Wishful thinking
a model of optimal reality denial

(preliminary)

Augustin Landier*

september 2000

Abstract

This paper presents a model of non bayesian beliefs dynamics. Beliefs have a hedonic component through savoring (anticipation utility). This determines optimistic strategies for interpretation and memorization of information. Several applications are considered: savings behavior, health care, political economics, mispricing and illiquidity on the housing market.

We develop in this framework an asset pricing model where heterogeneity of beliefs emerges as an endogenous outcome of the general equilibrium, without asymmetric information.

1 introduction

Overconfidence is one of the most robust and documented rationality biases. This paper is an attempt to model overconfidence as an optimal response of the self to its environment : we argue that the structure of preferences confronts the self to a trade-off between present psychological well-being and the

*MIT and NBER

efficiency of future decisions. This trade-off, which is qualitatively recognized by the psychological literature, leads to specific patterns of beliefs dynamics. The advantages of the modelization we propose lie in the possibility to intimately relate non bayesian aspects of beliefs dynamics to the underlying structure of preferences (anticipation utility) and uncertainty (irreversibility, distribution of risk).

Contrary to most of the literature on overconfidence, we don't take the bias as exogenous, but rather describe it as the endogenous result of the interaction between preferences and the environment and unlike papers such as Carillo-Mariotti(98), Benabou-Tirole (99) or Koszegi(00), which investigate the effects of a motivation to preserve self-estim on the avoidance of potentially informational situations, we depart from bayesian updating: our point of view is that current economic modelizations of intrapersonal belief management largely underestimates the ability of agents to simply underweight disturbing information (and vice-versa) for hedonic purposes. Akerlof-Dickens () which is explicitely based on cognitive disonnance is certainly the closest paper to ours. However, they don't use anticipation utility (beliefs enter directly the utility function) and their model doesn't consider the dynamics of learning.

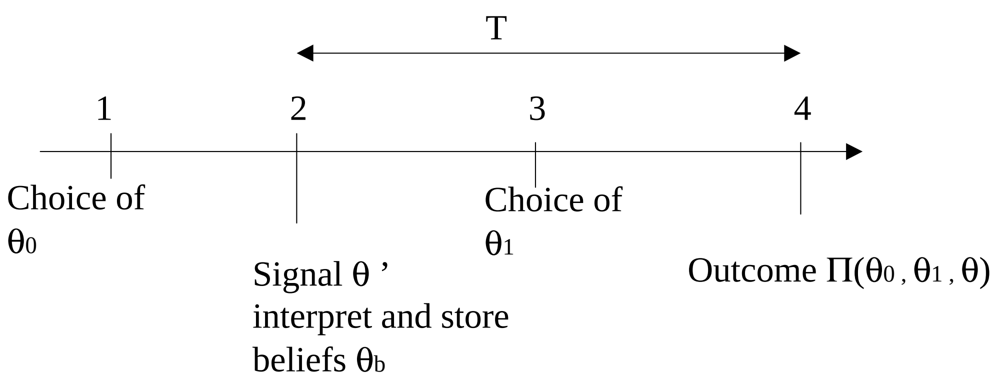
The structure of the paper is as follows: Part 1 reviews empirical evidence brought by the psychological literature about wishful thinking and denial. Part 2 describes in a simple framework the trade-off that we capture. Part 3 presents different applications of the model to topics such as savings behavior, health, political economics, innovation. In part 4, we develop a general equilibrium asset pricing model with endogenous heterogeneity of beliefs among agents.

One of the messages of this paper is that the notion of *optimality* (in some sense the very core of the neo-classical economics framework) might not have to be given up, when incorporating rationality imperfections becomes the issue: irrational behavior can (and should) in many cases be described as the optimal response under a given (codetermined) structure of feelings and preferences. How this structure simultaneously shapes belief dynamics and decision-making is the dimension we try to investigate.

2 A first simple model

In order to give a first description of the concepts we are dealing with, we look at a simple strategic game where irreversible choices interplay with the efficiency of new choices: willing to believe they didn't make too bad a choice at the beginning, agents tend to underreact when the possibility of a partial adjustment arises. The extent of irreversibility plays a crucial role in this set-up.

2.1 structure of the game



The game has 4 periods.

- In period 1, the agent has to choose a parameter $\theta_0 \in \mathbb{R}$, his best guess of an underlying unknown parameter θ distributed, according to her prior beliefs, on $\mathbb{R}$ following a density f. (for example, one can think to θ as being a metaphor for a technological choice).
- In period 2, a signal about θ is made observable and the agent stores his own interpretation of the corresponding signal in memory, which corresponds to a belief θ_b.
- In period 3, the agent chooses θ_1,according to his beliefs, to maximize his payoff of period 4.
- In period 4, θ is realised and the agent recieves the (negative) payoff:

$$\Pi = -i(\theta_0 - \theta)^2 - (1 - i)(\theta_1 - \theta)^2$$

The parameter $i \in [0,1]$ is a measure of the importance of irreversibility in the agent's decisional environment: for example, if $i = 1$, the adjusted choice θ_1 is simply irrelevant, wheras when $i = 0$, the initial choice doesn't matter (full reversibility).

The quadratic form of the payoff function is just a way to expose the concepts with clarity and by no means a crucial assumption.

- We denote by T the number of time units that elapse between period 3 (where new beliefs are formed and stored in memory) and period 4 where the actual payoff is received.

In a classical framework, the first period strategy would be of course to update beliefs with regard to the signal in a bayesian way, and choose θ_1 accordingly, to maximize the expected payoff (precisely, the optimal choice is $\theta_1 = E\theta$, where the expectation is taken with regard to bayesian beliefs).

In our framework, the reluctance of agents to accept the (ex post) non optimality of their first choice will lead them to find some compromise with reality through "stubborn beliefs".

2.2 structure of preferences and decision-making

To articulate the within-self conflict (between the reality principle and fantasy), we need to build a concept of *feelings* which extends the notion of instantaneous utility. In what follows, we are partly inspired by Loewenstein(87) (modelization of savoring).

We define feelings in the following way[1]:

$$feel(t) = u(c_t) + \alpha E_{B_t} \sum_{t_i > t} \frac{u(c_{t_i})}{(1+r)^{(t_i - t)}}$$

B_t denotes current beliefs and α can be interpreted as a propensity to fantasize about the future. In other words, my intantaneous feeling is a weighted average of the pleasure I get from present consumption and the discounted flow of my future utilities of consumption.

[1] We choose a discrete formalization: "feelings" occur every unit of time. The generalization to continuous time is straightforward.

For example, in our simple game, since consumption only occurs in the last period, we have:

$$feel(t) = \alpha \frac{E_{B_t} u(\pi)}{(1+r)^{(t_4 - t)}}$$

We now explict a series of hypothesis: we formulate them as general statements (that shall be used along the paper) about the structure of preferences and immediately look at how they apply to the model:

Hyp. 1 ***non cognitive choices (decisions):***

non cognitive choices maximize the discounted flow of feelings, expected under current beliefs (i.e. $E_{B_t} \sum_{t_i > t} \frac{feel(t_i)}{(1+r)^{(t_i - t)}}$*).*

This places us in the framework of savoring theory, as defined by Loewenstein(89). This concept is conform to the notion of "expectation utility" introduced by Bentham himself[2].

This first principle encompasses a notion of within-self coherence, in adequation with the theory of cognitive disonnance, that we shall formulate independently, since it deserves to be underlined:

Hyp. 2 ***principle of coherence:***

decisions are coherent with current beliefs.

So, decisions have to be optimal (in a sense depending on the structure of preferences) given the representation of the world of the agent. Conversely, his representation (i.e. beliefs) will be choosen "optimally", taking into account that the efficiency of his decision-making but also feelings are affected by representations, which we express as follows:

Hyp. 3 ***belief strategies :***

[2] "The pleasures of expectation are the pleasures that result from the contemplation of any sort of pleasure, referred to time *future,* and accompanied with the sentiment of *belief.*In contradistinction to these, all other pleasures may be termed pleasures of *enjoyment.*" Jeremy Bentham, *An introduction to the principles of morals and legislation*, chap. 5.

*When a signal is observed, it is instantaneously interpreted, and corresponding beliefs are formed and stored in memory to maximize the **rationally** expected flow of discounted feelings, taking into account the fact that beliefs impact actions.*

Both actions and feelings are affected by beliefs. Under some circumstances (*e.g.* irreversibility), this is likely to lead to a trade-off between feelings about the world and the efficiency of decision-making. Changing your representation of the world for a more relevant one can be rejected if it leads to a large cost in term of feelings for only slight improvements in choice efficiency.

Hyp. 4 *Agents are naive with regard to their memory (they ignore the fact that their beliefs emerge from a non-bayesian process leading to systematic biases for which they should correct).*

The formation of beliefs is strategic and forward-looking with regard to the future. Namely, the interpretation of a signal takes into account both the immediate hedonic impact and the consequences for future actions.

Though critical, the first assumption is by no way non-stantard. It is for example the view Mullainathan(98) takes in his modelization of the memory process. We would like to underline that we don't claim the formation of beliefs is a conscious intentional choice: we can think to it as an automatic process. Its optimality can for example be thought to in an evolutionist perspective.

The memory process:

More structure can be given in the description of the memory process: suppose there is uncertainty about a parameter θ. Consider the reception of a signal $\theta' = \theta + \epsilon$ where $\epsilon \sim N(0, \sigma)$. An interpretation of the signal is stored in memory in the form of a couple (s', σ') : in the furture, the agent does bayesian inference, treating naively the recalled signal (s', σ') as if it meant the reception of $s' = \theta + \epsilon'$ where $\epsilon' \sim N(0, \sigma')$. For example, a "disturbing information" will be stored with a high σ',which means it won't weight too much in the formation of beliefs. Depending on the nature of signals, it can make sense to impose no distortion of the mean, namely $s' = \theta'$. An alternative or complementary way to structure the memory process is to introduce a probability of recall p.

2.3 optimal belief/action strategy

Let's go back to the game. The interesting psychological action is taking place in period 2, where the agent interprets the signal, or equivalently, forms a belief on θ (*i.e.* a distribution $b(\theta)$) in accordance to which he makes his decision about a strategy θ_1. We describe the outcome of this situation when $u = Id$ (expected value case) in the following proposition, which will lead to a set of effects separate from any risk-aversion consideration.

From Hyp. 4, we know that beliefs are stable along $]t_2, t_4]$.From hyp. 1, we know that the decision taken in t_3 maximizes

$$\max E_{B_{t2}} \sum_{t=0}^{T} \frac{feel(t_2+t)}{(1+r)^t} \iff \max E_{B_{t2}} \left\{ \sum_{t=0}^{T-1} \frac{\alpha}{(1+r)^{T-t}} \frac{\pi}{(1+r)^t} + \frac{\pi}{(1+r)^T} \right\}$$
$$\iff \max E_{B_{t2}}(\pi) \iff \theta_1 = E_B(\theta)$$

From hyp. 3, The interpretation of the signal maximizes :

$$\max E_{B_{t2}} \left\{ \sum_{t=0}^{T-1} \frac{\alpha}{(1+r)^{T-t}} \frac{\pi[\theta(B_{t_2})]}{(1+r)^t} \right\} + E_{Rat.} \frac{\pi[\theta(B_{t_2})]}{(1+r)^T} \iff \max \alpha T E_{B_{t2}}(\pi) + E_{Bayes.}\pi$$

This leads to:

Proposition 1

- *beliefs take the form of a mass point (Dirac distribution), namely the agent will (choose to) believe in a θ_B with no uncertainty.*
- $\theta_1 = \theta_B$ *: actions are concordant with beliefs.*
- $$\theta_B = \theta_1 = \frac{(1-i)E_{Bayes}\theta + Ti\theta_0}{(1-i) + \alpha Ti}$$

proof:

The first point follows from standard considerations on concavity: given a belief distribution $b(\theta)$, θ_1 is chosen to minimize $E_B(\theta_1 - \theta)^2$, which leads to $\theta_1 = E_B\theta$. Now, beliefs are fixed to maximize the discounted flow of (belief)-expected feelings. To make the point again: since all consumption is supposed to occur in period 2, this means in terms of beliefs B:

$$\begin{aligned} & Max_B\{\alpha T E_B u_{(t=4)} + E_{Bayes.} u_{(t=4)}\} \\ s.c.\{\theta_1 &= E_B\theta\} \end{aligned}$$

The left of the expression incarnates the propensity to fantasy wheras the last term captures the force of realism. The trade-off is between enjoying the illusion that θ_0 is close to the true θ at the price of an inefficient second choice vs. accepting the pain of the non-optimality of θ_0 to make a good choice now.

We look for optimal beliefs $B = b(\theta)$ so that:

$$Min_B\{\alpha T E_B[i(\theta_0-\theta)^2+(1-i)(E_B\theta-\theta)^2]+E_{Bayes}[i(\theta_0-\theta)^2+(1-i)(E_B\theta-\theta)^2]\}$$

(in the second term of the expression, the expectation is "rational", in the sense that.beliefs are updated in a bayesian way with regard to the new signal).

This takes into acccount the fact that $\theta_1(B) = E_B\theta$.

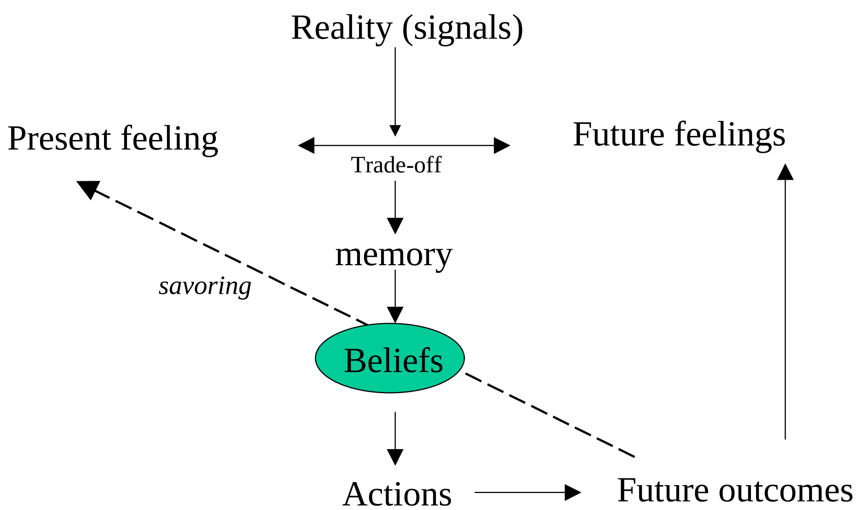
Given the concavity of the payoff function, there is no gain from uncertainty: a mean preserving spread only deteriorates the quality of your dreams, without improving your choices. So, the optimal belief system will be a Dirac distribution, putting probability 1 to a θ_B to which θ_1 will be equal. We then obtain θ_B by taking the first order condition arising from the optimization problem we have explicited.

interpretation:

Agents choose a θ_1 which is a weighted average of the true θ and the "fantasized" θ, namely θ_0.

When no irreversibility is present ($i = 0$) or no propensity to dream ($\alpha = 0$), agents believe "rationaly" and act efficiently ($\theta_1 = E_{Bayes}(\theta)$). The complementary interaction of fantasy and irreversibility is the source of the bias in beliefs and actions. This bias can be perceived as a *confirmatory bias*. In this simple framework, agents exhibit a (psychologically) optimal *conservatism*. They overestimate the quality of their previous choices (*overconfidence*) and underestimate the relevance of new information (*skepticism*).

The size of the bias increases with the intensity of anticipation utility (α) and with the length of the period of savoring, T: the longer the period affected by illusions, the more valuable it is to distort beliefs .



The basic trade-off

Remarks:

- In a quite more general perspective: the reluctance of people to admit their own errors can be due to the fact that admitting the weakness of their decision rule could make them feel less confident about the relevance of other choices they made (or will), still relevant for the future. Our model can be interpreted this way if θ is thought to as a "decision rule".

2.4 Initial uncertainty, irreversibility and the costs of reality denial

To simplify the formulation, let's consider the case where the first period signal fully reveals θ. Note tat our model is not degenerate in this case.

Proposition 2

- *The more uncertainty , and the more irreversibility there is (on the underlying parameter θ) at the beginning of the game, the larger will be the cost of the stubbornness exhibited by agents.*
- *More precisely, if σ_θ denotes the ex ante variance of θ the (ex ante) expectation of the costs of obstination is :*

$$(\frac{\alpha i}{\alpha i + (1-i)})^2 \sigma_\theta^2$$

proof: Suppose that initially θ is believed to be distributed following the density f and take this belief as "objective". The first choice of agents in this context will be $\theta_0 = E_f(\theta)$.

The objective (ex ante) costs in term of efficiency of agent's obstination are :

$$C = E_f(\theta_1 - \theta)^2 = (\frac{\alpha i}{\alpha i + (1-i)})^2 E_f[\theta - E_f\theta]^2$$

So, in conformity with intuition, the costs of obstination will be an important issue when the initial environment is risky. We believe that technological choices and innovation are typically the kind of environments subject to the effects we describe. Note also that the expected cost increases with irreversibility and propensity to fantasy, which comes to no surprise.

2.5 Conditional irreversibility

> *"One of the symptoms of an approaching nervous breakdown is the belief that one's work is terribly important".*
> Bertrand Russell.

We now want to turn to the case where the degree of irreversibility is itself (positively) dependent upon the importance of initial mistakes. If we want to put the emphasis on this case , it's because we think it encompasses the structure of many situations, especially concerning the creation process (i.e. a broad conception of innovation).

Think to a researcher beginning a paper on a new idea. After a few weeks of reflexion and writing, he lets the (embrionnary) paper circulate and gets some feedback from his collegues, which we will consider as objective for simplification and correctness. If the feedback is very negative, there is not much hope the idea can be improved: very bad a beginning lets no space for improvement or correction. On the contrary, if the idea is judged promising, the degree of irreversibility of the first incarnation of this idea is less irreversible: space is open for improvement, germination and correction.

To this extent, we believe that considering i as a decreasing function of $(\theta - \theta_0)^2$ encompasses an archetypic situation of conditional irreversibility.

Proposition 3

*if i increases with the initial mistake $(\theta - \theta_0)^2$, the adjustment rate $(\theta_0 - \theta_1)/(\theta_0 - R_{rat.}\theta)$ is itself **decreasing** with the size of the mistake.*

This means that in some sense error is self-enforcing in this framework. This pattern can be seen as one of "escalating commitment".

proof:

just note that

$$\frac{\theta_0 - E_{rat.}\theta}{\theta_0 - \theta_1} = 1 + \alpha \frac{i}{1-i}$$

Now, note the initial error $\epsilon = |\theta_0 - \theta|$ and let's take i of the form:

$$1 - i(\epsilon) = \frac{1}{1 + \epsilon^\gamma}$$

for a positive γ.

Then,

$$|\theta_0 - \theta_1| = \frac{\epsilon}{1 + \alpha\epsilon^\gamma} = f(\epsilon)$$

Proposition 4

For $\gamma > 1$, $|\theta_0 - \theta_1|$ *goes to* 0 *when* ϵ *goes to* 0.

In other words, a small adjustment can reveal either a very good initial choice, either a disastrous one. Moreover, since f *has for asymptot* 0*, realist parameters would make the second case more likely.*

proof:

for $\gamma > 1$, $f(+\infty) = 0 = f(0)$; f, first increases, then decreases towards 0.

The idea we emphasize here is that an external rational observer, seeing an agent who makes very small an adjusment in period 1 can (in this case of conditional irreversibility) infer that the initial choice has been very good or, possibly more likely, very bad, trapping the agent in delirious beliefs.

Another way to put it: *conservative behavior is likely to predict future poor performances.*

Let's go back to our researcher: if we see him discarding all critics and suggestions, and obstinatly focussing his research in the strict continuity of his first draft, we can infer either that his first version was incredibly great or more likely quite poor. The higher were his initial expectations, the more likely is this sad pattern to arise.

Note that that there is a maximal size of possible adjustments in this setup (f is bounded): agents will never adjust more than a certain level: this shows (like in Rabin(96)) how much "first impressions matter". Agents are determined by their first beliefs in the sense there are limits in the changes they can admit. These endogenous limits to the adaptations of the representation system of agents enlights the crucial role that education but also propaganda and endoctrinement can play, especially in term of long term effects.

3 Seniority and overconfidence

According to Griffin&Tversky(92), experts tend to be more prone to overconfidence than novices. Rather than invoking survival effects (Kyle&Wang), we can interpret it as the consequence of irreversibility rising with experience.

By a slight extension of the model, we can present a more precise view in term of life-cycle: novices limit their overconfidence to *preserve their ability to learn* in the next periods, since the rewards of learning can still be large for them. Getting more experienced, they liberate their confidence since on average their gains from learning become more limited. Sclerosis in beliefs and opinions comes with seniority : for example senior researshers whose work has entirely been written in the efficiency/RE framework are more likely to discard behavioral studies as a relevant contribution to economic theory.

Consider the following variation of the model, with now 2 signals:

In period 0, a signal (θ_0, σ_0) is given. It is interpreted and stored in memory in the form (θ_0, σ) and the choice θ_0 is made. In period 1, a new signal is released with probability λ; it leads to updating of beliefs and immediately, θ_1 is chosen.

The idea is that storing information with a small $\sigma < \sigma_0$, which means denying future risk as an immediate hedonic impact, but alleviates the quality of learning in the case where new information were to be released .

Proposition 5

- *When* $\lambda = 0, \sigma = 0$ *(total risk denial)*
- $\sigma < \sigma_0$
- σ *increases with lambda*

The last point gives the essence of the seniority effect: novices preserve doubt to preserve a high quality of learning, since they are more likely to benefit from a large amount of new informational content.

3.1 Remarks

- The managerial literature widely describes traps of reality denial, which our model can be read as a direct description of: managers are subject to patterns of escalating commitment to previous mistakes. They interpret information (signals of failure etc.) in a systematically biased way (attribution) that leads to suboptimal persistence in inefficient projects/decisions. Andy Grove's best-seller "only the paranoid survives" shows the crucial importance of "beliefs management" for a firm that has to make technological choices: implementing strategies for avoiding traps of denial is precisely the topic of the book.

- Our model provides a natural pattern of *procrastination*: take any real option framework, describing an exit decision in presence of uncertainty resolution. Exit occurs when the manager is indifferent between quitting now or quitting in dt unit of time (which has an opportunity cost) if good information doesn't arise in this interval. Wishful thinking naturally delays the occurence of this moment by putting more weight on the second option (the manager overestimates the probability that things are just about to turn well). [I have a note on that: "on the structure of destruction processes"].

- The use of a *prospect theory* utility function can be easily integrated in our framework. It allows for a convincing treatment of *sunk-cost* effects: a sunk-cost will accentuate denial/persistence given the high psychological price of failure. This enables to make prescriptions on the optimal timing of information release: for example, a researcher should receive feedback quickly in the creation process to avoid the initialization of a conservative beliefs dynamics.

- Our model of non bayesian beliefs dynamics is not without a certain consonance to the writings of the founders of pragmatism:

  " Nor can it be denied that a steady and immovable faith yields great peace of mind. It may, indeed, give rise to inconveniences, as if a man should resolutely continue to believe that fire would not burn him [...]. But then the man who adopts this method will not allow that its inconveniences are greater than its advantages. He will say, "I hold steadfastly to the truth and the truth is always wholesome." And in many cases it may very well be that the pleasure he derives from his calm faith overbalances any inconveniences resulting from its deceptive character."

  "When an ostrich buries its head in the sand as danger approaches, it very likely takes the happiest course. It hides the danger, and then calmly says there is no danger; and if it feels perfectly sure there is none, why should it raise its head to see?"

C. S. Peirce, The fixation of belief (1877), in Selected Writings, Dover Publications, 1966.

" A new opinion counts as true just in proportion as it gratifies the individual's desire to assimilate the novel in his experience to his beliefs in stock. It must both lean on old truth and grasp new fact; and its success in doing this is a matter for the individual's appreciation. When old truth grows, then, by new truth's addition, itb is for subjective reasons. We are in the process and obey the reasons. That new idea is truest which performs more felicitously its function of satisfying our double urgency." " The point I now urge you to observe particularly is the part played by the older truths. Their influence is absolutely controlling [...]. By far the most usual way of handling phenomena so novel that they would make for a serious rearrangement of our preconceptions is to ignore them altogether, or to abuse those who bear witness for them" William James, Pragmatism (1907), Harvard University Press, Cambridge Mass, 1975.

- We have restricted to economical costs of denial.It is clear though that there are psychological costs as well and that the possibility of denial is not uniform among signals.

- Why are people willing to buy negative expected value lotteries? Our theory gives a very natural answer: such gambles are a cheap way for agents to open spaces for dreams : the induced beliefs distortions don't lead to excessively inefficient behaviors.

4 Application 1: Savings behavior

Savings behavior is likely to be very affected by our cognitive dissonance based mechanism of wishful thinking: for example, optimist expectations about future income (career success, stocks returns etc.) are incompatible with a "rational" savings behavior now. The following model encapsulates this trade-off:

time	$]0,1]$	$]1,2]$
income flow	w_1	$\widetilde{w_2}$
consumption flow	$c_0 = w_1 - S$	$c_1 = \widetilde{w_2} + S$

Consider an agent living 2 periods. He receives an income flow w_0 in period 1, from which he saves S. For simplicity we take a zero discount and interest rate.w_1 is predetermined. In period 1, the agent consumes $c_1 = \widetilde{w_2}+S$. $\widetilde{w_2}$ is a random variable: $\widetilde{w_2} \sim N(w_2, \sigma)$ is its objective distribution. At the beginning of her life (time 0), the agent receives information about her income of period 2, namely the couple (w_2, σ) which means that the objective distribution of the future income is $\widetilde{w_2} \sim N(w_2, \sigma)$. The signal is instantenously interpretred and stored in memory as (w_b, σ_b). This memorized interpretation determines ulterior beliefs about her future income; these beliefs have 2 effects: first a hedonic savoring effect (anticipation utility). It feels good now to expect a bright future. Secondly, it affects savings behavior: the agent maximizes her flow of feelings with regard to his beliefs.

We take the utility function $u(x) = -\exp(-\gamma x)$.

We introduce the following notations for the rational and perceived certainty equivalents:

$$\mu = w_2 - \frac{\gamma}{2}\sigma^2$$

$$\mu_b = w_b - \frac{\gamma}{2}\sigma_b^2$$

As we will see, μ_b is a suficient statistics in our problem regarding the interpretation of the signal and the subsequent beliefs.

Proposition 6 *For given beliefs, the savings behavior is determined by:*

$$2S = w_1 - \mu_b + \frac{\ln(1+\alpha)}{\gamma}$$

The level of undersaving (as compared to an agent that wouldn't be subject to beliefs distortions) is thus:

$$S - S_{rat} = -(\mu_b - \mu)/2$$

(The savoring effect in the first formula is nothing more than a negative discounting effect.)

proof:

$\max_S \{u(w_1 - S) + (1+\alpha)E_b u(\widetilde{w_2} + S)\}$

leads to:

$\exp(-\gamma(w_1 - S)) = (1+\alpha)\exp(-\gamma(\mu_b + S))$•

Proposition 7 *Beliefs exhibit a bias towards optimism; the strategic interpretation/encoding of the signal leads to a systematic overestimation of the certainty equivalent:*

$$\mu_b - \mu = \frac{\ln(1+2\alpha)}{\gamma}$$

proof:

The strategic formation of beliefs results in the maximization :

$$\max_b \{u(w_1 - S) + \alpha E_b u(\widetilde{w_2} + S) + E_{Rat.} u(\widetilde{w_2} + S)\}$$

under the constraint

$$2S = w_1 - \mu_b + \frac{\ln(1+\alpha)}{\gamma}$$

i.e.

$$\max_{\mu_b} - \left\{e^{-\gamma(\mu_b + S)}[(1+2\alpha) + e^{-\gamma(\mu - \mu_b)}]\right\}$$

under the constraint

$$2(\mu_b + S) = w_1 + \mu_b + \frac{\ln(1+\alpha)}{\gamma}$$

Replacing and maximizing the log, we get the result•

So we have a systematic pattern of undersaving, decreasing with risk-aversion and increasing with the savoring coefficient.

Symmetrically, a natural pattern of *over*saving for retired people can be generated in this model: it is based on the hedonic denial of mortality probability.

immediate extensions: introduce interest/discount rate r, length of period 1, T.

5 Application2: Political economics and the management of beliefs

The survival of diictatorial regimes is often highly dependent upon the ability to maintain a high level of reality denial among people.

2 ideas:

- The sunk-cost effect predicts under our model that a high level of privation makes the level of denial higher and the immunity of the illusion: people are very reluctant to admitt they suffered for nothing.

-Switch in beliefs: strategic complementarity: the probability of a change in regime is negatively correlated with the level of aggregate denial. Possibility of multiple equilibria: if denial of the population is strong, an individual expects the regime to lasts and thus has an incentive to deny.

6 Application 3: Health/Insurance

-This model prescribes public intervention on prevention/diagnosis and insurance: the government can avoid to agents the anxiety of consistent beliefs or the costs of denials by imposing certain behaviors regarding medical tests, security standards or insurance.

7 Application 4: pricing anomalies on the housing market. Illiquidity and wishful thinking.

Houses are a typical illiquid asset. People invest in houses for relatively long periods of time. This should make their beliefs about the value of the asset biased towards optimism (the cost of belief distortion is not so high since the probability to sell the asset in a short time is small). This should lead to price downwards rigidity and and excessive waiting time in the process of search for a buyer.

8 Application 5: optimizing announcement of news: interpersonal management of beliefs

Our model makes precise predictions regarding *optimal announcement of news*: suppose a firm has two pieces of bad news to announce and wants to minimize the negative impact on shareholder's beliefs. The optimal announcement policy is to announce the news successively: 2 successive bad signals have less impact than a unique one (informationnaly equivalent). The reason is that when interpreting the first one, the agent can afford to be more optimistic since there is a probability that the next one is good. On the contrary, good news should be announced in blocks. There is actually evidence this is the way firm deal with earnings announcements.Naturally, when formalizing, it becomes an issue whether agents are aware firms try to manipulate their beliefs. But (even) a bayesian accounting of this last fact only limits the effect.

9 Application 6: An asset pricing model with symmetric information and endogenous heterogeneity of beliefs

The structure of uncertainty on the stock market is likely to lead to the patterns of overconfidence we have seen previously: the trade-off is now between diversification and the psychological recomfort brought by the negation of risk. We are going to describe a specific market which we believe captures important features of the technology stocks at the end of the 90's. There are

2 assets, 2 periods, 2 states of the world and 2 agents. The risk-free gross return is R. Agents have wealth w_0, which they allocate in period 0 between 2 risky assets (in limited supply) and a riskless asset in infinite supply. The value of the risky assets in period 2 is given by the following matrix:

	asset 1	asset 2	objective proba	subjective proba
state 1	K	0	π	π_1^i
state 2	0	K	π	$\pi_2^i = 2\pi - \pi_1^i$
state 3	0	0	0	0

These stocks are thus complementary in the sense that at most one of them is likely to be a "winner". This gives a motive for diversification.

Partial equilibrium:

We start by looking at the behavior of a price-taker agent, confronted to *symmetric prices*: both stocks are worth P in period 1. We put some constraints on the patterns of distortion which our agent is subject to, namely, we suppose that the only distortion is on the probabilities (not the structure of uncertainty, nor K). So the beliefs can be represented by (π_1, π_2). We also impose $\pi_1 + \pi_2 = 2\pi$, which means that there is no distortion on the probability that one of the 2 stocks is a winner. The last assumption is not crucial; it is a natural way to disentangle misallocation between stocks of the same risk class from inter-risk misallocation (the last can easily be studied with only one risky asset). Both assumptions have some intuitive appeal: For example, the value of amazon.com and buy.com is driven by the small probability that they become great empires. in our model, the agent doesn't distort the probability that one of them become an empire, but relative probabilities. Take the system of beliefs as given. The portfolio allocation of the agent results from the following maximization (with natural notations):

$$\max_{x_1,x_2} -e^{-\gamma(w_0-x_1-x_2)R}\left[(1-\pi_1-\pi_2)+\pi_1 e^{-\gamma K x_1}+\pi_2 e^{-\gamma K x_2}\right]$$

We focus on the case where an interior solution exists. It is then determined by the first order condition:

Proposition 8 *For $i = 1, 2$:*

$$R[(1-\pi_1-\pi_2)+\pi_1 e^{-\gamma K x_1/P_1}+\pi_2 e^{-\gamma K x_2/P_2}] = \frac{K}{P_i}\pi_i e^{-\gamma K x_i/P_i}$$

in particular:

$$\frac{\pi_1}{P_1}e^{-\gamma K x_1/P_1} = \frac{\pi_2}{P_2}e^{-\gamma K x_2/P_2}$$

In the case of symmetric prices, $P_1 = P_2 = P$,

$$x_1 - x_2 = \frac{\ln(\pi_1)-\ln(\pi_2)}{\gamma K}P$$

and:

$$R(1-2\pi) = (K/P-2R)\pi_i e^{-\gamma K x_i}$$

We now have $x_i(\pi_1)$, implicitely defined by $\pi_i e^{-\gamma K x_i} = A$. For notational simplicity, we normalize prices $P = 1$, (so that, $A = R(1-2\pi)/(K-2R)$)

We now turn to the process of beliefs formation: the relevant trade-off is between efficiency of diversification and the fear of risk: believing that $\pi_1 = 2\pi$ alleviates the discomfort resulting from the anticipation of a risky outcome but creates a gap to the rational level of diversification. A bayesian agent will share the risky part of her portfolio equally between the 2 assets. A wishful thinker however is likely to end-up with dissymmetric beliefs and a disbalanced portfolio. Formally, beliefs are formed as a result of the following maximization:

$$\max_{(\pi_1,\pi_2=1-\pi_1)} \alpha E_{beliefs}U + E_{Bayes.}U$$

$$\min_{\{\pi_1,\pi_2=1-\pi_1\}} e^{-\gamma(w_0-x_1-x_2)R}\{\alpha[(1-2\pi)+\pi_1 e^{-\gamma K x_1}+\pi_2 e^{-\gamma K x_2}]+[(1-2\pi)+\pi e^{-\gamma K x_1}+\pi e^{-\gamma K x_2}]\}$$

Using the fact that $e^{-\gamma R x_i} = (e^{-\gamma K x_i})^{R/K} = (A/\pi_i)^{R/K}$:

$$\min_{\{\pi_1,\pi_2=1-\pi_1\}} [\pi_1\pi_2]^{R/K}[(1+\alpha)(1-2\pi)+(\alpha+\pi/\pi_1)A+(\alpha+\pi/\pi_2)A]$$

Or, noting $\delta = \pi_1 - \pi$:

$$\min_{\{\delta\in]-\pi,\pi[\}} [(\pi+\delta)(\pi-\delta)]^{R/K}[(1+\alpha)(1-2\pi)+(\alpha+\frac{\pi}{\pi+\delta})A+(\alpha+\frac{\pi}{\pi-\delta})A]$$

The problem is symmetric with regard to 0.

Proposition 9

- *When* $\alpha = 0$, *no distortion of beliefs occur:* $\pi_1 = \pi_2 = \pi$. *This remains true in the neighbourhood of 0.*
- *When* $\alpha \to +\infty, \lim(\pi_1^b) = 2\pi - \lim(\pi_2^b) \in \{0, 2\pi\}$
- *For an interval of intermediate values of* α, *there are 2 symmetric interior solutions.*

general equilibrium:

We use a concept of "rational expectations" equilibrium where beliefs are part of the action set:

Definition 1 *An equilibrium for the game we consider is defined as a REE where actions are optimized with regard to beliefs and prices and where beliefs are optimal with regard to the structure of preferences and prices.*

Proposition 10

- *In the neigbourhood of* $\alpha = 0$, *there is no distortion: beliefs are symmetric and rational.*
- *For* α *big enough, the equilibrium has symmetric prices and distorted and antisymmetric beliefs:* $\pi_1^i = 2\pi - \pi_1^j \neq \pi$.

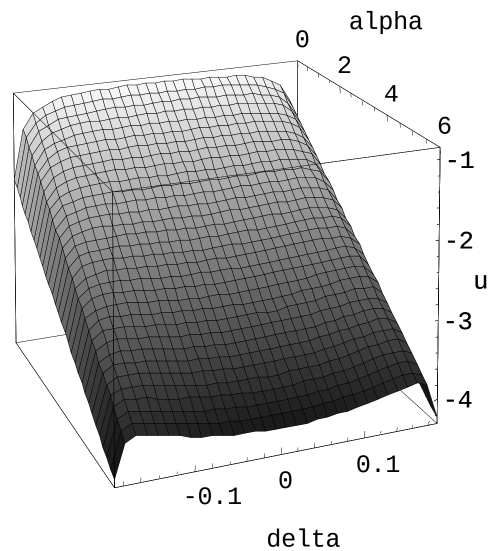


Figure 1: Utility, as a function of beliefs distortion and α.

- *heterogeneity (polarization) of beliefs increases with* α

proof: comes from strategic substituability of beliefs.

Proposition 11 *In the absence of (binding) short sales constraints, the equilibrium exhibits underpricing with regard to the bayesian REE.*

. This result comes from the concavity in probability of demand functions. it relies strongly on the condition $\pi_1^i + \pi_2^i = 2\pi$.

This hypothesis enabled us to describe a pure intra-risk category (mis)allocation effect. The essence of our results in this part is that cognitive dissonance gives a natural limit to optimal diversification: agents don't allocate symmetrically between risk-similar stocks: on the contrary, they tend to "bet" on specific stocks, at the expense of portfolio diversification. This tendency to favor certain stocks independently on any specific information leads in general equilibrium to endogenous heterogeneity of beliefs. This opens a path for a pure *disagreement* based model of asset pricing, where the interaction between the objective structure of risk and the allocation of beliefs in equilibrium is shaped by the structure of preferences independently of information asymmetries. Excess volume and price impact of news can be studied in this framework.

We now turn to "inter-risk" allocation effects of wishful thinking[3]

9.1 inter-risk allocation effects

	risk free asset	risky asset	objective proba	subjective proba
state 1	R	K	π	π_b
state 2	R	0	$1-\pi$	$1-\pi_b$

Price of the risky asset is normalized to 1.

The amount x invested in the risky asset for beliefs π_b is given by the first order condition:

$$R(1-\pi_b) = (K-R)\pi_b e^{-\gamma K x}$$

Now, beliefs result of the maximization:

$$\min_{\{\pi_b\}} e^{-\gamma(w_0 - x(\pi_b))R}\{\alpha[(1-\pi_b)+\pi_b e^{-\gamma K x(\pi_b)}]+[(1-\pi)+\pi e^{-\gamma K x(\pi_b)}]\}$$

$$\min_{\{\pi_b\}}[\frac{1-\pi_b}{\pi_b}]^{-\frac{R}{K}}[(1-\pi)+\frac{1-\pi_b}{K-R}(\alpha K + R\pi/\pi_b)]$$

Proposition 12 • *When $\alpha = 0, \pi_b = \pi$.*

- *for $\alpha > 0, \pi_b > \pi$ (overconfidence), and π_b increases with α*
- *The risky asset is overpriced with regard to the the bayesian REE.*

Proposition 13 • *The objective function is not always concave. As a result, π_b can exhibit discontinuities in the form of jumps to full optimism ($\pi_b = 1$).Excess volatility of beliefs and price.*

[3] which is in some sense equivalent to ask what other effects arise if we relax the constraint $\pi_1^i + \pi_2^i = 2\pi$.

- *Misperception of risk is in proportion more important for small probabilities/large returns assets. Those assets are relatively overvalued: they play thew role of lotteries, namely a channel for providing a channel for dreams.*

Price impact of information in this setup._neg covariations old economics/new economics (multiplier)?