VALUATION: PACKET 3
REAL OPTIONS, ACQUISITION VALUATION AND VALUE ENHANCEMENT

Aswath Damodaran

Updated: January 2024

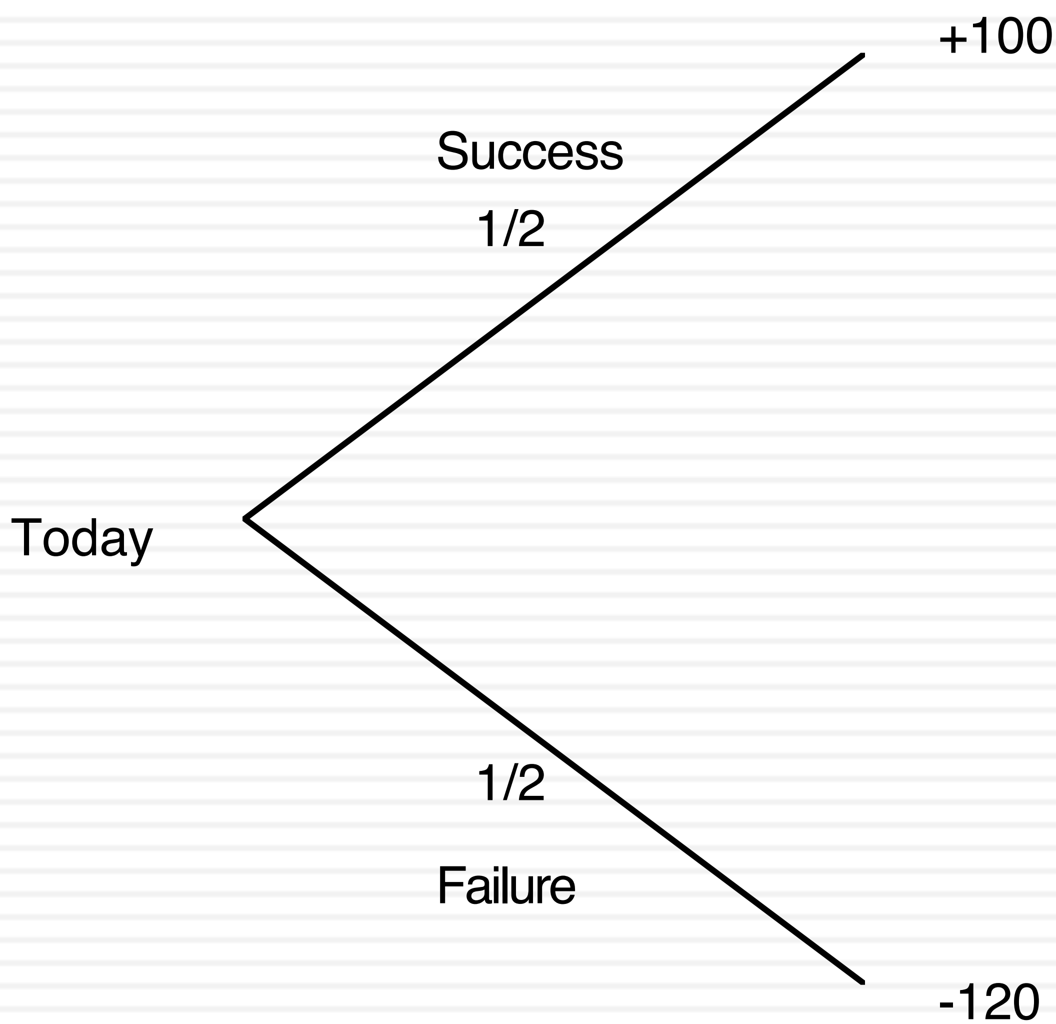
REAL OPTIONS: FACT AND FANTASY

Aswath Damodaran

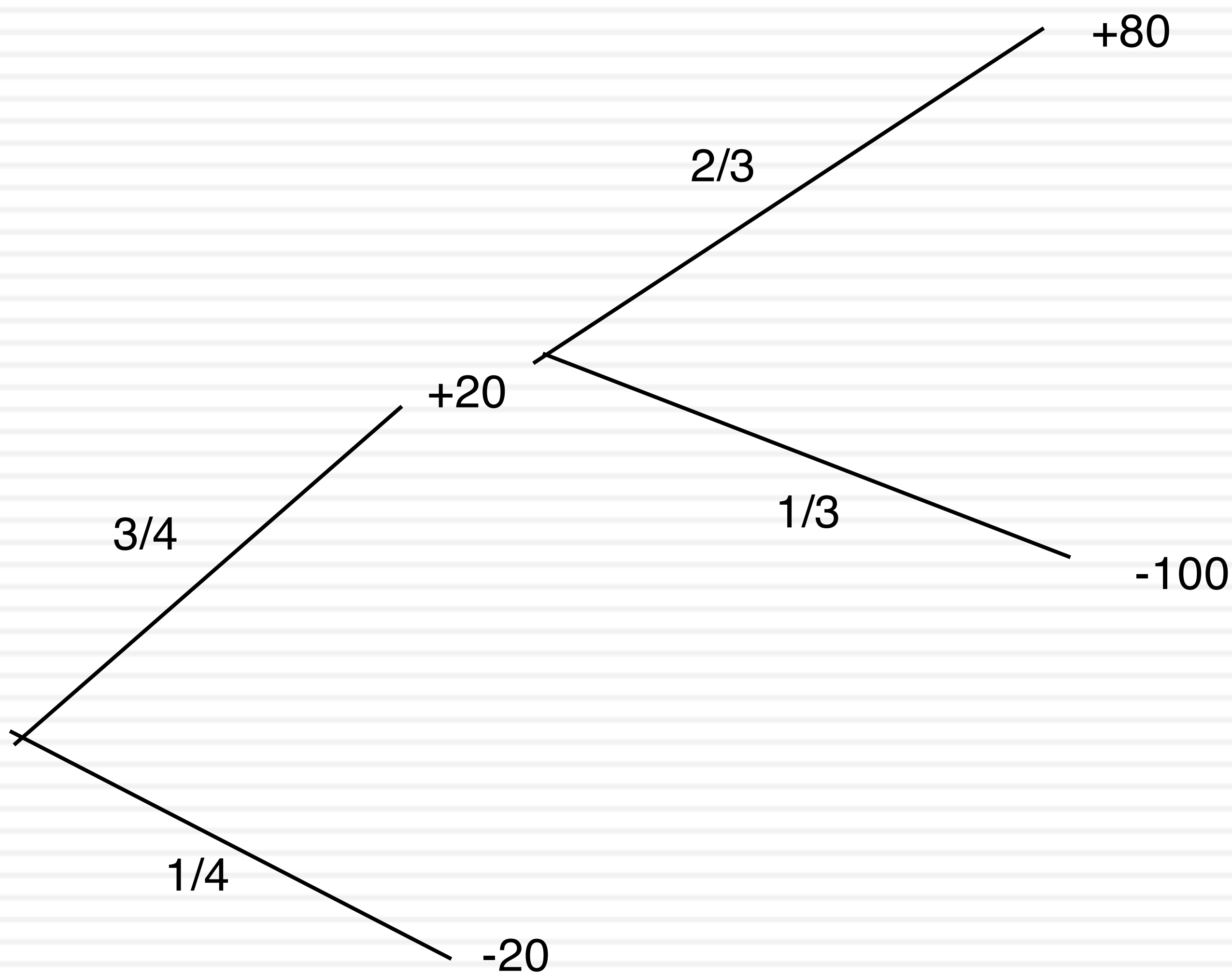
Underlying Theme: Searching for an Elusive Premium

- Traditional discounted cashflow models underestimate the value of investments, where there are options embedded in the investments to
  - Delay or defer making the investment (delay)
  - Adjust or alter production schedules as price changes (flexibility)
  - Expand into new markets or products at later stages in the process, based upon observing favorable outcomes at the early stages (expansion)
  - Stop production or abandon investments if the outcomes are unfavorable at early stages (abandonment)
- Put another way, real option advocates believe that you should be paying a premium on discounted cashflow value estimates.

A bad investment…



Becomes a good one…



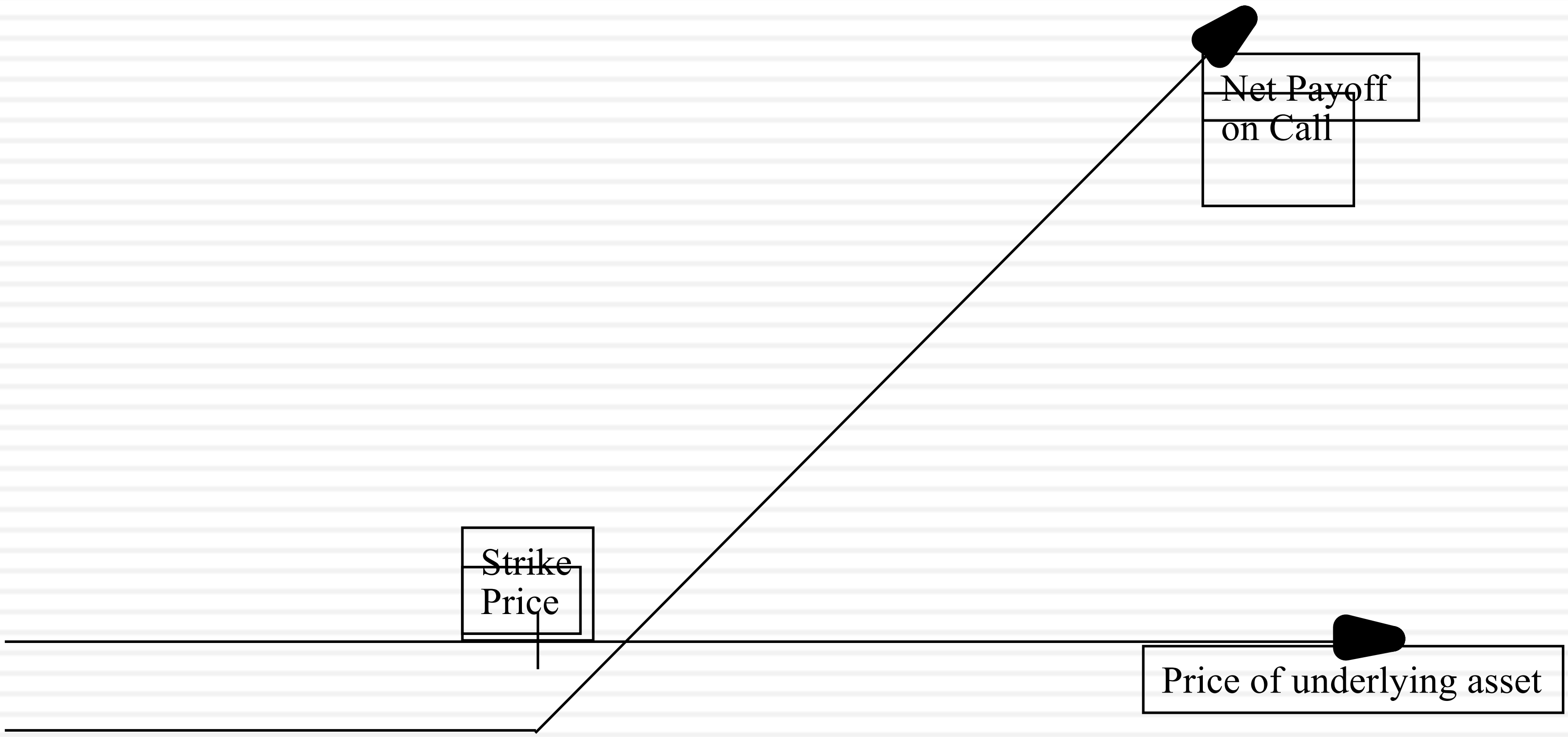
Three Basic Questions

1. When is there a real option embedded in a decision or an asset?
2. When does that real option have significant economic value?
3. Can that value be estimated using an option pricing model?

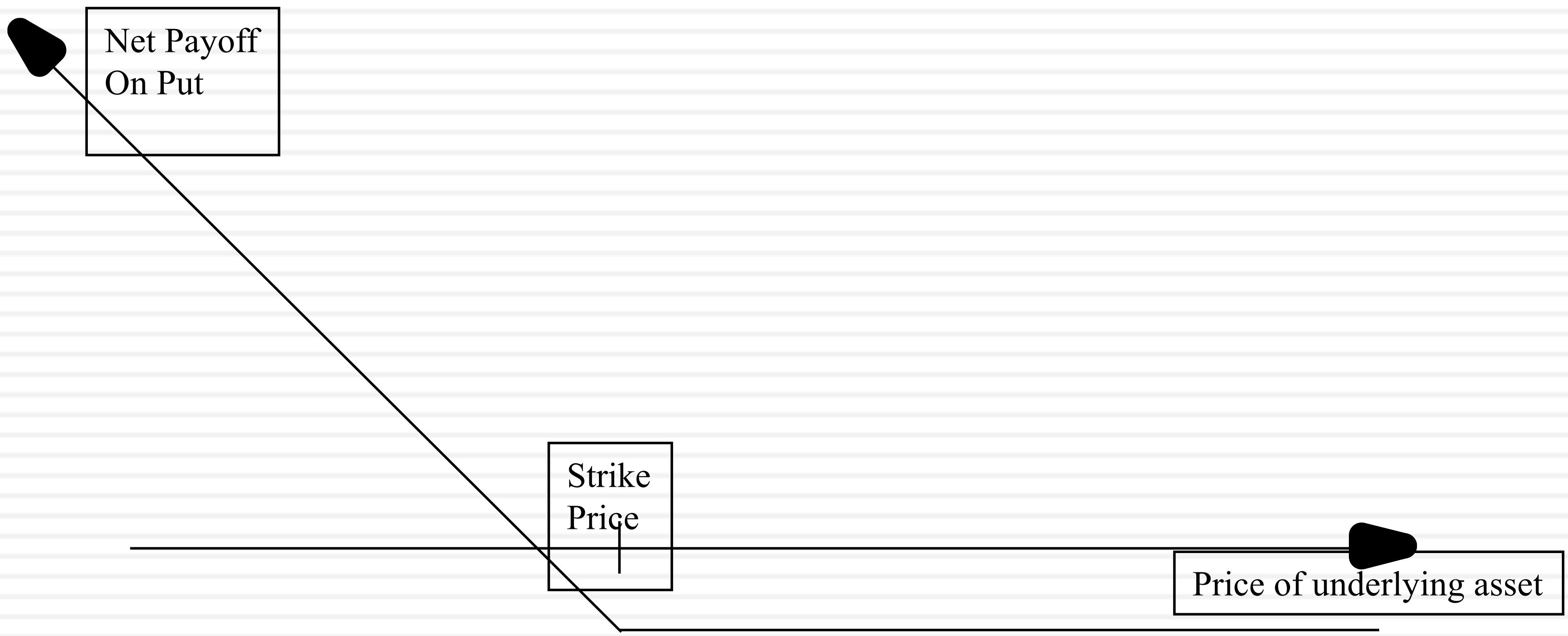
When is there an option embedded in an action?

- An option provides the holder with the *right* to buy or sell a specified quantity of an underlying asset at a *fixed price* (called a strike price or an exercise price) at or before the expiration date of the option.
- There has to be a *clearly defined underlying asset* whose value changes over time in unpredictable ways.
- The payoffs on this asset (real option) have to be *contingent on a specified event* occurring within a finite period.

Payoff Diagram on a Call



Payoff Diagram on Put Option



When does the option have significant economic value?

- For an option to have significant economic value, there has to be a restriction on competition in the event of the contingency. In a perfectly competitive product market, no contingency, no matter how positive, will generate positive net present value.
- At the limit, real options are most valuable when you have exclusivity - you and only you can take advantage of the contingency. They become less valuable as the barriers to competition become less steep.

Determinants of option value

- Variables Relating to Underlying Asset
  - Value of Underlying Asset; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
  - Variance in that value; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
  - Expected dividends on the asset, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- Variables Relating to Option
  - Strike Price of Options; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
  - Life of the Option; both calls and puts benefit from a longer life.
- Level of Interest Rates; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

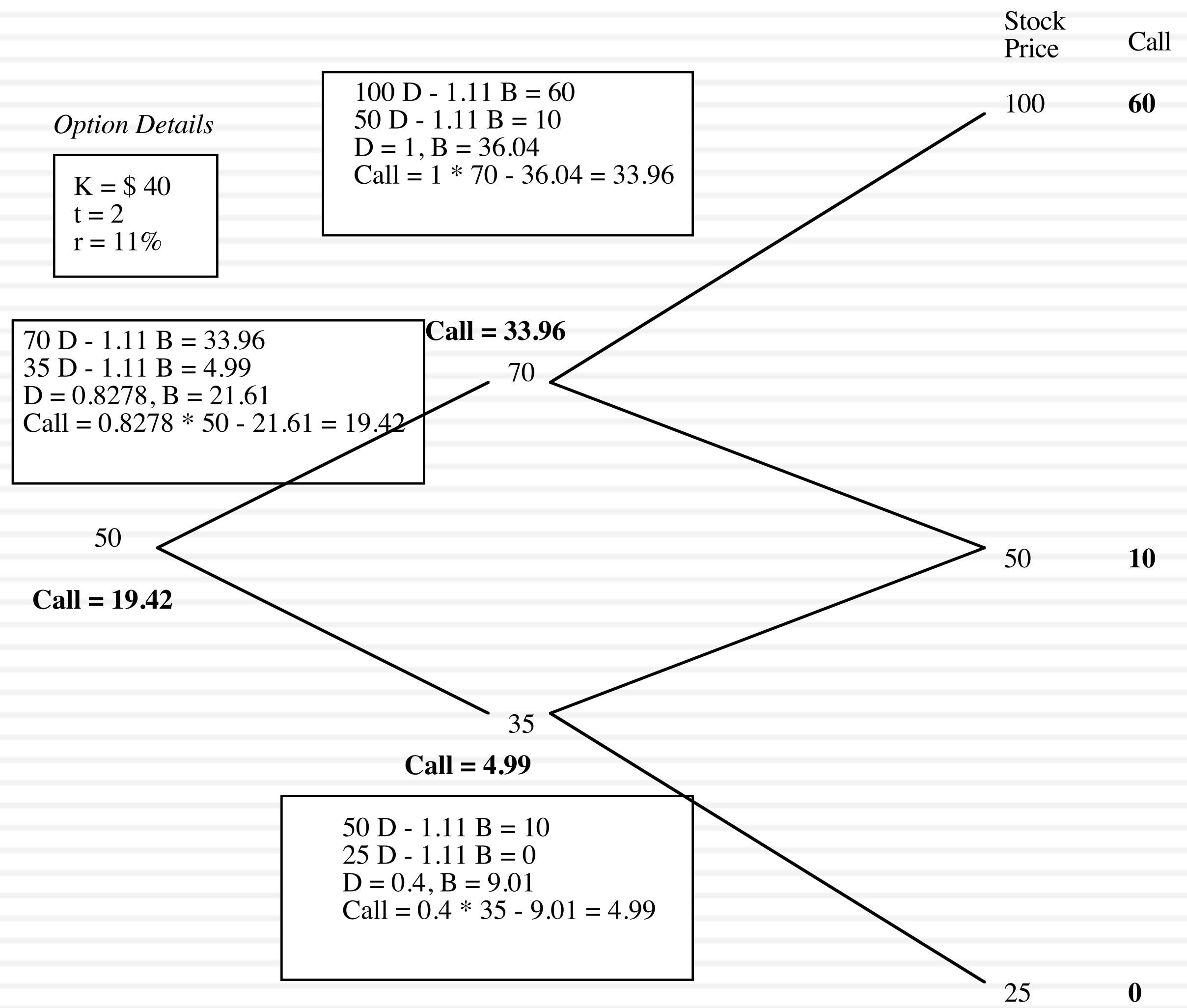
When can you use option pricing models to value real options?

- The notion of a replicating portfolio that drives option pricing models makes them most suited for valuing real options where
  - The underlying asset is traded - this yield not only observable prices and volatility as inputs to option pricing models but allows for the possibility of creating replicating portfolios
  - An active marketplace exists for the option itself.
  - The cost of exercising the option is known with some degree of certainty.
- When option pricing models are used to value real assets, we have to accept the fact that
  - The value estimates that emerge will be far more imprecise.
  - The value can deviate much more dramatically from market price because of the difficulty of arbitrage.

Creating a replicating portfolio

- The objective in creating a replicating portfolio is to use a combination of riskfree borrowing/lending and the underlying asset to create the same cashflows as the option being valued.
  - Call = Borrowing + Buying D of the Underlying Stock
  - Put = Selling Short D on Underlying Asset + Lending
  - The number of shares bought or sold is called the option delta.
- The principles of arbitrage then apply, and the value of the option has to be equal to the value of the replicating portfolio.

The Binomial Option Pricing Model



The Limiting Distributions….

- As the time interval is shortened, the limiting distribution, as t -> 0, can take one of two forms.
  - If as t -> 0, price changes become smaller, the limiting distribution is the normal distribution and the price process is a continuous one.
  - If as t->0, price changes remain large, the limiting distribution is the poisson distribution, i.e., a distribution that allows for price jumps.
- The Black-Scholes model applies when the limiting distribution is the normal distribution , and explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

Black and Scholes…

- The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected.
- The value of a call option in the Black-Scholes model can be written as a function of the following variables:
  - S = Current value of the underlying asset
  - K = Strike price of the option
  - t = Life to expiration of the option
  - r = Riskless interest rate corresponding to the life of the option
  - σ^2 = Variance in the ln(value) of the underlying asset

The Black Scholes Model

Value of call = S N (d1) - K e^{-rt} N(d2)

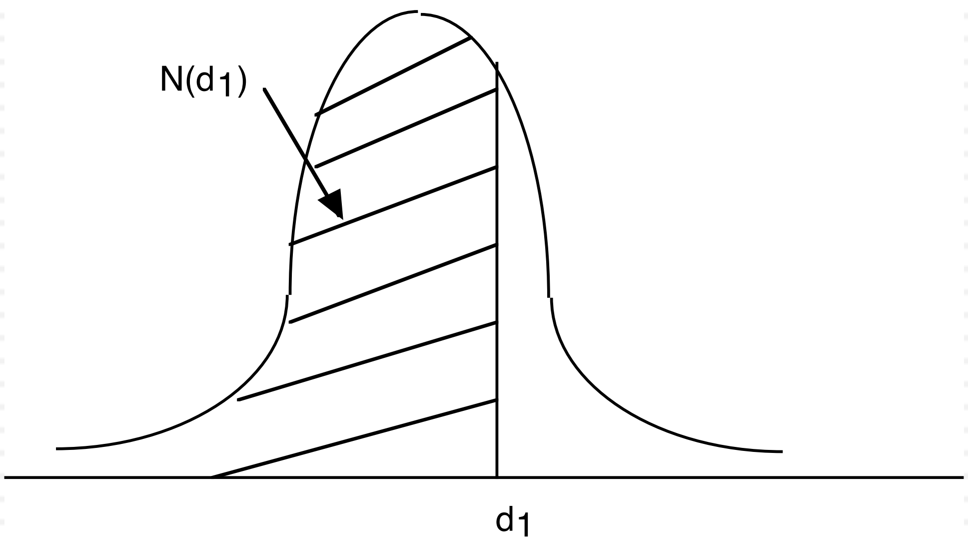
where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})\, t}{\sigma \sqrt{t}}$$

$d2 = d1 - \sigma \sqrt{t}$

- The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to
  - Buy N(d1) shares of stock; N(d1) is called the option delta
  - Borrow K e^{-rt} N(d2)

The Normal Distribution



d	N(d)	d	N(d)	d	N(d)
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

Adjusting for Dividends

- If the dividend yield (y = dividends/ Current value of the asset) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.
- $C = S\,e^{-yt}\,N(d1) - K\,e^{-rt}\,N(d2)$

where,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$d2 = d1 - \sigma \sqrt{t}$

- The value of a put can also be derived:
- $P = K\,e^{-rt}\,(1-N(d2)) - S\,e^{-yt}\,(1-N(d1))$

Dividend Yield = Cost of Delay

- Options have time premiums, and when they are traded, you very seldom get early exercise, with one exception being calls before big ex-dividend dates. The trade off that drives early exercise is:
  - Loss of the time premium of the option from exercising early (against)
  - Dividends you will receive, if you exercise early
  - If the dividend exceeds the time premium, you will see early exercise.
- Put differently, the dividends foregone become the cost of delaying exercise and leaving the option live.
- Thus, having a cost of delay in an option will require that you use an dividend-adjusted version of the option pricing model.