Value of Biogen's Future R&D

- Biogen continued to fund research into new products, spending about $ 100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years, and 5% thereafter.
- It was assumed that every dollar invested in research would create $ 1.25 in value in patents (valued using the option pricing model described above) for the next 10 years, and break even after that (i.e., generate $ 1 in patent value for every $ 1 invested in R&D).
- There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.

Value of Future R&D

Yr	Value of Patents	R&D Cost	Excess Value	PV (at 15%)
1	$ 150.00	$ 120.00	$ 30.00	$ 26.09
2	$ 180.00	$ 144.00	$ 36.00	$ 27.22
3	$ 216.00	$ 172.80	$ 43.20	$ 28.40
4	$ 259.20	$ 207.36	$ 51.84	$ 29.64
5	$ 311.04	$ 248.83	$ 62.21	$ 30.93
6	$ 373.25	$ 298.60	$ 74.65	$ 32.27
7	$ 447.90	$ 358.32	$ 89.58	$ 33.68
8	$ 537.48	$ 429.98	$ 107.50	$ 35.14
9	$ 644.97	$ 515.98	$ 128.99	$ 36.67
10	$ 773.97	$ 619.17	$ 154.79	$ 38.26
				$ 318.30

Value of Biogen

- The value of Biogen as a firm is the sum of all three components – the present value of cash flows from existing products, the value of Avonex (as an option) and the value created by new research:

  Value = Existing products + Existing Patents + Value: Future R&D

  = $ 397.13 million + $ 907 million + $ 318.30 million

  = $1622.43 million

- Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.50 million) to arrive at a value per share:
  - Value per share = $ 1,622.43 million / 35.5 = $ 45.70

The Real Options Test: Patents and Technology

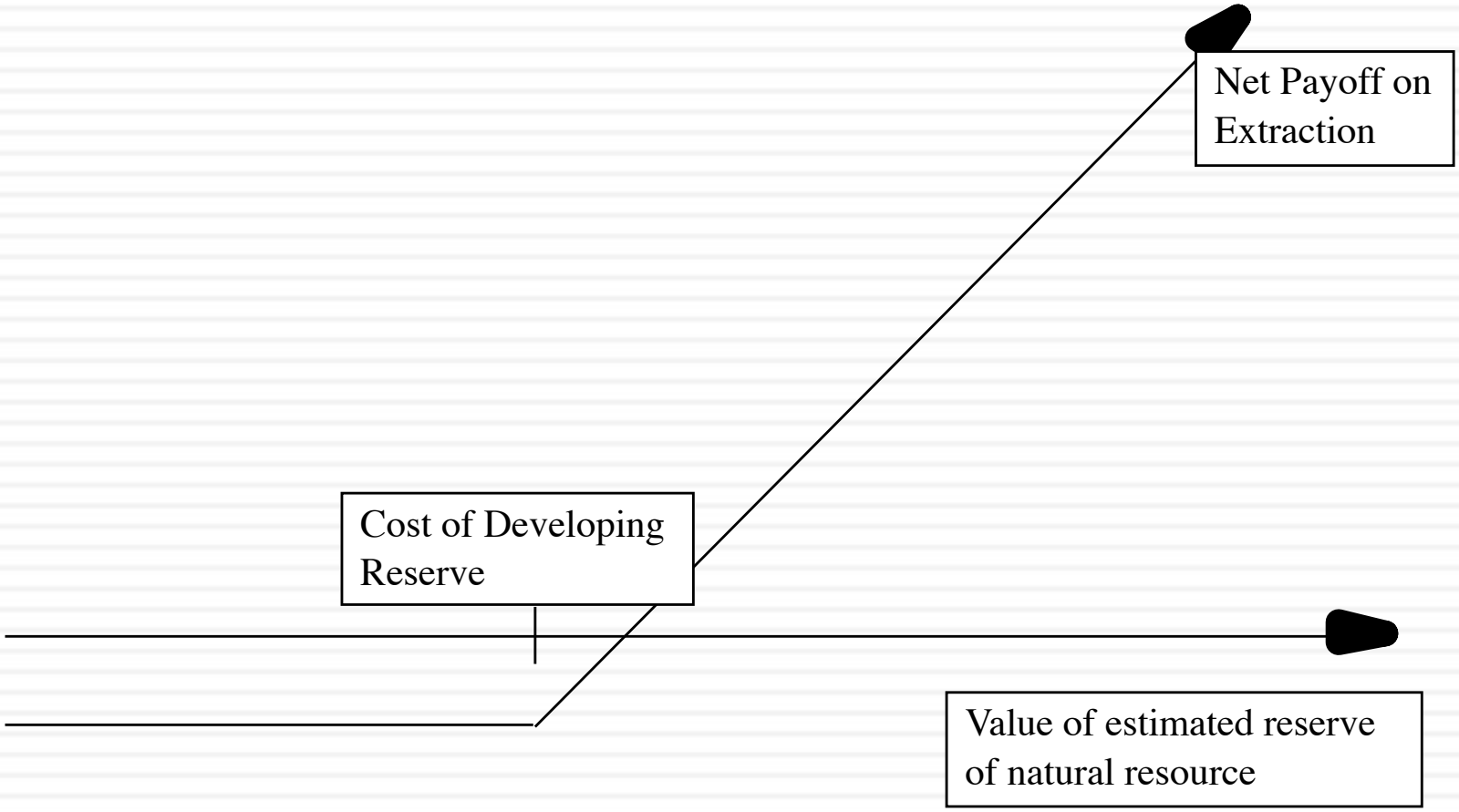
- The Option Test:
  - Underlying Asset: Product that would be generated by the patent
  - Contingency:
    - If PV of CFs from development > Cost of development: PV - Cost
    - If PV of CFs from development < Cost of development: 0
- The Exclusivity Test:
  - Patents restrict competitors from developing similar products
  - Patents do not restrict competitors from developing other products to treat the same disease.
- The Pricing Test
  - Underlying Asset: Patents are not traded. Not only do you therefore have to estimate the present values and volatilities yourself, you cannot construct replicating positions or do arbitrage.
  - Option: Patents are bought and sold, though not as frequently as oil reserves or mines.
  - Cost of Exercising the Option: This is the cost of converting the patent for commercial production. Here, experience does help and drug firms can make fairly precise estimates of the cost.
- Conclusion: You can estimate the value of the real option but the quality of your estimate will be a direct function of the quality of your capital budgeting. It works best if you are valuing a publicly traded firm that generates most of its value from one or a few patents - you can use the market value of the firm and the variance in that value then in your option pricing model.

Example 2: Valuing Natural Resource Options

- In a natural resource investment, the underlying asset is the resource and the value of the asset is based upon two variables - the quantity of the resource that is available in the investment and the price of the resource.
- In most such investments, there is a cost associated with developing the resource, and the difference between the value of the asset extracted and the cost of the development is the profit to the owner of the resource.
- Defining the cost of development as X, and the estimated value of the resource as V, the potential payoffs on a natural resource option can be written as follows:

Payoff on natural resource investment	= V - X	if V > X
	= 0	if V≤ X

Payoff Diagram on Natural Resource Firms



Estimating Inputs for Natural Resource Options

Input	Estimation Process
1. Value of Available Reserves of the Resource	• Expert estimates (Geologists for oil..); The present value of the after-tax cash flows from the resource are then estimated.
2. Cost of Developing Reserve (Strike Price)	• Past costs and the specifics of the investment
3. Time to Expiration	• Relinqushment Period: if asset has to be relinquished at a point in time. • Time to exhaust inventory - based upon inventory and capacity output.
4. Variance in value of underlying asset	• based upon variability of the price of the resources and variability of available reserves.
5. Net Production Revenue (Dividend Yield)	• Net production revenue every year as percent of market value.
6. Development Lag	• Calculate present value of reserve based upon the lag.

Valuing Gulf Oil

- Gulf Oil was the target of a takeover in early 1984 at $70 per share (It had 165.30 million shares outstanding, and total debt of $9.9 billion).
  - It had estimated reserves of 3038 million barrels of oil and the average cost of developing these reserves was estimated to be $10 a barrel in present value dollars (The development lag is approximately two years).
  - The average relinquishment life of the reserves is 12 years.
  - The price of oil was $22.38 per barrel, and the production cost, taxes and royalties were estimated at $7 per barrel.
  - The bond rate at the time of the analysis was 9.00%.
  - Gulf was expected to have net production revenues each year of approximately 5% of the value of the developed reserves. The variance in oil prices is 0.03.

Valuing Undeveloped Reserves

- Inputs for valuing undeveloped reserves
  - Value of underlying asset = Value of estimated reserves discounted back for period of development lag= 3038 * ($ 22.38 - $7) / 1.05^2 = $42,380.44
  - Exercise price = Estimated development cost of reserves = 3038 * $10 = $30,380 million
  - Time to expiration = Average length of relinquishment option = 12 years
  - Variance in value of asset = Variance in oil prices = 0.03
  - Riskless interest rate = 9%
  - Dividend yield = Net production revenue/ Value of developed reserves = 5%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:

d1 = 1.6548 N(d1) = 0.9510

d2 = 1.0548 N(d2) = 0.8542

Call Value= 42,380.44 $\exp^{(-0.05)(12)}$ (0.9510) -30,380 ($\exp^{(-0.09)(12)}$ (0.8542)

= $ 13,306 million

Valuing Gulf Oil

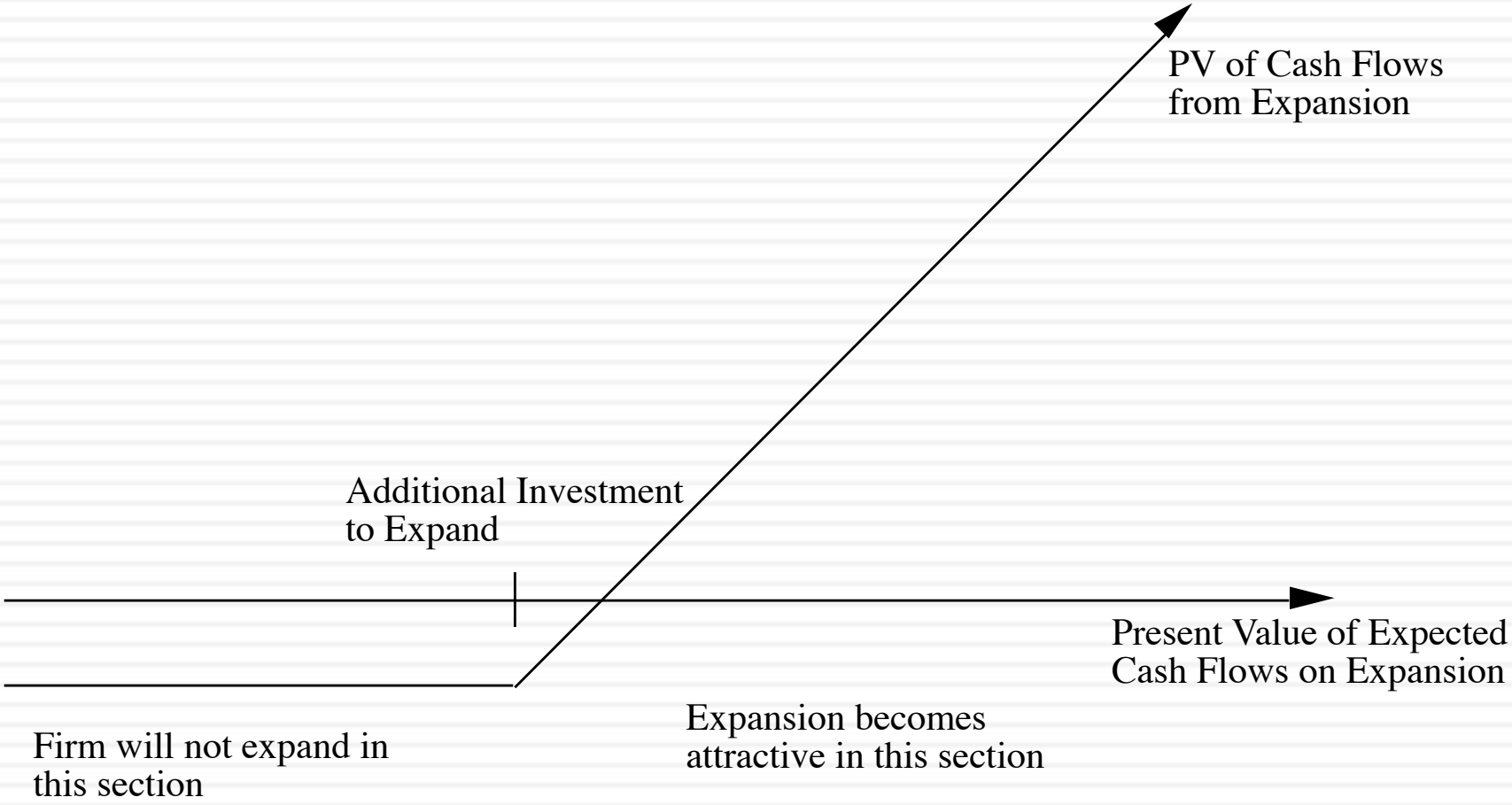
- In addition, Gulf Oil had free cashflows to the firm from its oil and gas production of $915 million from already developed reserves and these cashflows are likely to continue for ten years (the remaining lifetime of developed reserves).
- The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:
  - Value of already developed reserves = 915 (1 - 1.125^{-10})/.125 = $5065.83
- Adding the value of the developed and undeveloped reserves

Value of undeveloped reserves		= $ 13,306 million
Value of production in place		= $ 5,066 million
Total value of firm		= $ 18,372 million
Less Outstanding Debt		= $ 9,900 million
Value of Equity		= $ 8,472 million
Value per share	= $ 8,472/165.3	= $51.25

B. The Option to Expand/Take Other Projects

- Taking a project today may allow a firm to consider and take other valuable projects in the future.
- Thus, even though a project may have a negative NPV, it may be a project worth taking if the option it provides the firm (to take other projects in the future) provides a more-than-compensating value.
- These are the options that firms often call "strategic options" and use as a rationale for taking on "negative NPV" or even "negative return" projects.

The Option to Expand



The option to expand: Valuing a young, start-up company

- You have complete a DCF valuation of a small anti-virus software company, Secure Mail, and estimated a value of $115 million.
- Assume that there is the possibility that the company could use the customer base that it develops for the anti-virus software and the technology on which the software is based to create a database software program sometime in the next 5 years.
  - It will cost Secure Mail about $500 million to develop a new database program, if they decided to do it today.
  - Based upon the information you have now on the potential for a database program, the company can expect to generate about $ 40 million a year in after-tax cashflows for ten years. The cost of capital for private companies that provide database software is 12%.
  - The annualized standard deviation in firm value at publicly traded database companies is 50%.
  - The five-year treasury bond rate is 3%.

Valuing the Expansion Option

S = Value of entering the database software market
= PV of $40 million for 10 years @12% = $226 million

K = Exercise price
= Cost of entering the database software market = $ 500 million

t = Period over which you have the right to enter the market
= 5 years

σ = Standard deviation of stock prices of database firms = 50%

r = Riskless rate = 3%

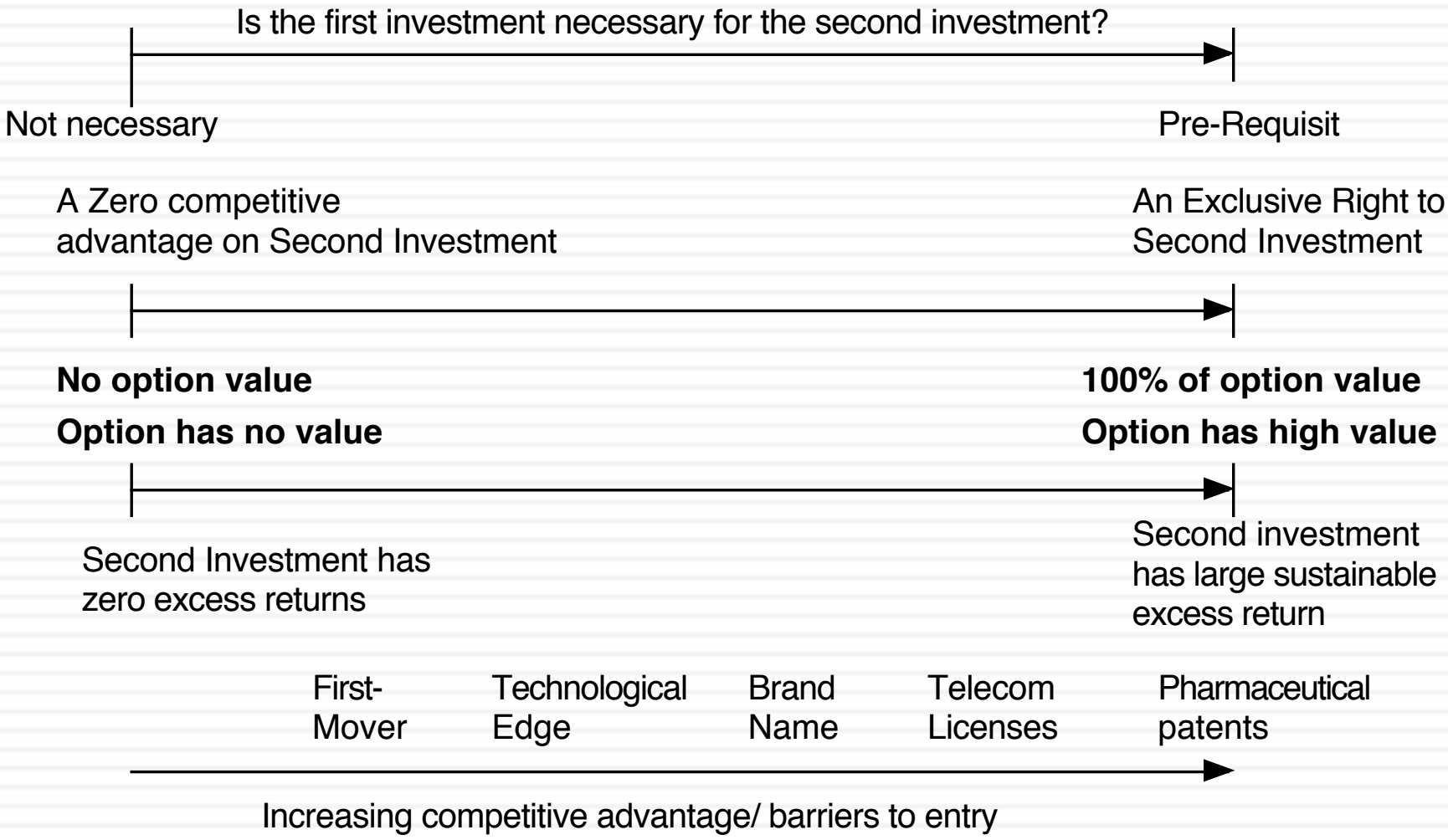
- Call Value= $ 56 Million

DCF valuation of the firm = $ 115 million

Value of Option to Expand to Database market = $ 56 million

Value of the company with option to expand = $ 171 million

A note of caution: Opportunities are not options…

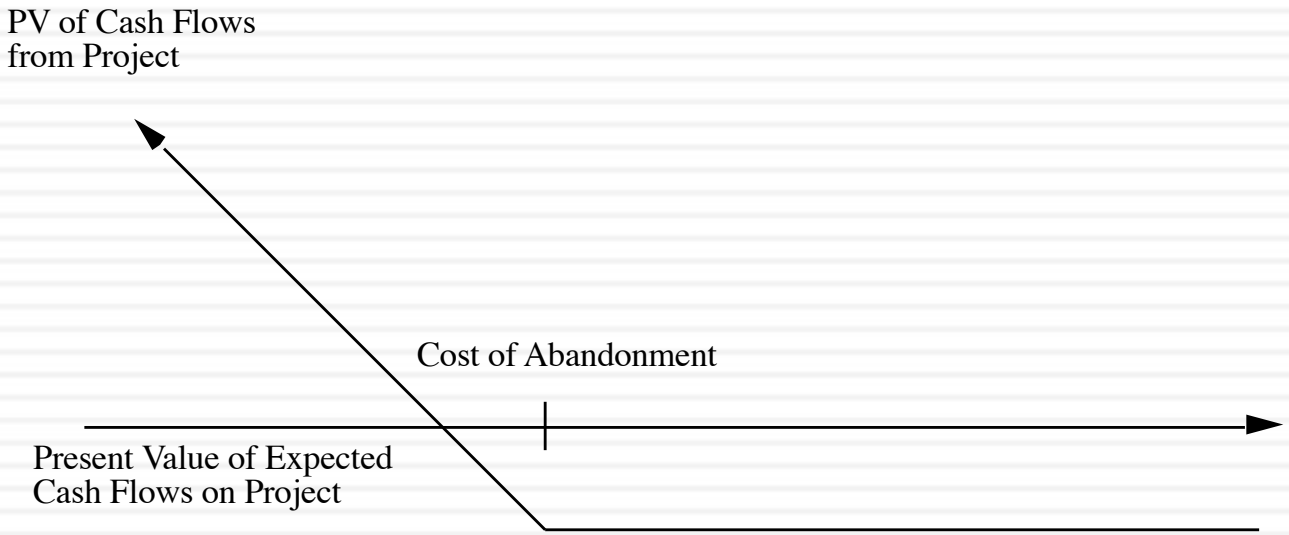


The Real Options Test for Expansion Options

- The Options Test
  - Underlying Asset: Expansion Project
  - Contingency
  - If PV of CF from expansion > Expansion Cost: PV - Expansion Cost
  - If PV of CF from expansion < Expansion Cost: 0
- The Exclusivity Test
  - Barriers may range from strong (exclusive licenses granted by the government) to weaker (brand name, knowledge of the market) to weakest (first mover).
- The Pricing Test
  - Underlying Asset: As with patents, there is no trading in the underlying asset and you have to estimate value and volatility.
  - Option: Licenses are sometimes bought and sold, but more diffuse expansion options are not.
  - Cost of Exercising the Option: Not known with any precision and may itself evolve over time as the market evolves.
- Using option pricing models to value expansion options will not only yield extremely noisy estimates, but may attach inappropriate premiums to discounted cashflow estimates.

C. The Option to Abandon

- A firm may sometimes have the option to abandon a project, if the cash flows do not measure up to expectations.
- If abandoning the project allows the firm to save itself from further losses, this option can make a project more valuable.



Valuing the Option to Abandon

- Airbus is considering a joint venture with Lear Aircraft to produce a small commercial airplane (capable of carrying 40-50 passengers on short haul flights)
  - Airbus will have to invest $ 500 million for a 50% share of the venture
  - Its share of the present value of expected cash flows is 480 million.
- Lear Aircraft, which is eager to enter into the deal, offers to buy Airbus's 50% share of the investment anytime over the next five years for $ 400 million, if Airbus decides to get out of the venture.
- A simulation of the cash flows on this time share investment yields a variance in the present value of the cash flows from being in the partnership is 0.16.
- The project has a life of 30 years.

Project with Option to Abandon

- Value of the Underlying Asset (S) = PV of Cash Flows from Project = $ 480 million
- Strike Price (K) = Salvage Value from Abandonment = $ 400 million
- Variance in Underlying Asset's Value = 0.16
- Time to expiration = Life of the Project =5 years
- Dividend Yield = 1/Life of the Project = 1/30 = 0.033 (We are assuming that the project's present value will drop by roughly 1/n each year into the project)
- Assume that the five-year riskless rate is 6%. The value of the put option can be estimated.

Should Airbus enter into the joint venture?

Value of Put $= Ke^{-rt}(1-N(d2)) - Se^{-yt}(1-N(d1))$

$= 400 \exp^{(-0.06)(5)}(1-0.4624) - 480 \exp^{(-0.033)(5)}(1-0.7882)$

= $ 73.23 million

- The value of this abandonment option has to be added on to the net present value of the project of -$ 20 million, yielding a total net present value with the abandonment option of $ 53.23 million.

Implications for Investment Analysis/ Valuation

- Having a option to abandon a project can make otherwise unacceptable projects acceptable.
- Other things remaining equal, you would attach more value to companies with
  - More cost flexibility, that is, making more of the costs of the projects into variable costs as opposed to fixed costs.
  - Fewer long-term contracts/obligations with employees and customers, since these add to the cost of abandoning a project.
- These actions will undoubtedly cost the firm some value, but this has to be weighed off against the increase in the value of the abandonment option.

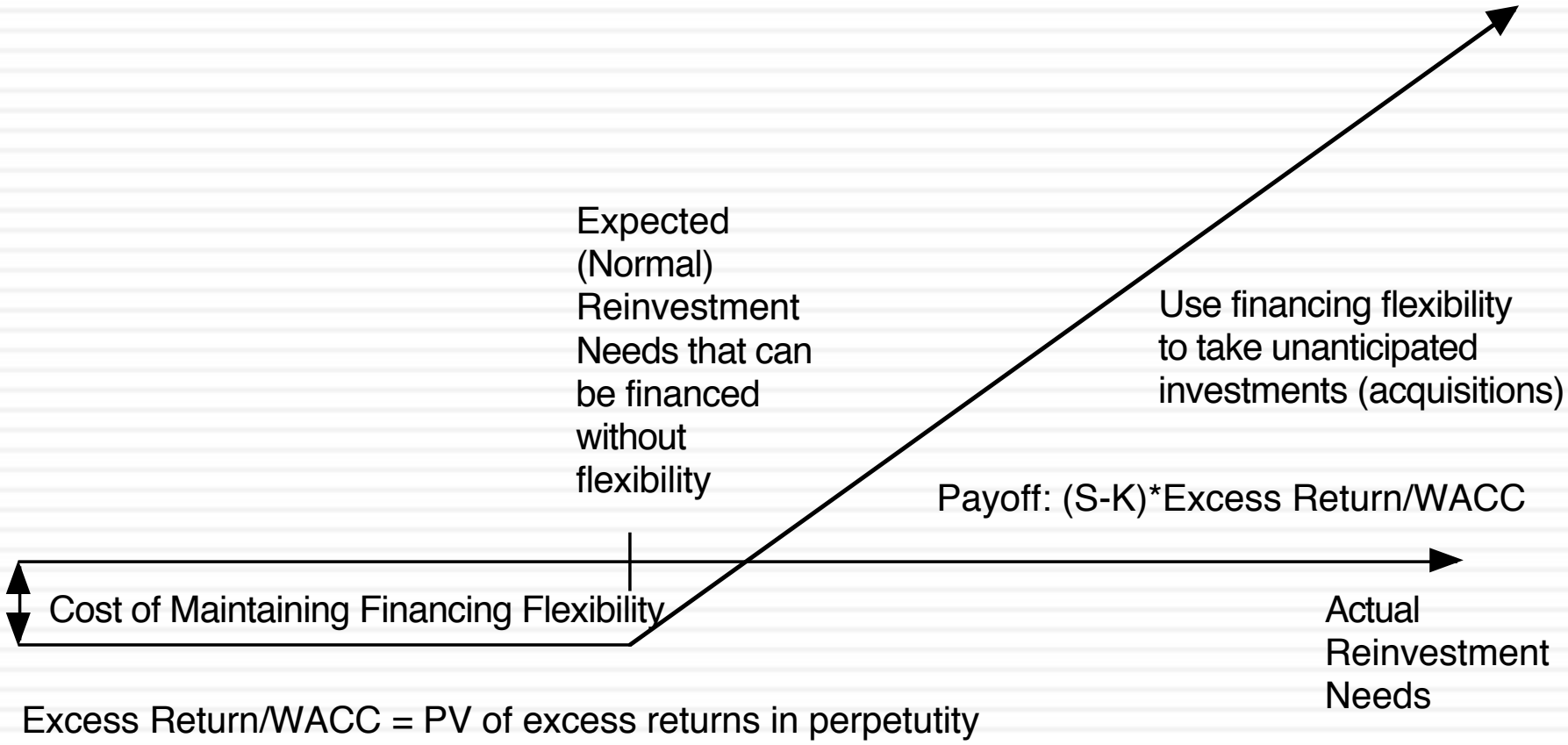
D. Options in Capital Structure

- The most direct applications of option pricing in capital structure decisions is in the design of securities. In fact, most complex financial instruments can be broken down into some combination of a simple bond/common stock and a variety of options.
  - If these securities are to be issued to the public, and traded, the options have to be priced.
  - If these are non-traded instruments (bank loans, for instance), they still have to be priced into the interest rate on the instrument.
- The other application of option pricing is in valuing flexibility. Often, firms preserve debt capacity or hold back on issuing debt because they want to maintain flexibility.

The Value of Flexibility

- Firms maintain excess debt capacity or larger cash balances than are warranted by current needs, to meet unexpected future requirements.
- While maintaining this financing flexibility has value to firms, it also has a cost; the excess debt capacity implies that the firm is giving up some value and has a higher cost of capital.
- The value of flexibility can be analyzed using the option pricing framework; a firm maintains large cash balances and excess debt capacity in order to have the option to take projects that might arise in the future.

The Value of Flexibility



Excess Return/WACC = PV of excess returns in perpetutity

Disney's Optimal Debt Ratio

Debt Ratio	Cost of Equity	Cost of Debt	Cost of Capital
0.00%	13.00%	4.61%	13.00%
10.00%	13.43%	4.61%	12.55%
Current:18%	13.85%	4.80%	12.22%
20.00%	13.96%	4.99%	12.17%
30.00%	14.65%	5.28%	11.84%
40.00%	15.56%	5.76%	11.64%
50.00%	16.85%	6.56%	11.70%
60.00%	18.77%	7.68%	12.11%
70.00%	21.97%	7.68%	11.97%
80.00%	28.95%	7.97%	12.17%
90.00%	52.14%	9.42%	13.69%

Inputs to Option Valuation Model- Disney

Model input	Estimated as	In general…	For Disney
S	Expected annual reinvestment needs (as % of firm value)	Measures magnitude of reinvestment needs	Average of Reinvestment/ Value over last 5 years = 5.3%
σ^2	Variance in annual reinvestment needs	Measures how much volatility there is in investment needs.	Variance over last 5 years in ln(Reinvestment/Value) =0.375
K	(Internal + Normal access to external funds)/ Value	Measures the capital constraint	Average over last 5 years = 4.8%
T	1 year	Measures an annual value for flexibility	T =1

Valuing Flexibility at Disney

- The value of an option with these characteristics is 1.6092%. You can consider this the value of the option to take a project, but the overall value of flexibility will still depend upon the quality of the projects taken. In other words, the value of the option to take a project is zero if the project has zero net present value.
- Disney earns 18.69% on its projects has a cost of capital of 12.22%. The excess return (annually) is 6.47%. Assuming that they can continue to generate these excess returns in perpetuity:

  Value of Flexibility (annual)= 1.6092%(.0647/.1222) = 0.85 % of value
- Disney's cost of capital at its optimal debt ratio is 11.64%. The cost it incurs to maintain flexibility is therefore 0.58% annually (12.22%-11.64%). It therefore pays to maintain flexibility.

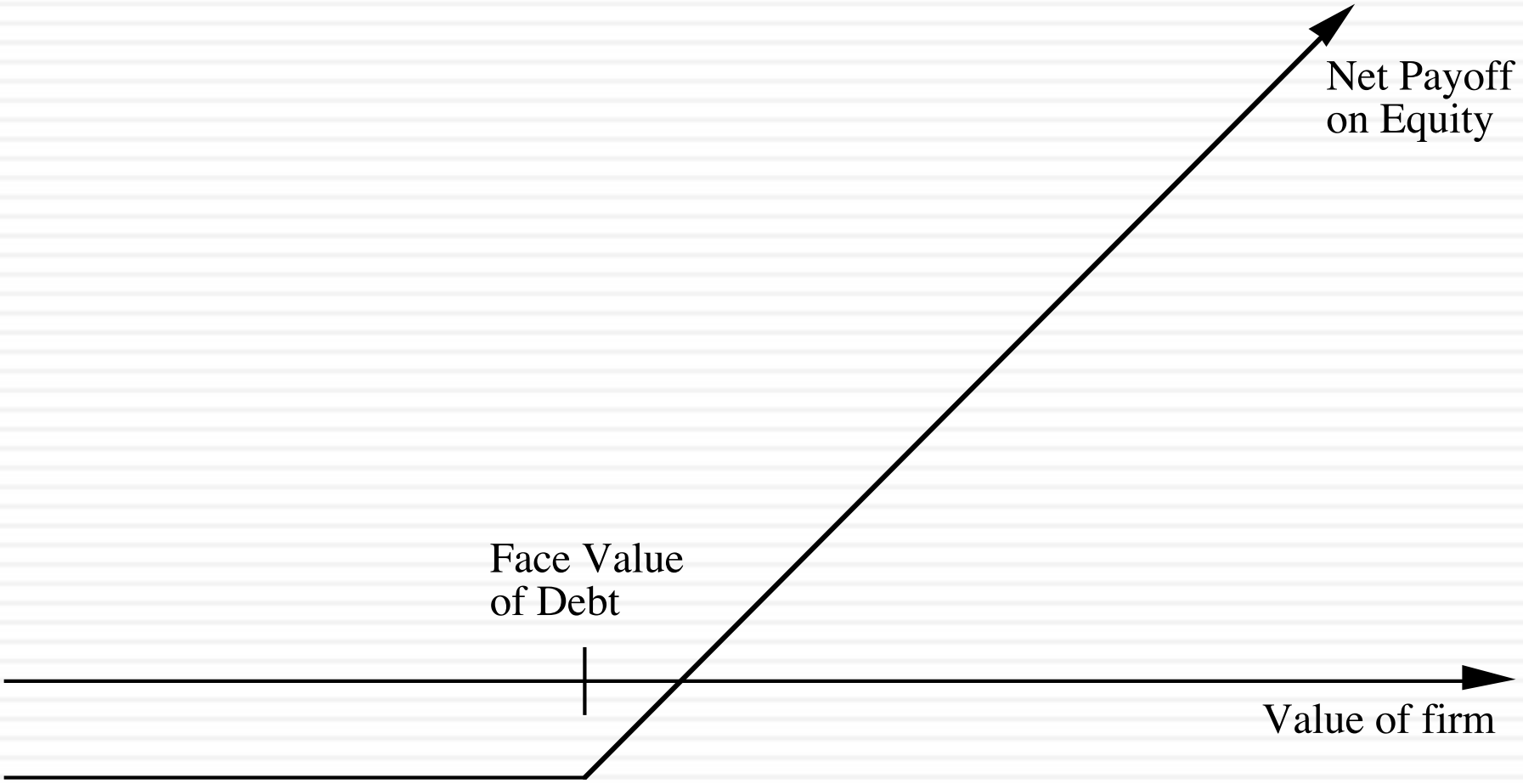
Determinants of the Value of Flexibility

- Capital Constraints (External and Internal): The greater the capacity to raise funds, either internally or externally, the less the value of flexibility.
  - 1.1: Firms with significant internal operating cash flows should value flexibility less than firms with small or negative operating cash flows.
  - 1.2: Firms with easy access to financial markets should have a lower value for flexibility than firms without that access.
- Unpredictability of reinvestment needs: The more unpredictable the reinvestment needs of a firm, the greater the value of flexibility.
- Capacity to earn excess returns: The greater the capacity to earn excess returns, the greater the value of flexibility.
  - 1.3: Firms that do not have the capacity to earn or sustain excess returns get no value from flexibility.

E. Valuing Equity as an option

- The equity in a firm is a residual claim, i.e., equity holders lay claim to all cashflows left over after other financial claim-holders (debt, preferred stock etc.) have been satisfied.
- If a firm is liquidated, the same principle applies, with equity investors receiving whatever is left over in the firm after all outstanding debts and other financial claims are paid off.
- The principle of limited liability, however, protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm.

Payoff Diagram for Liquidation Option



Application to valuation: A simple example

- Assume that you have a firm whose assets are currently valued at $100 million and that the standard deviation in this asset value is 40%.
- Further, assume that the face value of debt is $80 million (It is zero coupon debt with 10 years left to maturity).
- If the ten-year treasury bond rate is 10%,
  - how much is the equity worth?
  - What should the interest rate on debt be?

Model Parameters

- Value of the underlying asset = S
  - Value of the firm = $ 100 million
- Exercise price = K
  - Face Value of outstanding debt = $ 80 million
- Life of the option = t
  - Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2
  - Variance in firm value = 0.16
- Riskless rate = r
  - Treasury bond rate corresponding to option life = 10%

Valuing Equity as a Call Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:

  d1 = 1.5994 N(d1) = 0.9451

  d2 = 0.3345 N(d2) = 0.6310

- Value of the call = 100 (0.9451) - 80 $\exp^{(-0.10)(10)}$ (0.6310) = $75.94 million
- Value of the outstanding debt = $100 - $75.94 = $24.06 million
- Interest rate on debt = ($ 80 / $24.06)$^{1/10}$ -1 = 12.77%

I. The Effect of Catastrophic Drops in Value

- Assume now that a catastrophe wipes out half the value of this firm (the value drops to $ 50 million), while the face value of the debt remains at $ 80 million. What will happen to the equity value of this firm?
  a. It will drop in value to $ 25.94 million [$ 50 million - market value of debt from previous page]
  b. It will be worth nothing since debt outstanding > Firm Value
  c. It will be worth more than $ 25.94 million

Valuing Equity in the Troubled Firm

- Value of the underlying asset = S
  - Value of the firm = $ 50 million
- Exercise price = K
  - Face Value of outstanding debt = $ 80 million
- Life of the option = t
  - Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2
  - Variance in firm value = 0.16
- Riskless rate = r
  - Treasury bond rate corresponding to option life = 10%

The Value of Equity as an Option

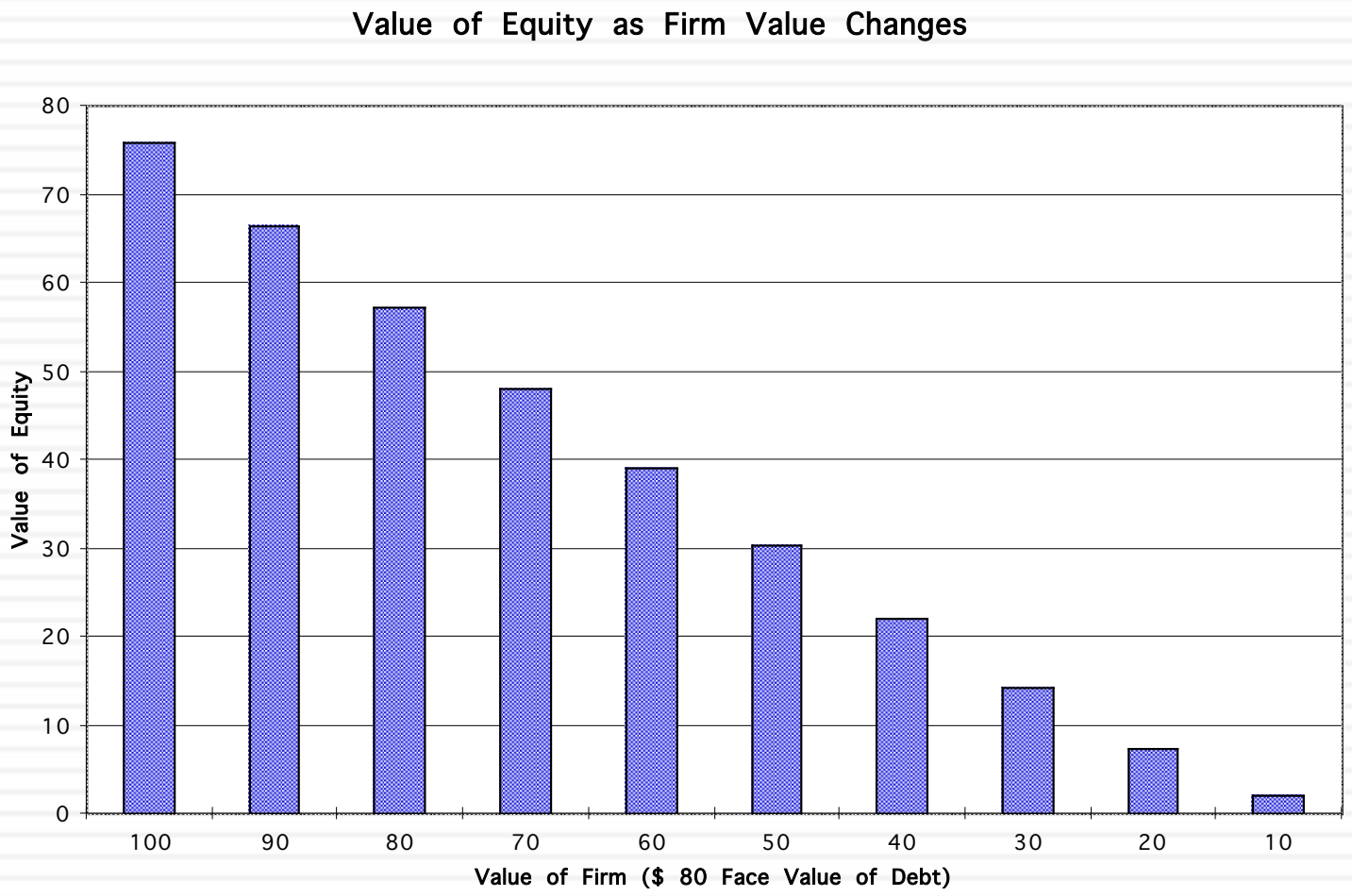
- Based upon these inputs, the Black-Scholes model provides the following value for the call:

  d1 = 1.0515 N(d1) = 0.8534

  d2 = -0.2135 N(d2) = 0.4155

- Value of the call = 50 (0.8534) - 80 $\exp^{(-0.10)(10)}$ (0.4155) = $30.44 million
- Value of the bond= $50 - $30.44 = $19.56 million
- The equity in this firm drops by $45.50 million, less than the overall drop in value of $50 million, because of the option characteristics of equity.
- This might explain why stock in firms, which are in Chapter 11 and essentially bankrupt, still has value.

Equity value persists ..



II. The conflict between stockholders and bondholders

- Consider again the firm described in the earlier example , with a value of assets of $100 million, a face value of zero-coupon ten-year debt of $80 million, a standard deviation in the value of the firm of 40%. The equity and debt in this firm were valued as follows:
  - Value of Equity = $75.94 million
  - Value of Debt = $24.06 million
  - Value of Firm == $100 million
- Now assume that the stockholders have the opportunity to take a project with a negative net present value of -$2 million, but assume that this project is a very risky project that will push up the standard deviation in firm value to 50%. Would you invest in this project?
  a. Yes
  b. No

Valuing Equity after the Project

- Value of the underlying asset = S
  - Value of the firm = $ 100 million - $2 million = $ 98 million (The value of the firm is lowered because of the negative net present value project)
- Exercise price = K
  - Face Value of outstanding debt = $ 80 million
- Life of the option = t
- Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2
  - Variance in firm value = 0.25
- Riskless rate = r
  - Treasury bond rate corresponding to option life = 10%

Option Valuation

- Option Pricing Results for Equity and Debt Value
  - Value of Equity = $77.71
  - Value of Debt = $20.29
  - Value of Firm = $98.00
- The value of equity rises from $75.94 million to $ 77.71 million , even though the firm value declines by $2 million. The increase in equity value comes at the expense of bondholders, who find their wealth decline from $24.06 million to $20.19 million.

Effects of an Acquisition

- Assume that you are the manager of a firm and that you buy another firm, with a fair market value of $ 150 million, for exactly $ 150 million. In an efficient market, the stock price of your firm will
  a. Increase
  b. Decrease
  c. Remain Unchanged

Effects on equity of a conglomerate merger

- You are provided information on two firms, which operate in unrelated businesses and hope to merge.

	Firm A	Firm B
Value of the firm	$100 million	$ 150 million
Face Value of Debt (10 yr zeros)	$ 80 million	$ 50 million
Maturity of debt	10 years	10 years
Std. Dev. in value	40 %	50 %
Correlation between cashflows	0.4	

  - The ten-year bond rate is 10%.
- The variance in the value of the firm after the acquisition can be calculated as follows:

Variance in combined firm value $= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2$

$= (0.4)^2 (0.16) + (0.6)^2 (0.25) + 2 (0.4) (0.6) (0.4) (0.4) (0.5)$

$= 0.154$

Valuing the Combined Firm

- The values of equity and debt in the individual firms and the combined firm can then be estimated using the option pricing model:

	Firm A	Firm B	Combined firm
Value of equity in the firm		$75.94	$134.47	$ 207.43
Value of debt in the firm		$24.06	$ 15.53	$ 42.57
Value of the firm		$100.00	$150.00	$ 250.00

- The combined value of the equity prior to the merger is $ 210.41 million and it declines to $207.43 million after.
- The wealth of the bondholders increases by an equal amount.
- There is a transfer of wealth from stockholders to bondholders, as a consequence of the merger. Thus, conglomerate mergers that are not followed by increases in leverage are likely to see this redistribution of wealth occur across claim holders in the firm.

Obtaining option pricing inputs - Some real world problems

- The examples that have been used to illustrate the use of option pricing theory to value equity have made some simplifying assumptions. Among them are the following:

  (1) There were only two claim holders in the firm - debt and equity.

  (2) There is only one issue of debt outstanding and it can be retired at face value.

  (3) The debt has a zero coupon and no special features (convertibility, put clauses etc.)

  (4) The value of the firm and the variance in that value can be estimated.

Real World Approaches to Valuing Equity in Troubled Firms: Getting Inputs

Input	Estimation Process
Value of the Firm	• Cumulate market values of equity and debt (or) • Value the assets in place using FCFF and WACC (or) • Use cumulated market value of assets, if traded.
Variance in Firm Value	• If stocks and bonds are traded, $\sigma^2_{firm} = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2 w_e w_d \rho_{ed} \sigma_e \sigma_d$ where σ_e^2 = variance in the stock price w_e = MV weight of Equity σ_d^2 = the variance in the bond price w_d = MV weight of debt • If not traded, use variances of similarly rated bonds. • Use average firm value variance from the industry in which company operates.
Value of the Debt	• If the debt is short term, you can use only the face or book value of the debt. • If the debt is long term and coupon bearing, add the cumulated nominal value of these coupons to the face value of the debt.
Maturity of the Debt	• Face value weighted duration of bonds outstanding (or) • If not available, use weighted maturity

Valuing Equity as an option - Eurotunnel in early 1998

- Eurotunnel has been a financial disaster since its opening
  - In 1997, Eurotunnel had earnings before interest and taxes of -£56 million and net income of -£685 million
  - At the end of 1997, its book value of equity was -£117 million
- It had £8,865 million in face value of debt outstanding
  - The weighted average duration of this debt was 10.93 years

Debt Type	Face Value	Duration
Short term	935	0.50
10 year	2435	6.7
20 year	3555	12.6
Longer	1940	18.2
Total	£8,865 mil	10.93 years

The Basic DCF Valuation

- The value of the firm estimated using projected cashflows to the firm, discounted at the weighted average cost of capital was £2,312 million.
- This was based upon the following assumptions –
  - Revenues will grow 5% a year in perpetuity.
  - The COGS which is currently 85% of revenues will drop to 65% of revenues in yr 5 and stay at that level.
  - Capital spending and depreciation will grow 5% a year in perpetuity.
  - There are no working capital requirements.
  - The debt ratio, which is currently 95.35%, will drop to 70% after year 5. The cost of debt is 10% in high growth period and 8% after that.
  - The beta for the stock will be 1.10 for the next five years, and drop to 0.8 after the next 5 years.
  - The long term bond rate is 6%.

Other Inputs

- The stock has been traded on the London Exchange, and the annualized std deviation based upon ln (prices) is 41%.
- There are Eurotunnel bonds, that have been traded; the annualized std deviation in ln(price) for the bonds is 17%.
  - The correlation between stock price and bond price changes has been 0.5. The proportion of debt in the capital structure during the period (1992-1996) was 85%.
  - Annualized variance in firm value

$= (0.15)^2 (0.41)^2 + (0.85)^2 (0.17)^2 + 2 (0.15) (0.85)(0.5)(0.41)(0.17) = 0.0335$

- The 15-year bond rate is 6%. (I used a bond with a duration of roughly 11 years to match the life of my option)

Valuing Eurotunnel Equity and Debt

- Inputs to Model
  - Value of the underlying asset = S = Value of the firm = £2,312 million
  - Exercise price = K = Face Value of outstanding debt = £8,865 million
  - Life of the option = t = Weighted average duration of debt = 10.93 years
  - Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.0335
  - Riskless rate = r = Treasury bond rate corresponding to option life = 6%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:
  - d1 = -0.8337 N(d1) = 0.2023
  - d2 = -1.4392 N(d2) = 0.0751
- Value of the call = 2312 (0.2023) - 8,865 $\exp^{(-0.06)(10.93)}$ (0.0751) = £122 million
- Appropriate interest rate on debt = $(8865/2190)^{(1/10.93)}$-1= 13.65%

In Closing…

- There are real options everywhere.
- Most of them have no significant economic value because there is no exclusivity associated with using them.
- When options have significant economic value, the inputs needed to value them in a binomial model can be used in more traditional approaches (decision trees) to yield equivalent value.
- The real value from real options lies in
  - Recognizing that building in flexibility and escape hatches into large decisions has value
  - Insights we get on understanding how and why companies behave the way they do in investment analysis and capital structure choices.