CHAPTER 13

Dividend Discount Models

In the strictest sense, the only cash flow you receive from a firm when you buy publicly traded stock in it is a dividend. The simplest model for valuing equity is the dividend discount model—the value of a stock is the present value of expected dividends on it. While many analysts have turned away from the dividend discount model and view it as outmoded, much of the intuition that drives discounted cash flow valuation is embedded in the model. In fact, there are companies where the dividend discount model remains a useful tool for estimating value.

This chapter explores the general model as well as specific versions of it tailored for different assumptions about future growth. It also examines issues in using the dividend discount model and the results of studies that have looked at its efficacy.

THE GENERAL MODEL

When an investor buys stock, he or she generally expects to get two types of cash flows—dividends during the period the stock is held and an expected price at the end of the holding period. Since this expected price is itself determined by future dividends, the value of a stock is the present value of dividends through infinity:

$$\text{Value per share of stock} = \sum_{t=1}^{t=\infty} \frac{E(DPS_t)}{(1+k_e)^t}$$

where DPS_t = Expected dividends per share
k_e = Cost of equity

The rationale for the model lies in the present value rule—the value of any asset is the present value of expected future cash flows, discounted at a rate appropriate to the riskiness of the cash flows being discounted.

There are two basic inputs to the model—expected dividends and the cost on equity. To obtain the expected dividends, we make assumptions about expected future growth rates in earnings and payout ratios. The required rate of return on a stock is determined by its riskiness, measured differently in different models—the market beta in the capital asset pricing model (CAPM) and the factor betas in the arbitrage and multifactor models. The model is flexible enough to allow for time-varying discount rates, where the time variation is because of expected changes in interest rates or risk across time.

VERSIONS OF THE MODEL

Since projections of dollar dividends cannot be made through infinity, several versions of the dividend discount model have been developed based on different assumptions about future growth. We will begin with the simplest—a model designed to value stock in a stable growth firm that pays out what it can afford to in dividends—and then look at how the model can be adapted to value companies in high growth that may be paying little or no dividends.

The Gordon Growth Model

The Gordon growth model can be used to value a firm that is in "steady state" with dividends growing at a rate that can be sustained forever.

The Model The Gordon growth models relates the value of a stock to its expected dividends in the next time period, the cost of equity, and the expected growth rate in dividends.

$$\text{Value of stock} = \frac{\text{DPS}_1}{k_e - g}$$

where DPS_1 = Expected dividends next year
k_e = Cost of equity
g = Growth rate in dividends forever

What Is a Stable Growth Rate? While the Gordon growth model provides a simple approach to valuing equity, its use is limited to firms that are growing at a stable growth rate. There are two insights worth keeping in mind when estimating a stable growth rate. First, since the growth rate in the firm's dividends is expected to last forever, the firm's other measures of performance (including earnings) can also be expected to grow at the same rate. To see why, consider the consequences in the long term of a firm whose earnings grow 6 percent a year forever, while its dividends grow at 8 percent. Over time, the dividends will exceed earnings. If a firm's earnings grow at a faster rate than dividends in the long term, the payout ratio, in the long term, will converge toward zero, which is also not a steady state. Thus, though the model's requirement is for the expected growth rate in dividends, analysts should be able to substitute in the expected growth rate in earnings and get precisely the same result, if the firm is truly in steady state.

The second issue relates to what growth rate is reasonable as a stable growth rate. As noted in Chapter 12, this growth rate has to be less than or equal to the growth rate of the economy in which the firm operates. This does not, however, imply that analysts will always agree about what this rate should be even if they agree that a firm is a stable growth firm for three reasons:

1. Given the uncertainty associated with estimates of expected inflation and real growth in the economy, there can be differences in the benchmark growth rate used by different analysts (i.e., analysts with higher expectations

of inflation in the long term may project a nominal growth rate in the economy that is higher).

2. The growth rate of a company cannot be greater than the stable growth rate but can be less. Firms can become smaller over time relative to the economy.
3. There is another instance in which an analyst may be stray from a strict limit imposed on the stable growth rate. If a firm is likely to maintain a few years of “above-stable” growth rates, an approximate value for the firm can be obtained by adding a premium to the stable growth rate, to reflect the above-average growth in the initial years. Even in this case, the flexibility that the analyst has is limited. The sensitivity of the model to growth implies that the stable growth rate cannot be more than 1 percent or 2 percent above the growth rate in the economy. If the deviation become larger, the analyst will be better served using a two-stage or a three-stage model to capture the supernormal or above-average growth, and restricting the Gordon growth model to when the firm becomes truly stable.

Limitations of the Model The Gordon growth model is extremely sensitive to the inputs for the growth rate. Used incorrectly, it can yield misleading or even absurd results since as the growth rate converges on the discount rate, the value goes to infinity. Consider a stock with an expected dividend per share next period of $2.50, a cost of equity of 15 percent, and an expected growth rate of 5 percent forever. The value of this stock is:

$$\text{Value} = 2.50/(.15 - .05) = \$25$$

Note, however, the sensitivity of this value to estimates of the growth rate in Figure 13.1. As the growth rate approaches the cost of equity, the value per share ap-

DOES A STABLE GROWTH RATE HAVE TO BE CONSTANT OVER TIME?

The assumption that the growth rate in dividends has to be constant over time is a difficult assumption to meet, especially given the volatility of earnings. If a firm has an average growth rate that is close to a stable growth rate, the model can be used with little real effect on value. Thus a cyclical firm that can be expected to have year-to-year swings in growth rates, but has an average growth rate that is 5 percent, can be valued using the Gordon growth model, without a significant loss of generality. There are two reasons for this result. First, since dividends are smoothed even when earnings are volatile, they are less likely to be affected by year-to-year changes in earnings growth. Second, the mathematical effects on present value of using year-specific growth rates rather than a constant growth rate are small.

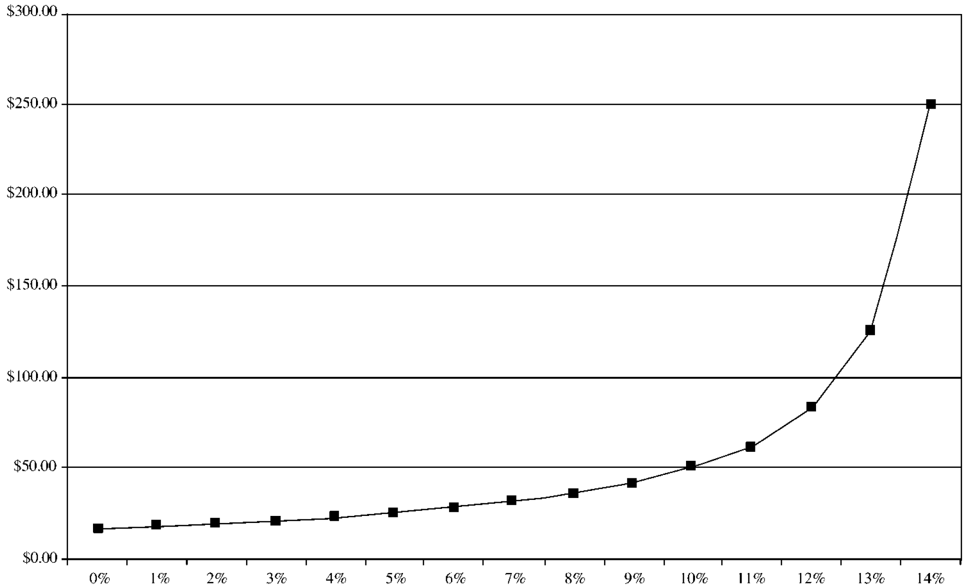

FIGURE 13.1 Value per Share and Expected Growth Rate

proaches infinity. If the growth rate exceeds the cost of equity, the value per share becomes negative.

This issue is tied to the question of what comprises a stable growth rate. If an analyst follows the constraints discussed in the previous chapter in estimating stable growth rates, this will never happen. In this example, for instance, an analyst who uses a 14 percent growth rate and obtains a $250 value would have been violating a basic rule on what comprises stable growth.

Firms Model Works Best For In summary, the Gordon growth model is best suited for firms growing at a rate equal to or lower than the nominal growth in the economy and which have well established dividend payout policies that they intend to continue into the future. The dividend payout of the firm has to be consistent with the assumption of stability, since stable firms generally pay substantial dividends.[1] In particular, this model will underestimate the value of the stock in firms that consistently pay out less than they can afford to and accumulate cash in the process.

[1]The average payout ratio for large stable firms in the United States is about 60%.

ILLUSTRATION 13.1: Regulated Firm: Consolidated Edison in May 2001

Consolidated Edison is the electric utility that supplies power to homes and businesses in New York City and its environs. It is a monopoly whose prices and profits are regulated by the state of New York.

Rationale for Using the Model

- The firm is in stable growth based on its size and the area that it serves. Its rates are also regulated; it is unlikely that the regulators will allow profits to grow at extraordinary rates.
- The firm is in a stable business and regulation is likely to restrict expansion into new businesses.
- The firm is in stable leverage.
- The firm pays out dividends that are roughly equal to FCFE.
  - Average annual FCFE between 1996 and 2000 = $551 million
  - Average annual dividends between 1996 and 2000 = $506 million
  - Dividends as % of FCFE = 91.54%

Background Information

Earnings per share in 2000 = $3.13
Dividend payout ratio in 2000 = 69.97%
Dividends per share in 2000 = $2.19
Return on equity = 11.63%

Estimates

We first estimate the cost of equity, using a bottom-up levered beta for electric utilities of 0.90, a risk-free rate of 5.40% and a market risk premium of 4%:

$$\text{Con Ed beta} = 0.90$$

$$\text{Cost of equity} = 5.4\% + 0.90 \times 4\% = 9\%$$

We estimate the expected growth rate from fundamentals:

$$\begin{aligned}\text{Expected growth rate} &= (1 - \text{Payout ratio})\text{Return on equity} \\ &= (1 - .6997).1163 = 3.49\%\end{aligned}$$

Valuation

We now use the Gordon growth model to value the equity per share at Con Ed:

$$\begin{aligned}\text{Value of equity} &= \text{Expected dividends next year}/(\text{Cost of equity} - \text{Expected growth rate}) \\ &= \$2.19(1.0349)/(.09 - .0349) = \$41.15\end{aligned}$$

Con Ed was trading for $36.59 on the day of this analysis (May 14, 2001). Based on this valuation, the stock would have been undervalued.

IMPLIED GROWTH RATE

The value for Con Ed is different from the market price, and this is likely to be the case with almost any company that you value. There are three possible explanations for this deviation. One is that you are right and the market is wrong. While this may be the correct explanation, you should probably make sure that the other two explanations do not hold—that the market is right and you are wrong or that the difference is too small to draw any conclusions.

To examine the magnitude of the difference between the market price and your estimate of value, you can hold the other variables constant and change the growth rate in your valuation until the value converges on the price. Figure 13.2 estimates value as a function of the expected growth rate (assuming a beta of 0.90 and current dividends per share of $2.19). Solving for the expected growth rate that provides the current price, we get:

$$\$36.59 = \$2.19(1 + g)/(.09 - g)$$

The growth rate in earnings and dividends would have to be 2.84% a year to justify the stock price of $36.59. This growth rate is called an implied growth rate. Since we estimate growth from fundamentals, this allows us to estimate an implied return on equity:

$$\text{Implied return on equity} = \text{Implied growth rate/Retention ratio}$$
$$= .0284/.3003 = 9.47\%$$

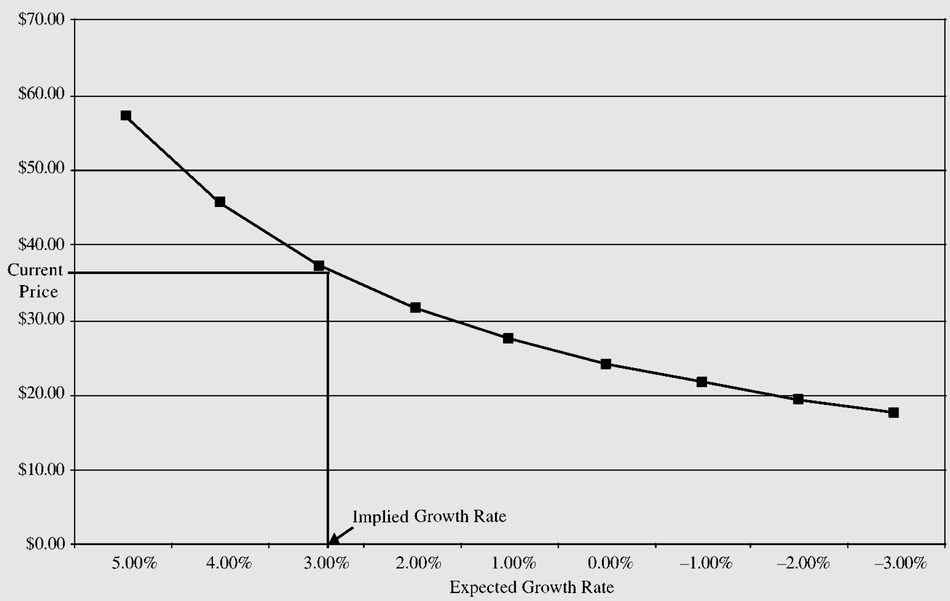

FIGURE 13.2 Value per Share versus Growth

ILLUSTRATION 13.2: Real Estate Investment Trust: Vornado REIT

Real estate investment trusts (REITs) were created in the early 1970s by a law that allowed these entities to invest in real estate and pass the income, tax-free, to their investors. In return for the tax benefit, however, REITs are required to return 95% of their earnings as dividends. Thus, they provide an interesting case study in dividend discount model valuation. Vornado Realty Trust owns and has investments in real estate in the New York area including Alexander's, the Hotel Pennsylvania, and other ventures.

Rationale for Using the Model

Since the firm is required to pay out 95% of its earnings as dividends, the growth in earnings per share will be modest,[2] making it a good candidate for the Gordon growth model.

Background Information

In 2000, Vornado paid dividends per share of $2.12 on earnings per share of $2.22. The estimated payout ratio is:

$$\text{Expected payout ratio} = 2.12/2.22 = 95.50\%$$

The firm had a return on equity of 12.29%.

Estimates

We use the average beta for real estate investment trusts of 0.69, a risk-free rate of 5.4% and a risk premium of 4% to estimate a cost of equity:

$$\text{Cost of equity} = 5.4\% + 0.69(4\%) = 8.16\%$$

The expected growth rate is estimated from the dividend payout ratio and the return on equity:

$$\text{Expected growth rate} = (1 - .955)(.1229) = 0.55\%$$

Valuation

$$\text{Value per share} = 2.12(1.0055)/(.0816 - .0055) = \$28.03$$

It is particularly important with REITs when estimating per-share value that we steer away from net income growth, which may be much higher. On May 14, 2001, Vornado Realty was trading at $36.57, which would make it overvalued.

DDMst.xls: **This spreadsheet allows you to value a stable growth firm, with stable firm characteristics (beta and return on equity) and dividends that roughly match cash flows.**

[2]Growth in net income may be much higher, since REITs can still issue new equity for investing in new ventures.

Two-Stage Dividend Discount Model

The two-stage growth model allows for two stages of growth—an initial phase where the growth rate is not a stable growth rate and a subsequent steady state where the growth rate is stable and is expected to remain so for the long term. While, in most cases, the growth rate during the initial phase is higher than the stable growth rate, the model can be adapted to value companies that are expected to post low or even negative growth rates for a few years and then revert back to stable growth.

The Model The model is based on two stages of growth, an extraordinary growth phase that lasts n years, and a stable growth phase that lasts forever after that:

Extraordinary growth rate: g% each year for n years — Stable growth: g_n forever

Value of the stock = PV of dividends during extraordinary phase + PV of terminal price

$$P_0 = \sum_{t=1}^{t=n} \frac{DPS_t}{\left(1+k_{e,hg}\right)^t} + \frac{P_n}{\left(1+k_{e,hg}\right)^n}$$

$$\text{where } P_n = \frac{DPS_{n+1}}{\left(k_{e,st} - g_n\right)}$$

where DPS_t = Expected dividends per share in year t
k_e = Cost of equity (hg: high growth period; st: stable growth period)
P_n = Price at the end of year n
g = Extraordinary growth rate for the first n years
g_n = Growth rate forever after year n

In the case where the extraordinary growth rate (g) and payout ratio are unchanged for the first n years, this formula can be simplified as follows:

$$P_0 = \frac{DPS_0 \times (1+g) \times \left[1 - \frac{(1+g)^n}{\left(1+k_{e,hg}\right)^n}\right]}{k_{e,hg} - g} + \frac{DPS_{n+1}}{\left(k_{e,st} - g_n\right)\left(1+k_{e,hg}\right)^n}$$

where the inputs are as defined previously.

Calculating the Terminal Price The same constraint that applies to the growth rate for the Gordon growth model (i.e., that the growth rate in the firm is comparable to the nominal growth rate in the economy) applies for the terminal growth rate (g_n) in this model as well.

In addition, the payout ratio has to be consistent with the estimated growth rate. If the growth rate is expected to drop significantly after the initial growth phase, the payout ratio should be higher in the stable phase than in the growth phase. A stable firm can pay out more of its earnings in dividends than a growing firm. One way of estimating this new payout ratio is to use the fundamental growth model described in Chapter 12:

$$\text{Expected growth} = \text{Retention ratio} \times \text{Return on equity}$$
$$= (1 - \text{Payout ratio}) \times \text{Return on equity}$$

Algebraic manipulation yields the following stable period payout ratio:

$$\text{Stable payout ratio} = 1 - \text{Stable growth rate/Stable period return on equity}$$

Thus a firm with a 5 percent growth rate and a return on equity of 15 percent will have a stable period payout ratio of 66.67 percent.

The other characteristics of the firm in the stable period should be consistent with the assumption of stability. For instance, it is reasonable to assume that a high growth firm has a beta of 2.0, but unreasonable to assume that this beta will remain unchanged when the firm becomes stable. In fact, the rule of thumb that we developed in the last chapter—that stable period betas betas be between 0.8 and 1.2—is worth repeating here. Similarly, the return on equity, which can be high during the initial growth phase, should come down to levels commensurate with a stable firm in the stable growth phase. What is a reasonable stable period return on equity? The industry average return on equity and the firm's own stable period cost of equity provide useful information to make this judgment.

Limitations of the Model There are three problems with the two-stage dividend discount model; the first two would apply to any two-stage model, and the third is specific to the dividend discount model.

1. The first practical problem is in defining the length of the extraordinary growth period. Since the growth rate is expected to decline to a stable level after this period, the value of an investment will increase as this period is made longer. While we did develop criteria that might be useful in making this judgment in Chapter 12, it is difficult in practice to convert these qualitative considerations into a specific time period.
2. The second problem with this model lies in the assumption that the growth rate is high during the initial period and is transformed overnight to a lower stable rate at the end of the period. While these sudden transformations in growth can happen, it is much more realistic to assume that the shift from high growth to stable growth happens gradually over time.
3. The focus on dividends in this model can lead to skewed estimates of value for firms that are not paying out what they can afford to in dividends. In particular, we will underestimate the value of firms that accumulate cash and pay out too little in dividends.

Firms Model Works Best For Since the two-stage dividend discount model is based on two clearly delineated growth stages—high growth and stable growth—it is best suited for firms that are in high growth and expect to maintain that growth rate for a specific time period, after which the sources of the high growth are expected to disappear. One scenario, for instance, where this may apply is when a company has patent rights to a very profitable product for the next few years, and is expected to enjoy supernormal growth during this period. Once the patent expires, it is expected to settle back into stable growth. Another scenario where it may be reasonable to make this assumption about growth is when a firm is in an industry that is enjoying supernormal growth because there are significant barriers to entry (either legal or as a consequence of infrastructure requirements), which can be expected to keep new entrants out for several years.

The assumption that the growth rate drops precipitously from its level in the initial phase to a stable rate also implies that this model is more appropriate for firms with modest growth rates in the initial phase. For instance, it is more reasonable to assume that a firm growing at 12 percent in the high growth period will see its growth rate drops to 6 percent after than it is for a firm growing at 40 percent in the high-growth period.

Finally, the model works best for firms that maintain a policy of paying out residual cash flows (i.e., cash flows left over after debt payments and reinvestment needs have been met) as dividends.

ILLUSTRATION 13.3: Valuing a Firm with the Two-Stage Dividend Discount Model: Procter & Gamble

Procter & Gamble (P&G) manufactures and markets consumer products all over the world. Some of its best-known brand names include Pampers diapers, Tide detergent, Crest toothpaste, and Vicks cough/cold medicines.

RATIONALE FOR USING THE MODEL

- *Why two-stage?* While P&G is a firm with strong brand names and an impressive track record on growth, it faces two problems. The first is the saturation of the domestic U.S. market, which represents about half of P&G's revenues. The second is the increased competition from generics across all of its product lines. We will assume that the firm will continue to grow but restrict the growth period to five years.
- *Why dividends?* P&G has a reputation for paying high dividends, and it has not accumulated large amounts of cash over the previous decade.

BACKGROUND INFORMATION

Earnings per share in 2000 = \$3.00
Dividends per share in 2000 = \$1.37
Payout ratio in 2000 = 1.37/3.00 = 45.67%
Return on equity in 2000 = 29.37%

ESTIMATES

We will first estimate the cost of equity for P&G, based on a bottom-up beta of 0.85 (estimated using the unlevered beta for consumer product firms and P&G's debt-to-equity ratio), a risk-free rate of 5.4%, and a risk premium of 4%:

$$\text{Cost of equity} = 5.4\% + 0.85(4\%) = 8.8\%$$

To estimate the expected growth in earnings per share over the five-year high growth period, we use the retention ratio in the most recent financial year (2000) but lower the expected return on equity to 25%:

$$\begin{aligned}\text{Expected growth rate} &= \text{Retention ratio} \times \text{Return on equity}\\ &= (1 - 1.37/3.00)(.25) = 13.58\%\end{aligned}$$

In stable growth, we will estimate that the beta for the stock will rise to 1, leading to a cost of equity of 9.40%:

Cost of equity in stable growth = 5.4% + 4% = 9.4%

The expected growth rate will be assumed to be equal to the growth rate of the economy (5%) and the return on equity will drop to 15%, which is lower than the current industry average (17.4%) but higher than the cost of equity estimated above. The retention ratio in stable growth can then be written as:

Retention ratio in stable growth = g/ROE = 5%/15% = 33.33%

The payout ratio in stable growth is therefore 66.67%.

Estimating the Value

The first component of value is the present value of the expected dividends during the high growth period. Based on the current earnings ($3.00), the expected growth rate (13.58%), and the expected dividend payout ratio (45.67%), the expected dividends can be computed for each year in the high-growth period:

Year	*EPS*	*DPS*	*Present Value*
1	$3.41	$1.56	$1.43
2	$3.87	$1.77	$1.49
3	$4.40	$2.01	$1.56
4	$4.99	$2.28	$1.63
5	$5.67	$2.59	$1.70
Sum			$7.81

The present value is computed using the cost of equity of 8.8% for the high-growth period.

Cumulative present value of dividends during high growth (@8.8%) = $7.81

The present value of the dividends can also be computed in shorthand using the following computation:

$$\text{PV of dividends} = \frac{\$1.37 \times 1.1358 \times \left(1 - \dfrac{1.1358^5}{1.088^5}\right)}{.088 - .1358} = \$7.81$$

The value at the end of the high-growth phase (end of year 5), can be estimated using the constant growth model.

Terminal price = Expected dividends per share$_{n+1}$/($k_{e,st} - g_n$)
Expected earnings per share$_6$ = 3.00 × 1.1358^5 × 1.05 = $5.96
Expected dividends per share$_6$ = EPS$_6$ × Stable period payout ratio
= $5.96 × 0.6667 = $3.97
Terminal price = Dividends$_6$/ ($k_{e,st}$ – g) = $3.97/(.094 – .05) = $90.23

The present value of the terminal price can be then written as:

$$\text{PV of terminal price} = \frac{\$90.23}{1.088^5} = \$59.18$$

The cumulated present value of dividends and the terminal price can then be calculated as follows:

$$P_0 = \frac{\$1.37 \times 1.1358 \times \left(1 - \dfrac{1.1358^5}{1.088^5}\right)}{.088 - .1358} + \frac{\$90.23}{1.088^5} = \$7.81 + \$59.18 = \$66.99$$

P&G was trading at $63.90 at the time of this analysis on May 14, 2001.

DDM2st.xls: **This spreadsheet allows you to value a growth firm, with an initial period of high growth and stable growth thereafter, using expected dividends.**

A TROUBLESHOOTING GUIDE: WHAT IS WRONG WITH THIS VALUATION? (TWO-STAGE DDM)

If This Is Your Problem	*This May Be the Solution*
• If you get a extremely low value from the two-stage DDM, the likely culprits are:	
The stable period payout ratio is too low for a stable firm (< 40%).	If using fundamentals, use a higher ROE. If entering directly, enter a higher payout.
The beta in the stable period is too high for a stable firm.	Use a beta closer to 1.
The two-stage model is being used when the three-stage model is more appropriate.	Use a three-stage model.
• If you get an extremely high value:	
The growth rate in the stable growth period is too high for a stable firm.	Use a growth rate closer to GNP growth, and make sure that your retention ratio is consistently estimated.

Modifying the Model to Include Stock Buybacks In recent years, firms in the United States have increasingly turned to stock buybacks as a way of returning cash to stockholders. Figure 13.3 presents the cumulative amounts paid out by firms in the form of dividends and stock buybacks from 1960 to 1998. The trend toward stock buybacks is very strong, especially in the 1990s.

What are the implications for the dividend discount model? Focusing strictly on dividends paid as the only cash returned to stockholders exposes us to the risk that we might be missing significant cash returned to stockholders in the form of stock buybacks. The simplest way to incorporate stock buybacks into a dividend discount model is to add them on to the dividends and compute a modified payout ratio:

$$\text{Modified dividend payout ratio} = (\text{Dividends} + \text{Stock buybacks})/\text{Net income}$$

While this adjustment is straightforward, the resulting ratio for any one year can be skewed by the fact that stock buybacks, unlike dividends, are not smoothed out. In other words, a firm may buy back \$3 billion in stock in one year, and not buy back stock for the next three years. Consequently, a much better estimate of the modified payout ratio can be obtained by looking at the average value over a

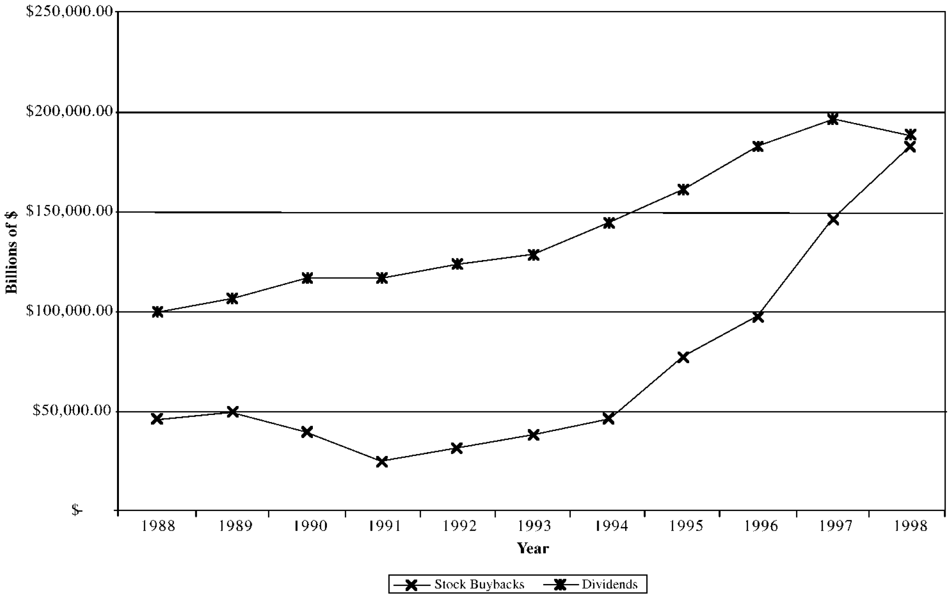

FIGURE 13.3 Stock Buybacks and Dividends: Aggregate for U.S. Firms, 1989–1998
Source: Compustat.

four- or five-year period. In addition, firms may sometimes buy back stock as a way of increasing financial leverage. We could adjust for this by netting out new debt issued from the earlier calculation:

$$\text{Modified dividend payout} = (\text{Dividends} + \text{Stock buybacks} - \text{Long-term debt issues})/\text{Net income}$$

Adjusting the payout ratio to include stock buybacks will have ripple effects on estimated growth and the terminal value. In particular, the modified growth rate in earnings per share can be written as:

$$\text{Modified growth rate} = (1 - \text{Modified payout ratio}) \times \text{Return on equity}$$

Even the return on equity can be affected by stock buybacks. Since the book value of equity is reduced by the market value of equity bought back, a firm that buys back stock can reduce its book equity (and increase its return on equity) dramatically. If we use this return on equity as a measure of the marginal return on equity (on new investments), we will overstate the value of a firm. Adding back stock buybacks in recent years to the book equity and reestimating the return on equity can sometimes yield a more reasonable estimate of the return on equity on investments.

ILLUSTRATION 13.4: Valuing a Firm with Modified Dividend Discount Model: Procter & Gamble

Consider our earlier valuation of Procter & Gamble that used the current dividends as the basis for projections. Note that over the past four years P&G has had significant stock buybacks each period. The following table summarizes the dividends and buybacks:

	1997	*1998*	*1999*	*2000*	*Total*
Net income	3,415	3,780	3,763	3,542	14,500
Dividends	1,329	1,462	1,626	1,796	6,213
Buybacks	2,152	391	1,881	–1021	3,403
Dividends + buybacks	3,481	1,853	3,507	775	9,616
Payout ratio	38.92%	38.68%	43.21%	50.71%	42.85%
Modified payout ratio	101.93%	49.02%	93.20%	21.88%	66.32%
Buybacks	1,652	1,929	2,533	1,766	
Net long-term debt issued	–500	1,538	652	2,787	
Buybacks net of debt	2,152	391	1,881	–1,021	

Over the four-year period, P&G had significant buybacks but it also increased its leverage dramatically in the last three years. Summing up the total cash returned to stockholders over the past four years, we arrive at a payout ratio of 66.32 percent. If we substitute this payout ratio into the valuation in Illustration 13.3, the expected growth rate over the next five years drops to 8.42%:

$$\text{Expected growth rate} = (1 - \text{Modified payout ratio})\text{ROE} = (1 - .6632)(.25) = 8.42\%$$

We will still assume a five-year high-growth period and that the parameters in stable growth remain unchanged. The value per share can be estimated as follows:

$$P_0 = \frac{\$3.00 \times .6632 \times (1.0842) \times \left(1 - \frac{(1.0842)^5}{(1.0880)^5}\right)}{.0880 - .0842} + \frac{\$71.50}{(1.0880)^5} = \$56.75$$

Note that the drop in earnings growth reduces earnings in the terminal year and the terminal value.

This value is lower than that obtained in Illustration 13.3, and it reflects our expectation that P&G does not have as many new profitable new investments (earning a return on equity of 25%).

Valuing an Entire Market Using the Dividend Discount Model All our examples of the dividend discount model so far have involved individual companies, but there is no reason why we cannot apply the same model to value a sector or even the entire market. The market price of the stock would be replaced by the cumulative market value of all of the stocks in the sector or market. The expected dividends would be the cumulated dividends of all these stocks, and could be expanded to include stock buybacks by all firms. The expected growth rate would be the growth rate in cumulated earnings of the index. There would be no need for a beta or betas, since you are looking at the entire market (which should have a beta of 1), and you could add the risk premium (or premiums) to the risk-free rate to estimate a cost of equity. You could use a two-stage model, where this growth rate is greater than the growth rate of the economy, but you should be cautious about setting the growth rate too high or the growth period too long, because it will be difficult for cumulated earnings growth of all firms in an economy to run ahead of the growth rate in the economy for extended periods.

Consider a simple example. Assume that you have an index trading at 700, and that the average dividend yield of stocks in the index is 5 percent. Earnings and dividends can be expected to grow at 4 percent a year forever, and the riskless rate is 5.4 percent. If you use a market risk premium of 4 percent, the value of the index can be estimated as follows:

Cost of equity = Riskless rate + Risk premium = 5.4% + 4% = 9.4%

Expected dividends next year = (Dividend yield × Value of the index)(1 + expected growth rate)
= (.05 × 700)(1.04) = 36.4

Value of the index = Expected dividends next year /(Cost of equity – Expected growth rate)
= 36.4/(.094 – .04) = 674

At its existing level of 700, the market is slightly overpriced.

ILLUSTRATION 13.5: Valuing the S&P 500 Using a Dividend Discount Model: January 1, 2001

On January 1, 2001, the S&P 500 index was trading at 1,320. The dividend yield on the index was only 1.43%, but including stock buybacks increases the yield to 2.50%. Analysts were estimating that the earnings of the stocks in the index would increase 7.5% a year for the next five years. Beyond year 5, the expected growth rate is expected to be 5%, the nominal growth rate in the economy. The Treasury bond rate was 5.1%, and we will use a market risk premium of 4%, leading to a cost of equity of 9.1%:

Cost of equity = 5.1% = 4% = 9.1%

The expected dividends (and stock buybacks) on the index for the next five years can be estimated from the current dividends and expected growth of 7.50%:

Current dividends = 2.50% of 1,320 = 33.00

	1	*2*	*3*	*4*	*5*
Expected dividends	$35.48	$38.14	$41.00	$44.07	$47.38
Present value	$32.52	$32.04	$31.57	$31.11	$30.65

The present value is computed by discounting back the dividends at 9.1%. To estimate the terminal value, we estimate dividends in year 6 on the index:

Expected dividends in year 6 = $47.38(1.05) = $49.74

Terminal value of the index = Expected dividends$_6$/(r – g) = $49.74/(.091 – .05) = $1,383.11

Present value of terminal value = $1,383.11/1.091^5 = $894.81

The value of the index can now be computed:

Value of index = Present value of dividends during high growth + Present value of terminal value
= $32.52 + $32.04 + $31.57 + $31.11 + $30.65 + $894.81 = $1,052.69

Based on this, we would have concluded that the index was overvalued at 1,320.

The Value of Growth

Investors pay a price premium when they acquire companies with high growth potential. This premium takes the form of higher price-earnings or price–book value ratios. While no one will contest the proposition that growth is valuable, it is possible to pay too much for growth. In fact, empirical studies that show low price-earnings ratio stocks earning return premiums over high price-earnings ratio stocks in the long term support the notion that investors overpay for growth. This section uses the two-stage dividend discount model to examine the value of growth, and it provides a benchmark that can be used to compare the actual prices paid for growth.

Estimating the Value of Growth The value of the equity in any firm can be written in terms of three components:

$$P_0 = \underbrace{\left\{ \frac{DPS_0 \times (1+g) \times \left[1 - \frac{(1+g)^n}{(1+r)^n}\right]}{k_{e,hg} - g} + \frac{DPS_{n+1}}{(k_{e,st} - g_n)(1+k_{e,hg})^n} - \frac{DPS_1}{k_{e,st} - g_n} \right\}}_{\text{Extraordinary growth}}$$

$$+ \underbrace{\left(\frac{DPS_1}{k_{e,st} - g_n} - \frac{DPS_0}{k_{e,st}} \right)}_{\text{Stable growth}} + \underbrace{\frac{DPS_0}{k_{e,st}}}_{\text{Assets in place}}$$

where DPS_t = Expected dividends per share in year t
k_e = Cost of equity
g_n = Growth rate forever after year n

Value of extraordinary growth = Value of the firm with extraordinary growth in first n years – Value of the firm as a stable growth firm[3]

Value of stable growth = Value of the firm as a stable growth firm – Value of firm with no growth

Assets in place = Value of firm with no growth

In making these estimates, though, we have to remain consistent. For instance, to value assets in place, you would have to assume that the entire earnings could be paid out in dividends, while the payout ratio used to value stable growth should be a stable period payout ratio.

[3]The payout ratio used to calculate the value of the firm as a stable firm can be either the current payout ratio, if it is reasonable, or the new payout ratio calculated using the fundamental growth formula.

ILLUSTRATION 13.6: The Value of Growth: P&G in May 2001

In Illustration 13.3, we valued P&G using a two-stage dividend discount model at $66.99. We first value the assets in place using current earnings ($3.00) and assume that all earnings are paid out as dividends. We also use the stable growth cost of equity as the discount rates.

Value of assets in place = Current EPS/r = $3.00/.094 = $31.91

To estimate the value of stable growth, we assume that the expected growth rate will be 5% and that the payout ratio is the stable period payout ratio of 66.67%:

$$\text{Value of stable growth} = \text{Current EPS} \times \text{Stable payout ratio} \times (1 + g_n)/(r - g_n) - \text{Value of assets in place}$$
$$= (\$3.00 \times 0.6667 \times 1.05)/(.094 - .05) - \$31.91 = \$15.81$$

Value of extraordinary growth = $66.99 – $31.91 – $15.81 = $19.26

Note that $66.99 was our estimate of value per share in Illustration 13.3.

DETERMINANTS OF THE VALUE OF GROWTH

- *Growth rate during extraordinary period.* The higher the growth rate in the extraordinary period, the higher is the estimated value of growth. If the growth rate in the extraordinary growth period had been raised to 20% for the Procter & Gamble valuation, the value of extraordinary growth would have increased from $19.26 to $39.45. Conversely, the value of high growth companies can drop precipitously if the expected growth rate is reduced, either because of disappointing earnings news from the firm or as a consequence of external events.
- *Length of the extraordinary growth period.* The longer the extraordinary growth period, the greater the value of growth. At an intuitive level, this is fairly simple to illustrate. The value of $19.26 obtained for extraordinary growth in P&G is predicated on the assumption that high growth will last for five years. If this is revised to last 10 years, the value of extraordinary growth will increase to $43.15.
- *Profitability of projects.* The profitability of projects determines both the growth rate in the initial phase and the terminal value. As projects become more profitable, they increase both growth rates, and the resulting value from extraordinary growth will be greater.
- *Riskiness of the firm/equity.* The riskiness of a firm determines the discount rate at which cash flows in the initial phase are discounted. Since the discount rate increases as risk increases, the present value of the extraordinary growth will decrease.

H Model for Valuing Growth

The H model is a two-stage model for growth, but unlike the classic two-stage model, the growth rate in the initial growth phase is not constant but declines linearly over time to reach the stable growth rate in steady state. This model was presented in Fuller and Hsia (1984).

The Model The model is based on the assumption that the earnings growth rate starts at a high initial rate (g_a) and declines linearly over the extraordinary growth period (which is assumed to last 2H periods) to a stable growth rate (g_n). It also assumes that the dividend payout and cost of equity are constant over time, and are not affected by the shifting growth rates. Figure 13.4 graphs the expected growth over time in the H model.

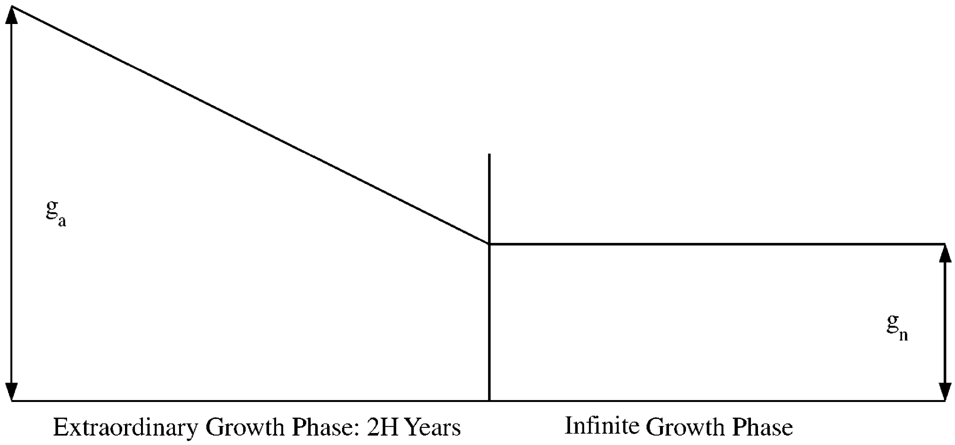

FIGURE 13.4 Expected Growth in the H Model

The value of expected dividends in the H model can be written as follows:

$$P_0 = \underbrace{\frac{DPS_0 \times (1+g_n)}{k_e - g_n}}_{\text{Stable growth}} + \underbrace{\frac{DPS_0 \times H \times (g_a - g_n)}{k_e - g_n}}_{\text{Extraordinary growth}}$$

where P_0 = Value of the firm now per share
DPS_t = DPS in year t
k_e = Cost of equity
g_a = Growth rate initially
g_n = Growth rate at end of 2H years, applies forever after that

Limitations This model avoids the problems associated with the growth rate dropping precipitously from the high-growth to the stable growth phase, but it does so at a cost. First, the decline in the growth rate is expected to follow the strict structure laid out in the model—it drops in linear increments each year based on the initial growth rate, the stable growth rate, and the length of the extraordinary growth period. While small deviations from this assumption do not affect the value significantly, large deviations can cause problems. Second, the assumption that the payout ratio is constant through both phases of growth exposes the analyst to an inconsistency—as growth rates decline, the payout ratio usually increases.

Firms Model Works Best For The allowance for a gradual decrease in growth rates over time may make this a useful model for firms that are growing rapidly right now, but where the growth is expected to decline gradually over time as the firms get larger and the differential advantage they have over their competitors declines. The assumption that the payout ratio is constant, however, makes this an inappropriate model to use for any firm that has low or no dividends currently. Thus, the model, by requiring a combination of high growth and high payout, may be quite limited[4] in its applicability.

ILLUSTRATION 13.7: Valuing with the H Model: Alcatel

Alcatel, a French telecommunications firm, paid dividends per share of 0.72 Ffr on earnings per share of 1.25 Ffr in 2000. The firm's earnings per share had grown at 12% over the prior five years but the growth rate is expected to decline linearly over the next 10 years to 5%, while the payout ratio remains unchanged. The beta for the stock is 0.8, the risk-free rate is 5.1%, and the market risk premium is 4%.

$$\text{Cost of equity} = 5.1\% + 0.8 \times 4\% = 8.30\%$$

The stock can be valued using the H model:

$$\text{Value of stable growth} = \frac{0.72 \times (1.05)}{.083 - .05} = 22.91 \text{ Ffr}$$

$$\text{Value of extraordinary growth} = \frac{0.72 \times (10/2) \times (.12 - .05)}{.083 - .05} = 7.64 \text{ Ffr}$$

$$\text{Value of stock} = 22.91 + 7.64 = 30.55 \text{ Ffr}$$

The stock was trading at 33.40 Ffr in May 2001.

***DDMH.xls:* This spreadsheet allows you to value a firm, with an initial period when the high growth declines to stable growth, using expected dividends.**

Three-Stage Dividend Discount Model

The three-stage dividend discount model combines the features of the two-stage model and the H model. It allows for an initial period of high growth, a transitional period where growth declines, and a final stable growth phase. It is the most general of the models because it does not impose any restrictions on the payout ratio.

The Model This model assumes an initial period of stable high growth, a second period of declining growth, and a third period of stable low growth that lasts forever. Figure 13.5 graphs the expected growth over the three time periods.

The value of the stock is then the present value of expected dividends during the high-growth and the transitional periods, and of the terminal price at the start of the final stable growth phase.

[4]Proponents of the model would argue that using a steady-state payout ratio for firms that pay little or no dividends is likely to cause only small errors in the valuation.

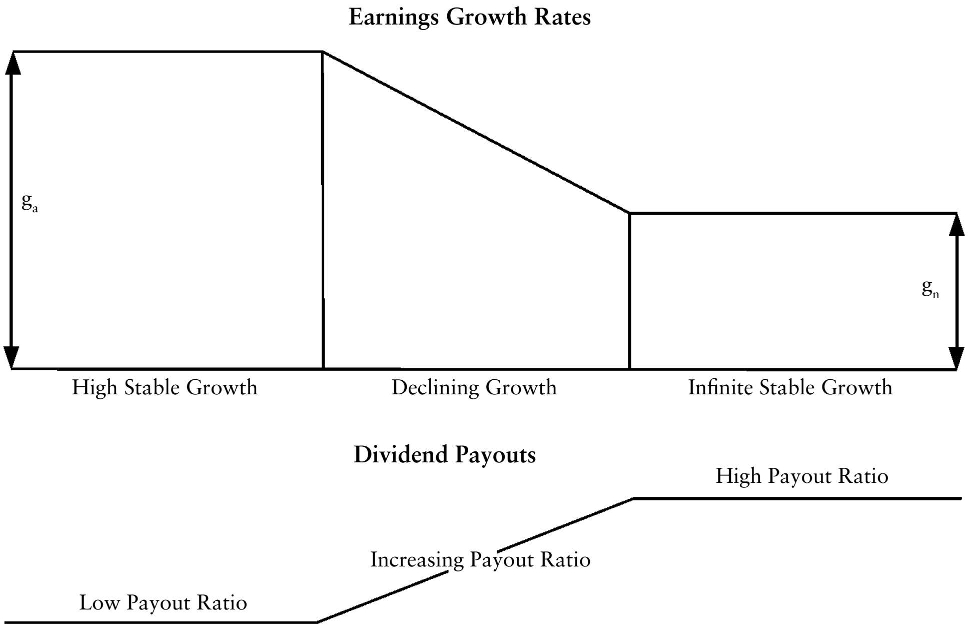

FIGURE 13.5 Expected Growth in the Three-Stage Dividend Discount Model

$$P_0 = \underbrace{\sum_{t=1}^{t=n1} \frac{EPS_0 \times (1+g_a)^t \times \Pi_a}{(1+k_{e,hg})^t}}_{\text{High-growth phase}} + \underbrace{\sum_{t=n1+1}^{t=n2} \frac{DPS_t}{(1+k_{e,t})^t}}_{\text{Transition}} + \underbrace{\frac{EPS_{n2}(1+g_n) \times \Pi_n}{(k_{e,st}-g_n)(1+r)^n}}_{\text{Stable growth phase}}$$

where EPS_t = Earnings per share in year t
DPS_t = Dividends per share in year t
g_a = Growth rate in high-growth phase (lasts n1 periods)
g_n = Growth rate in stable phase
Π_a = Payout ratio in high-growth phase
Π_n = Payout ratio in stable growth phase
k_e = Cost of equity in high growth (hg), transition (t), and stable growth (st)

Assumptions This model removes many of the constraints imposed by other versions of the dividend discount model. In return, however, it requires a much larger number of inputs—year-specific payout ratios, growth rates, and betas. For firms where there is substantial noise in the estimation process, the errors in these inputs can overwhelm any benefits that accrue from the additional flexibility in the model.

Firms Model Works Best For This model's flexibility makes it a useful model for any firm that in addition to changing growth over time is expected to change on other dimensions as well—in particular, payout policies and risk. It is best suited for firms that are growing at an extraordinary rate now and are expected to maintain this rate for an initial period, after which the differential advantage of the firm is expected to deplete leading to gradual declines in the growth rate to a stable growth rate. Practically speaking, this may be the more appropriate model to use for a firm whose earnings are growing at very high rates,[5] are expected to continue growing at those rates for an initial period, but are expected to start declining gradually toward a stable rate as the firm become larger and loses its competitive advantages.

ILLUSTRATION 13.8: Valuing with the Three-Stage DDM Model: Coca-Cola

Coca-Cola, the owner of the most valuable brand name in the world according to Interbrand (a consulting firm), was able to increase its market value tenfold in the 1980s and 1990s. Growth has leveled off in the past few years, but the firm is still expanding into both other products and other markets.

Rationale for Using the Three-Stage Dividend Discount Model

- *Why three-stage?* Coca-Cola is still in high growth, but its size and dominant market share will cause growth to slide in the second phase of the high-growth period. The high-growth period is expected to last five years, and the transition period is expected to last an additional five years.
- *Why dividends?* The firm has had a track record of paying out large dividends to its stockholders, and these dividends tend to mirror free cash flows to equity.
- The financial leverage is stable.

Background Information

Current earnings/dividends

Earnings per share in 2000 = $1.56
Dividends per share in 2000 = $0.69
Payout ratio in 2000 = 44.23%
Return on equity = 23.37%

Estimate

Cost of Equity: We will begin by estimating the cost of equity during the high-growth phase, expected. We use a bottom-up levered beta of 0.80 and a risk-free rate of 5.4%. We use a risk premium of 5.6%, significantly higher than the mature market premium of 4% that we have used in the valuations so far, to reflect Coca-Cola's exposure in Latin America, Eastern Europe, and Asia. The cost of equity can then be estimated for the high-growth period.

$$\text{Cost of equity}_{\text{high growth}} = 5.4\% + 0.8(5.6\%) = 9.88\%$$

In stable growth, we assume that the beta will remain 0.80, but reduce the risk premium to 5% to reflect the expected maturing of many emerging markets.

$$\text{Cost of equity}_{\text{stable growth}} = 5.4\% + 0.8(5.0\%) = 9.40\%$$

During the transition period, the cost of equity will linearly decline from 9.88% in year 5 to 9.40% in year 10.

[5]The definition of a "very high" growth rate is largely subjective. As a rule of thumb, growth rates over 25 percent would qualify as very high when the stable growth rate is 6 to 8 percent.

Expected Growth and Payout Ratios: The expected growth rate during the high-growth phase is estimated using the current return on equity of 23.37% and payout ratio of 44.23%.

$$\text{Expected growth rate} = \text{Retention ratio} \times \text{Return on equity} = (1 - 0.4423)(0.2337) = 13.03\%$$

During the transition phase, the expected growth rate declines linearly from 13.03% to a stable growth rate of 5.5%. To estimate the payout ratio in stable growth, we assume a return on equity of 20% for the firm:

$$\text{Stable period payout ratio} = 1 - \frac{g}{\text{ROE}} = 1 - \frac{5.5\%}{20\%} = 72.5\%$$

During the transition phase, the payout ratio adjusts upward from 44.23% to 72.5% in linear increments.

Estimating the Value

These inputs are used to estimated expected earnings per share (EPS), dividends per share (DPS), and costs of equity for both the high-growth transition, and the stable periods. The present values are also shown in the last column of the following table:

Year	*Expected Growth*	*EPS*	*Payout Ratio*	*DPS*	*Cost of Equity*	*Present Value*
High-Growth Stage						
1	13.03%	$1.76	44.23%	$0.78	9.88%	$0.71
2	13.03%	$1.99	44.23%	$0.88	9.88%	$0.73
3	13.03%	$2.25	44.23%	$1.00	9.88%	$0.75
4	13.03%	$2.55	44.23%	$1.13	9.88%	$0.77
5	13.03%	$2.88	44.23%	$1.27	9.88%	$0.79
Transition Stage						
6	11.52%	$3.21	49.88%	$1.62	9.78%	$0.91
7	10.02%	$3.53	55.54%	$1.96	9.69%	$1.02
8	8.51%	$3.83	61.19%	$2.34	9.59%	$1.11
9	7.01%	$4.10	66.85%	$2.74	9.50%	$1.18
10	5.50%	$4.33	72.50%	$3.14	9.40%	$1.24

Since the costs of equity change each year, the present value has to be calculated using the cumulated cost of equity. Thus in year 7 the present value of dividends is:

$$\text{PV of year 7 dividend} = \frac{\$1.96}{(1.0988)^5(1.0978)(1.0969)} = \$1.02$$

The terminal price at the end of year 10 can be calculated based on the earnings per share in year 11, the stable growth rate of 5%, a cost of equity of 9.40%, and the payout ratio of 72.50%:

$$\text{Terminal price} = \frac{\$4.33(1.055)(0.725)}{0.094 - 0.055} = \$84.83$$

The components of value are as follows:

Present value of dividends in high-growth phase	$ 3.76
Present value of dividends in transition phase	$ 5.46
Present value of terminal price at end of transition	$33.50
Value of Coca-Cola stock	$42.72

Coca-Cola was trading at $46.29 on May 21, 2001.

DDM3st.xls: **This spreadsheet allows you to value a firm with a period of high growth followed by a transition period where growth declines to a stable growth rate.**

A TROUBLESHOOTING GUIDE: WHAT IS WRONG WITH THIS MODEL? (THREE-STAGE DDM)

If This Is Your Problem	*This May Be the Solution*
• If you are getting too low a value from this model:	
The stable period payout ratio is too low for a stable firm (< 40%).	If using fundamentals, use a higher ROE. If entering directly, enter a higher payout.
The beta in the stable period is too high for a stable firm.	Use a beta closer to 1.
• If you get an extremely high value:	
The growth rate in the stable growth period is too high for stable firm.	Use a growth rate closer to gross national product (GNP) growth.
The period of growth (high + transition) is too high.	Use shorter high growth and transition periods.

ISSUES IN USING THE DIVIDEND DISCOUNT MODEL

The dividend discount model's primary attraction is its simplicity and its intuitive logic. There are many analysts, however, who view its results with suspicion because of limitations that they perceive it to possess. The model, they claim, is not really useful in valuation except for a limited number of stable, high-dividend-paying stocks. This section examines some of the areas where the dividend discount model is perceived to fall short.

Valuing Non-Dividend-Paying or Low-Dividend-Paying Stocks

The conventional wisdom is that the dividend discount model cannot be used to value a stock that pays low or no dividends. It is wrong. If the dividend payout ratio is adjusted to reflect changes in the expected growth rate, a reasonable value can be obtained even for non-dividend-paying firms. Thus, a high-growth firm, paying no dividends currently, can still be valued based on dividends that it is expected to pay out when the growth rate declines. If the payout ratio is not adjusted to reflect changes in the growth rate, however, the dividend discount model will underestimate the value of non-dividend-paying or low-dividend-paying stocks.

Is the Model Too Conservative in Estimating Value?

A standard critique of the dividend discount model is that it provides too conservative an estimate of value. This criticism is predicated on the notion that the value is determined by more than the present value of expected dividends. For instance, it is argued that the dividend discount model does not reflect the value of "unutilized assets." There is no reason, however, that these unutilized assets cannot be valued separately and added on to the value from the dividend discount model. Some of the assets that are supposedly ignored by the dividend discount model, such as the value of brand names, can be dealt with fairly simply within the context of the model.

A more legitimate criticism of the model is that it does not incorporate other ways of returning cash to stockholders (such as stock buybacks). If you use the modified version of the dividend discount model, this criticism can also be countered.

Contrarian Nature of the Model

The dividend discount model is also considered by many to be a contrarian model. As the market rises, fewer and fewer stocks, they argue, will be found to be undervalued using the dividend discount model. This is not necessarily true. If the market increase is due to an improvement in economic fundamentals, such as higher expected growth in the economy and/or lower interest rates, there is no reason, a priori, to believe that the values from the dividend discount model will not increase by an equivalent amount. If the market increase is not due to fundamentals, the dividend discount model values will not follow suit, but that is more a sign of strength than weakness. The model is signaling that the market is overvalued relative to dividends and cash flows, and the cautious investor will pay heed.

TESTS OF THE DIVIDEND DISCOUNT MODEL

The ultimate test of a model lies in how well it works at identifying undervalued and overvalued stocks. The dividend discount model has been tested and the results indicate that it does, in the long term, provide for excess returns. It is unclear, however, whether this is because the model is good at finding undervalued stocks or because it proxies for well-known empirical irregularities in returns relating to price-earnings ratios and dividend yields.

Simple Test of the Dividend Discount Model

A simple study of the dividend discount model was conducted by Sorensen and Williamson, where they valued 150 stocks from the S&P 400 in December 1980 using the dividend discount model. They used the difference between the market price at that time and the model value to form five portfolios based on the degree of under or over valuation. They made fairly broad assumptions in using the dividend discount model:

- The average of the earnings per share between 1976 and 1980 was used as the current earnings per share.
- The cost of equity was estimated using the CAPM.

- The extraordinary growth period was assumed to be five years for all stocks, and the I/B/E/S consensus forecast of earnings growth was used as the growth rate for this period.
- The stable growth rate, after the extraordinary growth period, was assumed to be 8 percent for all stocks.
- The payout ratio was assumed to be 45 percent for all stocks.

The returns on these five portfolios were estimated for the following two years (January 1981 to January 1983) and excess returns were estimated relative to the S&P 500 index using the betas estimated at the first stage and the CAPM. Figure 13.6 illustrates the excess returns earned by the portfolio that was undervalued by the dividend discount model relative to both the market and the overvalued portfolio.

The undervalued portfolio had a positive excess return of 16 percent per annum between 1981 and 1983, while the overvalued portfolio had a negative excess return of 15 percent per annum during the same time period. Other studies that focus only on the dividend discount model come to similar conclusions. In the long term, undervalued and overvalued stocks from the dividend discount model outperform and underperform, respectively, the market index on a risk-adjusted basis.

Caveats on the Use of the Dividend Discount Model

The dividend discount model provides impressive results in the long term. There are, however, three considerations in generalizing the findings from these studies:

The Dividend Discount Model Does Not Beat the Market Every Year The dividend discount model outperforms the market over five-year time periods, but there have been individual years where the model has significantly underperformed the mar-

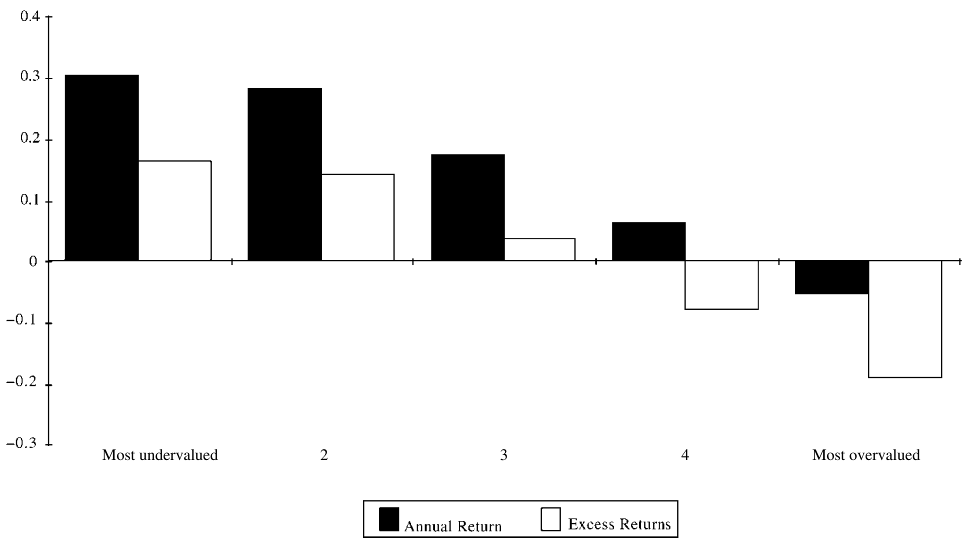

FIGURE 13.6 Performance of the Dividend Discount Model, 1981–1983

ket. Haugen reports on the results of a fund that used the dividend discount model to analyze 250 large capitalization firms and to classify them into five quintiles from the first quarter of 1979 to the last quarter of 1991. The betas of these quintiles were roughly equal. The valuation was done by six analysts who estimated an extraordinary growth rate for the initial high-growth phase, the length of the high-growth phase, and a transitional phase for each of the firms. The returns on the five portfolios, as well as the returns on all 250 stocks and the S&P 500 from 1979 to 1991, are reported in Table 13.1. The undervalued portfolio earned significantly higher returns than the overvalued portfolio and the S&P 500 for the 1979–1991 period, but it underperformed the market in 6 of the 13 years and the overvalued portfolio in 4 of the 13 years.

Is the Model Just Proxying for Low PE Ratios and Dividend Yields? The dividend discount model weights expected earnings and dividends in near periods more than earnings and dividends in far periods, and is biased toward finding low price-earnings ratio stocks with high dividend yields to be undervalued and high price-earnings ratio stocks with low or no dividend yields to be overvalued. As noted in Chapter 6, studies of market efficiency indicate that low-PE-ratio stocks have outperformed (in terms of excess returns) high-PE-ratio stocks over extended time periods. Similar conclusions have been drawn about high-dividend-yield stocks relative to low-dividend-yield stocks. Thus, the valuation findings of the model are consistent with empirical irregularities observed in the market.

It is unclear how much the model adds in value to investment strategies that use PE ratios or dividend yields to screen stocks. Jacobs and Levy (1988b) indicate that the marginal gain is relatively small.

TABLE 13.1 Returns on Quintiles: Dividend Discount Model

	Quintile						
	Undervalued	2	3	4	Over-valued	250 Stocks	S&P 500
1979	35.07%	25.92%	18.49%	17.55%	20.06%	23.21%	18.57%
1980	41.21%	29.19%	27.41%	38.43%	26.44%	31.86%	32.55%
1981	12.12%	10.89%	1.25%	–5.59%	–8.51%	28.41%	24.55%
1982	19.12%	12.81%	26.72%	28.41%	35.54%	24.53%	21.61%
1983	34.18%	21.27%	25.00%	24.55%	14.35%	24.10%	22.54%
1984	15.26%	5.50%	6.03%	–4.20%	–7.84%	3.24%	6.12%
1985	38.91%	32.22%	35.83%	29.29%	23.43%	33.80%	31.59%
1986	14.33%	11.87%	19.49%	12.00%	20.82%	15.78%	18.47%
1987	0.42%	4.34%	8.15%	4.64%	–2.41%	2.71%	5.23%
1988	39.61%	31.31%	17.78%	8.18%	6.76%	20.62%	16.48%
1989	26.36%	23.54%	30.76%	32.60%	35.07%	29.33%	31.49%
1990	–17.32%	–8.12%	–5.81%	2.09%	–2.65%	–6.18%	–3.17%
1991	47.68%	26.34%	33.38%	34.91%	31.64%	34.34%	30.57%
1979–1991	1,253%	657%	772%	605%	434%	722%	654%

Attribute	*Average Excess Return per Quarter: 1982–1987*
Dividend discount model	0.06% per quarter
Low P/E ratio	0.92% per quarter
Book/price ratio	0.01% per quarter
Cash flow/price	0.18% per quarter
Sales/price ratio	0.96% per quarter
Dividend yield	−0.51% per quarter

This suggests that using low PE ratios to pick stocks adds 0.92 percent to your quarterly returns, whereas using the dividend discount model adds only a further 0.06 percent to quarterly returns. If, in fact, the gain from using the dividend discount model is that small, screening stocks on the basis of observables (such as PE ratio or cash flow measures) may provide a much larger benefit in terms of excess returns.

Tax Disadvantages from High-Dividend Stocks Portfolios created with the dividend discount model are generally characterized by high dividend yield, which can create a tax disadvantage if dividends are taxed at a rate greater than capital gains or if there is a substantial tax timing liability associated with dividends.[6] Since the excess returns uncovered in the studies presented above are pretax to the investor, the introduction of personal taxes may significantly reduce or even eliminate these excess returns.

In summary, the dividend discount model's impressive results in studies looking at past data have to be considered with caution. For a tax-exempt investment with a long time horizon, the dividend discount model is a good tool (though it may not be the only one) to pick stocks. For a taxable investor, the benefits are murkier, since the tax consequences of the strategy have to be considered. For investors with shorter time horizons, the dividend discount model may not deliver on its promised excess returns because of the year-to-year volatility in its performance.

CONCLUSION

When you buy stock in a publicly traded firm, the only cash flow you receive directly from this investment in expected dividends. The dividend discount model builds on this simple proposition and argues that the value of a stock then has to be the present value of expected dividends over time. Dividend discount models can range from simple growing perpetuity models such as the Gordon growth model, where a stock's value is a function of its expected dividends next year, the cost of equity, and the stable growth rate, to complex three-stage models, where payout ratios and growth rates change over time. While the model is often criticized as being of limited value, it has proven to be surprisingly adaptable and useful in a wide range of circumstances. It may be a conservative model that finds fewer and fewer undervalued firms as market prices rise relative to fundamentals (earnings, dividends, etc.); but that can also be viewed as a strength. Tests of the model also seem to indicate its usefulness in gauging value, though much of its effectiveness may be derived from its finding low-PE-ratio, high-dividend-yield stocks to be undervalued.

[6]Investors do not have a choice of when they receive dividends, whereas they have a choice on the timing of capital gains.

QUESTIONS AND SHORT PROBLEMS

1. Respond true or false to the following statements relating to the dividend discount model:
   a. The dividend discount model cannot be used to value a high-growth company that pays no dividends.
      True ____ False ____
   b. The dividend discount model will undervalue stocks, because it is too conservative.
      True ____ False ____
   c. The dividend discount model will find more undervalued stocks when the overall stock market is depressed.
      True ____ False ____
   d. Stocks that are undervalued using the dividend discount model have generally made significant positive excess returns over long time periods (five years or more).
      True ____ False ____
   e. Stocks that pay high dividends and have low price-earnings ratios are more likely to come out as undervalued using the dividend discount model.
      True ____ False ____
2. Ameritech Corporation paid dividends per share of $3.56 in 1992, and dividends are expected to grow 5.5% a year forever. The stock has a beta of 0.90, and the Treasury bond rate is 6.25%. (Risk premium is 5.5%.)
   a. What is the value per share, using the Gordon growth model?
   b. The stock was trading for $80 per share. What would the growth rate in dividends have to be to justify this price?
3. Church & Dwight, a large producer of sodium bicarbonate, reported earnings per share of $1.50 in 1993 and paid dividends per share of $0.42. In 1993, the firm also reported the following:

   Net income = $30 million
   Interest expense = $0.8 million
   Book value of debt = $7.6 million
   Book value of equity = $160 million

   The firm faced a corporate tax rate of 38.5%. (The market value debt-to-equity ratio is 5%. The Treasury bond rate is 7%.)

   The firm expected to maintain these financial fundamentals from 1994 to 1998, after which it was expected to become a stable firm, with an earnings growth rate of 6%. The firm's financial characteristics were expected to approach industry averages after 1998. The industry averages were as follows:

   Return on capital = 12.5%
   Debt/equity ratio = 25%
   Interest rate on debt = 7%

   Church & Dwight had a beta of 0.85 in 1993, and the unlevered beta was not expected to change over time.
   a. What is the expected growth rate in earnings, based on fundamentals, for the high-growth period (1994 to 1998)?

b. What is the expected payout ratio after 1998?
c. What is the expected beta after 1998?
d. What is the expected price at the end of 1998?
e. What is the value of the stock, using the two-stage dividend discount model?
f. How much of this value can be attributed to extraordinary growth? To stable growth?

4. Oneida Inc, the world's largest producer of stainless steel and silverplated flatware, reported earnings per share of $0.80 in 1993, and paid dividends per share of $0.48 in that year. The firm was expected to report earnings growth of 25% in 1994, after which the growth rate was expected to decline linearly over the following six years to 7% in 1999. The stock was expected to have a beta of 0.85. (The Treasury bond rate is 6.25%, and the risk premium is 5.5%.)
   a. Estimate the value of stable growth, using the H model.
   b. Estimate the value of extraordinary growth, using the H model.
   c. What are the assumptions about dividend payout in the H model?
5. Medtronic Inc., the world's largest manufacturer of implantable biomedical devices, reported earnings per share in 1993 of $3.95, and paid dividends per share of $0.68. Its earnings were expected to grow 16% from 1994 to 1998, but the growth rate was expected to decline each year after that to a stable growth rate of 6% in 2003. The payout ratio was expected to remain unchanged from 1994 to 1998, after which it would increase each year to reach 60% in steady state. The stock was expected to have a beta of 1.25 from 1994 to 1998, after which the beta would decline each year to reach 1.00 by the time the firm becomes stable. (The Treasury bond rate is 6.25%, and the risk premium is 5.5%.)
   a. Assuming that the growth rate declines linearly (and the payout ratio increases linearly) from 1999 to 2003, estimate the dividends per share each year from 1994 to 2003.
   b. Estimate the expected price at the end of 2003.
   c. Estimate the value per share, using the three-stage dividend discount model.
6. Yuletide Inc. is a manufacturer of Christmas ornaments. The firm earned $100 million last year and paid out 20% of its earnings as dividends. The firm also has bought back $180 million of stock over the past four years, in varying amounts each year. The firm is in stable growth, expects to grow 5% a year in perpetuity, and has a cost of equity of 12%.
   a. Assuming that the dividend payout ratio will not change over time, estimate the value of equity.
   b. How would your answer change if your dividend payout ratio is modified to include stock buybacks?

CHAPTER 14

Free Cash Flow to Equity Discount Models

The dividend discount model is based on the premise that the only cash flows received by stockholders are dividends. Even if we use the modified version of the model and treat stock buybacks as dividends, we may misvalue firms that consistently fail to return what they can afford to their stockholders.

This chapter uses a more expansive definition of cash flows to equity as the cash flows left over after meeting all financial obligations, including debt payments, and after covering capital expenditure and working capital needs. It discusses the reasons for differences between dividends and free cash flows to equity, and presents the discounted free cash flow to equity model for valuation.

MEASURING WHAT FIRMS CAN RETURN TO THEIR STOCKHOLDERS

Given what firms are returning to their stockholders in the form of dividends or stock buybacks, how do we decide whether they are returning too much or too little? We measure how much cash is available to be paid out to stockholders after meeting reinvestment needs and compare this amount to the amount actually returned to stockholders.

Free Cash Flows to Equity

To estimate how much cash a firm can afford to return to its stockholders, we begin with the net income—the accounting measure of the stockholders' earnings during the period—and convert it to a cash flow by subtracting out a firm's reinvestment needs. First, any capital expenditures, defined broadly to include acquisitions, are subtracted from the net income, since they represent cash outflows. Depreciation and amortization, on the other hand, are added back in because they are noncash charges. The difference between capital expenditures and depreciation (net capital expenditures) is usually a function of the growth characteristics of the firm. High-growth firms tend to have high net capital expenditures relative to earnings, whereas low-growth firms may have low, and sometimes even negative, net capital expenditures.

Second, increases in working capital drain a firm's cash flows, while decreases in working capital increase the cash flows available to equity investors. Firms that are growing fast, in industries with high working capital requirements (retailing, for instance), typically have large increases in working capital. Since we are interested in the cash flow effects, we consider only changes in noncash working capital in this analysis.

Finally, equity investors also have to consider the effect of changes in the levels of debt on their cash flows. Repaying the principal on existing debt represents a cash outflow, but the debt repayment may be fully or partially financed by the issue of new debt, which is a cash inflow. Again, netting the repayment of old debt against the new debt issues provides a measure of the cash flow effects of changes in debt.

Allowing for the cash flow effects of net capital expenditures, changes in working capital, and net changes in debt on equity investors, we can define the cash flows left over after these changes as the free cash flow to equity (FCFE):

$$\begin{aligned}\text{Free cash flow to equity} = {} & \text{Net income} - (\text{Capital expenditures} - \text{Depreciation}) \\ & - (\text{Change in noncash working capital}) \\ & + (\text{New debt issued} - \text{Debt repayments})\end{aligned}$$

This is the cash flow available to be paid out as dividends.

This calculation can be simplified if we assume that the net capital expenditures and working capital changes are financed using a fixed mix[1] of debt and equity. If δ is the proportion of the net capital expenditures and working capital changes that is raised from debt financing, the effect on cash flows to equity of these items can be represented as follows:

Equity cash flows associated with meeting capital expenditure needs

$$= -(\text{Capital expenditures} - \text{Depreciation})(1 - \delta)$$

Equity cash flows associated with meeting working capital needs

$$= -(\Delta\ \text{Working capital})(1 - \delta)$$

Accordingly, the cash flow available for equity investors after meeting capital expenditure and working capital needs is:

$$\begin{aligned}\text{Free cash flow to equity} = {} & \text{Net income} - (\text{Capital expenditures} - \text{Depreciation}) \\ & \times (1 - \delta) - (\Delta\ \text{Working capital})(1 - \delta)\end{aligned}$$

Note that the net debt payment item is eliminated, because debt repayments are financed with new debt issues to keep the debt ratio fixed. It is particularly useful to assume that a specified proportion of net capital expenditures and working capital needs will be financed with debt if the target or optimal debt ratio of the firm is used to forecast the free cash flow to equity that will be available in future periods. Alternatively, in examining past periods, we can use the firm's average debt ratio over the period to arrive at approximate free cash flows to equity.

[1]The mix has to be fixed in book value terms. It can be varying in market value terms.

WHAT ABOUT PREFERRED DIVIDENDS?

In both the long and short formulations of free cash flows to equity described in the preceding section, we have assumed that there are no preferred dividends paid. Since the equity that we value is only common equity, you would need to modify the formulas slightly for the existence of preferred stock and dividends. In particular, you would subtract the preferred dividends to arrive at the free cash flow to equity:

Free cash flow to equity = Net income – (Capital expenditures
– Depreciation) – (Change in noncash WC)
– (Preferred dividends + New preferred stock issued)
+ (New debt issued – Debt repayments)

In the short form, you would obtain the following:

Free cash flow to equity = Net income – Preferred dividend
– (Capital expenditures – Depreciation)
$\times$ (1 – δ) – (Δ Working capital)(1 – δ)

The debt ratio (δ) would then have to include the expected financing from new preferred stock issues.

ILLUSTRATION 14.1: Estimating Free Cash Flows to Equity: The Home Depot and Boeing

In this illustration, we estimate the free cash flows to equity for the Home Depot, the home improvement retail giant, and Boeing. We begin by estimating the free cash flow to equity for the Home Depot each year from 1989 to 1998 in the table, using the full calculation described in the last section.

Year	*Net Income*	*Depreciation*	*Capital Spending*	*Change in Noncash Working Capital*	*Net Debt Issued*	*FCFE*
1989	$ 111.95	$ 21.12	$ 190.24	$ 6.20	$181.88	$118.51
1990	$ 163.43	$ 34.36	$ 398.11	$ 10.41	$228.43	$ 17.70
1991	$ 249.15	$ 52.28	$ 431.66	$ 47.14	–$ 1.94	($179.31)
1992	$ 362.86	$ 69.54	$ 432.51	$ 93.08	$802.87	$709.68
1993	$ 457.40	$ 89.84	$ 864.16	$153.19	–$ 2.01	($472.12)
1994	$ 604.50	$129.61	$1,100.65	$205.29	$ 97.83	($474.00)
1995	$ 731.52	$181.21	$1,278.10	$247.38	$497.18	($115.57)
1996	$ 937.74	$232.34	$1,194.42	$124.25	$470.24	$321.65
1997	$1,160.00	$283.00	$1,481.00	$391.00	–$ 25.00	($454.00)
1998	$1,615.00	$373.00	$2,059.00	$131.00	$238.00	$ 36.00
Average	$ 639.36	$146.63	$ 942.99	$140.89	$248.75	($ 49.15)

As the table indicates, the Home Depot had negative free cash flows to equity in 5 of the 10 years, largely as a consequence of significant capital expenditures. The average net debt issued during the period was $248.75 million, and the average net capital expenditure and working capital needs amounted to $937.25 million ($942.99 – 146.63 + 140.89), resulting in a debt ratio of 26.54%. Using the approximate formulation for FCFE yields the following results for FCFE for the same period:

Year	*Net Income*	*Net Capital Expenditures (1 – DR)*	*Change in Noncash Working Capital (1 – DR)*	*FCFE*
1989	$ 111.95	$ 124.24	$ 4.55	($ 16.84)
1990	$ 163.43	$ 267.21	$ 7.65	($111.43)
1991	$ 249.15	$ 278.69	$ 34.63	($ 64.17)
1992	$ 362.86	$ 266.64	$ 68.38	$ 27.85
1993	$ 457.40	$ 568.81	$112.53	($223.95)
1994	$ 604.50	$ 713.32	$150.81	($259.63)
1995	$ 731.52	$ 805.77	$181.72	($255.98)
1996	$ 937.74	$ 706.74	$ 91.27	$139.72
1997	$1,160.00	$ 880.05	$287.23	($ 7.28)
1998	$1,615.00	$1,238.53	$ 96.23	$280.24
Average	$ 639.36	$ 585.00	$103.50	($ 49.15)

DR = Average debt ratio during the period = 26.54%

Note that the approximate formulation yields the same average FCFE for the period. Since new debt issues are averaged out over the 10 years in the approach, it also smooths out the annual FCFE, since actual debt issues are much more unevenly spread over time.

A similar estimation of FCFE was done for Boeing from 1989 to 1998 in the following table:

Year	*Net Income*	*Net Capital Expenditures (1 – DR)*	*Change in Noncash Working Capital (1 – DR)*	*FCFE*
1989	$ 973.00	$423.80	$333.27	$ 215.93
1990	$1,385.00	$523.55	$113.59	$ 747.86
1991	$1,567.00	$590.44	($ 55.35)	$1,031.92
1992	$ 552.00	$691.34	($555.26)	$ 415.92
1993	$1,244.00	$209.88	$268.12	$ 766.00
1993	$ 856.00	($200.08)	$ 6.34	$1,049.74
1995	$ 393.00	($232.95)	($340.77)	$ 966.72
1996	$1,818.00	($155.68)	($ 21.91)	$1,995.59
1997	($ 178.00)	$516.63	($650.98)	($ 43.65)
1998	$1,120.00	$754.77	$107.25	$ 257.98
Average	$ 973.00	$312.17	($ 79.57)	$ 740.40

DR = Average debt ratio during the period = 42.34%

During the period, Boeing financed a high proportion of its reinvestment needs with debt, and its market debt ratio increased from about 1% to approximately 20%. The average free cash flow to equity during the period was $740.40 million. Note that the 1997 and 1998 capital expenditures include the amount spent by Boeing to acquire McDonnell Douglas.

Comparing Dividends to Free Cash Flows to Equity

The conventional measure of dividend policy—the dividend payout ratio—gives us the value of dividends as a proportion of earnings. Our approach measures the total cash returned to stockholders as a proportion of the free cash flow to equity:

Dividend payout ratio = Dividends/Earnings

Cash to stockholders to FCFE ratio = (Dividends + Equity repurchases)/FCFE

The ratio of cash to stockholders to FCFE shows how much of the cash available to be paid out to stockholders is actually returned to them in the form of dividends and stock buybacks. If this ratio, over time, is equal or close to 1, the firm is paying out all that it can to its stockholders. If it is significantly less than 1, the firm is paying out less than it can afford to and is using the difference to increase its cash balance or to invest in marketable securities. If it is significantly over 1, the firm is paying out more than it can afford and is either drawing on an existing cash balance or issuing new securities (stocks or bonds).

We can observe the tendency of firms to pay out less to stockholders than they have available in free cash flows to equity by examining cash returned to stockholders paid as a percentage of free cash flow to equity. In 1998, for instance, the average dividend to free cash flow to equity ratio across all firms on the New York Stock Exchange was 51.55%. Figure 14.1 shows the distribution of cash returned as a percent of FCFE across all firms.

A percentage less than 100 percent means that the firm is paying out less in dividends than it has available in free cash flows and that it is generating surplus cash. For those firms, this cash surplus appears as an increase in the cash balance. A percentage greater than 100 percent indicates that the firm is paying out more in dividends than it has available in cash flow. These firms have to finance these dividend payments either out of existing cash balances or by making new stock issues.

The implications for valuation are simple. If we use the dividend discount model and do not allow for the buildup of cash that occurs when firms pay out less than they can afford, we will underestimate the value of equity in firms. If we use the model to value firms that pay out more dividends than they have available, we will overvalue the firm. The rest of this chapter is designed to correct for this limitation.

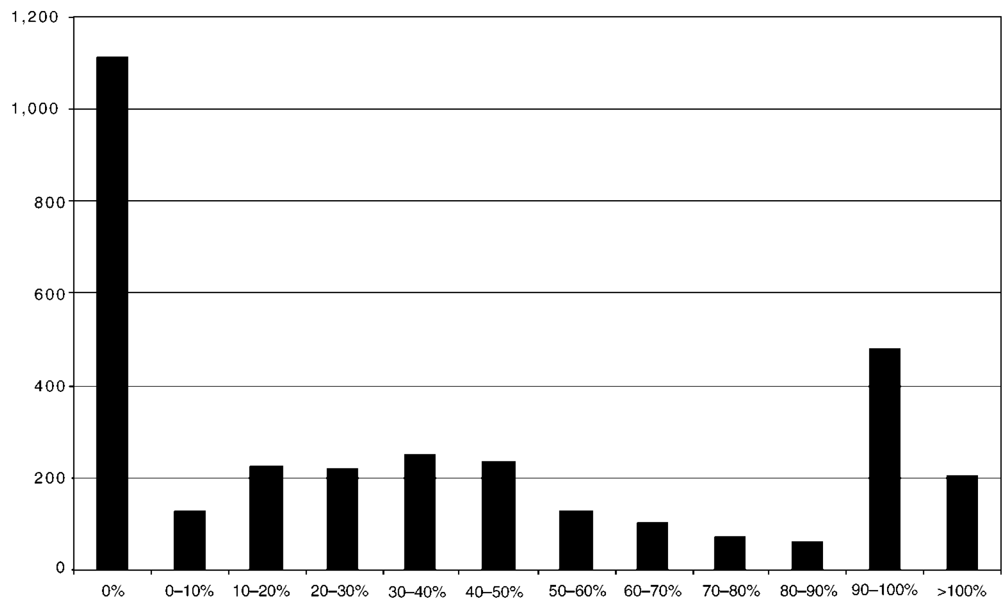

FIGURE 14.1 Cash Returned as Percent of FCFE
Source: Compustat database 1998.

dividends.xls: **This spreadsheet allows you to estimate the free cash flow to equity and the cash returned to stockholders for a period of up to 10 years.**

divfcfe.xls: **This dataset on the Web summarizes dividends, cash returned to stockholders, and free cash flows to equity, by sector, in the United States.**

Why Firms May Pay Out Less than Is Available

Many firms pay out less to stockholders, in the form of dividends and stock buybacks, than they have available in free cash flows to equity. The reasons vary from firm to firm.

Desire for Stability Firms are generally reluctant to change dividends, and dividends are considered "sticky" because the variability in dividends is significantly lower than the variability in earnings or cash flows. The unwillingness to change dividends is accentuated when firms have to reduce dividends, and empirically, increases in dividends outnumber cuts in dividends by at least a five-to-one margin in most periods. As a consequence of this reluctance to cut dividends, firms will often refuse to increase dividends even when earnings and FCFE go up, because they are uncertain about their capacity to maintain these higher dividends. This leads to a lag between earnings increases and dividend increases. Similarly, firms frequently keep dividends unchanged in the face of declining earnings and FCFE. Figure 14.2 reports the number of dividend changes (increases, decreases, no changes) between 1989 and 1998.

The number of firms increasing dividends outnumbers those decreasing dividends seven to one. The number of firms, however, that do not change dividends

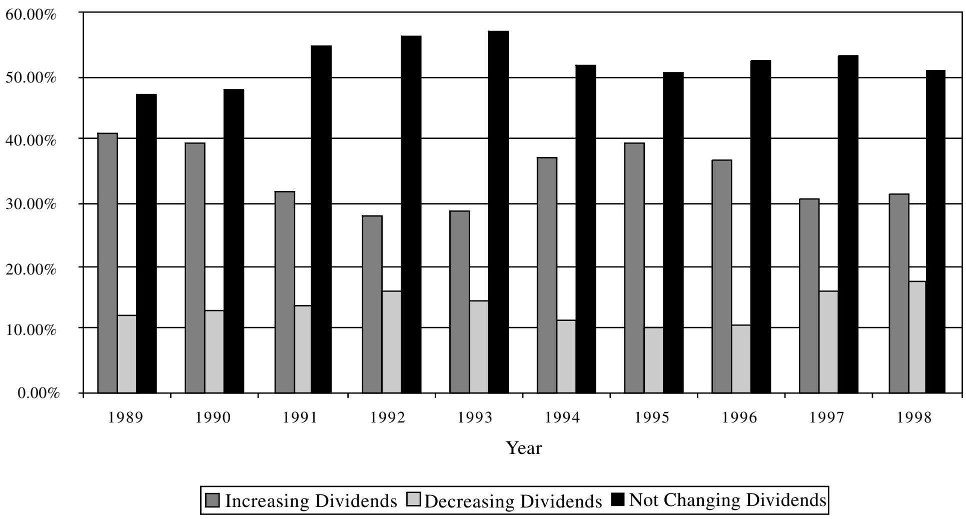

FIGURE 14.2 Dividend Changes, 1989–1998
Source: Compustat.

outnumbers firms that do about four to one. Dividends are also less variable than either FCFE or earnings, but this reduced volatility is a result of keeping dividends significantly below the FCFE.

Future Investment Needs A firm might hold back on paying its entire FCFE as dividends if it expects substantial increases in capital expenditure needs in the future. Since issuing stocks is expensive (from a flotation cost standpoint), it may choose to keep the excess cash to finance these future needs. Thus, to the degree that a firm may be unsure about its future financing needs, it may retain some cash to take on unexpected investments or meet unanticipated needs.

Tax Factors If dividends are taxed at a higher tax rate than capital gains, a firm may choose to retain the excess cash and pay out much less in dividends than it has available. This is likely to be accentuated if the stockholders in the firm are in high tax brackets, as is the case with many family-controlled firms. If, however, investors in the firm like dividends or tax laws favor dividends, the firm may pay more out in dividends than it has available in FCFE, often borrowing or issuing new stock to do so.

Signaling Prerogatives Firms often use dividends as signals of future prospects, with increases in dividends being viewed as positive signals and decreases as negative signals. The empirical evidence is consistent with this signaling story, since stock prices generally go up on dividend increases and down on dividend decreases. The use of dividends as signals may lead to differences between dividends and FCFE.

Managerial Self-interest The managers of a firm may gain by retaining cash rather than paying it out as a dividend. The desire for empire building may make increasing the size of the firm an objective on its own. Or management may feel the need to build up a cash cushion to tide over periods when earnings may dip; in such periods, the cash cushion may reduce or obscure the earnings drop and may allow managers to remain in control.

FCFE VALUATION MODELS

The free cash flow to equity model does not represent a radical departure from the traditional dividend discount model. In fact, one way to describe a free cash flow to equity model is that it represents a model where we discount potential dividends rather than actual dividends. Consequently, the three versions of the FCFE valuation model presented in this section are simple variants on the dividend discount model, with one significant change—free cash flows to equity replace dividends in the models.

Underlying Principle

When we replace the dividends with FCFE to value equity, we are doing more than substituting one cash flow for another. We are implicitly assuming that the FCFE will be paid out to stockholders. There are two consequences:

1. There will be no future cash buildup in the firm, since the cash that is available after debt payments and reinvestment needs is paid out to stockholders each period.
2. The expected growth in FCFE will include growth in income from operating assets and not growth in income from increases in marketable securities. This follows directly from the last point.

How does discounting free cash flows to equity compare with the modified dividend discount model, where stock buybacks are added back to dividends and discounted? You can consider stock buybacks to be the return of excess cash accumulated largely as a consequence of not paying out their FCFE as dividends. Thus, FCFE represents a smoothed-out measure of what companies can return to their stockholders over time in the form of dividends and stock buybacks.

Estimating Growth in FCFE

Free cash flows to equity, like dividends, are cash flows to equity investors and you could use the same approach that you used to estimate the fundamental growth rate in dividends per share:

$$\text{Expected growth rate} = \text{Retention ratio} \times \text{Return on equity}$$

The use of the retention ratio in this equation implies that whatever is not paid out as dividends is reinvested back into the firm. There is a strong argument to be made, though, that this is not consistent with the assumption that free cash flows to equity are paid out to stockholders, which underlies FCFE models. It is far more consistent to replace the retention ratio with the equity reinvestment rate, which measures the percent of net income that is invested back into the firm.

$$\text{Equity reinvestment rate} = 1 - (\text{Net cap ex} + \text{Change in working capital} - \text{Net debt issues})/\text{Net income}$$

The return on equity may also have to be modified to reflect the fact that the conventional measure of the return includes interest income from cash and marketable securities in the numerator and the book value of equity also includes the value of the cash and marketable securities. In the FCFE model, there is no excess cash left in the firm and the return on equity should measure the return on noncash investments. You could construct a modified version of the return on equity that measures this:

$$\text{Noncash ROE} = \frac{\text{Net income} - \text{After-tax income from cash and marketable securities}}{\text{Book value of equity} - \text{Cash and marketable securities}}$$

The product of the equity reinvestment rate and the modified ROE will yield the expected growth rate in FCFE:

$$\text{Expected growth in FCFE} = \text{Equity reinvestment rate} \times \text{Noncash ROE}$$

Constant Growth FCFE Model

The constant growth FCFE model is designed to value firms that are growing at a stable growth rate and are hence in steady state.

The Model The value of equity, under the constant growth model, is a function of the expected FCFE in the next period, the stable growth rate, and the required rate of return.

$$\text{Value} = \frac{\text{FCFE}_1}{k_e - g_n}$$

where Value = Value of stock today
FCFE_1 = Expected FCFE next year
k_e = Cost of equity of the firm
g_n = Growth rate in FCFE for the firm forever

Caveats The model is very similar to the Gordon growth model in its underlying assumptions and works under some of the same constraints. The growth rate used in the model has to be reasonable, relative to the nominal growth rate in the economy in which the firm operates. As a general rule, a stable growth rate cannot exceed the growth rate of the economy in which the firm operates.

The assumption that a firm is in steady state also implies that it possesses other characteristics shared by stable firms. This would mean, for instance, that capital expenditures are not disproportionately large, relative to depreciation, and the firm is of average risk. (If the capital asset pricing model is used, the beta of the equity should be close to 1.) To estimate the reinvestment for a stable growth firm, you can use one of two approaches:

You can use the typical reinvestment rates for firms in the industry to which the firm belongs. A simple way to do this is to use the average capital expenditure to depreciation ratio for the industry (or better still, just stable firms in the industry) to estimate a normalized capital expenditure for the firm.

Alternatively, you can use the relationship between growth and fundamentals to estimate the required reinvestment. The expected growth in net income can be written as:

$$\text{Expected growth rate in net income} = \text{Equity reinvestment rate} \times \text{Return on equity}$$

This allows us to estimate the equity reinvestment rate:

$$\text{Equity reinvestment rate} = \text{Expected growth rate/Return on equity}$$

To illustrate, a firm with a stable growth rate of 4 percent and a return on equity of 12 percent would need to reinvest about one-third of its net income back into net capital expenditures and working capital needs. Put another way, the free cash flows to equity should be two-thirds of net income.

Best Suited for Firms This model, like the stable growth dividend discount model, is best suited for firms growing at a rate comparable to or lower than the nominal growth in the economy. It is, however, the better model to use than the dividend discount model for stable firms that pay out dividends that are unsustainably high (because they exceed FCFE by a significant amount) or are significantly lower than the FCFE. Note, though, that if the firm is stable, and pays out its FCFE as dividend, the value obtained from this model will be the same as the one obtained from the Gordon growth model.

ILLUSTRATION 14.2: FCFE Stable Growth Model: Singapore Airlines

Rationale for Using the Model

- Singapore Airlines is a large firm in a mature industry. Given the competition for air passengers and the limited potential for growth, it seems reasonable to assume stable growth for the future. Singapore Airline's revenues have grown about 3% a year for the past five years.
- Singapore Airlines has maintained a low book debt ratio historically, and its management seems inclined to keep leverage low.

Background Information

In the financial year ended March 2001, Singapore Airlines reported net income of S$1,164 million on revenues of S$7,816 million, and earned a noncash return on equity of 10% for the year. The capital expenditures during the year amounted to S$2,214 million, but the average capital expenditures between 1997 and 2000 were S$1,520 million. The depreciation in 2000 was S$1,205 million. The noncash working capital increased by $303 million in 2000. The book value debt to capital ratio at the end of 2000 was 5.44%.

Estimation

We begin by estimating a normalized free cash flow to equity for the current year. We will assume that earnings will grow 5% over the next year. To estimate net capital expenditures, we will use the average capital expenditures between 1997 and 2000 (to smooth out the year-to-year jumps) and the depreciation from the most recent year. Finally, we will assume that the 5.44% of future reinvestment needs will come from debt, reflecting the firm's current book debt ratio:[2]

Net income next year	$1,164 million
Net cap ex (1 – Debt ratio) = (1,520 – 1,205)(1 – .0544)	$298 million
Change in working capital (1 – Debt ratio) = 303 (1 – .0544)	$287 million
Normalized FCFE for current year	$579 million

As a check, we also computed the equity reinvestment rate that Singapore Airlines would need to maintain to earn a growth of 5%, based on its return on equity of 10%:

$$\text{Equity reinvestment rate} = g/\text{ROE} = 50\%$$

With this reinvestment rate, the free cash flows to equity would have been half the net income. The reinvestment we used in the calculation above is very close to this value:

$$\text{Equity reinvestment rate used} = (289 + 287)/1{,}164 = 50.2\%$$

[2]In making estimates for the future, you can go with either book or market debt ratios, depending on what you think about the firm's financing policy.

To estimate the cost of equity, we used the bottom-up unlevered beta for airlines (0.81), Singapore Airlines' market debt to equity ratio of 3.63% and tax rate of 38%.

$$\text{Levered beta} = 0.81[1 + (1 - .38)(.0363)] = 0.83$$

Using a riskless rate of 6% based on a 10-year S$-denominated bond issued by the Singapore government, and using a risk premium of 5% (4% for mature market risk plus 1% for additional country risk), we estimate a cost of equity:

$$\text{Cost of equity} = 6\% + 0.83 \times (5\%) = 10.14\%$$

VALUATION

With the normalized FCFE estimated above, a perpetual growth rate of 5%, and a cost of equity of 10.14%, we can estimate the value of equity:

$$\text{Value of equity} = \text{Expected FCFE next year}/(\text{Cost of equity} - \text{Expected growth})$$
$$= 579(1.05)/(.1014 - .05) = \text{S\$}11{,}833 \text{ million}$$

The equity in the firm had a market value of S$14,627 million in May 2001.

FCFEst.xls: **This spreadsheet allows you to value the equity in a firm in stable growth, with all of the inputs of a stable growth firm.**

LEVERAGE, FCFE, AND EQUITY VALUE

Embedded in the FCFE computation seems to be the makings of a free lunch. Increasing the debt ratio increases free cash flow to equity because more of a firm's reinvestment needs will come from borrowing and less is needed from equity investors. The released cash can be paid out as additional dividends or used for stock buybacks. In the case for Singapore Airlines, for instance, the free cash flow to equity is shown as a function of the debt to capital ratio in Figure 14.3.

If the free cash flow to equity increases as the leverage increases, does it follow that the value of equity will also increase with leverage? Not necessarily. The discount rate used is the cost of equity, which is estimated based on a beta or betas. As leverage increases, the beta will also increase, pushing up the cost of equity. In fact, in the levered beta equation that we introduced in Chapter 8 the levered beta is:

$$\text{Levered beta} = \text{Unlevered beta } [1 + (1 - \text{Tax rate})(\text{Debt/Equity})]$$

This, in turn, will have a negative effect on equity value. The net effect on value will then depend on which effect—the increase in cash flows or the increase in betas—dominates. Figure 14.4 graphs out the value of Singapore Airlines as a function of the debt-to-capital ratio. The value of equity is maximized at a debt ratio of 30 percent, but beyond that level debt's costs outweigh its benefits.

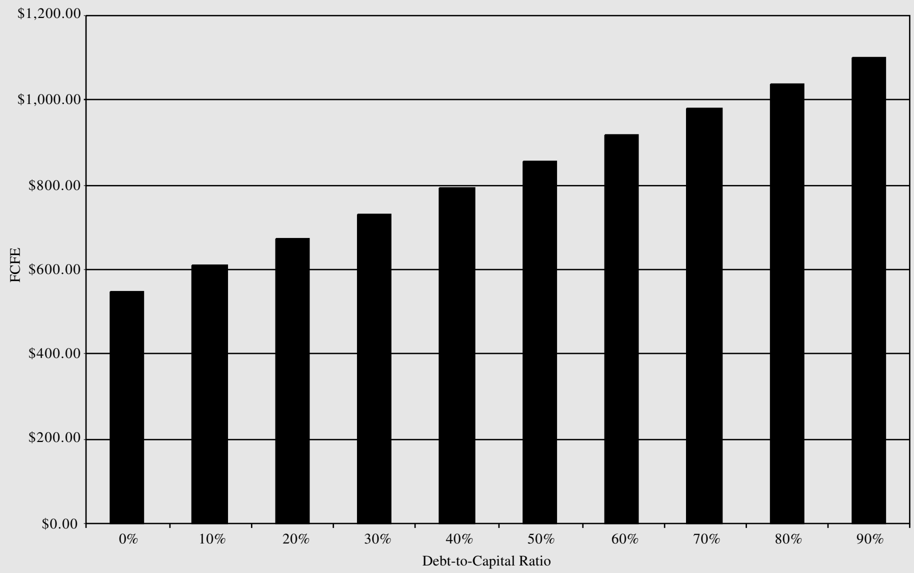

FIGURE 14.3 FCFE and Leverage—Singapore Airlines

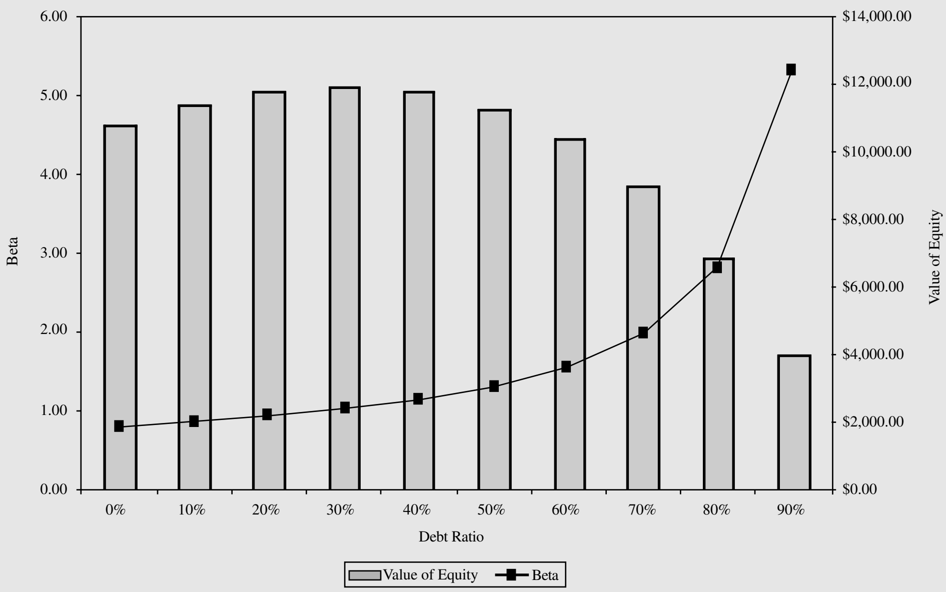

FIGURE 14.4 Singapore Air—Leverage and Value of Equity

A TROUBLESHOOTING GUIDE: WHAT IS WRONG WITH THIS VALUATION? (CONSTANT GROWTH FCFE MODEL)

If This Is Your Problem	*This May Be the Solution*
• If you get a low value from this model, it may be because:	
Capital expenditures are too high relative to depreciation.	Use a smaller cap ex or use the two-stage model.
Working capital as a percent of revenues is too high.	Normalize this ratio, using historical averages.
The beta is high for a stable firm.	Use a beta closer to 1.
• If you get too high a value, it is because:	
Capital expenditures are lower than depreciation.	Estimate an appropriate reinvestment rate = g/ROE.
Working capital ratio as percent of revenue is negative.	Set equal to zero.
The expected growth rate is too high for a stable firm.	Use a growth rate less than or equal to GNP growth.

Two-Stage FCFE Model

The two-stage FCFE model is designed to value a firm that is expected to grow much faster than a stable firm in the initial period and at a stable rate after that.

The Model The value of any stock is the present value of the FCFE per year for the extraordinary growth period plus the present value of the terminal price at the end of the period.

$$\text{Value} = \text{PV of FCFE} + \text{PV of terminal price}$$

$$= \sum_{t=1}^{t=n} \text{FCFE}_t / (1 + k_{e,hg})^t + P_n / (1 + k_{e,hg})^n$$

where FCFE_t = Free cash flow to equity in year t
P_n = Price at the end of the extraordinary growth period
k_e = Cost of equity in high growth (hg) and stable growth (st) periods

The terminal price is generally calculated using the infinite growth rate model:

$$P_n = \text{FCFE}_{n+1} / (k_{e,st} - g_n)$$

where g_n = Growth rate after the terminal year forever

Calculating the Terminal Price The same caveats that apply to the growth rate for the stable growth rate model, described in the previous section, apply here as well. In addition, the assumptions made to derive the free cash flow to equity after the terminal year have to be consistent with this assumption of stability. For instance, while capital spending may be much greater than depreciation in the initial high-growth phase, the difference should narrow as the firm enters its stable growth phase. We can use the two approaches described for the stable growth model—industry average capital expenditure requirements or the fundamental growth equation (equity reinvestment rate = g/ROE) to make this estimate.

The beta and debt ratio may also need to be adjusted in stable growth to reflect the fact that stable growth firms tend to have average risk (betas closer to 1) and use more debt than high-growth firms.

ILLUSTRATION 14.3: Capital Expenditure, Depreciation, and Growth Rates

Assume you have a firm that is expected to have earnings growth of 20% for the next five years and 5% thereafter. The current earnings per share is \$2.50. Current capital spending is \$2.00, and current depreciation is \$1.00. If we assume that capital spending and depreciation grow at the same rate as earnings and there are no working capital requirements or debt:

Earnings in year 5 = $2.50 \times (1.20)^5$	\$6.22
Capital spending in year 5 = $2.00 \times (1.20)^5$	\$4.98
Depreciation in year 5 = $1.00 \times (1.20)^5$	\$2.49
Free cash flow to equity in year 5 = \$6.22 + \$2.49 – \$4.98	\$3.73

If we use the infinite growth rate model, but fail to adjust the imbalance between capital expenditures and depreciation, the free cash flow to equity in the terminal year is:

$$\text{Free cash flow to equity in year 6} = 3.73 \times 1.05 = \$3.92$$

This free cash flow to equity can then be used to compute the value per share at the end of year 5, but it will understate the true value. There are two ways in which you can adjust for this:

1. Adjust capital expenditures in year 6 to reflect industry average capital expenditure needs: Assume, for instance, that capital expenditures are 150% of depreciation for the industry in which the firm operates. You could compute the capital expenditures in year 6 as follows:

$$\text{Depreciation in year 6} = 2.49(1.05) = \$2.61$$

$$\begin{aligned}\text{Capital expenditures in year 6} &= \text{Depreciation in year 6} \\ &\quad \times \text{Industry average capital expenditures as \% of depreciation} \\ &= \$2.61 \times 1.50 = \$3.92\end{aligned}$$

$$\text{FCFE in year 6} = \$6.53 + \$2.61 - \$3.92 = \$5.23$$

2. Estimate the equity reinvestment rate in year 6, based on expected growth and the firm's return on equity. For instance, if we assume that this firm's return on equity will be 15% in stable growth, the equity reinvestment rate would need to be:

$$\text{Equity reinvestment rate} = g/\text{ROE} = 5\%/15\% = 33.33\%$$

$$\begin{aligned}\text{Net capital expenditures in year 6} &= \text{Equity reinvestment rate} \times \text{Earnings per share} \\ &= .3333 \times \$6.53 = \$2.18\end{aligned}$$

$$\begin{aligned}\text{Capital expenditures in year 6} &= \text{Net capital expenditures} + \text{Depreciation} \\ &= \$2.18 + \$2.61 = \$4.79\end{aligned}$$

$$\text{FCFE in year 6} = \$6.53 + \$2.61 - \$4.79 = \$4.35$$

Firms Model Works Best For This model makes the same assumptions about growth as the two-stage dividend discount model (i.e., that growth will be high and constant in the initial period and drop abruptly to stable growth after that). It is different because of its emphasis on FCFE rather than dividends. Consequently, it provides much better results than the dividend discount model when valuing firms which either have dividends which are unsustainable (because they are higher than FCFE) or which pay less in dividends than they can afford to (i.e., dividends are less than FCFE).

ILLUSTRATION 14.4: Two-Stage FCFE Model: Nestlé

Nestlé has operations all over the world, with 97% of its revenues coming from markets outside Switzerland, where it is headquartered. The firm, like many large European corporations, has a weak corporate governance system, and stockholders have little power over managers.

Rationale for Using the Model

- *Why two-stage?* Nestlé has a long and impressive history of growth, and while we believe that its growth will be moderate, we assume that it will be able to maintain high growth for 10 years.
- *Why FCFE?* Given its weak corporate governance structure and a history of accumulating cash, the dividends paid by Nestlé bear little resemblance to what the firm could have paid out.

Background Information

Current net income = Sfr 5,763 million
Current capital spending = Sfr 5,058 million
Current depreciation = Sfr 3,330 million
Current revenues = Sfr 81,422 million
Noncash working capital= Sfr 5,818 million
Change in working capital = Sfr 368 million
Net debt issues = Sfr 272 million

Earnings per share = Sfr 148.33
Capital expenditures/share = Sfr 130.18
Depreciation/share = Sfr 85.71
Revenue/share = Sfr 2,095.64
Working capital/share = Sfr 149.74
Change in working capital/share = Sfr 9.47

Estimates

We will begin by estimating the cost of equity for Nestlé during the high growth period in Swiss francs. We will use the 10-year Swiss government Sfr bond rate of 4% as the risk-free rate. To estimate the risk premium, we used the breakdown of Nestlé's revenues by region:

Region	*Revenues (in Billions Sfr)*	*Weight*	*Risk Premium*
North America	20.21	24.82%	4.00%
South America	4.97	6.10%	12.00%
Switzerland	1.27	1.56%	4.00%
Germany/France/United Kingdom	21.25	26.10%	4.00%
Italy/Spain	7.39	9.08%	5.50%
Asia	6.70	8.23%	9.00%
Rest of Western Europe	15.01	18.44%	4.00%
Eastern Europe	4.62	5.67%	8.00%
Total	81.42	100.00%	5.26%

The risk premiums for each region represent an average of the risk premiums of the countries in the region. Using a bottom-up beta of 0.85 for Nestlé, we estimated a cost of equity of:

$$\text{Cost of equity} = 4\% + 0.85(5.26\%) = 8.47\%$$

To estimate the expected growth rate in free cash flows to equity, we first computed the free cash flows to equity in the current year:

FCFE = Net income – (Cap ex – Depreciation) – Change in working capital + Net debt issues
= 5,763 – (5,058 – 3,330) – 368 + 272 = Sfr 3,939 million

The equity reinvestment rate can be estimated from this value:

Equity reinvestment rate = 1 – FCFE/Net income = 1 – 3,939/5,763 = 31.65%

The return on equity in 2000 was estimated using the net income from 2000 and the book value of equity from the end of the previous year:

Return on equity = 5,763/25,078 = 22.98%

The expected growth rate in FCFE is a product of the equity reinvestment rate and the return on equity:

Expected growth in FCFE = Equity reinvestment rate $\times$ Return on equity = .3165 $\times$ .2298 = 7.27%

We will assume that net capital expenditures and working capital will grow at the same rate as earnings and that the firm will raise 33.92% of its reinvestment needs from debt (which is its current book value debt-to-capital ratio).

In stable growth, we assume a growth rate of 4%. We also assume that the cost of equity remains unchanged but that the return on equity drops to 15%. The equity reinvestment rate in stable growth can be estimated as follows:

Equity reinvestment in stable growth = g/ROE = 4%/15% = 26.67%

Valuation

The first component of value is the present value of the expected FCFE during the high-growth period, (see table) assuming earnings, net capital expenditures, and working capital grow at 7.27% and 33.92% of reinvestment needs come from debt:

Year	*Earnings per Share*	*Net Cap Ex per Share*	*Change in Working Capital per Share*	*Reinvestment per Share*	*Equity Reinvestment per Share*	*FCFE per Share*	*Present Value*
1	159.12	47.71	10.89	58.60	38.72	120.39	110.99
2	170.69	51.18	11.68	62.86	41.54	129.15	109.76
3	183.10	54.90	12.53	67.44	44.56	138.54	108.55
4	196.42	58.90	13.44	72.34	47.80	148.62	107.35
5	210.71	63.18	14.42	77.60	51.28	159.43	106.17
6	226.03	67.77	15.47	83.25	55.01	171.02	105.00
7	242.47	72.70	16.60	89.30	59.01	183.46	103.84
8	260.11	77.99	17.80	95.80	63.30	196.81	102.69
9	279.03	83.67	19.10	102.76	67.91	211.12	101.56
10	299.32	89.75	20.49	110.24	72.85	226.48	100.44
		Sum of present value of FCFE					1,056.34

Note that the change in working capital each year is computed based on the existing working capital of Sfr 149.74 per share, and that the present value is computed using the cost of equity of 8.47%.

To estimate the terminal value, we first estimate the free cash flows to equity in year 11:

Expected earnings per share in year 11 = $EPS_{10}(1 + g)$ = 299.32(1.04) = 311.30
Equity reinvestment in year 11 = $EPS_{11} \times$ Stable equity reinvestment rate = 311.30 × .2667 = 83.02
Expected FCFE in year 11 = EPS_{11} – Equity $reinvestment_{11}$ = 311.30 – 83.02 = 228.28
Terminal value of equity per share = $FCFE_{11}$/(Cost of $equity_{11}$ – g) = 228.28/(.0847 – .04) = 5,105.88

The value per share can be estimated as the sum of the present value of FCFE during the high growth phase and the present value of the terminal value of equity:

Value per share = PV of dividend during high-growth phase + Terminal price/$(1 + k_e)^n$
= 1,056.34 + 5,105.88/1.0847^{10} = 3,320.65 Sfr

The stock was trading at 3,390 Sfr per share in May 2001 at the time of this valuation.

***FCFE2st.xls:* This spreadsheet allows you to value a firm with a temporary period of high growth in FCFE, followed by stable growth.**

REINVESTMENT ASSUMPTIONS, TERMINAL VALUE, AND EQUITY VALUE

We have repeatedly emphasized the importance of linking growth assumptions to assumptions about reinvestment, and especially so in stable growth. A very common assumption in many discounted cash flow valuations is that capital expenditures offset depreciation in stable growth. When combined with the assumption of no working capital changes, this translates into zero reinvestment. While this may be a reasonable assumption for a year or two, it is not consistent with the assumption that operating income will grow in perpetuity. How much of a difference can one assumption make? In the Nestlé valuation, we reestimated terminal value of equity per share assuming no reinvestment:

Estimated terminal value of equity per share = 311.30/(.0847 – .04) = 6,962.57

Keeping all of our other assumptions intact, this results in a value of equity per share of 4,144 Sfr per share—an increase in value of approximately 22 percent.

E Model—A Three-Stage FCFE Model

The E model is designed to value firms that are expected to go through three stages of growth—an initial phase of high growth rates, a transitional period where the growth rate declines, and a steady-state period where growth is stable.

The Model The E model calculates the present value of expected free cash flow to equity over all three stages of growth:

$$P_0 = \sum_{t=1}^{t=n1} \frac{FCFE_t}{(1+k_e)^t} + \sum_{t=n1+1}^{t=n2} \frac{FCFE_t}{(1+k_e)^t} + \frac{P_{n2}}{(1+k_e)^{n2}}$$

where P_0 = Value of the stock today
$FCFE_t$ = FCFE in year t
k_e = Cost of equity
P_{n2} = Terminal price at the end of transitional period = $FCFE_{n2+1}/(k_e - g_n)$
n1 = End of initial high-growth period
n2 = End of transition period

Caveats in Using Model Since the model assumes that the growth rate goes through three distinct phases—high growth, transitional growth, and stable growth—it is important that assumptions about other variables are consistent with these assumptions about growth.

Capital Spending versus Depreciation It is reasonable to assume that as the firm goes from high growth to stable growth, the relationship between capital

A TROUBLESHOOTING GUIDE: WHAT IS WRONG WITH THIS VALUATION? (TWO-STAGE FCFE MODEL)

If This Is Your Problem	*This May Be the Solution*
• If you get a extremely low value from the two-stage FCFE, the likely culprits are:	
Earnings are depressed due to some reason (economy, etc.).	Use normalized earnings.
Capital expenditures are significantly higher than depreciation in stable growth phase.	Reduce the difference for stable growth period. (Compute the appropriate reinvestment rate—you might need a higher ROE.)
The beta in the stable period is too high for a stable firm.	Use a beta closer to 1.
Working capital as percent of revenue is too high to sustain.	Use a working capital ratio closer to industry.
The use of the two-stage model when the three-stage model is more appropriate.	Use a three-stage model.
• If you get an extremely high value:	
Earnings are inflated above normal levels.	Use normalized earnings.
Capital expenditures offset or lag depreciation during high-growth period.	Compute the appropriate reinvestment rate = g/ROE.
The growth rate in the stable growth period is too high for stable firm.	Use a growth rate closer to GNP growth.

spending and depreciation will change. In the high-growth phase, capital spending is likely to much larger than depreciation. In the transitional phase, the difference is likely to narrow and the difference between capital spending and depreciation will be lower still in stable growth, reflecting the lower expected growth rate. (See Figure 14.5.)

Risk As the growth characteristics of a firm change, so do its risk characteristics. In the context of the CAPM, as the growth rate declines the beta of the firm can be expected to change. The tendency of betas to converge toward one in the long term has been confirmed by empirical observation of portfolios of firms with high betas. Over time, as these firms get larger and more diversified, the average betas of these portfolios move toward 1.

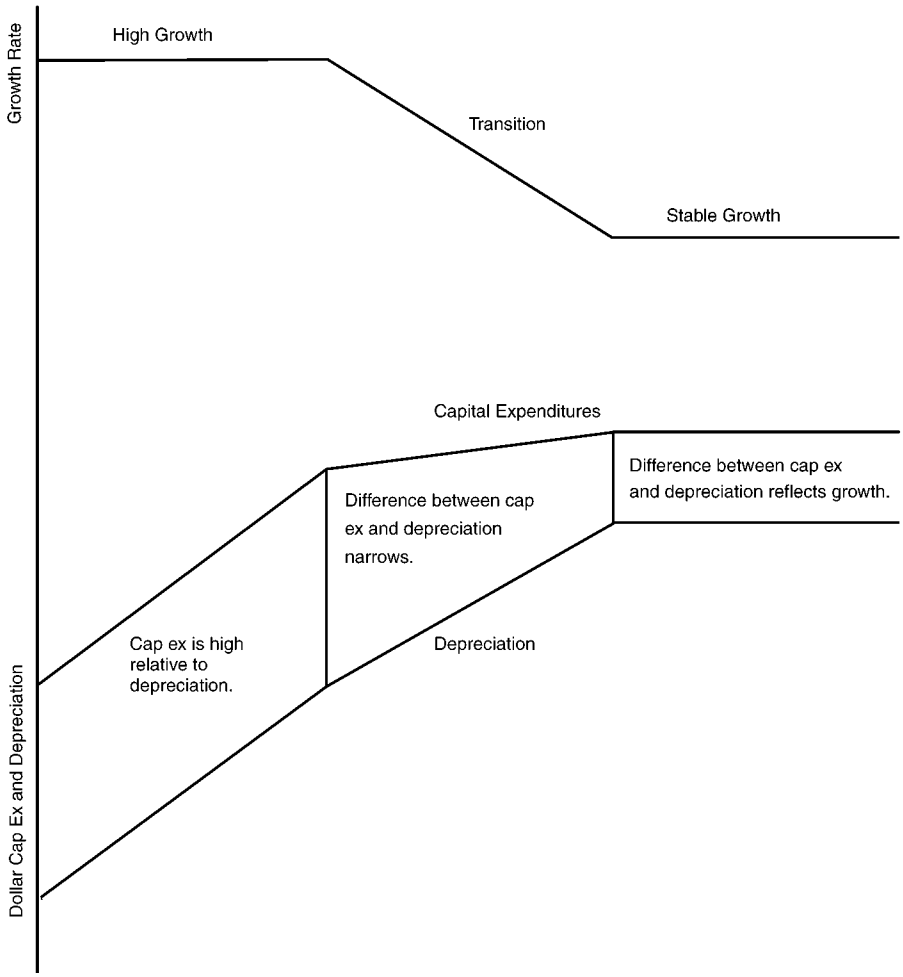

FIGURE 14.5 Three-Stage FCFE Model: Reinvestment Needs

Firms Model Works Best For Since the model allows for three stages of growth and for a gradual decline from high to stable growth, it is the appropriate model to use to value firms with very high growth rates currently. The assumptions about growth are similar to the ones made by the three-stage dividend discount model, but the focus is on FCFE instead of dividends, making it more suited to value firms whose dividends are significantly higher or lower than the FCFE.

ILLUSTRATION 14.5: Three-Stage FCFE Model: Tsingtao Breweries (China)

Tsingtao Breweries produces and distributes beer and other alcoholic beverages in China and around the world under the Tsingtao brand name. The firm has 653.15 million shares listed on the Shanghai and Hong Kong exchanges.

Rationale for Using the Three-Stage FCFE Model

- *Why three-stage?* Tsingtao is a small firm serving a huge and growing market—China, in particular, and the rest of Asia in general. The firm's current return on equity is low, and we anticipate that it will improve over the next five years. As it increases, earnings growth will be pushed up.
- *Why FCFE?* Corporate governance in China tends to be weak and dividends are unlikely to reflect free cash flow to equity. In addition, the firm consistently funds a portion of its reinvestment needs with new debt issues.

Background Information

In 2000, Tsingtao Breweries earned 72.36 million CY (Chinese yuan) in net income on a book value of equity of 2,588 million CY, giving it a return on equity of 2.80%. The firm had capital expenditures of 335 million CY and depreciation of 204 million CY during the year, and noncash working capital dropped by 1.2 million CY during the year. The total reinvestment in 2000 was therefore:

$$\begin{aligned}\text{Total reinvestment} &= \text{Capital expenditures} - \text{Depreciation} + \text{Change in noncash working capital}\\ &= 335 - 204 - 1.2 = 129.8 \text{ million}\end{aligned}$$

The working capital changes over the past four years have been volatile, and we normalize the change using noncash working capital as a percent of revenues in 2000:

$$\begin{aligned}\text{Normalized change in noncash working capital} &= (\text{Noncash working capital}_{2000}/\text{Revenues}_{2000})\\ &\quad \times (\text{Revenues}_{2000} - \text{Revenues}_{1999})\\ &= (180/2{,}253) \times (2{,}253 - 1{,}598) = 52.3 \text{ million CY}\end{aligned}$$

The normalized reinvestment in 2000 can then be estimated as follows:

$$\begin{aligned}\text{Normalized reinvestment} &= \text{Capital expenditures} - \text{Depreciation}\\ &\quad + \text{Normalized change in noncash working capital}\\ &= 335 - 204 + 52.3 = 183.3 \text{ million CY}\end{aligned}$$

As with working capital, debt issues have been volatile. We estimate the firm's book debt to capital ratio of 40.94% at the end of 2000 and use it to estimate the normalized equity reinvestment in 2000:

$$\text{Equity reinvestment in 2000} = \text{Reinvestment}(1 - \text{Debt ratio}) = 183.3(1 - .4094) = 108.27 \text{ million CY}$$

As a percent of net income,

$$\text{Equity reinvestment rate in 2000} = 108.27/72.36 = 149.97\%$$

ESTIMATION

To estimate free cash flows to equity for the high-growth period, we make the assumption that the return on equity, which is 2.80% today, will drift up to 12% by the fifth year. In addition, we will assume that new investments from now on will earn a return on equity of 12%. Finally, we will assume that the equity reinvestment rate will remain at its current level (149.97%) each year for the next five years. The expected growth rate over the next five years can then be estimated as follows:

$$\begin{aligned}\text{Expected growth rate—next five years} &= \text{Equity reinvestment rate} \times ROE_{new} \\ &\quad + [(ROE_{new} - ROE_{today})/ROE_{today}]^{1/5} - 1 \\ &= 1.4997 \times .12 + \{[(.12 - .028)/.028]^{1/5} - 1\} = 44.91\%\end{aligned}$$

After year 5, we will assume that the expected growth rate declines linearly each year from years 6 through 10 to reach a stable growth rate of 10% in year 10. (Note that the growth rate is in nominal CY; the higher stable growth rate reflects the higher expected inflation in that currency.) As the growth rate declines, the equity reinvestment rate also drops off to a stable period equity reinvestment rate of 50%, estimated using the 10% stable growth rate and an assumed return on equity in stable growth of 20%.

$$\text{Stable period equity reinvestment rate} = g/ROE = 10\%/20\% = 50\%$$

To estimate the cost of equity, we used a risk-free rate of 10% (in nominal CY), a risk premium of 6.28% (4% for mature market risk and 2.28% as the country risk premium for China) and a beta of 0.75 (reflecting the bottom-up beta for breweries):

$$\text{Cost of equity} = 10\% + 0.75(6.28\%) = 14.71\%$$

In stable growth, we assume that the beta will drift up to 0.80 and that the country risk premium will drop to 0.95%:

$$\text{Cost of equity} = 10\% + 0.80(4.95\%) = 13.96\%$$

The cost of equity adjusts in linear increments from 14.71% in year 5 to 13.96% in year 10.

VALUATION To value Tsingtao, we will begin by projecting the free cash flows to equity during the high growth and transition phases, using an expected growth rate of 44.91% in net income and an equity reinvestment rate of 149.97% for the first five years. The next five years represent a transition period, where the growth drops in linear increments from 44.91% to 10% and the equity reinvestment rate drops from 149.97% to 50%. The resulting free cash flows to equity are shown in the following table:

Year	*Expected Growth*	*Net Income*	*Equity Reinvestment Rate*	*FCFE*	*Cost of Equity*	*Present Value*
Current		CY72.36	149.97%			
1	44.91%	CY104.85	149.97%	(CY52.40)	14.71%	(CY45.68)
2	44.91%	CY151.93	149.97%	(CY75.92)	14.71%	(CY57.70)
3	44.91%	CY220.16	149.97%	(CY110.02)	14.71%	(CY72.89)
4	44.91%	CY319.03	149.97%	(CY159.43)	14.71%	(CY92.08)
5	44.91%	CY462.29	149.97%	(CY231.02)	14.71%	(CY116.32)
6	37.93%	CY637.61	129.98%	(CY191.14)	14.56%	(CY84.01)
7	30.94%	CY834.92	109.98%	(CY83.35)	14.41%	(CY32.02)
8	23.96%	CY1,034.98	89.99%	CY103.61	14.26%	CY34.83
9	16.98%	CY1,210.74	69.99%	CY363.29	14.11%	CY107.04
10	10.00%	CY1,331.81	50.00%	CY665.91	13.96%	CY172.16

Sum of the present values of FCFE during high growth = ($186.65)

To estimate the terminal value of equity, we use the net income in the year 11, reduce it by the equity reinvestment needs in that year, and then assume a perpetual growth rate to get to a value.

Expected stable growth rate =10%
Equity reinvestment rate in stable growth = 50%
Cost of equity in stable growth = 13.96%
Expected FCFE in year 11 = Net income$_{11}$ × (1 – Stable period equity reinvestment rate)
= CY1,331.81(1.10)(1 – .5) = CY732.50 million
Terminal value of equity in Tsingtao Breweries = FCFE$_{11}$/(Stable period cost of equity – Stable growth rate) = 732.5/(.1396 – .10)
= CY18,497 million

To estimate the value of equity today, we sum up the present value of the FCFE over the high-growth period and add to it the present value of the terminal value of equity:

Value of equity = PV of FCFE during the high-growth period + PV of terminal value
= –CY186.65 + CY18,497/($1.1471^5 \times 1.1456 \times 1.1441 \times 1.1426 \times 1.1411 \times 1.1396$) = CY4,596 million

Value of equity per share = Value of equity/Number of shares = CY4,596/653.15 = CY7.04 per share

The stock was trading at 10.10 yuan per share, which would make it overvalued based on this valuation.

NEGATIVE FCFE, EQUITY DILUTION, AND VALUE PER SHARE

Unlike dividends, free cash flows to equity can be negative. This can occur either because net income is negative or because a firm's reinvestment needs are significant; this is the case with Tsingtao in Illustration 14.5. The resulting net capital expenditure and working capital needs may be much larger than the net income. In fact, this is likely to occur fairly frequently with high-growth firms.

The FCFE model is flexible enough to deal with this issue. The free cash flows to equity will be negative as the firm reinvests substantial amounts to generate high growth. As the growth declines, the reinvestment needs also drop off and free cash flows to equity turn positive.

Intuitively, though, consider what a negative free cash flow to equity implies. It indicates that the firm does not generate enough cash flows from current operations to meet its reinvestment needs. Since the free cash flow to equity is after net debt issues, the firm will have to issue new equity in years where the cash flow is negative. This expected dilution in future years will reduce the value of equity per share today. In the FCFE model, the negative free cash flows to equity in the earlier years will reduce the estimated value of equity today. Thus the dilution effect is captured in the present value, and no additional consideration is needed of new stock issues in future years and the effect on value per share today.

A TROUBLESHOOTING GUIDE: WHAT IS WRONG WITH THIS VALUATION? (THREE-STAGE FCFE MODEL)

If This Is Your Problem	*This May Be the Solution*
• If you get a extremely low value from the three-stage FCFE, the likely culprits are:	
Capital expenditures are significantly higher than depreciation in stable growth phase.	Reduce net cap ex in stable growth. Cap ex grows slower than depreciation during transition period.
The beta in the stable period is too high for a stable firm.	Use a beta closer to 1.
Working capital as percent of revenue is too high to sustain.	Use working capital ratio closer to industry average.
• If you get an extremely high value:	
Capital expenditures offset depreciation during high-growth period.	Capital expenditures should be set higher.
Capital expenditures are less than depreciation.	(Calculate reinvestment rate = g/ROC)
Growth period (high growth and transition) is too long.	Use a shorter growth period.
The growth rate in the stable growth period is too high for stable firm.	Use a growth rate closer to GNP growth.

FCFE3st.xls: **This spreadsheet allows you to value a firm with a temporary period of high growth in FCFE, followed by a transition period, followed by stable growth.**

FCFE VALUATION VERSUS DIVIDEND DISCOUNT MODEL VALUATION

The discounted cash flow model that uses FCFE can be viewed as an alternative to the dividend discount model. Since the two approaches sometimes provide different estimates of value, it is worth examining when they provide similar estimates of value, when they provide different estimates of value, and what the difference tells us about the firm.

When They Are Similar

There are two conditions under which the value from using the FCFE in discounted cash flow valuation will be the same as the value obtained from using the dividend discount model. The first is the obvious one, where the dividends are equal to the FCFE. The second condition is more subtle, where the FCFE is greater than dividends, but the excess cash (FCFE minus dividends) is invested in projects with net present value of zero. (For instance, investing in financial assets that are fairly priced should yield a net present value of zero.)

When They Are Different

There are several cases where the two models will provide different estimates of value. First, when the FCFE is greater than the dividend and the excess cash either earns below-market interest rates or is invested in negative net present value projects, the value from the FCFE model will be greater than the value from the dividend discount model. There is reason to believe that this is not as unusual as it would seem at the outset. There are numerous case studies of firms, which having accumulated large cash balances, by paying out low dividends relative to FCFE, have chosen to use this cash to finance unwise takeovers (where the price paid is greater than the value received from the takeover). Second, the payment of smaller dividends than can be afforded to be paid out by a firm lowers debt-to-equity ratios and may lead the firm to become underleveraged, causing a loss in value.

In the cases where dividends are greater than FCFE, the firm will have to issue either new stock or new debt to pay these dividends leading to at least three negative consequences for value. One is the flotation cost on these security issues, which can be substantial for equity issues, creates an unnecessary expenditure that decreases value. Second, if the firm borrows the money to pay the dividends, the firm may become overlevered (relative to the optimal) leading to a loss in value. Finally, paying too much in dividends can lead to capital rationing constraints where good projects are rejected, resulting in a loss of wealth.

There is a third possibility and it reflects different assumptions about reinvestment and growth in the two models. If the same growth rate is used in the dividend discount and FCFE models, the FCFE model will give a higher value than the dividend discount model whenever FCFE are higher than dividends and a lower value when dividends exceed FCFE. In reality, the growth rate in FCFE should be different from the growth rate in dividends, because the free cash flow to equity is assumed to be paid out to stockholders. This will affect the reinvestment rate of the firm. In addition, the return on equity used in the FCFE model should reflect the return on equity on noncash investments, whereas the return on equity used in the dividend discount model should be the overall return on equity. Table 14.1 summarizes the differences in assumptions between the two models.

In general, when firms pay out much less in dividends than they have available in FCFE, the expected growth rate and terminal value will be higher in the dividend discount model, but the year-to-year cash flows will be higher in the FCFE model. The net effect on value will vary from company to company.

TABLE 14.1 Differences between DDM and FCFE Models

	Dividend Discount Model	FCFE Model
Implicit assumption	Only dividends are paid. Remaining portions of earnings are invested back into the firm, some in operating assets and some in cash and marketable securities.	The FCFE is paid out to stockholders. The remaining earnings are invested only in operating assets.
Expected growth	Measures growth in income from both operating and cash assets. In terms of fundamentals, it is the product of the retention ratio and the return on equity.	Measures growth only in income from operating assets. In terms of fundamentals, it is the product of the equity reinvestment rate and the noncash return on equity.
Dealing with cash and marketable securities	The income from cash and marketable securities is built into earnings and ultimately into dividends. Therefore, cash and marketable securities do not need to be added in.	You have two choices: 1. Build in income from cash and marketable securities into projections of income, and estimate the value of equity. 2. Ignore income from cash and marketable securities, and add their value to equity value in model.

What Does It Mean When They Are Different?

When the value using the FCFE model is different from the value using the dividend discount model, with consistent growth assumptions, there are two questions that need to be addressed: What does the difference between the two models tell us? Which of the two models is the appropriate one to use in evaluating the market price?

The more common occurrence is for the value from the FCFE model to exceed the value from the dividend discount model. The difference between the value from the FCFE model and the value using the dividend discount model can be considered one component of the value of controlling a firm—it measures the value of controlling dividend policy. In a hostile takeover, the bidder can expect to control the firm and change the dividend policy (to reflect FCFE), thus capturing the higher FCFE value.

As for which of the two values is the more appropriate one for use in evaluating the market price, the answer lies in the openness of the market for corporate control. If there is a sizable probability that a firm can be taken over or its management changed, the market price will reflect that likelihood, and the appropriate benchmark to use is the value from the FCFE model. As changes in corporate control become more difficult because of a firm's size and/or legal or market restrictions on takeovers, the value from the dividend discount model will provide the appropriate benchmark for comparison.

ILLUSTRATION 14.6: Comparing the DDM and FCFE Models: Coca-Cola

In Chapter 13, we valued Coca-Cola using a three-stage dividend discount model at $42.72 a share. Here, we will value Coca-Cola using a three-stage free cash flow to equity model.

Rationale for Using Three-Stage FCFE Model

- *Why three-stage?* Coca-Cola's strong brand name will allow it to overcome some of the constraints that may exist on its high growth rate—the saturation of its domestic market and its high market share in these markets. However, we believe that this growth will come under assault from competition in future years, leading us to allow for a transition to stable growth.
- *Why FCFE?* While the firm does have a history of returning cash to stockholders, we wanted to examine the differences in value, if any, estimated with the dividend and FCFE models.
- The firm has used debt a little more liberally in the past few years, but it remains a firm that uses equity for much of its reinvestment needs.

Background Information

Net income = $3,879.77
Number of shares outstanding = 2,487.03
Current capital expenditures = $992.00
Current depreciation = $773.00
Increase in noncash working capital in most recent year = $852.00
Net debt issued (paid) during the year = ($585.00)

Based on these values, we can estimate the free cash flows to equity in the most recent year as follows:

Free cash flow to equity = Net income – (Cap expenditures – Depreciation)
– Change in noncash working capital + Net debt issued
= 3,878 – (992 – 773) – 852 + (–585) = $2,222 million

The return on equity in the most recent year was estimated to be 23.37% in the dividend discount model. We reestimated the return on equity excluding the income from cash and marketable securities from net income[3] and the value of the cash and marketable securities from book equity:

Modified return on equity = (Net income – After-tax interest income from cash)
/(Book value of equity – Cash and marketable securities)
= (2,177 – 91)/(9,317 – 1,822) = 27.83%

Estimation

We assume that the cost of equity for Coca-Cola will be 9.99% for the five-year high-growth period, declining in linear increments to 9.40% in year 10 and stable growth beyond. The slightly higher cost of equity results from the use of beta of 0.82 in the high-growth period. (In the DDM we used a beta of 0.80.)

The capital expenditures, working capital requirements and the debt ratio for Coca-Cola have been volatile over the past five years. To normalize changes over time, we decided to do the following:

First, we computed the net capital expenditures as a percent of earnings before interest and taxes each year for the past five years:

[3]As in the dividend discount model, we used a normalized net income ($2,177 million) just for this computation. The rest of the valuation is based on the actual net income prior to extraordinary items.

	–5	*–4*	*–3*	*–2*	*Current*	*Average*
Net cap ex	$1,391.00	$1,485.00	$1,996.00	$2,332.00	$ 219.00	$1,484.60
EBIT	$4,833.00	$5,001.00	$4,967.00	$3,982.00	$5,134.00	$4,783.40
Average net cap ex/EBIT =						31.04%

Normalized net capital expenditure = Average net cap ex as % of EBIT over past five years
$\times$ EBIT in most recent year = .3104 $\times$ 5,134 = $1,593 million

Then we estimated noncash working capital as a percent of revenues in the most recent year and used this to estimate the change in noncash working capital over the last year:

Noncash working capital in current year = $223 million
Revenues in current year = $20,458 million
Revenues last year = $19,805 million
Normalized change in noncash working capital last year = (223/20,458)(20,458 – 19,805)
= $7.12 million

Finally, we normalized the net debt issued by assuming that Coca-Cola would continue to fund its reinvestment needs with its market debt-to-capital ratio. To estimate the market debt-to-capital ratio, we used the total interest bearing debt outstanding at the end of 2000 and the current market value of equity:

Debt ratio = Interest-bearing debt/(Interest-bearing debt + Market value of equity)
= 5,651/(5,651 + 115,125) = 4.68%

Normalized debt issued in current year = (Normalized net capital expenditures
+ Normalized change in noncash working capital)
$\times$ Debt ratio = (1,593 + 7.12) $\times$ (.0468) = $74.89 million

The normalized free cash flow to equity can then be computed:

Normalized FCFE = Net income – Normalized net cap ex – Normalized change in working capital
+ Normalized net debt issued = 3,878 – 1,593 – 7.12 + 74.89 = $2,353 million

This normalized FCFE also lets us compute the equity reinvestment rate for the firm:

Equity reinvestment rate = 1 – FCFE/Net income = 1 – 2,353/3,878 = 39.3%

With the current return on equity of 27.83%, this yields an expected growth rate in noncash net income at Coca-Cola of 10.94%.

Expected growth = Equity reinvestment rate $\times$ Return on equity = .393 $\times$ .2783 = .1094

In stable growth, we assume that the return on equity drops to 20% and that the growth rate in perpetuity in net income is 5.5%. The equity reinvestment rate can then be estimated as follows:

Equity reinvestment rate in stable growth = g/ROE = 5.5%/20% = 27.5%

VALUATION

To value Coca-Cola, we will begin by projecting the free cash flows to equity during the high growth and transition phases, using an expected growth rate of 10.94% in noncash net income and an equity reinvestment rate of 39.3% for the first five years.

Noncash net income = Net income – After-tax interest income from cash and marketable securities
= $3,878 million – $91 million = $3,789 million

The next five years represent a transition period, where the growth drops in linear increments from 10.94% to 5% and the equity reinvestment rate drops from 39.3% to 25%. The resulting free cash flows to equity are shown in the following table:

Year	*Expected Growth*	*Net Income*	*Equity Reinvestment Rate*	*FCFE*	*Cost of Equity*	*Present Value*
1	10.94%	$4,203.28	39.32%	$2,550.42	9.99%	$ 2,318.73
2	10.94%	$4,663.28	39.32%	$2,829.53	9.99%	$ 2,338.80
3	10.94%	$5,173.61	39.32%	$3,139.18	9.99%	$ 2,359.03
4	10.94%	$5,739.79	39.32%	$3,482.72	9.99%	$ 2,379.44
5	10.94%	$6,367.93	39.32%	$3,863.86	9.99%	$ 2,400.03
6	9.85%	$6,995.48	36.96%	$4,410.06	9.87%	$ 2,493.13
7	8.77%	$7,608.71	34.59%	$4,976.57	9.76%	$ 2,563.34
8	7.68%	$8,192.87	32.23%	$5,552.37	9.64%	$ 2,608.54
9	6.59%	$8,732.68	29.86%	$6,124.69	9.52%	$ 2,627.34
10	5.50%	$9,212.97	27.50%	$6,679.40	9.40%	$ 2,619.11
	Sum of the present values of FCFE during high growth					$24,707.49

To estimate the terminal value of equity, we use the net income in the terminal year (year 11), reduce it by the equity reinvestment needs in that year, and then assume a perpetual growth rate to get to a value.

Expected stable growth rate = 5.5%
Equity reinvestment rate in stable growth = 27.5%
Cost of equity in stable growth = 9.40%
Expected FCFE in year 11 = Net income$_{11}$ × (1 – Stable period equity reinvestment rate)
= $9,213(1.055)(1 – .275) = $7,047 million
Value of equity in Coca-Cola = FCFE$_{11}$/(Stable period cost of equity – Stable growth rate)
= 7,047/(.094 – .055) = $180,686

To estimate the value of equity today, we sum up the present value of the FCFE over the high-growth period and add to it the present value of the terminal value of equity:

Value of equity = PV of FCFE during the high-growth period + PV of terminal value
= $24,707 + $180,686/($1.0999^5 \times 1.0987 \times 1.0976 \times 1.0964 \times 1.0952 \times 1.094$)
= $95,558 million

Adding in the value of the cash and marketable securities that Coca-Cola had on hand at the end of 2001, we obtain the total value of equity:

Value of equity including cash = $95,588 + $1,892 = $97,447 million

Value of equity per share = Value of equity/Number of shares = $97,447/2,487.03 = $39.19

The FCFE model yields a slightly lower value than the dividend discount model value of $42.72 a share. This may seem surprising since the FCFE each year for the high-growth period are greater than the dividends, but this effect is more than offset by the decline in the expected growth rate, which is generated by the equity reinvestment rate being lower than the retention ratio. This valuation is probably more realistic than the dividend discount model because it keeps investments in cash and marketable securities separate from investments in operating assets. The dividend discount model overstates the expected growth rate because it does not consider the fact that the low return earned by cash investments will bring the return on equity down over time (and growth down with it).

CONCLUSION

The primary difference between the dividend discount models described in the previous chapter and the free cash flow to equity models described in this one lies in the definition of cash flows; the dividend discount model uses a strict definition of cashflow to equity (i.e., the expected dividends on the stock), while the FCFE model uses an expansive definition of cash flow to equity as the residual cash flow after meeting all financial obligations and investment needs. When firms have dividends that are different from the FCFE, the values from the two models will be different. In valuing firms for takeovers or in valuing firms where there is a reasonable chance of changing corporate control, the value from the FCFE model provides the better estimate of value.

QUESTIONS AND SHORT PROBLEMS

1. Respond true or false to the following statements relating to the calculation and use of FCFE:
   a. The free cash flow to equity will generally be more volatile than dividends.
      True ____ False ____
   b. The free cash flow to equity will always be higher than dividends.
      True ____ False ____
   c. The free cash flow to equity will always be higher than net income.
      True ____ False ____
   d. The free cash flow to equity can never be negative.
      True ____ False ____
2. Kimberly-Clark, a household product manufacturer, reported earnings per share of $3.20 in 1993 and paid dividends per share of $1.70 in that year. The firm reported depreciation of $315 million in 1993, and capital expenditures of $475 million. (There were 160 million shares outstanding, trading at $51 per share.) This ratio of capital expenditures to depreciation is expected to be maintained in the long term. The working capital needs are negligible. Kimberly-Clark had debt outstanding of $1.6 billion, and intended to maintain its current financing mix (of debt and equity) to finance future investment needs. The firm was in steady state and earnings were expected to grow 7% a year. The stock had a beta of 1.05. (The Treasury bond rate was 6.25%, and the risk premium was 5.5%.)
   a. Estimate the value per share, using the dividend discount model.
   b. Estimate the value per share, using the FCFE model.
   c. How would you explain the difference between the two models, and which one would you use as your benchmark for comparison to the market price?
3. Ecolab Inc. sells chemicals and systems for cleaning, sanitizing, and maintenance. It reported earnings per share of $2.35 in 1993, and expected earnings growth of 15.5% a year from 1994 to 1998, and 6% a year after that. The capital expenditure per share was $2.25, and depreciation was $1.125 per share in 1993. Both were expected to grow at the same rate as earnings from 1994 to 1998. Working capital was expected to remain at 5% of revenues, and revenues, which were $1 billion in 1993, were expected to increase 6% a year from 1994 to 1998, and 4% a year after that. The firm had has a debt ratio [D/(D + E)] of 5%, but planned to finance future investment needs (including working capital investments) using a debt ratio of 20%. The stock was expected to have a beta of 1 for the period of

the analysis, and the Treasury bond rate was 6.50%. (There were 63 million shares outstanding, and the market risk premium was 5.5%.)

a. Assuming that capital expenditures and depreciation offset each other after 1998, estimate the value per share. Is this a realistic estimate?
b. Assuming that capital expenditures continue to be 200% of depreciation even after 1998, estimate the value per share.
c. What would the value per share have been, if the firm had continued to finance new investments with its old financing mix (5%)? Is it fair to use the same beta for this analysis?

4. Dionex Corporation, a leader in the development and manufacture of ion chromography systems (used to identify contaminants in electronic devices), reported earnings per share of $2.02 in 1993, and paid no dividends. These earnings were expected to grow 14% a year for five years (1994 to 1998) and 7% a year after that. The firm reported depreciation of $2 million in 1993 and capital spending of $4.20 million, and had 7 million shares outstanding. The working capital was expected to remain at 50% of revenues, which were $106 million in 1993, and were expected to grow 6% a year from 1994 to 1998 and 4% a year after that. The firm was expected to finance 10% of its capital expenditures and working capital needs with debt. Dionex had a beta of 1.20 in 1993, and this beta was expected to drop to 1.10 after 1998. (The Treasury bond rate was 7%, and the market risk premium was 5.5%.)
   a. Estimate the expected free cash flow to equity from 1994 to 1998, assuming that capital expenditures and depreciation grow at the same rate as earnings.
   b. Estimate the terminal price per share (at the end of 1998). Stable firms in this industry have capital expenditures which are 150% of revenues, and maintain working capital at 25% of revenues.
   c. Estimate the value per share today, based on the FCFE model.
5. Biomet Inc., which designs, manufactures, and markets reconstructive and trauma devices, reported earnings per share of $0.56 in 1993, on which it paid no dividends (it had revenues per share in 1993 of $2.91). It had capital expenditures of $0.13 per share in 1993, and depreciation in the same year of $0.08 per share. The working capital was 60% of revenues in 1993 and were expected to remain at that level from 1994 to 1998, while earnings and revenues were expected to grow 17% a year. The earnings growth rate was expected to decline linearly over the following five years to a rate of 5% in 2003. During the high-growth and transition periods, capital spending and depreciation were expected to grow at the same rate as earnings, but capital spending would be 120% of depreciation when the firm reaches steady state. Working capital was expected to drop from 60% of revenues during the 1994–1998 period to 30% of revenues after 2003. The firm had no debt currently, but planned to finance 10% of its net capital investment and working capital requirements with debt.

   The stock was expected to have a beta of 1.45 for the high-growth period (1994–1998), and it was expected to decline to 1.10 by the time the firm goes into steady state (in 2003). The Treasury bond rate is 7%, and the market risk premium is 5.5%.
   a. Estimate the value per share, using the FCFE model.
   b. Estimate the value per share, assuming that working capital stays at 60% of revenues forever.

c. Estimate the value per share, assuming that the beta remains unchanged at 1.45 forever.

6. Will the following firms be likely to have a higher value from the dividend discount model, a higher value from the FCFE model, or the same value from both models?
   a. A firm that pays out less in dividends than it has available in FCFE, but which invests the balance in treasury bonds.
   b. A firm that pays out more in dividends than it has available in FCFE, and then issues stock to cover the difference.
   c. A firm that pays out, on average, its FCFE as dividends.
   d. A firm that pays out less in dividends that it has available in FCFE, but which uses the cash at regular intervals to acquire other firms with the intent of diversifying.
   e. A firm that pays out more in dividends than it has available in FCFE, but borrows money to cover the difference. (The firm is overlevered to begin with.)
7. You have been asked to value Oneida Steel, a midsize steel company. The firm reported $80 million in net income, $50 million in capital expenditures, and $20 million in depreciation in the just-completed financial year. The firm reported that its noncash working capital increased by $20 million during the year and that total debt outstanding increased by $10 million during the year. The book value of equity at Oneida Steel at the beginning of the last financial year was $400 million. The cost of equity is 10%.
   a. Estimate the equity reinvestment rate, return on equity, and expected growth rate for Oneida Steel. (You can assume that the firm will continue to maintain the same debt ratio that it used last year to finance its reinvestment needs.)
   b. If this growth rate is expected to last five years and then drop to a 4% stable growth rate after that and the return on equity after year 5 is expected to be 12%, estimate the value of equity today, using the projected free cash flows to equity.
8. Luminos Corporation, a manufacturer of lightbulbs, is a firm in stable growth. The firm reported net income of $100 million on a book value of equity of $1 billion. However, the firm also had a cash balance of $200 million on which it earned after-tax interest income of $10 million last year. (This interest income is included in the net income, and the cash is part of the book value of equity.) The cost of equity for the firm is 9%.
   a. Estimate the noncash return on equity at Luminos Corporation.
   b. If you expect the cash flows from the operating assets of Luminos to increase 3% a year in perpetuity, estimate the value of equity at Luminos.