Cash Flows and Discount Rates

- Assume that you are analyzing a company with the following cashflows for the next five years.

Year	CF to Equity	Interest Expense (1-t)	CF to Firm
1	$ 50	$ 40	$ 90
2	$ 60	$ 40	$ 100
3	$ 68	$ 40	$ 108
4	$ 76.2	$ 40	$ 116.2
5	$ 83.49	$ 40	$ 123.49
Terminal Value	$ 1603.0		$ 2363.008

- Assume also that the cost of equity is 13.625% and the firm can borrow long term at 10%. (The tax rate for the firm is 50%.)
- The current market value of equity is $1,073 and the value of debt outstanding is $800.

Equity versus Firm Valuation

- Method 1: Discount CF to Equity at Cost of Equity to get value of equity
  - Cost of Equity = 13.625%
  - Value of Equity = $50/1.13625 + 60/1.13625^2 + 68/1.13625^3 + 76.2/1.13625^4 + (83.49+1603)/1.13625^5$ = **$1073**
- Method 2: Discount CF to Firm at Cost of Capital to get value of firm
  - Cost of Debt = Pre-tax rate (1- tax rate) = 10% (1-.5) = 5%

Cost of Capital = 13.625% (1073/1873) + 5% (800/1873) = 9.94%

  - PV of Firm = $90/1.0994 + 100/1.0994^2 + 108/1.0994^3 + 116.2/1.0994^4 + (123.49+2363)/1.0994^5$ = $1873
  - Value of Equity = Value of Firm - Market Value of Debt

= $ 1873 - $ 800 = **$1073**

First Principle of Valuation

- Discounting Consistency Principle: Never mix and match cash flows and discount rates.
- Mismatching cash flows to discount rates is deadly.
  - Discounting cashflows after debt cash flows (equity cash flows) at the weighted average cost of capital will lead to an upwardly biased estimate of the value of equity
  - Discounting pre-debt cashflows (cash flows to the firm) at the cost of equity will yield a downward biased estimate of the value of the firm.

The Effects of Mismatching Cash Flows and Discount Rates

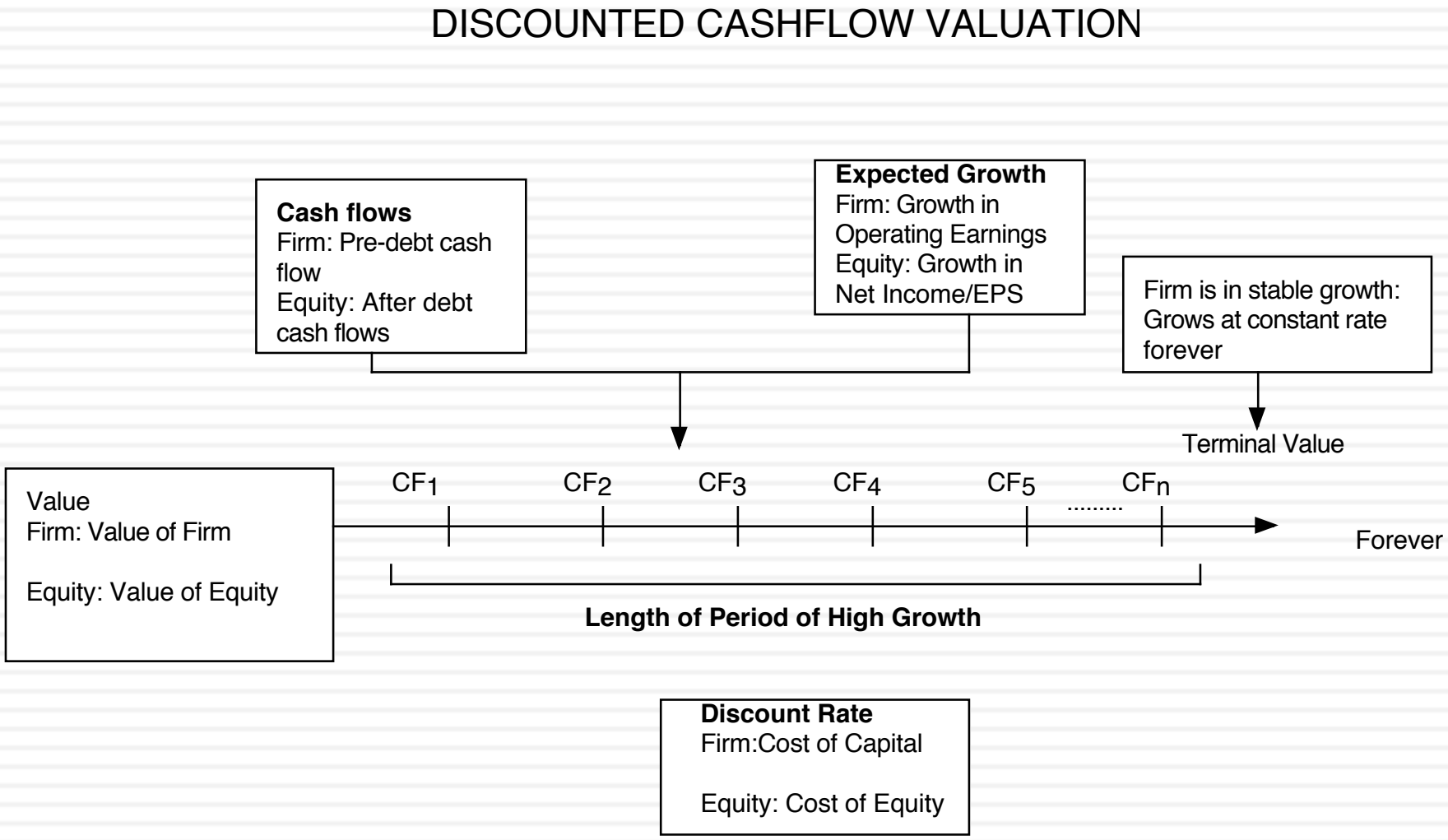
- Error 1: Discount CF to Equity at Cost of Capital to get equity value
  - PV of Equity = $50/1.0994 + 60/1.0994^2 + 68/1.09943 + 76.2/1.0994^4 + (83.49+1603)/1.0994^5 = \$1248$
  - Value of equity is overstated by \$175.
- Error 2: Discount CF to Firm at Cost of Equity to get firm value
  - PV of Firm = $90/1.13625 + 100/1.13625^2 + 108/1.13625^3 + 116.2/1.13625^4 + (123.49+2363)/1.13625^5 = \$1613$
  - PV of Equity = \$1612.86 - \$800 = \$813
  - Value of Equity is understated by \$ 260.
- Error 3: Discount CF to Firm at Cost of Equity, forget to subtract out debt, and get too high a value for equity
  - Value of Equity = \$ 1613
  - Value of Equity is overstated by \$ 540

DCF: First Steps

Discounted Cash Flow Valuation: The Steps

1. Estimate the discount rate or rates to use in the valuation
    1. Discount rate can be either a cost of equity (if doing equity valuation) or a cost of capital (if valuing the firm)
    2. Discount rate can be in nominal terms or real terms, depending upon whether the cash flows are nominal or real
    3. Discount rate can vary across time.
2. Estimate the current earnings and cash flows on the asset, to either equity investors (CF to Equity) or to all claimholders (CF to Firm)
3. Estimate the future earnings and cash flows on the firm being valued, generally by estimating an expected growth rate in earnings.
4. Estimate when the firm will reach "stable growth" and what characteristics (risk & cash flow) it will have when it does.
5. Choose the right DCF model for this asset and value it.

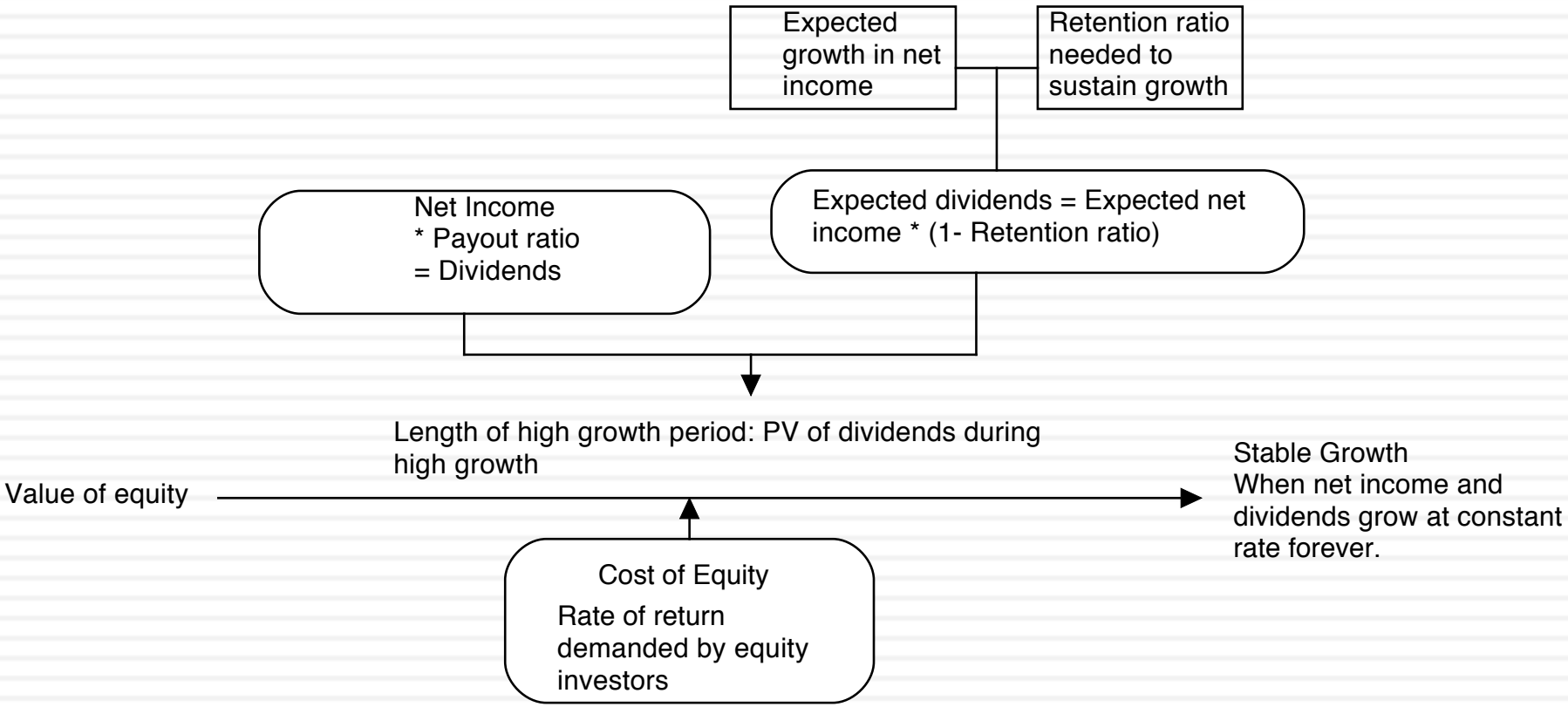
Generic DCF Valuation Model



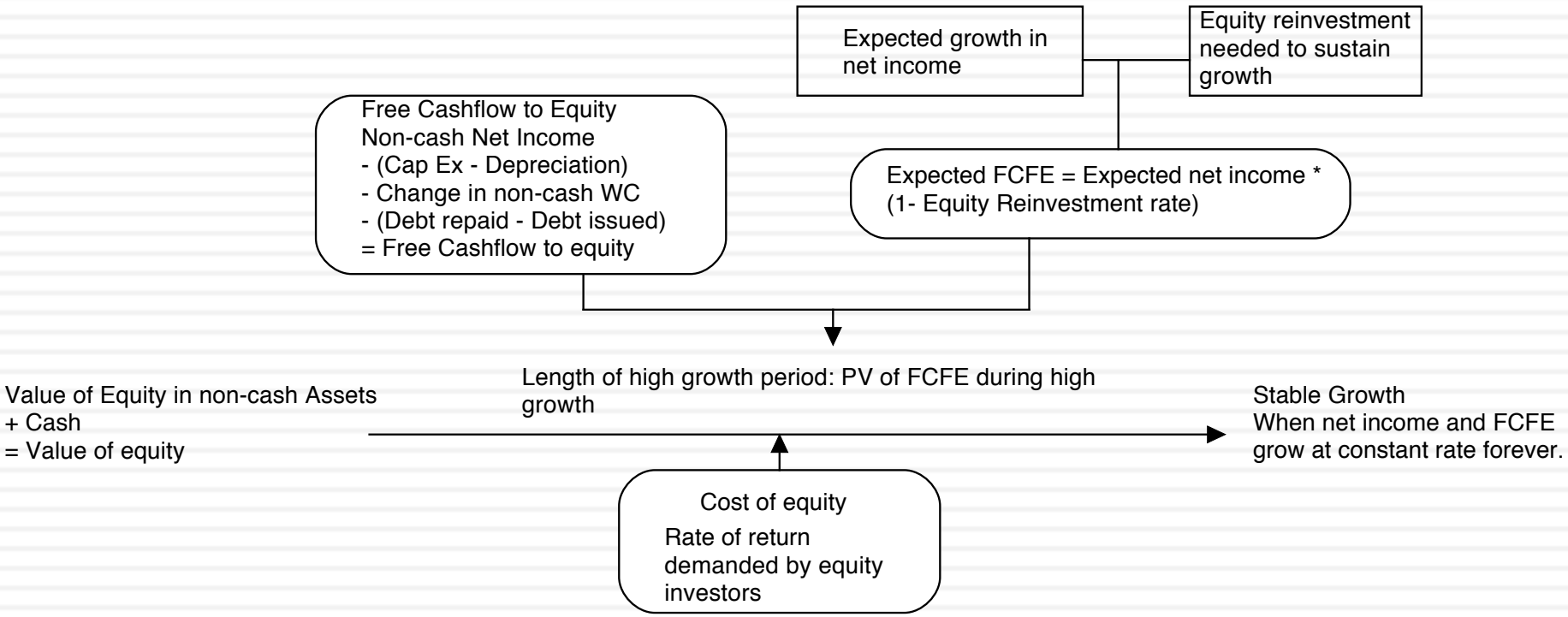
Same ingredients, different approaches…

Input	Dividend Discount Model	FCFE (Potential dividend) discount model	FCFF (firm) valuation model
Cash flow	Dividend	Potential dividends = FCFE = Cash flows after taxes, reinvestment needs and debt cash flows	FCFF = Cash flows before debt payments but after reinvestment needs and taxes.
Expected growth	In equity income and dividends	In equity income and FCFE	In operating income and FCFF
Discount rate	Cost of equity	Cost of equity	Cost of capital
Steady state	When dividends grow at constant rate forever	When FCFE grow at constant rate forever	When FCFF grow at constant rate forever

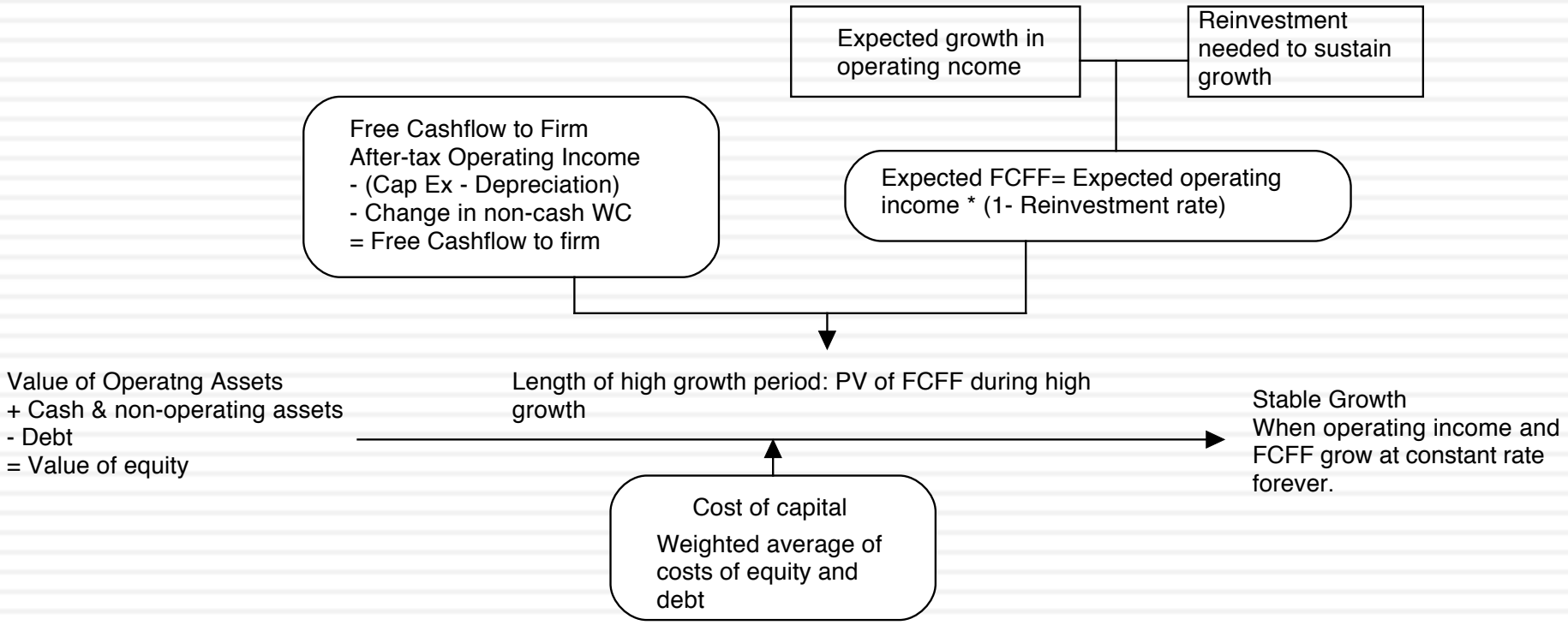
Start easy: The Dividend Discount Model



Moving on up: The "potential dividends" or FCFE model



To valuing the entire business: The FCFF model



DISCOUNT RATES

The D in the DCF..

Estimating Inputs: Discount Rates

- While discount rates obviously matter in DCF valuation, they don't matter as much as most analysts think they do.
- At an intuitive level, the discount rate used should be consistent with both the riskiness and the type of cashflow being discounted.
  - Equity versus Firm: If the cash flows being discounted are cash flows to equity, the appropriate discount rate is a cost of equity. If the cash flows are cash flows to the firm, the appropriate discount rate is the cost of capital.
  - Currency: The currency in which the cash flows are estimated should also be the currency in which the discount rate is estimated.
  - Nominal versus Real: If the cash flows being discounted are nominal cash flows (i.e., reflect expected inflation), the discount rate should be nominal

Risk in the DCF Model

Expectation of cash flows across all scenarios, good and bad. Incorporates all risks that affect the asset / business.

$$\frac{\text{Expected Cash Flows}}{\text{Risk Adjusted Discount Rate}}$$

Discount rate should reflect the risk perceived by the marginal investor in the company

Risk Adjusted Cost of equity	=	Risk free rate in the currency of analysis	+	Relative risk of company/equity in questiion	X	Equity Risk Premium required for average risk equity

Not all risk is created equal…

- Estimation versus Economic uncertainty
  - Estimation uncertainty reflects the possibility that you could have the "wrong model" or estimated inputs incorrectly within this model.
  - Economic uncertainty comes the fact that markets and economies can change over time and that even the best models will fail to capture these unexpected changes.
- Micro uncertainty versus Macro uncertainty
  - Micro uncertainty refers to uncertainty about the potential market for a firm's products, the competition it will face and the quality of its management team.
  - Macro uncertainty reflects the reality that your firm's fortunes can be affected by changes in the macro economic environment.
- Discrete versus continuous uncertainty
  - Discrete risk: Risks that lie dormant for periods but show up at points in time. (Examples: A drug working its way through the FDA pipeline may fail at some stage of the approval process or a company in Venezuela may be nationalized)
  - Continuous risk: Risks changes in interest rates or economic growth occur continuously and affect value as they happen.

Risk and Cost of Equity: The role of the marginal investor

- Not all risk counts: While the notion that the cost of equity should be higher for riskier investments and lower for safer investments is intuitive, what risk should be built into the cost of equity is the question.
- Risk through whose eyes? While risk is usually defined in terms of the variance of actual returns around an expected return, risk and return models in finance assume that the risk that should be rewarded (and thus built into the discount rate) in valuation should be the risk perceived by the marginal investor in the investment
- The diversification effect: Most risk and return models in finance also assume that the marginal investor is well diversified, and that the only risk that he or she perceives in an investment is risk that cannot be diversified away (i.e, market or non-diversifiable risk). In effect, it is primarily economic, macro, continuous risk that should be incorporated into the cost of equity.

The Cost of Equity: Competing " Market Risk" Models

Model	Expected Return	Inputs Needed
CAPM	$E(R) = Rf + \beta (R_m - R_f)$	Riskfree Rate Beta relative to market portfolio Market Risk Premium
APM	$E(R) = Rf + \Sigma\beta_j (R_j - R_f)$	Riskfree Rate; # of Factors; Betas relative to each factor Factor risk premiums
Multi factor	$E(R) = Rf + \Sigma\beta_j (R_j - R_f)$	Riskfree Rate; Macro factors Betas relative to macro factors Macro economic risk premiums
Proxy	$E(R) = a + \Sigma \beta_j Y_j$	Proxies Regression coefficients

Classic Risk & Return: Cost of Equity

- In the CAPM, the cost of equity:

  Cost of Equity = Riskfree Rate + Equity Beta * (Equity Risk Premium)

- In APM or Multi-factor models, you still need a risk free rate, as well as betas and risk premiums to go with each factor.
- To use any risk and return model, you need
  - A risk free rate as a base
  - A single equity risk premium (in the CAPM) or factor risk premiums, in the the multi-factor models
  - A beta (in the CAPM) or betas (in multi-factor models)

Discount Rates I

The Riskfree Rate

The Risk Free Rate: Laying the Foundations

- On a riskfree investment, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have
  - No default risk
  - No reinvestment risk
- It follows then that if asked to estimate a risk free rate:

1. Time horizon matters: Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time.
2. Currencies matter: A risk free rate is currency-specific and can be very different for different currencies.
3. Not all government securities are riskfree: Some governments face default risk and the rates on bonds issued by them will not be riskfree.

Test 1: A riskfree rate in US dollars!

- In valuation, we estimate cash flows forever (or at least for very long time periods). The right risk free rate to use in valuing a company in US dollars would be
  a. A three-month Treasury bill rate (0.09%)
  b. A ten-year Treasury bond rate (0.93%)
  c. A thirty-year Treasury bond rate (1.4%)
  d. A TIPs (inflation-indexed treasury) rate (-1.0%)
  e. None of the above

  What are we implicitly assuming about the US treasury when we use any of the treasury numbers?

Test 2: A Riskfree Rate in Euros?

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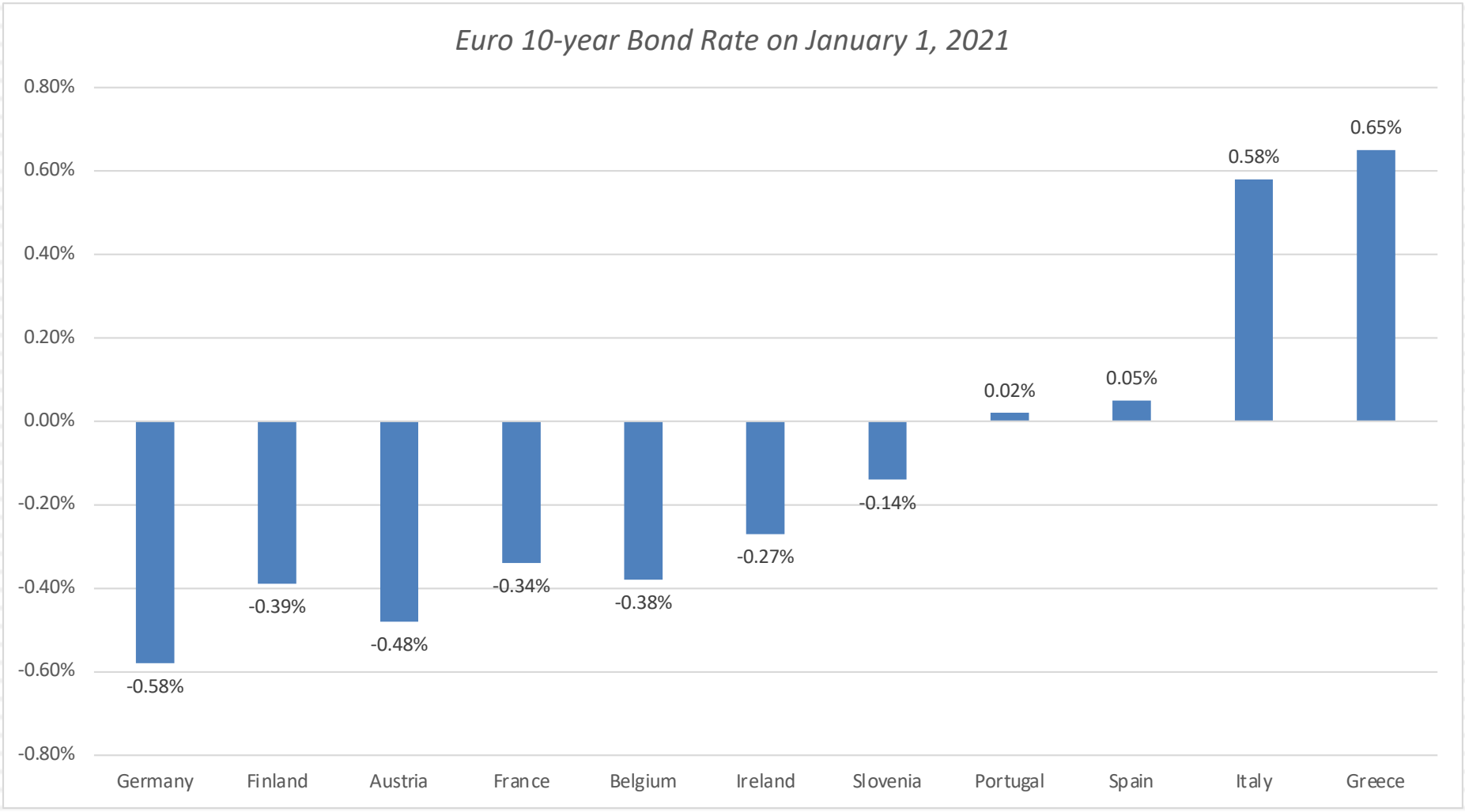


Test 3: A Riskfree Rate in Indian Rupees

- The Indian government had 10-year Rupee bonds outstanding, with a yield to maturity of about 5.92% on January 1, 2021.
- In January 2021, the Indian government had a local currency sovereign rating of Baa3. The typical default spread (over a default free rate) for Baa3 rated country bonds in early 2021 was 1.95%. The riskfree rate in Indian Rupees is
  a. The yield to maturity on the 10-year bond (5.92%)
  b. The yield to maturity on the 10-year bond + Default spread (7.87%)
  c. The yield to maturity on the 10-year bond – Default spread 3.97%)
  d. None of the above

Sovereign Default Spread: Three paths to the same destination…

- Sovereign dollar or euro denominated bonds: Find sovereign bonds denominated in US dollars, issued by an emerging sovereign.
  - Default spread = Emerging Govt Bond Rate (in US $) – US Treasury Bond rate with same maturity.
- CDS spreads: Obtain the traded value for a sovereign Credit Default Swap (CDS) for the emerging government.
  - Default spread = Sovereign CDS spread (with perhaps an adjustment for CDS market frictions).
- Sovereign-rating based spread: For countries which don't issue dollar denominated bonds or have a CDS spread, you have to use the average spread for other countries with the same sovereign rating.

Local Currency Government Bond Rates – January 2021

Currency	Govt Bond Rate 12/31/20	Currency	Govt Bond Rate 12/31/20	Currency	Govt Bond Rate 12/31/20
Australian $	1.05%	Indian Rupee	5.92%	Qatari Dinar	1.69%
Brazilian Reai	7.02%	Indonesian Rupiah	6.24%	Romanian Lev	3.50%
British Pound	0.82%	Israeli Shekel	0.86%	Russian Ruble	5.82%
Bulgarian Lev	0.40%	Japanese Yen	0.02%	Singapore $	0.92%
Canadian $	0.77%	Kenyan Shilling	11.90%	South African Rand	8.94%
Chilean Peso	2.79%	Korean Won	1.65%	Swedish Krona	0.01%
Chinese Yuan	3.35%	Malyasian Ringgit	2.78%	Swiss Franc	-0.53%
Colombian Peso	4.95%	Mexican Peso	5.53%	Taiwanese $	0.29%
Croatian Kuna	0.85%	Nigerian Naira	7.27%	Thai Baht	1.27%
Czech Koruna	1.29%	Norwegian Krone	0.89%	Turkish Lira	12.99%
Danish Krone	-0.47%	NZ $	0.98%	US $	0.93%
Euro	-0.58%	Pakistani Rupee	9.90%	Vietnamese Dong	2.55%
HK $	0.72%	Peruvian Sol	4.55%	Zambian kwacha	34.00%
Hungarian Forint	2.30%	Phillipine Peso	2.94%		
Iceland Krona	3.08%	Polish Zloty	1.37%		

Approach 1: Default spread from Government Bonds

Country	$ Bond Rate	Riskfree Rate	Default Spread
	$ Bonds		
Peru	3.66%	0.93%	2.73%
Brazil	2.98%	0.93%	2.05%
Colombia	1.93%	0.93%	1.00%
Poland	1.33%	0.93%	0.40%
Turkey	6.12%	0.93%	5.19%
Mexico	2.21%	0.93%	1.28%
Russia	2.43%	0.93%	1.50%
	Euro Bonds		
Bulgaria	1.00%	-0.58%	1.58%

Approach 2: CDS Spreads – January 2021

Country	1/1/21	CDS Spread net of US	Country	1/1/21	CDS Spread net of US	Country	1/1/21	CDS Spread net of US
Abu Dhabi	0.69%	0.46%	Greece	1.61%	1.38%	Greece	1.61%	1.38%
Algeria	1.10%	0.87%	Guatamela	2.15%	1.92%	Guatamela	2.15%	1.92%
Angola	7.50%	7.27%	Hong Kong	0.73%	0.50%	Hong Kong	0.73%	0.50%
Australia	0.23%	0.00%	Hungary	0.94%	0.71%	Hungary	0.94%	0.71%
Austria	0.18%	0.00%	Iceland	0.85%	0.62%	Iceland	0.85%	0.62%
Bahrain	3.18%	2.95%	India	1.24%	1.01%	India	1.24%	1.01%
Belgium	0.27%	0.04%	Indonesia	1.28%	1.05%	Indonesia	1.28%	1.05%
Brazil	2.15%	1.92%	Iraq	6.98%	6.75%	Iraq	6.98%	6.75%
Bulgaria	0.70%	0.47%	Ireland	0.32%	0.09%	Ireland	0.32%	0.09%
Cameroon	5.87%	5.64%	Israel	0.77%	0.54%	Israel	0.77%	0.54%
Canada	0.42%	0.19%	Italy	1.43%	1.20%	Italy	1.43%	1.20%
Chile	0.90%	0.67%	Japan	0.28%	0.05%	Japan	0.28%	0.05%
China	0.56%	0.33%	Kazakhstan	0.96%	0.73%	Kazakhstan	0.96%	0.73%
Colombia	1.52%	1.29%	Kenya	4.06%	3.83%	Kenya	4.06%	3.83%
Costa Rica	6.13%	5.90%	Korea	0.42%	0.19%	Korea	0.42%	0.19%
Croatia	1.28%	1.05%	Kuwait	0.75%	0.52%	Kuwait	0.75%	0.52%
Cyprus	1.19%	0.96%	Latvia	0.93%	0.70%	Latvia	0.93%	0.70%
Czech Republic	0.51%	0.28%	Lithuania	0.90%	0.67%	Lithuania	0.90%	0.67%
Denmark	0.16%	0.00%	Malaysia	0.70%	0.47%	Malaysia	0.70%	0.47%
Dubai	1.46%	1.23%	Mexico	1.45%	1.22%	Mexico	1.45%	1.22%
Ecuador	10.36%	10.13%	Morocco	1.56%	1.33%	Morocco	1.56%	1.33%
Egypt	4.08%	3.85%	Netherlands	0.20%	0.00%	Netherlands	0.20%	0.00%
El Salvador	7.78%	7.55%	New Zealand	0.25%	0.02%	New Zealand	0.25%	0.02%
Estonia	0.70%	0.47%	Nicaragua	4.52%	4.29%	Nicaragua	4.52%	4.29%
Finland	0.25%	0.02%	Nigeria	3.59%	3.36%	Nigeria	3.59%	3.36%
France	0.32%	0.09%	Norway	0.23%	0.00%	Norway	0.23%	0.00%
Germany	0.23%	0.00%	Oman	3.90%	3.67%	Oman	3.90%	3.67%

Approach 3: Typical Default Spreads: January 2021

S&P Sovereign Rating	Moody's Sovereign Rating	Default Spread
AAA	Aaa	0.00%
AA+	Aa1	0.35%
AA	Aa2	0.44%
AA-	Aa3	0.53%
A+	A1	0.62%
A	A2	0.75%
A-	A3	1.06%
BBB+	Baa1	1.41%
BBB	Baa2	1.68%
BBB-	Baa3	1.95%
BB+	Ba1	2.21%
BB	Ba2	2.65%
BB	Ba3	3.18%
B+	B1	3.98%
B	B2	4.86%
B-	B3	5.75%
CCC+	Caa1	6.63%
CCC	Caa2	7.96%
CCC-	Caa3	8.83%
CC+	Ca1	10.60%
CC	Ca2	13.76%
CC-	Ca3	15.00%
C+	C1	16.00%
C	C2	17.50%
C-	C3	20.00%

Getting to a risk free rate in a currency: Example

- The Brazilian government bond rate in nominal reais on January 1, 2021 was 7.02%. To get to a riskfree rate in nominal reais, we can use one of three approaches.
  - Approach 1: Government Bond spread
    - The 2030 Brazil bond, denominated in US dollars, has a spread of 2.05% over the US treasury bond rate.
    - Riskfree rate in $R = 7.02% - 2.05% = 4.97%
  - Approach 2: The CDS Spread
    - The CDS spread for Brazil, adjusted for the US CDS spread was 1.92%.
    - Riskfree rate in $R = 7.02% - 1.92% = 5.10%
  - Approach 3: The Rating based spread
    - Brazil has a Ba2 local currency rating from Moody's. The default spread for that rating is 2.65%
    - Riskfree rate in $R = 7.02% - 2.65% = 4.47%

Test 4: A Real Riskfree Rate

- In some cases, you may want a riskfree rate in real terms (in real terms) rather than nominal terms.
- To get a real riskfree rate, you would like a security with no default risk and a guaranteed real return. Treasury indexed securities offer this combination.
- In January 2020, the yield on a 10-year indexed treasury bond was 0.40%. Which of the following statements would you subscribe to?
  a. This (0.40%) is the real riskfree rate to use, if you are valuing US companies in real terms.
  b. This (0.40%) is the real riskfree rate to use, anywhere in the world

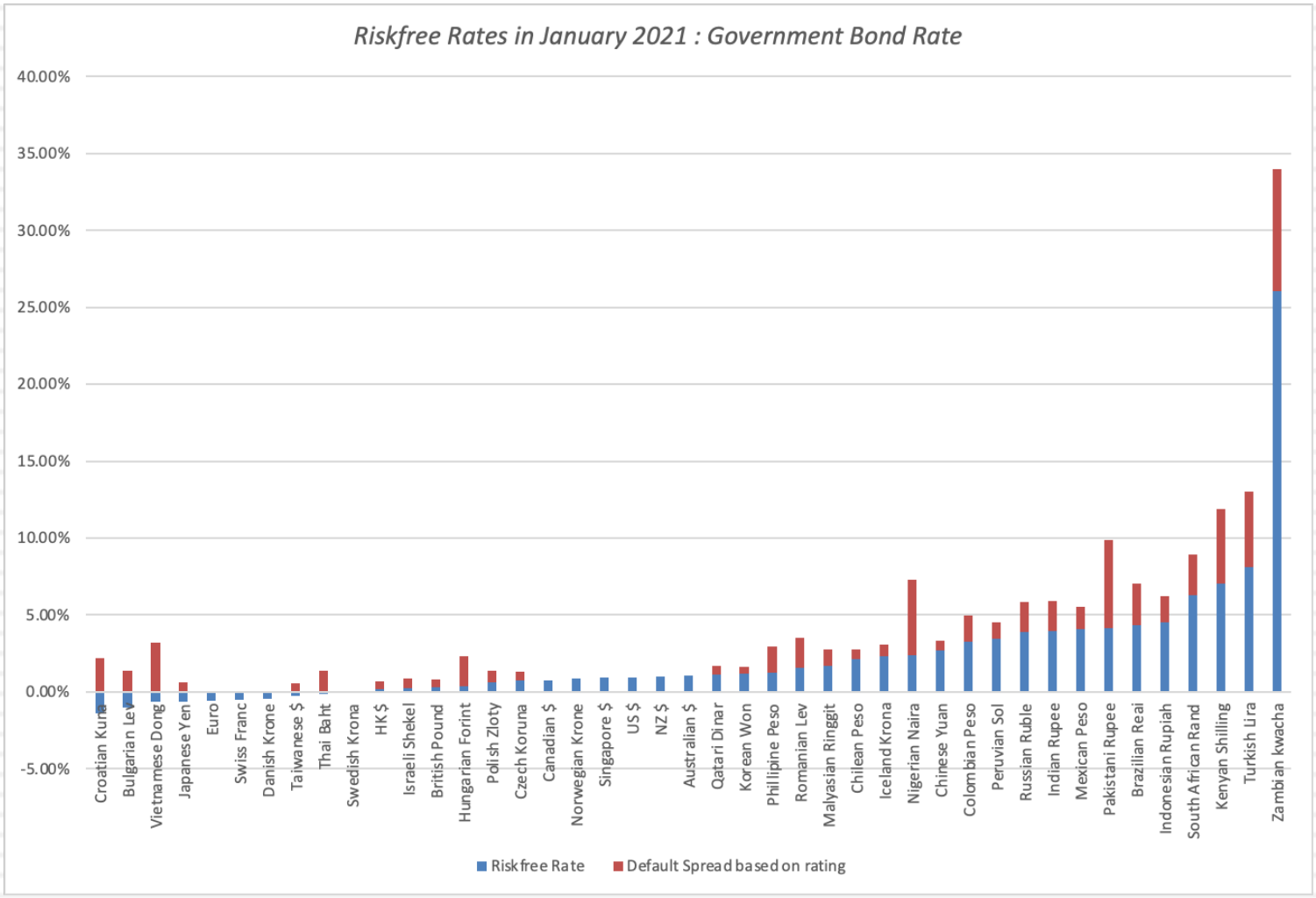
  Explain.

No default free entity: Choices with riskfree rates….

- Estimate a range for the riskfree rate in local terms:
  - Approach 1: Subtract default spread from local government bond rate:

    Government bond rate in local currency terms - Default spread for Government in local currency
  - Approach 2: Use forward rates and the riskless rate in an index currency (say Euros or dollars) to estimate the riskless rate in the local currency.
- Do the analysis in real terms (rather than nominal terms) using a real riskfree rate, which can be obtained in one of two ways –
  - from an inflation-indexed government bond, if one exists
  - set equal, approximately, to the long term real growth rate of the economy in which the valuation is being done.
- Do the analysis in a currency where you can get a riskfree rate, say US dollars or Euros.

Why do risk free rates vary across currencies? January 2021 Risk free rates



Risk free Rate: Don't have or trust the government bond rate?

1. Build up approach: The risk free rate in any currency can be written as the sum of two variables:

Risk free rate = Expected Inflation in currency + Expected real interest rate

Thus, if the expected inflation rate in a country is expected to be 15% and the TIPs rate is 1%, the risk free rate is 16%.

2. US $ rate & Differential Inflation: Alternatively, you can scale up the US $ risk free rate by the differential inflation between the US $ and the currency in question:

Risk free rate$_{Currency}$= $(1 + Riskfree\ rate_{US\ \$}) \frac{(1 + Expected\ Inflation_{Foreign\ Currency})}{(1 + Expected\ Inflation_{US\ \$})} - 1$

Thus, if the US $ risk free rate is 2.00%, the inflation rate in the foreign currency is 15% and the inflation rate in US $ is 1.5%, the foreign currency risk free rate is as follows:

Risk free rate = $(1.02)\frac{(1.15)}{(1.015)} - 1 = 15.57\%$

One more test on riskfree rates…

- On January 1, 2021, the 10-year treasury bond rate in the United States was 0.93%, low by historic standards. Assume that you were valuing a company in US dollars then, but were wary about the risk free rate being too low. Which of the following should you do?
  a. Replace the current 10-year bond rate with a more reasonable normalized riskfree rate (the average 10-year bond rate over the last 30 years has been about 5-6%)
  b. Use the current 10-year bond rate as your riskfree rate but make sure that your other assumptions (about growth and inflation) are consistent with the riskfree rate.
  c. Something else…