

IV. An Intermediate Problem

Private to VC to Public offering...

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- Assume that you have a private business operating in a sector, where publicly traded companies have an average beta of 1 and where the average correlation of firms with the market is 0.25. Consider the cost of equity at three stages (Riskfree rate = 4%; ERP = 5%):
- Stage 1: The nascent business, with a private owner, who is fully invested in that business.
Perceived Beta = $1 / 0.25 = 4$
Cost of Equity = $4\% + 4 (5\%) = 24\%$
- Stage 2: Angel financing provided by specialized venture capitalist, who holds multiple investments, in high technology companies. (Correlation of portfolio with market is 0.5)
Perceived Beta = $1 / 0.5 = 2$
Cost of Equity = $4\% + 2 (5\%) = 14\%$
- Stage 3: Public offering, where investors are retail and institutional investors, with diversified portfolios:
Perceived Beta = 1
Cost of Equity = $4\% + 1 (5\%) = 9\%$

To value this company...

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Assume that this company will be fully owned by its current owner for two years, will access the technology venture capitalist at the start of year 3 and that is expected to either go public or be sold to a publicly traded firm at the end of year 5.

	1	2	3	4	5	Terminal year	Growth rate 2% forever after year 5
E(Cash flow)	\$100	\$125	\$150	\$165	\$170	\$175	
Market beta	1	1	1	1	1	1	
Correlation	0.25	0.25	0.5	0.5	0.5	1	
Beta used	4	4	2	2	2	1	
Cost of equity	24.00%	24.00%	14.00%	14.00%	14.00%	9.00%	
Terminal value					\$2,500		175/ (.09-.02)
Cumulated COE	1.2400	1.5376	1.7529	1.9983	2.2780	2.4830	
PV	\$80.65	\$81.30	\$85.57	\$82.57	\$1,172.07		

Value of firm	\$1,502	(Correct value, using changing costs of equity)
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Value of firm	\$1,221	(using 24% as cost of equity forever. You will undervalue firm)
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Value of firm	\$2,165	(Using 9% as cost of equity forever. You will overvalue firm)
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Implications

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- Proposition 1: The value of a private business that is expected to transition to a publicly traded company will be higher than the value of an otherwise similar private business that does not expect to make this transition.
 - ▣ Private businesses in sectors that are “hot” in terms of going public (social media in 2014) will be worth more than private businesses in less sexy sectors.
 - ▣ As IPOs boom (bust) private company valuations will increase (decrease).
 - ▣ Private companies in countries that have easy access to public markets will have higher value than companies in countries without that access.
- Proposition 2: The value of a private business that expects to make the transition to a public company sooner will be higher than the value of an otherwise similar company that will take longer.
 - ▣ Private businesses will be worth more if companies are able to go public earlier in their life cycle.

Private company valuation: Closing thoughts

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- The value of a private business will depend on the potential buyer.
- If you are the seller of a private business, you will maximize value, if you can sell to
 - ▣ A long term investor
 - ▣ Who is well diversified (or whose investors are)
 - ▣ And does not think too highly of you (as a person)
- If you are valuing a private business for legal purposes (tax or divorce court), the assumptions you use and the value you arrive at will depend on which side of the legal divide you are on.
- As a final proposition, always keep in mind that the owner of a private business has the option of investing his wealth in publicly traded stocks. There has to be a relationship between what you can earn on those investments and what you demand as a return on your business.



VALUATION: PACKET 3 REAL OPTIONS, ACQUISITION VALUATION AND VALUE ENHANCEMENT

Aswath Damodaran
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REAL OPTIONS: FACT AND FANTASY

Aswath Damodaran

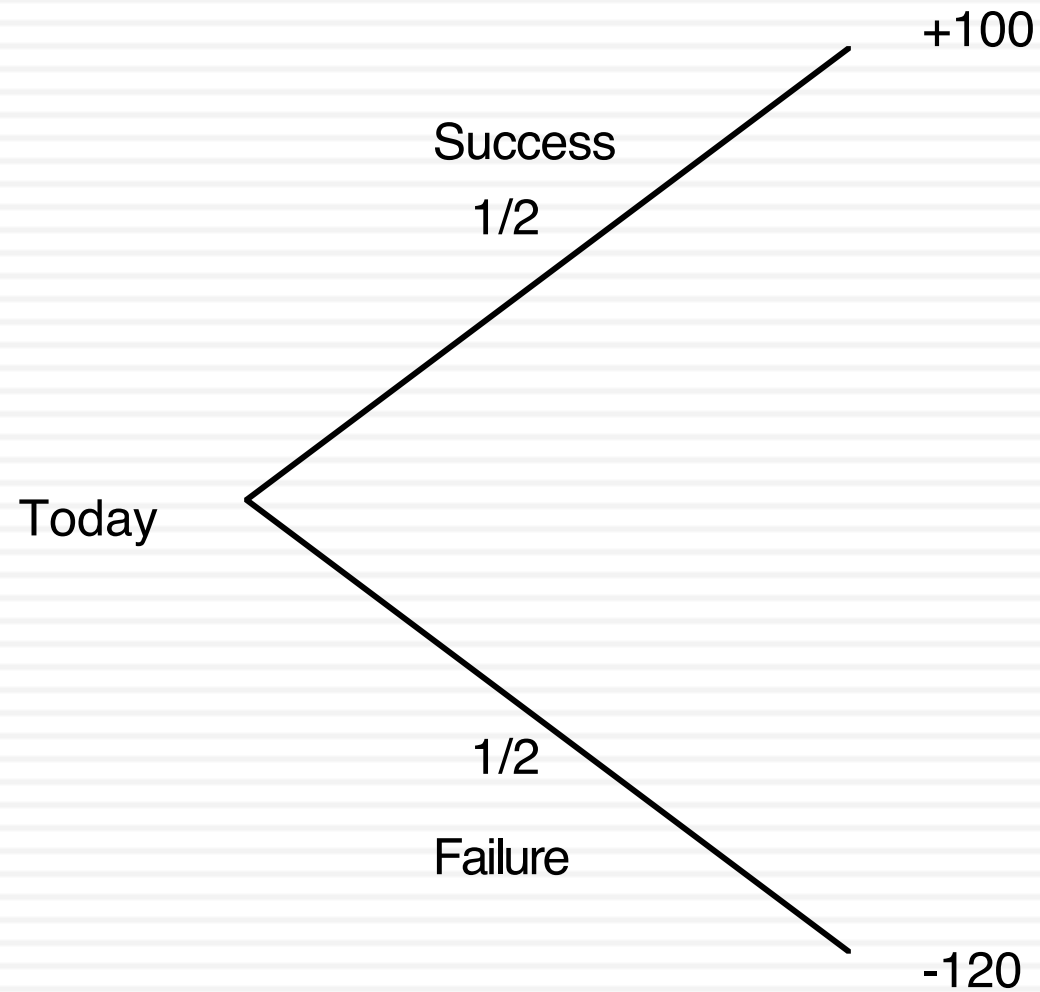
Underlying Theme: Searching for an Elusive Premium

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- Traditional discounted cashflow models under estimate the value of investments, where there are options embedded in the investments to
 - ▣ Delay or defer making the investment (delay)
 - ▣ Adjust or alter production schedules as price changes (flexibility)
 - ▣ Expand into new markets or products at later stages in the process, based upon observing favorable outcomes at the early stages (expansion)
 - ▣ Stop production or abandon investments if the outcomes are unfavorable at early stages (abandonment)
- Put another way, real option advocates believe that you should be paying a premium on discounted cashflow value estimates.

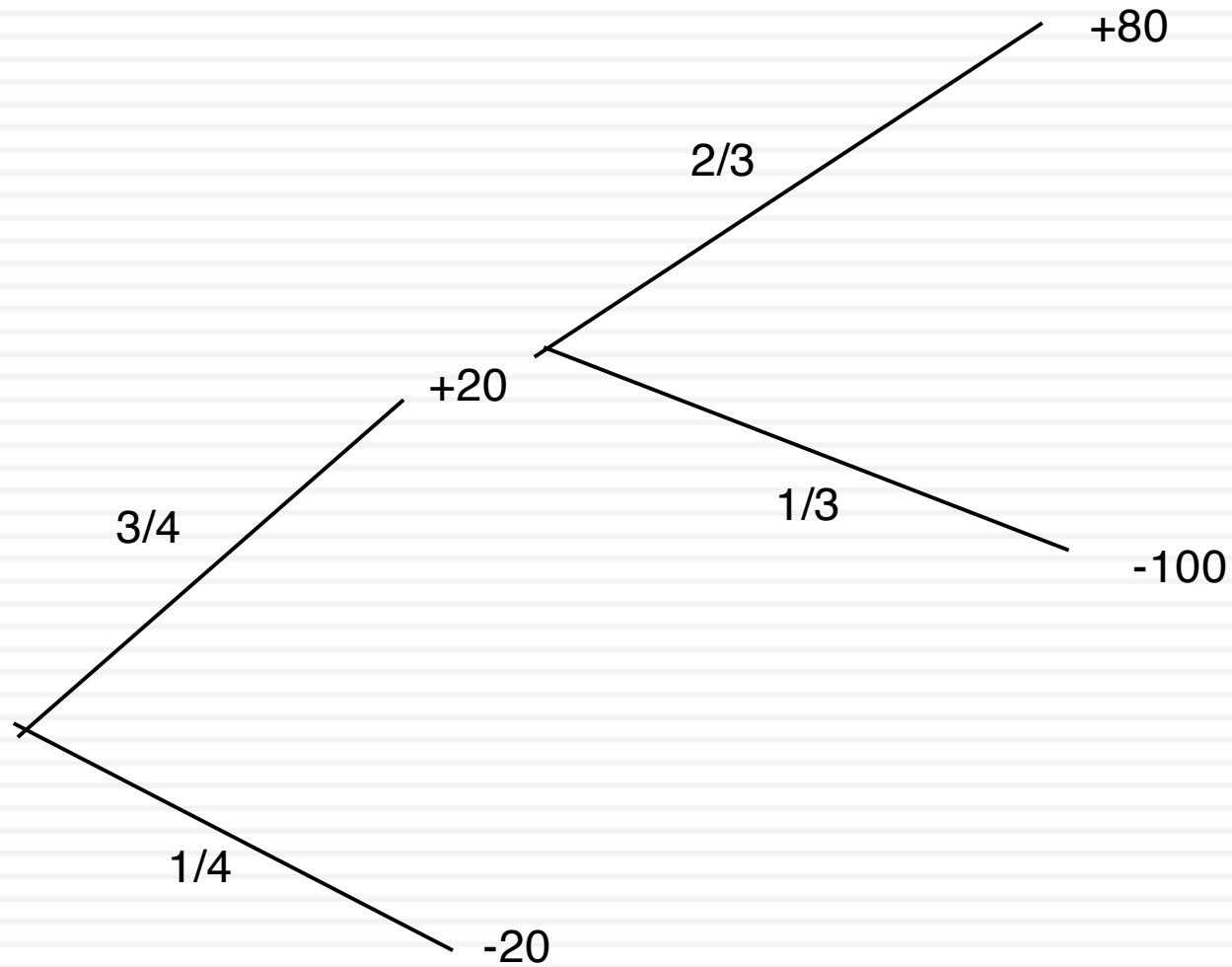
A bad investment...

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Becomes a good one...

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Three Basic Questions

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- When is there a real option embedded in a decision or an asset?
- When does that real option have significant economic value?
- Can that value be estimated using an option pricing model?

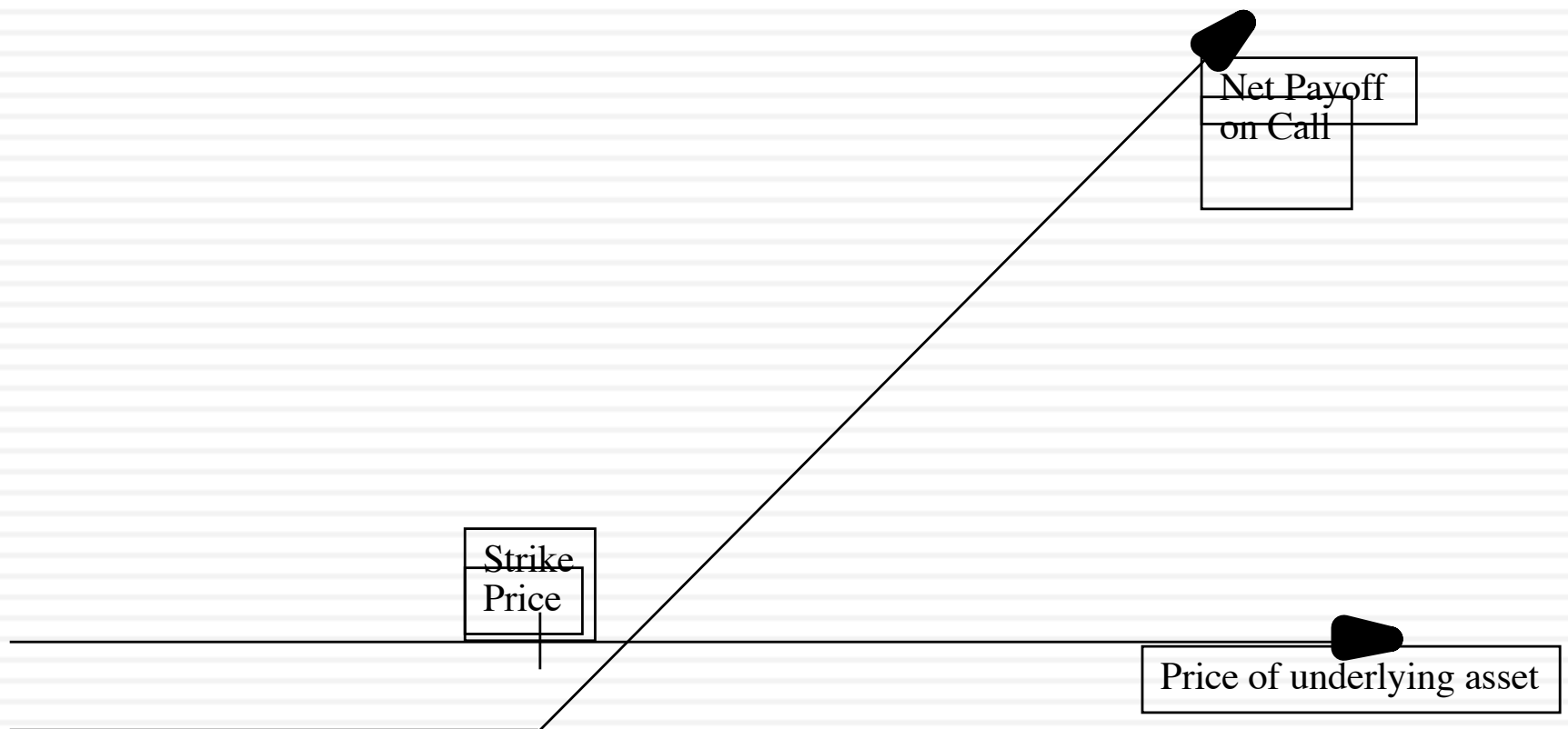
When is there an option embedded in an action?

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- An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- There has to be a clearly defined underlying asset whose value changes over time in unpredictable ways.
- The payoffs on this asset (real option) have to be contingent on an specified event occurring within a finite period.

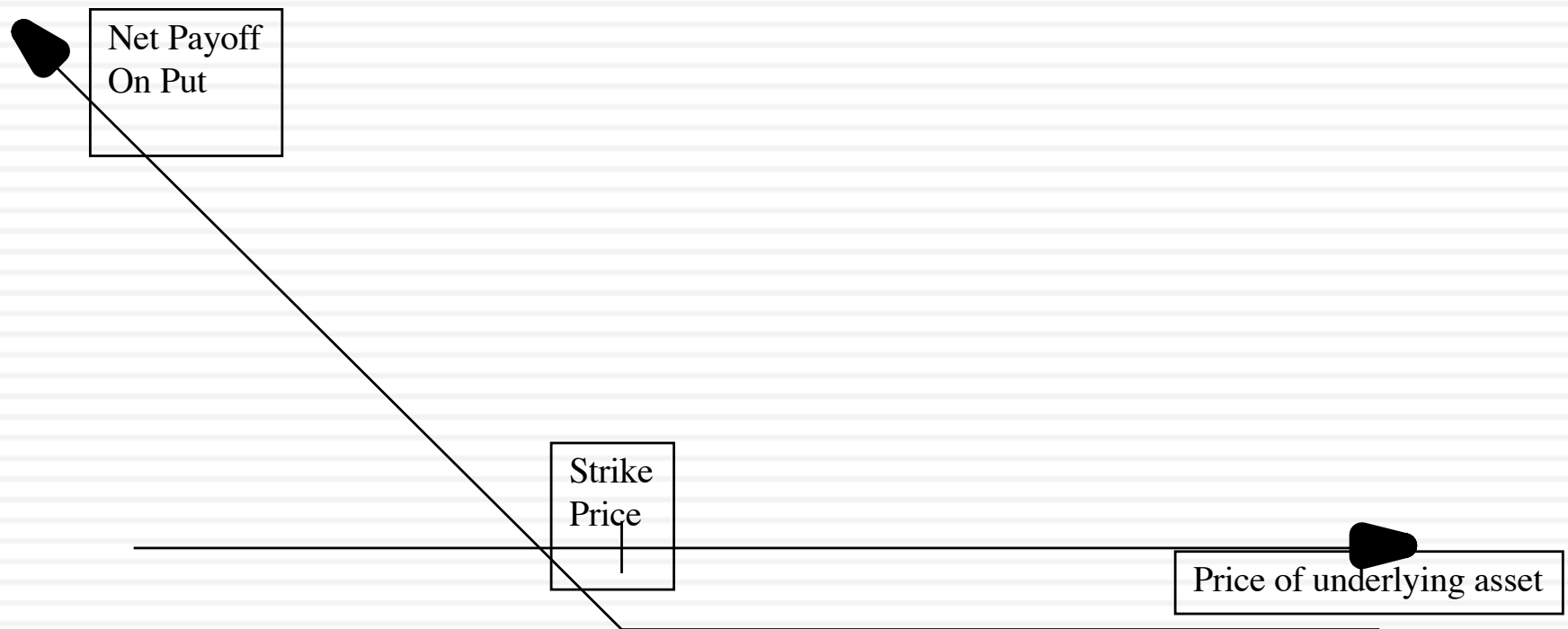
Payoff Diagram on a Call

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Payoff Diagram on Put Option

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When does the option have significant economic value?

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- For an option to have significant economic value, there has to be a restriction on competition in the event of the contingency. In a perfectly competitive product market, no contingency, no matter how positive, will generate positive net present value.
- At the limit, real options are most valuable when you have exclusivity - you and only you can take advantage of the contingency. They become less valuable as the barriers to competition become less steep.

Determinants of option value

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- Variables Relating to Underlying Asset
 - Value of Underlying Asset; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
 - Variance in that value; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
 - Expected dividends on the asset, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- Variables Relating to Option
 - Strike Price of Options; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
 - Life of the Option; both calls and puts benefit from a longer life.
- Level of Interest Rates; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

When can you use option pricing models to value real options?

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- The notion of a replicating portfolio that drives option pricing models makes them most suited for valuing real options where
 - ▣ The underlying asset is traded - this yields not only observable prices and volatility as inputs to option pricing models but allows for the possibility of creating replicating portfolios
 - ▣ An active marketplace exists for the option itself.
 - ▣ The cost of exercising the option is known with some degree of certainty.
- When option pricing models are used to value real assets, we have to accept the fact that
 - ▣ The value estimates that emerge will be far more imprecise.
 - ▣ The value can deviate much more dramatically from market price because of the difficulty of arbitrage.

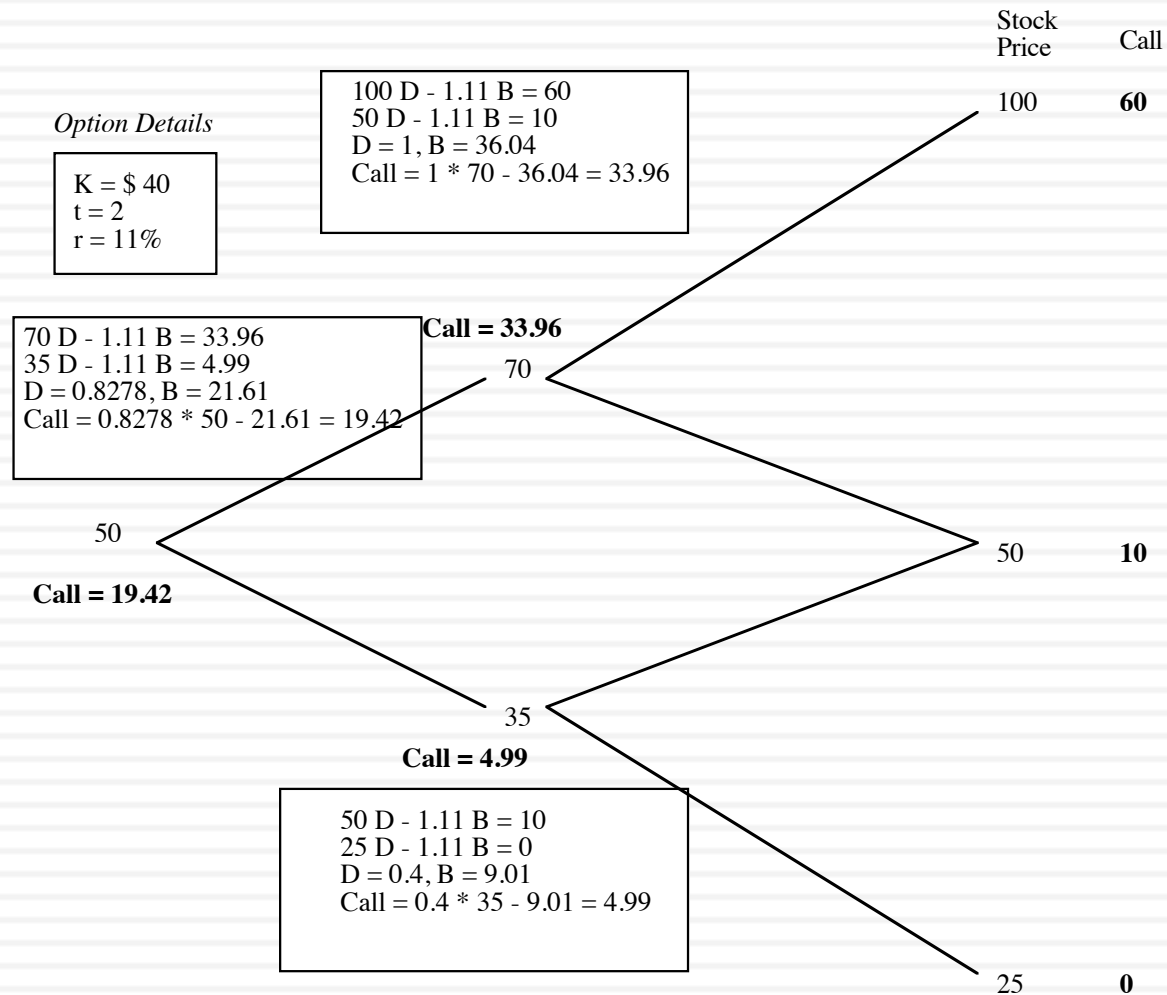
Creating a replicating portfolio

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- The objective in creating a replicating portfolio is to use a combination of riskfree borrowing/lending and the underlying asset to create the same cashflows as the option being valued.
 - ▣ Call = Borrowing + Buying D of the Underlying Stock
 - ▣ Put = Selling Short D on Underlying Asset + Lending
 - ▣ The number of shares bought or sold is called the option delta.
- The principles of arbitrage then apply, and the value of the option has to be equal to the value of the replicating portfolio.

The Binomial Option Pricing Model

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The Limiting Distributions....

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- As the time interval is shortened, the limiting distribution, as $t \rightarrow 0$, can take one of two forms.
 - If as $t \rightarrow 0$, price changes become smaller, the limiting distribution is the normal distribution and the price process is a continuous one.
 - If as $t \rightarrow 0$, price changes remain large, the limiting distribution is the poisson distribution, i.e., a distribution that allows for price jumps.
- The Black-Scholes model applies when the limiting distribution is the normal distribution, and explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

Black and Scholes...

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- The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected.
- The value of a call option in the Black-Scholes model can be written as a function of the following variables:
 - ▣ S = Current value of the underlying asset
 - ▣ K = Strike price of the option
 - ▣ t = Life to expiration of the option
 - ▣ r = Riskless interest rate corresponding to the life of the option
 - ▣ σ^2 = Variance in the $\ln(\text{value})$ of the underlying asset

The Black Scholes Model

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Value of call = $S N(d_1) - K e^{-rt} N(d_2)$

where

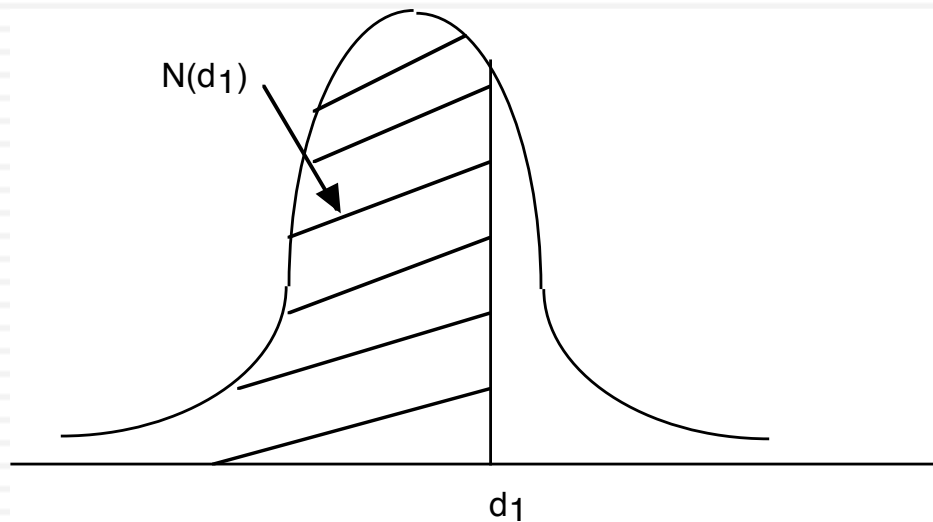
$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

- The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to
 - Buy $N(d_1)$ shares of stock; $N(d_1)$ is called the option delta
 - Borrow $K e^{-rt} N(d_2)$

The Normal Distribution

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d	$N(d)$	d	$N(d)$	d	$N(d)$
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

Adjusting for Dividends

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- If the dividend yield ($y = \text{dividends} / \text{Current value of the asset}$) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

- $C = S e^{-yt} N(d_1) - K e^{-rt} N(d_2)$

where,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r - y + \frac{\sigma^2}{2}) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

- The value of a put can also be derived:

- $P = K e^{-rt} (1 - N(d_2)) - S e^{-yt} (1 - N(d_1))$

Choice of Option Pricing Models

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- Most practitioners who use option pricing models to value real options argue for the binomial model over the Black-Scholes and justify this choice by noting that
 - ▣ Early exercise is the rule rather than the exception with real options
 - ▣ Underlying asset values are generally discontinuous.
- If you can develop a binomial tree with outcomes at each node, it looks a great deal like a decision tree from capital budgeting. The question then becomes when and why the two approaches yield different estimates of value.

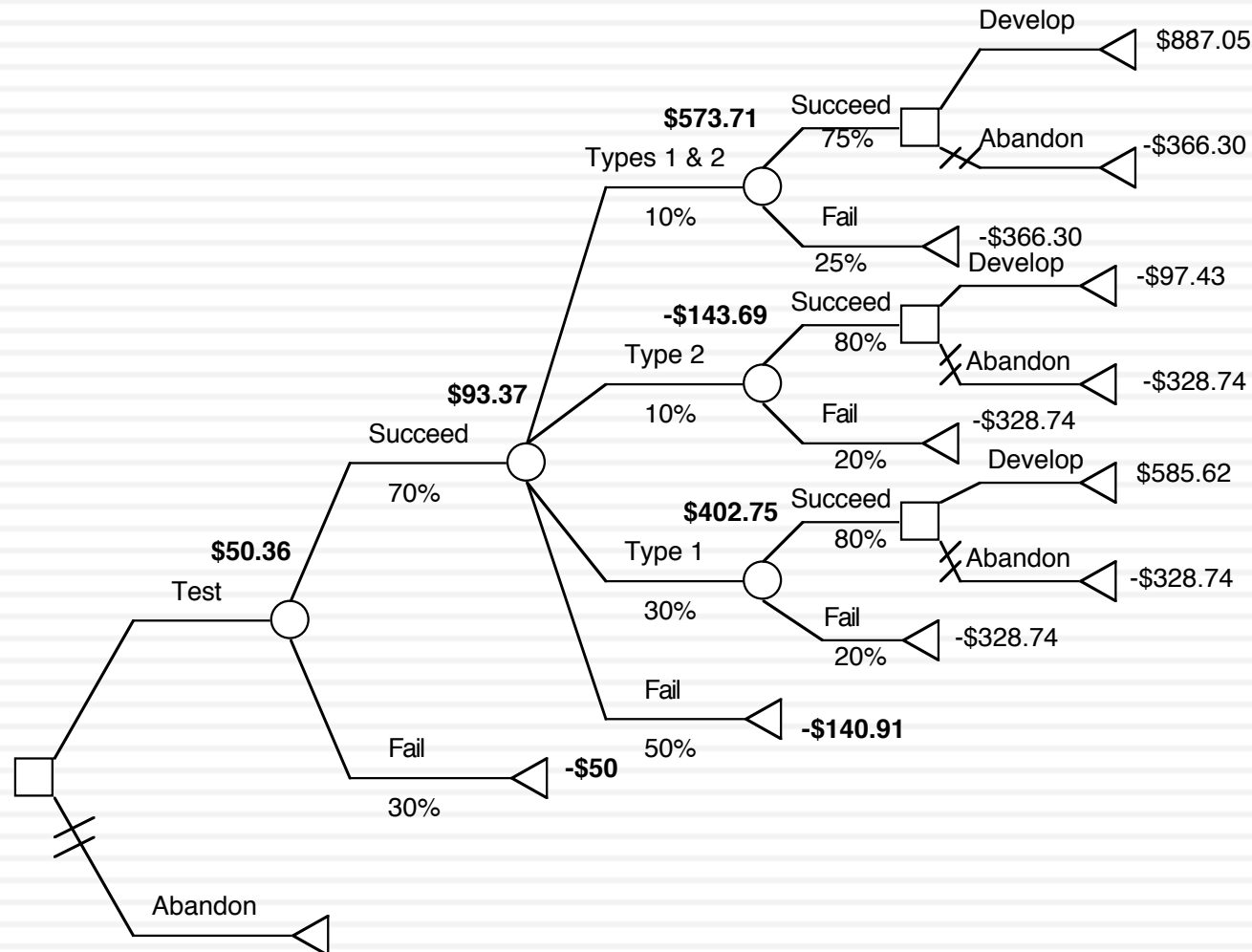
The Decision Tree Alternative

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- Traditional decision tree analysis tends to use
 - ▣ One cost of capital to discount cashflows in each branch to the present
 - ▣ Probabilities to compute an expected value
 - ▣ These values will generally be different from option pricing model values
- If you modified decision tree analysis to
 - ▣ Use different discount rates at each node to reflect where you are in the decision tree (This is the Copeland solution) (or)
 - ▣ Use the riskfree rate to discount cashflows in each branch, estimate the probabilities to estimate an expected value and adjust the expected value for the market risk in the investment
- Decision Trees could yield the same values as option pricing models

A decision tree valuation of a pharmaceutical company with one drug in the FDA pipeline...

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Key Tests for Real Options

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- Is there an option embedded in this asset/ decision?
 - ▣ Can you identify the underlying asset?
 - ▣ Can you specify the contingency under which you will get payoff?
- Is there exclusivity?
 - ▣ If yes, there is option value.
 - ▣ If no, there is none.
 - ▣ If in between, you have to scale value.
- Can you use an option pricing model to value the real option?
 - ▣ Is the underlying asset traded?
 - ▣ Can the option be bought and sold?
 - ▣ Is the cost of exercising the option known and clear?

I. Options in Projects/Investments/Acquisitions

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- One of the limitations of traditional investment analysis is that it is static and does not do a good job of capturing the options embedded in investment.
 - The first of these options is the option to delay taking a investment, when a firm has exclusive rights to it, until a later date.
 - The second of these options is taking one investment may allow us to take advantage of other opportunities (investments) in the future
 - The last option that is embedded in projects is the option to abandon a investment, if the cash flows do not measure up.
- These options all add value to projects and may make a “bad” investment (from traditional analysis) into a good one.

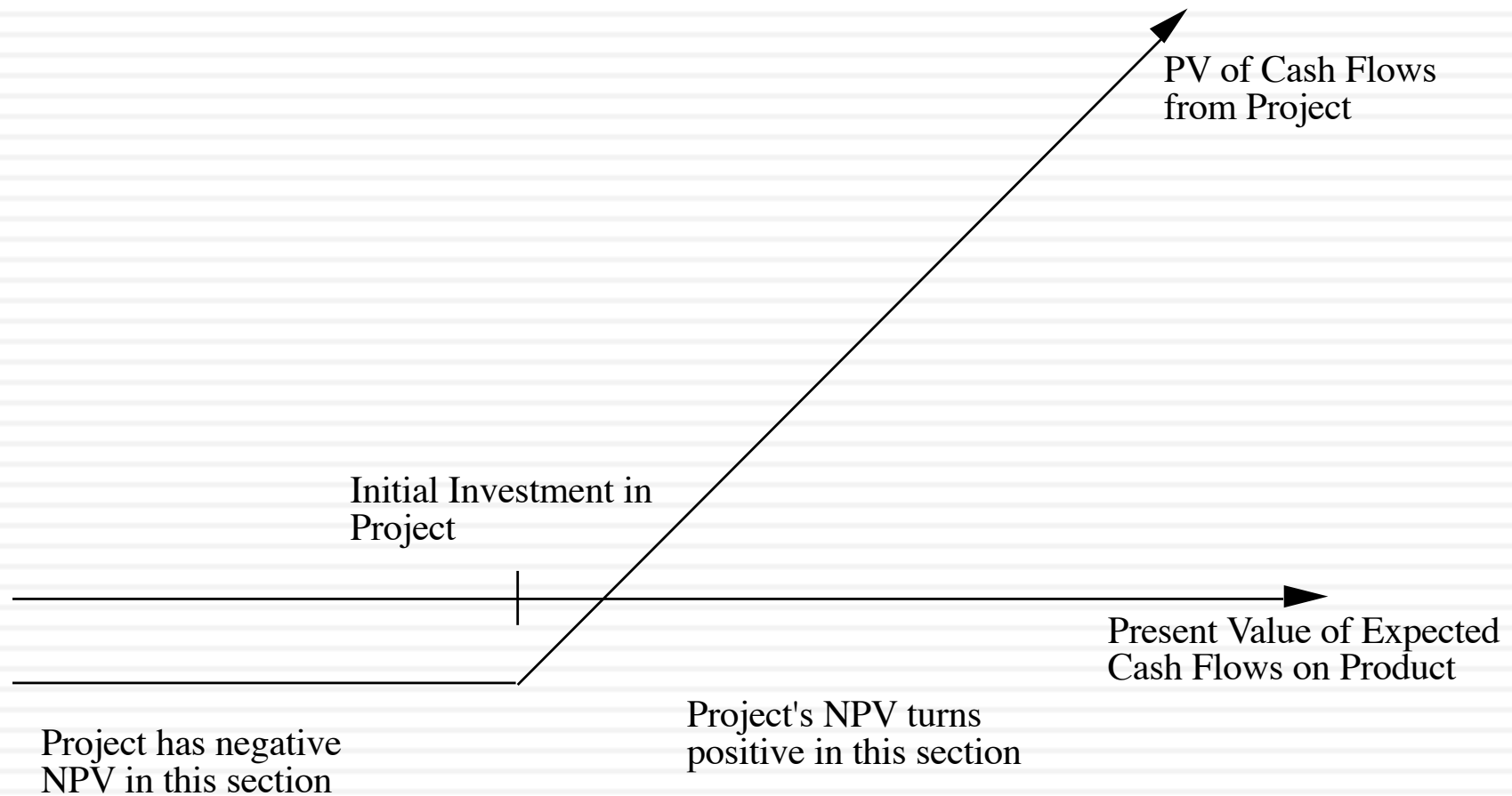
A. The Option to Delay

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- When a firm has exclusive rights to a project or product for a specific period, it can delay taking this project or product until a later date.
- A traditional investment analysis just answers the question of whether the project is a “good” one if taken today.
- Thus, the fact that a project does not pass muster today (because its NPV is negative, or its IRR is less than its hurdle rate) does not mean that the rights to this project are not valuable.

Valuing the Option to Delay a Project

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Example 1: Valuing product patents as options

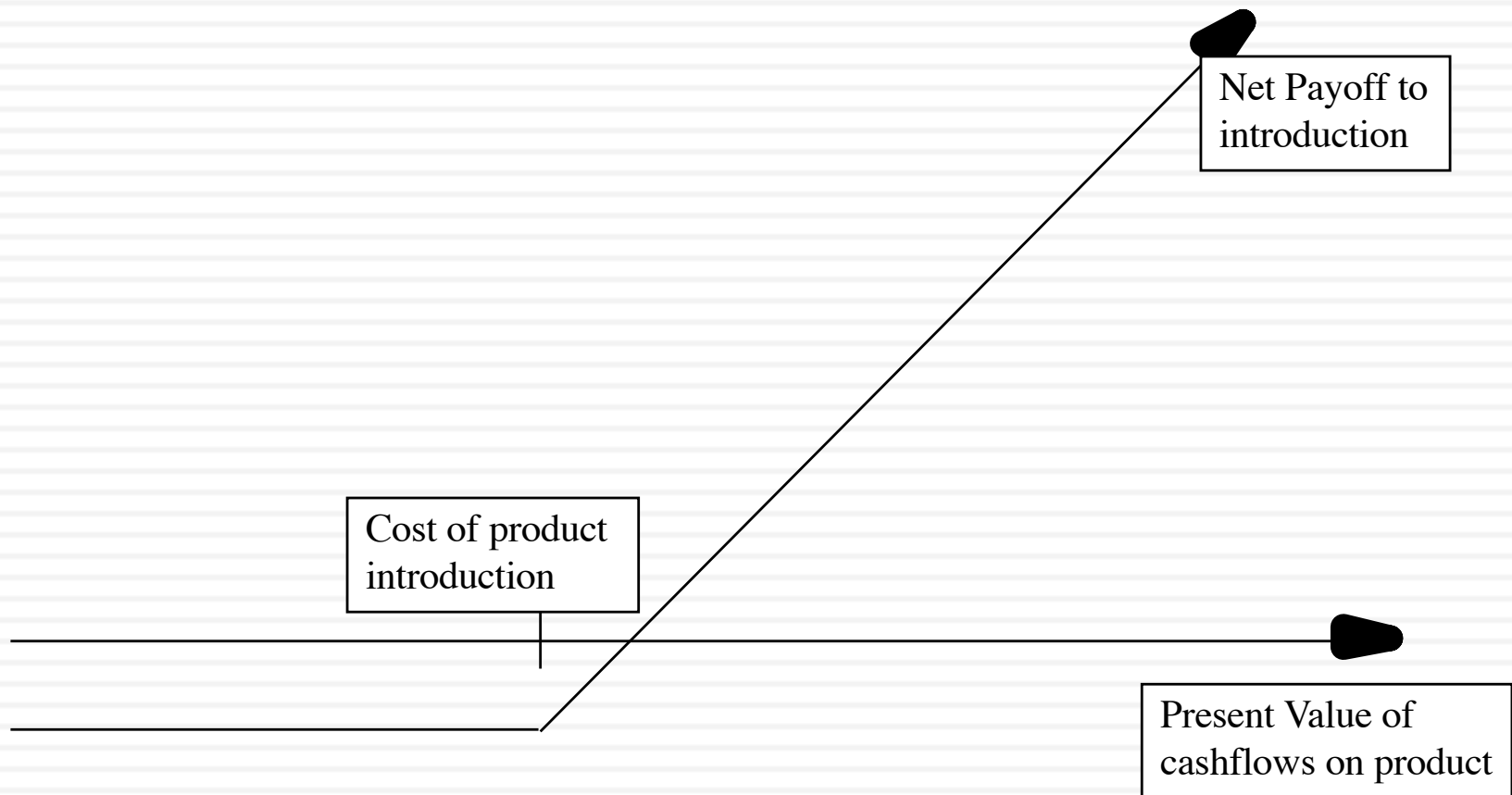
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- A product patent provides the firm with the right to develop the product and market it.
- It will do so only if the present value of the expected cash flows from the product sales exceed the cost of development.
- If this does not occur, the firm can shelve the patent and not incur any further costs.
- If I is the present value of the costs of developing the product, and V is the present value of the expected cashflows from development, the payoffs from owning a product patent can be written as:

$$\begin{aligned} \text{Payoff from owning a product patent} &= V - I && \text{if } V > I \\ &= 0 && \text{if } V \leq I \end{aligned}$$

Payoff on Product Option

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Obtaining Inputs for Patent Valuation

Input	Estimation Process
1. Value of the Underlying Asset	<ul style="list-style-type: none">• Present Value of Cash Inflows from taking project now• This will be noisy, but that adds value.
2. Variance in value of underlying asset	<ul style="list-style-type: none">• Variance in cash flows of similar assets or firms• Variance in present value from capital budgeting simulation.
3. Exercise Price on Option	<ul style="list-style-type: none">• Option is exercised when investment is made.• Cost of making investment on the project ; assumed to be constant in present value dollars.
4. Expiration of the Option	<ul style="list-style-type: none">• Life of the patent
5. Dividend Yield	<ul style="list-style-type: none">• Cost of delay• Each year of delay translates into one less year of value-creating cashflows $\text{Annual cost of delay} = \frac{1}{n}$

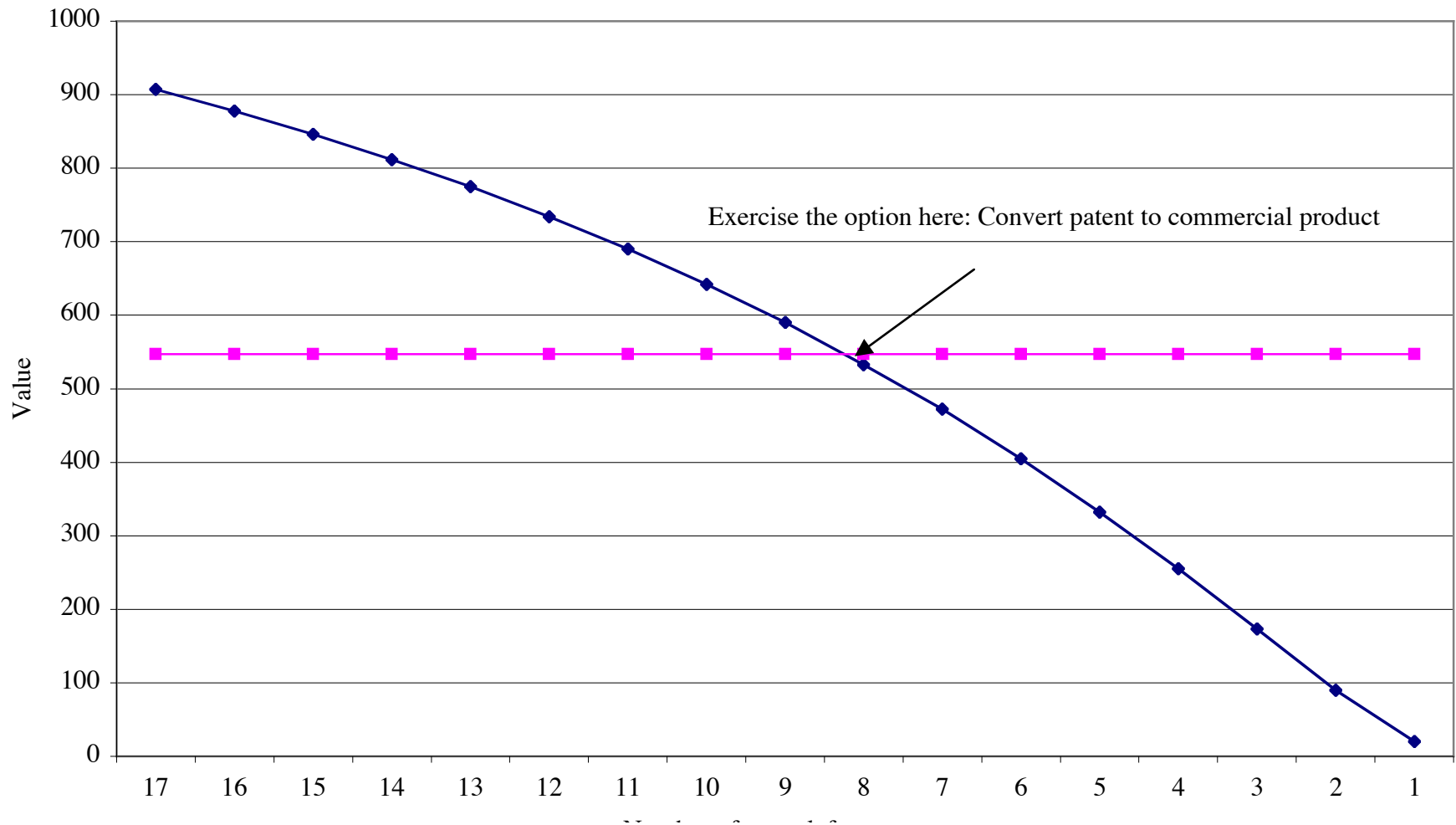
Valuing a Product Patent: Avonex

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- Biogen, a bio-technology firm, has a patent on Avonex, a drug to treat multiple sclerosis, for the next 17 years, and it plans to produce and sell the drug by itself.
 - The key inputs on the drug are as follows:
 - ▣ PV of Cash Flows from Introducing the Drug Now = S = \$ 3.422 billion
 - ▣ PV of Cost of Developing Drug for Commercial Use = K = \$ 2.875 billion
 - ▣ Patent Life = t = 17 years Riskless Rate = r = 6.7% (17-year T.Bond rate)
 - ▣ Variance in Expected Present Values = σ^2 = 0.224 (Industry average firm variance for bio-tech firms)
 - ▣ Expected Cost of Delay = y = 1/17 = 5.89%
 - The output from the option pricing model
 - ▣ d1 = 1.1362 N(d1) = 0.8720
 - ▣ d2 = -0.8512 N(d2) = 0.2076
- Call Value= $3,422 \exp^{(-0.0589)(17)} (0.8720) - 2,875 \exp^{(-0.067)(17)} (0.2076) = \$ 907 \text{ million}$

The Optimal Time to Exercise

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Valuing a firm with patents

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- The value of a firm with a substantial number of patents can be derived using the option pricing model.

Value of Firm = Value of commercial products (using DCF value
+ Value of existing patents (using option pricing)
+ (Value of New patents that will be obtained in the
future – Cost of obtaining these patents)

- The last input measures the efficiency of the firm in converting its R&D into commercial products. If we assume that a firm earns its cost of capital from research, this term will become zero.
- If we use this approach, we should be careful not to double count and allow for a high growth rate in cash flows (in the DCF valuation).

Value of Biogen's existing products

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- Biogen had two commercial products (a drug to treat Hepatitis B and Intron) at the time of this valuation that it had licensed to other pharmaceutical firms.
- The license fees on these products were expected to generate \$ 50 million in after-tax cash flows each year for the next 12 years.
- To value these cash flows, which were guaranteed contractually, the pre-tax cost of debt of the guarantors was used:

$$\begin{aligned}\text{Present Value of License Fees} &= \$ 50 \text{ million } (1 - (1.07)^{-12}) / .07 \\ &= \$ 397.13 \text{ million}\end{aligned}$$

Value of Biogen's Future R&D

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- Biogen continued to fund research into new products, spending about \$ 100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years, and 5% thereafter.
- It was assumed that every dollar invested in research would create \$ 1.25 in value in patents (valued using the option pricing model described above) for the next 10 years, and break even after that (i.e., generate \$ 1 in patent value for every \$ 1 invested in R&D).
- There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.

Value of Future R&D

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Yr	Value of Patents	R&D Cost	Excess Value	PV (at 15%)
1	\$ 150.00	\$ 120.00	\$ 30.00	\$ 26.09
2	\$ 180.00	\$ 144.00	\$ 36.00	\$ 27.22
3	\$ 216.00	\$ 172.80	\$ 43.20	\$ 28.40
4	\$ 259.20	\$ 207.36	\$ 51.84	\$ 29.64
5	\$ 311.04	\$ 248.83	\$ 62.21	\$ 30.93
6	\$ 373.25	\$ 298.60	\$ 74.65	\$ 32.27
7	\$ 447.90	\$ 358.32	\$ 89.58	\$ 33.68
8	\$ 537.48	\$ 429.98	\$ 107.50	\$ 35.14
9	\$ 644.97	\$ 515.98	\$ 128.99	\$ 36.67
10	\$ 773.97	\$ 619.17	\$ 154.79	\$ 38.26
				\$ 318.30

Value of Biogen

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- The value of Biogen as a firm is the sum of all three components – the present value of cash flows from existing products, the value of Avonex (as an option) and the value created by new research:

$$\begin{aligned}\text{Value} &= \text{Existing products} + \text{Existing Patents} + \text{Value: Future R\&D} \\ &= \$ 397.13 \text{ million} + \$ 907 \text{ million} + \$ 318.30 \text{ million} \\ &= \$1622.43 \text{ million}\end{aligned}$$

- Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.50 million) to arrive at a value per share:

- ▣ Value per share = \$ 1,622.43 million / 35.5 = \$ 45.70