

## Lecture Notes 4b

# Equities: Positions and Portfolio Returns

- I. Readings and Suggested Practice Problems**
- II. Stock Transactions Involving Credit**
- III. Portfolio Returns**
- IV. Appendix**
- V. Additional Readings**

***Buzz Words:***     *Going Long, Buying on Margin, Selling Short, Percentage Margin, Maintenance Margin, Portfolio Weights*

## I. Readings and Suggested Practice Problems

BKM, Chapter 3: Sections 3.6-3.7.

*Suggested Problems*, Chapter 3: 2-4, 13, 14.

## II. Stock Transactions involving Credit

Use the stock of XYZ corporation as an example.

The following are the dividends and closing prices for XYZ:

<i>Date</i>	<i>Dividend: XYZ</i>	<i>Closing Price:XYZ</i>
end Jan	0	8
end Feb	0	10
end Mar	0	7

Suppose you have \$8000 cash at the end of January.

Your balance sheet at the end of January looks like:

<i>Assets</i>		<i>Liabilities</i>	
Cash	8000	Net Worth	8000
Total Assets	8000	Total Liab & Net W.	8000

Assume your broker requires interest of 1% per month on any funds borrowed from her. Any funds placed with your broker also earn 1%.

We consider this rate (EAR=12.68%) as the *riskless rate* (known in advance with certainty). Note that XYZ's stock is a risky asset because its price over time is not known in advance.

### ***A. Long the Stock***

*Long the Stock: Means buying the stock with your own funds.*

#### **Example**

At the end of January, you want to invest your \$8000 in XYZ shares at the closing price; so buy 1000 shares:

<i>Assets</i>		<i>Liabilities</i>	
1000 XYZ sh @ \$8	8000	Net Worth	8000
Total Assets	8000	Total Liab & Net W.	8000

By the end of February, XYZ's price has gone up:

<i>Assets</i>		<i>Liabilities</i>	
1000 XYZ sh @ \$10	10000	Net Worth	10000
Total Assets	10000	Total Liab & Net W.	10000

By the end of March, XYZ's price has plunged:

<i>Assets</i>		<i>Liabilities</i>	
1000 XYZ sh @ \$7	7000	Net Worth	7000
Total Assets	7000	Total Liab & Net W.	7000

## ***B. Buying Stock on Margin (Leveraging)***

*Buying Stock on Margin: Means buying stock using funds borrowed from a broker.*

*The client is charged an interest on the borrowed funds (plus a fee).*

*Percentage margin:* refers to net worth (value of the stock less amount borrowed) as a percentage of the value of the stock:

$$\text{Percentage Margin} = \frac{\text{Net Worth}}{\text{Value of Stock}} \times 100\%$$

1. The Board of Governors of the Federal Reserve System (that has the authority to set the margin-credit limit by the Securities Exchange Act of 1934) has set the minimum initial margin at 50%.  
(*Regulation T, part 220 – Credit by Brokers and Dealers.*)

So the borrowed amount must be less than 50% of the value of the stock purchased.

Usually, the investor is not allowed to let the portion of debt rise higher than 75% of the value of the stocks in the account (Percentage Margin must remain above 25%).

2. If the percentage margin falls below the *maintenance margin* set by the broker (which may be above 25%), the customer has to put up enough collateral to satisfy the maintenance margin.

At the end of January, you want to buy 1400 shares in XYZ.

You thus need to borrow  $1400 \times \$8 - \$8000 = \$3200$  from your broker.

Percent Margin =  $8000/11200 = 71.43\% > 50\%$ :

<i>Assets</i>		<i>Liabilities</i>	
1400 XYZ sh @ \$8	11200	Loan	3200
		Net Worth	8000
Total Assets	11200	Total Liab & Net W.	11200

By the end of February, XYZ's price is up:

Percent Margin =  $10768/14000 = 76.91\%$ .

<i>Assets</i>		<i>Liabilities</i>	
1400 XYZ sh @ \$10	14000	Loan (\$3200@1%)	3232
		Net Worth	10768
Total Assets	14000	Total Liab & Net W.	14000

By the end of March, XYZ's price has plunged:

Percent Margin =  $6536/9800 = 66.69\%$ .

<i>Assets</i>		<i>Liabilities</i>	
1400 XYZ sh @ \$7	9800	Loan (\$3232@1%)	3264
		Net Worth	6536
Total Assets	9800	Total Liab & Net W.	9800

### ***C. Short Selling Stock***

*Short Selling Stock: Means borrowing the stock and selling it.*

Proceeds from the sale must remain with the broker.

The short-seller will return the stock back when asked to do so by the broker, or when he/she chooses to do so, whichever occurs first. If the stock pays dividends, the short-seller must pay them to the original owner.

*Percentage margin:* refers to net worth as a percentage of the value of the stock borrowed:

$$\text{Percentage Margin} = \frac{\text{Net Worth}}{\text{Value of Stock Borrowed}} \times 100\%$$

If the percentage margin falls below the maintenance margin set by the broker, the customer has to put up enough collateral to satisfy the *maintenance margin*.

**Example**

1. At the end of January, you have \$8000 in cash with your broker, and you want to short-sell 1400 shares of XYZ.

- a. You borrow 1400 shares of XYZ.  
The loan is denominated in shares of XYZ not in dollars.

<i>Assets</i>		<i>Liabilities</i>	
1400 XYZ sh @ \$8	11200	Loan (1400 sh @ \$8)	11200
Cash	8000	Net Worth	8000
Total Assets	19200	Total Liab & Net W.	19200

- b. You sell the borrowed shares at the closing price at the end of January:  
Percent Margin =  $8000/11200 = 71.43\%$

<i>Assets</i>		<i>Liabilities</i>	
Cash	19200	Loan (1400 sh @ \$8)	11200
		Net Worth	8000
Total Assets	19200	Total Liab & Net W.	19200

2. By the end of February, XYZ's price has gone up:

Percent Margin =  $5392/14000 = 38.51\%$

<i>Assets</i>		<i>Liabilities</i>	
Cash (\$19200@1%)	19392	Loan (1400 sh@\$10)	14000
		Net Worth	5392
Total Assets	19392	Total Liab & Net W.	19392

2. By the end of March, XYZ's price has plummeted:

$$\text{Percent Margin} = 9786/9800 = 99.86\%$$

<i>Assets</i>		<i>Liabilities</i>	
Cash (\$19392@1%)	19586	Loan (1400 sh@\$7)	9800
		Net Worth	9786
Total Assets	19586	Total Liab & Net W.	19586

3. To close out the position at the end of March, take the following steps:

a. purchase 1400 shares of stock:

<i>Assets</i>		<i>Liabilities</i>	
Cash	9786	Loan (1400 sh@\$7)	9800
1400 sh XYZ @\$7	9800	Net Worth	9786
Total Assets	19586	Total Liab & Net W.	19586

b. repay the stock loan:

<i>Assets</i>		<i>Liabilities</i>	
Cash	9786	Net Worth	9786
Total Assets	9786	Total Liab & Net W.	9786

### ***D. Comparison of Positions***

*What is the monthly return, each month, for the above three holding strategies?*

Recall that in the absence of any dividend payment:

$$\text{Return} = [\text{Price (end)} / \text{Price (start)}] - 1.$$

So ,

$$\text{Portfolio Return} = [\text{Portfolio Value(end)} / \text{Portfolio Value(start)}] - 1$$

where in our example the *Portfolio Value* is given by *Net Worth*.

	<i>Long 1000 shares of XYZ</i>		<i>Buy 1400 shares of XYZ on Margin</i>		<i>Short Sell 1400 shares of XYZ</i>	
<i>Date</i>	<i>Net Worth</i>	<i>Return</i>	<i>Net Worth</i>	<i>Return</i>	<i>Net Worth</i>	<i>Return</i>
Jan	8000		8000		8000	
Feb	10000	25%	10768	34.6%	5392	-32.6 %
Mar	7000	-30%	6536	-39.3%	9786	81.5%

1. ***Buying on Margin vs Going Long:***

Instead of using starting net worth to buy a firm's stock, the investor uses it to buy a ***larger*** number of the firm's shares on margin (the portfolio is "***leveraged***").

- a. A given price *decline* causes a *larger reduction* in net worth.
- b. A given price *increase* causes a *larger increase* in net worth.
- c. So the investor's net worth is more sensitive to changes in the stock price.

2. ***Short-Selling vs Buying the Stock:***

Short-selling causes net worth to have a negative sensitivity to changes in the stock price.

- a. A price *decline* causes an *increase* in net worth.
- b. A price *increase* causes a *decrease* in net worth.

### III. Portfolio Returns

#### *A. Portfolio Return Formula*

The following formula can be used to calculate the return on a portfolio, in period  $t$ , given the returns on the assets that comprise the portfolio:

$$r_p(t) = w_{1,p} r_1(t) + w_{2,p} r_2(t) + \dots + w_{N,p} r_N(t)$$

where

- $N$  is the number of assets in the portfolio;
- $r_i(t)$  is the return on asset  $i$  in period  $t$ ;
- $w_{i,p}$  is the weight of asset  $i$  in portfolio  $p$ ;
- $r_p(t)$  is the return on portfolio  $p$  in period  $t$ .

Remarks:

1. Portfolio weights must sum to 1.
2. The notation  $r_f$  or  $R_f$  is usually reserved for the riskless rate (also referred to as the *risk-free* rate)

#### *Example*

Suppose you have \$8000 to invest at the end of January. You decide to invest \$6000 in XYZ and \$2000 in the Riskless Asset (i.e., the Money-Market account, which yields 1%).

You have invested:

\$6000/\$8000=0.75 or 75% of your money in XYZ.  
You have a weight  $w_{XYZ,p} = 0.75$  in stocks.

\$2000/\$8000=0.25 or 25% in the riskless asset.  
You have a weight  $w_{f,p} = 0.25$  in a riskless asset.

Your portfolio's return in February can be calculated using the portfolio return formula. (Instead of calculating portfolio's dollar value using balance sheets for the return calculation, it is simpler to use the returns and the weights of individual assets within the portfolio).

$$\begin{aligned}r_p(\text{Feb}) &= w_{XYZ,p} r_{XYZ}(\text{Feb}) + w_{f,p} r_f(\text{Feb}) \\ &= 0.75 \times 25\% + 0.25 \times 1\% \\ &= 19\%.\end{aligned}$$

## ***B. Interpretation of Portfolio Weights $w$***

### ***1. Buying a Stock on Margin: $w_{Stock} > 1$***

When 1400 shares of XYZ were bought on margin at the end of January, the weight on XYZ in the portfolio was  $>1$ .

The value of the portfolio at the end of January was \$8000, of which  $1400 \times \$8 = \$11200$  was invested in XYZ and  $-\$3200$  was invested in the riskless asset.

So,

$$\begin{aligned}w_{XYZ,p} &= \$11200/\$8000 = 1.4 \\ w_{f,p} &= -\$3200/\$8000 = -0.4\end{aligned}$$

(note the weights sum to 1.)

Then can calculate the portfolio return for February 2000:

$$\begin{aligned}r_p(\text{Feb}) &= w_{XYZ,p} r_{XYZ}(\text{Feb}) + w_{f,p} r_f(\text{Feb}) \\ &= 1.4 \times 25\% + (-0.4) \times 1\% \\ &= 34.6\%\end{aligned}$$

which agrees with the return calculated on page 10.

**2. Short selling:  $w_{Stock} < 0$**

When 1400 shares of XYZ were short sold at the end of January 2000, the weight on XYZ in the portfolio was  $< 0$ .

The value of the portfolio at the end of January 2000 was \$8000, of which  $-1400 \times \$8 = -\$11200$  was invested in XYZ and  $\$8000 + \$11200 = \$19200$  was invested in the riskless asset.

So,

$$\begin{aligned}w_{XYZ,p} &= -\$11200/\$8000 = -1.4 \\w_{f,p} &= \$19200/\$8000 = 2.4\end{aligned}$$

(note the weights sum to 1.)

Then can calculate the portfolio return for February 2000:

$$\begin{aligned}r_p(\text{Feb}) &= w_{XYZ,p} r_{XYZ}(\text{Feb}) + w_{f,p} r_f(\text{Feb}) \\&= -1.4 \times 25\% + 2.4 \times 1\% \\&= -32.6\%\end{aligned}$$

which agrees with the return calculated on page 10.

## IV. Appendix: More on the Portfolio Return Formula

This appendix is to assure the skeptics among you, that there is no “magic” in the portfolio return formula. Instead, it is just a useful shortcut.

*You may skip this appendix.*

Since you are still reading... to illustrate the idea, consider a portfolio of two assets: Stock 1 and Stock 2. You are interested in the portfolio’s return over the period from  $t-1$  to  $t$ :

- At the time  $t-1$  you have:  
 $N_1$  shares of Stock 1, with price per share  $P_1(t-1)$   
 $N_2$  shares of Stock 2, with price per share  $P_2(t-1)$   
Portfolio value at  $t-1$  is

$$P(t-1) = N_1 P_1(t-1) + N_2 P_2(t-1)$$

- At time  $t$ , the price of Stock 1 is  $P_1(t)$   
and the price of Stock 2 is  $P_2(t)$ ,  
and you still have  $N_1$  shares of Stock 1 and  $N_2$  shares of Stock 2, and the portfolio value is

$$P(t) = N_1 P_1(t) + N_2 P_2(t)$$

- You want to verify that the portfolio return  $r_P(t)$  is given by the formula

$$r_P(t) = W_{1,P} r_1(t) + W_{2,P} r_2(t)$$

where the weights are set at time  $t-1$  as

$$W_{1,P} = \frac{N_1 P_1(t-1)}{P(t-1)} \qquad W_{2,P} = \frac{N_2 P_2(t-1)}{P(t-1)}$$

and the returns on each stock are, respectively,

$$r_1(t) = \frac{P_1(t) - P_1(t-1)}{P_1(t-1)} \qquad r_2(t) = \frac{P_2(t) - P_2(t-1)}{P_2(t-1)}$$

assuming there are no dividends paid during the studied period.

- To verify the formula for the portfolio return  $r_P(t)$ , just use the formula

$$r_P(t) = \frac{P(t) - P(t-1)}{P(t-1)}$$

and express the portfolio value at each time, explicitly:

$$\begin{aligned} r_P(t) &= \frac{P(t) - P(t-1)}{P(t-1)} = \\ &= \frac{[(N_1 P_1(t) + N_2 P_2(t)) - (N_1 P_1(t-1) + N_2 P_2(t-1))]}{P(t-1)} = \\ &= \frac{[N_1 (P_1(t) - P_1(t-1)) + N_2 (P_2(t) - P_2(t-1))]}{P(t-1)} = \\ &= \left[ N_1 P_1(t-1) \frac{P_1(t) - P_1(t-1)}{P_1(t-1)} + N_2 P_2(t-1) \frac{P_2(t) - P_2(t-1)}{P_2(t-1)} \right] / P(t-1) = \\ &= \frac{N_1 P_1(t-1)}{P(t-1)} r_1(t) + \frac{N_2 P_2(t-1)}{P(t-1)} r_2(t) = \\ &= W_{1,P} r_1(t) + W_{2,P} r_2(t) \end{aligned}$$

Clearly, the above two-asset logic applies to  $N$  assets as well, as in the general formula.

## V. Additional Readings

The following articles discuss:

- *The ongoing debate about Margin Debt*

To put things in historic perspective: At the height of the Bull Market, Margin debt in January 2000 rose to 1.57% of market value, equal to its peak in the fall of 1987 (ahead of the October 1987 stock-market crash). That ratio approached 30% in 1929!

- *The Practice of Short-Selling*

You can short stocks, bonds, and other financial assets; the ability to short (and leverage) is also one important distinction between mutual funds and hedge funds.