CHAPTER 4

# The Basics of Risk

When valuing assets and firms, we need to use discount rates that reflect the riskiness of the cash flows. In particular, the cost of debt has to incorporate a default spread for the default risk in the debt, and the cost of equity has to include a risk premium for equity risk. But how do we measure default and equity risk? More importantly, how do we come up with the default and equity risk premiums?

This chapter lays the foundations for analyzing risk in valuation. It presents alternative models for measuring risk and converting these risk measures into acceptable hurdle rates. It begins with a discussion of equity risk and presents the analysis in three steps. In the first step, risk is defined in statistical terms to be the variance in actual returns around an expected return. The greater this variance, the more risky an investment is perceived to be. The next step, the central one, is to decompose this risk into risk that can be diversified away by investors and risk that cannot. The third step looks at how different risk and return models in finance attempt to measure this nondiversifiable risk. It compares the most widely used model, the capital pricing asset model (CAPM), with other models and explains how and why they diverge in their measures of risk and the implications for the equity risk premium.

The final part of this chapter considers default risk and how it is measured by ratings agencies. By the end of the chapter, we should have a way of estimating the equity risk and default risk for any firm.

## WHAT IS RISK?

Risk, for most of us, refers to the likelihood that in life's games of chance we will receive an outcome that we will not like. For instance, the risk of driving a car too fast is getting a speeding ticket or, worse still, getting into an accident. *Merriam-Webster's Collegiate Dictionary*, in fact, defines the verb to risk as "to expose to hazard or danger." Thus risk is perceived almost entirely in negative terms.

In finance, our definition of risk is both different and broader. Risk, as we see it, refers to the likelihood that we will receive a return on an investment that is different from the return we expect to make. Thus, risk includes not only the bad outcomes (returns that are lower than expected), but also good outcomes (returns that are higher than expected). In fact, we can refer to the former as downside risk and the latter as upside risk, but we consider both when measuring risk. In fact, the spirit of our definition of risk in finance is captured best by the Chinese symbols for risk:

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The first symbol is the symbol for "danger," while the second is the symbol for "opportunity," making risk a mix of danger and opportunity. It illustrates very clearly the trade-off that every investor and business has to make—between the higher rewards that come with the opportunity and the higher risk that has to be borne as a consequence of the danger.

Much of this chapter can be viewed as an attempt to come up with a model that best measures the danger in any investment, and then attempts to convert this into the opportunity that we would need to compensate for the danger. In finance terms, we term the danger to be "risk" and the opportunity to be "expected return."

What makes the measurement of risk and expected return so challenging is that it can vary depending on whose perspective we adopt. When analyzing the risk of a firm, for instance, we can measure it from the viewpoint of the firm's managers. Alternatively, we can argue that the firm's equity is owned by its stockholders, and that it is their perspective on risk that should matter. A firm's stockholders, many of whom hold the stock as one investment in a larger portfolio, might perceive the risk in the firm very differently from the firm's managers, who might have the bulk of their capital, human and financial, invested in the firm.

This chapter will argue that risk in an investment has to be perceived through the eyes of investors in the firm. Since firms often have thousands of investors, often with very different perspectives, it can be asserted that risk has to be measured from the perspective of not just any investor in the stock, but of the marginal investor, defined to be the investor most likely to be trading on the stock at any given point in time. The objective in corporate finance is the maximization of firm value and stock price. If we want to stay true to this objective, we have to consider the viewpoint of those who set the stock prices, and they are the marginal investors.

## EQUITY RISK AND EXPECTED RETURN

To demonstrate how risk is viewed in finance, risk analysis is presented here in three steps: first, defining risk in terms of the distribution of actual returns around an expected return; second, differentiating between risk that is specific to one or a few investments and risk that affects a much wider cross section of investments (in a market where the marginal investor is well diversified, it is only the latter risk, called market risk, that will be rewarded); and third, alternative models for measuring this market risk and the expected returns that go with it.

### Defining Risk

Investors who buy assets expect to earn returns over the time horizon that they hold the asset. Their actual returns over this holding period may be very different from the expected returns, and it is this difference between actual and expected returns that is a source of risk. For example, assume that you are an investor with a one-year time horizon buying a one-year Treasury bill (or any other default-free one-year bond) with a 5 percent expected return. At the end of the one-year holding period, the actual return on this investment will be 5 percent, which is equal to the expected return. The return distribution for this investment is shown in Figure 4.1. This is a riskless investment.

To provide a contrast to the riskless investment, consider an investor who buys stock in a firm, say Boeing. This investor, having done her research, may conclude that she can make an expected return of 30 percent on Boeing over her one-year holding period. The actual return over this period will almost certainly not be equal to 30 percent; it might be much greater or much lower. The distribution of returns on this investment is illustrated in Figure 4.2.

In addition to the expected return, an investor now has to consider the following. First, note that the actual returns, in this case, are different from the expected return. The spread of the actual returns around the expected return is measured by the variance or standard deviation of the distribution; the greater the deviation of the actual returns from the expected return, the greater the variance. Second, the bias toward positive or negative returns is represented by the skewness of the distribution. The distribution in Figure 4.2 is positively skewed, since there is a higher probability of large positive returns than large negative returns. Third, the shape of the tails of the distribution is measured by the kurtosis of the distribution; fatter tails lead to higher kurtosis. In investment terms, this represents the tendency of the price of this investment to jump (up or down from current levels) in either direction.

In the special case where the distribution of returns is normal, investors do not have to worry about skewness and kurtosis, since there is no skewness (normal distributions are symmetric) and a normal distribution is defined to have a kurtosis of zero. Figure 4.3 illustrates the return distributions on two investments with symmetric returns.

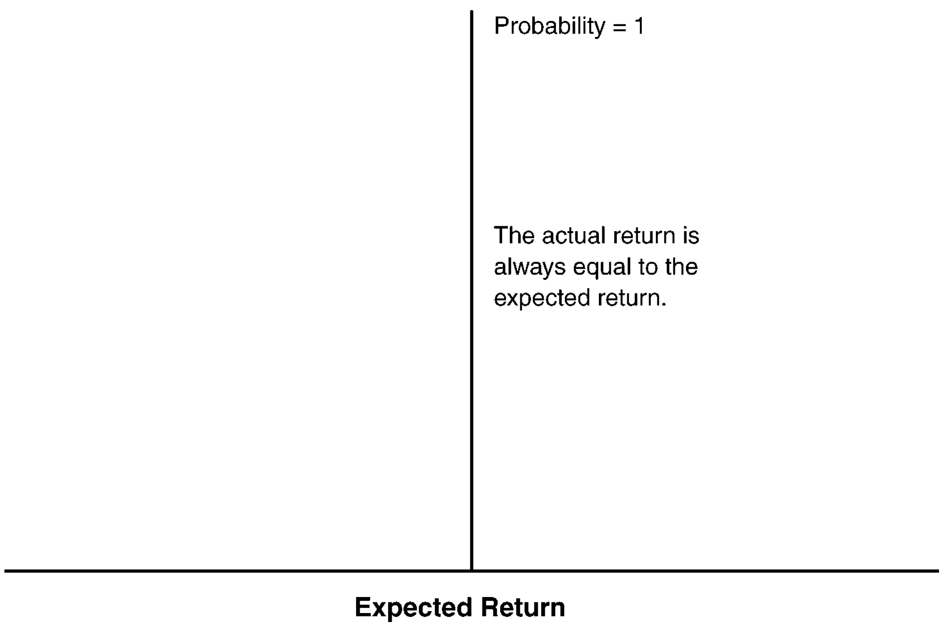



**FIGURE 4.1** Probability Distribution of Returns on a Risk-Free Investment

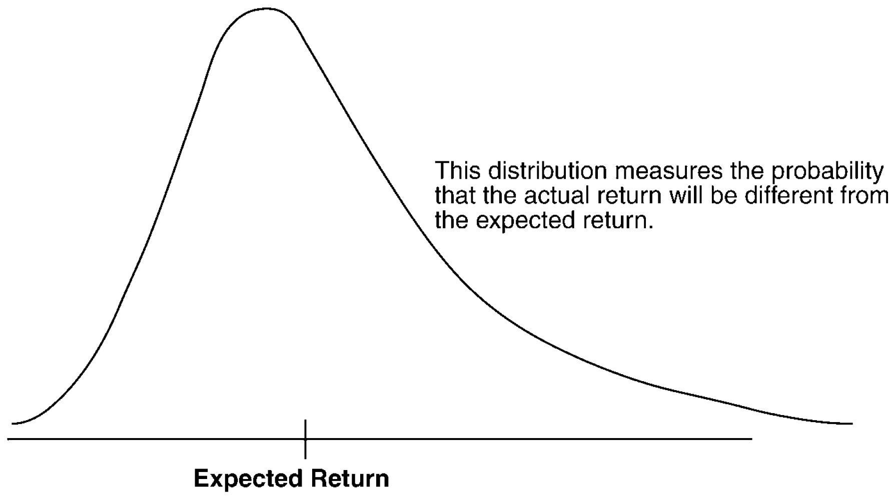



**FIGURE 4.2** Return Distribution for Risky Investment

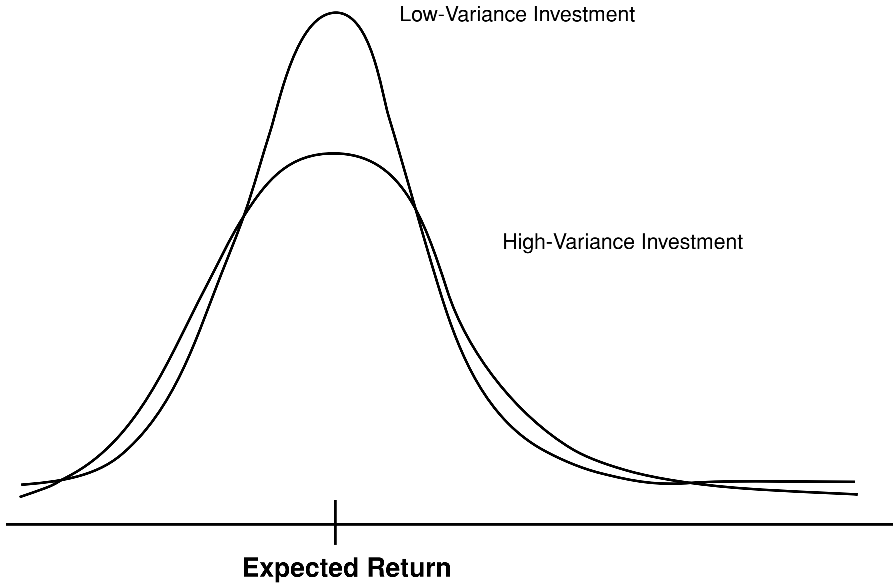



**FIGURE 4.3** Return Distribution Comparisons

When return distributions take this form, the characteristics of any investment can be measured with two variables—the expected return, which represents the opportunity in the investment, and the standard deviation or variance, which represents the danger. In this scenario, a rational investor, faced with a choice between two investments with the same standard deviation but different expected returns, will always pick the one with the higher expected return.

In the more general case, where distributions are neither symmetric nor normal, it is still conceivable that investors will choose between investments on the basis of only the expected return and the variance, if they possess utility functions that allow them to do so.[1] It is far more likely, however, that they prefer positive skewed distributions to negatively skewed ones, and distributions with a lower likelihood of jumps (lower kurtosis) over those with a higher likelihood of jumps (higher kurtosis). In this world, investors will trade off the good (higher expected returns and more positive skewness) against the bad (higher variance and kurtosis) in making investments.

In closing, it should be noted that the expected returns and variances that we run into in practice are almost always estimated using past returns rather than future returns. The assumption made when using historical variances is that past return distributions are good indicators of future return distributions. When this assumption is violated, as is the case when the asset's characteristics have changed significantly over time, the historical estimates may not be good measures of risk.

**ILLUSTRATION 4.1: Calculation of Standard Deviation Using Historical Returns: Boeing and the Home Depot**

We will use Boeing and the Home Depot as our investments to illustrate how standard deviations and variances are computed. To make our computations simpler, we will look at returns on an annual basis from 1991 to 1998. To begin the analysis, we first estimate returns for each company for each of these years, in percentage terms, incorporating both price appreciation and dividends into these returns:

$$\text{Return in year n} = \frac{\text{Price at end of year n} - \text{Price at beginning of year n} + \text{Dividend in year n}}{\text{Price at beginning of year n}}$$

[1]A utility function is a way of summarizing investor preferences into a generic term called "utility" on the basis of some choice variables. In this case, for instance, the investors' utility or satisfaction is stated as a function of wealth. By doing so, we effectively can answer questions such as, Will investors be twice as happy if they have twice as much wealth? Does each marginal increase in wealth lead to less additional utility than the prior marginal increase? In one specific form of this function, the quadratic utility function, the entire utility of an investor can be compressed into the expected wealth measure and the standard deviation in that wealth.

The following table summarizes returns on the two companies:

| | *Return on Boeing* | *Return on the Home Depot* |
|---|---|---|
| 1991 | 5.00% | 161.00% |
| 1992 | –16.00% | 50.30% |
| 1993 | 7.80% | –22.00% |
| 1994 | 8.70% | 16.50% |
| 1995 | 66.80% | 3.80% |
| 1996 | 35.90% | 5.00% |
| 1997 | –8.10% | 76.20% |
| 1998 | –33.10% | 107.90% |
| Sum | 67.00% | 398.70% |

We compute the average and standard deviation in these returns for the two firms, using the information in the table (there are eight years of data):

$$\text{Average return on Boeing}_{91-98} = 67.00\%/8 = 8.38\%$$

$$\text{Average return on the Home Depot}_{91-98} = 398.70\%/8 = 49.84\%$$

The variance is measured by looking at the deviations of the actual returns in each year, for each stock, from the average return. Since we consider both better-than-expected and worse-than-expected deviations in measuring variance, we square the deviations:[2]

| | *Return on Boeing* | *Return on the Home Depot* | *$[R_B - \text{Average}(R_B)]^2$* | *$[R_{HD} - \text{Average}(R_{HD})]^2$* |
|---|---|---|---|---|
| 1991 | 5.00% | 161.00% | 0.00113906 | 1.23571014 |
| 1992 | –16.00% | 50.30% | 0.05941406 | 2.1391E-05 |
| 1993 | 7.80% | –22.00% | 3.3063E-05 | 0.51606264 |
| 1994 | 8.70% | 16.50% | 1.0562E-05 | 0.11113889 |
| 1995 | 66.80% | 3.80% | 0.34134806 | 0.21194514 |
| 1996 | 35.90% | 5.00% | 0.07576256 | 0.20104014 |
| 1997 | –8.10% | 76.20% | 0.02714256 | 0.06949814 |
| 1998 | –33.10% | 107.90% | 0.17201756 | 0.33712539 |
| Sum | | | 0.67686750 | 2.68254188 |

Following the standard practice for estimating the variances of samples, the variances in returns at the two firms can be estimated by dividing the sum of the squared deviation columns by (n – 1), where n is the number of observations in the sample; the standard deviations can be computed to be the squared root of the variances:

| | *Boeing* | *Home Depot* |
|---|---|---|
| Variance | 0.6768675/(8 – 1) = .0967 | 2.68254188/(8 – 1) = .3832 |
| Standard Deviation | $\sqrt{0.0967} = .311$ or 31.1% | $\sqrt{0.3832} = .619$ or 61.9% |

Based on this data, the Home Depot looks like it was two times more risky than Boeing between 1991 and 1998. What does this tell us? By itself, it provides a measure of how much each these companies' returns in the past have deviated from the average. If we assume that the past is a good indicator of the future, the Home Depot is a more risky investment than Boeing.

[2]If we do not square the deviations, the sum of the deviations will be zero.

***optvar.xls:*** **This is a dataset on the Web that summarizes standard deviations and variances of stocks in various sectors in the United States.**

### Diversifiable and Nondiversifiable Risk

Although there are many reasons why actual returns may differ from expected returns, we can group the reasons into two categories: firm-specific and marketwide. The risks that arise from firm-specific actions affect one or a few investments, while the risks arising from marketwide reasons affect many or all investments. This distinction is critical to the way we assess risk in finance.

**Components of Risk** When an investor buys stock or takes an equity position in a firm, he or she is exposed to many risks. Some risk may affect only one or a few firms, and this risk is categorized as firm-specific risk. Within this category, we would consider a wide range of risks, starting with the risk that a firm may have misjudged the demand for a product from its customers; we call this project risk. For instance, consider Boeing's investment in a Super Jumbo jet. This investment is based on the assumption that airlines want a larger airplane and are willing to pay a high price for it. If Boeing has misjudged this demand, it will clearly have an impact on Boeing's earnings and value, but it should not have a significant effect on other firms in the market. The risk could also arise from competitors proving to be stronger or weaker than anticipated, called competitive risk. For instance, assume that Boeing and Airbus are competing for an order from Qantas, the Australian airline. The possibility that Airbus may win the bid is a potential source of risk to Boeing and perhaps some of its suppliers, but again, few other firms will be affected by it. Similarly, the Home Depot recently launched an online store to sell its home improvement products. Whether it succeeds is clearly important to the Home Depot and its competitors, but it is unlikely to have an impact on the rest of the market. In fact, risk measures can be extended to include risks that may affect an entire sector but are restricted to that sector; we call this sector risk. For instance, a cut in the defense budget in the United States will adversely affect all firms in the defense business, including Boeing, but there should be no significant impact on other sectors. What is common across the three risks described—project, competitive, and sector risk—is that they affect only a small subset of firms.

There is another risk that is much more pervasive and affects many if not all investments. For instance, when interest rates increase, all investments are negatively affected, albeit to different degrees. Similarly, when the economy weakens, all firms feel the effects, though cyclical firms (such as automobiles, steel, and housing) may feel it more. We term this risk market risk.

Finally, there are risks that fall in a gray area, depending on how many assets they affect. For instance, when the dollar strengthens against other currencies, it has a significant impact on the earnings and values of firms with international operations. If most firms in the market have significant international operations, it could well be categorized as market risk. If only a few do, it would be closer to firm-specific risk. Figure 4.4 summarizes the spectrum of firm-specific and market risks.

**Why Diversification Reduces or Eliminates Firm-Specific Risk: An Intuitive Explanation**
As an investor, you could invest all your portfolio in one asset. If you do so, you are exposed to both firm-specific and market risk. If, however, you expand your portfolio to include other assets or stocks, you are diversifying, and by doing so you can reduce

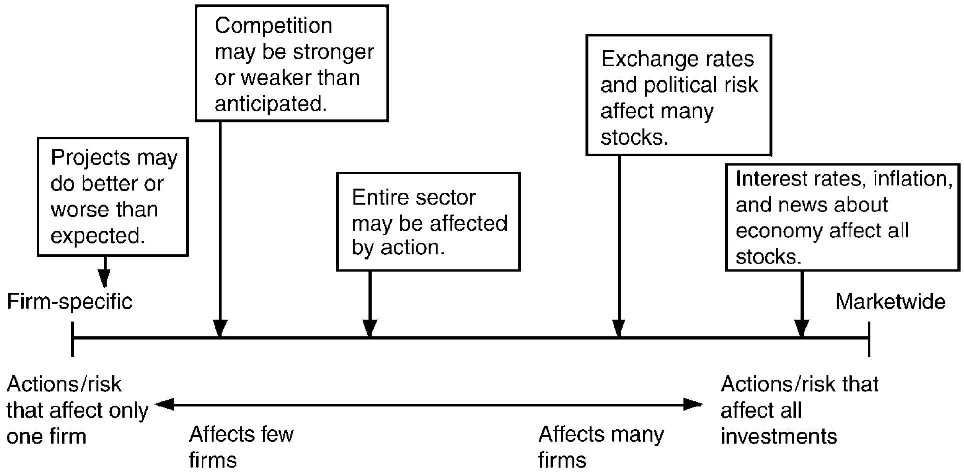



**FIGURE 4.4** Breakdown of Risk

your exposure to firm-specific risk. There are two reasons why diversification reduces or, at the limit, eliminates firm-specific risk. The first is that each investment in a diversified portfolio is a much smaller percentage of that portfolio than would be the case if you were not diversified. Any action that increases or decreases the value of only that investment or a small group of investments will have only a small impact on your overall portfolio, whereas undiversified investors are much more exposed to changes in the values of the investments in their portfolios. The second reason is that the effects of firm-specific actions on the prices of individual assets in a portfolio can be either positive or negative for each asset for any period. Thus, in very large portfolios this risk will average out to zero and will not affect the overall value of the portfolio.

In contrast, the effects of marketwide movements are likely to be in the same direction for most or all investments in a portfolio, though some assets may be affected more than others. For instance, other things being equal, an increase in interest rates will lower the values of most assets in a portfolio. Being more diversified does not eliminate this risk.

**A Statistical Analysis of Diversification-Reducing Risk** The effects of diversification on risk can be illustrated fairly dramatically by examining the effects of increasing the number of assets in a portfolio on portfolio variance. The variance in a portfolio is partially determined by the variances of the individual assets in the portfolio and partially by how they move together; the latter is measured statistically with a correlation coefficient or the covariance across investments in the portfolio. It is the covariance term that provides an insight into why diversification will reduce risk and by how much.

Consider a portfolio of two assets. Asset A has an expected return of $\mu_A$ and a variance in returns of $\sigma^2_A$, while asset B has an expected return of $\mu_B$ and a variance in returns of $\sigma^2_B$. The correlation in returns between the two assets, which measures how the assets move together, is $\rho_{AB}$. The expected returns and variances of a two-asset portfolio can be written as a function of these inputs and the proportion of the portfolio going to each asset.

$$\mu_{portfolio} = w_A \mu_A + (1 - w_A)\mu_B$$
$$\sigma^2_{portfolio} = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2w_A(1 - w_A)\rho_{AB}\sigma_A\sigma_B$$

where $w_A$ = Proportion of the portfolio in asset A

The last term in the variance formulation is sometimes written in terms of the covariance in returns between the two assets, which is:

$$\sigma_{AB} = \rho_{AB}\sigma_A\sigma_B$$

The savings that accrue from diversification are a function of the correlation coefficient. Other things remaining equal, the higher the correlation in returns between the two assets, the smaller are the potential benefits from diversification.

### Models Measuring Market Risk

While most risk and return models in use in corporate finance agree on the first two steps of the risk analysis process (i.e., that risk comes from the distribution of

**WHY IS THE MARGINAL INVESTOR ASSUMED TO BE DIVERSIFIED?**

The argument that diversification reduces an investor's exposure to risk is clear both intuitively and statistically, but risk and return models in finance go further. They look at risk through the eyes of the investor most likely to be trading on the investment at any point in time—the marginal investor. They argue that this investor, who sets prices for investments, is well diversified; thus, the only risk that he or she cares about is the risk added to a diversified portfolio or market risk. This argument can be justified simply. The risk in an investment will always be perceived to be higher for an undiversified investor than for a diversified one, since the latter does not shoulder any firm-specific risk and the former does. If both investors have the same expectations about future earnings and cash flows on an asset, the diversified investor will be willing to pay a higher price for that asset because of his or her perception of lower risk. Consequently, the asset, over time, will end up being held by diversified investors.

This argument is powerful, especially in markets where assets can be traded easily and at low cost. Thus, it works well for a stock traded in the United States, since investors can become diversified at fairly low cost. In addition, a significant proportion of the trading in U.S. stocks is done by institutional investors, who tend to be well diversified. It becomes a more difficult argument to sustain when assets cannot be easily traded or the costs of trading are high. In these markets, the marginal investor may well be undiversified, and firm-specific risk may therefore continue to matter when looking at individual investments. For instance, real estate in most countries is still held by investors who are undiversified and have the bulk of their wealth tied up in these investments.

actual returns around the expected return and that risk should be measured from the perspective of a marginal investor who is well diversified), they part ways when it comes to measuring nondiversifiable or market risk. This section will discuss the different models that exist in finance for measuring market risk and why they differ. It begins with what still is the standard model for measuring market risk in finance—the capital asset pricing model (CAPM)—and then discusses the alternatives to this model that have developed over the past two decades. While the discussion will emphasize the differences, it will also look at what the models have in common.

**Capital Asset Pricing Model** The risk and return model that has been in use the longest and is still the standard in most real-world analyses is the capital asset pricing model (CAPM). This section will examine the assumptions on which the model is based and the measures of market risk that emerge from these assumptions.

**Assumptions** While diversification reduces the exposure of investors to firm-specific risk, most investors limit their diversification to holding only a few assets. Even large mutual funds rarely hold more than a few hundred stocks, and many of them hold as few as 10 to 20. There are two reasons why investors stop diversifying. One is that an investor or mutual fund manager can obtain most of the benefits of diversification from a relatively small portfolio, because the marginal benefits of diversification become smaller as the portfolio gets more diversified. Consequently, these benefits may not cover the marginal costs of diversification, which include transactions and monitoring costs. Another reason for limiting diversification is that many investors (and funds) believe they can find undervalued assets and thus choose not to hold those assets that they believe to be fairly valued or overvalued.

The capital asset pricing model assumes that there are no transaction costs, all assets are traded, and investments are infinitely divisible (i.e., you can buy any fraction of a unit of the asset). It also assumes that everyone has access to the same information and that investors therefore cannot find under- or overvalued assets in the marketplace. By making these assumptions, it allows investors to keep diversifying without additional cost. At the limit, their portfolios will not only include every traded asset in the market but will have identical weights on risky assets (based on their market value).

The fact that this portfolio includes all traded assets in the market is the reason it is called the market portfolio, which should not be a surprising result, given the benefits of diversification and the absence of transaction costs in the capital asset pricing model. If diversification reduces exposure to firm-specific risk and there are no costs associated with adding more assets to the portfolio, the logical limit to diversification is to hold a small proportion of every traded asset in the economy. If this seems abstract, consider the market portfolio to be an extremely well diversified mutual fund that holds stocks and real assets. In the CAPM, all investors will hold combinations of the riskier asset and the same mutual fund.[3]

[3]The significance of introducing the riskless asset into the choice mix and the implications for portfolio choice were first noted in Sharpe (1964) and Lintner (1965). Hence, the model is sometimes called the Sharpe-Lintner model.

**Investor Portfolios in the CAPM** If every investor in the market holds the identical market portfolio, how exactly do investors reflect their risk aversion in their investments? In the capital asset pricing model, investors adjust for their risk preferences in their allocation decision, where they decide how much to invest in a riskless asset and how much in the market portfolio. Investors who are risk averse might choose to put much or even all of their wealth in the riskless asset. Investors who want to take more risk will invest the bulk or even all of their wealth in the market portfolio. Investors who invest all their wealth in the market portfolio and are desirous of taking on still more risk would do so by borrowing at the riskless rate and investing in the same market portfolio as everyone else.

These results are predicated on two additional assumptions. First, there exists a riskless asset, where the expected returns are known with certainty. Second, investors can lend and borrow at the riskless rate to arrive at their optimal allocations. While lending at the riskless rate can be accomplished fairly simply by buying Treasury bills or bonds, borrowing at the riskless rate might be more difficult for individuals to do. There are variations of the CAPM that allow these assumptions to be relaxed and still arrive at conclusions that are consistent with the model.

**Measuring the Market Risk of an Individual Asset** The risk of any asset to an investor is the risk added by that asset to the investor's overall portfolio. In the CAPM world, where all investors hold the market portfolio, the risk to an investor of an individual asset will be the risk that this asset adds to the market portfolio. Intuitively, if an asset moves independently of the market portfolio, it will not add much risk to the market portfolio. In other words, most of the risk in this asset is firm-specific and can be diversified away. In contrast, if an asset tends to move up when the market portfolio moves up and down when it moves down, it will add risk to the market portfolio. This asset has more market risk and less firm-specific risk. Statistically, this added risk is measured by the covariance of the asset with the market portfolio.

**Measuring the Nondiversifiable Risk** In a world in which investors hold a combination of only two assets—the riskless asset and the market portfolio—the risk of any individual asset will be measured relative to the market portfolio. In particular, the risk of any asset will be the risk it adds to the market portfolio. To arrive at the appropriate measure of this added risk, assume that $\sigma^2_m$ is the variance of the market portfolio prior to the addition of the new asset and that the variance of the individual asset being added to this portfolio is $\sigma^2_i$. The market value portfolio weight on this asset is $w_i$, and the covariance in returns between the individual asset and the market portfolio is $\sigma_{im}$. The variance of the market portfolio prior to and after the addition of the individual asset can then be written as:

$$\text{Variance prior to asset i being added} = \sigma^2_m$$
$$\text{Variance after asset i is added} = \sigma^2_{m'} = w_i^2\sigma^2_i + (1 - w_i)^2\sigma^2_m + 2w_i(1 - w_i)\sigma_{im}$$

The market value weight on any individual asset in the market portfolio should be small, since the market portfolio includes all traded assets in the economy. Consequently, the first term in the equation should approach zero, and the second term should approach $\sigma^2_m$, leaving the third term ($\sigma_{im}$, the covariance) as the measure of the risk added by asset i.

**Standardizing Covariances** The covariance is a percentage value, and it is difficult to pass judgment on the relative risk of an investment by looking at this value. In other words, knowing that the covariance of Boeing with the market portfolio is 55 percent does not provide us a clue as to whether Boeing is riskier or safer than the average asset. We therefore standardize the risk measure by dividing the covariance of each asset with the market portfolio by the variance of the market portfolio. This yields a risk measure called the beta of the asset:

$$\text{Beta of asset i} = \frac{\text{Covariance of asset i with market portfolio}}{\text{Variance of the market portfolio}} = \frac{\sigma_{im}}{\sigma_m^2}$$

Since the covariance of the market portfolio with itself is its variance, the beta of the market portfolio (and, by extension, the average asset in it) is 1. Assets that are riskier than average (using this measure of risk) will have betas that exceed 1, and assets that are safer than average will have betas that are lower than 1. The riskless asset will have a beta of zero.

**Getting Expected Returns** The fact that every investor holds some combination of the riskless asset and the market portfolio leads to the next conclusion, which is that the expected return on an asset is linearly related to the beta of the asset. In particular, the expected return on an asset can be written as a function of the risk-free rate and the beta of that asset:

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

where $E(R_i)$ = Expected return on asset i
$R_f$ = Risk-free rate
$E(R_m)$ = Expected return on market portfolio
$\beta_i$ = Beta of asset i

To use the capital asset pricing model, we need three inputs. While the next chapter will look at the estimation process in far more detail, each of these inputs is estimated as follows:

- The riskless asset is defined to be an asset for which the investor knows the expected return with certainty for the time horizon of the analysis.
- The risk premium is the premium demanded by investors for investing in the market portfolio, which includes all risky assets in the market, instead of investing in a riskless asset.
- The beta, defined as the covariance of the asset divided by the market portfolio, measures the risk added by an investment to the market portfolio.

In summary, in the capital asset pricing model all the market risk is captured in one beta measured relative to a market portfolio, which at least in theory should include all traded assets in the marketplace held in proportion to their market value.

**Arbitrage Pricing Model** The restrictive assumptions on transaction costs and private information in the capital asset pricing model, and the model's dependence on the market portfolio, have long been viewed with skepticism by both academics and practitioners. Ross (1976) suggested an alternative model for measuring risk called the arbitrage pricing model (APM).

**Assumptions** If investors can invest risklessly and earn more than the riskless rate, they have found an arbitrage opportunity. The premise of the arbitrage pricing model is that investors take advantage of such arbitrage opportunities, and in the process eliminate them. If two portfolios have the same exposure to risk but offer different expected returns, investors will buy the portfolio that has the higher expected returns and sell the portfolio with the lower expected returns, and earn the difference as a riskless profit. To prevent this arbitrage from occurring, the two portfolios have to earn the same expected return.

Like the capital asset pricing model, the arbitrage pricing model begins by breaking risk down into firm-specific and market risk components. As in the capital asset pricing model, firm-specific risk covers information that affects primarily the firm. Market risk affects many or all firms and would include unanticipated changes in a number of economic variables, including gross national product, inflation, and interest rates. Incorporating both types of risk into a return model, we get:

$$R = E(R) + m + \varepsilon$$

where R is the actual return, E(R) is the expected return, m is the marketwide component of unanticipated risk, and ε is the firm-specific component. Thus, the actual return can be different from the expected return, because of either market risk or firm-specific actions.

**Sources of Marketwide Risk** While both the capital asset pricing model and the arbitrage pricing model make a distinction between firm-specific and marketwide risk, they measure market risk differently. The CAPM assumes that market risk is captured in the market portfolio, whereas the arbitrage pricing model allows for multiple sources of marketwide risk and measures the sensitivity of investments to changes in each source. In general, the market component of unanticipated returns can be decomposed into economic factors:

$$\begin{aligned} R &= E(R) + m + \varepsilon \\ &= R + (\beta_1 F_1 + \beta_2 F_2 + \ldots + \beta_n F_n) + \varepsilon \end{aligned}$$

where $\beta_j$ = Sensitivity of investment to unanticipated changes in factor j
$F_j$ = Unanticipated changes in factor j

Note that the measure of an investment's sensitivity to any macroeconomic factor takes the form of a beta, called a factor beta. In fact, this beta has many of the same properties as the market beta in the CAPM.

**Effects of Diversification** The benefits of diversification were discussed earlier, in the context of the breakdown of risk into market and firm-specific risk. The primary point of that discussion was that diversification eliminates firm-specific risk. The arbitrage pricing model uses the same argument and concludes that the return on a portfolio will not have a firm-specific component of unanticipated returns. The return on a portfolio can be written as the sum of two weighted averages—that of the anticipated returns in the portfolio and that of the market factors:

$$R_p = (w_1R_1 + w_2R_2 + \ldots + w_nR_n) + (w_1\beta_{1,1} + w_2\beta_{1,2} + \ldots + w_n\beta_{1,n})F_1 + (w_1\beta_{2,1} + w_2\beta_{2,2} + \ldots + w_n\beta_{2,n})F_2 \ldots$$

where $w_j$ = Portfolio weight on asset j (where there are n assets)
$R_j$ = Expected return on asset j
$\beta_{i,j}$ = Beta on factor i for asset j

**Expected Returns and Betas** The final step in this process is estimating an expected return as a function of the betas just specified. To do this, we should first note that the beta of a portfolio is the weighted average of the betas of the assets in the portfolio. This property, in conjunction with the absence of arbitrage, leads to the conclusion that expected returns should be linearly related to betas. To see why, assume that there is only one factor and three portfolios. Portfolio A has a beta of 2.0 and an expected return of 20 percent; portfolio B has a beta of 1.0 and an expected return of 12 percent; and portfolio C has a beta of 1.5 and an expected return of 14 percent. Note that investors can put half of their wealth in portfolio A and half in portfolio B and end up with portfolios with a beta of 1.5 and an expected return of 16 percent. Consequently no investor will choose to hold portfolio C until the prices of assets in that portfolio drop and the expected return increases to 16 percent. By the same rationale, the expected returns of every portfolio should be a linear function of the beta. If they were not, we could combine two other portfolios, one with a higher beta and one with a lower beta, to earn a higher return than the portfolio in question, creating an opportunity for arbitrage. This argument can be extended to multiple factors with the same results. Therefore, the expected return on an asset can be written as:

$$E(R) = R_f + \beta_1[E(R_1) - R_f] + \beta_2[E(R_2) - R_f] \ldots + \beta_K[E(R_K) - R_f]$$

where $R_f$ = Expected return on a zero-beta portfolio
$E(R_j)$ = Expected return on a portfolio with a factor beta of 1 for factor j, and zero for all other factors (where j = 1, 2, . . . , K factors)

The terms in the brackets can be considered to be risk premiums for each of the factors in the model.

The capital asset pricing model can be considered to be a special case of the arbitrage pricing model, where there is only one economic factor driving marketwide returns, and the market portfolio is the factor.

$$E(R) = R_f + \beta_m[E(R_m) - R_f]$$

**The APM in Practice** The arbitrage pricing model requires estimates of each of the factor betas and factor risk premiums in addition to the riskless rate. In practice, these are usually estimated using historical data on asset returns and a factor analysis. Intuitively, in a factor analysis, we examine the historical data looking for common patterns that affect broad groups of assets (rather than just one sector or a few assets). A factor analysis provides two output measures:

1. It specifies the number of common factors that affected the historical return data.
2. It measures the beta of each investment relative to each of the common factors and provides an estimate of the actual risk premium earned by each factor.

The factor analysis does not, however, identify the factors in economic terms. In summary, in the arbitrage pricing model the market risk is measured relative to multiple unspecified macroeconomic variables, with the sensitivity of the investment relative to each factor being measured by a beta. The number of factors, the factor betas, and the factor risk premiums can all be estimated using the factor analysis.

**Multifactor Models for Risk and Return** The arbitrage pricing model's failure to identify the factors specifically in the model may be a statistical strength, but it is an intuitive weakness. The solution seems simple: Replace the unidentified statistical factors with specific economic factors, and the resultant model should have an economic basis while still retaining much of the strength of the arbitrage pricing model. That is precisely what multifactor models try to do.

**Deriving a Multifactor Model** Multifactor models generally are determined by historical data rather than by economic modeling. Once the number of factors has been identified in the arbitrage pricing model, their behavior over time can be extracted from the data. The behavior of the unnamed factors over time can then be compared to the behavior of macroeconomic variables over that same period, to see whether any of the variables is correlated, over time, with the identified factors.

For instance, Chen, Roll, and Ross (1986) suggest that the following macroeconomic variables are highly correlated with the factors that come out of factor analysis: industrial production, changes in default premium, shifts in the term structure, unanticipated inflation, and changes in the real rate of return. These variables can then be correlated with returns to come up with a model of expected returns, with firm-specific betas calculated relative to each variable.

$$E(R) = R_f + \beta_{GNP}\,[E(R_{GNP}) - R_f] + \beta_I[E(R_I) - R_f] \ldots + \beta_\delta[E(R_\delta) - R_f]$$

where $\beta_{GNP}$ = Beta relative to changes in industrial production
$E(R_{GNP})$ = Expected return on a portfolio with a beta of one on the industrial production factor and zero on all other factors
$\beta_I$ = Beta relative to changes in inflation
$E(R_I)$ = Expected return on a portfolio with a beta of one on the inflation factor and zero on all other factors

The costs of going from the arbitrage pricing model to a macroeconomic multifactor model can be traced directly to the errors that can be made in identifying the factors. The economic factors in the model can change over time, as will the risk premium associated with each one. For instance, oil price changes were a significant economic factor driving expected returns in the 1970s but are not as significant in other time periods. Using the wrong factor or missing a significant factor in a multifactor model can lead to inferior estimates of expected return.

In summary, multifactor models, like the arbitrage pricing model, assume that market risk can be captured best using multiple macroeconomic factors and betas relative to each. Unlike the arbitrage pricing model, multifactor models do attempt to identify the macroeconomic factors that drive market risk.

**Regression or Proxy Models** All the models described so far begin by defining market risk in broad terms and then developing models that might best measure this market risk. All of them, however, extract their measures of market risk (betas) by looking at historical data. There is a final class of risk and return models that start with the returns, and try to explain differences in returns across stocks over long time periods, using characteristics such as a firm's market value or price multiples.[4] Proponents of these models argue that if some investments earn consistently higher returns than other investments, they must be riskier. Consequently, we could look at the characteristics that these high-return investments have in common and consider these characteristics to be indirect measures or proxies for market risk.

Fama and French (1992), in a highly influential study of the capital asset pricing model, noted that actual returns between 1963 and 1990 have been highly correlated with book-to-price ratios[5] and size. High-return investments, over this period, tended to be investments in companies with low market capitalization and high book-to-price ratios. Fama and French suggested that these measures be used as proxies for risk and reported the following regression for monthly returns on stocks on the New York Stock Exchange (NYSE):

$$R_t = 1.77\% - 0.11 \ln(\text{MV}) + 0.35 \ln(\text{BV/MV})$$

where ln = Natural log
MV = Market value of equity
BV/MV = Book value of equity/market value of equity

The values for market value of equity and book-to-price ratios for individual firms, when plugged into this regression, should yield expected monthly returns.

## A COMPARATIVE ANALYSIS OF RISK AND RETURN MODELS

Figure 4.5 summarizes all the risk and return models in finance, noting their similarities in the first two steps and the differences in the way they define market risk.

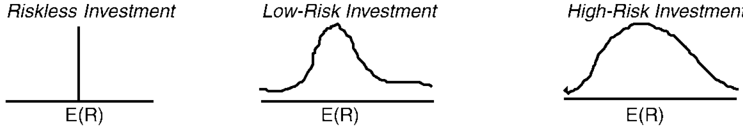
As noted in Figure 4.5, all the risk and return models developed in this chapter make some assumptions in common. They all assume that only market risk is rewarded, and they derive the expected return as a function of measures of this risk. The capital asset pricing model makes the most restrictive assumptions about how markets work but arrives at the simplest model, with only one factor driving risk and requiring estimation. The arbitrage pricing model makes fewer assumptions but arrives at a more complicated model, at least in terms of the pa-

[4]A price multiple is obtained by dividing the market price by its earnings or its book value. Studies indicate that stocks that have low price-earnings multiples or low price–book value multiples earn higher returns than other stocks.
[5]The book-to-price ratio is the ratio of the book value of equity to the market value of equity.

**Step 1: Defining Risk**

**The risk in an investment can be measured by the variance in actual returns around an expected return.**

*Riskless Investment* *Low-Risk Investment* *High-Risk Investment*




**Step 2: Differentiating between Rewarded and Unrewarded Risk**

Risk that is specific to investment (firm-specific risk) can be diversified away in a diversified portfolio.
1. Each investment is a small proportion of portfolio.
2. Risk averages out across investments in portfolio.

Risk that affects all investments (market risk) cannot be diversified away since most assets are affected by it.

**The marginal investor is assumed to hold a diversified portfolio. Thus, only market risk will be rewarded and priced.**

**Step 3: Measuring Market Risk**

| **The CAPM** | **The APM** | **Multifactor Models** | **Proxy Models** |
|---|---|---|---|
| If there is<br>1. no private information<br>2. no transactions cost<br>the optimal diversified portfolio includes every traded asset. Everyone will hold this market portfolio.<br>**Market Risk = Risk added by any investment to the market portfolio** | If there are no arbitrage opportunities, then the market risk of any asset must be captured by betas relative to factors that affect all investments.<br>**Market Risk = Risk exposures of any asset to market factors** | Since market risk affects most or all investments, it must come from macroeconomic factors.<br>**Market Risk = Risk exposures of any asset to macro-economic factors** | In an efficient market, differences in returns across long periods must be due to market risk differences. Looking for variables correlated with returns should then give us proxies for this risk.<br>**Market Risk = Captured by the proxy variable(s)** |
| Betas of assets relative to market portfolio (from a regression) | Betas of assets relative to unspecified market factors (from a factor analysis) | Betas of assets relative to specified macro-economic factors (from a regression) | Equation relating returns to proxy variables (from a regression) |

**FIGURE 4.5** Risk and Return Models in Finance

rameters that require estimation. The capital asset pricing model can be considered a specialized case of the arbitrage pricing model, where there is only one underlying factor and it is completely measured by the market index. In general, the CAPM has the advantage of being a simpler model to estimate and to use, but it will underperform the richer APM when an investment is sensitive to economic factors not well represented in the market index. For instance, oil company stocks, which derive most of their risk from oil price movements, tend to have low CAPM betas and low expected returns. Using an arbitrage pricing model, where one of the factors may measure oil and other commodity price movements, will yield a better estimate of risk and higher expected return for these firms.[6]

Which of these models works best? Is beta a good proxy for risk, and is it correlated with expected returns? The answers to these questions have been debated widely in the past two decades. The first tests of the CAPM suggested that betas

[6]Weston and Copeland (1992) used both approaches to estimate the cost of equity for oil companies in 1989 and came up with 14.4 percent with the CAPM and 19.1 percent using the arbitrage pricing model.

and returns were positively related, though other measures of risk (such as variance) continued to explain differences in actual returns. This discrepancy was attributed to limitations in the testing techniques. In 1977, Roll, in a seminal critique of the model's tests, suggested that since the market portfolio could never be observed, the CAPM could never be tested, and all tests of the CAPM were therefore joint tests of both the model and the market portfolio used in the tests. In other words, all that any test of the CAPM could show was that the model worked (or did not) given the proxy used for the market portfolio. It could therefore be argued that in any empirical test that claimed to reject the CAPM, the rejection could be of the proxy used for the market portfolio rather than of the model itself. Roll noted that there was no way ever to prove that the CAPM worked and thus there was no empirical basis for using the model.

Fama and French (1992) examined the relationship between betas and returns between 1963 and 1990 and concluded that there is no relationship. These results have been contested on three fronts. First, Amihud, Christensen, and Mendelson (1992) used the same data, performed different statistical tests, and showed that differences in betas did in fact explain differences in returns during the time period. Second, Kothari and Shanken (1995) estimated betas using annual data instead of the shorter intervals used in many tests, and concluded that betas do explain a significant proportion of the differences in returns across investments. Third, Chan and Lakonishok (1993) looked at a much longer time series of returns from 1926 to 1991 and found that the positive relationship between betas and returns broke down only in the period after 1982. They also found that betas are a useful guide to risk in extreme market conditions, with the riskiest firms (the 10 percent with highest betas) performing far worse than the market as a whole in the 10 worst months for the market between 1926 and 1991. (See Figure 4.6.)

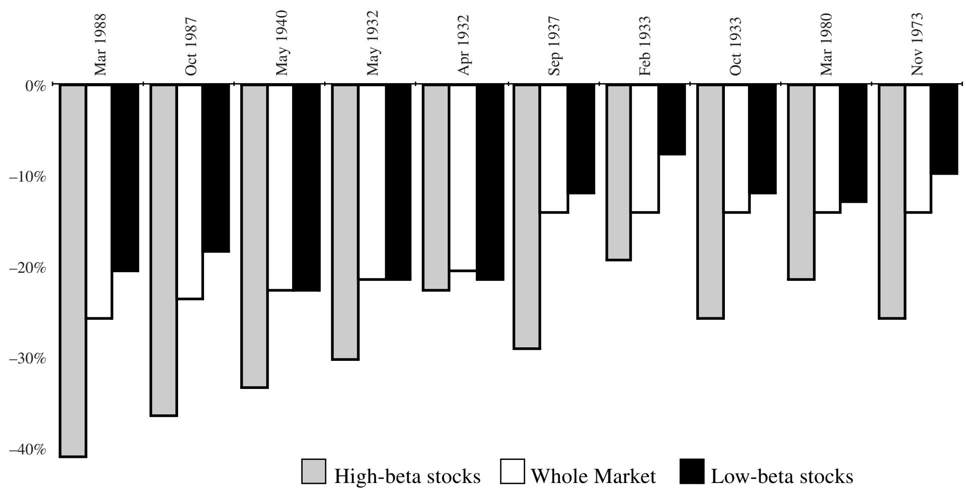



**FIGURE 4.6** Returns and Betas: Ten Worst Months between 1926 and 1991
*Source:* Chan and Lakonishok.

While the initial tests of the APM suggested that they might provide more promise in terms of explaining differences in returns, a distinction has to be drawn between the use of these models to explain differences in past returns and their use to predict expected returns in the future. The competitors to the CAPM clearly do a much better job of explaining past returns since they do not constrain themselves to one factor, as the CAPM does. This extension to multiple factors does become more of a problem when we try to project expected returns into the future, since the betas and premiums of each of these factors now have to be estimated. Because the factor premiums and betas are themselves volatile, the estimation error may eliminate the benefits that could be gained by moving from the CAPM to more complex models. The regression models that were offered as an alternative also have an estimation problem, since the variables that work best as proxies for market risk in one period (such as market capitalization) may not be the ones that work in the next period.

Ultimately, the survival of the capital asset pricing model as the default model for risk in real-world applications is a testament to both its intuitive appeal and the failure of more complex models to deliver significant improvement in terms of estimating expected returns. It would seem that a judicious use of the capital asset pricing model, without an overreliance on historical data, is still the most effective way of dealing with risk in modern corporate finance.

## MODELS OF DEFAULT RISK

The risk discussed so far in this chapter relates to cash flows on investments being different from expected cash flows. There are some investments, however, in which the cash flows are promised when the investment is made. This is the case, for instance, when you lend to a business or buy a corporate bond; the borrower may default on interest and principal payments on the borrowing. Generally speaking, borrowers with higher default risk should pay higher interest rates on their borrowing than those with lower default risk. This section examines the measurement of default risk and the relationship of default risk to interest rates on borrowing.

In contrast to the general risk and return models for equity, which evaluate the effects of market risk on expected returns, models of default risk measure the consequences of firm-specific default risk on promised returns. While diversification can be used to explain why firm-specific risk will not be priced into expected returns for equities, the same rationale cannot be applied to securities that have limited upside potential and much greater downside potential from firm-specific events. To see what is meant by limited upside potential, consider investing in the bond issued by a company. The coupons are fixed at the time of the issue, and these coupons represent the promised cash flow on the bond. The best-case scenario for you as an investor is that you receive the promised cash flows; you are not entitled to more than these cash flows even if the company is wildly successful. All other scenarios contain only bad news, though in varying degrees, with the delivered cash flows being less than the promised cash flows. Consequently, the expected return on a corporate bond is likely to reflect the firm-specific default risk of the firm issuing the bond.

### Determinants of Default Risk

The default risk of a firm is a function of two variables. The first is the firm's capacity to generate cash flows from operations, and the second is its financial obligations—including interest and principal payments.[7] Firms that generate high cash flows relative to their financial obligations should have lower default risk than do firms that generate low cash flows relative to obligations. Thus, firms with significant existing investments that generate high cash flows will have lower default risk than will firms that do not have such investments.

In addition to the magnitude of a firm's cash flows, the default risk is also affected by the volatility in these cash flows. The more stability there is in cash flows, the lower is the default risk in the firm. Firms that operate in predictable and stable businesses will have lower default risk than will otherwise similar firms that operate in cyclical or volatile businesses.

Most models of default risk use financial ratios to measure the cash flow coverage (i.e., the magnitude of cash flows relative to obligations) and control for industry effects in order to evaluate the variability in cash flows.

### Bond Ratings and Interest Rates

The most widely used measure of a firm's default risk is its bond rating, which is generally assigned by an independent ratings agency. The two best known are Standard & Poor's and Moody's. Thousands of companies are rated by these two agencies, and their views carry significant weight with financial markets.

**The Ratings Process** The process of rating a bond starts when the issuing company requests a rating from a bond ratings agency. The ratings agency then collects information from both publicly available sources, such as financial statements, and the company itself and makes a decision on the rating. If the company disagrees with the rating, it is given the opportunity to present additional information. This process is presented schematically for one ratings agency, Standard & Poor's (S&P), in Figure 4.7.

The ratings assigned by these agencies are letter ratings. A rating of AAA from Standard & Poor's and Aaa from Moody's represents the highest rating, granted to firms that are viewed as having the lowest default risk. As the default risk increases, the ratings decrease toward D for firms in default (Standard & Poor's). A rating above BBB by Standard & Poor's is categorized as above investment grade, reflecting the view of the ratings agency that there is relatively little default risk in investing in bonds issued by these firms.

**Determinants of Bond Ratings** The bond ratings assigned by ratings agencies are primarily based on publicly available information, though private information con-

[7]Financial obligation refers to any payment that the firm has legally obligated itself to make, such as interest and principal payments. It does not include discretionary cash flows, such as dividend payments or new capital expenditures, which can be deferred or delayed without legal consequences, though there may be economic consequences.

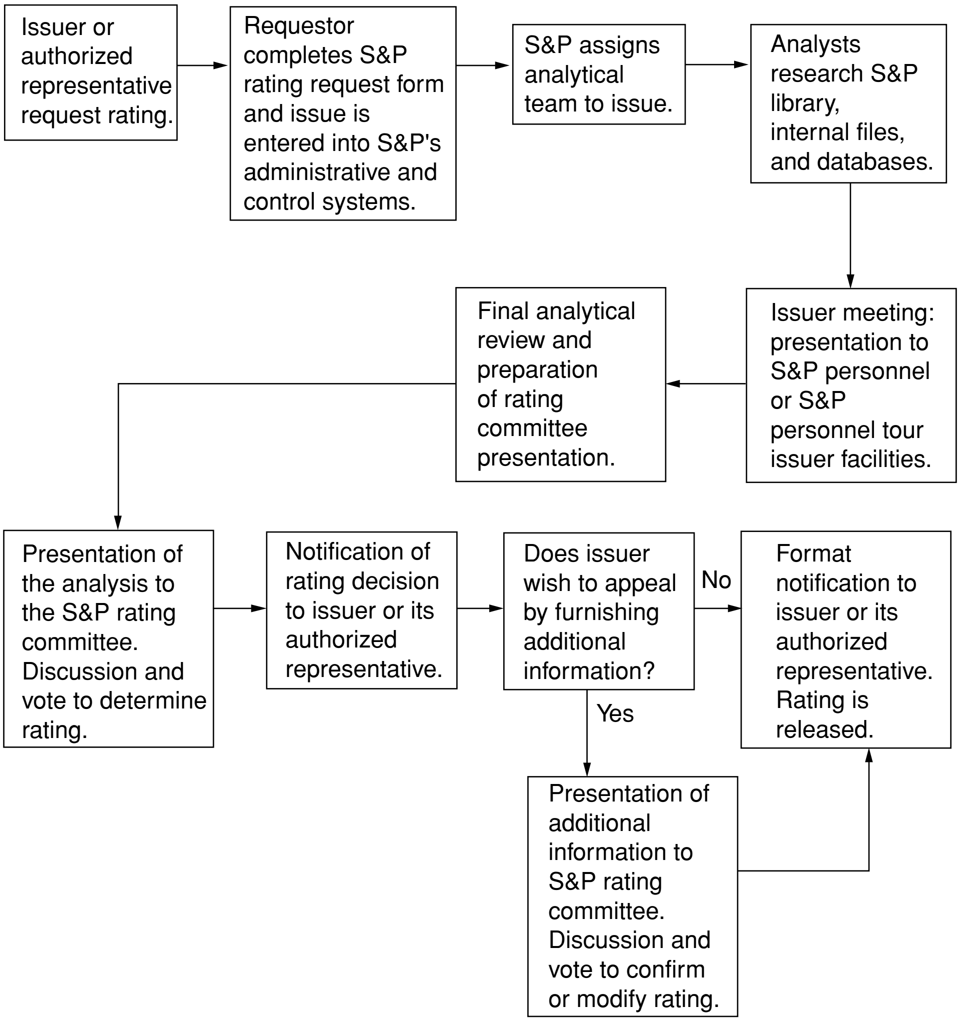



**FIGURE 4.7** The Ratings Process

veyed by the firm to the rating agency does play a role. The rating assigned to a company's bonds will depend in large part on financial ratios that measure the capacity of the company to meet debt payments and generate stable and predictable cash flows. While a multitude of financial ratios exist, Table 4.1 summarizes some of the key ratios used to measure default risk.

There is a strong relationship between the bond rating a company receives and its performance on these financial ratios. Table 4.2 provides a summary of the median ratios[8] from 1997 to 1999 for different S&P ratings classes for manufacturing firms.

Not surprisingly, firms that generate income and cash flows significantly higher than debt payments, that are profitable, and that have low debt ratios are more likely to be highly rated than are firms that do not have these characteristics. There

[8]See the Standard & Poor's online site (www.standardandpoors.com/ratings/criteria/index.htm).

**TABLE 4.1** Financial Ratios Used to Measure Default Risk

| Ratio | Description |
|---|---|
| Pretax interest coverage | = (Pretax income from continuing operations + Interest expense)/Gross interest |
| EBITDA interest coverage | = EBITDA/Gross interest |
| Funds from operations/total debt | = (Net income from continuing operations + depreciation)/Total debt |
| Free operating cash flow/total debt | = (Funds from operations – Capital expenditures – Change in working capital)/Total debt |
| Pretax return on permanent capital | = (Pretax income from continuing operations + Interest expense)/Average of beginning of the year and end of the year of long- and short-term debt, minority interest, and shareholders' equity |
| Operating income/sales (%) | = (Sales – Cost of goods sold before depreciation – Selling expenses – Administrative expenses – R&D expenses)/Sales |
| Long-term debt capital | = Long-term debt/(Long-term debt + Equity) |
| Total debt/capitalization | $= \dfrac{\text{Total debt}}{(\text{Total debt} + \text{Equity})}$ |

*Source:* Standard & Poor's.

**TABLE 4.2** Three-Year (1997 to 1999) Medians

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| EBIT int. cov. (X) | 17.5 | 10.8 | 6.8 | 3.9 | 2.3 | 1.0 | 0.2 |
| EBITDA int. cov. (X) | 21.8 | 14.6 | 9.6 | 6.1 | 3.8 | 2.0 | 1.4 |
| Funds flow % total debt | 105.8 | 55.8 | 46.1 | 30.5 | 19.2 | 9.4 | 5.8 |
| Free oper. cash flow/total debt (%) | 55.4 | 24.6 | 15.6 | 6.6 | 1.9 | (4.5) | (14.0) |
| Return on cap. (%) | 28.2 | 22.9 | 19.9 | 14.0 | 11.7 | 7.2 | 0.5 |
| Oper. inc. % sales | 29.2 | 21.3 | 18.3 | 15.3 | 15.4 | 11.2 | 13.6 |
| Long-term debt/cap. (%) | 15.2 | 26.4 | 32.5 | 41.0 | 55.8 | 70.7 | 80.3 |
| Total debt % cap. | 26.9 | 35.6 | 40.1 | 47.4 | 61.3 | 74.6 | 89.4 |
| Companies | 10 | 34 | 150 | 234 | 276 | 240 | 23 |

*Source:* Standard & Poor's.
*Note:* Pretax interest coverage ratio and EBITDA interest coverage ratio are stated in terms of times interest earned; the other ratios are stated in percentage terms.

***ratingfins.xls:*** **This is a dataset on the Web that summarizes key financial ratios by bond rating class for the United States in the most recent period for which the data is available.**

will be individual firms whose ratings are not consistent with their financial ratios, however, because the ratings agency does add subjective judgments into the final mix. Thus a firm that performs poorly on financial ratios but is expected to improve its performance dramatically over the next period may receive a higher rating than is justified by its current financials. For most firms, however, the financial ratios should provide a reasonable basis for guessing at the bond rating.

**Bond Ratings and Interest Rates** The interest rate on a corporate bond should be a function of its default risk, which is measured by its rating. If the rating is a good measure of the default risk, higher-rated bonds should be priced to yield lower interest rates than those of lower-rated bonds. In fact, the difference between the interest rate on a bond with default risk and a default-free government bond is the default spread. This default spread will vary by maturity of the bond and can also change from period to period, depending on economic conditions. Chapter 7 will consider how best to estimate these default spreads and how they might vary over time.

## CONCLUSION

Risk, as defined in finance, is measured based on deviations of actual returns on an investment from its expected returns. There are two types of risk. The first, called equity risk, arises in investments where there are no promised cash flows, but there are expected cash flows. The second, default risk, arises on investments with promised cash flows.

On investments with equity risk, the risk is best measured by looking at the variance of actual returns around the expected returns, with greater variance indicating greater risk. This risk can be broken down into risk that affects one or a few investments, called firm-specific risk, and risk that affects many investments, refered to as market risk. When investors diversify, they can reduce their exposure to firm-specific risk. By assuming that the investors who trade at the margin are well diversified, we conclude that the risk we should be looking at with equity investments is the market risk. The different models of equity risk introduced in this chapter share this objective of measuring market risk, but they differ in the way they do it. In the capital asset pricing model, exposure to market risk is measured by a market beta, which estimates how much risk an individual investment will add to a portfolio that includes all traded assets. The arbitrage pricing model and the multifactor model allow for multiple sources of market risk and estimate betas for an investment relative to each source. Regression or proxy models for risk look for firm characteristics, such as size, that have been correlated with high returns in the past and use these to measure market risk. In all these models, the risk measures are used to estimate the expected return on an equity investment. This expected return can be considered the cost of equity for a company.

On investments with default risk, risk is measured by the likelihood that the promised cash flows might not be delivered. Investments with higher default risk should have higher interest rates, and the premium that we demand over a riskless rate is the default spread. For most U.S. companies, default risk is measured by rating agencies in the form of a company rating; these ratings determine, in large part, the interest rates at which these firms can borrow. Even in the absence of ratings, interest rates will include a default spread that reflects the lenders' assessments of default risk. These default-risk-adjusted interest rates represent the cost of borrowing or debt for a business.

## QUESTIONS AND SHORT PROBLEMS

1. The following table lists the stock prices for Microsoft from 1989 to 1998. The company did not pay any dividends during the period.

| *Year* | *Price* |
|---|---|
| 1989 | $ 1.20 |
| 1990 | $ 2.09 |
| 1991 | $ 4.64 |
| 1992 | $ 5.34 |
| 1993 | $ 5.05 |
| 1994 | $ 7.64 |
| 1995 | $10.97 |
| 1996 | $20.66 |
| 1997 | $32.31 |
| 1998 | $69.34 |

   a. Estimate the average annual return you would have made on your investment.
   b. Estimate the standard deviation and variance in annual returns.
   c. If you were investing in Microsoft today, would you expect the historical standard deviations and variances to continue to hold? Why or why not?

2. Unicom is a regulated utility serving northern Illinois. The following table lists the stock prices and dividends on Unicom from 1989 to 1998.

| *Year* | *Price* | *Dividends* |
|---|---|---|
| 1989 | $36.10 | $3.00 |
| 1990 | $33.60 | $3.00 |
| 1991 | $37.80 | $3.00 |
| 1992 | $30.90 | $2.30 |
| 1993 | $26.80 | $1.60 |
| 1994 | $24.80 | $1.60 |
| 1995 | $31.60 | $1.60 |
| 1996 | $28.50 | $1.60 |
| 1997 | $24.25 | $1.60 |
| 1998 | $35.60 | $1.60 |

   a. Estimate the average annual return you would have made on your investment.
   b. Estimate the standard deviation and variance in annual returns.
   c. If you were investing in Unicom today, would you expect the historical standard deviations and variances to continue to hold? Why or why not?

3. The following table summarizes the annual returns you would have made on two companies—Scientific Atlanta, a satellite and data equipment manufacturer, and AT&T, the telecommunications giant—from 1989 to 1998.

| *Year* | *Scientific Atlanta* | *AT&T* |
|---|---|---|
| 1989 | 80.95% | 58.26% |
| 1990 | –47.37% | –33.79% |
| 1991 | 31.00% | 29.88% |
| 1992 | 132.44% | 30.35% |
| 1993 | 32.02% | 2.94% |
| 1994 | 25.37% | –4.29% |
| 1995 | –28.57% | 28.86% |
| 1996 | 0.00% | –6.36% |
| 1997 | 11.67% | 48.64% |
| 1998 | 36.19% | 23.55% |

a. Estimate the average annual return and standard deviation in annual returns in each company.
b. Estimate the covariance and correlation in returns between the two companies.
c. Estimate the variance of a portfolio composed, in equal parts, of the two investments.

4. You are in a world where there are only two assets, gold and stocks. You are interested in investing your money in one, the other, or both assets. Consequently you collect the following data on the returns on the two assets over the past six years.

| | *Gold* | *Stock Market* |
|---|---|---|
| Average return | 8% | 20% |
| Standard deviation | 25% | 22% |
| Correlation | –0.4 | |

a. If you were constrained to pick just one, which one would you choose?
b. A friend argues that this is wrong. He says that you are ignoring the big payoffs that you can get on the other asset. How would you go about alleviating his concern?
c. How would a portfolio composed of equal proportions in gold and stocks do in terms of mean and variance?
d. You now learn that GPEC (a cartel of gold-producing countries) is going to vary the amount of gold it produces in relation to stock prices in the United States. (GPEC will produce less gold when stock markets are up and more when they are down.) What effect will this have on your portfolio? Explain.

5. You are interested in creating a portfolio of two stocks—Coca-Cola and Texas Utilities. Over the past decade, an investment in Coca-Cola stock would have earned an average annual return of 25%, with a standard deviation in returns of 36%. An investment in Texas Utilities stock would have earned an average annual return of 12%, with a standard deviation of 22%. The correlation in returns across the two stocks is 0.28.

a. Assuming that the average return and standard deviation, estimated using past returns, will continue to hold in the future, estimate the future average returns and standard deviation of a portfolio composed 60% of Coca-Cola and 40% of Texas Utilities stock.

b. Now assume that Coca-Cola's international diversification will reduce the correlation to 0.20, while increasing Coca-Cola's standard deviation in returns to 45%. Assuming all of the other numbers remain unchanged, estimate one standard deviation of the portfolio in (a).

6. Assume that you have half your money invested in Times Mirror, the media company, and the other half invested in Unilever, the consumer product company. The expected returns and standard deviations on the two investments are:

| | *Times Mirror* | *Unilever* |
|---|---|---|
| Expected return | 14% | 18% |
| Standard deviation | 25% | 40% |

Estimate the variance of the portfolio as a function of the correlation coefficient (start with –1 and increase the correlation to +1 in 0.2 increments).

7. You have been asked to analyze the standard deviation of a portfolio composed of the following three assets:

| | *Expected Return* | *Standard Deviation* |
|---|---|---|
| Sony Corporation | 11% | 23% |
| Tesoro Petroleum | 9% | 27% |
| Storage Technology | 16% | 50% |

You have also been provided with the correlations across these three investments:

| | *Sony Corporation* | *Tesoro Petroleum* | *Storage Technology* |
|---|---|---|---|
| Sony Corporation | 1.00 | –0.15 | 0.20 |
| Tesoro Petroleum | –0.15 | 1.00 | –0.25 |
| Storage Technology | 0.20 | –0.25 | 1.00 |

Estimate the variance of a portfolio, equally weighted across all three assets.

8. Assume that the average variance of return for an individual security is 50 and that the average covariance is 10. What is the expected variance of a portfolio of 5, 10, 20, 50, and 100 securities? How many securities need to be held before the risk of a portfolio is only 10% more than the minimum?

9. Assume you have all your wealth (a million dollars) invested in the Vanguard 500 index fund, and that you expect to earn an annual return of 12%, with a standard deviation in returns of 25%. Since you have become more risk averse, you decide to shift $200,000 from the Vanguard 500 index fund to Treasury bills. The T-bill rate is 5%. Estimate the expected return and standard deviation of your new portfolio.

10. Every investor in the capital asset pricing model owns a combination of the market portfolio and a riskless asset. Assume that the standard deviation of the market portfolio is 30% and that the expected return on the portfolio is 15%. What proportion of the following investors' wealth would you suggest investing in the market portfolio and what proportion in the riskless asset? (The riskless asset has an expected return of 5%.)

a. An investor who desires a portfolio with no standard deviation.
b. An investor who desires a portfolio with a standard deviation of 15%.
c. An investor who desires a portfolio with a standard deviation of 30%.
d. An investor who desires a portfolio with a standard deviation of 45%.
e. An investor who desires a portfolio with an expected return of 12%.

11. The following table lists returns on the market portfolio and on Scientific Atlanta, each year from 1989 to 1998.

| *Year* | *Scientific Atlanta* | *Market Portfolio* |
|---|---|---|
| 1989 | 80.95% | 31.49% |
| 1990 | –47.37% | –3.17% |
| 1991 | 31.00% | 30.57% |
| 1992 | 132.44% | 7.58% |
| 1993 | 32.02% | 10.36% |
| 1994 | 25.37% | 2.55% |
| 1995 | –28.57% | 37.57% |
| 1996 | 0.00% | 22.68% |
| 1997 | 11.67% | 33.10% |
| 1998 | 36.19% | 28.32% |

a. Estimate the covariance in returns between Scientific Atlanta and the market portfolio.
b. Estimate the variances in returns on both investments.
c. Estimate the beta for Scientific Atlanta.

12. United Airlines has a beta of 1.5. The standard deviation in the market portfolio is 22%, and United Airlines has a standard deviation of 66%.
a. Estimate the correlation between United Airlines and the market portfolio.
b. What proportion of United Airlines' risk is market risk?

13. You are using the arbitrage pricing model to estimate the expected return on Bethlehem Steel, and have derived the following estimates for the factor betas and risk premium:

| *Factor* | *Beta* | *Risk Premium* |
|---|---|---|
| 1 | 1.2 | 2.5% |
| 2 | 0.6 | 1.5% |
| 3 | 1.5 | 1.0% |
| 4 | 2.2 | 0.8% |
| 5 | 0.5 | 1.2% |

a. Which risk factor is Bethlehem Steel most exposed to? Is there any way, within the arbitrage pricing model, to identify the risk factor?
b. If the risk-free rate is 5%, estimate the expected return on Bethlehem Steel.
c. Now assume that the beta in the capital asset pricing model for Bethlehem Steel is 1.1, and that the risk premium for the market portfolio is 5%. Estimate the expected return using the CAPM.
d. Why are the expected returns different using the two models?

14. You are using the multifactor model to estimate the expected return on Emerson Electric, and have derived the following estimates for the factor betas and risk premiums:

| *Macroeconomic Factor* | *Measure* | *Beta* | *Risk Premium* $(R_{factor} - R_f)$ |
|---|---|---|---|
| Level of interest rates | T-bond rate | 0.5 | 1.8% |
| Term structure | T-bond rate—T-bill rate | 1.4 | 0.6% |
| Inflation rate | Consumer price index | 1.2 | 1.5% |
| Economic growth | Gross national product growth rate | 1.8 | 4.2% |

With a riskless rate of 6%, estimate the expected return on Emerson Electric.

15. The following equation is reproduced from the study by Fama and French of returns between 1963 and 1990.

$$R_t = 1.77 - 0.11 \ln(MV) + 0.35 \ln(BV/MV)$$

where MV is the market value of equity in hundreds of millions of dollars and BV is the book value of equity in hundreds of millions of dollars. The return is a monthly return.

a. Estimate the expected annual return on Lucent Technologies if the market value of its equity is \$180 billion and the book value of its equity is \$73.5 billion.
b. Lucent Technologies has a beta of 1.55. If the riskless rate is 6% and the risk premium for the market portfolio is 5.5%, estimate the expected return.
c. Why are the expected returns different under the two approaches?

# CHAPTER 5

# Option Pricing Theory and Models

In general, the value of any asset is the present value of the expected cash flows on that asset. This section will consider an exception to that rule when it looks at assets with two specific characteristics:

1. The assets derive their value from the values of other assets.
2. The cash flows on the assets are contingent on the occurrence of specific events.

These assets are called options, and the present value of the expected cash flows on these assets will understate their true value. This section will describe the cash flow characteristics of options, consider the factors that determine their value, and examine how best to value them.

## BASICS OF OPTION PRICING

An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to expire. There are two types of options—call options and put options.

### Call and Put Options: Description and Payoff Diagrams

A call option gives the buyer of the option the right to buy the underlying asset at the strike price or the exercise price at any time prior to the expiration date of the option. The buyer pays a price for this right. If at expiration the value of the asset is less than the strike price, the option is not exercised and expires worthless. If, however, the value of the asset is greater than the strike price, the option is exercised—the buyer of the option buys the stock at the exercise price, and the difference between the asset value and the exercise price comprises the gross profit on the investment. The net profit on the investment is the difference between the gross profit and the price paid for the call initially.

A payoff diagram illustrates the cash payoff on an option at expiration. For a call, the net payoff is negative (and equal to the price paid for the call) if the value of the underlying asset is less than the strike price. If the price of the underlying asset exceeds the strike price, the gross payoff is the difference between the value of the underlying asset and the strike price, and the net payoff is the difference between the gross payoff and the price of the call. This is illustrated in Figure 5.1.

A put option gives the buyer of the option the right to sell the underlying asset at a fixed price, again called the strike or exercise price, at any time prior to the expiration date of the option. The buyer pays a price for this right. If the price of the underlying asset is greater than the strike price, the option will not be exercised and will expire worthless. But if the price of the underlying asset is less than the strike price, the owner of the put option will exercise the option and sell the stock at the strike price, claiming the difference between the strike price and the market value of the asset as the gross profit. Again, netting out the initial cost paid for the put yields the net profit from the transaction.

A put has a negative net payoff if the value of the underlying asset exceeds the strike price, and has a gross payoff equal to the difference between the strike price and the value of the underlying asset if the asset value is less than the strike price. This is summarized in Figure 5.2.

## DETERMINANTS OF OPTION VALUE

The value of an option is determined by six variables relating to the underlying asset and financial markets.

1. *Current value of the underlying asset.* Options are assets that derive value from an underlying asset. Consequently, changes in the value of the underlying asset affect the value of the options on that asset. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increases.
2. *Variance in value of the underlying asset.* The buyer of an option acquires the right to buy or sell the underlying asset at a fixed price. The higher the variance

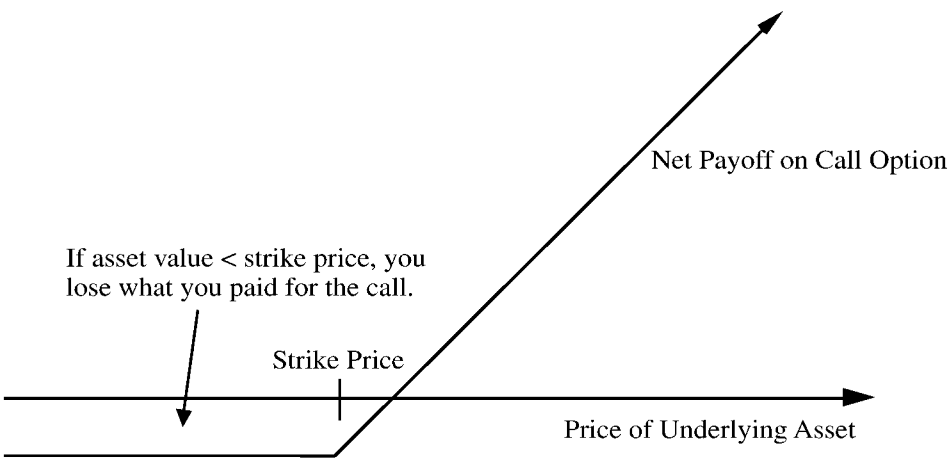



**FIGURE 5.1** Payoff on Call Option

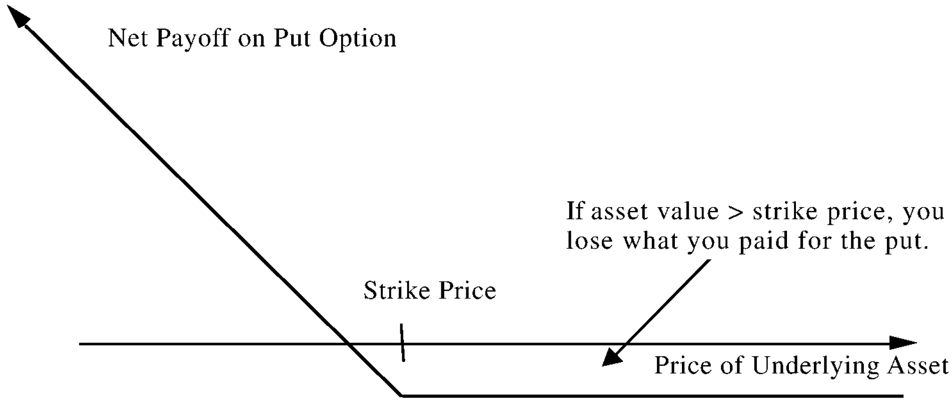



**FIGURE 5.2** Payoff on Put Option

in the value of the underlying asset, the greater the value of the option.[1] This is true for both calls and puts. While it may seem counterintuitive that an increase in a risk measure (variance) should increase value, options are different from other securities since buyers of options can never lose more than the price they pay for them; in fact, they have the potential to earn significant returns from large price movements.

3. *Dividends paid on the underlying asset.* The value of the underlying asset can be expected to decrease if dividend payments are made on the asset during the life of the option. Consequently, the value of a call on the asset is a *decreasing* function of the size of expected dividend payments, and the value of a put is an *increasing* function of expected dividend payments. A more intuitive way of thinking about dividend payments, for call options, is as a cost of delaying exercise on in-the-money options. To see why, consider an option on a traded stock. Once a call option is in-the-money (i.e., the holder of the option will make a gross payoff by exercising the option), exercising the call option will provide the holder with the stock and entitle him or her to the dividends on the stock in subsequent periods. Failing to exercise the option will mean that these dividends are forgone.
4. *Strike price of the option.* A key characteristic used to describe an option is the strike price. In the case of calls, where the holder acquires the right to buy at a fixed price, the value of the call will decline as the strike price increases. In the case of puts, where the holder has the right to sell at a fixed price, the value will increase as the strike price increases.
5. *Time to expiration on the option.* Both calls and puts are more valuable the greater the time to expiration. This is because the longer time to expiration provides more time for the value of the underlying asset to move, increasing the value of both types of options. Additionally, in the case of a call, where the

[1]Note, though, that higher variance can reduce the value of the underlying asset. As a call option becomes more in-the-money, the more it resembles the underlying asset. For very deep in-the-money call options, higher variance can reduce the value of the option.

buyer has to pay a fixed price at expiration, the present value of this fixed price decreases as the life of the option increases, increasing the value of the call.

6. *Riskless interest rate corresponding to life of the option.* Since the buyer of an option pays the price of the option up front, an opportunity cost is involved. This cost will depend on the level of interest rates and the time to expiration of the option. The riskless interest rate also enters into the valuation of options when the present value of the exercise price is calculated, since the exercise price does not have to be paid (received) until expiration on calls (puts). Increases in the interest rate will increase the value of calls and reduce the value of puts.

Table 5.1 summarizes the variables and their predicted effects on call and put prices.

### American versus European Options: Variables Relating to Early Exercise

A primary distinction between American and European options is that an American option can be exercised at any time prior to its expiration, while European options can be exercised only at expiration. The possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. There is one compensating factor that enables the former to be valued using models designed for the latter. In most cases, the time premium associated with the remaining life of an option and transaction costs make early exercise suboptimal. In other words, the holders of in-the-money options generally get much more by selling the options to someone else than by exercising the options.

## OPTION PRICING MODELS

Option pricing theory has made vast strides since 1972, when Fischer Black and Myron Scholes published their pathbreaking paper that provided a model for valuing dividend-protected European options. Black and Scholes used a "replicating portfolio"—a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued—and the notion of arbitrage to come

**TABLE 5.1** Summary of Variables Affecting Call and Put Prices

| | Effect On | |
|---|---|---|
| **Factor** | **Call Value** | **Put Value** |
| Increase in underlying asset's value | Increases | Decreases |
| Increase in variance of underlying asset | Increases | Increases |
| Increase in strike price | Decreases | Increases |
| Increase in dividends paid | Decreases | Increases |
| Increase in time to expiration | Increases | Increases |
| Increase in interest rates | Increases | Decreases |

up with their final formulation. While their derivation is mathematically complicated, there is a simpler binomial model for valuing options that draws on the same logic.

### Binomial Model

The binomial option pricing model is based on a simple formulation for the asset price process in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial path is shown in Figure 5.3. In this figure, S is the current stock price; the price moves up to Su with probability p and down to Sd with probability 1 – p in any time period.

**Creating a Replicating Portfolio** The objective in creating a replicating portfolio is to use a combination of risk-free borrowing/lending and the underlying asset to create the same cash flows as the option being valued. The principles of arbitrage apply here, and the value of the option must be equal to the value of the replicating portfolio. In the case of the general formulation shown in Figure 5.3, where stock prices can move either up to Su or down to Sd in any time period, the replicating portfolio for a call with strike price K will involve borrowing \$B and acquiring $\Delta$ of the underlying asset, where:

$$\Delta = \text{Number of units of the underlying asset bought} = \frac{C_u - C_d}{Su - Sd}$$

where $C_u$ = Value of the call if the stock price is Su
$C_d$ = Value of the call if the stock price is Sd

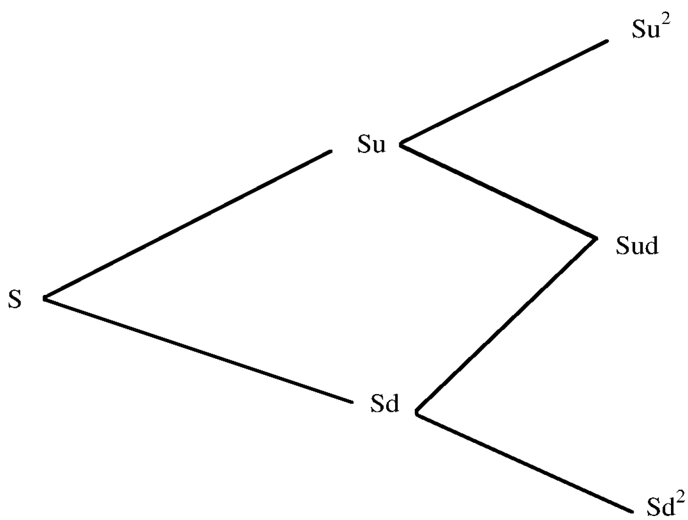


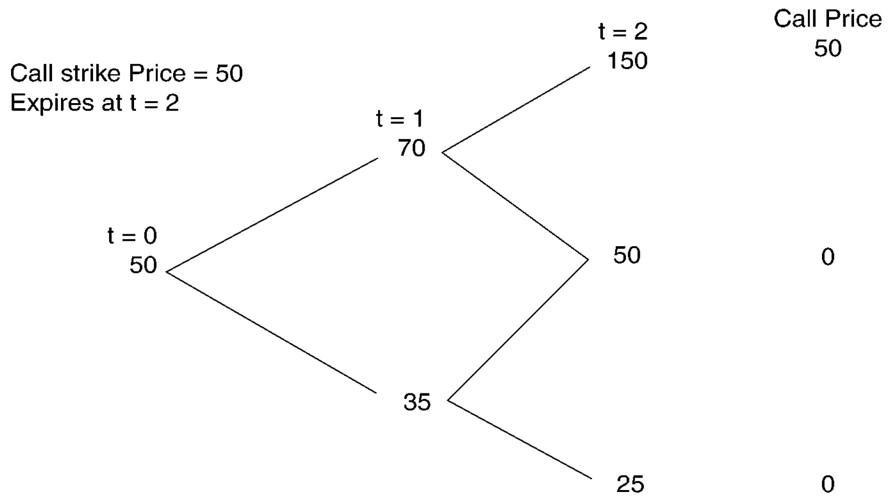

**FIGURE 5.3** General Formulation for Binomial Price Path

In a multiperiod binomial process, the valuation has to proceed iteratively (i.e., starting with the final time period and moving backward in time until the current point in time). The portfolios replicating the option are created at each step and valued, providing the values for the option in that time period. The final output from the binomial option pricing model is a statement of the value of the option in terms of the replicating portfolio, composed of $\Delta$ shares (option delta) of the underlying asset and risk-free borrowing/lending.

$$\text{Value of the call} = \text{Current value of underlying asset} \times \text{Option delta} - \text{Borrowing needed to replicate the option}$$

**ILLUSTRATION 5.1: Binomial Option Valuation**

Assume that the objective is to value a call with a strike price of \$50, which is expected to expire in two time periods, on an underlying asset whose price currently is \$50 and is expected to follow a binomial process:



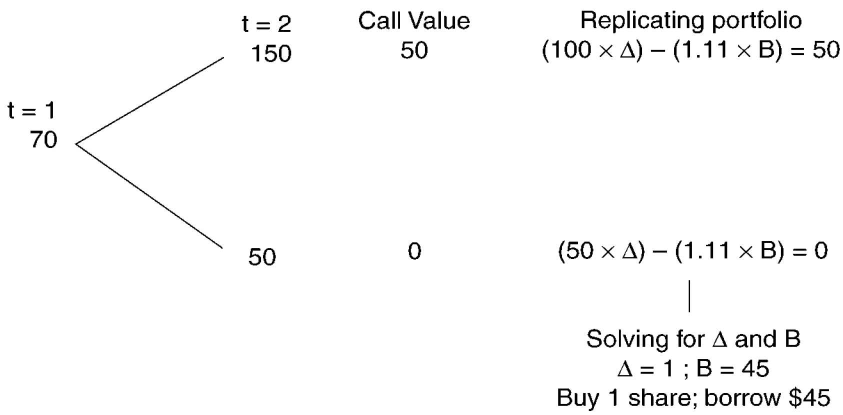

Now assume that the interest rate is 11%. In addition, define:

$\Delta$ = Number of shares in the replicating portfolio

B = Dollars of borrowing in replicating portfolio

The objective is to combined $\Delta$ shares of stock and B dollars of borrowing to replicate the cash flows from the call with a strike price of \$50. This can be done iteratively, starting with the last period and working back through the binomial tree.

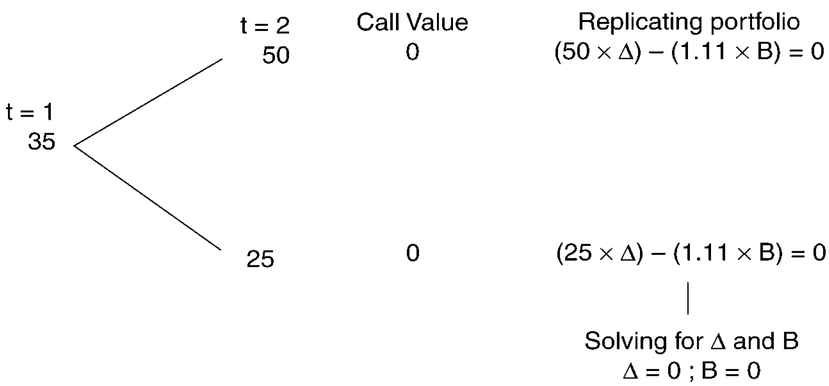
**Step 1:** Start with the end nodes and work backward:




Thus, if the stock price is $70 at t = 1, borrowing $45 and buying one share of the stock will give the same cash flows as buying the call. The value of the call at t = 1, if the stock price is $70, is therefore:

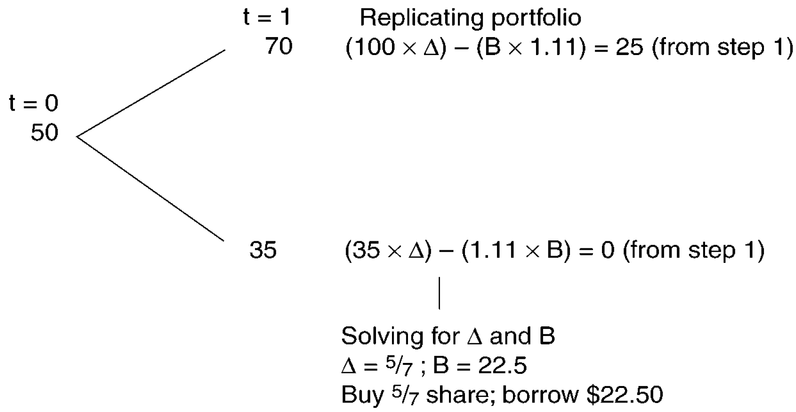
$$\text{Value of call} = \text{Value of replicating position} = 70\,\Delta - B = 70 - 45 = 25$$

Considering the other leg of the binomial tree at t = 1,




If the stock price is $35 at t = 1, then the call is worth nothing.

**STEP 2:** Move backward to the earlier time period and create a replicating portfolio that will provide the cash flows the option will provide.




In other words, borrowing $22.50 and buying five-sevenths of a share will provide the same cash flows as a call with a strike price of $50. The value of the call therefore has to be the same as the cost of creating this position.

$$\text{Value of call} = \text{Value of replicating position} = \left(\frac{5}{7}\right)\times \text{Current stock price} - \text{Borrowing}$$

$$= \left(\frac{5}{7}\right)(50) - 22.5 = \$13.21$$

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**The Determinants of Value** The binomial model provides insight into the determinants of option value. The value of an option is not determined by the *expected* price of the asset but by its *current* price, which, of course, reflects expectations about the future. This is a direct consequence of arbitrage. If the option value deviates from the value of the replicating portfolio, investors can create an arbitrage position (i.e., one that requires no investment, involves no risk, and delivers positive returns). To illustrate, if the portfolio that replicates the call costs more than the call does in the market, an investor could buy the call, sell the replicating portfolio, and be guaranteed the difference as a profit. The cash flows on the two positions will offset each other, leading to no cash flows in subsequent periods. The call option value also increases as the time to expiration is extended, as the price movements (u and d) increase, and with increases in the interest rate.

While the binomial model provides an intuitive feel for the determinants of option value, it requires a large number of inputs, in terms of expected future prices at each node. As time periods are made shorter in the binomial model, it becomes possible to make one of two assumptions about asset prices. It can be assumed that price changes become smaller as periods get shorter; this leads to price changes becoming infinitesimally small as time periods approach zero, leading to a continuous price process. Alternatively, it can be assumed that price changes stay large even as the period gets shorter; this leads to a jump price process, where prices can jump in any period. This section will consider the option pricing models that emerge with each of these assumptions.

### Black-Scholes Model

When the price process is continuous (i.e., price changes become smaller as time periods get shorter), the binomial model for pricing options converges on the Black-Scholes model. The model, named after its cocreators, Fischer Black and Myron Scholes, allows us to estimate the value of any option using a small number of inputs, and has been shown to be remarkably robust in valuing many listed options.

**The Model** While the derivation of the Black-Scholes model is far too complicated to present here, it is based on the idea of creating a portfolio of the underlying asset and the riskless asset with the same cash flows, and hence the same cost, as the option being valued. The value of a call option in the Black-Scholes model can be written as a function of the five variables:

$S$ = Current value of the underlying asset

$K$ = Strike price of the option

$t$ = Life to expiration of the option

$r$ = Riskless interest rate corresponding to the life of the option

$\sigma^2$ = Variance in the ln(value) of the underlying asset

The value of a call is then:

$$\text{Value of call} = S\,N(d_1) - K\,e^{-rt}\,N(d_2)$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Note that $e^{-rt}$ is the present value factor, and reflects the fact that the exercise price on the call option does not have to be paid until expiration. $N(d_1)$ and $N(d_2)$ are probabilities, estimated by using a cumulative standardized normal distribution, and the values of $d_1$ and $d_2$ obtained for an option. The cumulative distribution is shown in Figure 5.4.

In approximate terms, these probabilities yield the likelihood that an option will generate positive cash flows for its owner at exercise (i.e., that $S > K$ in the case of a call option and that $K > S$ in the case of a put option). The portfolio that replicates the call option is created by buying $N(d_1)$ units of the underlying asset, and borrowing $Ke^{-rt}\,N(d_2)$. The portfolio will have the same cash flows as the call option, and thus the same value as the option. $N(d_1)$, which is the number of units of the underlying asset that are needed to create the replicating portfolio, is called the option delta.

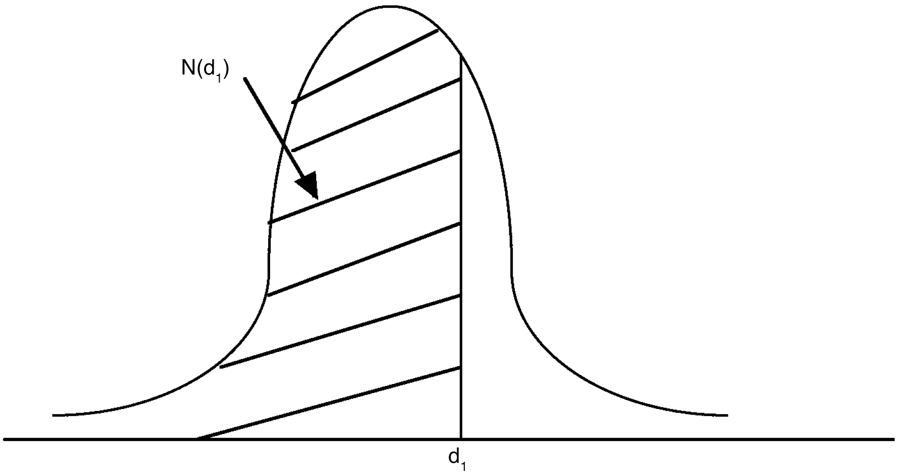


**FIGURE 5.4** Cumulative Normal Distribution

### A NOTE ON ESTIMATING THE INPUTS TO THE BLACK-SCHOLES MODEL

The Black-Scholes model requires inputs that are consistent on time measurement. There are two places where this affects estimates. The first relates to the fact that the model works in continuous time, rather than discrete time. That is why we use the continuous time version of present value ($\exp^{-rt}$) rather than the discrete version, $(1 + r)^{-t}$. It also means that the inputs such as the riskless rate have to be modified to make them continuous time inputs. For instance, if the one-year Treasury bond rate is 6.2 percent, the risk-free rate that is used in the Black-Scholes model should be:

$$\text{Continuous riskless rate} = \ln(1 + \text{Discrete riskless rate})$$
$$= \ln(1.062) = .06015 \text{ or } 6.015\%$$

The second relates to the period over which the inputs are estimated. For instance, the preceding rate is an annual rate. The variance that is entered into the model also has to be an annualized variance. The variance, estimated from ln(asset prices), can be annualized easily because variances are linear in time if the serial correlation is zero. Thus, if monthly or weekly prices are used to estimate variance, the variance is annualized by multiplying by 12 or 52, respectively.

**ILLUSTRATION 5.2: Valuing an Option Using the Black-Scholes Model**

On March 6, 2001, Cisco Systems was trading at $13.62. We will attempt to value a July 2001 call option with a strike price of $15, trading on the CBOE on the same day for $2. The following are the other parameters of the options:

- The annualized standard deviation in Cisco Systems stock price over the previous year was 81%. This standard deviation is estimated using weekly stock prices over the year, and the resulting number was annualized as follows:

$$\text{Weekly standard deviation} = 11.23\%$$

$$\text{Annualized standard deviation} = 11.23\% \times \sqrt{52} = 81\%$$

- The option expiration date is Friday, July 20, 2001. There are 103 days to expiration, and the annualized Treasury bill rate corresponding to this option life is 4.63%.

The inputs for the Black-Scholes model are as follows:

Current stock price (S) = $13.62

Strike price on the option = $15

Option life = 103/365 = 0.2822

Standard deviation in ln(stock prices) = 81%

Riskless rate = 4.63%

Inputting these numbers into the model, we get:

$$d_1 = \frac{\ln\left(\frac{13.62}{15.00}\right) + \left(.0463 + \frac{.81^2}{2}\right).2822}{.81\sqrt{.2822}} = .0212$$

$$d_2 = .0212 - .81\sqrt{.2822} = -.4091$$

Using the normal distribution, we can estimate the $N(d_1)$ and $N(d_2)$:

$$N(d_1) = .5085$$

$$N(d_2) = .3412$$

The value of the call can now be estimated:

$$\text{Value of Cisco call} = S\,N(d_1) - K\,e^{-rt}\,N(d_2)$$
$$= 13.62(.5085) - 15\,e^{-(.0463)(.2822)}(.3412) = \$1.87$$

Since the call is trading at $2, it is slightly overvalued, assuming that the estimate of standard deviation used is correct.

**IMPLIED VOLATILITY**

The only input on which there can be significant disagreement among investors is the variance. While the variance is often estimated by looking at historical data, the values for options that emerge from using the historical variance can be different from the market prices. For any option, there is some variance at which the estimated value will be equal to the market price. This variance is called an implied variance.

Consider the Cisco option valued in Illustration 5.2. With a standard deviation of 81 percent, the value of the call option with a strike price of $15 was estimated to be $1.87. Since the market price is higher than the calculated value, we tried higher standard deviations, and at a standard deviation 85.40 percent the value of the option is $2 (which is the market price). This is the implied standard deviation or implied volatility.

**Model Limitations and Fixes** The Black-Scholes model was designed to value options that can be exercised only at maturity and whose underlying assets do not pay dividends. In addition, options are valued based on the assumption that option exercise does not affect the value of the underlying asset. In practice, assets do pay dividends, options sometimes get exercised early, and exercising an option can affect the value of the underlying asset. Adjustments exist that, while not perfect, provide partial corrections to the Black-Scholes model.

**Dividends** The payment of a dividend reduces the stock price; note that on the ex-dividend day, the stock price generally declines. Consequently, call options become less valuable and put options more valuable as expected dividend payments increase. There are two ways of dealing with dividends in the Black-Scholes model:

1. *Short-term options.* One approach to dealing with dividends is to estimate the present value of expected dividends that will be paid by the underlying asset during the option life and subtract it from the current value of the asset to use as S in the model.

   Modified stock price = Current stock price
   – Present value of expected dividends
   during the life of the option

2. *Long-term options.* Since it becomes less practical to estimate the present value of dividends the longer the option life, an alternate approach can be used. If the dividend yield (y = Dividends/Current value of the asset) on the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S\, e^{-yt}\, N(d_1) - K\, e^{-rt}\, N(d_2)$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

From an intuitive standpoint, the adjustments have two effects. First, the value of the asset is discounted back to the present at the dividend yield to take into account the expected drop in asset value resulting from dividend payments. Second, the interest rate is offset by the dividend yield to reflect the lower carrying cost from holding the asset (in the replicating portfolio). The net effect will be a reduction in the value of calls estimated using this model.

**ILLUSTRATION 5.3: Valuing a Short-Term Option with Dividend Adjustments—The Black-Scholes Correction**

Assume that it is March 6, 2001, and that AT&T is trading at $20.50 a share. Consider a call option on the stock with a strike price of $20, expiring on July 20, 2001. Using past stock prices, the standard deviation in the log of stock prices for AT&T is estimated at 60%. There is one dividend, amounting to $0.15, and it will be paid in 23 days. The riskless rate is 4.63%.

Present value of expected dividend = $\$0.15/1.0463^{23/365} = \$0.15$

Dividend-adjusted stock price = $20.50 – $0.15 = $20.35

Time to expiration = 103/365 = 0.2822

Variance in ln(stock prices) = $0.6^2 = 0.36$

Riskless rate = 4.63%

The value from the Black-Scholes model is:

$d_1 = 0.2548$ $N(d_1) = 0.6006$

$d_2 = -0.0639$ $N(d_2) = 0.4745$

Value of call = $\$20.35\ (0.6006) - \$20\ \exp^{-(0.0463)(.2822)}(0.4745) = \$2.85$

The call option was trading at $2.60 on that day.

**ILLUSTRATION 5.4: Valuing a Long-Term Option with Dividend Adjustments—Primes and Scores**

In recent years, the CBOE has introduced longer-term call and put options on stocks. On March 6, 2001, for instance, you could have purchased an AT&T call expiring on January 17, 2003. The stock price for AT&T is $20.50 (as in the previous example). The following is the valuation of a call option with a strike price of $20. Instead of estimating the present value of dividends over the next two years, assume that AT&T's dividend yield will remain 2.51% over this period and that the risk-free rate for a two-year Treasury bond is 4.85%. The inputs to the Black-Scholes model are:

S = Current asset value = $20.50

K = Strike price = $20

Time to expiration = 1.8333 years

Standard deviation in ln(stock prices) = 60%

Riskless rate = 4.85%        Dividend yield = 2.51%

The value from the Black-Scholes model is:

$$d_1 = \frac{\ln\left(\frac{20.50}{20.00}\right) + \left(.0485 - .0251 + \frac{.6^2}{2}\right)1.8333}{.6\sqrt{1.8333}} = 0.4383 \quad N(d_1) = 0.6694$$

$$d_2 = .4383 - .6\sqrt{1.8333} = -.2387 \qquad N(d_2) = 0.4057$$

$$\text{Value of call } = \$20.50 \exp^{-(0.0251)(1.8333)}(0.6694) - \$20 \exp^{-(0.0485)(1.8333)}(0.4057) = \$6.63$$

The call was trading at $5.80 on March 8, 2001.

***stopt.xls:*** **This spreadsheet allows you to estimate the value of a short-term option when the expected dividends during the option life can be estimated.**

***ltops.xls:*** **This spreadsheet allows you to estimate the value of an option when the underlying asset has a constant dividend yield.**

**Early Exercise** The Black-Scholes model was designed to value options that can be exercised only at expiration. Options with this characteristic are called European options. In contrast, most options that we encounter in practice can be exercised at any time until expiration. These options are called American options. As mentioned earlier, the possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. In general, though, with traded options, it is almost always better to sell the option to someone else rather than exercise early, since options have a time premium (i.e., they sell for more than their exercise value). There are two exceptions. One occurs when the underlying asset pays large dividends, thus reducing the expected value of the asset. In this case, call options may be exercised *just be-*

*fore an ex-dividend date*, if the time premium on the options is less than the expected decline in asset value as a consequence of the dividend payment. The other exception arises when an investor holds both the underlying asset and *deep in-the-money puts* (i.e., puts with strike prices well above the current price of the underlying asset) on that asset at a time when interest rates are high. In this case, the time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

There are two basic ways of dealing with the possibility of early exercise. One is to continue to use the unadjusted Black-Scholes model and to regard the resulting value as a floor or conservative estimate of the true value. The other is to try to adjust the value of the option for the possibility of early exercise. There are two approaches for doing so. One uses the Black-Scholes model to value the option to each potential exercise date. With options on stocks, this basically requires that the investor values options to each ex-dividend day and chooses the maximum of the estimated call values. The second approach is to use a modified version of the binomial model to consider the possibility of early exercise. In this version, the up and the down movements for asset prices in each period can be estimated from the variance and the length of each period.[2]

### *Approach 1: Pseudo-American Valuation*

*Step 1:* Define when dividends will be paid and how much the dividends will be.

*Step 2:* Value the call option to each ex-dividend date using the dividend-adjusted approach described earlier, where the stock price is reduced by the present value of expected dividends.

*Step 3:* Choose the maximum of the call values estimated for each ex-dividend day.

#### ILLUSTRATION 5.5: Using Pseudo-American Option Valuation to Adjust for Early Exercise

Consider an option with a strike price of $35 on a stock trading at $40. The variance in the ln(stock prices) is 0.05, and the riskless rate is 4%. The option has a remaining life of eight months, and there are three dividends expected during this period:

| *Expected Dividend* | *Ex-Dividend Day* |
|---|---|
| $0.80 | In 1 month |
| $0.80 | In 4 months |
| $0.80 | In 7 months |

The call option is first valued to just before the first ex-dividend date:

[2]To illustrate, if $\sigma^2$ is the variance in ln(stock prices), the up and the down movements in the binomial can be estimated as follows:

$$u = \text{Exp}\left[(r - \sigma^2/2)(T/m) + \sqrt{(\sigma^2 T/m)}\right]$$

$$d = \text{Exp}\left[(r - \sigma^2/2)(T/m) - \sqrt{(\sigma^2 T/m)}\right]$$

where u and d are the up and down movements per unit time for the binomial, T is the life of the option, and m is the number of periods within that lifetime.

$S = \$40 \quad K = \$35 \quad t = 1/12 \quad \sigma^2 = 0.05 \quad r = 0.04$

The value from the Black-Scholes model is:

Value of call = $ 5.131

The call option is then valued to before the second ex-dividend date:

$$\text{Adjusted stock price} = \$40 - \$0.80/1.04^{1/12} = \$39.20$$

$K = \$35 \quad t = 4/12 \quad \sigma^2 = 0.05 \quad r = 0.04$

The value of the call based on these parameters is:

Value of call = $5.073

The call option is then valued to before the third ex-dividend date:

$$\text{Adjusted stock price} = \$40 - \$0.80/1.04^{1/12} - \$0.80/1.04^{4/12} = \$38.41$$

$K = \$35 \quad t = 7/12 \quad \sigma^2 = 0.05 \quad r = 0.04$

The value of the call based on these parameters is:

Value of call = $5.128

The call option is then valued to expiration:

$$\text{Adjusted stock price} = \$40 - \$0.80/1.04^{1/12} - \$0.80/1.04^{4/12} - \$0.80/1.04^{7/12} = \$37.63$$

$K = \$35 \quad t = 8/12 \quad \sigma^2 = 0.05 \quad r = 0.04$

The value of the call based on these parameters is:

Value of call = $4.757

Pseudo-American value of call = Maximum ($5.131, $5.073, $5.128, $4.757) = $5.131

***Approach 2: Using the Binomial Model*** The binomial model is much more capable of handling early exercise because it considers the cash flows at each time period, rather than just at expiration. The biggest limitation of the binomial model is determining what stock prices will be at the end of each period, but this can be overcome by using a variant that allows us to estimate the up and the down movements in stock prices from the estimated variance. There are four steps involved:

*Step 1:* If the variance in ln(stock prices) has been estimated for the Black-Scholes valuation, convert these into inputs for the binomial model:

$$u = e^{\sigma\sqrt{at} + \left(r - \frac{\sigma^2}{2}\right)dt}$$

$$d = e^{-\sigma\sqrt{at} + \left(r - \frac{\sigma^2}{2}\right)dt}$$

where u and d are the up and the down movements per unit time for the binomial, and dt is the number of periods within each year (or unit time).

*Step 2:* Specify the period in which the dividends will be paid and make the assumption that the price will drop by the amount of the dividend in that period.

*Step 3:* Value the call at each node of the tree, allowing for the possibility of early exercise just before ex-dividend dates. There will be early exercise if the remaining time premium on the option is less than the expected drop in option value as a consequence of the dividend payment.

*Step 4:* Value the call at time 0, using the standard binomial approach.

***bstobin.xls:* This spreadsheet allows you to estimate the parameters for a binomial model from the inputs to a Black-Scholes model.**

**Impact of Exercise on Underlying Asset Value** The Black-Scholes model is based on the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for some types of options. For instance, the exercise of warrants increases the number of shares outstanding and brings fresh cash into the firm, both of which will affect the stock price.[3] The expected negative impact (dilution) of exercise will decrease the value of warrants, compared to otherwise similar call options. The adjustment for dilution to the stock price is fairly simple in the Black-Scholes valuation. The stock price is adjusted for the expected dilution from the exercise of the options. In the case of warrants, for instance:

$$\text{Dilution-adjusted } S = (S\, n_s + W\, n_w)/(n_s + n_w)$$

where $S$ = Current value of the stock
$n_w$ = Number of warrants outstanding
$W$ = Value of warrants outstanding
$n_s$ = Number of shares outstanding

When the warrants are exercised, the number of shares outstanding will increase, reducing the stock price. The numerator reflects the market value of equity, including both stocks and warrants outstanding. The reduction in S will reduce the value of the call option.

There is an element of circularity in this analysis, since the value of the warrant is needed to estimate the dilution-adjusted S and the dilution-adjusted S is needed to estimate the value of the warrant. This problem can be resolved by starting the process off with an assumed value for the warrant (e.g., the exercise value or the current market price of the warrant). This will yield a value for the warrant, and this estimated value can then be used as an input to reestimate the warrant's value until there is convergence.

[3]Warrants are call options issued by firms, either as part of management compensation contracts or to raise equity.

### FROM BLACK-SCHOLES TO BINOMIAL

The process of converting the continuous variance in a Black-Scholes model to a binomial tree is a fairly simple one. Assume, for instance, that you have an asset that is trading at $30 currently and that you estimate the annualized standard deviation in the asset value to be 40 percent; the annualized riskless rate is 5 percent. For simplicity, let us assume that the option that you are valuing has a four-year life and that each period is a year. To estimate the prices at the end of each of the four years, we begin by first estimating the up and down movements in the binomial:

$$u = \exp^{.4\sqrt{1}+\left(.05-\frac{.4^2}{2}\right)1} = 1.4477$$

$$d = \exp^{-.4\sqrt{1}+\left(.05-\frac{.40^2}{2}\right)1} = 0.6505$$

Based on these estimates, we can obtain the prices at the end of the first node of the tree (the end of the first year):

Up price = $30(1.4477) = $43.43

Down price = $40(0.6505) = $19.52

Progressing through the rest of the tree, we obtain the following numbers:

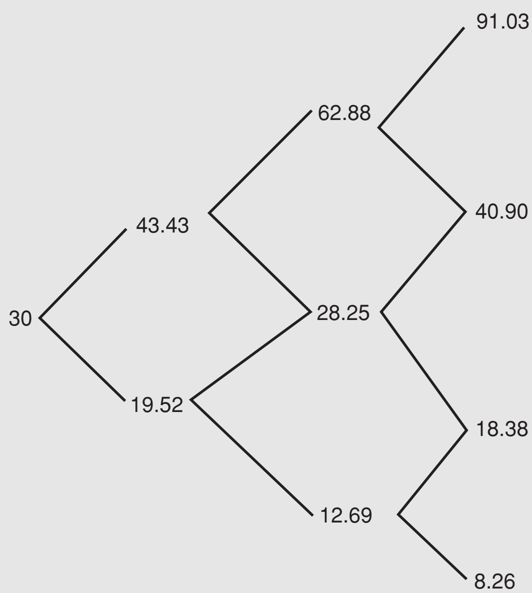

**ILLUSTRATION 5.6: Valuing a Warrant on Avatek Corporation**

Avatek Corporation is a real estate firm with 19.637 million shares outstanding, trading at \$0.38 a share. In March 2001 the company had 1.8 million options outstanding, with four years to expiration and with an exercise price of \$2.25. The stock paid no dividends, and the standard deviation in ln(stock prices) was 93%. The four-year Treasury bond rate was 4.9%. (The warrants were trading at \$0.12 apiece at the time of this analysis.)

The inputs to the warrant valuation model are as follows:

$S = (0.38 \times 19.637 + 0.12 \times 1.8)/(19.637 + 1.8) = 0.3544$

K = Exercise price on warrant = 2.25

t = Time to expiration on warrant = 4 years

r = Riskless rate corresponding to life of option = 4.9%

$\sigma^2$ = Variance in value of stock = $0.93^2$

y = Dividend yield on stock = 0.0%

The results of the Black-Scholes valuation of this option are:

$$d_1 = 0.0418 \qquad N(d_1) = 0.5167$$

$$d_2 = -1.8182 \qquad N(d_2) = 0.0345$$

$$\text{Value of warrant} = 0.3544(0.5167) - 2.25\ \exp^{-(0.049)(4)}(0.0345) = \$0.12$$

The warrants were trading at \$0.12 in March 2001. Since the value was equal to the price, there was no need for further iterations. If there had been a difference, we would have reestimated the adjusted stock price and warrant value.

***warrant.xls:* This spreadsheet allows you to estimate the value of an option when there is a potential dilution from exercise.**

**The Black-Scholes Model for Valuing Puts** The value of a put can be derived from the value of a call with the same strike price and the same expiration date:

$$C - P = S - K\ e^{-rt}$$

where C is the value of the call and P is the value of the put. This relationship between the call and put values is called put-call parity, and any deviations from parity can be used by investors to make riskless profits. To see why put-call parity holds, consider selling a call and buying a put with exercise price K and expiration date t, and simultaneously buying the underlying asset at the current price S. The payoff from this position is riskless and always yields K at expiration (t). To see this, assume that the stock price at expiration is S*. The payoff on each of the positions in the portfolio can be written as follows:

| *Position* | *Payoffs at t if S*>K* | *Payoffs at t if S*<K* |
|---|---|---|
| Sell call | $-(S^* - K)$ | 0 |
| Buy put | 0 | $K - S^*$ |
| Buy stock | $S^*$ | $S^*$ |
| Total | K | K |

Since this position yields K with certainty, the cost of creating this position must be equal to the present value of K at the riskless rate ($K\,e^{-rt}$).

$$S + P - C = K\,e^{-rt}$$

$$C - P = S - K\,e^{-rt}$$

Substituting the Black-Scholes equation for the value of an equivalent call into this equation, we get:

$$\text{Value of put} = K\,e^{-rt}\,[1 - N(d_2)] - S\,e^{-yt}\,[1 - N(d_1)]$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Thus, the replicating portfolio for a put is created by selling short $[1 - N(d_1)]$ shares of stock and investing $K\,e^{-rt}[1 - N(d_2)]$ in the riskless asset.

### ILLUSTRATION 5.7: Valuing a Put Using Put-Call Parity: Cisco Systems and AT&T

Consider the call that valued on Cisco Systems in Illustration 5.2. The call had a strike price of $15 on the stock, had 103 days left to expiration, and was valued at $1.87. The stock was trading at $13.62, and the riskless rate was 4.63%. The put can be valued as follows:

$$\text{Put value} = C - S + K\,e^{-rt} = \$1.87 - \$13.62 + \$15\,e^{-(.0463)(.2822)} = \$3.06$$

The put was trading at $3.38.

Also, a long-term call on AT&T was valued in Illustration 5.4. The call had a strike price of $20, 1.8333 years left to expiration, and a value of $6.63. The stock was trading at $20.50 and was expected to maintain a dividend yield of 2.51% over the period. The riskless rate was 4.85%. The put value can be estimated as follows:

$$\text{Put value} = C - S\,e^{-yt} + K\,e^{-rt} = \$6.63 - \$20.5\,e^{-(.0251)(1.8333)} + \$20\,e^{-(.0485)(1.8333)} = \$5.35$$

The put was trading at $3.80. Both the call and put trade at different prices from our estimates, which may indicate that we have not correctly estimated the stock's volatility.

### Jump Process Option Pricing Models

If price changes remain larger as the time periods in the binomial model are shortened, it can no longer be assumed that prices change continuously. When price changes remain large, a price process that allows for price jumps is much more realistic. Cox and Ross (1976) valued options when prices follow a pure jump process, where the jumps can only be positive. Thus, in the next interval, the stock price will either have a large positive jump with a specified probability or drift downward at a given rate.

Merton (1976) considered a distribution where there are price jumps superimposed on a continuous price process. He specified the rate at which jumps occur ($\lambda$) and the average jump size (k), measured as a percentage of the stock price. The model derived to value options with this process is called a jump diffusion model. In this model, the value of an option is determined by the five variables specified in the Black-Scholes model, and the parameters of the jump process ($\lambda$, k). Unfortunately, the estimates of the jump process parameters are so noisy for most firms that they overwhelm any advantages that accrue from using a more realistic model. These models, therefore, have seen limited use in practice.

## EXTENSIONS OF OPTION PRICING

All the option pricing models described so far—the binomial, the Black-Scholes, and the jump process models—are designed to value options with clearly defined exercise prices and maturities on underlying assets that are traded. However, the options we encounter in investment analysis or valuation are often on real assets rather than financial assets. Categorized as real options, they can take much more complicated forms. This section will consider some of these variations.

### Capped and Barrier Options

With a simple call option, there is no specified upper limit on the profits that can be made by the buyer of the call. Asset prices, at least in theory, can keep going up, and the payoffs increase proportionately. In some call options, though, the buyer is entitled to profits up to a specified price but not above it. For instance, consider a call option with a strike price of $K_1$ on an asset. In an unrestricted call option, the payoff on this option will increase as the underlying asset's price increases above

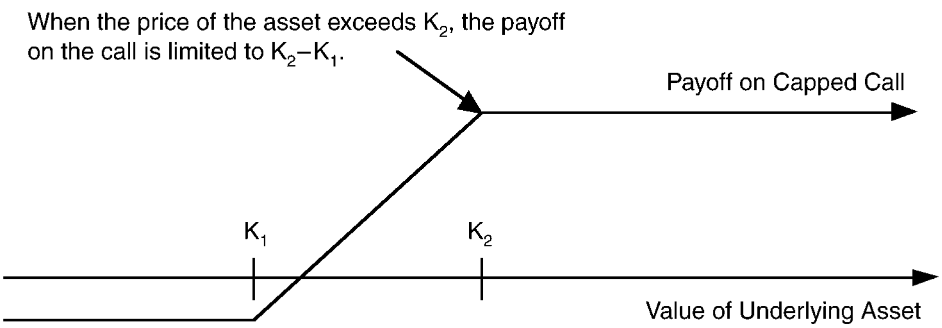


**FIGURE 5.5** Payoff on Capped Call

$K_1$. Assume, however, that if the price reaches $K_2$, the payoff is capped at $(K_2 - K_1)$. The payoff diagram on this option is shown in Figure 5.5.

This option is called a capped call. Notice, also, that once the price reaches $K_2$, there is no longer any time premium associated with the option, and the option will therefore be exercised. Capped calls are part of a family of options called barrier options, where the payoff on and the life of the option are a function of whether the underlying asset price reaches a certain level during a specified period.

The value of a capped call is always lower than the value of the same call without the payoff limit. A simple approximation of this value can be obtained by valuing the call twice, once with the given exercise price and once with the cap, and taking the difference in the two values. In the preceding example, then, the value of the call with an exercise price of $K_1$ and a cap at $K_2$ can be written as:

$$\text{Value of capped call} = \text{Value of call } (K = K_1) - \text{Value of call } (K = K_2)$$

Barrier options can take many forms. In a knockout option, an option ceases to exist if the underlying asset reaches a certain price. In the case of a call option, this knockout price is usually set below the strike price, and this option is called a down-and-out option. In the case of a put option, the knockout price will be set above the exercise price, and this option is called an up-and-out option. Like the capped call, these options are worth less than their unrestricted counterparts. Many real options have limits on potential upside, or knockout provisions, and ignoring these limits can result in the overstatement of the value of these options.

### Compound Options

Some options derive their value not from an underlying asset, but from other options. These options are called compound options. Compound options can take any of four forms—a call on a call, a put on a put, a call on a put, or a put on a call. Geske (1979) developed the analytical formulation for valuing compound options by replacing the standard normal distribution used in a simple option model with a bivariate normal distribution in the calculation.

Consider, for instance, the option to expand a project that will be discussed in Chapter 30. While we will value this option using a simple option pricing model, in reality there could be multiple stages in expansion, with each stage representing an option for the following stage. In this case, we will undervalue the option by considering it as a simple rather than a compound option.

Notwithstanding this discussion, the valuation of compound options becomes progressively more difficult as more options are added to the chain. In this case, rather than wreck the valuation on the shoals of estimation error, it may be better to accept the conservative estimate that is provided with a simple valuation model as a floor on the value.

### Rainbow Options

In a simple option, the uncertainty is about the price of the underlying asset. Some options are exposed to two or more sources of uncertainty, and these options are rainbow options. Using the simple option pricing model to value such options can lead to biased estimates of value. As an example, consider an undeveloped oil reserve as an option, where the firm that owns the reserve has the right to develop the reserve. Here there are two sources of uncertainty. The first is obviously the price of oil, and the second is the quantity of oil that is in the reserve. To value this undeveloped reserve, we can make the simplifying assumption that we know the quantity of oil in the reserve with certainty. In reality, however, uncertainty about the quantity will affect the value of this option and make the decision to exercise more difficult.[4]

## CONCLUSION

An option is an asset with payoffs that are contingent on the value of an underlying asset. A call option provides its holder with the right to buy the underlying asset at a fixed price, whereas a put option provides its holder with the right to sell at a fixed price, at any time before the expiration of the option. The value of an option is determined by six variables—the current value of the underlying asset, the variance in this value, the expected dividends on the asset, the strike price and life of the option, and the riskless interest rate. This is illustrated in both the binomial and the Black-Scholes models, which value options by creating replicating portfolios composed of the underlying asset and riskless lending or borrowing. These models can be used to value assets that have option like characteristics.

## QUESTIONS AND SHORT PROBLEMS

1. The following are prices of options traded on Microsoft Corporation, which pays no dividends.

| | *Call* | | *Put* | |
|---|---|---|---|---|
| | *K = 85* | *K = 90* | *K = 85* | *K = 90* |
| One-month | 2.75 | 1.00 | 4.50 | 7.50 |
| Three-month | 4.00 | 2.75 | 5.75 | 9.00 |
| Six-month | 7.75 | 6.00 | 8.00 | 12.00 |

   The stock is trading at $83, and the annualized riskless rate is 3.8%. The standard deviation in log stock prices (based on historical data) is 30%.
   a. Estimate the value of a three-month call with a strike price of $85.
   b. Using the inputs from the Black-Scholes model, specify how you would replicate this call.
   c. What is the implied standard deviation in this call?
   d. Assume now that you buy a call with a strike price of $85 and sell a call with a strike price of $90. Draw the payoff diagram on this position.
   e. Using put-call parity, estimate the value of a three-month put with a strike price of $85.
2. You are trying to value three-month call and put options on Merck with a strike price of $30. The stock is trading at $28.75, and the company expects to pay a

[4]The analogy to a listed option on a stock is the case where you do not know with certainty what the stock price is when you exercise the option. The more uncertain you are about the stock price, the more margin for error you have to give yourself when you exercise the option, to ensure that you are in fact earning a profit.

quarterly dividend per share of \$0.28 in two months. The annualized riskless interest rate is 3.6%, and the standard deviation in log stock prices is 20%.

a. Estimate the value of the call and put options, using the Black-Scholes model.
b. What effect does the expected dividend payment have on call values? On put values? Why?

3. There is the possibility that the options on Merck described in the preceding problem could be exercised early.
   a. Use the pseudo-American call option technique to determine whether this will affect the value of the call.
   b. Why does the possibility of early exercise exist? What types of options are most likely to be exercised early?
4. You have been provided the following information on a three-month call:

$$S = 95 \qquad K = 90 \qquad t = 0.25 \qquad r = 0.04$$
$$N(d_1) = 0.5750 \qquad N(d_2) = 0.4500$$

   a. If you wanted to replicate buying this call, how much money would you need to borrow?
   b. If you wanted to replicate buying this call, how many shares of stock would you need to buy?
5. Go Video, a manufacturer of video recorders, was trading at \$4 per share in May 1994. There were 11 million shares outstanding. At the same time, it had 550,000 one-year warrants outstanding, with a strike price of \$4.25. The stock has had a standard deviation of 60%. The stock does not pay a dividend. The riskless rate is 5%.
   a. Estimate the value of the warrants, ignoring dilution.
   b. Estimate the value of the warrants, allowing for dilution.
   c. Why does dilution reduce the value of the warrants?
6. You are trying to value a long-term call option on the NYSE Composite index, expiring in five years, with a strike price of 275. The index is currently at 250, and the annualized standard deviation in stock prices is 15%. The average dividend yield on the index is 3% and is expected to remain unchanged over the next five years. The five-year Treasury bond rate is 5%.
   a. Estimate the value of the long-term call option.
   b. Estimate the value of a put option with the same parameters.
   c. What are the implicit assumptions you are making when you use the Black-Scholes model to value this option? Which of these assumptions are likely to be violated? What are the consequences for your valuation?
7. A new security on AT&T will entitle the investor to all dividends on AT&T over the next three years, limiting upside potential to 20% but also providing downside protection below 10%. AT&T stock is trading at \$50, and three-year call and put options are traded on the exchange at the following prices:

| | *Call Options* | | *Put Options* | |
|---|---|---|---|---|
| *K* | *1-Year* | *3-Year* | *1-Year* | *3-Year* |
| 45 | \$8.69 | \$13.34 | \$1.99 | \$3.55 |
| 50 | \$5.86 | \$10.89 | \$3.92 | \$5.40 |
| 55 | \$3.78 | \$8.82 | \$6.59 | \$7.63 |
| 60 | \$2.35 | \$7.11 | \$9.92 | \$10.23 |

How much would you be willing to pay for this security?