

Lecture 5: The Intertemporal CAPM (ICAPM): a Multifactor Model.

- I. Reading.
- II. ICAPM Assumptions.
- III. When do individuals care about more than expected return and standard deviation?
- IV. Examples.
- V. Tastes and Preferences with a Long-term Investment Horizon.
- VI. Portfolio Choice.
- VII. Individual Assets.
- VIII. CAPM vs ICAPM
- IX. Numerical Example.

Lecture 5: The Intertemporal CAPM (ICAPM): a Multifactor Model.

- I. Reading.
 - A. BKM, Chapter 10, Section 10.4.
 - B. BKM, Chapter 27, Section 27.2.

- II. ICAPM Assumptions.
 - 1. Same as CAPM except can not represent individual tastes and preferences in $\{E[R], \sigma[R]\}$ space.

- III. When do individuals care about more than expected return and standard deviation?
 - A. single period setting:
 - 1. returns are not normally distributed *and* individual utility depends on more than expected portfolio return and standard deviation.
 - B. multiperiod setting:
 - 1. returns are not normally distributed *and* individual utility depends on more than expected portfolio return and standard deviation.
 - 2. individual preferences in the future depend on the state of the world at the end of this period;
 - 3. expected return and covariances of returns in future periods depends on the state of the world at the end of this period; e.g., predictable returns.

IV. Examples

A. Predictable Returns.

1. It has been empirically documented that expected stock returns over a period depend on variables known at the start of the period: e.g. dividend yield on the S&P 500 at the start of period t , $DP(\text{start } t)$: see Lecture 2.
2. A high S&P500 dividend yield at the start of this month implies high expected returns on stocks this month.
3. So a high S&P500 dividend yield at the end of this month implies high expected returns on stocks next month.
4. Thus, S&P500 dividend yield at the end of this month is a state variable that individuals care about when making portfolio decisions today.

B. Human capital value.

1. An unexpectedly poorer economy at the end of the month implies a larger negative shock to human capital value over the month
 - a. the negative shock to human wealth is due to an increased probability of a low bonus or, worse, job loss.
2. Thus, the state of the economy at the end of t is positively related to the shock to human capital value over t .
3. Suppose a macroeconomic indicator $MI(\text{end } t)$ summarizes the state of the economy at the end of t : the economy at the end of t is better for higher $MI(\text{end } t)$.
4. A sufficiently risk averse individual likes a portfolio whose return over t , $R_p(t)$, has a low or negative covariance with
 - a. the shock to the individual's human capital over t ;
 - b. the state of the economy at the end of t ;
 - c. $MI(\text{end } t)$.
5. The macroeconomic indicator, $MI(\text{end } t)$, is a state variable the individual cares about when making portfolio decisions at the start of t .

- V. Tastes and Preferences with a Long-term Investment Horizon.
- A. Example:
1. Today is the start of February.
 2. An unexpected low MI at the end of February implies an unexpectedly poor economy at the end of February.
 3. An unexpectedly poorer economy at the end of the February implies a larger negative shock to human capital value over the month.
 4. The macroeconomic indicator, $MI(\text{end Feb})$, is a state variable the individual cares about when making portfolio decisions at the start of February.
 5. A sufficiently risk averse individual likes a portfolio whose return over February, $R_p(\text{Feb})$, has a low or negative covariance with
 - a. the shock to the individual's human capital over February;
 - b. the state of the economy at the end of February;
 - c. $MI(\text{end Feb})$.
- B. In general, if an individual cares about a macroeconomic indicator $MI(\text{end } t)$ then can only fully represent an individual's tastes and preferences for her period t portfolio return using $\{E[R(t)], \sigma[R(t)], \sigma[R(t), MI(\text{end } t)]\}$.
- C. Even more generally, if individuals care about a set of K state variables $s_1(\text{end } t)$, ..., $s_K(\text{end } t)$, then can only fully represent an individual's tastes and preferences for her period t portfolio return using $E[R(t)], \sigma[R(t)], \sigma[R(t), s_1(\text{end } t)], \dots, \sigma[R(t), s_K(\text{end } t)]\}$.

VI. Portfolio Choice.

- A. Since individuals care about more than expected return and standard deviation of return, individuals no longer hold combinations of the riskfree asset and the tangency portfolio:
1. i.e., individuals no longer hold portfolios on efficient part of the MVF for the N risky assets and the riskless.
 2. i.e., individuals no longer hold portfolios on the Capital Allocation Line for the tangency portfolio.
- B. Thus, in the ICAPM, since individuals no longer necessarily hold combinations of the riskfree asset and the tangency portfolio, the market portfolio is no longer necessarily the tangency portfolio.
- C. Example (cont):
1. Today is the start of February.
 2. The tangency portfolio on the MVF for the N risky assets may have a high covariance with MI at the end of February.
 3. Thus, an individual may prefer to hold a portfolio in February not on capital allocation line for the tangency portfolio but which has a very low covariance with MI at the end of February.
- D. However, it is possible to show that in equilibrium all individuals irrespective of tastes and preferences hold a combination of:
1. the riskfree asset.
 2. the market portfolio.
 3. K hedging portfolios, $R_{h1}, R_{h2}, \dots, R_{hK}$, one for each state variable.
- E. Thus, the ICAPM is a generalization of the CAPM.
- F. Example (cont):
1. Today is the start of February.
 2. The tangency portfolio on the MVF may have a high covariance with MI at the end of February.
 3. All individuals hold combinations of
 - a. the riskfree asset.
 - b. the market portfolio.
 - c. a portfolio whose return $R_{MI}(\text{Feb})$ hedges shocks to human capital value over February.

VII. Individual Assets.

- A. Recall that the market portfolio is no longer necessarily the tangency portfolio: so the market need not lie on the positive sloped part of the MVF for the N risky assets.
- B. Minimum variance mathematics then tells us that there need not be a linear relation between expected return and Beta with respect to the market portfolio; i.e., assets need not all lie on the SML:

$$E[R_i] \neq R_f + \beta_{i,M} \{E[R_M] - R_f\}.$$

- C. Instead, if individuals care about the covariance of portfolio return with a set of state variables s_1, s_2, \dots, s_K , returns and the state variables are multivariate normally distributed then can show that the following holds for all assets and portfolios of assets:

$$E[R_i] = R_f + \beta_{i,M}^* \lambda_{M}^* + \beta_{i,s_1}^* \lambda_{s_1}^* + \beta_{i,s_2}^* \lambda_{s_2}^* + \dots + \beta_{i,s_K}^* \lambda_{s_K}^*$$

where:

$\lambda_{M}^*, \lambda_{s_1}^*, \lambda_{s_2}^*, \dots, \lambda_{s_K}^*$ are constants that are the same for all assets and portfolios; and

β_{i,s_k}^* for $k=1,2,\dots,K$, and $\beta_{i,M}^*$ are regression coefficients from a multivariate regression of R_i on R_M, s_1, s_2, \dots and s_K :

$$r_i = a_{i,0} + \beta_{i,M}^* r_M + \beta_{i,s_1}^* s_1 + \beta_{i,s_2}^* s_2 + \dots + \beta_{i,s_K}^* s_K + e_i$$

D. Note:

1. $r_i = R_i - R_f$ and $r_M = R_M - R_f$.
2. β_{i,s_k}^* for $k=1,2,\dots,K$, and $\beta_{i,M}^*$ are referred to as risk loadings and vary across assets; they measure the sensitivity of asset i to each of the risks that individuals care about.
3. $\lambda_{M}^*, \lambda_{s_1}^*, \lambda_{s_2}^*, \dots, \lambda_{s_K}^*$ are referred to as risk premia and measure the expected return compensation an individual must receive to bear one unit of the relevant risk.
4. $\lambda_{M}^* = E[R_M] - R_f = E[r_M]$ since when R_M is regressed on R_M, s_1, s_2, \dots and s_K get $\beta_{M,M}^* = 1$ and $\beta_{M,s_1}^* = \beta_{M,s_2}^* = \dots = \beta_{M,s_K}^* = 0$.

- E. Example (cont): If individuals care about covariance of portfolio return over t with MI(end t) and asset returns over t and MI(end t) are multivariate normally distributed, the following holds for all assets:

$$E[R_i(t)] = R_f + \beta_{i,M}^* \lambda_{M}^* + \beta_{i,MI}^* \lambda_{MI}^*$$

where:

$\lambda^*_M, \lambda^*_{DP}$ are constants that are the same for all assets and portfolios; and $\beta^*_{i,MI}$ and $\beta^*_{i,M}$ are regression coefficients from a multivariate regression of $r_i(t)$ on $r_M(t)$ and $MI(\text{end } t)$:

$$r_i(t) = a_{i,0} + \beta^*_{i,M} r_M(t) + \beta^*_{i,MI} MI(\text{end } t) + e_i(t).$$

- F. The K hedging portfolios can be used instead of the K state variables:
1. Replace s_1, s_2, \dots, s_K with $[R_{h1}-R_f], [R_{h2}-R_f], \dots, [R_{hK}-R_f]$ in the multiple regression.
 2. With this substitution, we get risk premia that satisfy:
 - a. $\lambda^*_{h1} = E[R_{h1}] - R_f, \lambda^*_{h2} = E[R_{h2}] - R_f, \dots, \lambda^*_{hK} = E[R_{hK}] - R_f.$
- G. Example (cont):
1. In the multiple regression to determine risk loadings, replace $MI(\text{end } t)$ with $r_{MI}(t) = [R_{MI}(t) - R_f]$, the excess return on the portfolio that hedges shocks to human capital value over t :

$$r_i(t) = a_{i,0} + \beta^h_{i,M} r_M(t) + \beta^h_{i,MI} r_{MI}(t) + e_i(t).$$

2. Then the following expression holds for all assets and portfolios of assets:

$$E[R_i] = R_f + \beta^h_{i,M} \lambda^*_M + \beta^h_{i,MI} \lambda^*_{hMI}$$

where:

$\lambda^*_M = E[R_M - R_f] = E[r_M]$ and $\lambda^*_{hMI} = E[R_{MI} - R_f] = E[r_{MI}]$ are constants that are the same for all assets and portfolios.

VIII. CAPM vs ICAPM

- A. It can easily be seen that the CAPM is a special case of this ICAPM model.
- B. In particular, the expression for expected return on any asset in VII.C above reduces to the CAPM when $K=0$; i.e., when individuals only care about $E[R]$ and $\sigma[R]$.

IX. Numerical Example. Let $GIP(\text{Jan})$ be the January growth rate of industrial production. Suppose each individual cares about $\{E[R_p(\text{Jan})], \sigma[R_p(\text{Jan})], \sigma[R_p(\text{Jan}), GIP(\text{Jan})]\}$ when forming his/her portfolio p for January. The following additional information is available:

i	$E[R_i(\text{Jan})]$	$\beta_{i,M}^*$	$\beta_{i,GIP}^*$
Pink	1.73%	1.3	0.25
Grey	1.34%	0.9	0.10
Black	?	0.9	0.05

where $\beta_{i,M}^*$ and $\beta_{i,GIP}^*$ are regression coefficients from a multiple regression (time-series) of $R_i(t)$ on $R_M(t)$ and $GIP(t)$:

$$R_i(t) = \phi_{i,0} + \beta_{i,M}^* R_M(t) + \beta_{i,GIP}^* GIP(t) + e_i(t).$$

Also know that riskless rate for January, $R_f(\text{Jan})$, 0.7%.

1. What is the risk premium for bearing $\beta_{i,M}^*$ risk?

Know ICAPM holds. So all assets lie on

$$E[R_i(\text{Jan})] = R_f(\text{Jan}) + \beta_{i,M}^* \lambda_M^* + \beta_{i,GIP}^* \lambda_{GIP}^*$$

where $\lambda_M^* = E[R_M(\text{Jan})] - R_f(\text{Jan})$.

Using this formula for Pink and Grey:

$$\text{Pink: } 1.73 = 0.7 + 1.3 \lambda_M^* + 0.25 \lambda_{GIP}^*$$

$$\text{Grey: } 1.34 = 0.7 + 0.9 \lambda_M^* + 0.10 \lambda_{GIP}^*$$

$$\text{Now Pink } \Rightarrow \lambda_{GIP}^* = 4 (1.03\% - 1.3 \lambda_M^*)$$

which can be substituted into Grey to obtain

$$1.34 = 0.7 + 0.9 \lambda_M^* + 0.10 \times 4 (1.03\% - 1.3 \lambda_M^*).$$

It follows that $\lambda_M^* = 0.6\%$ and $\lambda_{GIP}^* = 1\%$.

So the risk premium for bearing $\beta_{i,M}^*$ risk λ_M^* is 0.6%.

2. What is the expected January return on the market portfolio $E[R_M(\text{Jan})]$?

$$E[R_M(\text{Jan})] = \lambda_M^* + R_f(\text{Jan}) = 0.6\% + 0.7\% = 1.3\%.$$

3. What is the risk premium for bearing $\beta^*_{i,GIP}$ risk?

From above, the risk premium for bearing $\beta^*_{i,GIP}$ risk λ^*_{GIP} is 1%.

4. Is the market portfolio on the minimum variance frontier of the risky assets in the economy? Why or why not?

Not necessarily. The reason is that individuals care about more than just $E[R]$ and $\sigma[R]$.

5. What is the expected return on Black?

Know that Black satisfies:

$$\begin{aligned}
 E[R_{\text{Black}}(\text{Jan})] &= R_f(\text{Jan}) + \beta^*_{\text{Black},M} \lambda^*_M + \beta^*_{\text{Black},GIP} \lambda^*_{GIP} \\
 &= 0.7 + \beta^*_{\text{Black},M} 0.6 + \beta^*_{\text{Black},GIP} 1 \\
 &= 0.7 + 0.9 \times 0.6 + 0.05 \times 1 \\
 &= 1.29\%
 \end{aligned}$$

Lecture 5: Asset Pricing Models: Performance Measures and Empirical Evidence

- I. Reading.
- II. Performance Measurement in a Mean-Variance CAPM World.
- III. Limitations of CAPM Tests.
- IV. Asset Pricing Evidence.

Lecture 5: Asset Pricing Models: Performance Measures and Empirical Evidence

I. Reading.

- A. BKM, Chapter 24, Sections 24.1-24.2.
- B. BKM, Chapter 13, Sections 13.1-13.3.

II. Performance Measurement in a Mean-Variance CAPM World.

A. Relation between CAPM and the Market Model.

- 1. Market Model: Can always run the following regression for asset i :

$$r_i(t) = \alpha_{i,M} + \beta_{i,M} r_M(t) + e_{i,M}(t).$$

where

$$r_i(t) = R_i(t) - R_f.$$

$$\beta_{i,M} = \text{cov}[r_i(t), r_M(t)] / \text{var}[r_M(t)] = \text{cov}[R_i(t), R_M(t)] / \text{var}[R_M(t)].$$

- 2. Implications of CAPM for the Market Model

- a. CAPM Restriction : SML:

$$E[R_i] = R_f + \beta_{i,M} \{E[R_M] - R_f\}.$$

- b. Taking expectations of the market model regression.

$$E[r_i] = \alpha_{i,M} + \beta_{i,M} E[r_M].$$

- c. Rearranging SML gives

$$E[r_i] = 0 + \beta_{i,M} E[r_M].$$

- d. CAPM says all assets lie on the SML.
- e. Thus CAPM constrains $\alpha_{i,M} = 0$ for all i .

B. Jensen's Alpha.

- 1. The market model intercept $\alpha_{i,M}$ is known as Jensen's alpha:
 - a. $\alpha_{i,M} > 0$ implies asset i lies above the SML.
 - b. $\alpha_{i,M} = 0$ implies asset i lies on the SML.
 - c. $\alpha_{i,M} < 0$ implies asset i lies below the SML.
- 2. Note that Jensen's alpha can be calculated:

$$\alpha_{i,M} = E[r_i(t)] - \beta_{i,M} E[r_M(t)].$$

- 3. Jensen's alpha measures the performance of an asset as part of a CAPM-optimal portfolio of R_f and the market portfolio.
- 4. So Jensen's alpha can be used to measure the performance of a mutual

fund as an individual asset in a CAPM world.

5. Moreover, if an investor is combining the asset into a portfolio with the market portfolio and the riskfree then:
 - a. $\alpha_{i,M} > 0$ implies the asset has a positive weight in the portfolio.
 - b. $\alpha_{i,M} = 0$ implies the asset has a zero weight; the portfolio consists of the market and the riskfree.
 - c. $\alpha_{i,M} < 0$ implies the asset has a negative weight in the portfolio.

C. Sharpe ratio.

1. Earlier, investor's used the slope of the Capital Allocation Line to decide which risky asset to hold in combination with R_f .
2. The slope of the Capital Allocation Line for risky asset i is given by:

$$\text{slope}[CAL_i] = |E[R_i] - R_f| / \sigma[R_i].$$

3. The slope of the Capital Allocation Line (without the absolute value) is known as the Sharpe ratio for asset i :

$$\text{Sharpe}_i = E[r_i] / \sigma[R_i].$$

4. So the Sharpe ratio measures the performance of a fund as the only risky asset the investor holds (in combination with T-bills).

D. Example:

1. Evaluate Small and Microsoft using 5 years of data, ignoring DP, taking $R_f = 0.323\%$, and S&P 500 as the market proxy.
2. Know (using Lecture 2 pp.29-32)

$$E[r_{\text{Small}}] = 1.912 - 0.323 = 1.589.$$

$$E[r_{\text{Msft}}] = 3.126 - 0.323 = 2.803.$$

$$E[r_{\text{S\&P}}] = 1.326 - 0.323 = 1.003.$$

$$\sigma[R_{\text{Small}}] = 3.711.$$

$$\sigma[R_{\text{Msft}}] = 8.203.$$

$$\sigma[R_{\text{S\&P}}] = 2.886.$$

$$\sigma[R_{\text{Small}}, R_{\text{S\&P}}] = 6.647.$$

$$\sigma[R_{\text{Msft}}, R_{\text{S\&P}}] = 8.814.$$

3. Sharpe ratios: same as CAL slopes calculated in Lecture 3
 - a. $\text{Sharpe}_{\text{Small}} = E[r_{\text{Small}}] / \sigma[R_{\text{Small}}] = 1.589/3.711 = 0.428$;
 $\text{Sharpe}_{\text{Msft}} = E[r_{\text{Msft}}] / \sigma[R_{\text{Msft}}] = 2.803/8.203 = 0.342$;
 $\text{Sharpe}_{\text{Small}} > \text{Sharpe}_{\text{Msft}}$
 - b. Prefer to combine Small with T-bills rather than Microsoft and T-bills: an example of the benefits of diversification.
4. Jensen's alpha:
 - a. First need to calculate Beta:

$$\beta_{\text{Small,S\&P}} = \text{cov}[r_{\text{Small}}(t), r_{\text{S\&P}}(t)] / \text{var}[r_{\text{S\&P}}(t)]$$

$$= 6.647 / 2.886^2 = 0.798$$

$$\beta_{\text{Msft,S\&P}} = \text{cov}[r_{\text{Msft}}(t), r_{\text{S\&P}}(t)] / \text{var}[r_{\text{S\&P}}(t)]$$

$$= 8.814 / 2.886^2 = 1.058$$
 - b. Then can calculate Jensen's alpha:

$$\alpha_{\text{Small,M}} = E[r_{\text{Small}}(t)] - \beta_{\text{Small,S\&P}} E[r_{\text{S\&P}}(t)]$$

$$= 1.589 - 0.798 \times 1.003 = 0.789 > 0$$

$$\alpha_{\text{Msft,M}} = E[r_{\text{Msft}}(t)] - \beta_{\text{Msft,S\&P}} E[r_{\text{S\&P}}(t)]$$

$$= 2.803 - 1.058 \times 1.003 = 1.742 > 0$$
 - c. Both Small and Microsoft performed well over this 5 year period relative to the CAPM's SML but Microsoft performed particularly well.

- E. Morningstar reports both Jensen's alpha and the Sharpe ratio for each mutual fund.

III. Limitations of CAPM Tests.

1. Tests always use some kind of proxy for the market portfolio.
 - a. Market Portfolio is the value weighted portfolio of all assets which is unobservable (Roll [1977]'s critique).
2. Tests only use a subset of all available assets.
 - a. If the CAPM holds, every asset lies on the SML but not every asset is used in testing.

IV. Asset Pricing Evidence.

A. Risk Loading Proxies: Size and Book to Market.

1. Fama and French [1992].
2. Two sets of 100 portfolios:
 - a. first set: within each size decile form 10 portfolios on the basis of Beta with respect to the market.
 - b. second set: within each size decile form 10 portfolios on the basis of book-to-market.
3. Results.
 - a. Average return varies inversely with size (holding Beta fixed) but hardly varies with Beta (holding size fixed): inconsistent with CAPM.
 - b. Average return varies inversely with size (holding book-to-market fixed) and varies positively with book-to-market (holding size fixed): suggests that average returns vary across stocks with both size and book-to-market.
 - c. Results imply market proxy is not on the MVF for the individual stocks.
4. ICAPM context:
 - a. Interpret $size_i$ and $book-to-market_i$ for asset i as proxying for risk loadings ($\beta_{i,sk}^*$ s) on state variables that individuals care about.

- B. Candidate State Variables in ICAPM: The Fama and French 3-Factor Model.
1. Fama and French [1993].
 2. 25 portfolios:
 - a. quintile break-points calculated on the basis of size and book-to-market.
 - b. form 25 value-weighted portfolios based on these breakpoints.
 3. Two hedging portfolios:
 - a. SMB zero-investment portfolio: long small and short big stocks, while being book-to-market neutral.
 - b. HML zero-investment portfolio: long high and short low book-to-market stocks, while being size neutral.
 4. For each portfolio i , run the following multi-variate regression:

$$r_i(t) = \alpha_{i,3} + \beta_{i,M}^h r_M(t) + \beta_{i,SMB}^h r_{SMB}(t) + \beta_{i,HML}^h r_{HML}(t) + u_i(t)$$

where

$r_i(t)$ is the excess return on portfolio i in month t .

$r_M(t)$ is the excess return on market portfolio in month t .

$r_{SMB}(t)$ is the return on the SML portfolio in month t .

$r_{HML}(t)$ is the return on the HML portfolio in month t .

$\alpha_{i,3}$, $\beta_{i,M}^h$, $\beta_{i,SMB}^h$, and $\beta_{i,HML}^h$ are the regression coefficients.

5. ICAPM Implications.
 - a. Take $r_{SMB}(t)+R_f$ and $r_{HML}(t)+R_f$ to be the hedging portfolios for the two state variables that investors care about.
 - b. Can show that $\alpha_{i,3} = 0$ for all i .
 - (1) Use an argument similar to the one used for the market model above.
 - (2) The argument exploits the fact that hedging portfolios are being used as the two state variables.
6. Results.
 - a. The estimated $\alpha_{i,3}$ s are not economically different from zeros and are only marginally significant statistically.
 - b. The FF 3-factor model can explain other documented regularities in equity returns that CAPM cannot explain.
 - c. The FF 3-factor model can be used to generate better cost of capital estimates than the CAPM.

7. Example: How the FF 3 factor model can explain the value effect.
- Value effect: high book-to-market stocks have higher expected returns than low book-to-market stocks, after adjusting for differences in CAPM Beta.
 - FF 3 factor model implies (using their data):

$$\begin{aligned} \text{asset } i : E[r_i(t)] &= R_f + \beta_{i,M}^h E[r_M(t)] + \beta_{i,SMB}^h E[r_{SMB}(t)] + \beta_{i,HML}^h E[r_{HML}(t)] \\ &= 0.54\% + \beta_{i,M}^h 0.43\% + \beta_{i,SMB}^h 0.27\% + \beta_{i,HML}^h 0.40\%. \end{aligned}$$

- Consider two assets, one high book-to-market (high BM) and the other low book-to-market (lowBM) with the same risk loadings with respect to $r_M(t)$ and $r_{SMB}(t)$:

$$\begin{array}{lll} \beta_{\text{highBM},M}^h = 1 & \beta_{\text{highBM},SMB}^h = 0 & \beta_{\text{highBM},HML}^h = 0.7 \\ \beta_{\text{lowBM},M}^h = 1 & \beta_{\text{lowBM},SMB}^h = 0 & \beta_{\text{lowBM},HML}^h = -0.3 \end{array}$$

- Notice that the high BM portfolio loads positively on $r_{HML}(t)$ while the low BM portfolio loads negatively on $r_{HML}(t)$.
- Use the FF 3 factor model to calculate an expected return on each portfolio:

$$\begin{aligned} \text{highBM: } E[r_{\text{highBM}}(t)] &= 0.54\% + \beta_{\text{highBM},M}^h 0.43\% + \beta_{\text{highBM},SMB}^h 0.27\% + \beta_{\text{highBM},HML}^h 0.40\% \\ &= 0.54\% + 1 \times 0.43\% + 0 \times 0.27\% + 0.7 \times 0.40\% \\ &= 0.97\% + 0.28\% = 1.25\% \end{aligned}$$

$$\begin{aligned} \text{low BM: } E[r_{\text{lowBM}}(t)] &= 0.54\% + \beta_{\text{lowBM},M}^h 0.43\% + \beta_{\text{lowBM},SMB}^h 0.27\% + \beta_{\text{lowBM},HML}^h 0.40\% \\ &= 0.54\% + 1 \times 0.43\% + 0 \times 0.27\% + -0.3 \times 0.40\% \\ &= 0.97\% - 0.12\% = 0.85\% \end{aligned}$$

- So $E[r_{\text{highBM}}(t)] > E[r_{\text{lowBM}}(t)]$ which is the value effect.
8. Open questions.
- What are the state variables that investors are hedging with $r_{SMB}(t)$ and $r_{HML}(t)$?