from 1928 to 2000. Note that the premiums can range from 4.5 percent to 12.67 percent, depending on the choices made. In fact, these differences are exacerbated by the fact that many risk premiums that are in use today were estimated using historical data three, four, or even ten years ago.

***histretSP.xls:* There is a dataset on the Web that summarizes historical returns on stocks, T-bonds and T-bills in the United States going back to 1928.**

Historical Risk Premiums: Other Markets If it is difficult to estimate a reliable historical premium for the U.S. market, it becomes doubly so when looking at markets with short and volatile histories. This is clearly true for emerging markets, but it is also true for the European equity markets. While the economies of Germany, Italy, and France may be mature, their equity markets do not share the same characteristic. They tend to be dominated by a few large companies; many businesses remain private; and trading, until recently, tended to be thin except on a few stocks.

There are some practitioners who still use historical premiums for these markets. To capture some of the danger in this practice, Table 7.4 summarizes historical risk premiums[11] for major non-U.S. markets for 1970 to 1996.

TABLE 7.3 Historical Risk Premiums for the United States

	Stocks versus Treasury Bills		Stocks versus Treasury Bonds	
	Arithmetic	Geometric	Arithmetic	Geometric
1928–2000	8.41%	7.17%	6.53%	5.51%
1962–2000	6.41%	5.25%	5.30%	4.52%
1990–2000	11.42%	7.64%	12.67%	7.09%

Source: Federal Reserve Bank.

[10]In other words, good years are more likely to be followed by poor years, and vice versa. The evidence on negative serial correlation in stock returns over time is extensive, and can be found in Fama and French (1988). While they find that the one-year correlations are low, the five-year serial correlations are strongly negative for all size classes.

[11]This data is also from Ibbotson Associates, and can be obtained from their web site: www.ibbotson.com.

HISTORICAL RISK PREMIUM APPROACH: SOME CAVEATS

Given how widely the historical risk premium approach is used, it is surprising how flawed it is and how little attention these flaws have attracted. Consider first the underlying assumption that investors' risk premiums have not changed over time and that the average risk investment (in the market portfolio) has remained stable over the period examined. We would be hard-pressed to find anyone who would be willing to sustain this argument with fervor.

The obvious fix for this problem, which is to use a more recent time period, runs directly into a second problem, which is the large standard error associated with risk premium estimates. While these standard errors may be tolerable for very long time periods, they clearly are unacceptably high when shorter periods are used.

Finally, even if there is a sufficiently long time period of history available and investors' risk aversion has not changed in a systematic way over that period, there is a final problem. Markets that exhibit this characteristic, and let us assume that the U.S. market is one such example, represent so-called survivor markets. In other words, assume that one had invested in the 10 largest equity markets in the world in 1928, of which the United States was one. In the period extending from 1928 to 2000, investments in none of the other equity markets would have earned as large a premium as the U.S. equity market, and some of them (like Austria) would have resulted in investors earning little or even negative returns over the period. Thus, the survivor bias will result in historical premiums that are larger than expected premiums for markets like the United States, even assuming that investors are rational and factor risk into prices.

TABLE 7.4 Historical Risk Premiums in Non-U.S. Markets

	Equity			Bonds	
Country	Beginning	Ending	Annual Return	Annual Return	Risk Premium
Australia	100	898.36	8.47%	6.99%	1.48%
Canada	100	1,020.70	8.98%	8.30%	0.68%
France	100	1,894.26	11.51%	9.17%	2.34%
Germany	100	1,800.74	11.30%	12.10%	–0.80%
Hong Kong	100	14,993.06	20.39%	12.66%	7.73%
Italy	100	423.64	5.49%	7.84%	–2.35%
Japan	100	5,169.43	15.73%	12.69%	3.04%
Mexico	100	2,073.65	11.88%	10.71%	1.17%
Netherlands	100	4,870.32	15.48%	10.83%	4.65%
Singapore	100	4,875.91	15.48%	6.45%	9.03%
Spain	100	844.80	8.22%	7.91%	0.31%
Switzerland	100	3,046.09	13.49%	10.11%	3.38%
United Kingdom	100	2,361.53	12.42%	7.81%	4.61%

Data source: Ibbotson Associates.

Note that a couple of the countries have negative historical risk premiums, and a few others have risk premiums under 1 percent. Before an attempt is made to come up with rationale for why this might be so, it is worth noting that the standard errors on each and every one of these estimates is larger than 5 percent, largely because the estimation period includes only 26 years.

If the standard errors on these estimates make them close to useless, consider how much more noise there is in estimates of historical risk premiums for emerging market equity markets, which often have a reliable history of 10 years or less, and very large standard deviations in annual stock returns. Historical risk premiums for emerging markets may provide for interesting anecdotes, but they clearly should not be used in risk and return models.

Modified Historical Risk Premium While historical risk premiums for markets outside the United States cannot be used in risk models, we still need to estimate a risk premium for use in these markets. To approach this estimation question, let us start with the basic proposition that the risk premium in any equity market can be written as:

Equity risk premium = Base premium for mature equity market + Country premium

The country premium could reflect the extra risk in a specific market. This boils down our estimation to answering two questions:

1. What should the base premium for a mature equity market be?
2. Should there be a country premium, and if so, how do we estimate the premium?

To answer the first question, one can argue that the U.S. equity market is a mature market and that there is sufficient historical data in the United States to make a reasonable estimate of the risk premium. In fact, reverting back to our discussion of historical premiums in the U.S. market, we will use the geometric average premium earned by stocks over Treasury bonds of 5.51 percent between 1928 and 2000. We chose the long time period to reduce standard error, the Treasury bond to be consistent with our choice of a risk-free rate, and geometric averages to reflect our desire for a risk premium that we can use for longer-term expected returns.

On the issue of country premiums, there are some who argue that country risk is diversifiable and that there should be no country risk premium. After looking at the basis for their argument, and then considering the alternative view that there should be a country risk premium, we will present approaches for estimating country risk premiums, one based on country bond default spreads and one based on equity market volatility.

Should There Be a Country Risk Premium? Is there more risk in investing in a Malaysian or Brazilian stock than there is in investing in the United States? The answer, to most, seems to be obviously affirmative. That, however, does not answer the question of whether there should be an additional risk premium charged when investing in those markets.

Note that the only risk that is relevant for purposes of estimating a cost of equity is market risk or risk that cannot be diversified away. The key question then becomes whether the risk in an emerging market is diversifiable or nondiversifiable risk. If, in fact, the additional risk of investing in Malaysia or Brazil can be diversified away, then there should be no additional risk premium charged. If it cannot, then it makes sense to think about estimating a country risk premium.

But diversified away by whom? Equity in a Brazilian or Malaysian firm can be held by hundreds or thousands of investors, some of whom may hold only domestic stocks in their portfolio, whereas others may have more global exposure. For purposes of analyzing country risk, we look at the marginal investor—the investor most likely to be trading on the equity. If that marginal investor is globally diversified, there is at least the potential for global diversification. If the marginal investor does not have a global portfolio, the likelihood of diversifying away country risk declines substantially. Stulz (1999) made a similar point using different terminology. He differentiated between segmented markets, where risk premiums can be different in each market, because investors cannot or will not invest outside their domestic markets, and open markets, where investors can invest across markets. In a segmented market, the marginal investor will be diversified only across investments in that market, whereas in an open market, the marginal investor has the opportunity (even if he or she does not take it) to invest across markets.

Even if the marginal investor is globally diversified, there is a second test that has to be met for country risk not to matter. All or much of country risk should be country specific. In other words, there should be low correlation across markets. Only then will the risk be diversifiable in a globally diversified portfolio. If, however, the returns across countries have significant positive correlation, country risk has a market risk component, is not diversifiable, and can command a premium. Whether returns across countries are positively correlated is an empirical question. Studies from the 1970s and 1980s suggested that the correlation was low, and this was an impetus for global diversification. Partly because of the success of that sales pitch and partly because economies around the world have become increasingly intertwined over the past decade, more recent studies indicate that the correlation across markets has risen. This is borne out by the speed with which troubles in one market, say Russia, can spread to a market with little or no obvious relationship to it, say Brazil.

So where do we stand? We believe that while the barriers to trading across markets have dropped, investors still have a home bias in their portfolios and that markets remain partially segmented. While globally diversified investors are playing an increasing role in the pricing of equities around the world, the resulting increase in correlation across markets has resulted in a portion of country risk being nondiversifiable or market risk. The next section will consider how best to measure this country risk and build it into expected returns.

Measuring Country Risk Premiums If country risk matters and leads to higher premiums for riskier countries, the obvious follow-up question becomes how we measure this additional premium. This section will look at two approaches. The first builds on default spreads on country bonds issued by each country, whereas the second uses equity market volatility as its basis.

Default Risk Spreads While there are several measures of country risk, one of the simplest and most easily accessible is the rating assigned to a country's debt by a ratings agency; Standard & Poor's (S&P), Moody's Investors Service, and Fitch

IBCA all rate countries. These ratings measure default risk (rather than equity risk) but they are affected by many of the factors that drive equity risk—the stability of a country's currency, its budget and trade balances, and its political stability, for instance.[12] The other advantage of ratings is that they come with default spreads over the riskless rate. For instance, Table 7.5 summarizes the ratings and default spreads for Latin American countries as of June 2000.

The market spreads measure the difference between dollar-denominated bonds issued by the country and the U.S. Treasury bond rate. While this is a market rate and reflects current expectations, country bond spreads are extremely volatile and can shift significantly from day to day. To counter this volatility, typical spreads have been estimated by averaging the default spreads of all countries in the world with the specified rating over and above the appropriate riskless rate. These spreads tend to be less volatile and more reliable for long-term analysis.

Analysts who use default spreads as measures of country risk typically add them on to both the cost of equity and debt of every company traded in that country. For instance, the cost of equity for a Brazilian company, estimated in U.S. dollars, will be 4.83 percent higher than the cost of equity of an otherwise similar U.S. company. If we assume that the risk premium for the United States and other mature equity markets is 5.51%, the cost of equity for a Brazilian company with a beta of 1.2 can be estimated as follows (with a U.S. Treasury bond rate of 5 percent).

TABLE 7.5 Ratings and Default Spreads (in Basis Points): Latin America

Country	Rating[a]	Typical Spread[b]	Market Spread[c]
Argentina	B1	450	433
Bolivia	B1	450	469
Brazil	B2	550	483
Colombia	Ba2	300	291
Ecuador	Caa2	750	727
Guatemala	Ba2	300	331
Honduras	B2	550	537
Mexico	Baa3	145	152
Paraguay	B2	550	581
Peru	Ba3	400	426
Uruguay	Baa3	145	174
Venezuela	B2	550	571

[a]Ratings are foreign currency ratings from Moody's Investors Service.
[b]Typical spreads are estimated by looking at the default spreads on bonds issued by all countries with this rating, over and above a riskless rate (U.S. Treasury or German euro rate).
[c]Market spread measures the spread difference between dollar-denominated bonds issued by this country and the U.S. Treasury bond rate.

[12]The process by which country ratings are obtained in explained on the S&P web site at www.ratings.standardpoor.com/criteria/index.htm.

$$\text{Cost of equity} = \text{Risk-free rate} + \text{Beta} \times (\text{U.S. risk premium}) + \text{Default spread}$$
$$= 5\% + 1.2(5.51\%) + 4.83\% = 16.34\%$$

In some cases, analysts add the default spread to the U.S. risk premium and multiply it by the beta. This increases the cost of equity for high-beta companies and lowers it for low-beta firms.

While ratings provide a convenient measure of country risk, there are costs associated with using them as the only measure. First, ratings agencies often lag markets when it comes to responding to changes in the underlying default risk. Second, the ratings agency focus on default risk may obscure other risks that could still affect equity markets. What are the alternatives? There are numerical country risk scores that have been developed by some services as much more comprehensive measures of risk. The *Economist*, for instance, has a score that runs from 0 to 100 (where 0 is no risk, and 100 is most risky) that it uses to rank emerging markets. Alternatively, country risk can be estimated from the bottom up by looking at economic fundamentals in each country. This, of course, requires significantly more information than the other approaches. Finally, default spreads measure the risk associated with bonds issued by countries and not the equity risk in these countries. Since equities in any market are likely to be more risky than bonds, you could argue that default spreads understate equity risk premiums.

Relative Standard Deviations There are some analysts who believe that investors in equity markets choose between these markets based on their assessed riskiness and that the risk premiums should reflect the differences in equity risk. A conventional measure of equity risk is the standard deviation in stock prices; higher standard deviations are generally associated with more risk. If you scale the standard deviation of one market against another, you obtain a measure of relative risk.

$$\text{Relative standard deviation}_{\text{country X}} = \text{Standard deviation}_{\text{country X}}/\text{Standard deviation}_{\text{U.S.}}$$

This relative standard deviation, when multiplied by the premium used for U.S. stocks, should yield a measure of the total risk premium for any market.

THE DANGER OF DOUBLE COUNTING RISK

When assessing country risk, there is a substantial chance that the same risk may be counted more than once in a valuation. For instance, there are analysts who use the dollar-denominated bonds issued by a country—the Brazilian C-bond, for instance—as the risk-free rate when estimating cost of equity for Brazilian companies. The interest rate on this bond already incorporates the default spreads discussed in the preceding section. If the risk premium is also adjusted upward to reflect country risk, there has been a double counting of the risk. This effect is made worse when betas are adjusted upward and cash flows are adjusted downward (a process called "haircutting") because of country risk.

$$\text{Equity risk premium}_{\text{country X}} = \text{Risk premum}_{\text{U.S.}} \times \text{Relative standard deviation}_{\text{country X}}$$

Assume, for the moment, that you are using a mature market premium for the United States of 5.51 percent and that the annual standard deviation of U.S. stocks is 20 percent. If the annual standard deviation of Indonesian stocks is 35 percent, the estimate of a total risk premium for Indonesia would be:

$$\text{Equity risk premium}_{\text{Indonesia}} = 5.51\% \times (35\%/20\%) = 9.64\%$$

The country risk premium can be isolated as follows:

$$\text{Country risk premium}_{\text{Indonesia}} = 9.64\% - 5.51\% = 4.13\%$$

While this approach has intuitive appeal, there are problems with using standard deviations computed in markets with widely different market structures and liquidity. There are very risky emerging markets that have low standard deviations for their equity markets because the markets are illiquid. This approach will understate the equity risk premiums in those markets. The second problem is related to currencies, since the standard deviations are usually measured in local currency terms; the standard deviation in the U.S. market is a dollar standard deviation, whereas the standard deviation in the Indonesian market is a rupiah standard deviation. This is a relatively simple problem to fix, though, since the standard deviations can be measured in the same currency—you could estimate the standard deviation in dollar returns for the Indonesian market.

Default Spreads + Relative Standard Deviations The country default spreads that come with country ratings provide an important first step, but still only measure the premium for default risk. Intuitively, we would expect the country equity risk premium to be larger than the country default risk spread. To address the issue of how much higher, one can look at the volatility of the equity market in a country relative to the volatility of the country bond used to estimate the spread. This yields the following estimate for the country equity risk premium:

$$\text{Country risk premium} = \text{Country default spread} \times \left(\frac{\sigma_{\text{equity}}}{\sigma_{\text{country bond}}}\right)$$

To illustrate, consider the case of Brazil. In March 2000, Brazil was rated B2 by Moody's, resulting in a default spread of 4.83 percent. The annualized standard deviation in the Brazilian equity index over the previous year was 30.64 percent, while the annualized standard deviation in the Brazilian dollar-denominated C-bond was 15.28 percent. The resulting country equity risk premium for Brazil is as follows:

$$\text{Brazil's country risk premium} = 4.83\%(30.64\%/15.28\%) = 9.69\%$$

Note that this country risk premium will increase if the country rating drops or if the relative volatility of the equity market increases.

Why should equity risk premiums have any relationship to country bond spreads? A simple explanation is that an investor who can make 11 percent on a dollar-denominated Brazilian government bond would not settle for an expected return of 10.5 percent (in dollar terms) on Brazilian equity. Playing devil's advocate, however, a critic could argue that the interest rate on a country bond, from which default spreads are extracted, is not really an expected return since it is based on the promised cash flows (coupon and principal) on the bond rather than the expected cash flows. In fact, if we wanted to estimate a risk premium for bonds, we would need to estimate the expected return based on expected cash flows, allowing for the default risk. This would result in a much lower default spread and equity risk premium.

Both this approach and the previous one use the standard deviation in equity of a market to make a judgment about country risk premium, but they measure it relative to different bases. This approach uses the country bond as a base, whereas the previous one uses the standard deviation in the U.S. market. This approach assumes that investors are more likely to choose between Brazilian bonds and Brazilian equity, whereas the previous one approach assumes that the choice is across equity markets.

Choosing between the Approaches The three approaches to estimating country risk premiums will generally give you different estimates, with the bond default spread and relative equity standard deviation approaches yielding lower country risk premiums than the melded approach that uses both the country bond default spread and the equity standard deviation. We believe that the larger country risk premiums that emerge from the last approach are the most realistic for the immediate future, but that country risk premiums will decline over time. Just as companies mature and become less risky over time, countries can mature and become less risky as well.

One way to adjust country risk premiums over time is to begin with the premium that emerges from the melded approach and to adjust this premium down toward either the country bond default spread or the country premium estimated from equity standard deviations. Another way of presenting this argument is to note that the differences between standard deviations in equity and bond prices narrow over longer periods, and the resulting relative volatility will generally be smaller.[13] Thus, the equity risk premium will converge on the country bond spread as we look at longer-term expected returns. For example, the country risk premium for Brazil would be 9.69 percent for the next year but decline over time to either the 4.83 percent (country default spread) or 4.13 percent (relative standard deviation).

Estimating Asset Exposure to Country Risk Premiums Once country risk premiums have been estimated, the final question that has to be addressed relates to the exposure of individual companies within that country to country risk. There are three alternative views of country risk:

1. *Assume that all companies in a country are equally exposed to country risk.* Thus, for Brazil, with its estimated country risk premium of 9.69 percent, each

[13]Jeremy Siegel reports on the standard deviation in equity markets in his book *Stocks for the Very Long Run*, and notes that they tend to decrease with time horizon.

company in the market will have an additional country risk premium of 9.69 percent added to its expected returns. For instance, the cost of equity for Aracruz Celulose, a paper and pulp manufacturer listed in Brazil with a beta of 0.72, in U.S. dollar terms would be (assuming a U.S. Treasury bond rate of 5 percent and a mature market (U.S.) risk premium of 5.51 percent):

$$\text{Expected cost of equity} = 5.00\% + 0.72(5.51\%) + 9.69\% = 18.66\%$$

Note that the risk-free rate used is the U.S. Treasury bond rate and that the 5.51 percent is the equity risk premium for a mature equity market (estimated from historical data in the U.S. market). The biggest limitation of this approach is that it assumes that all firms in a country, no matter what their business or size, are equally exposed to country risk. To convert this dollar cost of equity into a cost of equity in the local currency, all that we need to do is to scale the estimate by relative inflation. To illustrate, if the Brazilian inflation rate is 10 percent and the U.S. inflation rate is 3 percent, the cost of equity for Aracruz in Brazilian real (BR) terms can be written as:

$$\text{Expected cost of equity}_{BR} = 1.1866(1.10/1.03) - 1 = 0.2672 \text{ or } 26.72\%$$

This will ensure consistency across estimates and valuations in different currencies.

2. *Assume that a company's exposure to country risk is proportional to its exposure to all other market risk, which is measured by the beta.* For Aracruz, this would lead to a cost of equity estimate of:

$$\text{Expected cost of equity} = 5.00\% + 0.72(5.51\% + 9.69\%) = 15.94\%$$

This approach does differentiate between firms, but it assumes that betas that measure exposure to all other market risk measure exposure to country risk as well. Thus, low-beta companies are less exposed to country risk than high-beta companies.

3. *The most general, and our preferred approach, is to allow for each company to have an exposure to country risk that is different from its exposure to all other market risk.* Measuring this exposure with λ, the cost of equity for any firm is estimated as follows:

$$\begin{aligned}\text{Expected return} = R_f &+ \text{Beta(Mature equity risk premium)}\\ &+ \lambda\text{(Country risk premium)}\end{aligned}$$

How can we best estimate λ? This question is considered in far more detail in the next chapter, but we would argue that commodity companies that get most of their revenues in U.S. dollars[14] by selling into a global market should be less exposed than manufacturing companies that service the local market. Using this rationale, Aracruz, which derives 80 percent or more of its revenues in the global paper market in U.S. dollars, should be less exposed than the typical Brazilian firm to country

[14]While I have categorized the revenues into dollar revenues and revenues in dollars, the analysis can be generalized to look at revenues in stable currencies (e.g., the dollar, euro, etc.) and revenues in risky currencies.

risk.[15] Using a λ of 0.25, for instance, we get a cost of equity in U.S. dollar terms for Aracruz of:

$$\text{Expected return} = 5\% + 0.72(5.51\%) + 0.25(9.69\%) = 11.39\%$$

Note that the third approach essentially converts our expected return model to a two-factor model, with the second factor being country risk, with λ measuring exposure to country risk. This approach also seems to offer the most promise in analyzing companies with exposures in multiple countries like Coca-Cola and Nestlé. While these firms are ostensibly developed market companies, they have substantial exposure to risk in emerging markets, and their costs of equity should reflect this exposure. We could estimate the country risk premiums for each country in which they operate and a λ relative to each country, and use these to estimate a cost of equity for either company.

***ctryprem.xls:* There is a dataset on the Web that contains the updated ratings for countries and the risk premiums associated with each.**

Alternative Approach: Implied Equity Premiums

There is an alternative to estimating risk premiums that does not require historical data or corrections for country risk, but does assume that the market, overall, is correctly priced. Consider, for instance, a very simple valuation model for stocks:

$$\text{Value} = \frac{\text{Expected dividends next period}}{(\text{Required return on equity} - \text{Expected growth rate})}$$

This is essentially the present value of dividends growing at a constant rate. Three of the four inputs in this model can be obtained externally—the current level of the market (value), the expected dividends next period, and the expected growth rate in earnings and dividends in the long term. The only unknown is then the required return on equity; when we solve for it, we get an implied expected return on stocks. Subtracting the risk-free rate will yield an implied equity risk premium.

To illustrate, assume that the current level of the S&P 500 index is 900, the expected dividend yield on the index is 2 percent, and the expected growth rate in earnings and dividends in the long term is 7 percent. Solving for the required return on equity yields the following:

$$900 = (.02 \times 900)/(r - .07)$$

Solving for r,

$$r = (18 + 63)/900 = 9\%$$

[15] λ_{Aracruz} = % from local market$_{\text{Aracruz}}$/% from local market$_{\text{average Brazilian firm}}$ = 0.20/0.80 = 0.25.

If the current risk-free rate is 6 percent, this will yield a premium of 3 percent.

This approach can be generalized to allow for high growth for a period, and extended to cover cash flow–based, rather than dividend, models. To illustrate this, consider the S&P 500 index as of December 31, 1999. The index was at 1,469, and the dividend yield on the index was roughly 1.68 percent. In addition, the consensus estimate[16] of growth in earnings for companies in the index was approximately 10 percent for the next five years. Since this is not a growth rate that can be sustained forever, we employ a two-stage valuation model, where we allow growth to continue at 10 percent for five years, and then lower the growth rate to the treasury bond rate of 6.5 percent after that.[17] The following table summarizes the expected cash flows for the next five years of high growth and the first year of stable growth thereafter:

Year	*Cash Flow on Index*
1	27.23
2	29.95
3	32.94
4	36.24
5	39.86
6	42.45

Cash flow in the first year = (Dividend yield)(Index)(1 + g) = (.0168)(1,469)(1.10).

If we assume that these are reasonable estimates of the cash flows and that the index is correctly priced, then:

$$\begin{aligned}\text{Level of the index} = 1{,}469 &= 27.23/(1+r) + 29.95/(1+r)^2 + 32.94/(1+r)^3 \\ &\quad + 36.24/(1+r)^4 + [39.86 + 42.45/(r - .065)]/(1+r)^5\end{aligned}$$

Note that the last term in the equation is the terminal value of the index, based on the stable growth rate of 6.5 percent, discounted back to the present. Solving for r in this equation yields us the required return on equity of 8.6 percent. The Treasury bond rate on December 31, 1999, was approximately 6.5 percent, yielding an implied equity premium of 2.10 percent.

The advantage of this approach is that it is market-driven and current, and does not require any historical data. Thus, it can be used to estimate implied equity premiums in any market. It is, however, bounded by whether the model used for the valuation is the right one and the availability and reliability of the inputs to that model. For instance, the equity risk premium for the Argentine market on September 30, 1998, was estimated from the following inputs. The index (Merval) was at 687.50, and the current dividend yield on the index was 5.6 percent. Earnings in companies in the index are expected to grow 11 percent (in U.S. dollar terms) over

[16]We used the average of the analyst estimates for individual firms (bottom-up). Alternatively, we could have used the top-down estimate for the S&P 500 earnings (from economists).

[17]The Treasury bond rate is the sum of expected inflation and the expected real rate. If we assume that real growth is equal to the real rate, the long-term stable growth rate should be equal to the Treasury bond rate.

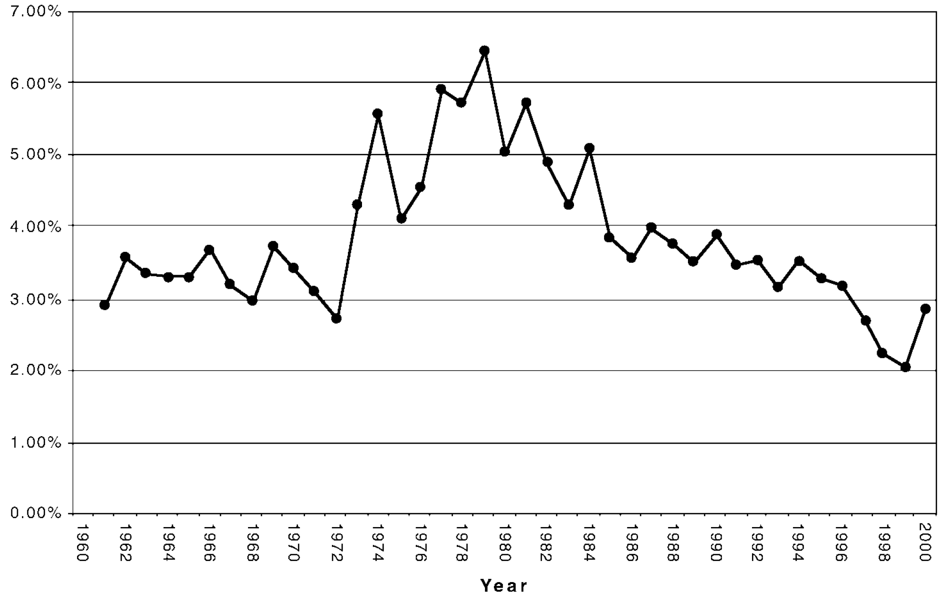

FIGURE 7.1 Implied Premium for U.S. Equity Market

the next five years, and 6 percent thereafter. These inputs yield a required return on equity of 10.59 percent, which when compared to the U.S. Treasury bond rate of 5.14 percent on that day results in an implied equity premium of 5.45 percent. For simplicity, we have used nominal dollar expected growth rates[18] and Treasury bond rates, but this analysis could have been done entirely in the local currency.

The implied equity premiums change over time as stock prices, earnings, and interest rates change. In fact, the contrast between these premiums and the historical premiums is best illustrated by graphing out the implied premiums in the S&P 500 going back to 1960 in Figure 7.1. In terms of mechanics, smoothed historical growth rates in earnings and dividends were used as projected growth rates, and a two-stage dividend discount model was used. Looking at these numbers, the following conclusions would be drawn:

- The implied equity premium has seldom been as high as the historical risk premium. Even in 1978, when the implied equity premium peaked, the estimate of 6.5 percent was well below what many practitioners use as the risk premium in their risk and return models. In fact, the average implied equity risk premium has been about 4 percent over the past 40 years. This is probably because of the survivor bias that pushes up historical risk premiums.
- The implied equity premium did increase during the 1970s as inflation increased. This does have interesting implications for risk premium estimation.

[18]The input that is most difficult to estimate for emerging markets is a long-term expected growth rate. For Argentine stocks, I used the average consensus estimate of growth in earnings for the largest Argentine companies that have American depository receipts (ADRs) listed on them. This estimate may be biased as a consequence.

Instead of assuming that the risk premium is a constant and unaffected by the level of inflation and interest rates, which is what we do with historical risk premiums, it may be more realistic to increase the risk premium as expected inflation and interest rates increase. In fact, an interesting avenue of research would be to estimate the fundamentals that determine implied risk premiums.

- Finally, the risk premium has been on a downward trend since the early 1980s, and the risk premium at the end of 1999 was a historical low. Part of the decline can be attributed to a decline in inflation uncertainty and lower interest rates, and part of it, arguably, may reflect other changes in investor risk aversion and characteristics over the period. There is, however, the very real possibility that the risk premium is low because investors had overpriced equity. In fact, the market correction in 2000 pushed the implied equity risk premium up to 2.87 percent by the end of 2000.

As a final point, there is a strong tendency toward mean reversion in financial markets. Given this tendency, it is possible that we can end up with a far better estimate of the implied equity premium by looking at not just the current premium but also at historical data. There are two ways in which we can do this:

- We can use the average implied equity premium over longer periods, say 10 to 15 years. Note that we do not need as many years of data here as we did with the historical premium estimate, because the standard errors tend to be smaller.
- A more rigorous approach would require relating implied equity risk premiums to fundamental macroeconomic data over the period. For instance, given that implied equity premiums tend to be higher during periods with higher inflation rates (and interest rates), we ran a regression of implied equity premiums against Treasury bond rates and a term structure variable between 1960 and 2000:

$$\text{Implied equity premium} = 1.87\% + \underset{[5.94]}{0.2903(\text{T-bond rate})} - \underset{[1.10]}{0.1162(\text{T-bond} - \text{T-bill})}$$

The regression has significant explanatory power, with an R-squared of 49 percent, and the t statistics (in brackets under the coefficients) indicate the statistical significance of the independent variables used. Substituting the current Treasury bond rate and bond-bill spread into this equation should yield an updated estimate[19] of the implied equity premium.

histimpl.xls: **This dataset on the Web shows the inputs used to calculate the premium in each year for the U.S. market.**

[19]On June 30, 2001, for instance, I substituted in the Treasury bond rate of 5 percent and a spread of 1.0 percent between the T-bond and T-bill rate into the regression equation to get:

$$.0182 + 0.2903(.05) - 0.1162(.01) = .032 \text{ or } 3.20\%$$

HISTORICAL VERSUS IMPLIED EQUITY PREMIUMS: EFFECT OF MARKET VIEWS

As you can see from the preceding discussion, historical premiums can be very different from implied equity premiums. At the end of 2000, the historical risk premium for stocks over bonds in the United States was 5.51%, whereas the implied equity risk premium was 2.87%. When doing discounted cash flow valuation, you have to decide which risk premium you will use in the valuation, and your choice will be determined by both your market views and your valuation mission.

Market Views: If you believe that the *market is right in the aggregate*, though it may make mistakes on individual stocks, the risk premium you should use is the implied equity risk premium (2.87% at the end of 2000). If you believe that the market often makes mistakes in the aggregate and that *risk premiums in markets tend to move back to historical norms* (mean reversion), you should go with the historical premium (5.5% at the end of 2000). A way to split the difference is to assume that *markets are right across time*, though they may make mistakes at individual points in time. If you make this assumption, you should use an average implied equity risk premium over time. The average implied equity risk premium from 1960 to 2000 is 4%. While this book will use the historical premium a few times in our valuations, we will stick with the average implied premium of 4% in most of the valuations.

Valuation Mission: If your valuation requires you to market neutral, you should use the implied equity risk premium. This is often the case if you are an equity research analyst or if you have to value a company for an acquisition.

implprem.xls: **This spreadsheet allows you to estimate the implied equity premium in a market.**

DEFAULT SPREADS ON BONDS

The interest rates on bonds are determined by the default risk that investors perceive in the issuer of the bonds. This default risk is often measured with a bond rating and the interest rate that corresponds to the rating is estimated by adding a default spread to the riskless rate. In Chapter 4, we examined the process used by rating agencies to rate firms. This chapter considers how to estimate default spreads for a given ratings class and why these spreads may change over time.

Estimating Default Spreads

The simplest way to estimate default spreads for each ratings class is to find a sampling of bonds within that ratings class and obtain the current market interest rate on these bonds. Why do we need a sampling rather than just one bond? A bond can be misrated or mispriced. Using a sample reduces or eliminates this problem. In obtaining this sample, you should try to focus on the most liquid bonds with as few special features attached to them as possible. Corporate bonds are often illiquid and the interest rates on such bonds may not reflect current market rates. The presence of special features on bonds such as convertibility can affect the pricing of these bonds and consequently the interest rates estimated on them.

Once a sample of bonds within each ratings class has been identified, you need to estimate the interest rate on these bonds. There are two measures that are widely used. The first is the yield on the bond, which is the coupon rate divided by the market price. The second is the yield to maturity on the bond, which is the interest rate that makes the present value of the coupons and face value of the bond equal to the market price. In general, it is the yield to maturity that better measures the market interest rate on the bond.

Having obtained the interest rates on the bonds in the sample, you have two decisions to make. The first relates to weighting. You could compute a simple average of the interest rates of the bonds in the sample or a weighted average, with the weights based upon the trading volume—more liquid bonds will be weighted more than less liquid bonds. The second relates to the index Treasury rate, since the average interest rate for a ratings class is compared to this rate to arrive at a default spread. In general, the maturity of the Treasury should match the average maturity of the corporate bonds chosen to estimate the average interest rate. Thus, the average interest rate for five-year BBB-rated corporate bonds should be compared to the average interest rate for five-year Treasuries to derive the spread for the BBB-rated bonds.

While publications like *Barron's* have historically provided interest rates on at least higher-rated bonds (BBB or higher), an increasing number of online services provide the same information today for all rated bonds. Table 7.6 is extracted from one such online service in early 2001 for 10-year bonds.

TABLE 7.6 Default Spreads by Ratings Class—January 2001 (T-Bond Rate = 5%)

Rating	Spread	Interest Rate on Debt
AAA	0.75%	5.75%
AA	1.00%	6.00%
A+	1.50%	6.50%
A	1.80%	6.80%
A–	2.00%	7.00%
BBB	2.25%	7.25%
BB	3.50%	8.50%
B+	4.75%	9.75%
B	6.50%	11.50%
B–	8.00%	13.00%
CCC	10.00%	15.00%
CC	11.50%	16.50%
C	12.70%	17.70%
D	14.00%	19.00%

Source: bondsonline.com.

Determinants of Default Spreads

Table 7.6 provides default spreads at a point in time, but default spreads not only vary across time, but they also can vary for bonds with the same rating but different maturities. This section considers how default spreads vary across time and for bonds with varying maturities.

Default Spreads and Bond Maturity Empirically, the default spread for corporate bonds of a given ratings class seems to increase with the maturity of the bond. Figure 7.2 presents the default spreads estimated for a AAA-, BBB-, and CCC-rated bond for maturities ranging from 1 to 10 years in January 2001.

For every rating, the default spread seems to widen for the longer maturities, and it widens more for the lower-rated bonds. Why might this be? It is entirely possible that default risk is multiplied as we look at longer maturities. A bond investor buying a 10-year bond in a CCC-rated company may feel more exposed to default risk than a bondholder buying a higher-rated bond.

Default Spreads over Time The default spreads presented in Table 7.6, after a year of declining markets and a slowing economy, were significantly higher than the default spreads a year earlier. This phenomenon is not new. Historically, default spreads for every ratings class have increased during recessions and decreased during economic booms. Figure 7.3 graphs the spread between 10-year Moody's Baa-rated bonds and the 10-year Treasury bond rate each year from 1960 to 2000. The default spreads did increase during periods of low economic growth; note the increase during 1973–1974

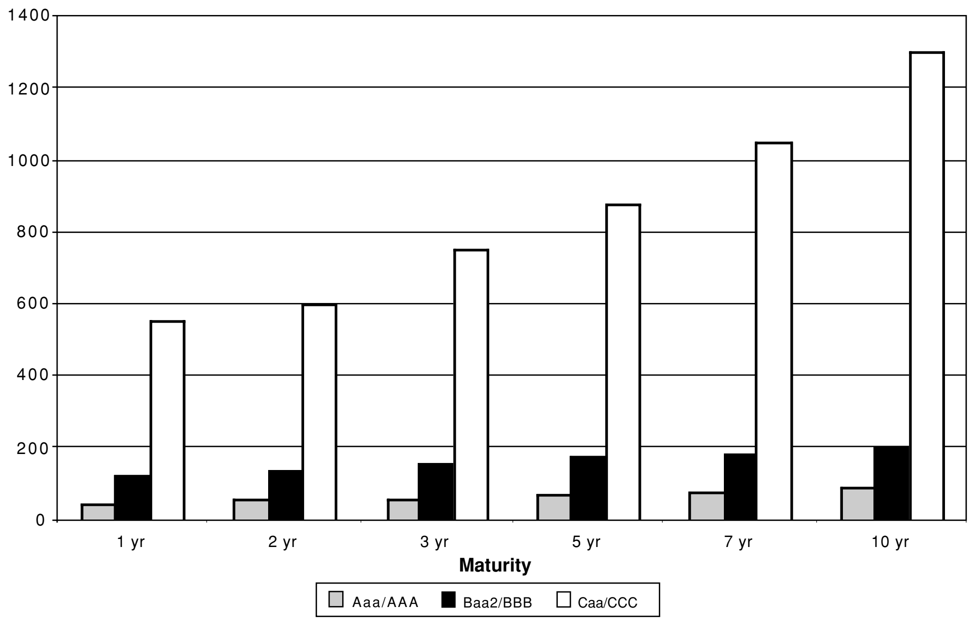

FIGURE 7.2 Default Spreads by Maturity—January 2001
Source: bondsonline.com.

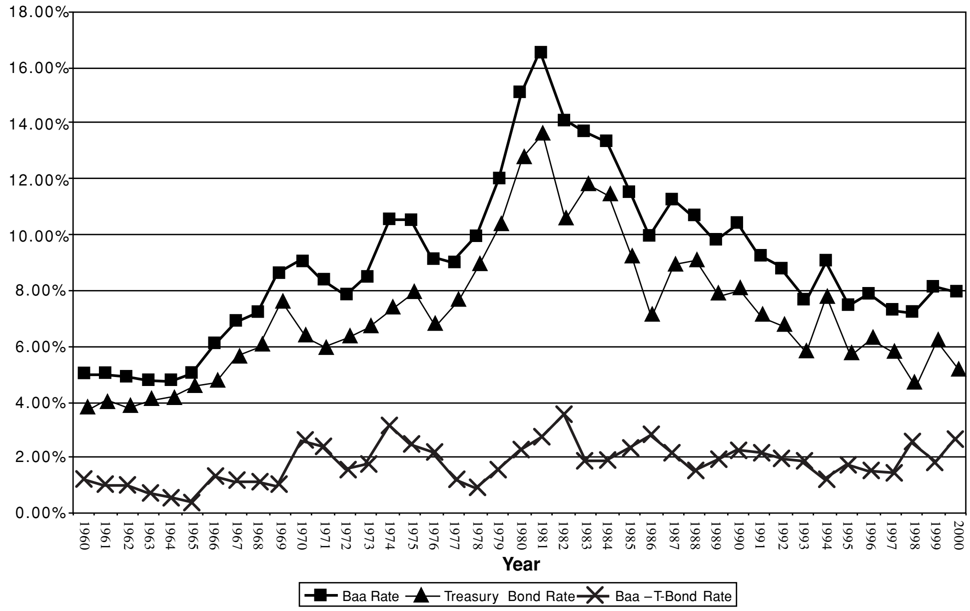

FIGURE 7.3 Default Spread—Baa versus Treasury Bond Rates from 1960 to 2000
Source: Federal Reserve.

and 1979–1981 in particular. In fact, a regression of default spreads each year against real economic growth that year bears out this conclusion:

$$\text{Default spread}_{\text{BBB-Treasury}} = 0.47 - \underset{[2.59]}{0.04}\ \text{GNP growth}_{\text{real}} \qquad R^2 = 41\%$$

After years of high real growth, default spreads tend to shrink.

The practical implication of this phenomenon is that default spreads for bonds have to be reestimated at regular intervals, especially if the economy shifts from low to high growth or vice versa.

***ratings.xls:* This dataset on the Web summarizes default spreads by bond rating class for the most recent period.**

CONCLUSION

The risk-free rate is the starting point for all expected return models. For an asset to be risk free, it has to be free of both default and reinvestment risk. Using these criteria, the appropriate risk-free rate to use to obtain expected returns should be a default-free (government) zero coupon rate that is matched up to when the cash flow that is being discounted occurs. In practice, however, it is usually appropriate to match up the duration of the risk-free asset to the duration of the cash flows being analyzed. In valuation, this will lead us toward long-term government bond rates as risk-free rates. It is also important that the risk-free rate be consistent with the cash flows being discounted. In particular, the currency in which the risk-free rate is denominated and whether it is a real or nominal risk-free rate should be determined by the currency in which the cash flows are estimated and whether the estimation is done in real or nominal terms.

The risk premium is a fundamental and critical component in portfolio management, corporate finance, and valuation. Given its importance, it is surprising that more attention has not been paid in practical terms to estimation issues. This chapter considered the conventional approach to estimating risk premiums, which is to use historical returns on equity and government securities, and evaluated some of its weaknesses. It also examined how to extend this approach to emerging markets, where historical data tends to be both limited and volatile. The alternative to historical premiums is to estimate the equity premium implied by equity prices. This approach does require that we start with a valuation model for equities, and estimate the expected growth and cash flows, collectively, on equity investments. It has the advantage of not requiring historical data and reflecting current market perceptions.

QUESTIONS AND SHORT PROBLEMS

1. Assume that you are valuing an Indonesian firm in U.S. dollars. What would you use as the riskless rate?
2. Explain why a six-month Treasury bill rate is not an appropriate riskless rate in discounting a five-year cash flow.
3. You have been asked to estimate a riskless rate in Indonesian rupiah. The Indonesian government has rupiah-denominated bonds outstanding, with an interest rate of 17%. S&P has a rating of BB on these bonds, and the typical spread for a BB-rated country is 5% over a riskless rate. Estimate the rupiah riskless rate.
4. You are valuing an Indian company in rupees. The current exchange rate is Rs 45 per dollar and you have been able to obtain a 10-year forward rate of Rs 70 per dollar. If the U.S. Treasury bond rate is 5%, estimate the riskless rate in Indian rupees.
5. You are attempting to do a valuation of a Chilean company in real terms. While you have been unable to get a real riskless rate in Latin America, you know that inflation-indexed Treasury bonds in the United States are yielding 3%. Could you use this as a real riskless rate? Why or why not? What are the alternatives?
6. Assume you have estimated the historical risk premium, based on 50 years of data, to be 6%. If the annual standard deviation in stock prices is 30%, estimate the standard error in the risk premium estimate.
7. When you use a historical risk premium as your expected future risk premium, what are the assumptions that you are making about investors and markets? Under what conditions would a historical risk premium give you too high a number (to use as an expected premium)?
8. You are trying to estimate a country equity risk premium for Poland. You find that S&P has assigned an A rating to Poland and that Poland has issued euro-denominated bonds that yield 7.6% in the market currently. (Germany, a AAA-rated country, has euro-denominated bonds outstanding that yield 5.1%.)
   a. Estimate the country risk premium, using the default spread on the country bond as the proxy.

b. If you were told that the standard deviation in the Polish equity market was 25% and that the standard deviation in the Polish euro bond was 15%, estimate the country risk premium.

9. The standard deviation in the Mexican Equity Index is 48%, and the standard deviation in the S&P 500 is 20%. You use an equity risk premium of 5.5% for the United States.
   a. Estimate the country equity risk premium for Mexico using relative equity standard deviations.
   b. Now assume that you are told that Mexico is rated BBB by Standard & Poor's and that it has dollar-denominated bonds outstanding that trade at a spread of about 3% above the Treasury bond rate. If the standard deviation in these bonds is 24%, estimate the country risk premium for Mexico.
10. The S&P 500 is at 1,400. The expected dividends and cash flows next year on the stocks in the index are expected to be 5% of the index. If the expected growth rate in dividends and cash flows over the long term are expected to be 6% and the riskless rate is 5.5%, estimate the implied equity risk premium.
11. The Bovespa (Brazilian equity index) is at 15,000. The dividends on the index last year were 5% of the index value, and analysts expect them to grow 15% a year in real terms for the next five years. After the fifth year, the growth is expected to drop to 5% in real terms in perpetuity. If the real riskless rate is 6%, estimate the implied equity risk premium in this market.
12. As stock prices go up, implied equity risk premiums will go down. Is this statement always true? If not, when is it not true?

CHAPTER 8

Estimating Risk Parameters and Costs of Financing

The preceding chapter laid the groundwork for estimating the costs of equity and capital for firms by looking at how best to estimate a riskless rate that operates as a base for all costs, an equity risk premium for estimating the cost of equity, and default spreads for estimating the cost of debt. It did not, however, consider how to estimate the risk parameters for individual firms. This chapter examines the process of estimating risk parameters for individual firms, for estimating both the cost of equity and the cost of debt.

For the cost of equity, we look at the standard process of estimating the beta for a firm and consider alternative approaches. For the cost of debt, we examine bond ratings as measures of default risk and the determinants of these ratings.

The chapter closes by bringing together the risk parameter estimates for individual firms and the economy-wide estimates of the risk-free rate and risk premiums to estimate a cost of capital for the firm. To do this, the sources of capital have to be weighted by their relative market values.

THE COST OF EQUITY AND CAPITAL

Firms raise money from both equity investors and lenders to fund investments. Both groups of investors make their investments expecting to make a return. Chapter 4 argued that the expected return for equity investors would include a premium for the equity risk in the investment. We label this expected return the cost of equity. Similarly, the expected return that lenders hope to make on their investments includes a premium for default risk, and we call that expected return the cost of debt. If we consider all of the financing that the firm takes on, the composite cost of financing will be a weighted average of the costs of equity and debt, and this weighted cost is the cost of capital.

The chapter begins by estimating the equity risk in a firm and using the equity risk to estimate the cost of equity, and follows up by measuring the default risk to estimate a cost of debt. It will conclude by determining the weights we should attach to each of these costs to arrive at a cost of capital.

COST OF EQUITY

The cost of equity is the rate of return investors require on an equity investment in a firm. The risk and return models described in Chapter 4 need a riskless rate and a risk premium (in the CAPM) or premiums (in the APM and multifactor models), which were estimated in the last chapter. They also need measures of a firm's exposure to market risk in the form of betas. These inputs are used to arrive at an expected return on an equity investment:

$$\text{Expected return} = \text{Riskless rate} + \text{Beta(Risk premium)}$$

This expected return to equity investors includes compensation for the market risk in the investment and is the cost of equity. This section will concentrate on the estimation of the beta of a firm. While much of the discussion is directed at the CAPM, it can be extended to apply to the arbitrage pricing and multifactor models, as well.

Betas

In the CAPM, the beta of an investment is the risk that the investment adds to a market portfolio. In the APM and multifactor model, the betas of the investment relative to each factor have to be measured. There are three approaches available for estimating these parameters: One is to use historical data on market prices for individual investments; the second is to estimate the betas from the fundamental characteristics of the investment; and the third is to use accounting data. All three approaches are described in this section.

Historical Market Betas The conventional approach for estimating the beta of an investment is a regression of returns on the investment against returns on a market index. For firms that have been publicly traded for a length of time, it is relatively straightforward to estimate returns that an investor would have made on investing in the firms' equity in intervals (such as a week or a month) over that period. In theory, these stock returns on the assets should be related to returns on a market portfolio (i.e., a portfolio that includes all traded assets) to estimate the betas of the assets. In practice, we tend to use a stock index such as the S&P 500 as a proxy for the market portfolio, and we estimate betas for stocks against the index.

Regression Estimates of Betas The standard procedure for estimating betas is to regress stock returns (R_j) against market returns (R_m):

$$R_j = a + b\, R_m$$

where a = Intercept from the regression
b = Slope of the regression = $\text{Covariance}(R_j, R_m)/\sigma^2_m$

The slope of the regression corresponds to the beta of the stock and measures the riskiness of the stock.

The intercept of the regression provides a simple measure of performance of the investment during the period of the regression, when returns are measured against the expected returns from the capital asset pricing model. To see why, consider the following rearrangement of the capital asset pricing model:

$$R_j = R_f + \beta(R_m - R_f)$$
$$= R_f(1 - \beta) + \beta\, R_m$$

Compare this formulation of the return on an investment to the return equation from the regression:

$$R_j = a + b\, R_m$$

Thus, a comparison of the intercept a to $R_f(1 - \beta)$ should provide a measure of the stock's performance, at least relative to the capital asset pricing model.[1] In summary, then:

If $a > R_f(1 - \beta)$ . . . Stock did better than expected during regression period.
$a = R_f(1 - \beta)$ . . . Stock did as well as expected during regression period.
$a < R_f(1 - \beta)$ . . . Stock did worse than expected during regression period.

The difference between a and $R_f(1 - \beta)$ is called Jensen's alpha[2] and provides a measure of whether the investment in question earned a return greater than or less than its required return, given both market performance and risk. For instance, a firm that earned 15 percent during a period when firms with similar betas earned 12 percent, will have earned an excess return of 3 percent; its intercept will also exceed $R_f(1 - \beta)$ by 3 percent.

The third statistic that emerges from the regression is the R-squared (R^2) of the regression. While the statistical explanation of the R-squared is that it provides a measure of the goodness of fit of the regression, the economic rationale is that it provides an estimate of the proportion of the risk of a firm that can be attributed to market risk; the balance $(1 - R^2)$ can then be attributed to firm-specific risk.

The final statistic worth noting is the standard error of the beta estimate. The slope of the regression, like any statistical estimate, may be different from the true value, and the standard error reveals just how much error there could be in the estimate. The standard error can also be used to arrive at confidence intervals for the "true" beta value from the slope estimate.

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[1]The regression is sometimes calculated using returns in excess of the riskless rate for both the stock and the market. In that case, the intercept of the regression should be zero if the actual returns equal the expected returns from the CAPM, greater than zero if the stock does better than expected, and less than zero if it does worse than expected.

[2]The terminology is confusing, since the intercept of the regression is sometimes also called the alpha and is sometimes compared to zero as a measure of risk-adjusted performance. The intercept can be compared to zero only if the regression is run with excess returns for both the stock and the index; the riskless rate has to be subtracted from the raw return in each month for both.

ILLUSTRATION 8.1: Estimating a Regression Beta for Boeing

Boeing Company is a dominant firm in both the aerospace and defense businesses, and has been traded on the New York Stock Exchange (NYSE) for decades. In assessing risk parameters for Boeing, we compute the returns on the stock and the market index in two steps:

1. The returns to a stockholder in Boeing are computed month by month from January 1996 to December 2000. These returns include both dividends and price appreciation and are defined as follows:

$$\text{Stock return}_{\text{Boeing, j}} = (\text{Price}_{\text{Boeing, j}} - \text{Price}_{\text{Boeing, j}-1} + \text{Dividends}_j)/\text{Price}_{\text{Boeing, j}-1}$$

where $\text{Stock return}_{\text{Boeing,j}}$ = Returns to a stockholder in Boeing in month j
$\text{Price}_{\text{Boeing, j}}$ = Price of Boeing stock at the end of month j
Dividends_j = Dividends on Boeing stock in month j

Dividends are added to the returns of the month in which stockholders are entitled to the dividend.[3]

2. The returns on the S&P 500 market index are computed for each month of the period, using the level of the index at the end of each month and the monthly dividend on stocks in the index.

$$\text{Market return}_j = (\text{Index}_j - \text{Index}_{j-1} + \text{Dividends}_j)/\text{Index}_{j-1}$$

where Index_j is the level of the index at the end of month j and Dividends_j is the dividends paid on the index in month j. While the S&P 500 and the NYSE Composite are the most widely used indexes for U.S. stocks, they are, at best, imperfect proxies for the market portfolio in the CAPM, which is supposed to include all assets.

Figure 8.1 graphs monthly returns on Boeing against returns on the S&P 500 index from January 1996 to December 2000.

The regression statistics for Boeing are as follows:

(a) Slope of the regression = 0.56. This is Boeing's beta, based on monthly returns from 1996 to 2000. Using a different time period for the regression or different return intervals (weekly or daily) for the same period can result in a different beta.

(b) Intercept of the regression = 0.54%. This is a measure of Boeing's performance, when it is compared with $R_f(1 - \beta)$. The monthly riskless rate (since the returns used in the regression are monthly returns) between 1996 and 2000 averaged 0.4%, resulting in the following estimate for the performance:

$$R_f(1 - \beta) = 0.4\%(1 - 0.56) = 0.18\%$$
$$\text{Intercept} - R_f(1 - \beta) = 0.54\% - 0.18\% = 0.36\%$$

This analysis suggests that Boeing performed 0.36% better than expected, when expectations are based on the CAPM, on a monthly basis between January 1996 and December 2000. This results in an annualized excess return of approximately 4.41%.

$$\text{Annualized excess return} = (1 + \text{Monthly excess return})^{12} - 1$$
$$= (1 - .0036)^{12} - 1 = 4.41\%$$

Note, however, that this does not imply that Boeing would be a good investment in the future.

[3]The stock has to be bought by a day called the ex-dividend day in order for investors to be entitled to dividends. The returns in a period include dividends if the ex-dividend day is in that period.

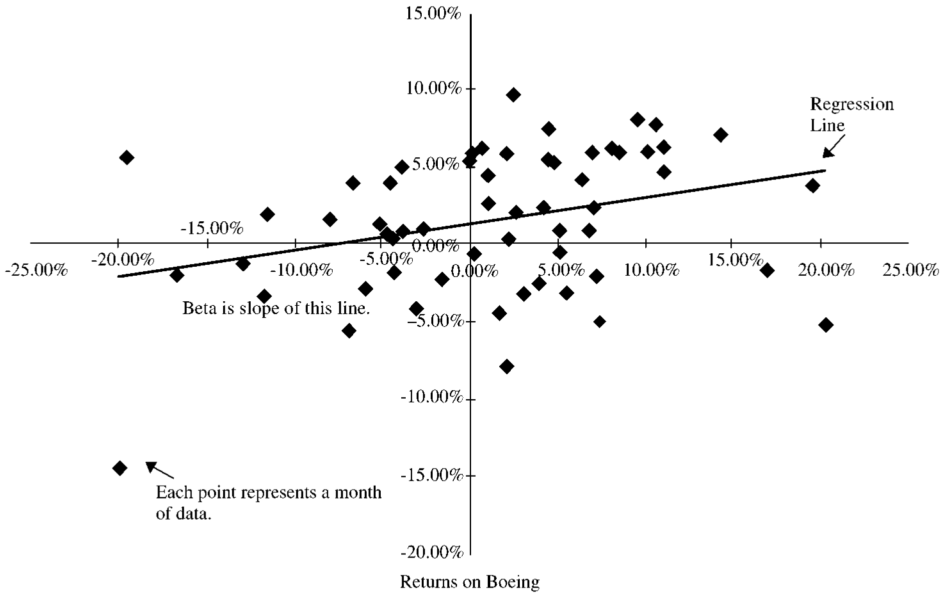

FIGURE 8.1 Boeing versus S&P 500 from 1996 to 2000

The performance measure also does not provide a breakdown of how much of this excess return can be attributed to the performance of the entire sector (aerospace and defense) and how much is specific to the firm. To make that breakdown, we would need to compute the excess over the same period for other firms in the aerospace and defense industry and compare them with Boeing's excess return. The difference would then be attributable to firm-specific actions. In this case, for instance, the average annualized excess return on other aerospace/defense firms between 1996 and 2000 was –0.85%, suggesting that the firm-specific component of performance for Boeing is actually 5.26% [firm-specific Jensen's alpha = 4.41% – (–0.85%)].

(c) R-squared of the regression = 9.43%. This statistic suggests that 9.43% of the risk (variance) in Boeing comes from market sources, and that the balance of 90.57% of the risk comes from firm-specific components. The latter risk should be diversifiable and therefore will not be rewarded with a higher expected return. Boeing's R-squared is lower than the median R-squared of companies listed on the New York Stock Exchange, which was approximately 19% in 2000.

(d) Standard Error of Beta Estimate = 0.23. This statistic implies that the true beta for Boeing could range from 0.33 to 0.79 (subtracting and adding one standard error to beta estimate of 0.56) with 67% confidence and from 0.10 to 1.02 (subtracting and adding two standard errors to beta estimate of 0.56) with 95% confidence. While these ranges may seem large, they are not unusual for most U.S. companies. This suggests that we should consider estimates of betas from regressions with caution.

Using a Service Beta Most of us who use betas obtain them from an estimation service; Merrill Lynch, Barra, Value Line, Standard & Poor's, Morningstar, and Bloomberg are some of the well-known services. All these services begin with the regression beta just described and adjust them to reflect what they feel are better estimates of future risk. Although many of these services do not reveal their estimation procedures, Bloomberg is an exception. Figure 8.2 is the beta calculation page

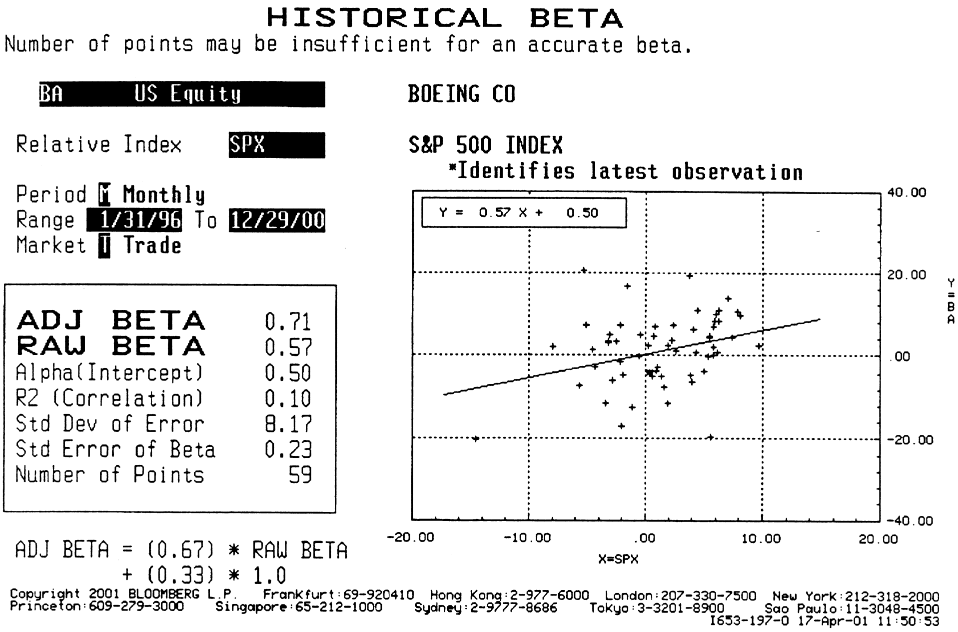

FIGURE 8.2 Beta Estimate for Boeing

from Bloomberg for Boeing, using the same period as our regression (January 1996 to December 2000).

While the time period used is identical to the one used in our earlier regression, there are subtle differences between this regression and the one in Figure 8.1. First, Bloomberg uses price appreciation in the stock and the market index in estimating betas and ignores dividends.[4] The fact that dividends are ignored does not make much difference for a company like Boeing, but it could make a difference for a company that either pays no dividends or pays significantly higher dividends than the market. This explains the mild differences in the intercept (.50% versus .54%) and the beta (.57 versus .56).

Second, Bloomberg also computes what it calls an adjusted beta, which is estimated as follows:

$$\text{Adjusted beta} = \text{Raw beta}(0.67) + 1.00(0.33)$$

These weights (0.67 and 0.33) do not vary across stocks, and this process pushes all estimated betas toward 1. Most services employ similar procedures to adjust betas toward 1. In doing so, they are drawing on empirical evidence that suggests that the betas for most companies, over time, tend to move toward the average beta, which is 1. This may be explained by the fact that firms get more diversified in their product mix and client base as they get larger. While we agree with the notion that betas move toward 1 over time, the weighting process used by most services strikes us as arbitrary and not particularly useful.

[4]This is done purely for computational convenience.

Estimation Choices for Beta Estimation There are three decisions that must be made in setting up the regression described earlier. The first concerns the length of the estimation period. Most estimates of betas, including those by Value Line and Standard & Poor's, use five years of data, while Bloomberg uses two years of data. The trade-off is simple: A longer estimation period provides more data, but the firm itself might have changed in its risk characteristics over the time period. Boeing, during the period of our analysis, acquired both Rockwell and McDonnell Douglas, changing its business mix and its basic risk characteristics.

The second estimation issue relates to the return interval. Returns on stocks are available on an annual, a monthly, a weekly, a daily, and even an intraday basis. Using daily or intraday returns increases the number of observations in the regression, but it exposes the estimation process to a significant bias in beta estimates related to nontrading.[5] For instance, the betas estimated for small firms, which are more likely to suffer from nontrading, are biased downward when daily returns are used. Using weekly or monthly returns can reduce the nontrading bias significantly.[6] In this case, using weekly returns for two years yields a beta estimate for Boeing of only 0.88, while the monthly beta estimate is 0.96.

The third estimation issue relates to the choice of a market index to be used in the regression. The standard practice used by most beta estimation services is to estimate the betas of a company relative to the index of the market in which its stock trades. Thus, the betas of German stocks are estimated relative to the Frankfurt DAX, British stocks relative to the FTSE, Japanese stocks relative to the Nikkei, and U.S. stocks relative to the NYSE Composite or the S&P 500. While this practice may yield an estimate that is a reasonable measure of risk for the domestic investor, it may not be the best approach for an international or cross-border investor, who would be better served with a beta estimated relative to an international index. For instance, Boeing's beta between 1996 and 2000 estimated relative to the Morgan Stanley Capital International (MSCI) index that is composed of stocks from different global markets yields a beta of 0.82.

To the extent that different services use different estimation periods, use different market indexes, and adjust the regression beta differently, they will often provide different beta estimates for the same firm at the same point in time. While these beta differences are troubling, note that the beta estimate delivered by each of these services comes with a standard error, and it is very likely that all the betas reported for a firm fall within the range of standard errors from the regressions.

Historical Beta Estimation for Companies in Smaller (or Emerging) Markets The process for estimating betas in markets with fewer stocks listed on them is no different from the process described earlier, but the estimation choices on return intervals, the market index, and the return period can make a much bigger difference in the estimate.

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[5]The nontrading bias arises because the returns in nontrading periods are zero (even though the market may have moved up or down significantly in those periods). Using these nontrading period returns in the regression will reduce the correlation between stock returns and market returns and the beta of the stock.

[6]The bias can also be reduced using statistical techniques suggested by Dimson and Scholes-Williams.

INDEX DOMINATION AND BETA ESTIMATES

There are a number of indexes that are dominated by one or a few stocks. One of the most striking cases was the Helsinki Stock Exchange (HEX) in the late 1990s. Nokia, the telecommunications giant, represented 75 percent of the Helsinki Index in terms of market value. Not surprisingly, a regression of Nokia against the HEX yielded the results shown in Figure 8.3.

The regression looks impeccable. In fact, the noise problem that we noted with Boeing, arising from the high standard errors, disappears. The beta estimate has a standard error of 0.03, but the results are deceptive. The low standard error is the result of a regression of Nokia on itself, since it dominates the index. The beta is meaningless to a typical investor in Nokia, who is likely to be diversified, if not globally, at least across European stocks. Worse still, the betas of all other Finnish stocks against the HEX become betas estimated against Nokia. In fact, the beta of every other Finnish stock at the time of this regression was less than 1. How is this possible, you might ask, if the average beta is 1? It is the weighted average beta that is 1, and if Nokia (which comprises three-quarters of the index) has a beta greater than 1 (which it does), every other stock in the index could well end up with a beta less than 1.

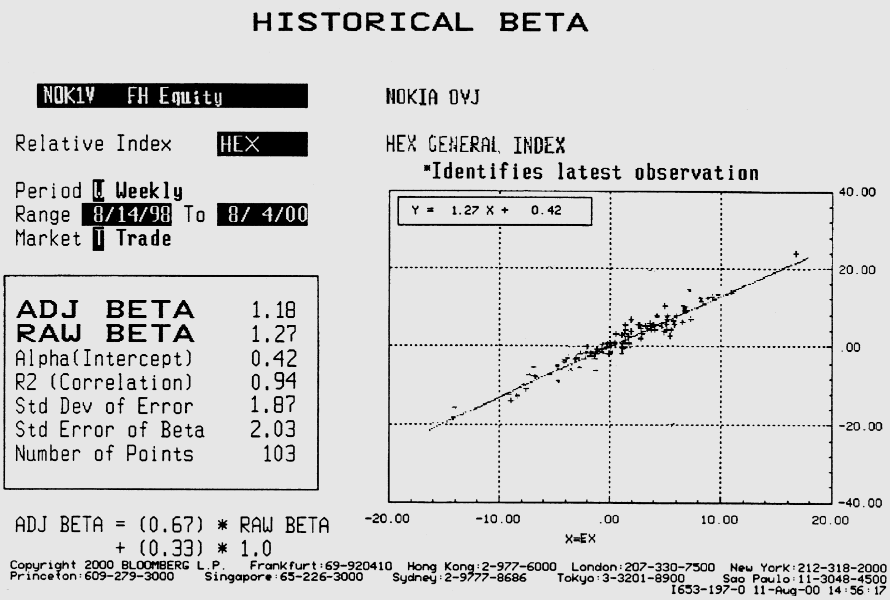

FIGURE 8.3 Beta Estimate for Nokia

- When liquidity is limited, as it often is in many stocks in emerging markets, the betas estimated using short return intervals tend to be much more biased. In fact, using daily or even weekly returns in these markets will tend to yield betas that are not good measures of the true market risk of the company.
- In many emerging markets, both the companies being analyzed and the market itself change significantly over short periods of time. Using five years of returns, as we did for Boeing, for a regression may yield a beta for a company (and market) that bears little resemblance to the company (and market) as it exists today.
- Finally, the indexes that measure market returns in many smaller markets tend to be dominated by a few large companies. For instance, the Bovespa (the Brazilian index) was dominated for several years by Telebras, which represented almost half the index. Nor is this just a problem with emerging markets. The DAX, the equity index for Germany, is dominated by Allianz, Deutsche Bank, Siemens, and Daimler. When an index is dominated by one or a few companies, the betas estimated against that index are unlikely to be true measures of market risk. In fact, the betas are likely to be close to 1 for the large companies that dominate the index and wildly variable for all other companies.

ILLUSTRATION 8.2: Estimating a Beta for Titan Cement Company

Titan Cement is a cement and construction company in Greece. Reproduced in Figure 8.4 is the beta estimate for Titan from April 1999 to April 2001 (using weekly returns) obtained from a beta service (Bloomberg). Note that the index used is the Athens Stock Exchange Index. Based on this regression, we arrive at the following equation:

$$\text{Returns}_{\text{Titan Cement}} = 0.31\% + 0.93 \ \text{Returns}_{\text{ASE}} \qquad \text{R-squared} = 57\%$$
$$[0.08]$$

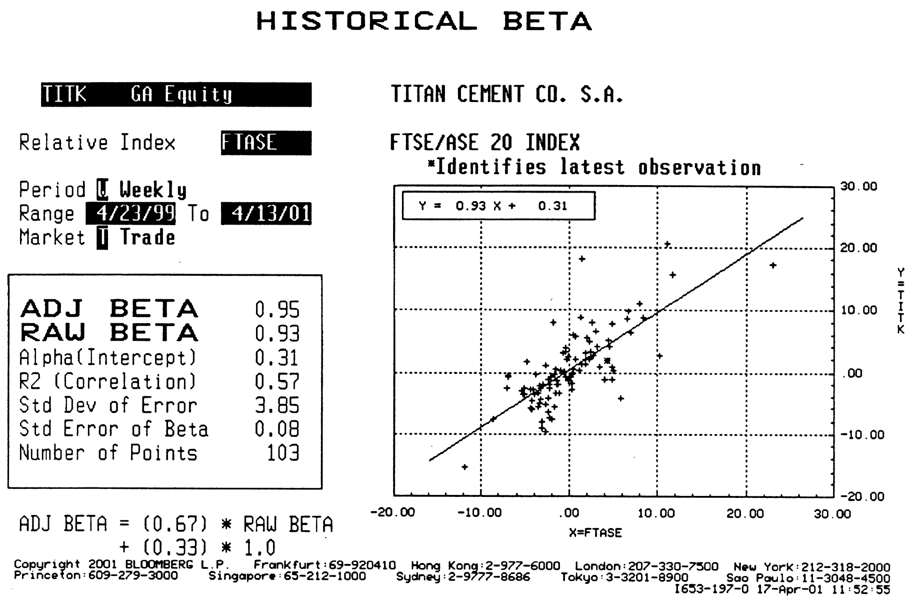

FIGURE 8.4 Beta Estimate for Titan Cement: Athens Stock Exchange Index

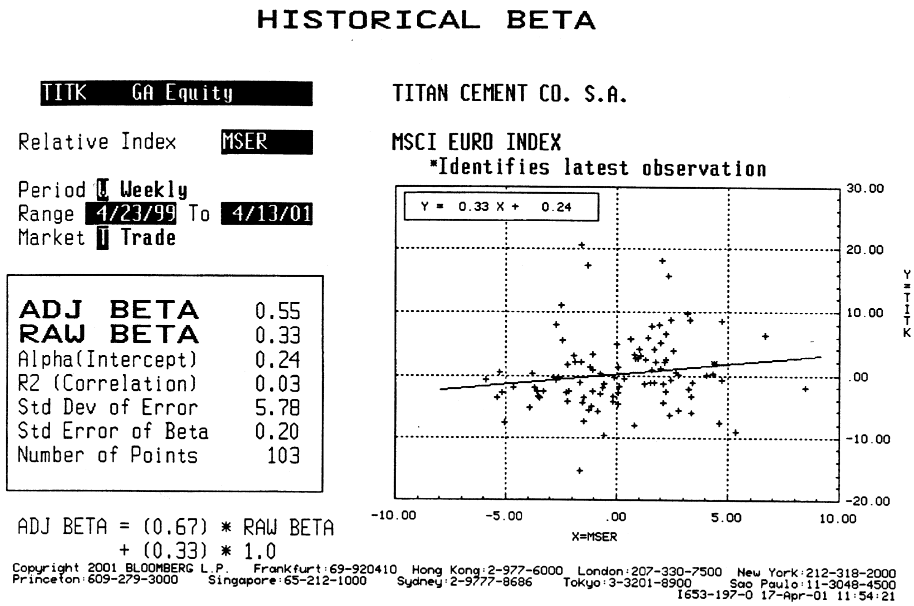

FIGURE 8.5 Beta Estimate for Titan Cement: MSCI European Index

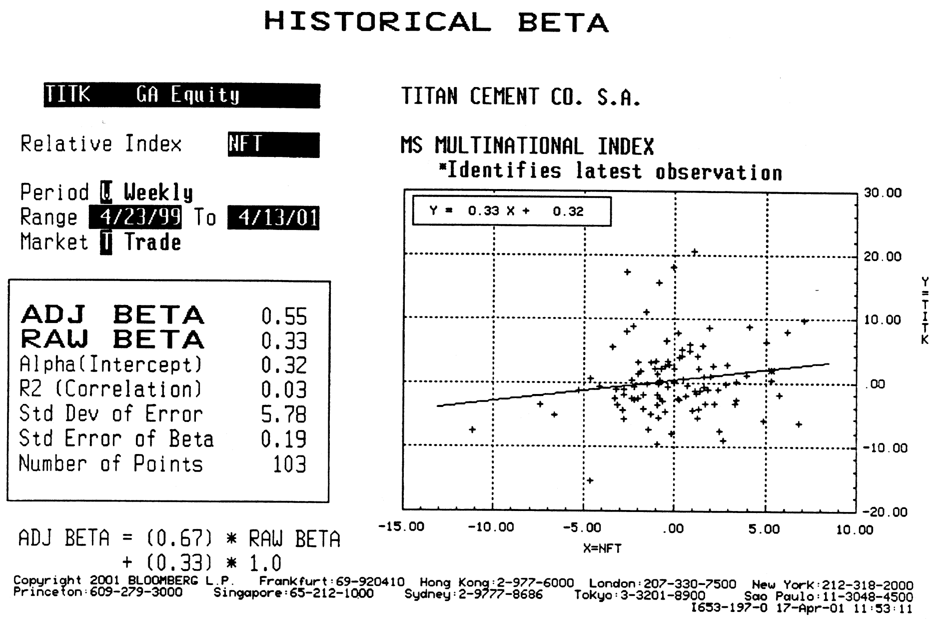

FIGURE 8.6 Beta Estimate for Titan Cement: MSCI Global Index

The beta for Titan Cement, based upon this regression, is 0.93. The standard error of the estimate, shown in brackets below, is only 0.08, but the caveats about narrow indexes applies to the Athens Stock Exchange Index.

Drawing on the arguments in the previous section, if the marginal investor in Titan Cement is, in fact, an investor diversified across European companies, the appropriate index would have been a European stock index. The Bloomberg beta calculation with the MSCI European index is reported in Figure 8.5. Note the decline in beta to 0.33 and the increase in the standard error of the beta estimate.

In fact, if the marginal investor is globally diversified, Titan Cement's beta (as well as Boeing's beta in Illustration 8.1) should have been estimated against a global index. Using the Morgan Stanley Capital International (MSCI) global index, we get a regression beta of 0.33 in Figure 8.6. In fact, the beta estimate and the standard error look very similar to the ones estimated against the European index.

Estimating the Historical Beta for Private Firms The historical approach to estimating betas works only for assets that have been traded and have market prices. Private companies do not have a market price history. Consequently, we cannot estimate a regression beta for these companies. Nevertheless, we still need estimates of cost of equity and capital for these companies.

You might argue that this is not an issue if you do not value private companies; but you will still be confronted with this issue even when valuing publicly traded firms. Consider, for instance, the following scenarios:

- If you have to value a private firm for an initial public offering, you will need to estimate discount rates for the valuation.
- Even after a firm has gone public, there will be a period of time lasting as long as two years when there will be insufficient data for a regression.
- If you are called upon to value the division of a publicly traded firm that is up for sale, you will not have past prices to draw on to run a regression.
- Finally, if your firm has gone through significant restructuring—divestitures or recapitalization—in the recent past, regression betas become meaningless because the company itself has changed its risk characteristics.

Thus regression betas are either unavailable or meaningless in a significant number of valuations.

Some analysts assume that discounted cash flow valuation is not feasible in these scenarios; instead they use multiples. Others make assumptions about discount rates based on rules of thumb. Neither approach is appealing. The next section develops an approach for estimating betas that is general enough to apply to all of these companies.

***risk.xls.* This spreadsheet allows you to run a regression of stock returns against market returns and estimate risk parameters.**

The Limitations of Regression Betas Much of what has been presented in this section represents an indictment of regression betas. In the case of Boeing, the biggest problem was that the beta had high standard error. In fact, this is not a problem unique to Boeing. Figure 8.7 presents the distribution of standard errors on beta estimates for U.S. companies.

With the Nokia regression, we seem to cure the standard error problem but at a very large cost. The low standard errors reflect the domination of the index by a stock and result in betas that may be precise but bear no resemblance to true risk.

Changing the market index, the return period, and the return interval offers no respite. If the index becomes a more representative index, the standard errors on betas will increase, reflecting the fact that more of the risk in the stock is firm-specific. If the beta changes as the return period or interval changes, it creates more uncertainty about the true beta of the company.

In short, regression betas will almost always be either too noisy or skewed by estimation choices to be useful measures of the equity risk in a company. The cost of equity is far too important an input into a discounted cash flow valuation to be left to statistical chance.

Fundamental Betas A second way to estimate betas is to look at the fundamentals of the business. The beta for a firm may be estimated from a regression, but it is determined by decisions the firm has made on what business to be in and how much operating leverage to use in the business, and by the degree to which the firm uses financial leverage. This section examines an alternative way of estimating betas, where we are less reliant on historical betas and more cognizant of their fundamental determinants.

Determinants of Betas The beta of a firm is determined by three variables: (1) the type of business or businesses the firm is in, (2) the degree of operating leverage of the firm, and (3) the firm's financial leverage. Although we will use these determinants to find betas in the capital asset pricing model, the same analysis can be used to calculate the betas for the arbitrage pricing and the multifactor models as well.

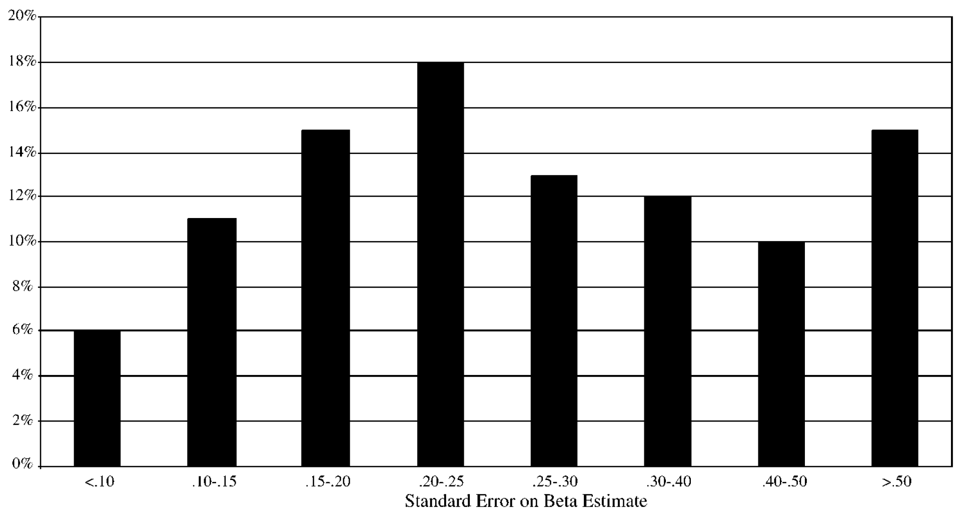

FIGURE 8.7 Distribution of Standard Errors on Beta Estimates
Data Source: Bloomberg.

Type of Business Since betas measure the risk of a firm relative to a market index, the more sensitive a business is to market conditions, the higher its beta. Thus, other things remaining equal, cyclical firms can be expected to have higher betas than noncyclical firms. Companies involved in housing and automobiles, two sectors of the economy that are very sensitive to economic conditions, should have higher betas than companies in food processing and tobacco, which are relatively insensitive to business cycles.

This view can be extended to a company's products. The degree to which a product's purchase is discretionary will affect the beta of the firm manufacturing the product. Firms whose products are much more discretionary to their customers—they can defer or delay buying these products—should have higher betas than firms whose products are viewed as necessary or less discretionary. Thus, the beta of Procter & Gamble, which sells diapers and daily household products, should be lower than the beta of Gucci, which manufactures luxury products.

Degree of Operating Leverage The degree of operating leverage is a function of the cost structure of a firm and is usually defined in terms of the relationship between fixed costs and total costs. A firm that has high fixed costs relative to total costs is said to have high operating leverage. A firm with high operating leverage will also have higher variability in operating income than would a firm producing a similar product with low operating leverage. Other things remaining equal, the higher variance in operating income will lead to a higher beta for the firm with high operating leverage.

Can firms change their operating leverage? While some of a firm's cost structure is determined by the business it is in (an energy utility has to build expensive power plants, and airlines have to buy or lease expensive planes), firms in the United States have become increasingly inventive in lowering the fixed cost component in their total costs. For instance, firms have made cost structures more flexible by:

- Negotiating labor contracts that emphasize flexibility and allow the firm to make its labor costs more sensitive to its financial success.
- Entering into joint venture agreements, where the fixed costs are borne by someone else.
- Subcontracting manufacturing and outsourcing, which reduce the need for expensive plant and equipment.

While the arguments for such actions may be couched in terms of offering competitive advantage and flexibility, they do also reduce the operating leverage of the firm and its exposure to market risk.

While operating leverage affects betas, it is difficult to measure the operating leverage of a firm, at least from the outside, since fixed and variable costs are often aggregated in income statements. It is possible to get an approximate measure of the operating leverage of a firm by looking at changes in operating income as a function of changes in sales.

Degree of operating leverage = % change in operating profit/% change in sales

For firms with high operating leverage, operating income should change more than proportionately when sales change.

SIZE, GROWTH, AND BETAS

Generally, smaller firms with higher growth potential are viewed as riskier than larger, more stable firms. While the rationale for this argument is clear when talking about total risk, it becomes more difficult to see when looking at market risk or betas. Should a smaller software firm have a higher beta than a larger software firm? One reason to believe that it should is operating leverage. If there is a setup cost associated with investing in infrastructure or economies of scale, smaller firms will have higher fixed costs than larger firms, leading in turn to higher betas for these firms.

With growth firms, the argument for higher betas rests on the notion of discretionary versus nondiscretionary purchases. For a high-growth firm to deliver on its growth, new customers have to adopt the product or existing customers have to buy more of the product. Whether they do so will depend, in large part, on how well-off they feel. This, in turn, will make the profits of high-growth firms much more dependent on how well the economy is doing, thus increasing their betas.

Degree of Financial Leverage Other things remaining equal, an increase in financial leverage will increase the beta of the equity in a firm. Intuitively, we would expect that the fixed interest payments on debt result in increasing income in good times and decreasing income in bad times. Higher leverage increases the variance in net income and makes equity investment in the firm riskier. If all the firm's risk is borne by the stockholders (i.e., the beta of debt is zero),[7] and debt has a tax benefit to the firm, then,

$$\beta_L = \beta_u[1 + (1 - t)(D/E)]$$

where β_L = Levered beta for equity in the firm
β_u = Unlevered beta of the firm (i.e., the beta of the firm without any debt)
t = Corporate tax rate
D/E = Debt-to-equity ratio (market value)

Intuitively, we expect that as leverage increases (as measured by the debt-to-equity ratio), equity investors bear increasing amounts of market risk in the firm, leading to higher betas. The tax factor in the equation measures the tax deductibility of interest payments.

The unlevered beta of a firm is determined by the types of the businesses in which

[7]This formula was originally developed by Hamada in 1972. There are two common modifications. One is to ignore the tax effects and compute the levered beta as:

$$\beta_L = \beta_u(1 + D/E)$$

If debt has market risk (i.e., its beta is greater than zero), the original formula can be modified to take this into account. If the beta of debt is β_D, the beta of equity can be written as:

$$\beta_L = \beta_u[1 + (1 - t)(D/E)] - \beta_D(1 - t)D/E$$

it operates and its operating leverage. It is often also referred to as the asset beta, since it is determined by the assets owned by the firm. Thus, the levered beta, which is also the beta for an equity investment in a firm, is determined both by the riskiness of the business it operates in by the amount of financial leverage risk it has taken on.

Since financial leverage multiplies the underlying business risk, it stands to reason that firms that have high business risk should be reluctant to take on financial leverage. It also stands to reason that firms that operate in stable businesses should be much more willing to take on financial leverage. Utilities, for instance, have historically had high debt ratios but have not had high betas, mostly because their underlying businesses have been stable and fairly predictable.

ILLUSTRATION 8.3: Effects of Leverage on Betas: Boeing

From the regression for the period from 1996 to 2000, Boeing had a historical beta of 0.56. Since this regression uses stock prices of Boeing over this period, we began by estimating the average debt-to-equity ratio between 1996 and 2000, using market values for debt and equity.

Average debt-to-equity ratio between 1996 and 2000 = 15.56%

The beta over the 1996–2000 period reflects this average leverage. To estimate the unlevered beta over the period, a marginal tax rate of 35% is used:

$$\begin{aligned}\text{Unlevered beta} &= \text{Current beta}/[1 + (1 - \text{Tax rate})(\text{Average debt/Equity})]\\ &= 0.56/[1 + (1 - 0.35)(0.1556)] = 0.51\end{aligned}$$

The unlevered beta for Boeing over the 1996–2000 period is 0.51. The levered beta at different levels of debt can then be estimated:

$$\text{Levered beta} = \text{Unlevered beta} \times [1 + (1 - \text{Tax rate})(\text{Debt/Equity})]$$

For instance, if Boeing were to increase its debt equity ratio to 10%, its equity beta will be:

$$\text{Levered beta (@10\% D/E)} = 0.51 \times [1 + (1 - 0.35)(0.10)] = 0.543$$

If the debt equity ratio were raised to 25%, the equity beta would be:

$$\text{Levered beta (@25\% D/E)} = 0.51 \times [1 + (1 - 0.35)(0.25)] = 0.59$$

The following table summarizes the beta estimates for different levels of financial leverage ranging from 0% to 90% debt.

Debt to Capital	*Debt/Equity Ratio*	*Beta*	*Effect of Leverage*
0%	0.00%	0.51	0.00
10%	11.11%	0.55	0.04
20%	25.00%	0.59	0.08
30%	42.86%	0.65	0.14
40%	66.67%	0.73	0.22
50%	100.00%	0.84	0.33
60%	150.00%	1.00	0.50
70%	233.33%	1.28	0.77
80%	400.00%	1.83	1.32
90%	900.00%	3.48	2.98

As Boeing's financial leverage increases, the beta increases concurrently.

***levbeta.xls.* This spreadsheet allows you to estimate the unlevered beta for a firm and compute the betas as a function of the leverage of the firm.**

Bottom-Up Betas Breaking down betas into their business risk and financial leverage components provides us with an alternative way of estimating betas, in which we do not need past prices on an individual firm or asset to estimate its beta.

To develop this alternative approach, we need to introduce an additional property of betas that proves invaluable. The beta of two assets put together is a weighted average of the individual asset betas, with the weights based on market value. Consequently, the beta for a firm is a weighted average of the betas of all the different businesses it is in. We can estimate the beta for a firm in five steps:

Step 1: Identify the business or businesses the firm operates in.

Step 2: Find other publicly traded firms in each business and obtain their regression betas, which we use to compute an average beta for the firms.

Step 3: Estimate the average unlevered beta for the business by unlevering the average beta for the firms by their average debt to equity ratio. Alternatively, we could estimate the unlevered beta for each firm and then compute the average of the unlevered betas. The first approach is preferable because unlevering an erroneous regression beta is likely to compound the error.

$$\text{Unlevered beta}_{\text{business}} = \text{Beta}_{\text{comparable firms}}/[1 + (1 - t)(\text{D/E ratio}_{\text{comparable firms}})]$$

Step 4: Estimate an unlevered beta for the firm being analyzed, taking a weighted average of the unlevered betas for the businesses it operates in, using the proportion of firm value derived from each business as the weights. If values are not available, use operating income or revenues as weights. This weighted average is called the bottom-up unlevered beta.

$$\text{Unlevered beta}_{\text{firm}} = \sum_{j=1}^{j=k} \left(\text{Unlevered beta}_j \times \text{Value weight}_j\right)$$

where the firm is assumed to operating in k different businesses.

Step 5: Finally, estimate the current market values of debt and equity at the firm and use this debt to equity ratio to estimate a levered beta.

The betas estimated using this processs are called bottom-up betas.

The Case for Bottom-Up Betas At first sight, the use of bottom-up betas may seem to leave us exposed to all of the problems noted with regression betas. After all, the betas for other publicly traded firms in the business are obtained from regressions. Notwithstanding this, bottom-up betas represent a significant improvement on regression betas for the following reasons:

- While each regression beta is estimated with standard error, the average across a number of regression betas has much lower standard error. The intuition is simple. A high standard error on a beta estimate indicates that it can be significantly higher or lower than the true beta. Averaging across these betas results in an average beta that is far more precise than the individual betas that went into it. In fact, if the estimation errors on individual firm betas are uncorrelated across firms, the savings in standard error can be stated as a function of the average standard error or beta estimates and the number of firms in the sample.

$$\text{Standard error}_{\text{bottom-up beta}} = \frac{\text{Average standard error}_{\text{comparable firms}}}{\sqrt{n}}$$

  where n is the number of firms in the sample. Thus, if the average standard error in beta estimates for software firms is 0.50 and the number of software firms is 100, the standard error of the average beta is only 0.05 ($0.50/\sqrt{100}$).
- A bottom-up beta can be adapted to reflect actual changes in a firm's business mix and expected changes in the future. Thus if a firm divested a major portion of its operations last week, the weights on the businesses can be modified to reflect the divestiture. The same can be done with acquisitions. In fact, a firm's strategic plans to enter new businesses in the future can be brought into the beta estimates for future periods.
- Firms do change their debt ratios over time. While regression betas reflect the average debt-to-equity ratio maintained by the firm during the regression period, bottom-up betas use the current debt-to-equity ratio. If a firm plans to change its debt-to-equity ratio in the future, the beta can be adjusted to show these changes.
- Finally, bottom-up betas wean us from our dependence on historical stock prices. While we do need these prices to get betas for comparable firms, all we need for the firm being analyzed is a breakdown of the businesses it is in. Thus, bottom-up betas can be estimated for private firms, divisions of business, and stocks that have just started trading in financial markets.

Computational Details While the idea behind bottom-up betas is fairly simple, there are several computational details that are deserving of attention.

- *Defining comparable firms.* First, we have to decide how narrowly we want to define a business. Consider, for instance, a firm that manufactures entertainment software. We could define the business as entertainment software and consider only companies that primarily manufacture entertainment software to be comparable firms. We could go even further and define comparable firms as firms manufacturing entertainment software with revenues similar to that of the company being analyzed. While there are benefits to narrowing the comparable firm definition, there is a large cost. Each additional criterion added to the definition of "comparable" will mean that fewer firms make the list, and the savings in standard error that comprise the biggest benefit to bottom-up betas become smaller. A commonsense principle should therefore come into play. If there are hundreds of firms in a business, as there are in the software sector, you can afford to be more selective. If there are relatively few firms, not only do you have to become less selective, you might have to broaden the definition of comparable to bring other firms into the mix.

- *Estimating betas.* Once the comparable firms in a business have been defined, you have to estimate the betas for these firms. Although it would be best to estimate the beta for each of these firms against a common and well-diversified equity index, it is usually easier to use service betas that are available for each of these firms. These service betas may be estimated against different indexes. For instance, if you define your business to be global telecommunications and obtain betas for global telecom firms from Bloomberg, these betas will be estimated against the local indexes. This is usually not a fatal problem, especially with large samples, since errors in the estimates tend to average out.
- *Averaging method.* The average beta for the firms in the sector can be computed in one of three ways. We could use market-weighted averages, but the savings in standard error that touted in the earlier section will be muted, especially if there are one or two very large firms in the sample. We could estimate the simple average of the betas of the companies, thus weighting all betas equally. The process weighs in the smallest firms in the sample disproportionately (to their market value), but the savings in standard error are likely to be maximized.
- *Controlling for differences.* In essence, when we use betas from comparable firms, we are assuming that all firms in the business are equally exposed to business risk and have similar operating leverage. Note that the process of levering and unlevering of betas allows us to control for differences in financial leverage. If there are significant differences in operating leverage—cost structure—across companies, the differences in operating leverage can be controlled for as well. This would require estimation of a business beta, where the effects of operating leverage are taken out from the unlevered beta:

  $$\text{Business beta} = \text{Unlevered beta}/[1 + (\text{Fixed costs}/\text{Variable costs})]$$

  Note the similarity to the adjustment for financial leverage; the only difference is that both fixed and variable costs are eligible for the tax deduction, and the tax rate is therefore no longer a factor. The business beta can then be relevered to reflect the differences in operating leverage across firms.

betas.xls: **This dataset on the Web has updated betas and unlevered betas by business sector in the United States.**

ILLUSTRATION 8.4: Estimating a Bottom-Up Beta for Vans Shoes—January 2001

Vans Shoes is a shoe manufacturing firm with a market capitalization of $191 million. To estimate the bottom-up beta for Vans Shoes, consider the betas of all publicly traded shoe companies in the following table:

Company Name	*Beta*	*Market D/E*	*Tax Rate*	*Fixed/ Variable*
Barry (R.G.)	1.00	40.51%	36.89%	75.66%
Brown Shoe	0.80	106.64%	37.06%	61.41%
Candie's Inc.	1.20	75.86%	0.00%	29.78%
Converse Inc.	0.60	653.46%	0.00%	39.64%
Deckers Outdoor Corp.	0.80	82.43%	0.00%	62.52%
Florsheim Group Inc.	0.65	96.79%	32.47%	79.03%
K-Swiss Inc.	0.65	0.69%	40.94%	56.92%
Kenneth Cole 'A'	1.05	0.29%	39.50%	56.97%
LaCrosse Footwear Inc.	0.55	81.15%	39.25%	30.36%
Maxwell Shoe Inc.	0.75	2.24%	33.28%	20.97%
Nike Inc. 'B'	0.90	9.47%	39.50%	46.07%
Reebok Int'l.	1.05	171.90%	32.28%	35.03%
Rocky Shoes & Boots Inc.	0.80	93.51%	0.00%	26.89%
Saucony Inc.	0.15	34.93%	31.11%	49.33%
Shoe Carnival	0.85	2.18%	39.97%	35.03%
Stride Rite Corp.	0.80	0.00%	36.80%	48.23%
Timberland Co. 'A'	1.10	15.23%	32.00%	49.50%
Vulcan Int'l.	0.65	3.38%	5.61%	11.92%
Wellco Enterprises Inc.	0.60	48.89%	0.00%	11.52%
Weyco Group	0.30	11.91%	35.74%	24.69%
Wolverine World Wide	1.35	44.37%	32.62%	32.31%
Average (Simple)	*0.79*	*75.04%*	*25.95%*	*42.08%*
Vans Shoes		*9.41%*	*34.06%*	*31.16%*

In addition to the betas for each firm, the table reports the market debt-to-equity ratio, the effective tax rate, and a measure of operating leverage obtained by dividing selling, general, and administrative (SG&A) expenses (which we consider fixed) by other operating expenses (which we consider variable). We can estimate the unlevered beta for the business using the averages for these values:

$$\text{Average beta} = 0.79$$
$$\text{Average debt-to-equity ratio} = 75.04\%$$

Using the average tax rate of 25.95%, we can estimate the unlevered beta.

$$\text{Unlevered beta} = 0.79/[1 + (1 - .2595).7504] = .5081$$

The beta for Vans Shoes can then be obtained using the firm's tax rate of 34.06% and its market debt to equity ratio of 9.41%.

$$\text{Levered beta}_{\text{Vans}} = 0.5081[1 + (1 - .3406).0941] = .5397$$

This levered beta is based on the implicit assumption that all shoe manufacturers have similar operating leverage. In fact, we could adjust the unlevered beta for the average fixed cost/variable cost ratio for the business and then relever back at the operating leverage for Vans Shoes:

Average fixed cost/Variable cost ratio = 42.08%

$$\begin{aligned}\text{Business beta} &= \text{Unlevered beta}/(1 + \text{Fixed costs/Variable costs})\\ &= .5081/1.4208 = .3576\end{aligned}$$

We can then use Vans' fixed cost to variable cost ratio of 31.16% to estimate an adjusted unlevered and levered beta.

$$\text{Unlevered beta}_{\text{Vans}} = 0.3576(1 + .3116) = .4691$$
$$\text{Levered beta} = .4691[1 + (1 - .3406).0941] = .4981$$

By having a debt-to-equity ratio and an operating leverage that is lower than the average for the industry, Vans Shoes ends up with a beta much lower than that of the industry.

ILLUSTRATION 8.5: Estimating a Bottom-Up Beta for Boeing—September 2000

Boeing has undergone a significant change in both its business mix and its financial leverage over the past five years. Not only did it acquire Rockwell and McDonnell Douglas, giving it a major foothold in the defense business, but it borrowed substantial amounts to make these acquisitions. Since these events have occurred over time, the historical regression beta does not fully reflect the effects of these changes. To estimate Boeing's beta today, we broke its business into two areas:

1. *Commercial aircraft*, which is Boeing's core business of manufacturing commercial jet aircraft and providing related support services.
2. *Information, space and defense systems (ISDS)*, which include research, development, production, and support of military aircraft, helicopters, and missile systems.

Each of these areas of business has very different risk characteristics, and the unlevered beta for each business was estimated by looking at comparable firms in each business. The following table summarizes these estimates.

Segment	*Revenues*	*Value/Sales Ratio for Segment*	*Estimated Value*	*Unlevered Beta*	*Segment Weight*	*Weighted Beta*
Commercial aircraft	\$26,929	1.12	\$30,160	0.91	70.39%	0.6405
ISDS	\$18,125	0.70	\$12,688	0.80	29.61%	0.2369
Boeing	\$45,054		\$42,848		100.00%	0.8774

For commercial aircraft there are no truly comparable firms. We looked at Boeing's own beta prior to its expansion in the defense business and computed the unlevered beta using this estimate. For ISDS, we used 17 firms that derived the bulk of their revenues from defense contracting, and computed the average beta and debt-to-equity ratio for these firms. The unlevered beta was computed using these averages. The values for each of the divisions were estimated using the revenues from each segment[8] and a typical revenue multiple[9] for that type of business. The unlevered beta for Boeing as a company in 2000 can be estimated by taking a value-weighted average of the betas of each of the different business areas. This is reported in the last column to be 0.8774.

The equity beta can then be estimated using the current financial leverage for Boeing as a firm. Combining the market value of equity of \$55.20 billion and the value of debt of \$7.85 billion, and using a 35% tax rate for the firm, we arrive at the current beta for Boeing.

$$\text{Equity beta for Boeing} = 0.8774[1 + (1 - .35)(7.85/55.2)] = 0.9585$$

This is very different from the historical beta of 0.56 that we obtained from the regression, but it is, in our view, a much truer reflection of the risk in Boeing.

[8]Note that Boeing breaks its business down in its financial statements into these two segments. We could have used operating income or EBITDA and a typical multiple to arrive at value.

[9]To estimate these multiples, we looked at the market value of publicly traded firms relative to their revenues. This is a ratio of enterprise value to revenues.

ILLUSTRATION 8.6: Estimating a Bottom-Up Beta for Titan Cements—January 2000

To estimate a beta for Titan Cement, we began by defining comparable firms as other cement companies in Greece but found only one comparable firm. When we expanded the list to include cement companies across Europe, we increased our sample to nine firms. Since we did not see any reason to restrict our comparison to just European firms, we decided to look at the average beta for cement companies globally. There were 108 firms in this sample, with an average beta of 0.99, an average tax rate of 34.2%, and an average debt-to-equity ratio of 27.06%. We used these numbers to arrive at an unlevered beta of 0.84.

$$\text{Unlevered beta for cement companies} = 0.99/[1 + (1 - .342)(.2706)] = 0.84$$

We then used Titan's market values of equity (566.95 million Gdr) and debt (13.38 million Gdr) to estimate a levered beta for its equity:

$$\text{Levered beta} = 0.84\ [1 + (1 - .2414)(13.38/566.95)] = 0.86$$

We used Titan's tax rate of 24.14% in this calculation.

HOW WELL DO BETAS TRAVEL?

Often, when analyzing firms in small or emerging markets, we have to estimate betas by looking at firms in the same business but traded on other markets. This is what we did when estimating the beta for Titan Cement. Is this appropriate? Should the beta for a steel company in the United States be comparable to that of a steel company in Indonesia? We see no reason why it should not be. But the company in Indonesia has much more risk, you might argue. We do not disagree, but the fact that we use similar betas does not mean that we believe that the costs of equity are identical across all steel companies. In fact, using the approach described in the preceding chapter, the risk premium used to estimate the cost of equity for the Indonesian company will incorporate a country risk premium, whereas the cost of equity for the U.S. company will not. Thus, even if the betas used for the two companies are identical, the cost of equity for the Indonesian company will be much higher.

There are a few exceptions to this proposition. Recall that one of the key determinants of betas is the degree to which a product or service is discretionary. It is entirely possible that products or services that are discretionary in one market (and command high betas) may be nondiscretionary in another market (and have low betas). For instance, phone service is viewed as a nondiscretionary product in most developing markets, but is a discretionary product in emerging markets. Consequently, the average beta estimated by looking at telecom firms in developed markets will understate the true beta of a telecom firm in an emerging market. For the latter beta, the comparable firms should be restricted to include only telecom firms in emerging markets.

Calculating Betas after a Major Restructuring The bottom-up process of estimating betas provides a solution when firms go through major restructurings that change both their business mix and their leverage. In these cases, the regression betas are misleading because they do not reflect fully the effects of these changes. Boeing's beta estimated using the bottom-up approach is likely to provide a more precise estimate than the historical beta from a regression of Boeing's stock prices, given Boeing's acquisitions of Rockwell and McDonnell Douglas and its increase in leverage. In fact, a firm's beta can be estimated using the bottom-up approach even before the restructuring becomes effective. Illustration 8.7, for instance, estimates Boeing's beta just before and after its acquisition of McDonnell Douglas, allowing for the changes in both the business mix and the leverage.

ILLUSTRATION 8.7: Beta of a Firm after an Acquisition: Boeing and McDonnell Douglas

In 1997, Boeing announced that it was acquiring McDonnell Douglas, another company involved in the aerospace and defense business. At the time of the acquisition, the two firms had the following market values and betas:

Company	*Beta*	*Debt*	*Equity*	*Firm Value*
Boeing	0.95	\$3,980	\$32,438	\$36,418
McDonnell Douglas	0.90	\$2,143	\$12,555	\$14,698

Note that the market values of equity used for the two firms reflect the market values after the acquisition announcement and reflect the acquisition price agreed on for McDonnell Douglas shares.

In order to evaluate the effects of the acquisition on Boeing's beta, we first examine the effects of the merger on the business risk of the combined firm by estimating the unlevered betas of the two companies and calculating the combined firm's unlevered beta.

$$\text{Boeing's unlevered beta} = 0.95/[1 + (1 - .35) \times (3{,}980/32{,}438)] = 0.88$$

$$\text{McDonnell Douglas' unlevered beta} = 0.90/[1 + (1 - .35) \times (2{,}143/12{,}555)] = 0.81$$

The unlevered beta for the combined firm can be calculated as the weighted average of the two unlevered betas, with the weights based on the market values of the two firms.

$$\begin{aligned}\text{Unlevered beta for combined firm} &= 0.88(36{,}418/51{,}116) + 0.81(14{,}698/51{,}116)\\ &= 0.86\end{aligned}$$

Boeing's acquisition of McDonnell Douglas was accomplished by issuing new stock in Boeing to cover the value of McDonnell Douglas' equity of \$12,555 million. Since no new debt was used to finance the deal, the debt outstanding in the firm after the acquisition is just the sum of the debt outstanding at the two companies before the acquisition.

$$\begin{aligned}\text{Debt} &= \text{McDonnell Douglas' old debt} + \text{Boeing's old debt}\\ &= \$3{,}980 + \$2{,}143 = \$6{,}123 \text{ million}\end{aligned}$$

$$\begin{aligned}\text{Equity} &= \text{Boeing's old equity} + \text{New equity used for acquisition}\\ &= \$32{,}438 + \$12{,}555 = \$44{,}993 \text{ million}\end{aligned}$$

The debt/equity ratio can then be computed as follows:

$$\text{D/E ratio} = 6{,}123/44{,}993 = 13.61\%$$

This debt/equity ratio in conjunction with the new unlevered beta for the combined firm yields a new beta of:

$$\text{New beta} = 0.86[1 + (1 - .35)(.1361)] = 0.94$$

Accounting Betas A third approach is to estimate the market risk parameters from accounting earnings rather than from traded prices. Thus, changes in earnings at a division or a firm, on a quarterly or an annual basis, can be related to changes in earnings for the market, in the same periods, to arrive at an estimate of a accounting beta to use in the CAPM. While the approach has some intuitive appeal, it suffers from three potential pitfalls. First, accounting earnings tend to be smoothed out relative to the underlying value of the company, as accountants spread expenses and income over multiple periods. This results in betas that are "biased down," especially for risky firms, or "biased up" for safer firms. In other words, betas are likely to be closer to 1 for all firms using accounting data.

Second, accounting earnings can be influenced by nonoperating factors, such as changes in depreciation or inventory methods, and by allocations of corporate expenses at the divisional level. Finally, accounting earnings are measured, at most, once every quarter, and often only once every year, resulting in regressions with few observations and not much explanatory power (low R-squared, high standard errors).

ILLUSTRATION 8.8: Estimating Accounting Betas: Defense Division of Boeing—1995

Having operated in the defense business for decades, Boeing has a record of its profitability. These profits are reported in the following table, together with earnings changes for companies in the S&P 500 going back to 1980.

Year	*S&P 500*	*Boeing's Defense Business*
1980	–2.10%	–12.70%
1981	–6.70%	–35.56%
1982	–45.50%	27.59%
1983	37.00%	159.36%
1984	41.80%	13.11%
1985	–11.80%	–26.81%
1986	7.00%	–16.83%
1987	41.50%	20.24%
1988	41.80%	18.81%
1989	2.60%	–29.70%
1990	–18.00%	–40.00%
1991	–47.40%	–35.00%
1992	64.50%	10.00%
1993	20.00%	–7.00%
1994	25.30%	11.00%

Regressing the changes in profits in the defense division (Δ Earnings$_{defense}$) against changes in profits for the S&P 500 (Δ Earnings$_{S\&P}$) yields the following:

$$\Delta \text{ Earnings}_{defense} = -0.03 + 0.65 \, \Delta \text{ Earnings}_{S\&P}$$

Based on this regression, the beta for the defense division is 0.65.

accbeta.xls: **This spreadsheet allows you to estimate the accounting beta on a division or firm.**

***spearn.xls:* This dataset on the Web has earnings changes, by year, for the S&P 500 going back to 1960.**

Market, Bottom-Up, and Accounting Betas: Which One Do We Use? For most publicly traded firms, betas can be estimated using accounting data or market data or from the bottom-up approach. Since the betas will almost never be the same, using these different approaches, the question is, which one do we use? We would almost never use accounting betas, for all the reasons specified earlier. We are almost as reluctant to use historical market betas for individual firms because of the standard errors in beta estimates, the failures of the local indexes (as is the case with most emerging market companies) and the inability of these regressions to reflect the effects of major changes in the business mix and financial risk at the firm. Bottom-up betas, in our view, provide us with the best beta estimates for three reasons:

1. They allow us to consider changes in business and financial mix, even before they occur.
2. They use average betas across large numbers of firms, which tend to be less noisy than individual firm betas.
3. They allow us to calculate betas by area of business for a firm, which is useful both in the context of investment analysis and in valuation.

Measuring Country Risk Exposure (Lambda) Chapter 7 introduced the concept of country risk exposure and the notion of lambda—a measure of a company's exposure to country risk. In this section, we would like to consider intuitively what factors determine this exposure and how best to estimate lambda. A company's exposure to country risk is affected by almost every aspect of its operations, beginning with where its factories are located and who its customers are and continuing with what currency its contracts are denominated in and how well it manages its exposure to exchange rate risk. Much of this information, however, is internal information and not available to someone valuing the firm from the outside. As a practical matter, then, we can estimate lambda using one of the following approaches:

- *Revenue breakdown.* The simplest way of estimating lambda is to use the proportion of a firm's revenues that are generated in a country and scale this to the proportion of the revenues generated by the average firm in that country.

$$\lambda = \frac{\text{Proportion of revenues in country}_{\text{firm}}}{\text{Proportion of revenues in country}_{\text{average firm}}}$$

  Thus, a firm that generates only 40 percent of its revenues in Indonesia when the average firm in Indonesia generates 80 percent of its revenues domestically will have a lambda of 0.5 for Indonesian country risk. Note, though, that if the remaining 60 percent of its revenues are in Thailand, you would have to estimate a lambda for Thai country risk and add that component to the cost of equity.
- *Regression versus country bond.* A second approach to estimating lambdas would be to run regressions of stock returns for each firm in the emerging market against the returns on the country bond. In Brazil, for instance, this would involve regressing returns on each Brazilian stock against returns on the Brazilian country bond (the C-bond). The slope of this regression should measure

how sensitive a stock is to changes in country risk (since country bond returns are direct measures of country risk) and thus yield a measure of lambda. For instance, assuming that regressing Embraer's stock returns against returns on the C-bond yields a slope of 0.30 and that the average slope across all Brazilian stocks is 0.75, the lambda would be 0.40 (0.30/0.75).

From Betas to Cost of Equity

Having estimated the riskless rate and the risk premium(s) in Chapter 7 and the beta(s) in this chapter, we can now estimate the expected return from investing in equity at any firm. In the CAPM, this expected return can be written as:

$$\text{Expected return} = \text{Riskless rate} + \text{Beta} \times \text{Expected risk premium}$$

where the riskless rate would be the rate on a long-term government bond, the beta would be either the historical, fundamental, or accounting betas described earlier, and the risk premium would be either the historical premium or an implied premium.

In the arbitrage pricing and multifactor model, the expected return would be written as follows:

$$\text{Expected return} = \text{Riskless rate} + \sum_{j-1}^{j=n} \beta_j \times \text{Risk premium}_j$$

where the riskless rate is the long-term government bond rate; β_j is the beta relative to factor j, estimated using historical data or fundamentals; and risk premium$_j$ is the risk premium relative to factor j, estimated using historical data.

The expected return on an equity investment in a firm, given its risk, has strong implications for both equity investors in the firm and the managers of the firm. For equity investors, it is the rate they need to earn to be compensated for the risk they have taken in investing in the equity of the firm. If, after analyzing an investment, they conclude they cannot make this return, they would not buy this investment; alternatively, if they decide they can make a higher return, they would make the investment. For managers in the firm, the return investors need to make to break even on their equity investments becomes the return they have to try to deliver to keep these investors from becoming restive and rebellious. Thus, it becomes the rate they have to beat in terms of returns on their equity investments in projects. In other words, this is the cost of equity to the firm.

ILLUSTRATION 8.9: Estimating the Cost of Equity for Boeing—December 2000

Now that we have an estimate of beta of 0.9585 for Boeing, based on the bottom-up estimates, we can estimate its cost of equity. To make the estimate, we used the prevailing Treasury bond rate of 5% and a historical risk premium of 5.51%.

$$\text{Cost of equity} = 5.00\% + 0.9585(5.51\%) = 10.28\%$$

There are two points making about this estimate. The first is that the cost of equity would have been significantly lower if we had chosen to use the implied equity premium on December 31, 2000, which was about 2.87% (see Chapter 7).

$$\text{Cost of equity} = 5.00\% + 0.9585(2.87\%) = 7.75\%$$

The second point is that we are not considering the exposure that Boeing has to emerging market risk from its business. If the exposure is significant, we should be adding a country risk premium to the cost of equity estimate.

ILLUSTRATION 8.10: Estimating the Cost of Equity for Embraer—March 2001

Embraer is a Brazilian aerospace firm. To estimate its cost of equity, we first estimated the unlevered beta by looking at aerospace firms globally.

$$\text{Unlevered beta for aerospace firms} = 0.87$$

Embraer's debt-to-equity ratio at the time of this analysis was 2.45%,[10] resulting in a levered beta for Embraer:

$$\text{Levered beta for Embraer} = 0.87[1 + (1 - .33).0245] = 0.88$$

To estimate the cost of equity for Embraer in U.S. dollar terms, we began with the Treasury bond rate of 5% at the time of the analysis, but incorporated the country risk associated with Brazil into the risk premium. Using the approach described in Chapter 7, we estimated a country risk premium of 10.24% in March 2001. In conjunction with a mature market risk premium of 5.51% estimated for the United States, this yields a cost of equity of 18.93%.

$$\text{Cost of equity for Embraer} = 5\% + 0.88(5.51\% + 10.24\%) = 18.86\%$$

Again, there are several points that are worth making on this estimate. The first is that this cost of equity can be expected to change over time as Brazil matures as a market and country risk declines. The second is that we have assumed that betas measure exposure to country risk. A company like Embraer that derives the bulk of its revenues outside Brazil could argue that it is less exposed to country risk. We could have derived λ as a measure of exposure to country risk for Embraer by looking at the proportion of its revenues that it derives in Brazil and comparing it to the proportion of revenues derived by a typical company in Brazil. In 2000, for instance, this would have yielded the following:

$$\lambda_{\text{Embraer}} = \frac{\text{Proportion of revenues from Brazil}_{\text{Embraer}}}{\text{Proportion of revenues from Brazil}_{\text{typical Brazilian firm}}} = \frac{9\%}{60\%} = 0.15$$

Using this measure of exposure to country risk, Embraer would have had a much lower cost of equity.

$$\begin{aligned}\text{Cost of equity in U.S. dollars} &= \text{Risk-free rate} + \text{Beta(Mature market risk premium)} \\ &\quad + \lambda\text{(Country risk premium)} \\ &= 5\% + 0.88(5.51\%) + 0.15(10.24\%) = 11.39\%\end{aligned}$$

The final point is that the cost of equity in dollar terms can be converted into a nominal Brazilian real (BR) cost of equity fairly simply by considering the differences in expected inflation rates in Brazil and the United States. For instance, if the expected inflation rate in Brazil is 10% and the expected inflation rate in the United States is 2%, the cost of equity in nominal BR is as follows:

$$\begin{aligned}\text{Cost of equity}_{\text{nominal BR}} &= (1 + \text{Cost of equity}_{\$})(\text{Inflation rate}_{\text{Brazil}} / \text{Inflation rate}_{\text{U.S.}}) - 1 \\ &= (1.1139)(1.10/1.02) - 1 = 20.12\%\end{aligned}$$

Implicitly, we assume that real risk-free rates around that world are the same with this approach and that the risk premium scales up with inflation as well. The alternative is to estimate a cost of equity from scratch, beginning with a nominal BR risk-free rate (which was 14% at the time of this analysis) and adding the premiums from before:

$$\begin{aligned}\text{Cost of equity}_{\text{nominal BR}} &= \text{Risk-free rate} + \text{Beta(Mature market risk premium)} \\ &\quad + \lambda\text{(Country risk premium)} \\ &= 14\% + 0.88(5.51\%) + 0.15(10.24\%) = 20.39\%\end{aligned}$$

Substituting in a real risk-free rate in the equation would yield a real cost of equity.

[10]We used net debt (the difference between gross debt and cash) in making this estimate. We discuss later in the chapter when this practice is appropriate and when it is not.

COST OF EQUITY AND A SMALL FIRM PREMIUM

Chapter 6 presented evidence of a small firm premium—small market-cap stocks earn higher returns than large market-cap stocks with equivalent betas. The magnitude and persistence of the small firm premium can be viewed as evidence that the capital asset pricing model understates the risk of smaller companies, and that a cost of equity based purely on a CAPM beta will therefore yield too low a number for these firms. There are some analysts who argue that you should therefore add a premium to the estimated cost of equity for smaller firms. Since small cap stocks have earned about 2 percent more than large cap stocks voer the past few decades, you could consider this a reasonable estimate for the small firm premium. To estimate the cost of equity for a small cap stock with a beta of 1.2 (assuming a risk-free rate of 5.1 percent and a market risk premium of 4 percent), for instance, you would do the following:

$$\begin{aligned}\text{Cost of equity for small cap stock} &= \text{Risk-free rate} + \text{Beta} \\ &\quad \times \text{Market risk premium} \\ &\quad + \text{Small cap premium} \\ &= 5.1\% + 1.2 \times 4\% + 2\% \\ &= 11.9\%\end{aligned}$$

We would introduce two notes of caution with this approach. First, it opens the door to a series of adjustments that you could make to the cost of equity, reflecting the numerous inefficiencies cited in Chapter 6. For instance, you could estimate a low PE premium, a low price-to-book premium, and a high dividend yield premium and add them all to the cost of equity. If our objective in valuation is to uncover market mistakes, it would be a mistake to start off with the presumption that markets are right in their assessments in the first place. Second, a better way of considering the small firm premium would be to identify the reasons for the premium and then develop more direct measures of risk. For instance, assume that the higher risk of small cap stocks comes from the higher operating leverage that these firms have, relative to their larger competitors. You could adjust the betas for operating leverage (as we did a few pages ago for Vans Shoes) and use the higher betas for small firms.

FROM COST OF EQUITY TO COST OF CAPITAL

While equity is undoubtedly an important and indispensable ingredient of the financing mix for every business, it is but one ingredient. Most businesses finance some or much of their operations using debt or some security that is a combination of equity and debt. The costs of these sources of financing are generally very different from the cost of equity, and the cost of financing for a firm should reflect their costs as well, in proportion to their use in the financing mix. Intuitively, the cost of capital is the weighted average of the costs of the different components of financing—including debt, equity, and hybrid securities—used by a firm to fund its financial requirements. This section examines the process of estimating the cost of financing other than equity, and the weights for computing the cost of capital.

Calculating the Cost of Debt

The cost of debt measures the current cost to the firm of borrowing funds to finance projects. In general terms, it is determined by the following variables:

- *The riskless rate.* As the riskless rate increases, the cost of debt for firms will also increase.
- *The default risk (and associated default spread) of the company.* As the default risk of a firm increases, the cost of borrowing money will also increase. Chapter 7 looked at how the default spread has varied across time and can vary across maturity.
- *The tax advantage associated with debt.* Since interest is tax deductible, the after-tax cost of debt is a function of the tax rate. The tax benefit that accrues from paying interest makes the after-tax cost of debt lower than the pretax cost. Furthermore, this benefit increases as the tax rate increases.

$$\text{After-tax cost of debt} = \text{Pretax cost of debt}(1 - \text{Tax rate})$$

This section will focus on how best to estimate the default risk in a firm and to convert that default risk into a default spread that can be used to come up with a cost of debt.

Estimating the Default Risk and Default Spread of a Firm The simplest scenario for estimating the cost of debt occurs when a firm has long-term bonds outstanding that are widely traded. The market price of the bond in conjunction with its coupon and maturity can serve to compute a yield that is used as the cost of debt. For instance, this approach works for a firm like AT&T that has dozens of outstanding bonds that are liquid and trade frequently.

Many firms have bonds outstanding that do not trade on a regular basis. Since these firms are usually rated, we can estimate their costs of debt by using their ratings and associated default spreads. Thus, Boeing with an AA rating can be expected to have a cost of debt approximately 1.00 percent higher than the Treasury bond rate, since this is the spread typically paid by AA-rated firms.

Some companies choose not to get rated. Many smaller firms and most private businesses fall into this category. While ratings agencies have sprung up in many emerging markets, there are still a number of markets where companies are not rated on the basis of default risk. When there is no rating available to estimate the cost of debt, there are two alternatives:

1. *Recent borrowing history.* Many firms that are not rated still borrow money from banks and other financial institutions. By looking at the most recent borrowings made by a firm, we can get a sense of the types of default spreads being charged the firm and use these spreads to come up with a cost of debt.
2. *Estimate a synthetic rating.* An alternative is to play the role of a ratings agency and assign a rating to a firm based on its financial ratios; this rating is called a synthetic rating. To make this assessment, we begin with rated firms and examine the financial characteristics shared by firms within each ratings class. To illustrate, Table 8.1 lists the range of interest coverage ratios for small manufacturing firms in each S&P ratings class.[11]

[11]This table was developed in early 2001 by listing out all rated firms with market capitalization lower than $2 billion and their interest coverage ratios, and then sorting firms based on their bond ratings. The ranges were adjusted to eliminate outliers and to prevent overlapping ranges.

TABLE 8.1 Interest Coverage Ratios and Ratings: Low-Market-Cap Firms

Interest Coverage Ratio	Rating	Spread
More than 12.5	AAA	0.75%
9.5 to 12.5	AA	1.00%
7.5 to 9.5	A+	1.50%
6 to 7.5	A	1.80%
4.5 to 6	A-	2.00%
3.5 to 4.5	BBB	2.25%
3 to 3.5	BB	3.50%
2.5 to 3	B+	4.75%
2 to 2.5	B	6.50%
1.5 to 2	B–	8.00%
1.25 to 1.5	CCC	10.00%
0.8 to 1.25	CC	11.50%
0.5 to 0.8	C	12.70%
Less than 0.5	D	14.00%

Source for raw data: Compustat.

Now consider a small firm that is not rated but has an interest coverage ratio of 6.15. Based on this ratio, a synthetic rating of A would be assessed for the firm.

The interest coverage ratios tend to be lower for larger firms for any given rating. Table 8.2 summarizes these ratios.

This approach can be expanded to allow for multiple ratios and qualitative

TABLE 8.2 Interest Coverage Ratios and Ratings: High-Market-Cap Firms

Interest Coverage Ratio	Rating	Spread
More than 8.5	AAA	0.75%
6.5 to 8.5	AA	1.00%
5.5 to 6.5	A+	1.50%
4.25 to 5.5	A	1.80%
3 to 4.25	A–	2.00%
2.5 to 3	BBB	2.25%
2 to 2.5	BB	3.50%
1.75 to 2	B+	4.75%
1.5 to 1.75	B	6.50%
1.25 to 1.5	B–	8.00%
0.8 to 1.25	CCC	10.00%
0.65 to 0.8	CC	11.50%
0.2 to 0.65	C	12.70%
Less than 0.2	D	14.00%

Source: Compustat.

EXTENDING THE SYNTHETIC RATINGS APPROACH

By basing the rating on the interest coverage ratio alone, we run the risk of missing the information that is available in the other financial ratios used by ratings agencies. The approach can be extended to incorporate other ratios. The first step would be to develop a score based on multiple ratios. For instance, the Altman Z score, which is used as a proxy for default risk, is a function of five financial ratios that are weighted to generate a Z score. The ratios used and their relative weights are usually based on empirical evidence on past defaults. The second step is to relate the level of the score to a bond rating, much as is done in Tables 8.1 and 8.2 with interest coverage ratios.

In making this extension, though, note that complexity comes at a cost. While credit or Z scores may, in fact, yield better estimates of synthetic ratings than those based on interest coverage ratios, changes in ratings arising from these scores are much more difficult to explain than those based on interest coverage ratios. That is a reason to prefer the flawed but simpler ratings derived from interest coverage ratios.

variables as well. Once a synthetic rating is assessed, it can be used to estimate a default spread which when added to the risk-free rate yields a pretax cost of debt for the firm.

Estimating a Tax Rate To estimate the after-tax cost of debt, consider the fact that interest expenses are tax deductible to the firm. While the computation is fairly simple and requires that the pretax cost be multiplied by (1 – Tax rate), the question of what tax rate to use can be a difficult one to answer because there are so many choices. For instance, firms often report an effective tax rate, estimated by dividing the taxes due by the taxable income. The effective tax rate, though, is usually very different from the marginal tax rate, which is the rate at which the last dollar of income is taxed. Since interest expenses save you taxes at the margin (they are deducted from your last dollar of income), the right tax rate to use is the marginal tax rate.

The other caveat to keep in mind is that interest creates a tax benefit only if a firm has enough income to cover the interest expenses. Firms that have operating losses will not get a tax benefit from interest expenses, at least in the year of the loss. The after-tax cost of debt will be equal to the pretax cost of debt in that year. If you expect the firm to make money in future years, you would need to adjust the after-tax cost of debt for taxes in those years.

The book will return to this issue and examine it in more detail in Chapter 10, where we look at the same issue in the context of estimating after-tax cash flows.

ILLUSTRATION 8.11: Estimating the Cost of Debt: Boeing in December 2000

Boeing is rated AA by S&P. Using the typical default spreads for AA-rated firms, we could estimate the pretax cost for Boeing by adding the default spread of 1.00%[12] to the riskless rate of 5%.

$$\text{Pretax cost of debt}_{\text{actual rating}} = 5\% + 1\% = 6\%$$

Boeing has an effective tax rate of 27%, but we use a marginal tax rate of 35%, which is the federal marginal corporate tax rate in the United States, to estimate the after-tax cost of debt for Boeing.

$$\text{After-tax cost of debt} = 6.00\%(1 - .35) = 3.90\%$$

We could also compute a synthetic rating for Boeing based on its interest coverage ratio from 1999. Based on its operating income of $1,720 million in 1999 and interest expense of $453 million in that year, we would have estimated an interest coverage ratio:

$$\text{Interest coverage ratio}_{\text{Boeing}} = 1{,}720/453 = 3.8$$

Using Table 8.2, we would have assigned a synthetic rating of A– to Boeing. Based on default spreads prevailing in December 2000, this would have resulted in a default spread of 2.00% and a pretax cost of debt of 7.00% for the firm.

Estimating the Cost of Debt for an Emerging Market Firm In general, there are three problems that we run into when assessing the cost of debt for emerging market firms. The first is that most of these firms are not rated, leaving us with no option but to estimate the synthetic rating (and associated costs). The second is that the synthetic ratings may be skewed by differences in interest rates between the emerging market and the United States. Interest coverage ratios will usually decline as interest rates increase and it may be far more difficult for a company in an emerging market to achieve the interest coverage ratios of companies in developed markets. Finally, the existence of country default risk hangs over the cost of debt of firms in that market.

The second problem can be fixed fairly simply by either modifying the tables developed using U.S. firms or restating the interest expenses (and interest coverage ratios) in dollar terms. The question of country risk is a thornier one. Conservative analysts often assume that companies in a country cannot borrow at a rate lower than the country itself can borrow at. With this reasoning, the cost of debt for an emerging market company will include the country default spread for the country.

$$\begin{aligned}\text{Cost of debt}_{\text{emerging market company}} &= \text{Riskless rate} + \text{Country default spread}_{\text{emerging market}} \\ &\quad + \text{Company default spread}_{\text{synthetic rating}}\end{aligned}$$

The counter to this argument is that companies may be safer than the countries in which they operate, and that they bear only a portion or perhaps even none of the country default spread.

[12]The default spread was obtained from Table 8.2.

ILLUSTRATION 8.12: Estimating the Cost of Debt: Embraer in March 2001

To estimate Embraer's cost of debt, we first estimated a synthetic rating for the firm. Based on its operating income of $810 million and interest expenses of $28 million in 2000, we arrived at an interest coverage ratio of 28.73 and an AAA rating. While the default spread for AAA-rated bonds was only 0.75% at the time, there is the added consideration that Embraer is a Brazilian firm. Since the Brazilian dollar-denominated government bond had a default spread of 5.37% at the time of the analysis, you could argue that every Brazilian company should pay this premium in addition to its own default spread. With this reasoning, the pretax cost of debt for Embraer in U.S. dollars (assuming a Treasury bond rate is 5%) can be calculated:

$$\text{Cost of debt} = \text{Risk-free rate} + \text{Default spread for country} + \text{Default spread for firm}$$
$$= 5\% + 5.37\% + 0.75\% = 11.12\%$$

Using a marginal tax rate of 33%, we can estimate an after-tax cost of debt for Embraer:

$$\text{After-tax cost of debt} = 11.12\%(1 - .33) = 7.45\%$$

With this approach, the cost of debt for a firm can never be lower than the cost of debt for the country in which it operates. Note, though, that Embraer gets a significant portion of its revenues in dollars from contracts with non-Brazilian airlines. Consequently, it could reasonably argue that it is less exposed to risk than is the Brazilian government and should therefore command a lower cost of debt.

ratings.xls: **This spreadsheet allows you to estimate the synthetic rating and cost of debt for any firm.**

Calculating the Cost of Hybrid Securities

While debt and equity represent the fundamental financing choices available for firms, there are some types of financing that share characteristics with both debt and equity. These are called hybrid securities. This section considers how best to estimate the costs of such securities.

Cost of Preferred Stock Preferred stock shares some of the characteristics of debt (the preferred dividend is prespecified at the time of the issue and is paid out before common dividend) and some of the characteristics of equity (the preferred dividend is not tax deductible). If preferred stock is viewed as perpetual (as it usually is), the cost of preferred stock can be written as follows:

$$k_{ps} = \text{Preferred dividend per share/Market price per preferred share}$$

This approach assumes the dividend is constant in dollar terms forever and that the preferred stock has no special features (convertibility, callability, etc.). If such special features exist, they will have to be valued separately to estimate the cost of preferred stock. In terms of risk, preferred stock is safer than common equity, because preferred dividends are paid before dividends on common equity. It is, however, riskier than debt since interest payments are made prior to preferred dividend payments. Consequently, on a pretax basis, it should command a higher cost than debt and a lower cost than equity.

ILLUSTRATION 8.13: Calculating the Cost of Preferred Stock: General Motors

In March 1995, General Motors had preferred stock that paid a dividend of $2.28 annually and traded at $26.38 per share. The cost of preferred stock can be estimated as follows:

Cost of preferred stock = Preferred dividend per share/Preferred stock price = $2.28/$26.38 = 8.64%

At the same time, GM's cost of equity, using the CAPM, was 13%, its pretax cost of debt was 8.25%, and its after-tax cost of debt was 5.28%. Not surprisingly, its preferred stock was less expensive than equity, but much more expensive than debt.

Calculating the Cost of Other Hybrid Securities A convertible bond is a bond that can be converted into equity at the option of the bondholder. A convertible bond can be viewed as a combination of a straight bond (debt) and a conversion option (equity). Instead of trying to calculate the cost of these hybrid securities individually, we can break down hybrid securities into their debt and equity components and treat the components separately.

ILLUSTRATION 8.14: Breaking Down a Convertible Bond into Debt and Equity Components: Amazon.com, Inc.

In 1999, Amazon.com, Inc., the online retailer, issued convertible bonds with a coupon rate of 4.75% and a 10-year maturity. Since the firm was losing money, it was rated CCC+ by S&P and would have had to pay 11% if it had issued straight bonds at the same time. The bonds were issued at a price that was 98% of par, and the total par value of the convertible bond issue was $1.25 billion. The convertible bond can be broken down into straight bond and conversion option components.

Straight bond component = Value of a straight 4.75% coupon bond due in 10 years with 11% interest rate
= $636 (assuming semiannual coupons)

Conversion option = $980 – $636 = $344

The straight bond component of $636 is treated as debt, and has the same cost as the rest of debt. The conversion option of $344 is treated as equity, with the same cost of equity as other equity issued by the firm. For the entire bond issue of $1.25 billion, the value of debt is $811 million, and the value of equity is $439 million.

Calculating the Weights of Debt and Equity Components

Now that we have the costs of debt, equity, and hybrid securities, we have to estimate the weights that should be attached to each. Before we discuss how best to estimate weights, we define what we include in debt. We then make the argument that weights used should be based on market value and not book value. This is so because the cost of capital measures the cost of issuing securities—stocks as well as bonds—to finance projects, and these securities are issued at market value, not at book value.

What Is Debt? The answer to this question may seem obvious since the balance sheet for a firm shows the outstanding liabilities of a firm. There are, however, limitations with using these liabilities as debt in the cost of capital computation. The first is that some of the liabilities on a firm's balance sheet, such as accounts payable and supplier credit, are not interest-bearing. Consequently, applying an after-tax cost of debt to these items can provide a misleading view of the true cost of capital for a firm. The second is that there are items off the balance sheet that create fixed commitments for the firm and provide the same tax deductions that interest payments on debt do. The most prominent of these off-balance sheet items are operating leases. Chapter 3 contrasted operating and capital leases and noted that operating leases are treated as operating expenses rather than financing expenses. Consider, though, what an operating lease involves. A retail firm leases a store space for 12 years and enters into a lease agreement with the owner of the space agreeing to pay a fixed amount each year for that period. We do not see much difference between this commitment and borrowing money from a bank and agreeing to pay off the bank loan over 12 years in equal annual installments.

There are therefore two adjustments we will make when we estimate how much debt a firm has outstanding.

1. We will consider only interest-bearing debt rather than all liabilities. We would include both short-term and long-term borrowings in debt.
2. We will also capitalize operating leases and treat them as debt.

Capitalizing Operating Leases Converting operating lease expenses into a debt equivalent is straightforward. The operating lease commitments in future years, which are revealed in the footnotes to the financial statements for U.S. firms, should be discounted back at a rate that reflects their status as unsecured and fairly risky debt. As an approximation, using the firm's current pretax cost of borrowing as the discount rate yields a good estimate of the value of operating leases.

Outside the United States, firms do not have to reveal their operating lease commitments in future periods. When this is the case, you can get a reasonably close estimate of the debt value of operating leases by estimating the present value of an annuity equal to the current year's payment for a period that reflects a typical lease period (8 to 10 years).

There is one final issue relating to capitalization. Earlier in this chapter it was stated that the interest coverage ratio could be used to estimate a synthetic rating for a firm that is not rated. For firms with little in terms of conventional debt and substantial operating leases, the interest coverage ratio used to estimate a synthetic rating has to be adapted to include operating lease expenses.

$$\text{Modified interest coverage ratio} = (\text{EBIT} + \text{Current year's operating lease expense}) \\ /(\text{Interest expenses} \\ + \text{Current year's operating lease expense})$$

This ratio can then be used in conjunction with Tables 8.1 and 8.2 to estimate a synthetic rating.

ILLUSTRATION 8.15: The Debt Value of Operating Leases: Boeing in December 2000

Boeing has both conventional debt and operating lease commitments. This illustration will estimate the "debt value" of Boeing's operating leases by taking the present value of operating lease expenses over time. To compute the present value of operating leases in the following table (in $millions), we use the pretax cost of borrowing for the firm, estimated in Illustration 8.11 to be 6%.

Year	*Operating Lease Expense*	*Present Value at 6%*
1	$205	$193.40
2	$167	$146.83
3	$120	$100.75
4	$ 86	$ 68.12
5	$ 61	$ 45.58
6 to15	$ —	$ 0.00
Present value of operating lease expenses		$556.48

Thus, Boeing has $556 million more in debt than is reported in the balance sheet.

***Oplease.xls:* This spreadsheet allows you to convert operating lease expenses into debt.**

Book Value versus Market Value Debt Ratios There are three standard arguments against using market value, and none of them is convincing. First, there are some financial managers who argue that book value is more reliable than market value because it is not as volatile. While it is true that book value does not change as much as market value, this is more a reflection of book value's weakness rather than its strength, since the true value of the firm changes over time as both firm-specific and market information is revealed. We would argue that market value, with its volatility, is a much better reflection of true value than is book value.[13]

Second, the defenders of book value also suggest that using book value rather than market value is a more conservative approach to estimating debt ratios. This assumes that market value debt ratios are always lower than book value debt ratios, an assumption not based on fact. Furthermore, even if the market value debt ratios are lower than the book value ratios, the cost of capital calculated using book value ratios will be lower than those calculated using market value ratios, making them less conservative estimates, not more. To illustrate this point, assume that the market value debt ratio is 10 percent, while the book value debt ratio is 30 percent, for a firm with a cost of equity of 15 percent and an after-tax cost of debt of 5 percent. The cost of capital can be calculated as follows:

$$\text{With market value debt ratios: } 15\%(.9) + 5\%(.1) = 14\%$$

$$\text{With book value debt ratios: } 15\%(.7) + 5\%(.3) = 12\%$$

[13]There are some who argue that stock prices are much more volatile than the underlying true value. Even if this argument is justified (and it has not conclusively been shown to be so), the difference between market value and true value is likely to be much smaller than the difference between book value and true value.

Third, it is claimed that lenders will not lend on the basis of market value, but this claim again seems to be based more on perception than on fact. Any homeowner who has taken a second mortgage on a house that has appreciated in value knows that lenders do lend on the basis of market value. It is true, however, that the greater the perceived volatility in the market value of an asset, the lower is the borrowing potential on that asset.

Estimating the Market Values of Equity and Debt The market value of equity is generally the number of shares outstanding times the current stock price. If there are other equity claims in the firm such as warrants and management options, these should also be valued and added to the value of the equity in the firm.

The market value of debt is usually more difficult to obtain directly, since very few firms have all their debt in the form of bonds outstanding trading in the market. Many firms have nontraded debt, such as bank debt, which is specified in book value terms but not market value terms. A simple way to convert book value debt into market value debt is to treat the entire debt on the books as one coupon bond, with a coupon set equal to the interest expenses on all the debt and the maturity set equal to the face-value weighted average maturity of the debt, and then to value this coupon bond at the current cost of debt for the company. Thus, the market value of \$1 billion in debt, with interest expenses of \$60 million and a maturity of six years, when the current cost of debt is 7.5 percent, can be estimated as follows:

$$\text{Estimated market value of debt} = 60\left(\frac{1-\dfrac{1}{1.075^6}}{.075}\right) + \frac{1{,}000}{1.075^6} = \$930$$

ILLUSTRATION 8.16: Difference between Market Value and Book Value Debt Ratios: Boeing in June 2000

This illustration contrasts the book values of debt and equity with the market values. For debt, we estimate the market value of debt using the book value of debt, the interest expense on the debt, the average maturity of the debt, and the pretax cost of debt for each firm. For Boeing, the book value of debt is \$6,972 million, the interest expense on the debt is \$453 million, the average maturity of the debt is 13.76 years, and the pretax cost of debt is 6.00%. The estimated market value is:

$$\text{Estimated MV of Boeing debt} = 453\left(\frac{1-\dfrac{1}{1.06^{13.76}}}{.06}\right) + \frac{6{,}972}{(1.06)^{13.76}} = \$7{,}291$$

To this, we need to add the present value of operating leases of \$556 million to arrive at a total market value for debt of \$7,847 million.

The book value of equity for Boeing was \$12,316 million while the market value of equity was \$55,197 million. The debt ratios in market value and book value terms are computed as follows:

	Market Value	*Book Value*
Debt to equity	7,847/55,197 = 14.22%	6,972/12,316 = 56.61%
Debt/(Debt + Equity)	7,847/(7,847 + 55,197) = 12.45%	6,972/(6,972 + 12,316) = 361.5%

The market debt ratio is significantly lower than the book debt ratio.

GROSS DEBT VERSUS NET DEBT

Gross debt refers to all debt outstanding in a firm. Net debt is the difference between gross debt and the cash balance of the firm. For instance, a firm with $1.25 billion in interest-bearing debt outstanding and a cash balance of $1 billion has a net cash balance of $250 million. The practice of netting cash against debt is common in both Latin America and Europe, and debt ratios are usually estimated using net debt.

It is generally safer to value a firm based on gross debt outstanding and to add the cash balance outstanding to the value of operating assets to arrive at the firm value. The interest payment on total debt is then entitled to the tax benefits of debt, and we can assess the effect of whether the company invests its cash balances efficiently on value.

In some cases, especially when firms maintain large cash balances as a matter of routine, analysts prefer to work with net debt ratios. If we choose to use net debt ratios, we have to be consistent all the way through the valuation. To begin, the beta for the firm should be estimated using a net debt-to-equity ratio rather than a gross debt-to-equity ratio. The cost of equity that emerges from the beta estimate can be used to estimate a cost of capital, but the market value weight on debt should be based on net debt. Once we discount the cash flows of the firm at the cost of capital, we should not add back cash. Instead, we should subtract the net debt outstanding to arrive at the estimated value of equity.

Implicitly, when we net cash against debt to arrive at net debt ratios, we are assuming that cash and debt have roughly similar risk. While this assumption may not be outlandish when analyzing highly rated firms, it becomes much shakier when debt becomes riskier. For instance, the debt in a BB-rated firm is much riskier than the cash balance in the firm, and netting out one against the other can provide a misleading view of the firm's default risk. In general, using net debt ratios will overstate the value of riskier firms.

wacccalc.xls: **This spreadsheet allows you to convert book values of debt into market values.**

Estimating the Cost of Capital

Since a firm can raise its money from three sources—equity, debt, and preferred stock—the cost of capital is defined as the weighted average of each of these costs. The cost of equity (k_e) reflects the riskiness of the equity investment in the firm, the after-tax cost of debt (k_d) is a function of the default risk of the firm, and the cost of preferred stock (k_{ps}) is a function of its intermediate standing in terms of risk between debt and equity. The weights on each of these components should reflect their market value proportions, since these proportions best measure how the existing firm is being financed. Thus if E, D, and PS are the market values of equity, debt, and preferred stock respectively, the cost of capital can be written as follows:

$$\text{Cost of capital} = k_e[E/(D + E + PS)] + k_d[D/(D + E + PS)] + k_{ps}[PS/(D + E + PS)]$$

ILLUSTRATION 8.17: Estimating Cost of Capital: Boeing in December 2000

Having estimated the costs of debt and equity in earlier illustrations, and the market value debt ratio in Illustration 8.16, we can put them together to arrive at a cost of capital for Boeing.

Cost of equity = 10.28% (from Illustration 8.9)
Cost of debt = 3.90% (from Illustration 8.11)
Market value debt ratio = 12.45% (from Illustration 8.16)
Cost of capital = 10.28%(.8755) + 3.90%(.1245) = 9.49%

ILLUSTRATION 8.18: Estimating Cost of Capital: Embraer in March 2001

To estimate a cost of capital for Embraer, we again draw on the estimates of cost of equity and cost of debt we obtained in prior illustrations. The cost of capital will be estimated using net debt all the way through (for the levered betas, interest coverage ratios, and debt ratios) and in U.S. dollars:

Cost of equity = 18.86% (from Illustration 8.10)
After-tax cost of debt = 7.45% (from Illustration 8.12)
Market value of debt = 1,328 million BR
Cash and marketable securities = 1,105 million BR
Market value of equity = 9,084 million BR

The cost of capital for Embraer is estimated as follows:

$$\text{Net debt} = 1{,}328 \text{ million BR} - 1{,}105 \text{ million BR} = 223 \text{ million BR}$$

$$\text{Cost of capital} = 18.86\%[9{,}084/(9{,}084 + 223)] + 7.45\%[223/(9{,}084 + 223)] = 18.59\%$$

To convert this into a nominal BR cost of capital, we would apply the differential inflation rates (10% in Brazil and 2% in the United States).

$$\begin{aligned}\text{Cost of capital}_{\text{nominal BR}} &= (1 + \text{Cost of capital}_{\$})(\text{Inflation rate}_{\text{Brazil}}/\text{Inflation rate}_{\text{U.S.}}) - 1\\ &= (1.1859)(1.10/1.02) - 1 = 27.89\%\end{aligned}$$

BEST PRACTICES AT FIRMS

We have spent this chapter discussing what firms should do when it comes to estimating the cost of capital. What do they actually do? Bruner, Eades, Harris, and Higgins surveyed 27 well-regarded corporations, and their findings are summarized in Table 8.3.

CONCLUSION

When we analyze the investments of a firm or assess its value, we need to know the cost that the firm faces in raising equity, debt, and capital. The risk and return models described in earlier chapters can be used to estimate the costs of equity and capital for a firm.

Building on the premise that the cost of equity should reflect the riskiness of equity to investors in the firm, there are three basic inputs we need to estimate the

TABLE 8.3 Current Practices for Estimating Cost of Capital

Cost of Capital Item	Current Practices
Cost of equity	• 81% of firms used the capital asset pricing model to estimate the cost of equity, 4% used a modified capital asset pricing model, and 15% were uncertain about how they estimated the cost of equity. • 70% of firms used 10-year Treasuries or longer as the riskless rate, 7% used 3- to 5-year Treasuries, and 4% used the Treasury bill rate. • 52% used a published source for a beta estimate, while 30% estimated it themselves. • There was wide variation in the market risk premium used, with 37% using a premium between 5% and 6%.
Cost of debt	• 52% of firms used a marginal borrowing rate and a marginal tax rate, while 37% used the current average borrowing rate and the effective tax rate.
Weights for debt and equity	• 59% used market value weights for debt and equity in the cost of capital, 15% used book value weights, and 19% were uncertain about what weights they used.

Source: Bruner, Eades, Harris, and Higgins (1998).

cost of equity for any firm. The riskless rate is the expected return on an investment with no default risk and no reinvestment risk. Since much of the analysis in corporate finance is long term, the riskless rate should be the interest rate on a long-term government bond. The risk premium measures what investors demand as a premium for investing in risky investments instead of riskless investments. This risk premium, which can vary across investors, can be estimated either by looking at past returns on stocks and government securities or by looking at how the market prices stocks currently. The beta for a firm is conventionally measured using a regression of returns on the firm's stock against returns on a market index. This approach yields imprecise beta estimates, and we are better off estimating betas by examining the betas of the businesses that the firm operates in.

The cost of capital is a weighted average of the costs of the different components of financing, with the weights based on the market values of each component. The cost of debt is the market rate at which the firm can borrow, adjusted for any tax advantages of borrowing. The cost of preferred stock, however, is the preferred dividend yield.

The cost of capital is useful at two levels. On a composite basis, it is what these firms have to make collectively on their investments to break even. It is also the appropriate discount rate to use to discount expected future cash flows to arrive at an estimate of firm value.

QUESTIONS AND SHORT PROBLEMS

1. In December 1995, Boise Cascade's stock had a beta of 0.95. The Treasury bill rate at the time was 5.8%, and the Treasury bond rate was 6.4%. The firm had debt outstanding of $1.7 billion and a market value of equity of $1.5 billion; the corporate marginal tax rate was 36%.

a. Estimate the expected return on the stock for a short-term investor in the company.
b. Estimate the expected return on the stock for a long-term investor in the company.
c. Estimate the cost of equity for the company.

2. Continuing problem 1, Boise Cascade also had debt outstanding of $1.7 billion and a market value of equity of $1.5 billion; the corporate marginal tax rate was 36%.
   a. Assuming that the current beta of 0.95 for the stock is a reasonable one, estimate the unlevered beta for the company.
   b. How much of the risk in the company can be attributed to business risk and how much to financial leverage risk?
3. Biogen Inc., a biotechnology firm, had a beta of 1.70 in 1995. It had no debt outstanding at the end of that year.
   a. Estimate the cost of equity for Biogen if the Treasury bond rate is 6.4%.
   b. What effect will an increase in long-term bond rates to 7.5% have on Biogen's cost of equity?
   c. How much of Biogen's risk can be attributed to business risk?
4. Genting Berhad is a Malaysian conglomerate with holdings in plantations and tourist resorts. The beta estimated for the firm relative to the Malaysian stock exchange is 1.15, and the long-term government borrowing rate in Malaysia is 11.5%. (The Malaysian risk premium is 12%.)
   a. Estimate the expected return on the stock.
   b. If you were an international investor, what concerns, if any, would you have about using the beta estimated relative to the Malaysian Index? If you do have concerns, how would you modify the beta?
5. You have just done a regression of monthly stock returns of HeavyTech Inc., a manufacturer of heavy machinery, on monthly market returns over the past five years and have come up with the following regression:

$$R_{HeavyTech} = 0.5\% + 1.2\ R_M$$

   The variance of the stock is 50%, and the variance of the market is 20%. The current T-bill rate is 3% (it was 5% one year ago). The stock is currently selling for $50, down $4 over the past year; it has paid a dividend of $2 during the past year and expects to pay a dividend of $2.50 over the next year. The NYSE Composite has gone down 8% over the past year, with a dividend yield of 3%. HeavyTech Inc. has a tax rate of 40%.
   a. What is the expected return on HeavyTech over the next year?
   b. What would you expect HeavyTech's price to be one year from today?
   c. What would you have expected HeavyTech's stock returns to be over the plast year?
   d. What were the actual returns on HeavyTech over the past year?
   e. HeavyTech has $100 million in equity and $50 million in debt. It plans to issue $50 million in new equity and retire $50 million in debt. Estimate the new beta.
6. Safecorp, which owns and operates grocery stores across the United States, currently has $50 million in debt and $100 million in equity outstanding. Its stock has a beta of 1.2. It is planning a leveraged buyout (LBO), where it will increase its debt-to-equity ratio of 8. If the tax rate is 40%, what will the beta of the equity in the firm be after the LBO?

7. Novell, which had a market value of equity of $2 billion and a beta of 1.50, announced that it was acquiring WordPerfect, which had a market value of equity of $1 billion and a beta of 1.30. Neither firm had any debt in its financial structure at the time of the acquisition, and the corporate tax rate was 40%.
   a. Estimate the beta for Novell after the acquisition, assuming that the entire acquisition was financed with equity.
   b. Assume that Novell had to borrow the $1 billion to acquire WordPerfect. Estimate the beta after the acquisition.
8. You are analyzing the beta for Hewlett Packard (HP) and have broken down the company into four broad business groups, with market values and betas for each group.

Business Group	*Market Value of Equity*	*Beta*
Mainframes	$2.0 billion	1.10
Personal Computers	$2.0 billion	1.50
Software	$1.0 billion	2.00
Printers	$3.0 billion	1.00

   a. Estimate the beta for Hewlett Packard as a company. Is this beta going to be equal to the beta estimated by regressing past returns on HP stock against a market index? Why or why not?
   b. If the Treasury bond rate is 7.5%, estimate the cost of equity for Hewlett Packard. Estimate the cost of equity for each division. Which cost of equity would you use to value the printer division?
   c. Assume that HP divests itself of the mainframe business and pays the cash out as a dividend. Estimate the beta for HP after the divestiture. (HP had $1 billion in debt outstanding.)
9. The following table summarizes the percentage changes in operating income, percentage changes in revenue, and betas for four pharmaceutical firms.

Firm	*% Change in Revenue*	*% Change in Operating Income*	*Beta*
PharmaCorp	27%	25%	1.00
SynerCorp	25%	32%	1.15
BioMed	23%	36%	1.30
Safemed	21%	40%	1.40

   a. Calculate the degree of operating leverage for each of these firms.
   b. Use the operating leverage to explain why these firms have different betas.
10. A prominent beta estimation service reports the beta of Comcast Corporation, a major cable TV operator, to be 1.45. The service claims to use weekly returns on the stock over the prior five years and the NYSE Composite as the market index to estimate betas. You replicate the regression using weekly returns over the same period and arrive at a beta estimate of 1.60. How would you reconcile the two estimates?
11. Battle Mountain is a mining company with gold, silver, and copper in mines in South America, Africa, and Australia. The beta for the stock is estimated to be 0.30. Given the volatility in commodity prices, how would you explain the low beta?
12. You have collected returns on AnaDone Corporation (AD Corp.), a large, diversified manufacturing firm, and the NYSE index for five years:

Year	*AD Corp.*	*NYSE*
1981	10%	5%
1982	5%	15%
1983	–5%	8%
1984	20%	12%
1985	–5%	–5%

a. Estimate the intercept (alpha) and slope (beta) of the regression.
b. If you bought stock in AD Corp. today, how much would you expect to make as a return over the next year? (The six-month T-bill rate is 6%.)
c. Looking back over the past five years, how would you evaluate AD Corp.'s performance relative to the market?
d. Assume now that you are an undiversified investor and that you have all of your money invested in AD Corp. What would be a good measure of the risk that you are taking on? How much of this risk would you be able to eliminate if you diversify?
e. AD Corp. is planning to sell off one of its divisions. The division under consideration has assets that comprise half of the book value of AD Corp. and 20% of the market value. Its beta is twice the average beta for AD Corp. (before divestment). What will the beta of AD Corp. be after divesting this division?

13. You run a regression of monthly returns of Mapco Inc., an oil- and gas-producing firm, on the S&P 500 index, and come up with the following output for the period 1991 to 1995:

    Intercept of the regression = 0.06%
    Slope of the regression = 0.46
    Standard error of X-coefficient = 0.20
    R-squared = 5%

    There are 20 million shares outstanding, and the current market price is $2 per share. The firm has $20 million in debt outstanding. (The firm has a tax rate of 36%.)
    a. What would an investor in Mapco's stock require as a return if the T-bond rate is 6%?
    b. What proportion of this firm's risk is diversifiable?
    c. Assume now that Mapco has three divisions of equal size (in market value terms). It plans to divest itself of one of the divisions for $20 million in cash and acquire another for $50 million (it will borrow $30 million to complete this acquisition). The division it is divesting is in a business line where the average unlevered beta is 0.20, and the division it is acquiring is in a business line where the average unlevered beta is 0.80. What will the beta of Mapco be after this acquisition?
14. You have just run a regression of monthly returns of American Airlines (AMR Corporation) against the S&P 500 over the past five years. You have misplaced some of the output and are trying to derive it from what you have.
    a. You know the R-squared of the regression is 0.36, and that your stock has a variance of 67%. The market variance is 12%. What is the beta of AMR?
    b. You also remember that AMR was not a very good investment during the period of the regression and that it did worse than expected (after adjusting

for risk) by 0.39% a month for the five years of the regression. During this period, the average risk-free rate was 4.84%. What was the intercept on the regression?

c. You are comparing AMR Inc. to another firm, which also has an R-squared of 0.48. Will the two firms have the same beta? If not, why not?

15. You have run a regression of monthly returns on Amgen, a large biotechnology firm, against monthly returns on the S&P 500 index, and come up with the following output:

$$R_{stock} = 3.28\% + 1.65\ R_{market} \qquad R^2 = 0.20$$

The current one-year Treasury bill rate is 4.8% and the current 30-year bond rate is 6.4%. The firm has 265 million shares outstanding, selling for $30 per share.

a. What is the expected return on this stock over the next year?
b. Would your expected return estimate change if the purpose was to get a discount rate to value the company?
c. An analyst has estimated, correctly, that the stock did 51.10% better than expected, annually, during the period of the regression. Can you estimate the annualized risk-free rate that she used for her estimate?
d. The firm has a debt/equity ratio of 3% and faces a tax rate of 40%. It is planning to issue $2 billion in new debt and acquire a new business for that amount, with the same risk level as the firm's existing business. What will the beta be after the acquisition?

16. You have just run a regression of monthly returns on MAD Inc., a newspaper and magazine publisher, against returns on the S&P 500, and arrived at the following result:

$$R_{MAD} = -0.05\% + 1.20\ R_{S\&P}$$

The regression has an R-squared of 22%. The current T-bill rate is 5.5%, and the current T-bond rate is 6.5%. The risk-free rate during the period of the regression was 6%. Answer the following questions relating to the regression:

a. Based on the intercept, how well or badly did MAD do, relative to expectations, during the period of the regression.
b. You now realize that MAD Inc. went through a major restructuring at the end of last month (which was the last month of your regression), and made the following changes:
   - The firm sold off its magazine division, which had an unlevered beta of 0.6, for $20 million.
   - It borrowed an additional $20 million, and bought back stock worth $40 million.

   After the sale of the division and the share repurchase, MAD Inc. had $40 million in debt and $120 million in equity outstanding. If the firm's tax rate is 40%, reestimate the beta after these changes.

17. Time Warner Inc., the entertainment conglomerate, has a beta of 1.61. Part of the reason for the high beta is the debt left over from the leveraged buyout of Time by Warner in 1989, which amounted to $10 billion in 1995. The market value of equity at Time Warner in 1995 was also $10 billion. The marginal tax rate was 40%.

a. Estimate the unlevered beta for Time Warner.
b. Estimate the effect of reducing the debt ratio by 10% each year for the next two years on the beta of the stock.

18. Chrysler, the automotive manufacturer, had a beta of 1.05 in 1995. It had $13 billion in debt outstanding in that year, and 355 million shares trading at $50 per share. The firm had a cash balance of $8 billion at the end of 1995. The marginal tax rate was 36%.
    a. Estimate the unlevered beta of the firm.
    b. Estimate the effect of paying out a special dividend of $5 billion on this unlevered beta.
    c. Estimate the beta for Chrysler after the special dividend.
19. You are trying to estimate the beta of a private firm that manufactures home appliances. You have managed to obtain betas for publicly traded firms that also manufacture home appliances.

Firm	*Beta*	*Debt*	*MV of Equity*
Black & Decker	1.40	$2,500	$3,000
Fedders Corp.	1.20	$ 5	$ 200
Maytag Corp.	1.20	$ 540	$2,250
National Presto	0.70	$ 8	$ 300
Whirlpool	1.50	$2,900	$4,000

    The private firm has a debt equity ratio of 25% and faces a tax rate of 40%. The publicly traded firms all have marginal tax rates of 40% as well.
    a. Estimate the beta for the private firm.
    b. What concerns, if any, would you have about using betas of comparable firms?
20. As the result of stockholder pressure, RJR Nabisco is considering spinning off its food division. You have been asked to estimate the beta for the division, and decide to do so by obtaining the beta of comparable publicly traded firms. The average beta of comparable publicly traded firms is 0.95, and the average debt-to-equity ratio of these firms is 35%. The division is expected to have a debt ratio of 25%. The marginal corporate tax rate is 36%.
    a. What is the beta for the division?
    b. Would it make any difference if you knew that RJR Nabisco had a much higher fixed cost structure than the comparable firms used here?
21. Southwestern Bell, a phone company, is considering expanding its operations into the media business. The beta for the company at the end of 1995 was 0.90, and the debt-to-equity ratio was 1. The media business is expected to be 30% of the overall firm value in 1999, and the average beta of comparable firms is 1.20; the average debt-to-equity ratio for these firms is 50%. The marginal corporate tax rate is 36%.
    a. Estimate the beta for Southwestern Bell in 1999, assuming that it maintains its current debt-to-equity ratio.
    b. Estimate the beta for Southwestern Bell in 1999, assuming that it decides to finance its media operations with a debt-to-equity ratio of 50%.
22. The chief financial officer of Adobe Systems, a growing software manufacturing firm, has approached you for some advice regarding the beta of his company. He subscribes to a service that estimates Adobe Systems' beta each year, and he has noticed that the beta estimates have gone down every year since 1991—from 2.35 in 1991 to 1.40 in 1995. He would like the answers to the following questions:

a. Is this decline in beta unusual for a growing firm?
b. Why would the beta decline over time?
c. Is the beta likely to keep decreasing over time?

23. You are analyzing Tiffany & Company, an upscale retailer, and find that the regression estimate of the firm's beta is 0.75; the standard error for the beta estimate is 0.50. You also note that the average unlevered beta of comparable specialty retailing firms is 1.15.
    a. If Tiffany has a debt/equity ratio of 20%, estimate the beta for the company based on comparable firms. (The tax rate is 40%.)
    b. Estimate a range for the beta from the regression.
    c. Assume that Tiffany is rated BBB and that the default spread for BBB-rated firms is 1% over the Treasury bond rate. If the Treasury bond rate is 6.5%, estimate the cost of capital for the firm.
24. You have been asked to estimate the cost of capital for NewTel, a telecom firm. The firm has the following characteristics:
    - There are 100 million shares outstanding, trading at $250 per share.
    - The firm has a book value of debt with a maturity of six years of $10 billion, and interest expenses of $600 million on the debt. The firm is not rated, but it had operating income of $2.5 billion last year. (Firms with an interest coverage ratio of 3.5 to 4.5 were rated BBB, and the default spread was 1%.)
    - The tax rate for the firm is 35%.

    The Treasury bond rate is 6%, and the unlevered beta of other telecom firms is 0.80.
    a. Estimate the market value of debt for this firm.
    b. Based on the synthetic rating, estimate the cost of debt for this firm.
    c. Estimate the cost of capital for this firm.