Not all risk is created equal…

- Estimation versus Economic uncertainty
  - Estimation uncertainty reflects the possibility that you could have the "wrong model" or estimated inputs incorrectly within this model.
  - Economic uncertainty comes the fact that markets and economies can change over time and that even the best models will fail to capture these unexpected changes.
- Micro uncertainty versus Macro uncertainty
  - Micro uncertainty refers to uncertainty about the potential market for a firm's products, the competition it will face and the quality of its management team.
  - Macro uncertainty reflects the reality that your firm's fortunes can be affected by changes in the macro economic environment.
- Discrete versus continuous uncertainty
  - Discrete risk: Risks that lie dormant for periods but show up at points in time. (Examples: A drug working its way through the FDA pipeline may fail at some stage of the approval process or a company in Venezuela may be nationalized)
  - Continuous risk: Risks changes in interest rates or economic growth occur continuously and affect value as they happen.

Risk and Cost of Equity: The role of the marginal investor

- Not all risk counts: While the notion that the cost of equity should be higher for riskier investments and lower for safer investments is intuitive, what risk should be built into the cost of equity is the question.
- Risk through whose eyes? While risk is usually defined in terms of the variance of actual returns around an expected return, risk and return models in finance assume that the risk that should be rewarded (and thus built into the discount rate) in valuation should be the risk perceived by the marginal investor in the investment
- The diversification effect: Most risk and return models in finance also assume that the marginal investor is well diversified, and that the only risk that he or she perceives in an investment is risk that cannot be diversified away (i.e, market or non-diversifiable risk). In effect, it is primarily economic, macro, continuous risk that should be incorporated into the cost of equity.

The Cost of Equity: Competing " Market Risk" Models

Model	Expected Return	Inputs Needed
CAPM	$E(R) = Rf + \beta (R_m - R_f)$	Riskfree Rate Beta relative to market portfolio Market Risk Premium
APM	$E(R) = Rf + \Sigma\beta_j (R_j - R_f)$	Riskfree Rate; # of Factors; Betas relative to each factor Factor risk premiums
Multi factor	$E(R) = Rf + \Sigma\beta_j (R_j - R_f)$	Riskfree Rate; Macro factors Betas relative to macro factors Macro economic risk premiums
Proxy	$E(R) = a + \Sigma \beta_j Y_j$	Proxies Regression coefficients

Classic Risk & Return: Cost of Equity

- In the CAPM, the cost of equity:

  Cost of Equity = Riskfree Rate + Equity Beta * (Equity Risk Premium)

- In APM or Multi-factor models, you still need a risk free rate, as well as betas and risk premiums to go with each factor.
- To use any risk and return model, you need
  - A risk free rate as a base
  - A single equity risk premium (in the CAPM) or factor risk premiums, in the the multi-factor models
  - A beta (in the CAPM) or betas (in multi-factor models)

Discount Rates I

The Riskfree Rate

The Risk Free Rate: Laying the Foundations

- On a riskfree investment, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have
  - No default risk
  - No reinvestment risk
- It follows then that if asked to estimate a risk free rate:

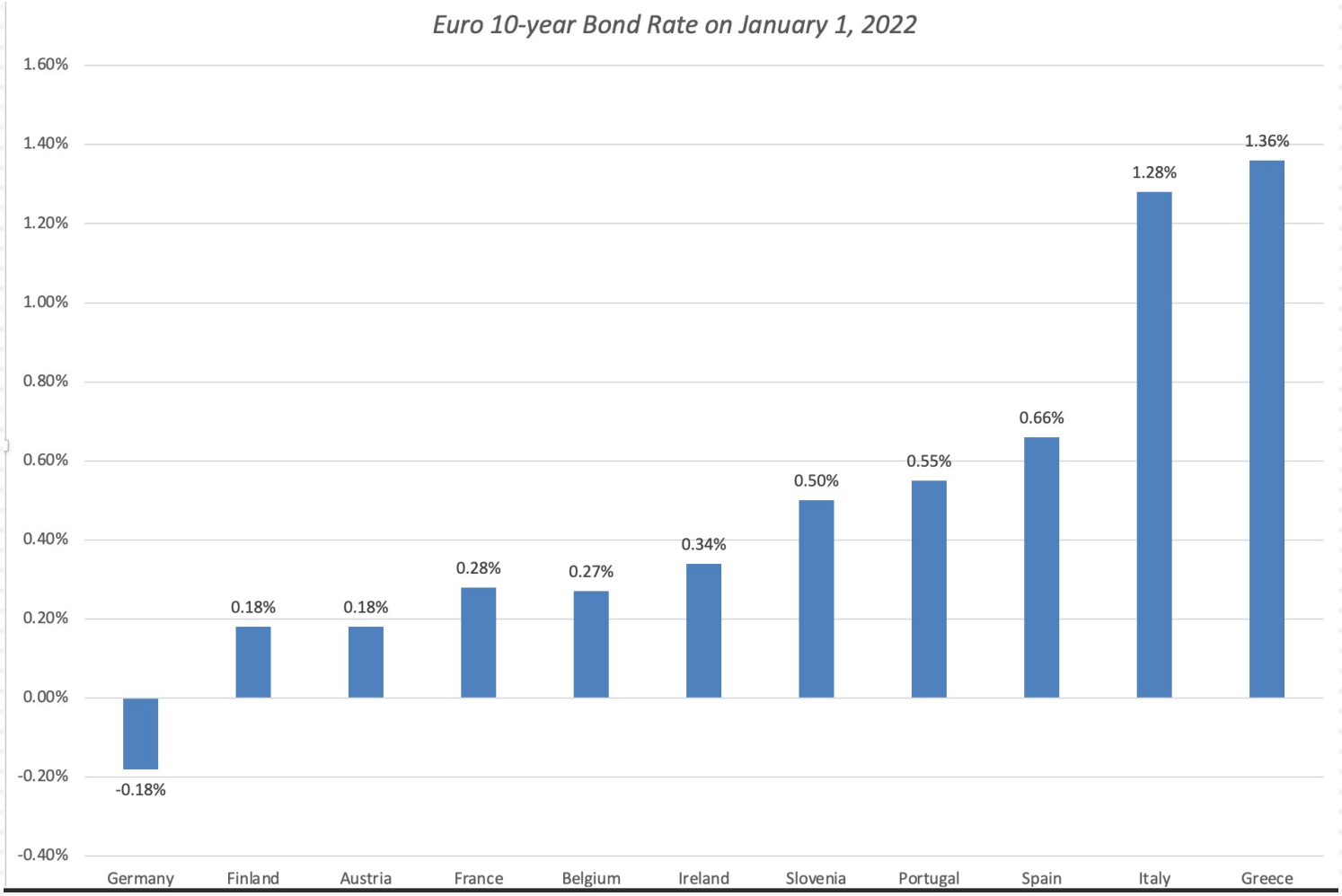
1. Time horizon matters: Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time.
2. Currencies matter: A risk free rate is currency-specific and can be very different for different currencies.
3. Not all government securities are riskfree: Some governments face default risk and the rates on bonds issued by them will not be riskfree.

Test 1: A riskfree rate in US dollars!

- In valuation, we estimate cash flows forever (or at least for very long time periods). The right risk free rate to use in valuing a company in US dollars would be
  a. A three-month Treasury bill rate (0.06%)
  b. A ten-year Treasury bond rate (1.51%)
  c. A thirty-year Treasury bond rate (2.0%)
  d. A TIPs (inflation-indexed treasury) rate (0.15%)
  e. None of the above

  What are we implicitly assuming about the US treasury when we use any of the treasury numbers?

Test 2: A Riskfree Rate in Euros?



Test 3: A Riskfree Rate in Indian Rupees

- The Indian government had 10-year Rupee bonds outstanding, with a yield to maturity of about 6.45% on January 1, 2022.
- In January 2022, the Indian government had a local currency sovereign rating of Baa3. The typical default spread (over a default free rate) for Baa3 rated country bonds in early 2022 was 1.87%. The risk free rate in Indian Rupees is
  a. The yield to maturity on the 10-year bond (6.45%)
  b. The yield to maturity on the 10-year bond + Default spread (8.32%)
  c. The yield to maturity on the 10-year bond – Default spread (4.58%)
  d. None of the above

Sovereign Default Spread: Three paths to the same destination…

- Sovereign dollar or euro denominated bonds: Find sovereign bonds denominated in US dollars, issued by an emerging sovereign.
  - Default spread = Emerging Govt Bond Rate (in US $) – US Treasury Bond rate with same maturity.
- CDS spreads: Obtain the traded value for a sovereign Credit Default Swap (CDS) for the emerging government.
  - Default spread = Sovereign CDS spread (with perhaps an adjustment for CDS market frictions).
- Sovereign-rating based spread: For countries which don't issue dollar denominated bonds or have a CDS spread, you have to use the average spread for other countries with the same sovereign rating.

Local Currency Government Bond Rates – January 2022

Currency	Govt Bond Rate 12/31/21	Currency	Govt Bond Rate 12/31/21	Currency	Govt Bond Rate 12/31/21
Australian $	1.67%	Indian Rupee	6.45%	Qatari Dinar	2.11%
Brazilian Reai	10.31%	Indonesian Rupiah	1.02%	Romanian Lev	5.21%
British Pound	0.97%	Israeli Shekel	1.02%	Russian Ruble	8.42%
Bulgarian Lev	0.70%	Japanese Yen	0.07%	Singapore $	1.67%
Canadian $	1.43%	Kenyan Shilling	12.55%	South African Rand	9.35%
Chilean Peso	5.72%	Korean Won	2.26%	Swedish Krona	0.21%
Chinese Yuan	2.85%	Malyasian Ringgit	3.59%	Swiss Franc	-0.14%
Colombian Peso	8.14%	Mexican Peso	7.56%	Taiwanese $	0.69%
Croatian Kuna	0.63%	Nigerian Naira	12.61%	Thai Baht	1.95%
Czech Koruna	2.98%	Norwegian Krone	1.72%	Turkish Lira	23.33%
Danish Krone	0.10%	NZ $	2.33%	US $	1.51%
Euro	-0.18%	Pakistani Rupee	11.54%	Vietnamese Dong	2.12%
HK $	1.45%	Peruvian Sol	5.87%	Zambian kwacha	31.50%
Hungarian Forint	4.58%	Phillipine Peso	4.90%		
Iceland Krona	4.15%	Polish Zloty	3.66%		

Approach 1: Default spread from Government Bonds

Country	$ Bond Rate	Riskfree Rate	Default Spread
	$ Bonds		
Peru	3.66%	1.51%	2.15%
Brazil	3.70%	1.51%	2.19%
Colombia	2.33%	1.51%	0.82%
Poland	1.80%	1.51%	0.29%
Turkey	5.43%	1.51%	3.92%
Mexico	2.01%	1.51%	0.50%
Russia	2.48%	1.51%	0.97%
	Euro Bonds		
Bulgaria	1.00%	-0.18%	1.18%

Approach 2: CDS Spreads – January 2022

Country	1/1/22	CDS Spread net of US	Country	1/1/22	CDS Spread net of US	Country	1/1/22	CDS Spread net of US
Abu Dhabi	0.77%	0.58%	Greece	1.69%	1.50%	Panama	1.26%	1.07%
Algeria	1.10%	0.91%	Guatamela	2.06%	1.87%	Peru	1.31%	1.12%
Angola	5.94%	5.75%	Hong Kong	0.41%	0.22%	Philippines	0.92%	0.73%
Argentina	23.32%	23.13%	Hungary	0.69%	0.50%	Poland	0.68%	0.49%
Australia	0.23%	0.04%	Iceland	0.73%	0.54%	Portugal	0.56%	0.37%
Austria	0.19%	0.00%	India	1.44%	1.25%	Qatar	0.74%	0.55%
Bahrain	3.40%	3.21%	Indonesia	1.36%	1.17%	Romania	1.24%	1.05%
Belgium	0.21%	0.02%	Iraq	5.63%	5.44%	Russia	1.70%	1.51%
Brazil	2.91%	2.72%	Ireland	0.27%	0.08%	Rwanda	3.36%	3.17%
Bulgaria	0.81%	0.62%	Israel	0.72%	0.53%	Saudi Arabia	0.88%	0.69%
Cameroon	3.56%	3.37%	Italy	1.41%	1.22%	Senegal	2.66%	2.47%
Canada	0.28%	0.09%	Japan	0.33%	0.14%	Serbia	1.37%	1.18%
Chile	1.25%	1.06%	Kazakhstan	0.99%	0.80%	Slovakia	0.63%	0.44%
China	0.74%	0.55%	Kenya	4.44%	4.25%	Slovenia	0.87%	0.68%
Colombia	2.77%	2.58%	Korea	0.35%	0.16%	South Africa	2.85%	2.66%
Costa Rica	3.92%	3.73%	Kuwait	0.86%	0.67%	Spain	0.60%	0.41%
Croatia	1.11%	0.92%	Latvia	0.74%	0.55%	Sri Lanka	19.69%	19.50%
Cyprus	0.74%	0.55%	Lebanon	NA	NA	Sweden	0.19%	0.00%
Czech Republic	0.47%	0.28%	Lithuania	0.79%	0.60%	Switzerland	0.11%	0.00%
Denmark	0.15%	0.00%	Malaysia	0.81%	0.62%	Thailand	0.52%	0.33%
Dubai	1.33%	1.14%	Mexico	1.58%	1.39%	Tunisia	8.82%	8.63%
Ecuador	7.57%	7.38%	Morocco	1.32%	1.13%	Turkey	5.51%	5.32%
Egypt	5.74%	5.55%	Netherlands	0.19%	0.00%	Ukraine	6.17%	5.98%
El Salvador	18.33%	18.14%	New Zealand	0.21%	0.02%	United Kingdom	0.18%	0.00%
Estonia	0.85%	0.66%	Nicaragua	4.36%	4.17%	United States	0.19%	0.00%
Ethiopia	20.40%	20.21%	Nigeria	5.53%	5.34%	Uruguay	1.46%	1.27%
Finland	0.20%	0.01%	Norway	0.19%	0.00%	Venezuela	NA	NA
France	0.34%	0.15%	Oman	3.19%	3.00%	Vietnam	1.56%	1.37%
Germany	0.18%	0.00%	Pakistan	3.67%	3.48%	Zambia	NA	NA

Approach 3: Typical Default Spreads: January 2022

S&P Sovereign Rating	Moody's Sovereign Rating	Default Spread
AAA	Aaa	0.00%
AA+	Aa1	0.34%
AA	Aa2	0.42%
AA-	Aa3	0.51%
A+	A1	0.60%
A	A2	0.72%
A-	A3	1.02%
BBB+	Baa1	1.36%
BBB	Baa2	1.62%
BBB-	Baa3	1.87%
BB+	Ba1	2.13%
BB	Ba2	2.56%
BB	Ba3	3.06%
B+	B1	3.83%
B	B2	4.68%
B-	B3	5.53%
CCC+	Caa1	6.38%
CCC	Caa2	7.66%
CCC-	Caa3	8.51%
CC+	Ca1	10.21%
CC	Ca2	12.88%
CC-	Ca3	14.50%
C+	C1	15.50%
C	C2	17.00%
C-	C3	19.00%

Getting to a risk free rate in a currency: Example

- The Brazilian government bond rate in nominal reais on January 1, 2022, was 10.31%. To get to a riskfree rate in nominal reais, we can use one of three approaches.
  - Approach 1: Government Bond spread
    - The 2032 Brazil bond, denominated in US dollars, has a spread of 2.19% over the US treasury bond rate.
    - Riskfree rate in $R = 10.31% - 2.19% = 8.12%
  - Approach 2: The CDS Spread
    - The CDS spread for Brazil, adjusted for the US CDS spread was 2.72%.
    - Riskfree rate in $R = 10.31% - 2.72% = 7.59%
  - Approach 3: The Rating based spread
    - Brazil has a Ba2 local currency rating from Moody's. The default spread for that rating is 2.56%
    - Riskfree rate in $R = 10.31% - 2.56% = 7.75%

Test 4: A Real Riskfree Rate

- In some cases, you may want a riskfree rate in real terms (in real terms) rather than nominal terms.
- To get a real riskfree rate, you would like a security with no default risk and a guaranteed real return. Treasury indexed securities offer this combination.
- In January 2022, the yield on a 10-year indexed treasury bond was 0.15%. Which of the following statements would you subscribe to?
  a. This (0.15%) is the real riskfree rate to use, if you are valuing US companies in real terms.
  b. This (0.15%) is the real riskfree rate to use, anywhere in the world

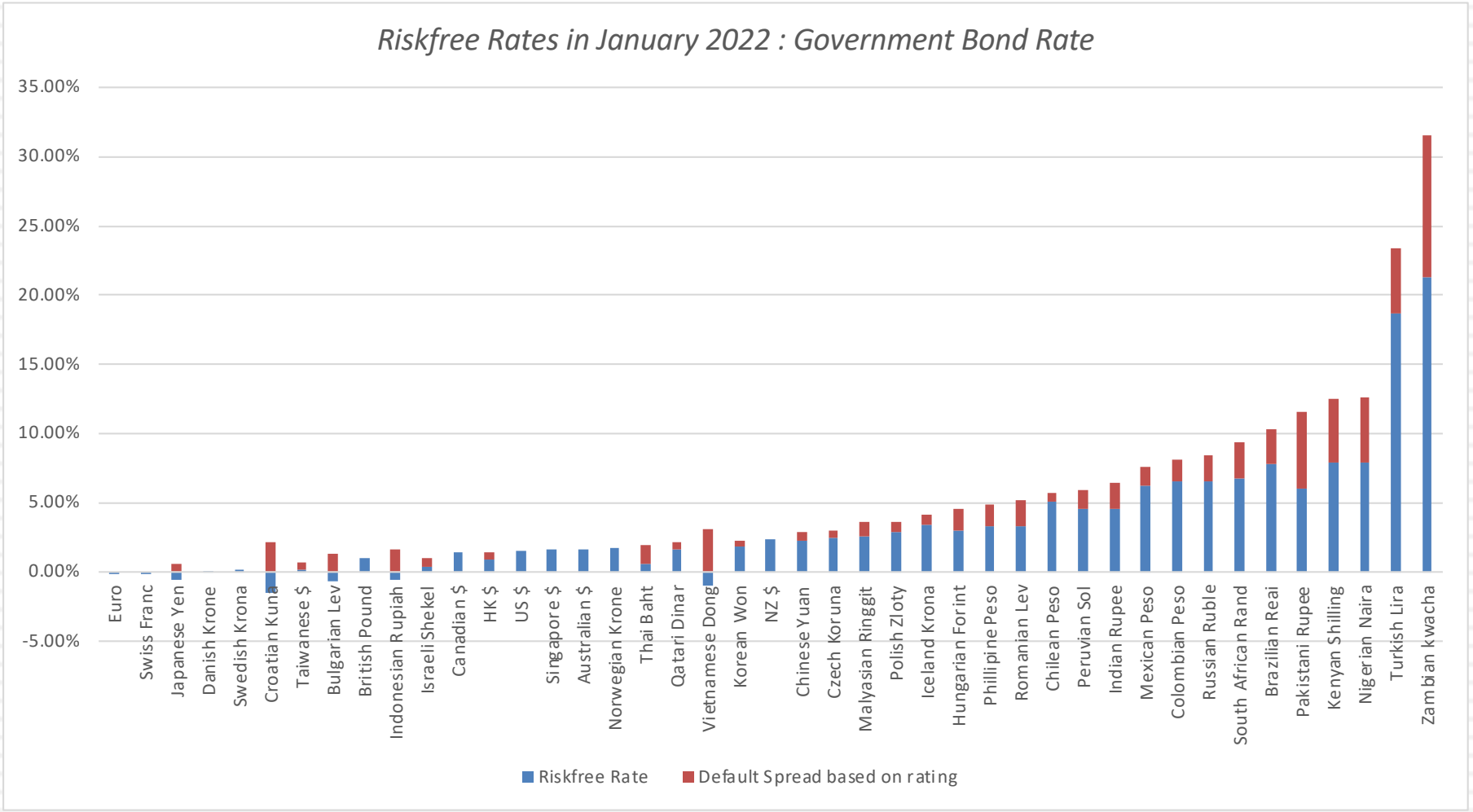
  Explain.

No default free entity: Choices with riskfree rates….

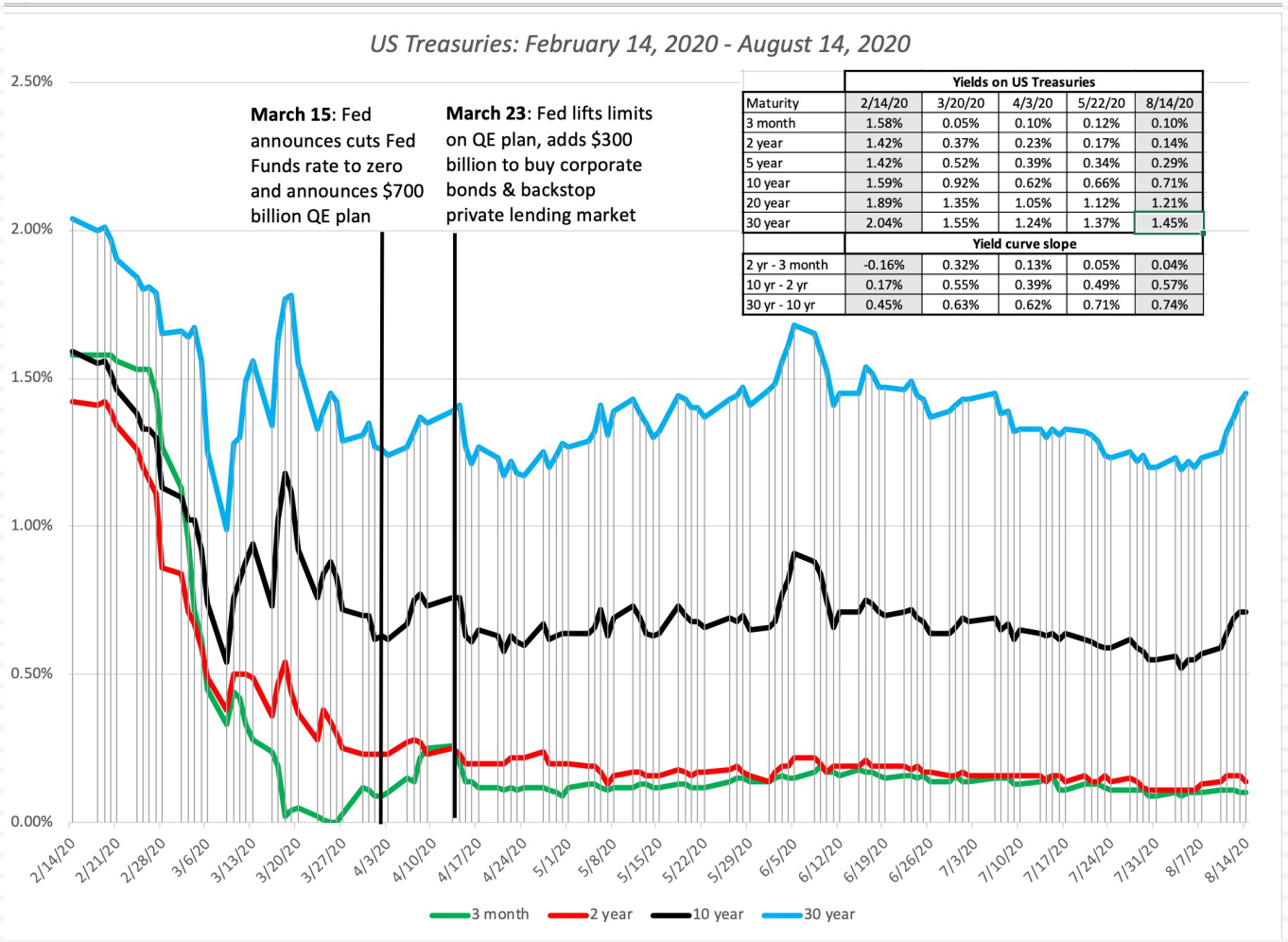
- Estimate a range for the riskfree rate in local terms:
  - Approach 1: Subtract default spread from local government bond rate:

    Government bond rate in local currency terms - Default spread for Government in local currency
  - Approach 2: Use forward rates and the riskless rate in an index currency (say Euros or dollars) to estimate the riskless rate in the local currency.
- Do the analysis in real terms (rather than nominal terms) using a real riskfree rate, which can be obtained in one of two ways –
  - from an inflation-indexed government bond, if one exists
  - set equal, approximately, to the long term real growth rate of the economy in which the valuation is being done.
- Do the analysis in a currency where you can get a riskfree rate, say US dollars or Euros.

Why do risk free rates vary across currencies? January 2022 Risk free rates



Or across time…



	Yields on US Treasuries				
Maturity	2/14/20	3/20/20	4/3/20	5/22/20	8/14/20
3 month	1.58%	0.05%	0.10%	0.12%	0.10%
2 year	1.42%	0.37%	0.23%	0.17%	0.14%
5 year	1.42%	0.52%	0.39%	0.34%	0.29%
10 year	1.59%	0.92%	0.62%	0.66%	0.71%
20 year	1.89%	1.35%	1.05%	1.12%	1.21%
30 year	2.04%	1.55%	1.24%	1.37%	1.45%
	Yield curve slope				
2 yr - 3 month	-0.16%	0.32%	0.13%	0.05%	0.04%
10 yr - 2 yr	0.17%	0.55%	0.39%	0.49%	0.57%
30 yr - 10 yr	0.45%	0.63%	0.62%	0.71%	0.74%

Risk free Rate: Don't have or trust the government bond rate?

1. Build up approach: The risk free rate in any currency can be written as the sum of two variables:

Risk free rate = Expected Inflation in currency + Expected real interest rate

Thus, if the expected inflation rate in a country is expected to be 15% and the TIPs rate is 1%, the risk free rate is 16%.

2. US $ rate & Differential Inflation: Alternatively, you can scale up the US $ risk free rate by the differential inflation between the US $ and the currency in question:

Risk free rate$_{Currency}$= $$(1 + Riskfree\ rate_{US\ \$}) \frac{\left(1 + Expected\ Inflation_{Foreign\ Currency}\right)}{(1 + Expected\ Inflation_{US\ \$})} - 1$$

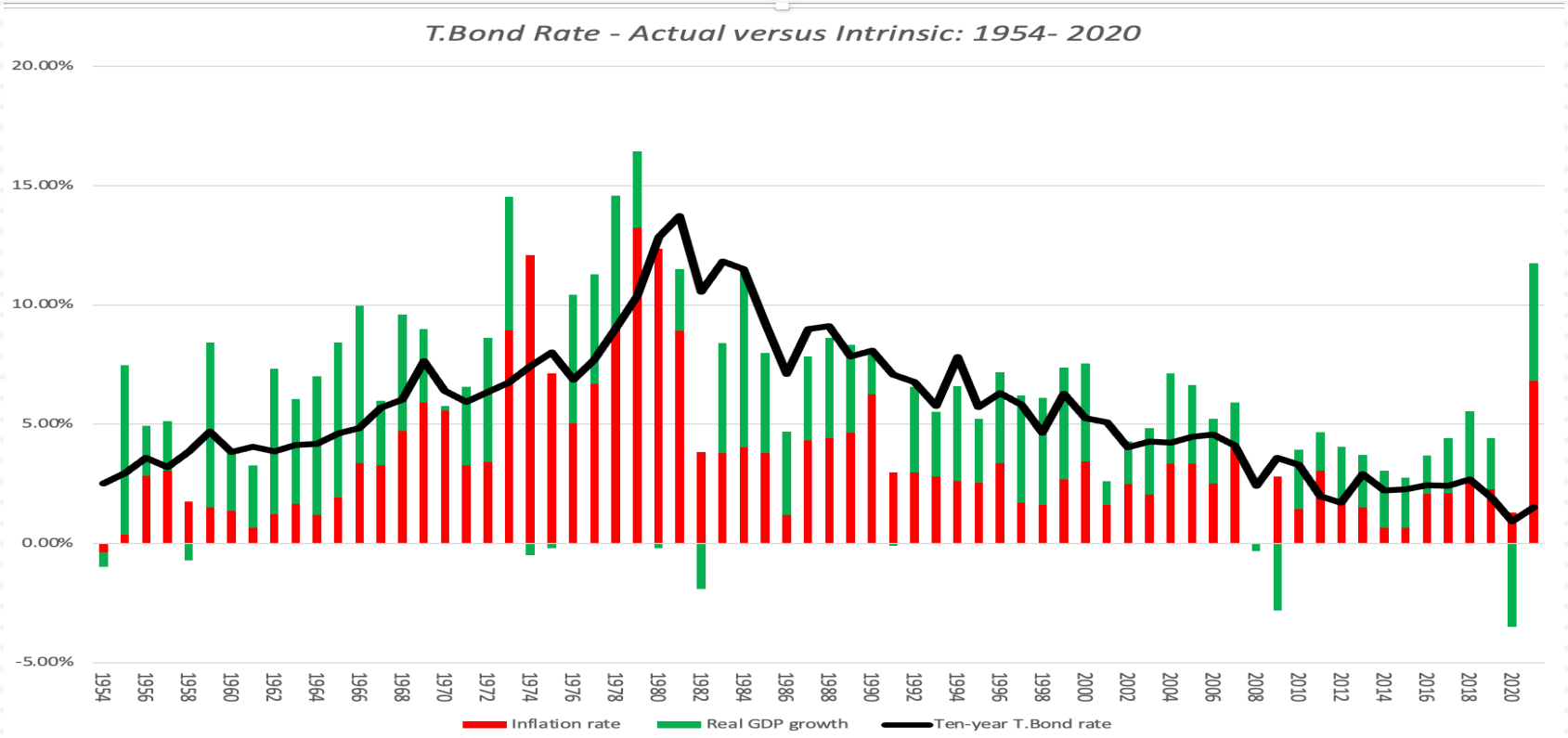
Thus, if the US $ risk free rate is 2.00%, the inflation rate in the foreign currency is 15% and the inflation rate in US $ is 1.5%, the foreign currency risk free rate is as follows:

Risk free rate = $$(1.02)\frac{(1.15)}{(1.015)} - 1 = 15.57\%$$

One more test on riskfree rates…

- On January 1, 2022, the 10-year treasury bond rate in the United States was 1.51%, low by historic standards. Assume that you are valuing a company in US dollars then, but are wary about the risk free rate being too low. Which of the following should you do?
  a. Replace the current 10-year bond rate with a more reasonable normalized riskfree rate (the average 10-year bond rate over the last 30 years has been about 5-6%)
  b. Use the current 10-year bond rate as your riskfree rate but make sure that your other assumptions (about growth and inflation) are consistent with the riskfree rate.
  c. Something else…

Some perspective on risk free rates



	Ten-year T.Bond rate	Inflation rate	Real GDP growth	Intrinsic riskfree rate
1954-2021	5.59%	3.55%	2.94%	6.50%
1954-1980	5.83%	4.49%	3.50%	7.98%
1981-2008	6.88%	3.26%	3.04%	6.30%
2011-2021	2.25%	1.76%	1.70%	3.46%

Negative Interest Rates?

- In 2022, there were at least three currencies (Swiss Franc, Japanese Yen, Euro) with negative interest rates. Using the fundamentals (inflation and real growth) approach, how would you explain negative interest rates?
  - How negative can rates get? (Is there a bound?)
  - Would you use these negative interest rates as risk free rates?
    - If no, why not and what would you do instead?
    - If yes, what else would you have to do in your valuation to be internally consistent?

Discount Rates: II

The Equity Risk Premium

II. The Equity Risk Premium
The ubiquitous historical risk premium

- The historical premium is the premium that stocks have historically earned over riskless securities.
- While the users of historical risk premiums act as if it is a fact (rather than an estimate), it is sensitive to
  - How far back you go in history…
  - Whether you use T.bill rates or T.Bond rates
  - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

	Arithmetic Average		Geometric Average	
	Stocks - T. Bills	Stocks - T. Bonds	Stocks - T. Bills	Stocks - T. Bonds
1928-2021	8.49%	6.71%	6.69%	5.13%
Std Error	2.05%	2.17%		
1972-2021	8.04%	5.47%	6.70%	4.47%
Std Error	2.44%	2.76%		
2012-2021	16.47%	14.39%	15.89%	14.00%
Std Error	3.88%	4.59%		

The perils of trusting the past…….

- Noisy estimates: Even with long time periods of history, the risk premium that you derive will have substantial standard error. For instance, if you go back to 1928 (about 90 years of history) and you assume a standard deviation of 20% in annual stock returns, you arrive at a standard error of greater than 2%:

  Standard Error in Premium = 20%/√90 = 2.1%

- Survivorship Bias: Using historical data from the U.S. equity markets over the twentieth century does create a sampling bias. After all, the US economy and equity markets were among the most successful of the global economies that you could have invested in early in the century.

Risk Premium for a Mature Market? Broadening the sample to 1900-2017

Country	*Geometric Mean*	*Standard Error*
Australia	5.00%	1.70%
Austria	2.90%	14.10%
Belgium	2.20%	1.90%
Canada	3.50%	1.70%
Denmark	2.20%	1.70%
Finland	5.20%	2.70%
France	3.10%	2.10%
Germany	5.10%	2.60%
Ireland	2.70%	1.80%
Italy	3.20%	2.70%
Japan	5.10%	3.00%
Netherlands	3.30%	2.00%
New Zealand	4.00%	1.60%
Norway	2.40%	2.50%
Portugal	5.30%	2.90%
South Africa	5.30%	1.80%
Spain	1.80%	1.90%
Sweden	3.10%	2.00%
Switzerland	2.20%	1.60%
U.K.	3.70%	1.60%
U.S.	4.40%	1.90%
Europe	3.00%	1.40%
World-ex U.S.	2.80%	1.30%
World	3.20%	1.40%