Liquidity and Volatility

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Liquidity and Volatility

1. Liquidity provision is a central function of the financial system
   - lowers the cost of buying and selling assets
2. We show that liquidity provision has a large negative beta to market volatility
   - when market volatility rises $\rightarrow$ liquidity providers lose
   - we show this theoretically and confirm empirically for stocks
3. Volatility beta + large volatility risk premium explains why:
   - liquidity provision earns a premium (the liquidity premium)
   - this premium increases with volatility (Nagel 2012)
   - liquidity supply contracts during crises
4. A new view of the risks and returns to liquidity provision

Intuition

1. Liquidity providers buy and sell from investors (absorb order flow)
   - protect against private info by making prices sensitive to order flow
   - sensitivity depends on amount of private info in the market
2. Key idea: liquidity providers don't know the amount of private info
   - set sensitivity of prices based on expected amount of private info
   - if private info higher than expected $\rightarrow$ prices weren't sensitive enough $\rightarrow$ liquidity providers lose
3. Information flows fluctuate with market volatility
   - volatility is the observed end-product of information flows
   - and asset volatilities covary strongly with market volatility
4. A positive shock to market volatility $\rightarrow$ all asset volatilities rise $\rightarrow$ private info is higher than was expected $\rightarrow$ prices adjust and liquidity providers lose

   $\Rightarrow$ liquidity providers have a negative market volatility beta

   $\Rightarrow$ liquidity premium $=$ volatility beta $\times$ volatility risk premium

Related literature

1. Liquidity and asymmetric information: Akerlof (1970); Grossman and Stiglitz (1980); Kyle (1985); Glosten and Milgrom (1985);Foster and Viswanathan (1990); Gorton and Pennacchi (1990);Collin-Dufresne and Fos (2016);

2. Volatility risk: Engle (1982); Andersen, Bollerslev, Diebold and Labys (2003); Carr and Wu (2008); Bollerslev, Tauchen and Zhou (2009); Drechsler and Yaron (2010); Todorov (2010); Drechsler (2013)

3. Asset prices and liquidity: Stoll(1978); Amihud and Mendelson (1986);Grossman and Miller (1988); Lehman (1990); Amihud (2002); Pástor and Stambaugh (2003);Easley and O'hara (2004); Acharya and Pedersen (2005); Duffie (2010); Nagel (2012)

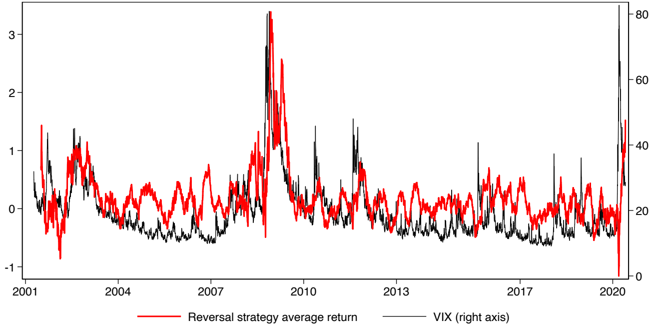
4. Macro finance and liquidity: Gromb and Vayanos (2002); Eisfeldt (2004); Brunnermeier and Pedersen (2009); Adrian and Shin (2010); Moreira and Savov (2017); Drechsler, Savov, Schnabl (2018)

Volatility and the returns to liquidity provision

1. Proxy for returns to liquidity provision using stock reversals
   - sort large-cap stocks into deciles by day's normalized return
   - buy lowest decile, sell highest decile, hold for 5 days (Nagel 2012)

⇒ Average return to liquidity provision 20 bps (10% annual, SR 0.5)
   - market neutral

2. Liquidity premium strongly increasing in market volatility:



⇒ Higher VIX → high liquidity premium (corr. 46%)

Volatility beta of liquidity provision

1. Estimate

$$R_{t,t+5} = \alpha + \beta_{VIX} \Delta VIX^2_{t,t+5} + \epsilon_{t,t+5}$$

2. Reversal strategy's $\beta_{VIX} = -0.2$
   - 77-bps drop for a 1 standard deviation increase in ΔVIX^2
   - large relative to 20-bps average return

3. Recover VIX premium (Variance Risk Premium) from option markets
   - VRP ≈ -100 bps per five days (for $\beta_{VIX} = 1$)

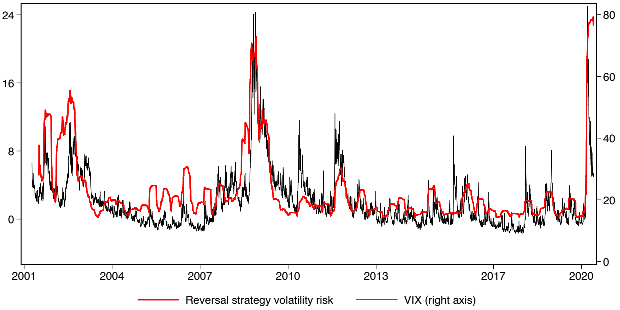
$\Rightarrow$ Volatility risk explains the liquidity premium:

$$\beta_{VIX} \times VRP \ = \ \text{Liquidity Premium}$$

$$-0.2 \times -100 \ bps \ = \ 20 \ bps$$

Volatility and the systematic risk of liquidity provision

1. Rolling estimate of volatility risk: $\sigma\left(\beta_{VIX}\Delta VIX^2\right)$
2. Volatility risk of liquidity provision is strongly increasing in volatility:

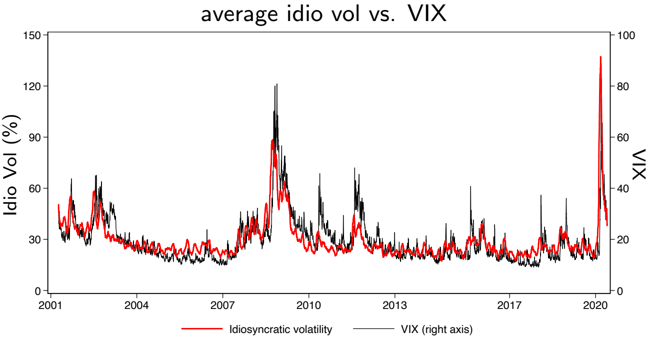


$\Rightarrow$ Higher VIX $\rightarrow$ liquidity provision riskier (corr. 58%)

3. Explains why liquidity premium is highly correlated with VIX

Idiosyncratic volatility and market volatility

1. Liquidity providers bet against idiosyncratic information
2. Though the information is idiosyncratic, its quantity covaries strongly across assets and with market vol



⇒ High VIX → high future idiosyncratic volatility (corr: 74%)

3. Liquidity provision exposed to systematic risk

Model

1. Kyle (1985) framework with uncertain quantity of private information
   - three dates: 0, $\tau \in (0, 1)$, and 1
   - three agents: informed trader, liquidity-demanders, liquidity providers
   - N assets: traded at time 0, pay off at date 1:

$$p_{i,1} = \overline{v}_i + v_i$$

   - $\overline{v}_i$ is known by all at $t = 0$
   - $v_i \sim N(0, \sigma_{v,i})$ is idiosyncratic; informed knows v_i at $t = 0$

2. Informed trader demands $y_i = \phi v_i$
3. Liquidity-demanders: demand $z_i \sim N(0, \sigma_{z,i})$
4. Liquidity providers: absorb net order flow $x_i = \phi v_i + z_i$
   - don't know v_i $\rightarrow$ try to infer it from x_i
   - competitive $\rightarrow$ set price to expected time-1 payoff using economy's SDF Λ_1:

$$p_{i,t} = E_t[\Lambda_1 \, p_{i,1}] = E_t^Q \left[p_{i,1}\right]$$

$\Rightarrow$ take aggregate risk premia into account

Model continued

5. Liquidity providers do *not* know quantity of private information $\sigma^2_{v,i}$

   - they estimate it: $E_t[\sigma^2_{v,i}]$
   - amounts of idiosyncratic and market information covary positively:

$$\sigma^2_{v,i} = k_i \sigma^2_m + \varsigma^2_{v,i}$$

   $\Rightarrow$ idio and market vols are highly correlated, as in the data
   $\Rightarrow$ news about σ^2_m provides information about $\sigma^2_{v,i}$

6. At date τ: public news causes investors to update expected market volatility $E_\tau[\sigma^2_m]$

   $\Rightarrow$ liquidity providers also update $E^Q_\tau[\sigma^2_{v,i}]$

Equilibrium pricing

1. At date 0: liquidity providers absorb $-x_i$ shares, set price to

$$p_{i,0} \;=\; E_0^Q\left[\,p_{i,1}\right|x_i] \;=\; \overline{v}_i + \phi\frac{E_0^Q\left[\sigma_{v,i}^2\right]}{\sigma_{x,i}^2}x_i$$

   - $p_{i,0}$ moves in direction of order flow x_i
   - higher $E_0^Q[\sigma_{v,i}^2] \rightarrow$ flow has more info $\rightarrow p_{i,0}$ more sensitive to x_i

2. Let $\Delta p_{i,0} = (p_{i,0} - \overline{v}_i)$ denote the time-0 price change. Then:

$$-x_i = -\frac{1}{\phi}\left(\frac{\sigma_{x,i}^2}{E_0^Q[\sigma_{v,i}^2]}\right)\Delta p_{i,0}$$

   ⇒ liquidity providers hold a portfolio of reversals
   ⇒ buy assets that are down and short assets that are up

3. We use reversals to proxy for liquidity provision empirically

Volatility risk exposure

1. At date τ: market volatility news $\rightarrow$ liquidity providers update $E^Q[\sigma^2_{v,i}]$ and hence informativeness of time-0 order flows

$$\Delta p_{i,\tau} \;=\; E^Q_\tau[p_{i,1}] - E^Q_0[p_{i,1}] \;=\; \frac{\phi x_i}{\sigma^2_{x,i}} \left(E^Q_\tau\left[\sigma^2_{v,i}\right] - E^Q_0\left[\sigma^2_{v,i}\right] \right)$$

$\Rightarrow$ a volatility increase drives $p_{i,t}$ further in the direction of x_i

$\Rightarrow$ longs go down, shorts go up (loses on both sides of portfolio)

2. Market vol beta β_{i,σ_m} of liquidity providers' position $-x_i \Delta p_{i,t}$ is:

$$\beta_{i,\sigma_m} \;=\; -\frac{k_i}{\phi} \left(\frac{\sigma_{x,i}}{E^Q_0\left[\sigma^2_{v,i}\right]} \Delta p_{i,0} \right)^2 < 0$$

- β_{i,σ_m} is always negative, magnitude increasing in $\Delta p_{i,0}$
- β_{i,σ_m} also increasing in k_i (loading of asset's idio vol on market vol)

$\Rightarrow$ Liquidity providers bear *undiversifiable* market volatility risk

- undiversifiable risk comes from correlation in assets' *second* moments

Liquidity premium and the variance risk premium

1. Market volatility risk commands a very large, negative price of risk
   - investors dislike times when market volatility goes up

$\Rightarrow$ Price of a market variance swap, $E_0^Q\left[\sigma_m^2\right]$, is substantially higher than its expected payoff $E_0\left[\sigma_m^2\right]$

   - the swap buyer's expected loss is the insurance premium they pay for hedging market volatility shocks

2. Liquidity providers' expected payoff on asset i from date 0 to 1 is:

$$E_0\left[-x_i \Delta p_{i,1}\right] = \beta_{i,\sigma_m}\left(E_0\left[\sigma_m^2\right] - E_0^Q\left[\sigma_m^2\right]\right) > 0$$

   - demand a positive risk premium for their negative beta to market volatility
   - should be the same premium in embedded in derivate prices if the markets are integrated

Predictions for the risks and returns of reversal portfolios

1. Market vol betas are negative: $\sum_{i=1}^{N} \beta_{i,\sigma_m} < 0$

2. Expected return (liquidity premium) is positive:

$$E_0\left[\sum_{i=1}^{N} -x_i \Delta p_{i,1}\right] = \beta_{\sigma_m}\Big(E_0\left[\sigma_m^2\right] - E_0^Q\left[\sigma_m^2\right]\Big) > 0$$

   - **Market integration**: price of market volatility beta is the same as in options/variance markets

3. Cross-sectional predictability:

   - larger $|\Delta p_{i,0}| \rightarrow$ more negative $\beta_{i,\sigma_m} \rightarrow$ higher average return
   - larger vol loading $k_i \rightarrow$ more negative $\beta_{i,\sigma_m} \rightarrow$ higher average return

4. Time-series predictability:

   - higher VIX $\rightarrow$ more volatility risk $\rightarrow$ higher avg. reversal returns
   - larger $|\Delta p_{i,0}| \rightarrow$ more negative $\beta_{i,\sigma_m} \rightarrow$ higher predictive loadings

Data and empirical strategy

1. Construct reversal portfolios
   - each day, sort stocks into quintiles by size and then deciles by return normalized by rolling standard deviation and weight by dollar volume, hold for five days as in Nagel (2012)
   - focus on period since decimilization: 4/9/2001 to 5/31/2020, when liquidity provision became competitive
   - drop stocks with share price in bottom 20%, and earnings announcements (public news events)
2. Look at the cross section of reversals:
   - buy low-return deciles, sell high-return deciles: 1–10 ("Lo–Hi"), 2–9, . . . , 5–6
   - portfolios capture most to least amounts of liquidity provision
3. Construct aggregate liquidity provider portfolio
   - aggregate all stocks using model-implied weights

Cross section of reversal returns

Average returns

	Lo–Hi	2–9	3–8	4–7	5–6
Small	0.78	0.46	0.20	−0.08	−0.10
2	0.40	0.15	0.06	0.08	−0.02
3	0.15	0.12	0.04	0.04	−0.02
4	0.16	0.19	0.11	0.12	0.02
Big	0.20	0.16	0.18	0.08	0.00

CAPM alphas

	Lo–Hi	2–9	3–8	4–7	5–6
Small	0.74	0.44	0.16	−0.10	−0.10
2	0.38	0.12	0.05	0.07	−0.02
3	0.12	0.10	0.03	0.03	−0.03
4	0.13	0.17	0.10	0.11	0.02
Big	0.17	0.14	0.16	0.07	−0.00

1. Large-caps avg. 5-day return: 20 bps (10% annual), Sharpe ratio 0.5
   - portfolios have $\approx$ 120 stocks, avg market cap of $80 Billion
   - Small & 2 reversal returns are larger, market cap $<$ 0.5% total
   - CAPM alphas $\approx$ average returns $\Rightarrow$ CAPM cannot price the reversals
2. Avg returns increase in amount of liquidity provision (5-6 to Lo-Hi)

Cross section of reversal volatility risk

$$R^p_{t,t+5} = \alpha_p + \beta^p_M R^M_{t,t+5} + \beta^p_{VIX} \Delta VIX^2_{t,t+5} + \epsilon^p_{t,t+5}$$

$\beta^{p,VIX}$ (controlling for market)

	Lo–Hi	2–9	3–8	4–7	5–6
Small	−0.18	−0.05	−0.15	−0.02	−0.05
2	−0.18	−0.10	−0.08	−0.10	−0.03
3	−0.19	−0.11	−0.11	−0.05	0.00
4	−0.17	−0.13	−0.06	−0.06	0.01
Big	−0.20	−0.11	−0.13	−0.07	−0.04

$t(\beta^{p,VIX})$

	Lo–Hi	2–9	3–8	4–7	5–6
Small	−3.48	−0.82	−1.91	−0.27	−0.80
2	−3.45	−2.45	−2.06	−3.80	−1.24
3	−3.89	−3.29	−4.55	−2.13	0.01
4	−3.98	−3.73	−2.23	−2.25	0.32
Big	−4.27	−3.31	−4.10	−3.30	−1.72

1. Reversal strategy has a large negative beta to ΔVIX^2
   - large-cap reversal drops by 100 bps per 5-point VIX^2 increase (1.3 standard deviations); big relative to 20bps average return
2. Beta magnitude is increasing in amount of liquidity provision

Price of risk from the cross section

$$R^p_{t,t+5} = \lambda_{0,t} + \lambda_{t,VIX}\beta^p_{VIX} + \lambda_{t,M}\beta^p_M + e^p_{t,t+5}.$$

	Constant	Market	VIX	Market + VIX
β_M		0.325***		0.152
		(0.122)		(0.125)
β_{VIX}			−0.570***	−1.079***
			(0.156)	(0.230)
Constant	0.205**	−0.434***	−0.527***	−0.251**
	(0.101)	(0.115)	(0.097)	(0.127)
N	50	50	50	50

1. Cross-sectional of reversal implies a large, negative, and highly significant variance risk premium (-108 bps per 5 days)

Pricing errors

$$R^p_{t,t+5} = \lambda_{0,t} + \lambda_{t,VIX}\beta^p_{VIX} + \lambda_{t,M}\beta^p_M + e^p_{t,t+5}.$$

Pricing errors

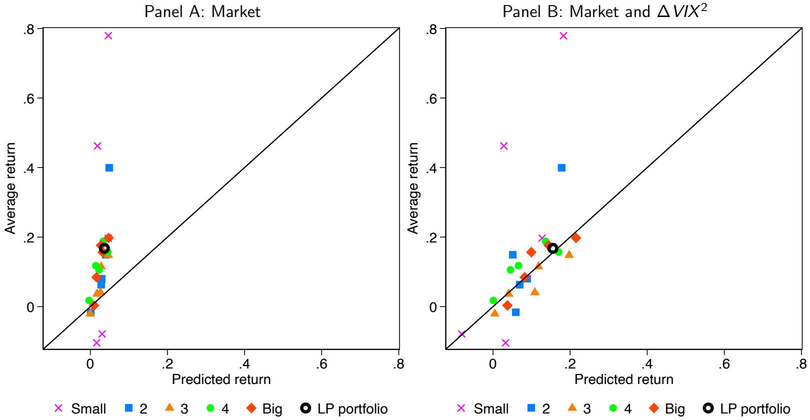
	Lo–Hi	2–9	3–8	4–7	5–6
Small	0.58	0.42	0.05	−0.00	−0.13
2	0.22	0.09	−0.01	−0.00	−0.08
3	−0.05	−0.01	−0.07	−0.01	−0.03
4	−0.02	0.05	0.06	0.05	0.02
Big	−0.02	0.06	0.03	0.00	−0.04

t statistics

	Lo–Hi	2–9	3–8	4–7	5–6
Small	6.70	5.43	0.60	−0.05	−1.58
2	4.08	1.85	−0.12	−0.05	−1.80
3	−1.11	−0.17	−1.80	−0.16	−0.96
4	−0.45	1.32	2.13	1.77	0.80
Big	−0.51	1.80	0.86	0.02	−1.65

1. VIX factor explains the reversal strategy returns of large- and mid-cap stocks
   - large cap return drops from 20bps to -2bps and is insignificant
   - Pricing errors for portfolios 1 and 2 still large

Cross-sectional fit



1. VIX factor explains the reversal strategy returns of large- and mid-cap stocks
2. Aggregate liquidity provider (LP) portfolio is priced exactly

Summary of cross-sectional pricing

	Constant	Market	VIX	Market+VIX
(ii) Long-short reversal strategies:				
R.m.s.e.	0.228	0.201	0.183	0.158
p-value	0.00	0.00	0.00	0.00
(iii) Long-short reversal strategies (ex small stocks):				
R.m.s.e.	0.123	0.094	0.065	0.040
p-value	0.00	0.00	0.01	0.02
(iv) Value-weighted reversal strategy:				
Pricing error	0.124	0.094	0.062	0.006
p-value	0.00	0.00	0.00	0.79
(v) Liquidity provider portfolio:				
Pricing error	0.168	0.127	0.084	0.008
p-value	0.00	0.00	0.00	0.78

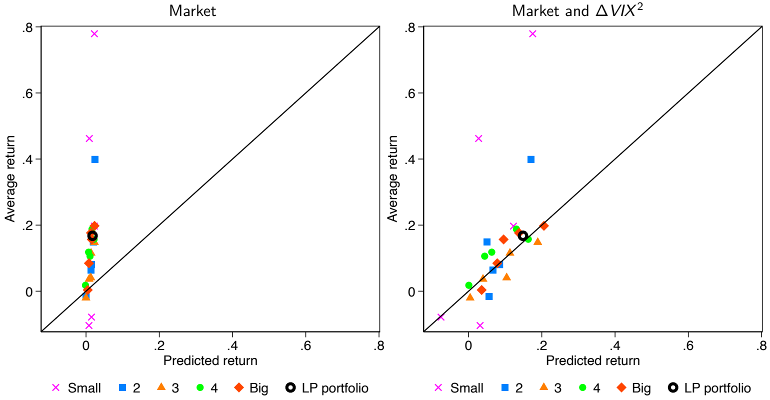
1. Pricing errors shrink to zero as we move towards the aggregate liquidity provision portfolio

Is the implied price of volatility risk consistent with option markets?

1. Volatility risk is traded directly in option markets
   - VIX is the price of a basket of options that replicates the realized variance of the S&P 500 over next 30 days
   - However, ΔVIX is not a return because basket changes daily
2. We replicate the VIX using S&P 500 options and use the change in the price of a given basket to get a VIX return
3. Average daily VIX return is -1.53%
   - in line with variance premium literature (e.g. Carr and Wu, 2008; Bollerslev, Tauchen, and Zhou, 2009, Drechsler and Yaron 2010)

$\Rightarrow$ **Implied price of risk is -103 bps for $\beta_{VIX} = 1$, very close to -108 bps from cross section of reversals**

Predicted returns with an options-based price of risk



1. Option-based price of VIX factor explains reversal returns of large- and mid-cap stocks
2. Returns to liquidity provision reflect risks priced in financial markets more broadly

Pricing regressions with an options-based price of risk

	Constant	Market	VIX	Market+VIX
(ii) Long-short reversal strategies:				
R.m.s.e.	0.228	0.212	0.158	0.159
p-value	0.00	0.00	0.00	0.00
(iii) Long-short reversal strategies (ex small stocks):				
R.m.s.e.	0.123	0.107	0.038	0.040
p-value	0.00	0.00	0.05	0.02
(iv) Value-weighted reversal strategy:				
Pricing error	0.124	0.108	0.012	0.011
p-value	0.00	0.00	0.59	0.62
(v) Liquidity provider portfolio:				
Pricing error	0.168	0.146	0.015	0.015
p-value	0.00	0.00	0.62	0.62

1. Restrictions to option-based priced of risk does not impact fit

Time-series Predictability

$$R^p_{t,t+5} = \alpha^p + \beta^p VIX_t + \epsilon^p_{t,t+5}$$

Predictive loadings (β^p)

	Lo–Hi	2–9	3–8	4–7	5–6
Small	9.81	7.69	9.51	−0.48	−2.60
2	7.71	4.34	3.70	1.68	−0.34
3	6.72	3.13	3.00	2.65	0.44
4	5.70	5.28	3.01	3.41	−0.54
Big	9.09	5.51	3.53	3.02	0.34

R^2

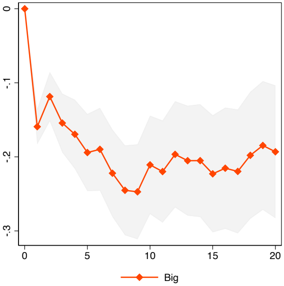
	Lo–Hi	2–9	3–8	4–7	5–6
Small	0.85	0.66	0.97	0.00	0.08
2	1.30	0.56	0.44	0.10	0.00
3	1.38	0.45	0.43	0.34	0.01
4	1.21	1.47	0.65	1.03	0.03
Big	3.45	1.94	1.09	1.06	0.02

1. *VIX* predicts reversal strategy returns
   - extends result of Nagel (2012) to cross section
   - predictive coefficients increasing in liquidity provision
   - very high R^2 for large stocks for a five-day horizon

Market segmentation (inventory frictions)

1. Nagel (2012) interprets the predictability of reversals by VIX as evidence of inventory costs
2. We extend the model to allow for inventory costs and show that
   - predicts a transitory volatility beta
   - → higher inventory cost induce fire-sale effect that reverts

$$R^p_{t,t+h} \quad = \quad \alpha_p + \beta^p_{VIX,h} \Delta VIX^2_{t,t+1} + \epsilon^p_{t,t+5},$$



3. Empirically, impact of vol shock does **not** revert for large caps

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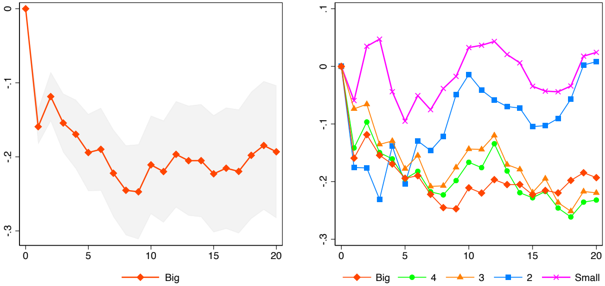


3. Empirically, impact of vol shock does **not** revert for large caps
   - permanent impact on mid to large caps (3 to 5)
   - transitory impact on small cap portfolios (1 and 2)
   - → small-caps mispricing consistent with inventory costs

Volatility co-movement portfolios

1. Model predicts market vol betas should increases with idio vol loading on market vol k_i

   - we estimate $\widehat{\sigma}^i_{t,t+5} = a + k_i VIX_t + \epsilon_{i,t}$

ΔVIX^2 betas (controlling for R^M)

	Lo–Hi	2–9	3–8	4–7	5–6
Low k	−0.07	−0.07	−0.09	−0.09	−0.02
2	−0.15	−0.06	−0.13	0.02	0.01
3	−0.20	−0.12	−0.09	−0.05	−0.05
4	−0.17	−0.17	−0.13	−0.01	−0.05
High k	−0.34	−0.03	−0.07	−0.01	0.02

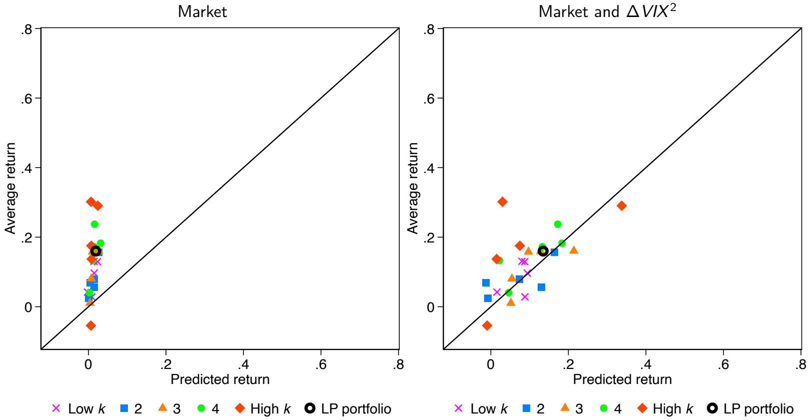
CAPM alphas

	Lo–Hi	2–9	3–8	4–7	5–6
Low k	0.10	0.11	0.08	0.02	0.04
2	0.13	0.06	0.04	0.06	0.02
3	0.14	0.14	0.15	0.07	0.01
4	0.15	0.22	0.16	0.12	0.04
High k	0.26	0.29	0.16	0.13	−0.06

2. VIX betas increasing in k_i
3. CAPM alphas increasing in k_i

Volatility co-movement portfolios

1. Impose option-based price of risk



2. Volatility betas do a great job in capturing variation in average returns

Takeaways

1. Fundamental connection between volatility and liquidity
2. Exposure to private information $\Rightarrow$ liquidity provision exposed to volatility risk
3. Explains level and variation of liquidity premium
4. Liquidity dry-ups such as the one we observed in March 2020 occur even if intermediaries well capitalized
5. A new, asset-pricing perspective on the risks and returns to liquidity provision