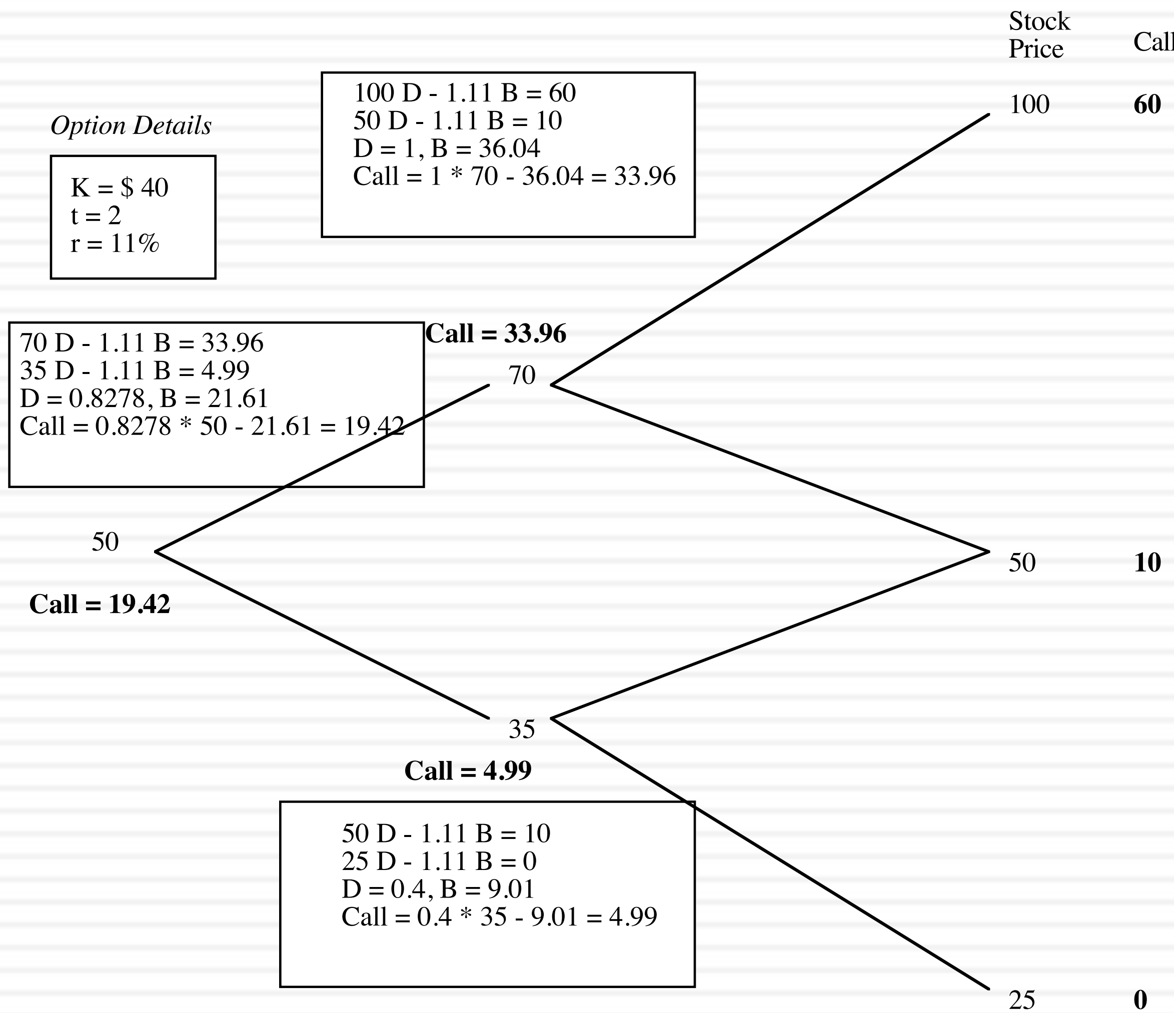
The Binomial Option Pricing Model



The Limiting Distributions....

- As the time interval is shortened, the limiting distribution, as t -> 0, can take one of two forms.
  - If as t -> 0, price changes become smaller, the limiting distribution is the normal distribution and the price process is a continuous one.
  - If as t->0, price changes remain large, the limiting distribution is the poisson distribution, i.e., a distribution that allows for price jumps.
- The Black-Scholes model applies when the limiting distribution is the normal distribution , and explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

Black and Scholes…

- The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected.
- The value of a call option in the Black-Scholes model can be written as a function of the following variables:
  - S = Current value of the underlying asset
  - K = Strike price of the option
  - t = Life to expiration of the option
  - r = Riskless interest rate corresponding to the life of the option
  - σ^2 = Variance in the ln(value) of the underlying asset

The Black Scholes Model

Value of call = S N (d1) - K e^{-rt} N(d2)

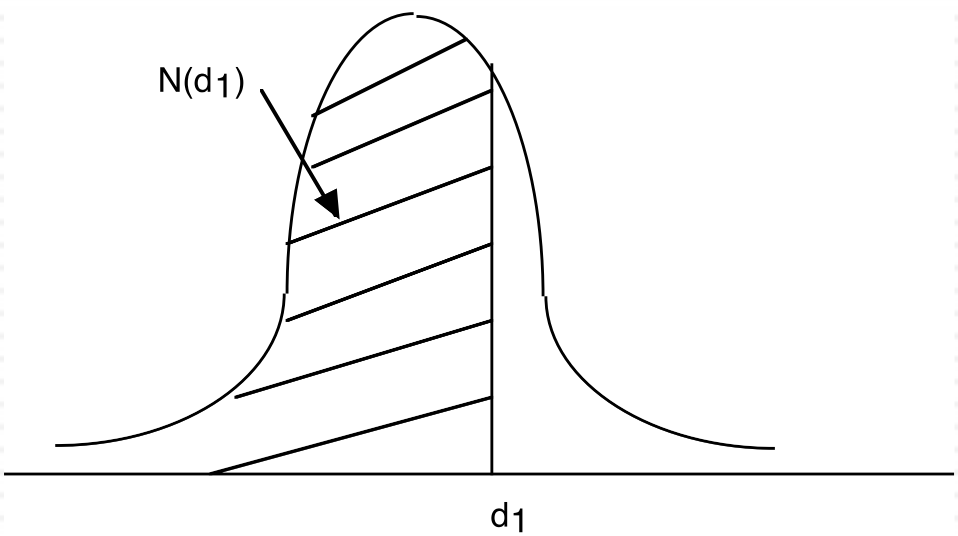
where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

d2 = d1 - ? √t

- The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to
  - Buy N(d1) shares of stock; N(d1) is called the option delta
  - Borrow K e^{-rt} N(d2)

The Normal Distribution



d	N(d)	d	N(d)	d	N(d)
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

Adjusting for Dividends

- If the dividend yield (y = dividends/ Current value of the asset) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.
- $C = S e^{-yt} N(d1) - K e^{-rt} N(d2)$

where,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r - y + \frac{\sigma^2}{2})\, t}{\sigma \sqrt{t}}$$

d2 = d1 - ? √t

- The value of a put can also be derived:
- $P = K e^{-rt} (1-N(d2)) - S e^{-yt} (1-N(d1))$

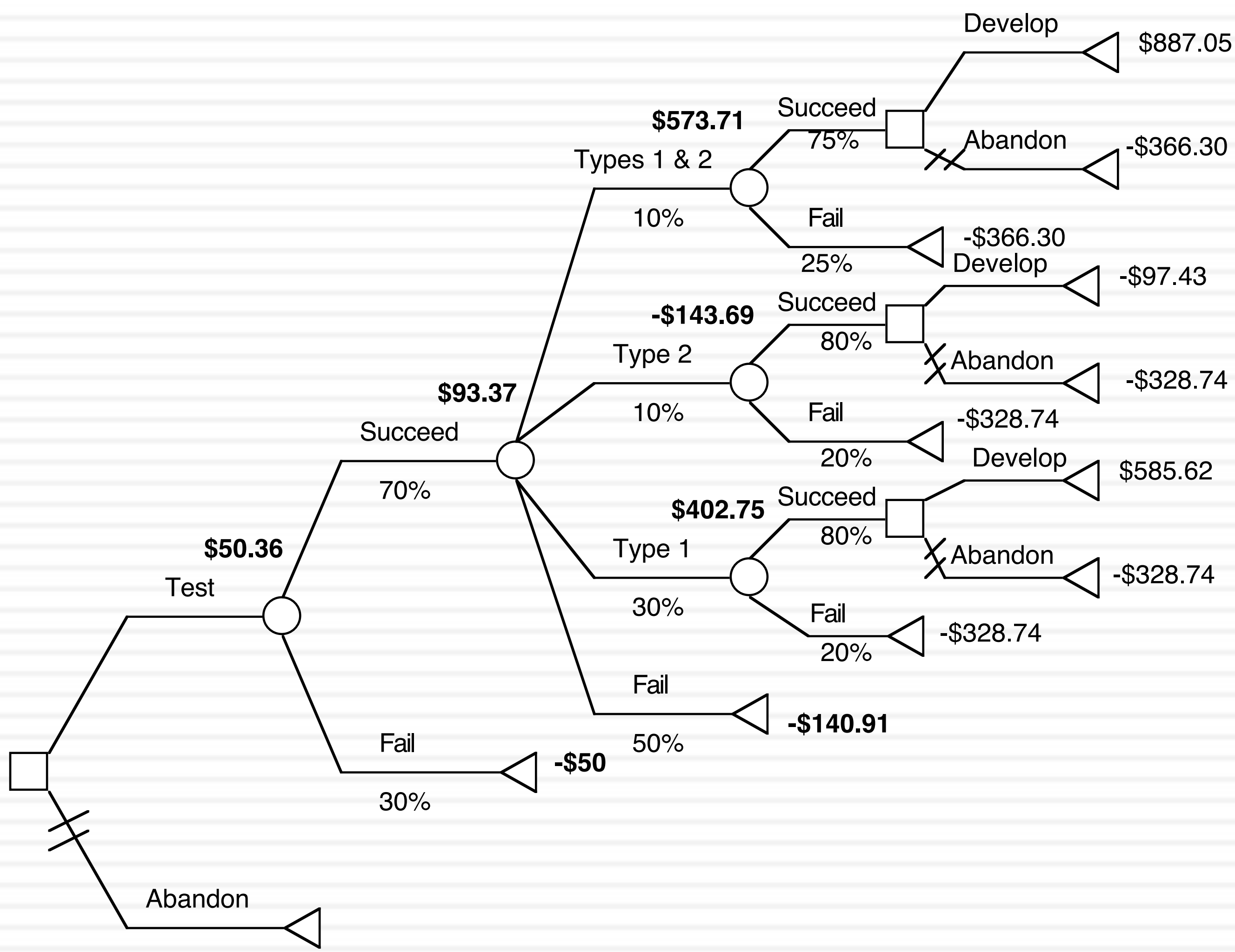
Choice of Option Pricing Models

- Most practitioners who use option pricing models to value real options argue for the binomial model over the Black-Scholes and justify this choice by noting that
  - Early exercise is the rule rather than the exception with real options
  - Underlying asset values are generally discontinous.
- If you can develop a binomial tree with outcomes at each node, it looks a great deal like a decision tree from capital budgeting. The question then becomes when and why the two approaches yield different estimates of value.

The Decision Tree Alternative

- Traditional decision tree analysis tends to use
  - One cost of capital to discount cashflows in each branch to the present
  - Probabilities to compute an expected value
  - These values will generally be different from option pricing model values
- If you modified decision tree analysis to
  - Use different discount rates at each node to reflect where you are in the decision tree (This is the Copeland solution) (or)
  - Use the riskfree rate to discount cashflows in each branch, estimate the probabilities to estimate an expected value and adjust the expected value for the market risk in the investment
- Decision Trees could yield the same values as option pricing models

A decision tree valuation of a pharmaceutical company with one drug in the FDA pipeline…



Key Tests for Real Options

- Is there an option embedded in this asset/ decision?
  - Can you identify the underlying asset?
  - Can you specify the contingency under which you will get payoff?
- Is there exclusivity?
  - If yes, there is option value.
  - If no, there is none.
  - If in between, you have to scale value.
- Can you use an option pricing model to value the real option?
  - Is the underlying asset traded?
  - Can the option be bought and sold?
  - Is the cost of exercising the option known and clear?

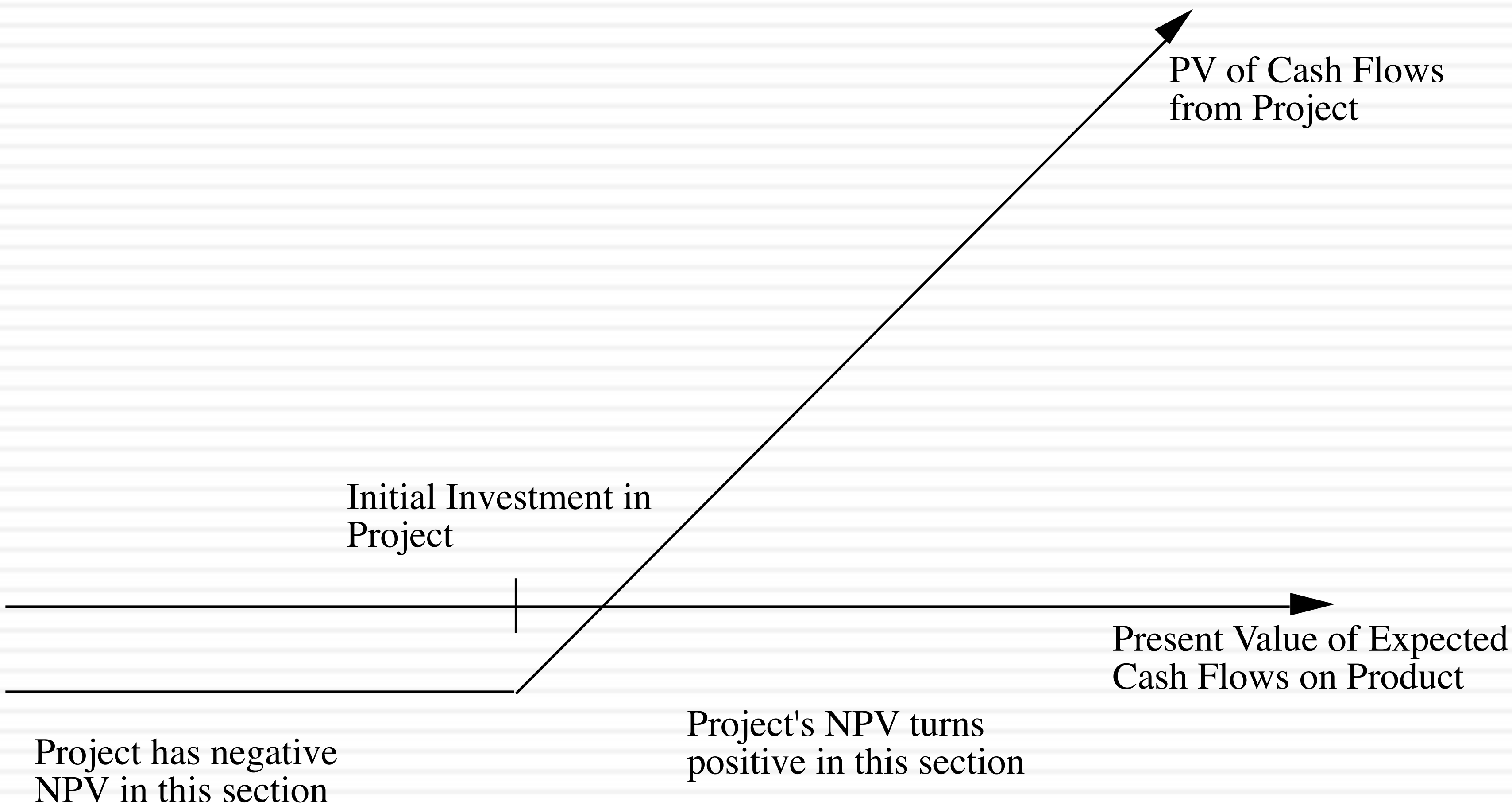
I. Options in Projects/Investments/Acquisitions

- One of the limitations of traditional investment analysis is that it is static and does not do a good job of capturing the options embedded in investment.
  - The first of these options is the option to delay taking a investment, when a firm has exclusive rights to it, until a later date.
  - The second of these options is taking one investment may allow us to take advantage of other opportunities (investments) in the future
  - The last option that is embedded in projects is the option to abandon a investment, if the cash flows do not measure up.
- These options all add value to projects and may make a “bad” investment (from traditional analysis) into a good one.

A. The Option to Delay

- When a firm has exclusive rights to a project or product for a specific period, it can delay taking this project or product until a later date.
- A traditional investment analysis just answers the question of whether the project is a "good" one if taken today.
- Thus, the fact that a project does not pass muster today (because its NPV is negative, or its IRR is less than its hurdle rate) does not mean that the rights to this project are not valuable.

Valuing the Option to Delay a Project



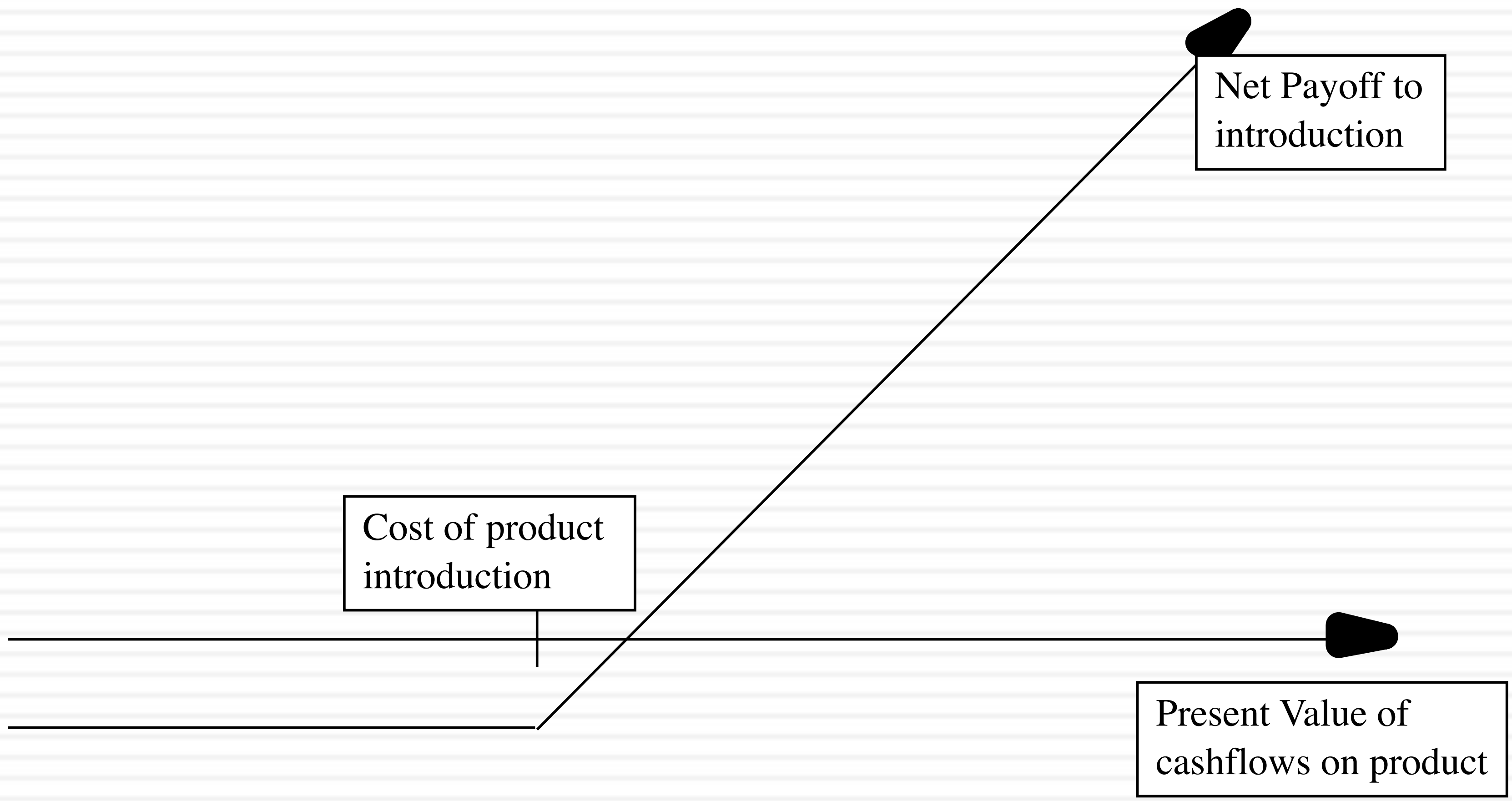
Example 1: Valuing product patents as options

- A product patent provides the firm with the right to develop the product and market it.
- It will do so only if the present value of the expected cash flows from the product sales exceed the cost of development.
- If this does not occur, the firm can shelve the patent and not incur any further costs.
- If I is the present value of the costs of developing the product, and V is the present value of the expected cashflows from development, the payoffs from owning a product patent can be written as:

Payoff from owning a product patent $= V - I$ if $V > I$

$= 0$ if $V \leq I$

Payoff on Product Option



Obtaining Inputs for Patent Valuation

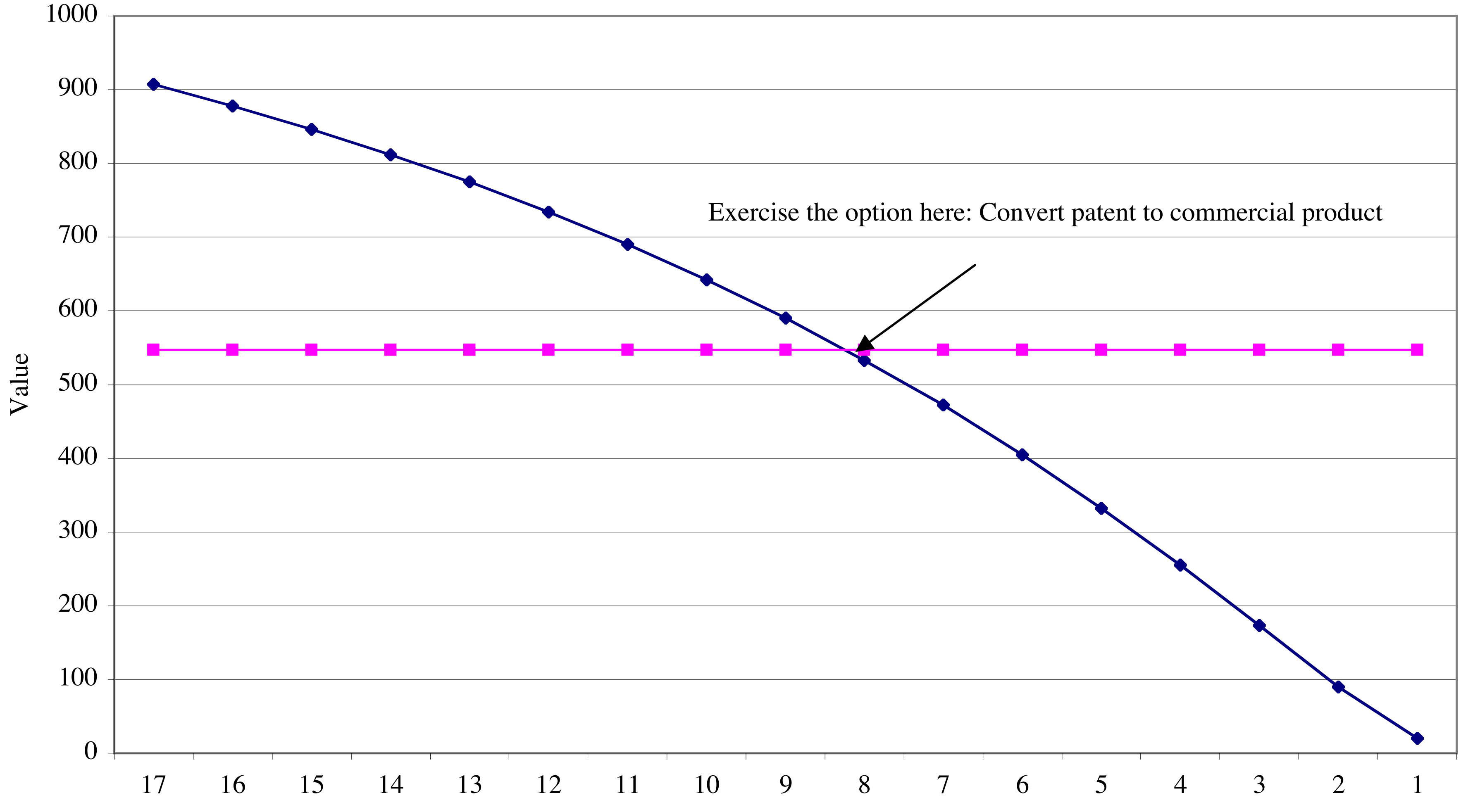
Input	Estimation Process
1. Value of the Underlying Asset	• Present Value of Cash Inflows from taking project now • This will be noisy, but that adds value.
2. Variance in value of underlying asset	• Variance in cash flows of similar assets or firms • Variance in present value from capital budgeting simulation.
3. Exercise Price on Option	• Option is exercised when investment is made. • Cost of making investment on the project ; assumed to be constant in present value dollars.
4. Expiration of the Option	• Life of the patent
5. Dividend Yield	• Cost of delay • Each year of delay translates into one less year of value-creating cashflows $\text{Annual cost of delay} = \frac{1}{n}$

Valuing a Product Patent: Avonex

- Biogen, a bio-technology firm, has a patent on Avonex, a drug to treat multiple sclerosis, for the next 17 years, and it plans to produce and sell the drug by itself.
- The key inputs on the drug are as follows:
  - PV of Cash Flows from Introducing the Drug Now = S = $ 3.422 billion
  - PV of Cost of Developing Drug for Commercial Use = K = $ 2.875 billion
  - Patent Life = t = 17 years   Riskless Rate = r = 6.7% (17-year T.Bond rate)
  - Variance in Expected Present Values = σ^2 = 0.224 (Industry average firm variance for bio-tech firms)
  - Expected Cost of Delay = y = 1/17 = 5.89%
- The output from the option pricing model
  - d1 = 1.1362   N(d1) = 0.8720
  - d2 = -0.8512   N(d2) = 0.2076

Call Value= 3,422 $\exp^{(-0.0589)(17)}$ (0.8720) - 2,875 $\exp^{(-0.067)(17)}$ (0.2076)= $ 907 million

The Optimal Time to Exercise



Valuing a firm with patents

- The value of a firm with a substantial number of patents can be derived using the option pricing model.

Value of Firm = Value of commercial products (using DCF value

+ Value of existing patents (using option pricing)

+ (Value of New patents that will be obtained in the future – Cost of obtaining these patents)

- The last input measures the efficiency of the firm in converting its R&D into commercial products. If we assume that a firm earns its cost of capital from research, this term will become zero.
- If we use this approach, we should be careful not to double count and allow for a high growth rate in cash flows (in the DCF valuation).

Value of Biogen's existing products

- Biogen had two commercial products (a drug to treat Hepatitis B and Intron) at the time of this valuation that it had licensed to other pharmaceutical firms.
- The license fees on these products were expected to generate $ 50 million in after-tax cash flows each year for the next 12 years.
- To value these cash flows, which were guaranteed contractually, the pre-tax cost of debt of the guarantors was used:

Present Value of License Fees = $\$ 50 \text{ million } (1 - (1.07)^{-12})/.07$

$= \$ 397.13 \text{ million}$

Value of Biogen's Future R&D

- Biogen continued to fund research into new products, spending about $ 100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years, and 5% thereafter.
- It was assumed that every dollar invested in research would create $ 1.25 in value in patents (valued using the option pricing model described above) for the next 10 years, and break even after that (i.e., generate $ 1 in patent value for every $ 1 invested in R&D).
- There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.