Lectures 21-22: Derivatives: Definitions and Payoffs.

I. Readings.
- A. Options:
  1. BKM, Chapter 20, Sections 20.1 - 20.4.
- B. Forward and Futures Contracts.
  1. BKM, Chapter 22, Sections 22.1 - 22.3.

II. Preliminaries.
- A. Payoff Diagrams.
  1. Let S(t) be the value of one unit of an asset at time t.
  2. Payoff diagram graphs the value of a portfolio at time T as a function of S(T).
  3. Long 1 unit of the asset is represented by a 45° line through the origin.
  4. Short 1 unit of the asset is represented by a -45° line through the origin.
  5. Long a discount bond with a face value of \$50 maturing at T is represented by a horizontal line at \$50: its value at T is \$50 irrespective of the value of the asset.
  6. Short a discount bond with a face value of \$50 maturing at T is represented by a horizontal line at -\$50.

III. Options.

A. Call Option.

1. Definition.

a. A call option gives its holder the right (but not the obligation) to buy the option's underlying asset at a specified price.

b. The specified price is known as the exercise or strike price.

c. A European option can only be exercised at the expiration date of the option. An American option can be exercised any time prior to the expiration of the option.

2. Payoff at the Expiration Date to the Holder of a European Option.

a. Let T be the expiration date, X=\$50 be the strike price and S(t) be the value of the option's underlying asset at time t.

b. Since the holder of the option is not obligated to buy the asset for \$50, she will only do so when the payoff from doing so is greater than zero.

(1) The price paid by the holder prior to T for the option is irrelevant to the decision to exercise since it is a sunk cost.

(2) If [S(T)-50] > 0, the holder wants to exercise the option as it allows her to buy for \$50 an asset worth more than \$50.

(3) If [S(T)-50] < 0, the holder does not want to exercise the option since she would be paying \$50 for an asset she could buy in the market for less than \$50. So her payoff is zero.

(4) Thus, the holder's payoff from the call option is max {S(T)-50, 0}.

(5) Since the payoff from the option is non-negative, no arbitrage tells you that the call option has a non-negative value at any time t, $C_{50}(t)$, prior to T.

(6) At the expiration date, $C_{50}(T) = \max \{S(T)-50, 0\}$.

3. Writer of the Option.

a. If the option holder exercises the option, the option writer must sell the asset to the holder for \$50. Her payoff is -[S(T)-50].

b. If the option holder does not exercise the option, the writer is not obliged to do anything. Her payoff is 0.

c. So the payoff to the option writer is just the negative of the payoff to the option holder.

d. In return, the option writer receives the option's value when she first writes the call option and sells it.

e. Suppose the call is written at time 0 and both the writer and the option buyer keep their positions until expiration. The cash flows of each given below in Table 1.

f. Can see that the sum of the writer's and the holder's payoff is zero.

B. Put Option.

1. Definition.
   a. A put option gives its holder the right (but not the obligation) to sell the option's underlying asset at a specified price (also called the exercise or strike price).
   b. European and American have the same conotations for puts as for calls.
2. Payoff at the Expiration Date to the Holder of the Option.
   a. Again, let T be the expiration date, X=$50 be the strike price and S(t) be the value of the option's underlying asset at time t.
   b. Since the holder of the option is not obligated to sell the asset for $50, she will only do so when the payoff from doing so is greater than zero.
      (1) The price paid by the holder prior to T for the option is irrelevant to the decision to exercise since it is a sunk cost.
      (2) If [50-S(T)] > 0, the holder wants to exercise the option since it allows her to sell for $50 an asset worth less than $50.
      (3) If [50-S(T)] < 0, the holder does not want to exercise the option since she would be receiving $50 for an asset she could sell in the market for more than $50. So her payoff is zero.
      (4) Thus, the holder's payoff from the put option is max {50-S(T), 0}.
      (5) Since the payoff from the option is non-negative, no arbitrage tells you that the put option has a non-negative value at any time t, $P_{50}(t)$, prior to T.
      (6) At the expiration date, $P_{50}(T) = \max \{50-S(T), 0\}$.
      (7) Note that holding a put is *not* the same as writing a call.
3. Writer of the Option.
   a. Suppose the put is written at time 0 and both the writer and the option buyer keep their positions until expiration. The cash flows of each given below in Table 1.
   b. Can see that the sum of the writer's and the holder's payoff is zero.

C. Markets for Options.
   1. Chicago Board Options Exchange (CBOE).
      a. Most options on individual stocks are traded on the CBOE which was started in 1973.
      b. TheCBOE is both a primary and secondary market for options.
   2. Which options have volume?
      a. Presently, there is very little trading or interest in options on individual stocks.
      b. Interest in options on indices, both exchange traded and over the counter is booming. Part of the reason is the growth of index funds.

D. Standardization: Publically traded options are standardized in several respects.
   1. Size of Contract.
      a. Traded options on stocks are usually for 100 shares of the stock, although prices are quoted on a per share basis.
   2. Maturities.
      a. Options expire on a three month cycle.
      b. The convention is that the option expires on the Saturday following the third Friday of the month.
   3. Exercise Prices.
      a. These are in multiples of $2½, $5 or $10 depending on the prevailing stock price.

E. Examples.
   1. Call and put options on Microsoft stock from the Bloomberg screen on 4/15/97.
   2. Call and put options on the S&P 500 index from the Bloomberg screen on 4/15/97.

IV. Forward and Futures Contracts.

A. Forward Contracts.

1. Definition.

a. A forward contract on an asset is an agreement between the buyer and seller to exchange cash for the asset at a predetermined price (the forward price) at a predetermined date (the settlement date).

b. The asset underlying a futures contract is often referred to the "underlying" and its current price is referred to as the "spot" price.

c. The buyer of the forward contract agrees today to buy the asset on the settlement date at the forward price. The seller agrees today to sell the asset at that price on that date.

d. No money changes hands until the settlement date. In fact, the forward price is set so that neither party needs to be paid any money today to enter into the agreement.

2. Payoff Diagram.

a. Let T be the settlement date, S(t) be the price of the asset at time t and $F_T(0)$ be the forward price agreed to at time 0 for delivery at time T.

b. At time T, can see that the buyer of the forward contract gets $S(T)-F_T(0)$; the seller gets $F_T(0)-S(T)$.

c. Can see that the payoff to both the buyer and seller at time T could be negative.

d. Can see that the sum of the payoffs to the buyer and the seller equal zero.

e. The cash flows to the buyer and the seller at time 0 and at time T are given in Table 1.

f. Can see that the forward price at the settlement date must equal the spot price: $F_T(T)=S(T)$.

3. Market Characteristics: Currency Markets.
   a. The most organized forward markets are currency markets.
   b. Example: WSJ. On Monday, 4/14/97, the British pound was quoted as $1.6200/£ for immediate delivery. For delivery 6-months forward, the quote was $1.6159/£. When a buyer and seller entered into this contract on 4/14/97, the buyer of the pounds agrees to pay the seller $1.6159 in 6 months (10/97) for each £ and the £'s are delivered at that time. This may differ from the spot exchange rate that prevails in 10/97. No money changes hands on 4/14/97.
   c. The forward market in FX is a telephone dealer market that exists as an adjunct to the spot market.
   d. It is only open to banks and other institutional players.
   e. All transactions are customized as to size, currency and delivery terms.
   f. The buyer and seller bear each others credit risk. So if one defaults the other has to bear the loss.

B. Futures Contracts vs Forward Contracts.

1. Futures contracts are sold on metals, agricultural commodities, oil, livestock and financial securities. Financial futures exist on stocks, bonds, T-bills, notes, Federal funds, Libor loans and municipal bonds.
2. Daily Resettlement.
   a. Forward and futures contracts are essentially the same except for the daily resettlement feature of futures contracts.
   b. With a forward contract, no money changes hands until the settlement date.
   c. With a futures contract:
      (1) on the day that the futures contract is bought or sold: no money is paid or received.
      (2) each day after the contract is bought or sold:
         (a) a positive change in the futures price from the previous day has to be paid by the seller to the buyer of the futures contract.
         (b) a negative change has to be paid by the buyer to the seller.
      (3) the futures price on the settlement day must equal the spot price on that day.

d. Example:

(1) Suppose that the £ is currently at \$1.57/£ and that a forward with a 3-day maturity is selling at \$1.60/£ and a future with a 3-day maturity is selling at \$1.60/£. (The \$1.60/£ is in both cases called the settlement price.)

(2) Over the next 3 days, the spot and futures prices and cash flows to a buyer and a seller of the forward and futures contract for 1£ looks like:

	Day 0	Day 1	Day 2	Day 3
Spot Price (for 1£)	\$1.57	\$1.62	\$1.62	\$1.61
Forward Price	\$1.60			\$1.61
Futures Price	\$1.60	\$1.65	\$1.63	\$1.61
Forward Contract: Buyer's cash flow.	\$0	\$0	\$0	\$1.61-\$1.60 = \$.01
Forward Contract: Seller's cash flow.	\$0	\$0	\$0	\$1.60-\$1.61 = -\$.01
Futures Contract: Buyer's cash flow.	\$0	\$1.65-\$1.60 = \$.05	\$1.63-\$1.65 = -\$.02	\$1.61-\$1.63 = -\$.02
Futures Contract: Seller's cash flow.	\$0	\$1.60-\$1.65 = -\$.05	\$1.65-\$1.63 = \$.02	\$1.63-\$1.61 = \$.02

(3) Notice that since the forward and futures prices on day 0 are the same, the sum of the cash flows to the futures contract buyer from day 0 to day 3 equals the cash flow on day 3 to the buyer of the forward contract; i.e., 0.05-0.02-0.02 = -0.01.

e. The effect of daily resettlement is to force the parties to take their profits and their losses on a daily basis. This prevents the buildup of credit risk. You know immediately if your counterparty is in trouble. At most, you might be out a day's profits.

f. Suppose the seller has $0.03 net worth in the following example:

	Day 0	Day 1	Day 2	Day 3
Spot Price (for 1£)	$1.57	$1.62	$1.68	$1.75
Forward Price	$1.60			$1.75
Futures Price	$1.60	$1.65	$1.70	$1.75
Forward Contract: Buyer's cash flow.	$0	$0	$0	$1.75-$1.60 = $.15
Forward Contract: Seller's cash flow.	$0	$0	$0	$1.60-$1.75 = -$.15
Futures Contract: Buyer's cash flow.	$0	$1.65-$1.60 = $.05	$1.70-$1.65 = $.05	$1.75-$1.70 = $.05
Futures Contract: Seller's cash flow.	$0	$1.60-$1.65 = -$.05	$1.65-$1.70 = -$.05	$1.70-$1.75 = -$.05

(1) Using a forward contract, the buyer loses $0.15-$0.03=$0.12.

(2) Using a futures contract, the loss is limited to $0.02 on Day 1 when the seller is unable to mark-to-market.

(3) On Day1, the seller's position would be closed out:

	Day 0	Day 1	Day 2	Day 3
Spot Price (for 1£)	$1.57	$1.62	$1.68	$1.75
Forward Price	$1.60			$1.75
Futures Price	$1.60	$1.65	$1.70	$1.75
Futures Contract: Seller's cash flow.	$0	$1.60-$1.65 = -$.05	$1.65-$1.70 = -$.05	$1.70-$1.75 = -$.05
Seller buys1 Futures Contract on Day 1		$0	$.05	$.05
Seller's Net Cash Flow: Day 2 onward			$0	$0

3. Example: Futures contract on the S&P 500 from the Bloomberg screen on 4/16/97.

4. Other differences: Future vs Forward Contract.
   a. Standardization.
      (1) Forward contracts are tailored to the needs of the parties involved.
      (2) Futures contracts traded on the CME (the "Merc") have standardized unit sizes and standardized maturity dates.
   b. Exchange traded.
      (1) Forward contracts are usually traded in dealer markets.
      (2) Merc futures contracts are traded only on the Merc. There is no trading off the Merc (by law). This is different from the situation with stocks: IBM, an NYSE-listed company often trades away from the NYSE.
   c. Credit risk is borne by a clearing house.
      (1) In a forward market, you must be concerned with the credit worthiness of your counterparty.
      (2) In a futures market, the clearing house (an affiliate of the exchange) interposes itself between the two parties. The clearing house sells to the buyer and buys from the seller.
         (a) The buyer's contract is with the clearing house; the seller's contract is with the clearing house.
         (b) The clearing house does not bear any risk other than the credit risk: it is long one contract and short one contract.
      (3) How does the clearing house manage the credit risk?
         (a) All exchange members are required to post funds with the clearing house.
         (b) An exchange member is responsible for its customers' transactions.
   d. Margin.
      (1) In a forward contract, no money changes hands until settlement.
      (2) In a futures transaction, the buyer and seller are each required to post a small amount of the face value of the contract with their brokers. In turn, their brokers are required to post margin with the clearing house.
5. Comment: the standardization, exchange trading facility and reduced credit risk serve to increase liquidity in the futures market, and make it accessible to many smaller investors who could not directly participate in

the forward market.

V. Features of Derivatives.

A. Zero Net Supply.

1. Stocks and bonds are said to be in positive net supply. They are claims on real assets. On balance the economy is long IBM stock, for example.

2. Derivatives (forwards, futures and options) are in zero net supply. For everyone who is long a forward contract, the opposite side is short. When you add up the long and short positions, they sum to zero.

B. Zero Sum Game.

1. As has been noted, if the long side of a derivative contract gains, the short side loses. One side's profit is the other's loss. The sum of the profits to both sides is zero.

VI. Motives for Holding Derivatives.

A. Speculation.

1. Example (cont). Today is 4/14/97. The buyer of the £'s for delivery on 10/97 may simply be betting that by the 10/97, the £ will have increased in value. Suppose the buyer agrees to buy £10000 on 10/97 (at a price of $16159). If she has no other exposure to £'s, she gains if the the spot price of the pound is high on 10/97; she loses if the spot price is low.

2. Example. Nicholas Leeson at Barings.

B. Hedging.

1. Example (cont). The buyer of the £'s may owe £10000 in October but is receiving a fixed number of $ ($16500) at that time. By entering into the forward contract on 4/14/97, she removes the exhange rate risk that she would otherwise face.

2. Example. Can combine the underlying asset with puts and calls to create a riskless position.

C. Risk Management.

1. Can use derivatives to manage the risks of a portfolio.

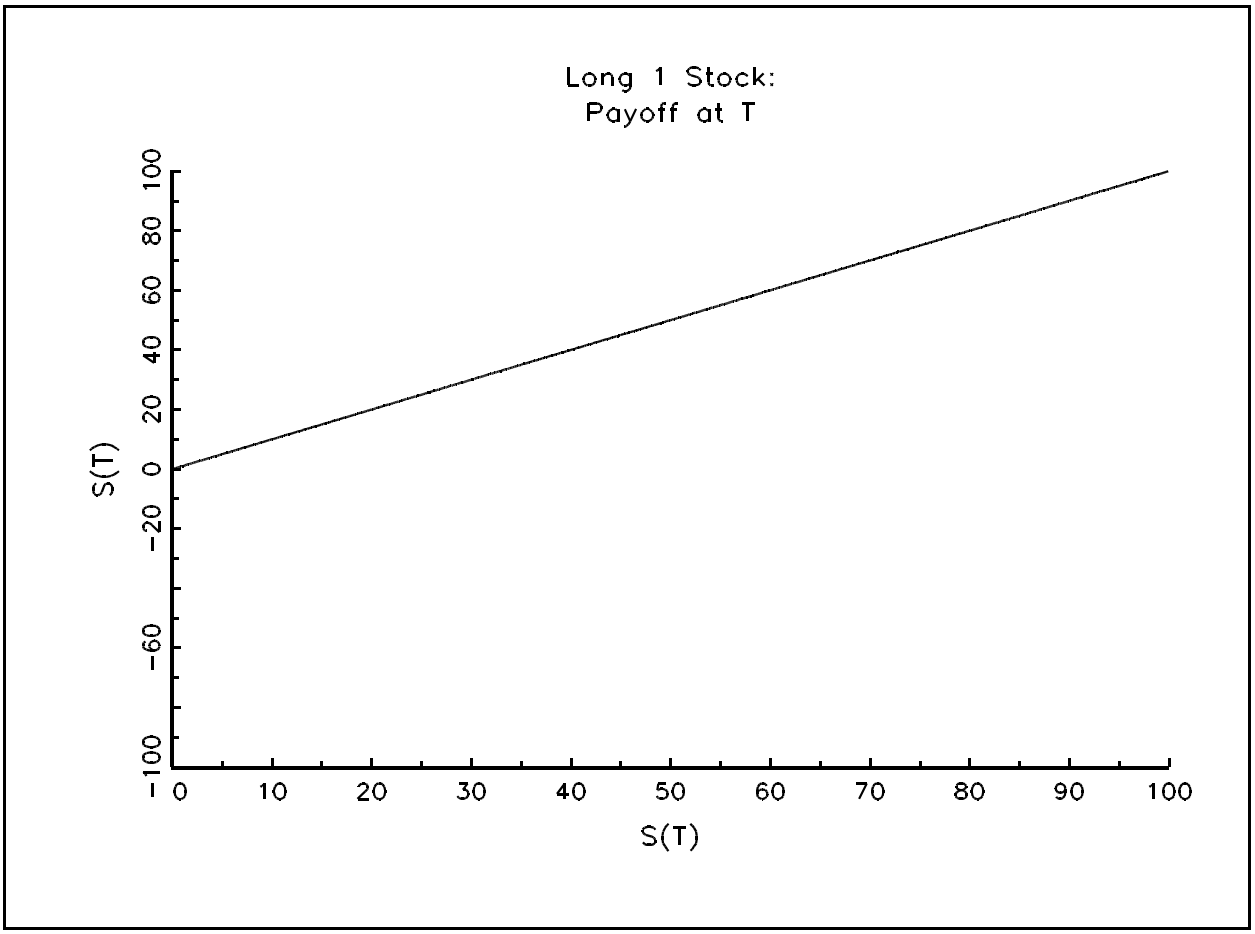
2. Portfolio Insurance.

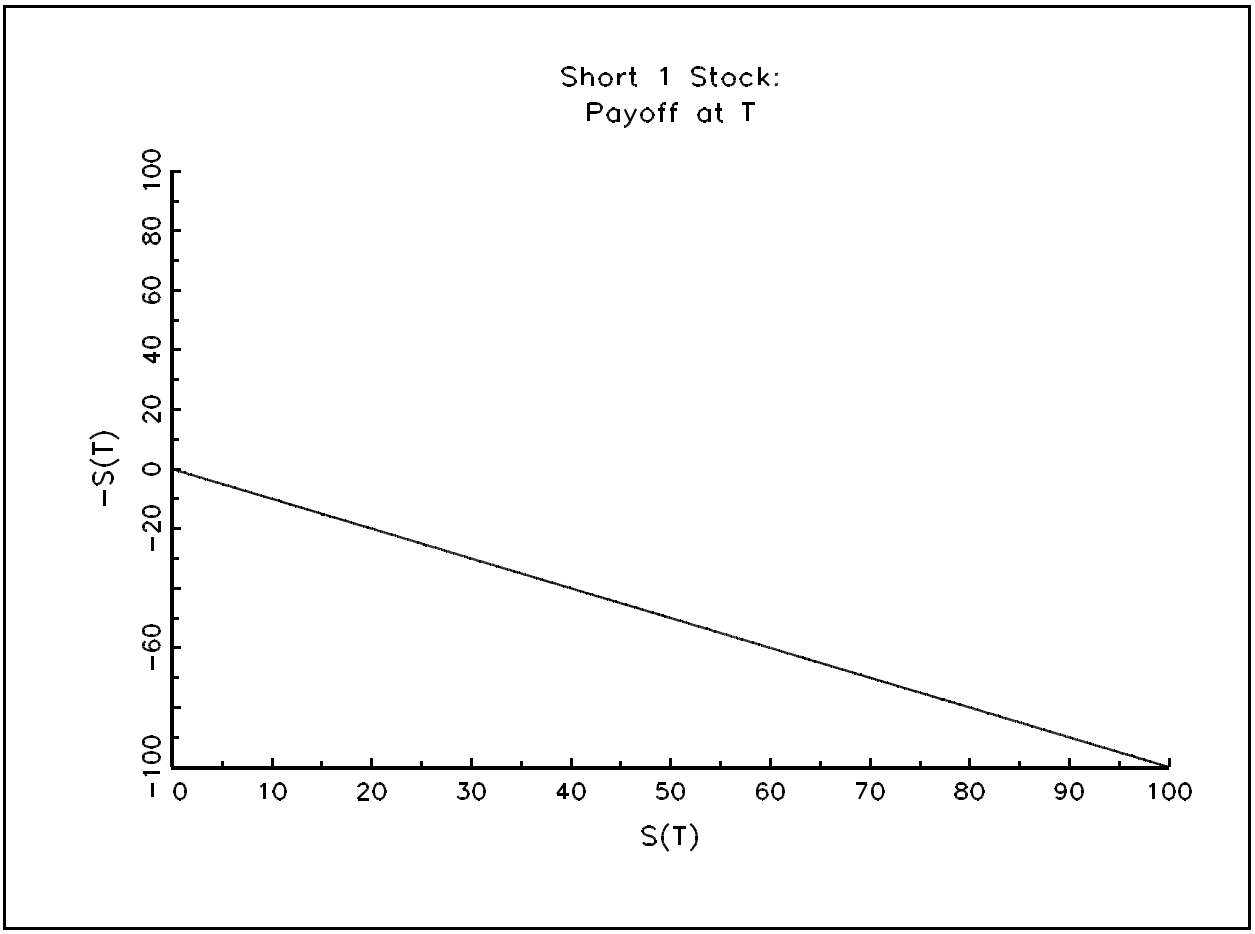
a. Can combine the underlying asset with a long position in puts on the asset to put a floor on the value of a position. This is known as portfolio insurance.

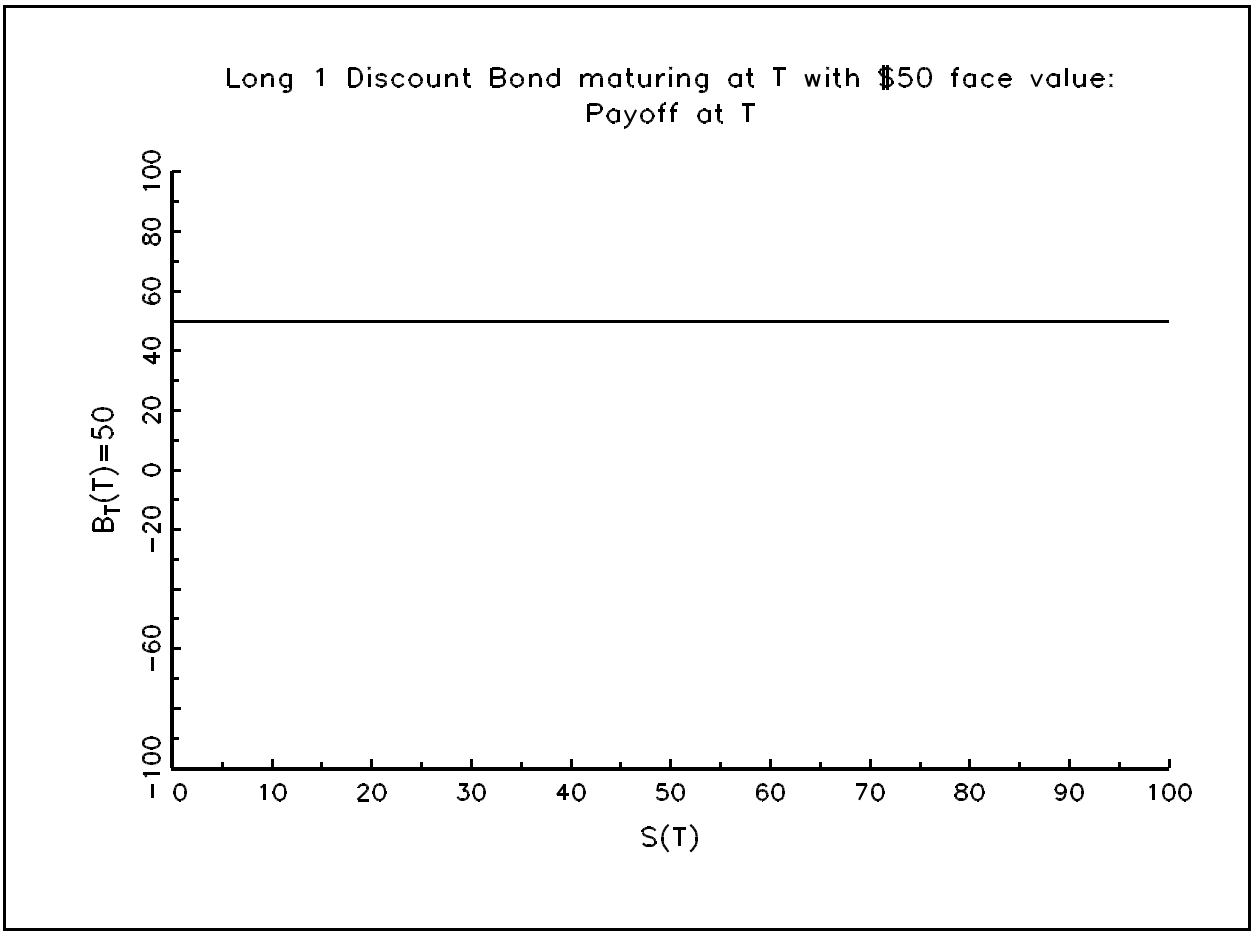
b. Example. Suppose you have 100 times the value of the S&P 500 index invested in the market at the start of 1997. When the S&P 500 hits 800 on 2/12/97, you decide you want to bound your losses so you buy a European put on 100 times the value of the S&P 500 with an exercise price of $800 and an expiry date of 12/97. You have implemented portfolio insurance.

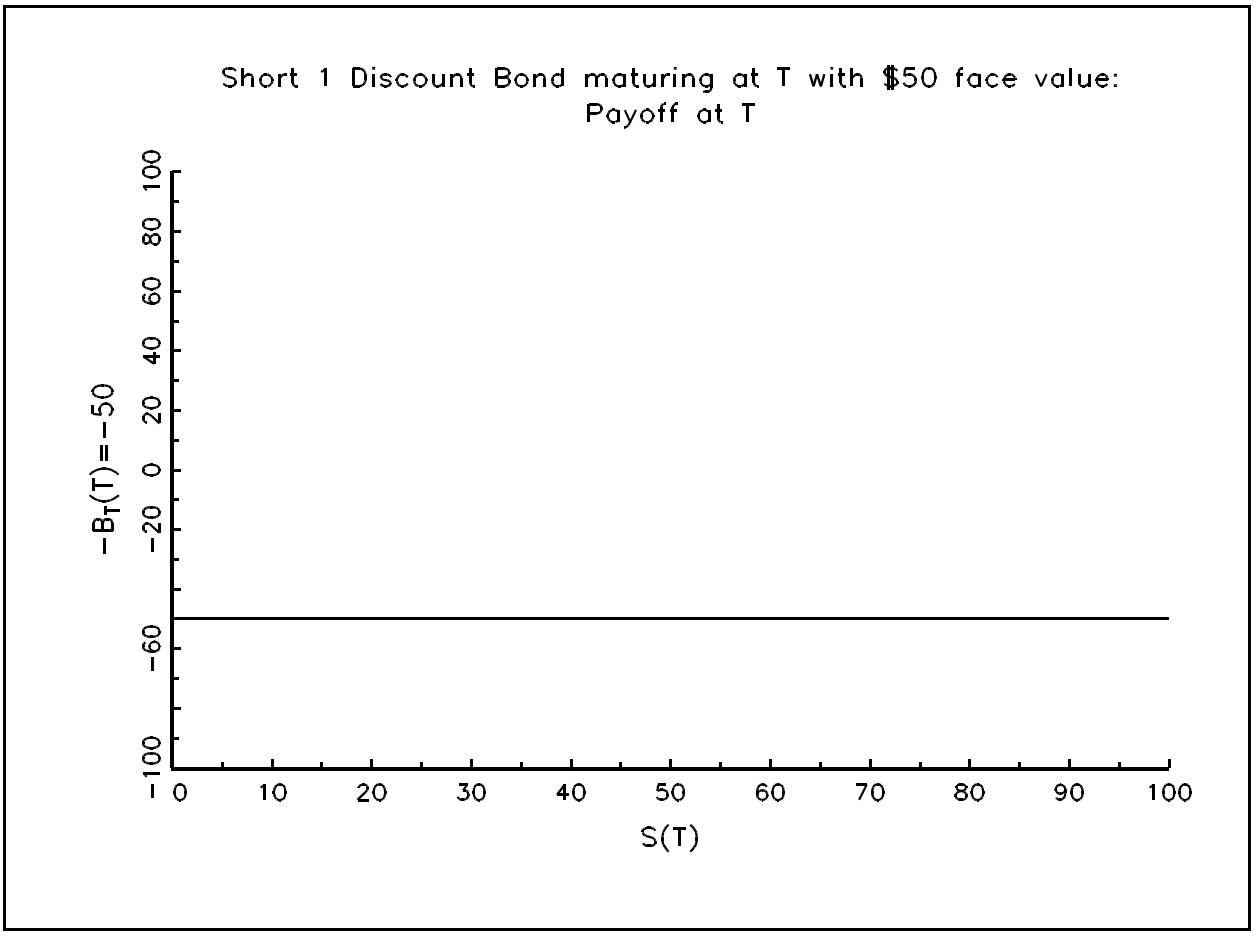
Table 1

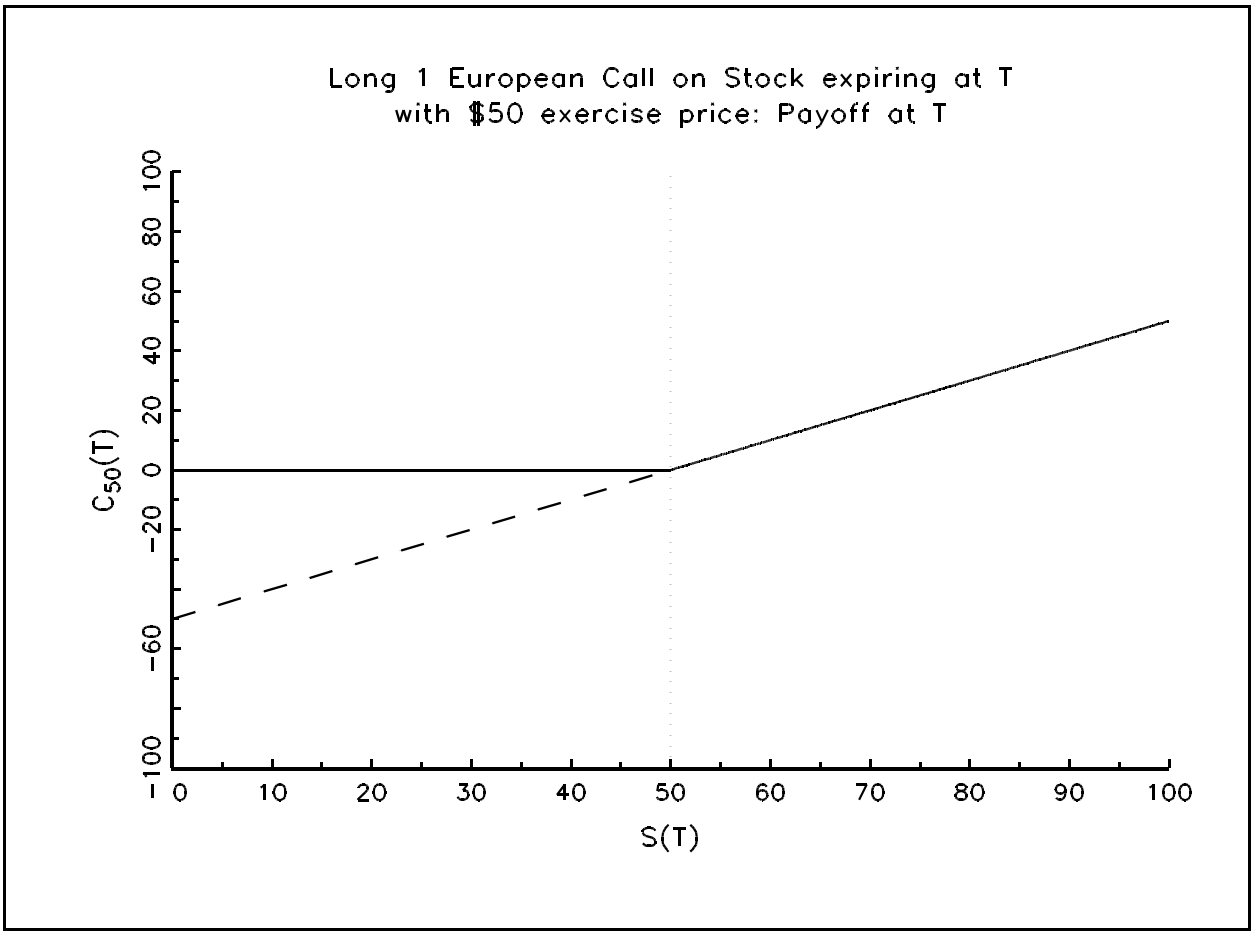
Position opened at time 0	Time 0	Time T		
		S(T)<50	S(T)≥50	
Long 1 Stock	-S(0)	S(T)	S(T)	S(T)
Short 1 Stock	S(0)	-S(T)	-S(T)	-S(T)
Long 1 Discount Bond maturing at T with face value of \$50	-50 $d_T(0)$	50	50	50
Short 1 Discount Bond maturing at T with face value of \$50	50 $d_T(0)$	-50	-50	-50
Long 1 European Call Option on the Stock expiring at T with an exercise price of \$50	$-C_{50,T}(0)$	0	S(T)-50	max [S(T)-50,0]
Short 1 European Call Option on the Stock expiring at T with an exercise price of \$50	$C_{50,T}(0)$	0	-[S(T)-50]	-max [S(T)-50,0]
Long 1 European Put Option on the Stock expiring at T with an exercise price of \$50	$-P_{50,T}(0)$	50-S(T)	0	max [50-S(T),0]
Short 1 European Put Option on the Stock expiring at T with an exercise price of \$50	$P_{50,T}(0)$	-[50-S(T)]	0	-max [50-S(T),0]
Long 1 Forward Contract on the Stock with delivery at T at a forward price $F_T(0)$	0	$S(T)-F_T(0)$	$S(T)-F_T(0)$	$S(T)-F_T(0)$
Short 1 Forward Contract on the Stock with delivery at T at a forward price $F_T(0)$	0	$F_T(0)-S(T)$	$F_T(0)-S(T)$	$F_T(0)-S(T)$

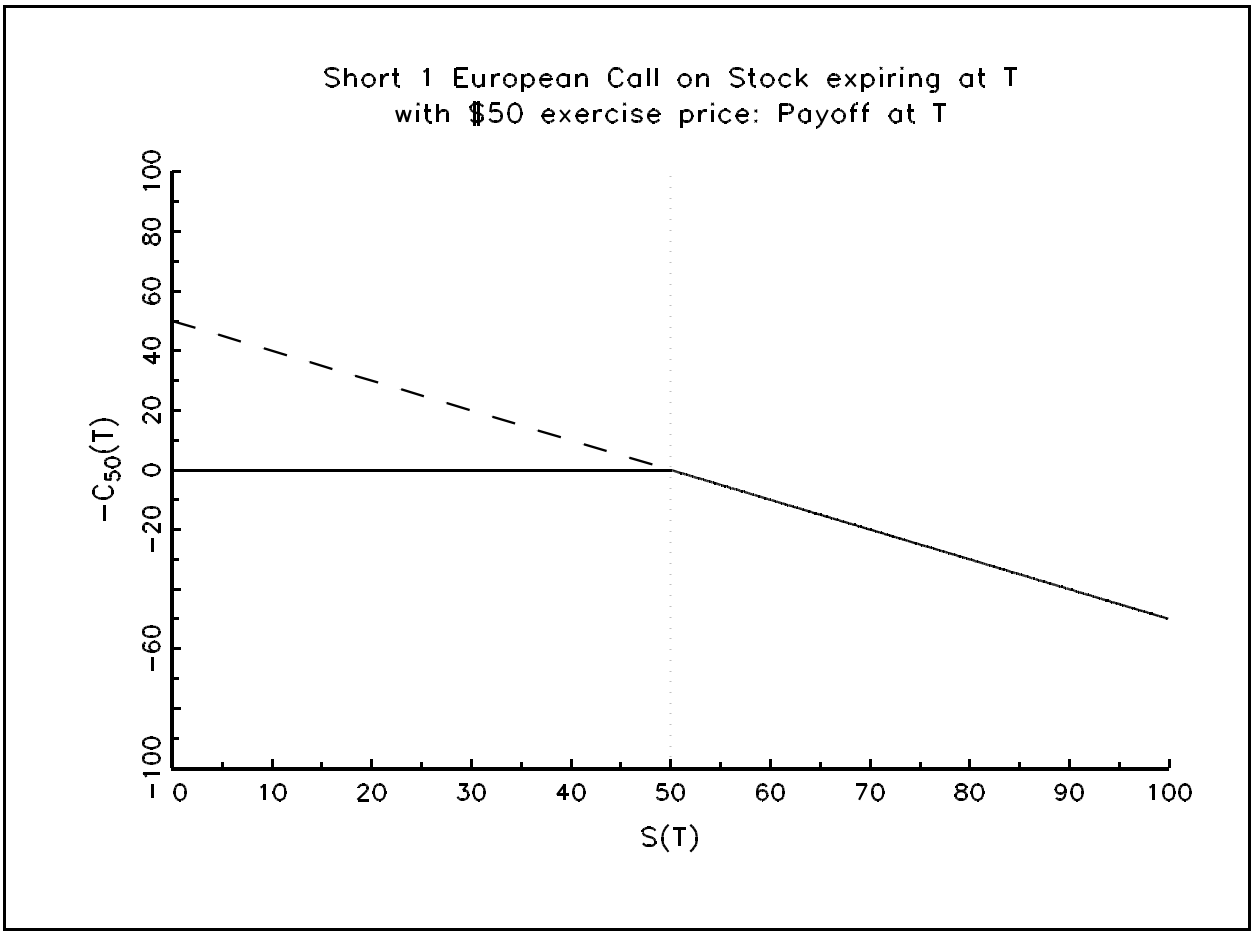

Long 1 Stock:
Payoff at T
S(T)
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)

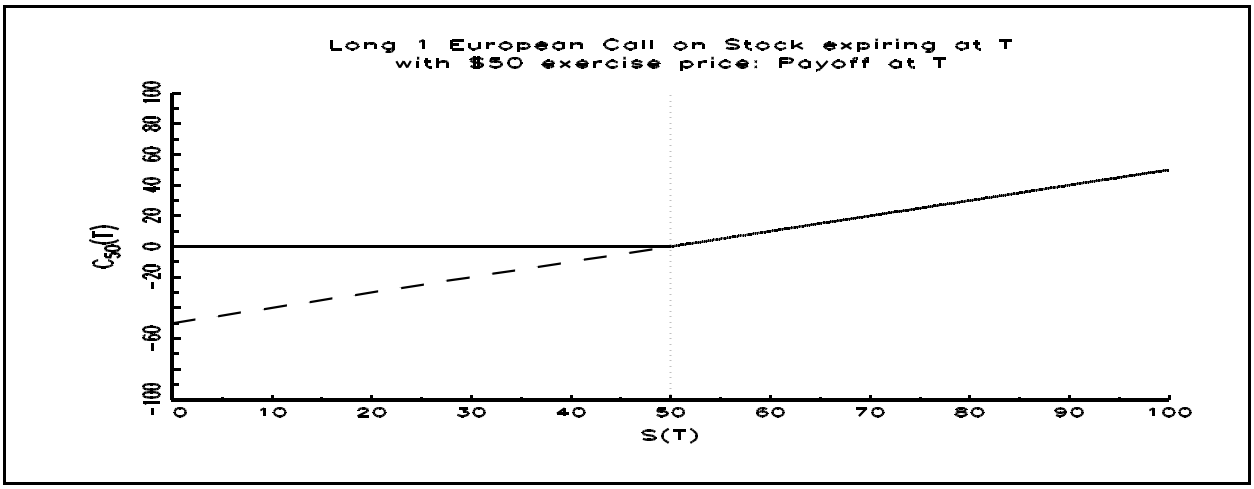

Short 1 Stock:
Payoff at T
-S(T)
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)

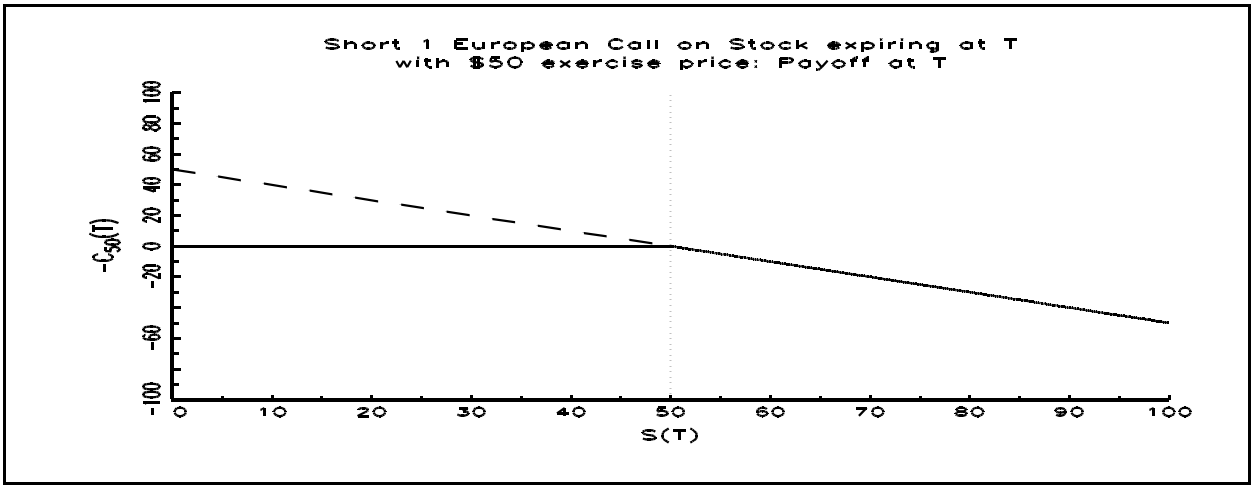

Long 1 Discount Bond maturing at T with $50 face value:
Payoff at T
B_T(T)=50
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)

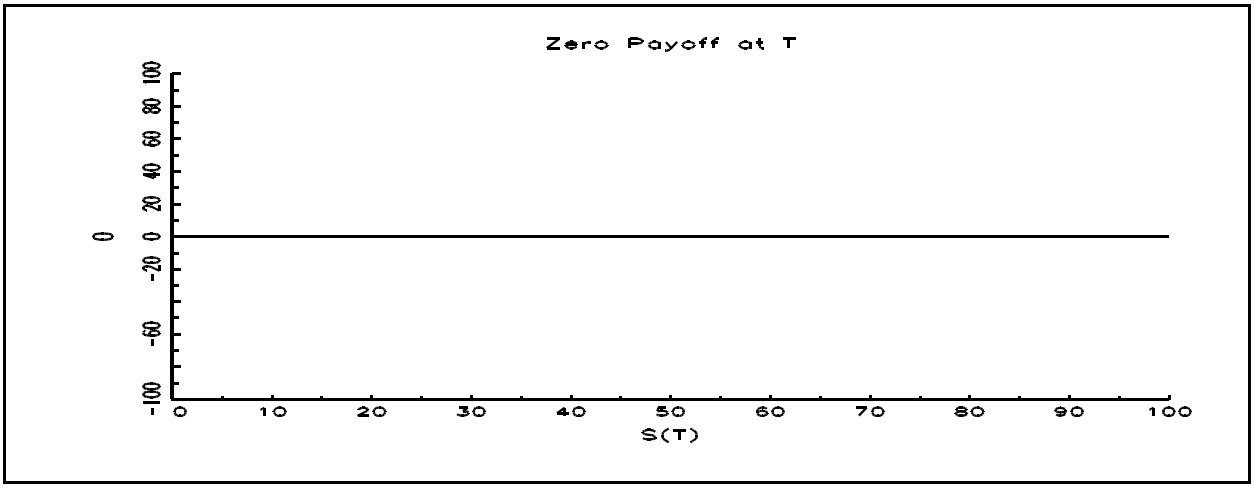

Short 1 Discount Bond maturing at T with $50 face value:
Payoff at T
-B_T(T)=-50
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)

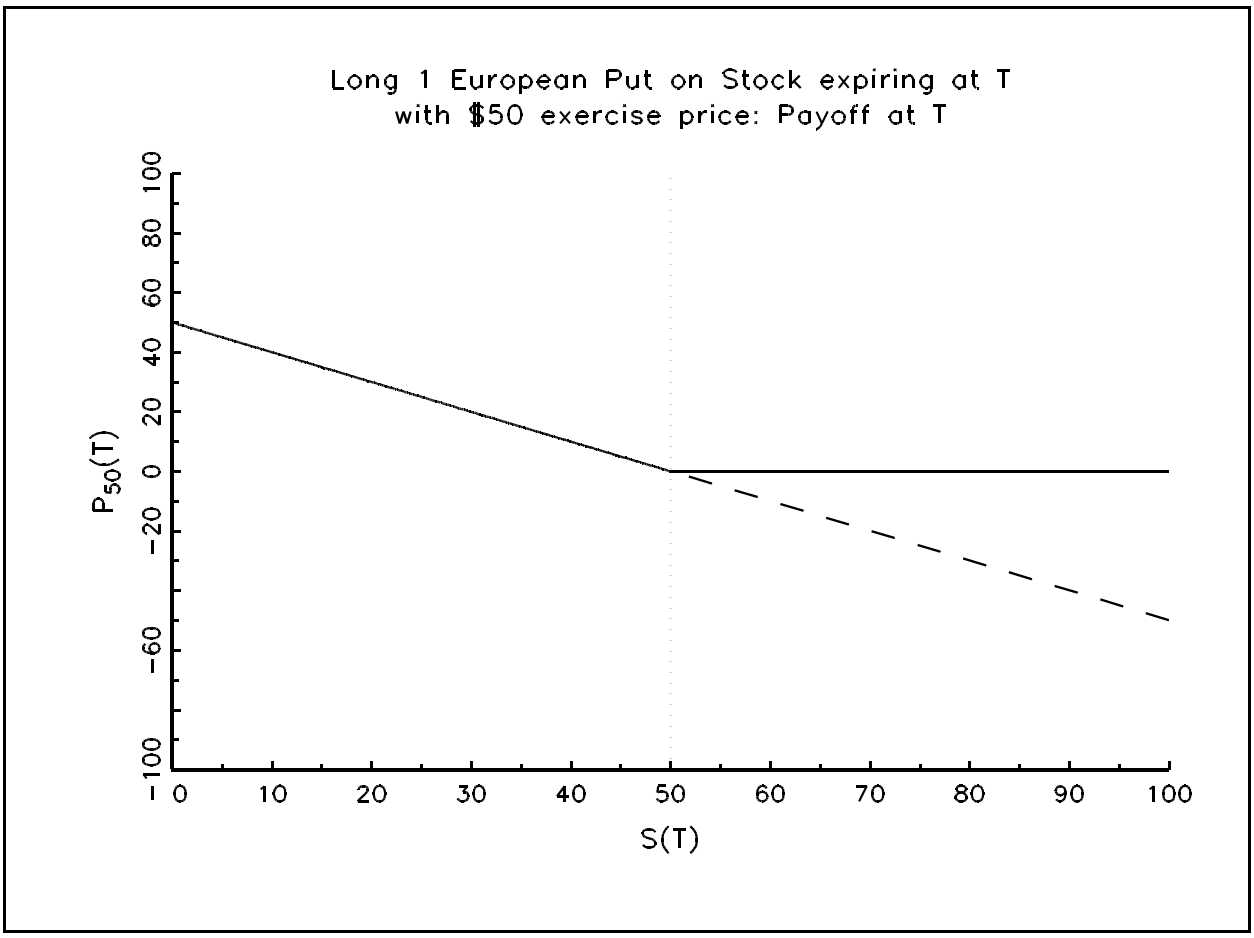

Long 1 European Call on Stock expiring at T
with $50 exercise price: Payoff at T
C50(T)
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)

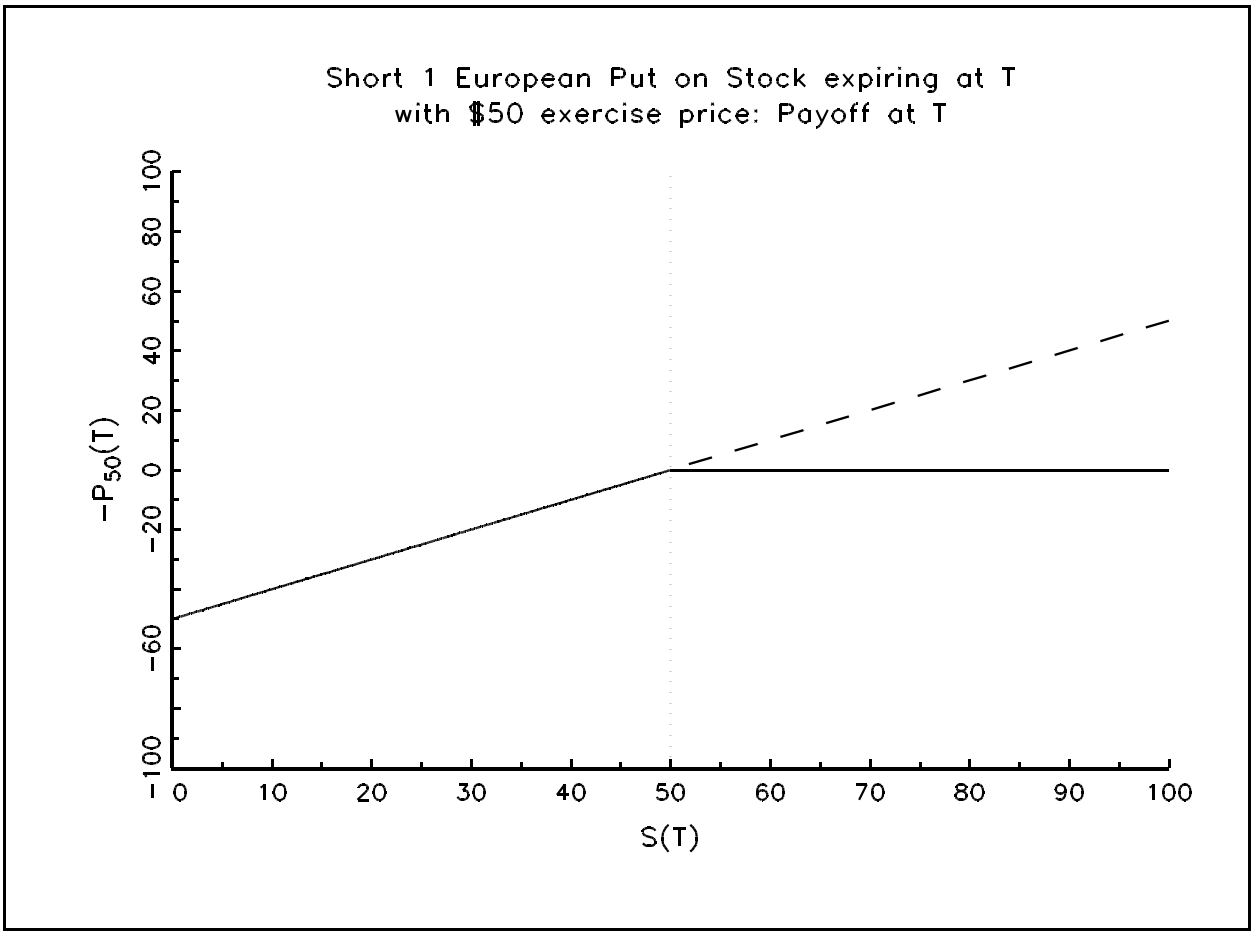

Short 1 European Call on Stock expiring at T
with $50 exercise price: Payoff at T
$-C_{50}(T)$
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)

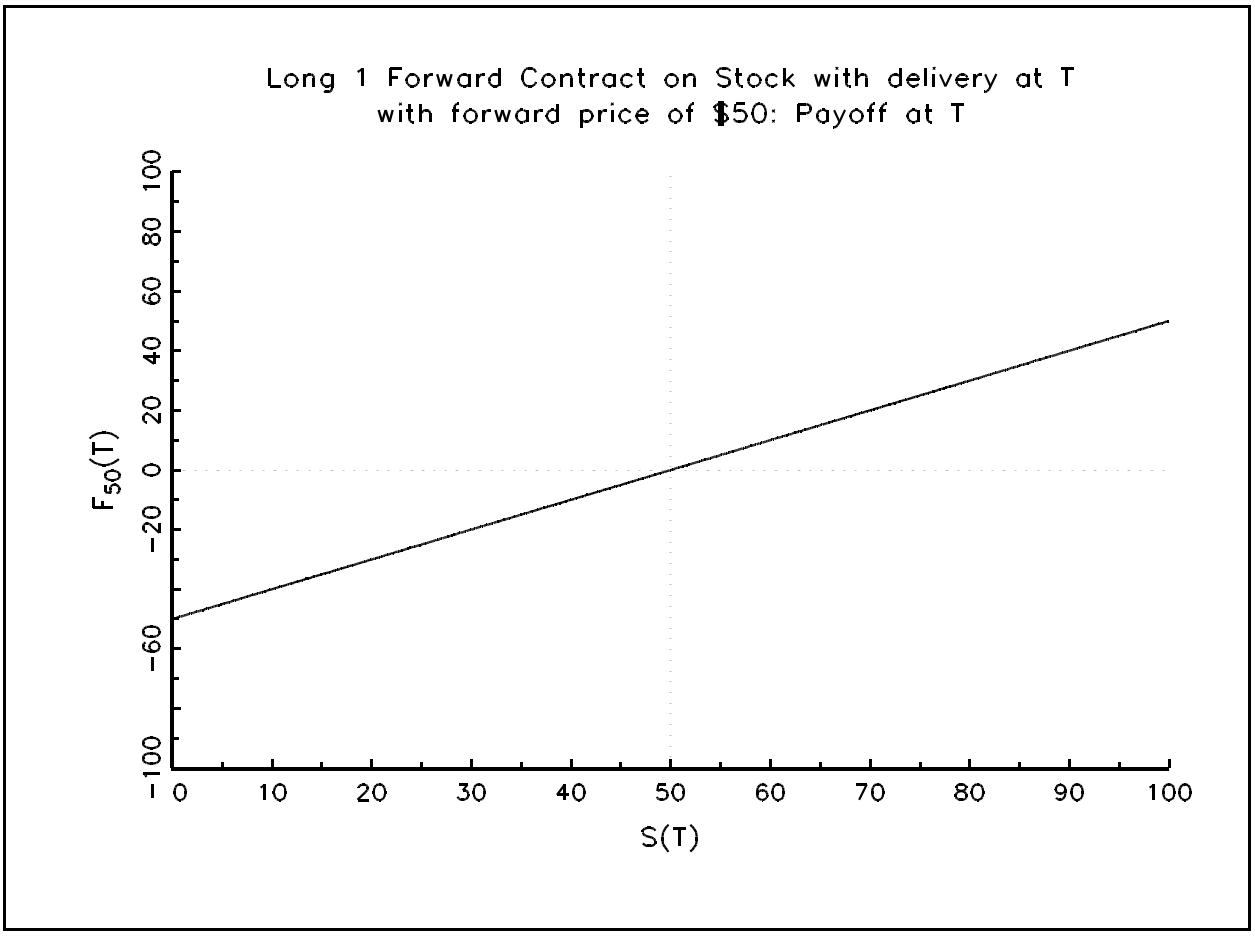

Long 1 European Call on Stock expiring at T
with $50 exercise price: Payoff at T
$C_{50}(T)$
S(T)

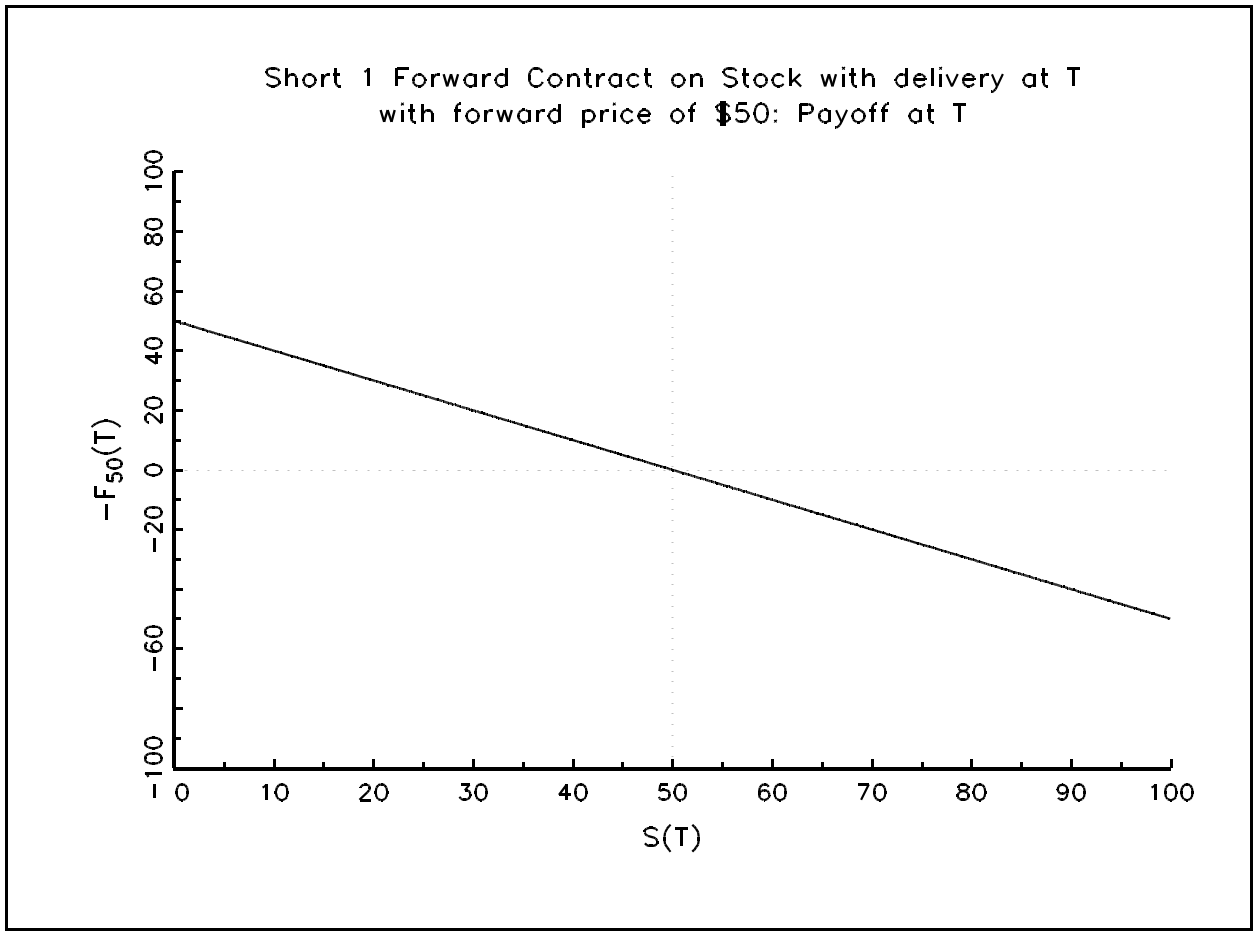

Short 1 European Call on Stock expiring at T
with $50 exercise price: Payoff at T
$-C_{50}(T)$
S(T)


Zero Payoff at T
0
S(T)


Long 1 European Put on Stock expiring at T
with $50 exercise price: Payoff at T
P50(T)
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)


Short 1 European Put on Stock expiring at T
with $50 exercise price: Payoff at T
$-P_{50}(T)$
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)


Long 1 Forward Contract on Stock with delivery at T
with forward price of $50: Payoff at T
F50(T)
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)


Short 1 Forward Contract on Stock with delivery at T
with forward price of $50: Payoff at T
$-F_{50}(T)$
100
80
60
40
20
0
-20
-60
-100
0
10
20
30
40
50
60
70
80
90
100
S(T)