Lecture Notes 9

The Capital Asset Pricing Model (CAPM)

Buzz Words: ***Equilibrium Process, Supply Equals Demand, Market Price of Risk, Cross-Section of Expected Returns, Risk Adjusted Expected Returns, Net Present Value and Cost of Equity Capital.***

I. Readings and Suggested Practice Problems

BKM, Chapter 9, Sections 2-4.
Suggested Problems, Chapter 9: 2, 4, 5, 13, 14, 15

Web: Visit www.morningstar.com, select a fund (e.g., Vanguard 500 Index VFINX), click on *Risk Measures*, and in the *Modern Portfolio Theory Statistics* section, view the beta.

II. Introduction: from Assumptions to Implications

A. Economic Equilibrium

1. Equilibrium analysis (unlike index models)

Assume economic behavior of individuals.

Then, draw conclusions about overall market prices, quantities, returns.

2. The CAPM is based on equilibrium analysis

Problems:

- There are many "dubious" assumptions.
- The main implication of the CAPM concerns expected returns, which can't be observed directly.

B. Implications of the CAPM: A Preview

If everyone believes this theory… then (as we will see next):

1. There is a central role for the ***market portfolio***:

   a. *This simplifies portfolio selection.*

   b. *Provides a rationale for a "market-indexing" investment strategy.*

2. There is explicit ***risk-return trade-off*** for individual stocks:

   The model specifies expected returns for use in capital budgeting, valuation, and regulation.

   Risk premium on an individual security is a function of its systematic risk, measured by the covariance with the market.

   a. *can use the model to <u>evaluate given estimates</u> of expected returns relative to risk*

   b. *Can <u>obtain estimates</u> of expected returns through estimates of risk. (This is more precise statistically than obtaining direct estimates of expected returns based on averages of past returns)*

III. The Market Portfolio

The market portfolio, M, as any other portfolio, is described by portfolio weights: $w_{1,M}, \ldots, w_{n,M}$.

The specific attribute of the market portfolio is that the weight on a stock is the fraction of that *stock's market value* relative to the *total market value of all stocks*:

***Stock's market value*:**

$$v_i = n_i \, p_i$$

where

p_i	price per share of company i's stock,
n_i	number of shares outstanding,
v_i	the market value of i's equity.

Example:

IBM has n_{IBM}=1730 Million shares outstanding.
As of 10/6/03, the price was p_{IBM}=\$91.18 per share.
So, IBM's market value is v_{IBM}=\$157.7 Billion

***The total market value of all stocks*:**

$$V = v_1+v_2+\ldots+v_n$$

The weight of stock i in the market portfolio

$$w_{i,M} = v_i \,/\, V.$$

Example

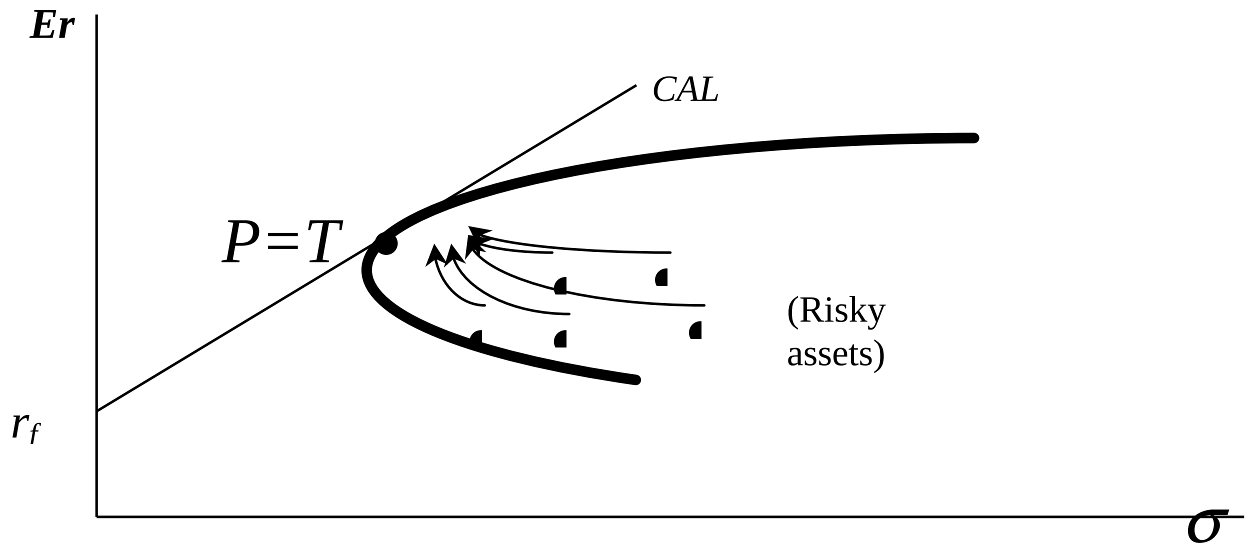
Suppose the weight of IBM is: $w_{\text{IBM}} = 1.5\%$
If we put \$100,000 in the market portfolio, \$1,500 should be invested in IBM.

IV. Assumptions Underlying the CAPM

- There are many investors. They behave competitively (price takers).
- All investors are looking ahead over the same (one period) planning horizon.
- All investors have equal access to all securities.
- No taxes.
- No commissions.
- Each investor cares only about Er_C and σ_C.
- All investors have the same beliefs about the investment opportunities: r_f, Er_1,. . .,Er_n, all $\sigma_{i,}$ and all correlations ("homogeneous beliefs") for the n risky assets.
- Investors can borrow and lend at the one riskfree rate.
- Investors can short any asset, and hold any fraction of an asset.

V. Portfolio Choice in the CAPM World

A. The investor's problem is to choose the "best" portfolio P. The solution: Choose T.



B. If T is the same for everybody (all investors agree on what are the tangent weights), then T is the Market portfolio (M).

That is, each asset's weight in the tangent portfolio, $w_{i,T}$, is simply its weight in the market portfolio: $w_{i,T} = w_{i,M}$

V = total market value of all stocks
= total funds invested in the tangent portfolio

v_i = market value of *i*'s equity
= total funds invested in firm *i*
= $w_{i,T} \times$ (total funds invested in the tangent portfolio)
= $w_{i,T} \times V$

Therefore,

$$w_{i,T} = \frac{w_{i,T} \times V}{V} = \frac{v_i}{V} = w_{i,M}$$

Example

Suppose based on the Mean-Variance analysis, *IBM*'s weight in the tangent portfolio is $w_{IBM,T} = 1\%$ (all investors agree on that), and all investors combined have $12 trillion to invest (so V= $12 trillion).

Then, *IBM*'s market value is
$v_{IBM} = w_{IBM,T} \times V = 1\% \times 12$ trillion = $120 billion

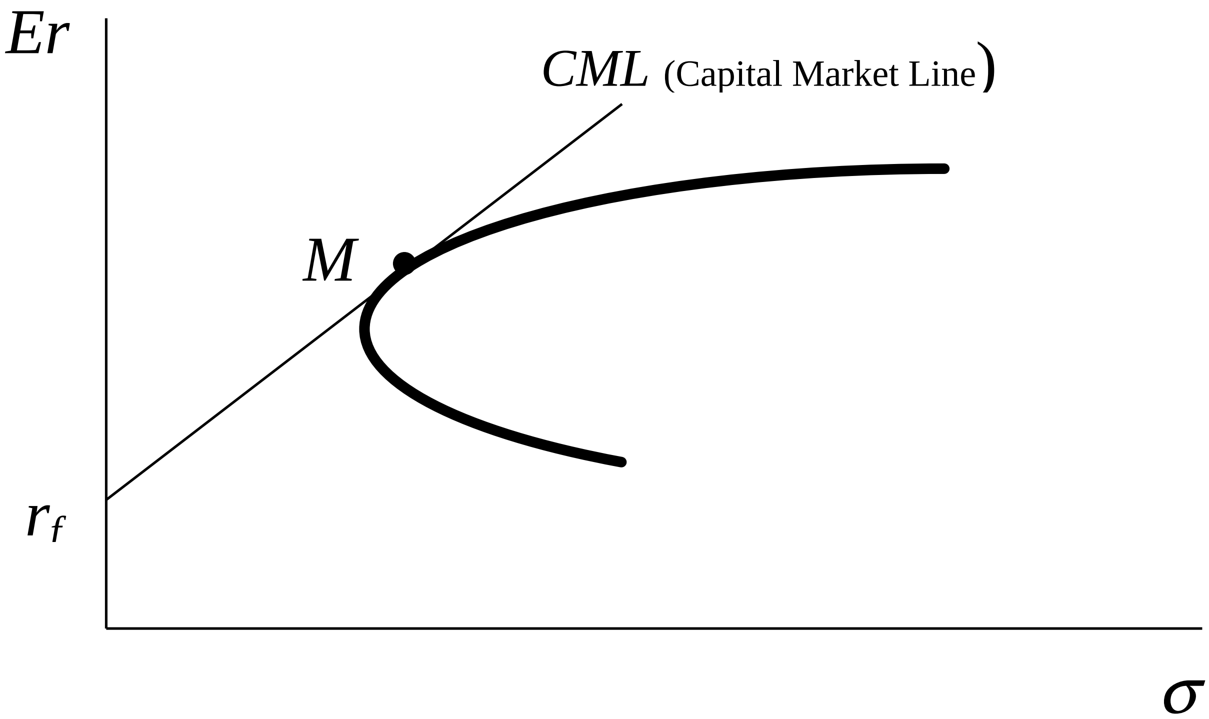
and *IBM*'s market weight is
$w_{IBM,M} = v_{IBM}/V$ = $120 billion/12 trillion = 1% = $w_{IBM,T}$

So everyone holds some combination of the value weighted market portfolio M and the riskless asset.

C. Capital Market Line (CML)

The CAL, which is obtained by combining the market portfolio and the riskless asset is known as the Capital Market Line (CML):

$$E r_C = r_f + \frac{E r_M - r_f}{\sigma_M} \sigma_C$$



D. Indexing

The portfolio strategy of matching your portfolio (of risky assets) to a popular index.

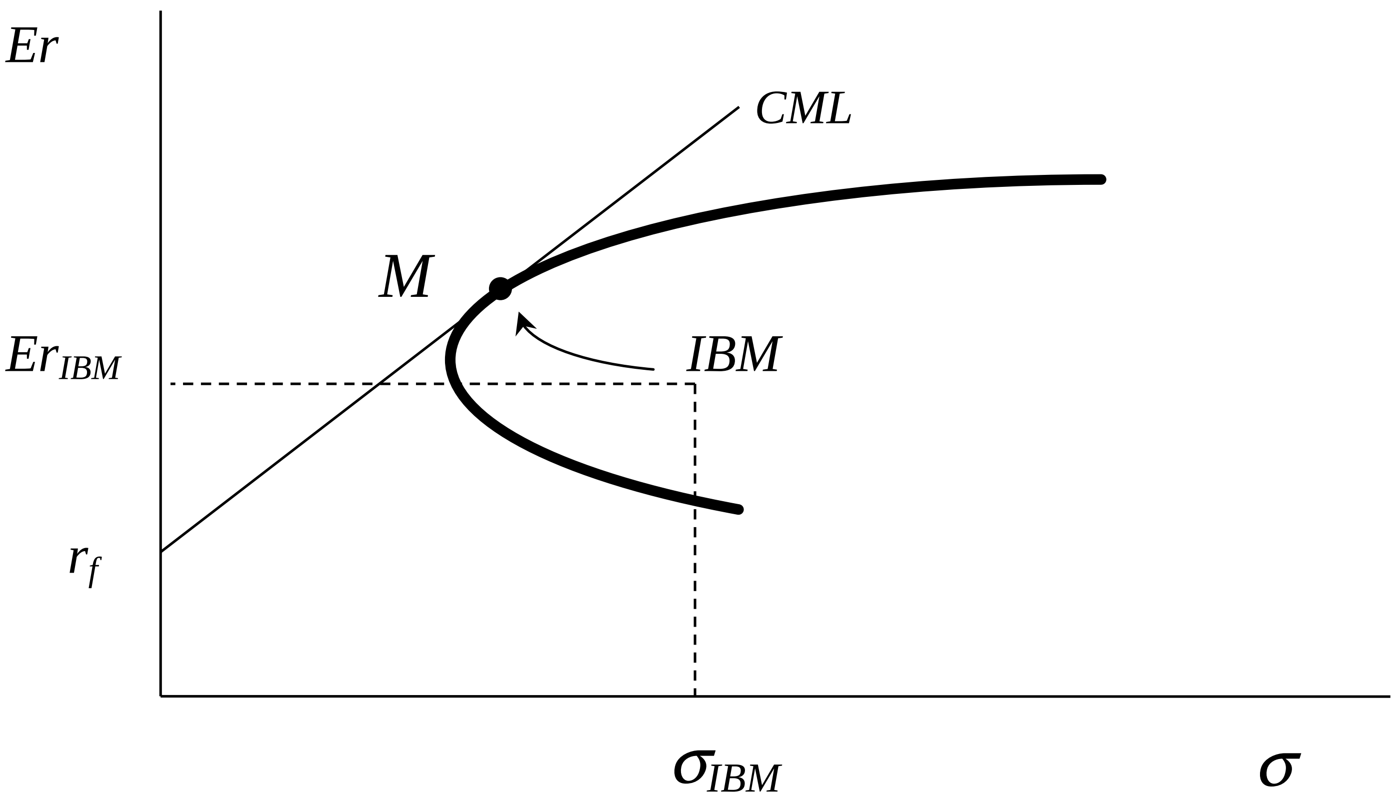
1. Indexing is a passive strategy. (No security analysis; no "market timing.")
2. Some stock indices (e.g., the S&P 500 index) use market value weights.
3. If the total value of the stocks in the index is close to the value of all stocks, then it may approximate the market portfolio.
4. Conclusion: the investor can approximate the market portfolio by matching a market index.

VI. The Risk-Return Tradeoff for Individual Stocks

A. The *CML* specifies the expected return, Er_C, for a given level of risk (σ_C) in our combined portfolios.

All possible combined portfolios lie on the *CML*, and all are Mean-Variance efficient portfolios.

Will an individual stock lie on the CML?



B. How should we measure the risk of an asset (IBM)?

As an investor, I care only about σ_C *(the risk in my combined portfolio). The risk in C derives entirely from the risk in the market portfolio M*: $\sigma_C = y\ \sigma_M$

What does *IBM* contribute to σ_M?

We can answer that using a "thought experiment", where we consider the covariance of the asset with the market, σ_{iM}, or equivalently its beta, because $\beta_{iM} = \sigma_{iM}/\sigma^2_M$.
[β_{iM} is often written simply as β_i (note: $\beta_M = 1$), and it measures how much an asset's return is driven by the market return.]

So now consider the following "marginal" portfolio formation scenario:

An investor holds the market portfolio.

A new stock is issued.

The investor is considering adding it to his portfolio.

If the stock has a *high positive* β*:*

- It will have large price swings driven by the market
- It will increase the risk of the investor's portfolio
  (in fact, will make the entire market more risky …)
- The investor will demand a *high Er* in compensation.

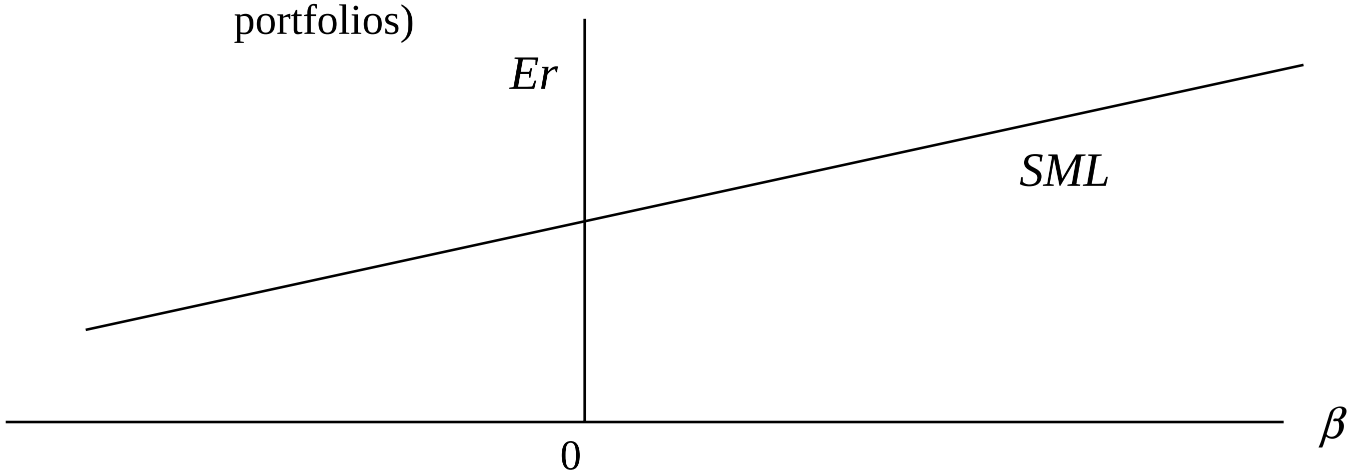
If the stock has a *negative* β:

- It moves "against" the market.
- It will decrease the risk of the market portfolio
- The investor will accept a *lower Er* (in exchange for the risk reduction, and *Er* can be negative).

D. The Pricing Implication of the CAPM for Any Asset

1. Security Market Line (SML)

In a "CAPM world," SML describes the relationship between β and *Er* (holds for individual assets and portfolios)

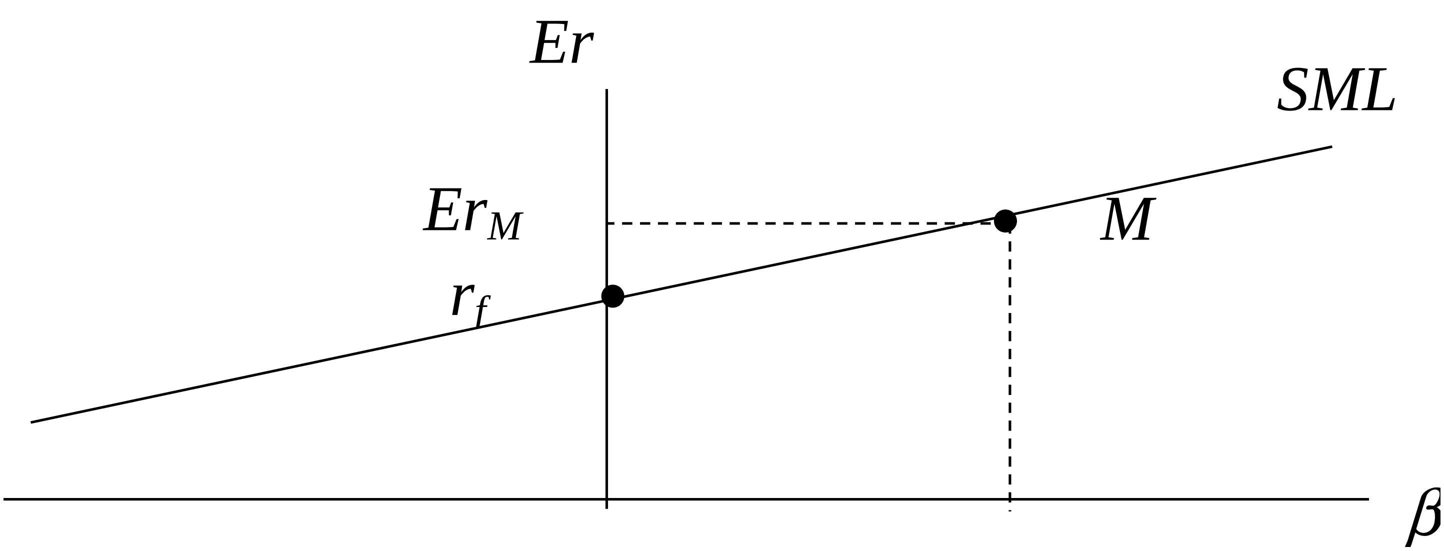


In order to fix the position of the SML, we need to know two points:

For the risk-free security, $\beta = 0$. (The risk-free return is constant; it isn't "driven" by the market.)

For the market itself, β=1. (If we run a regression of r_M vs r_M, the slope is 1.)

2. The CAPM equation



$$Er_i = r_f + \beta_i \left(Er_M - r_f \right),$$

where

Er_i is the expected return on an (arbitrary) stock or security *i*.

β_i is security *i*'s beta.

- The CAPM therefore states that in equilibrium, only the systematic (market) risk is priced, and not the total risk; investors do not require to be compensated for unique risk.

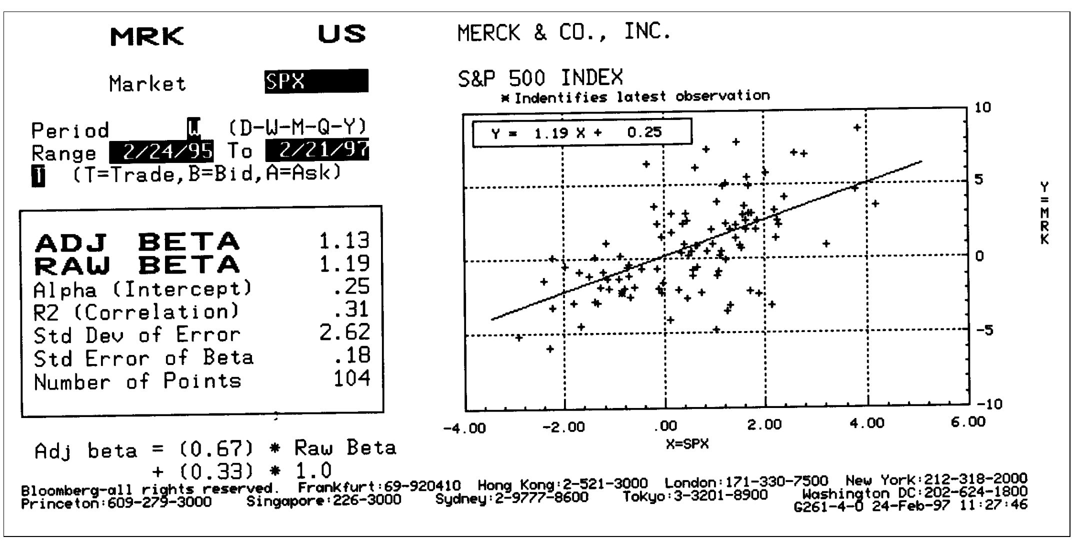
  (Although it is somewhat similar to what we saw in the market model, recall that in the market model the market beta determines the expected return of a security simply by construction, whereas here it is due to an economic argument that all investors hold the same tangent portfolio. Moreover in the market model the intercept varies across assets, and it is not clear what are all the determinants of expected returns.)

- Like the CML, the SML is a statement about *expected* returns. In any given year, a low-β stock could have a high return and a high-β stock could have a low return.

- In principle, a stock with $\beta<0$ will have $Er < r_f$. (If $\beta<<0$, could have $Er < 0$.) Most stocks have β's between 0 and 3.

3. Estimation

Example 1:



Note: this estimation uses weekly return data to get beta from the slope of the regression.

From a data source called SBBI (see BKM), Er_M-$r_f \approx 8.5\%$; also let's use $r_f = 5\%$;

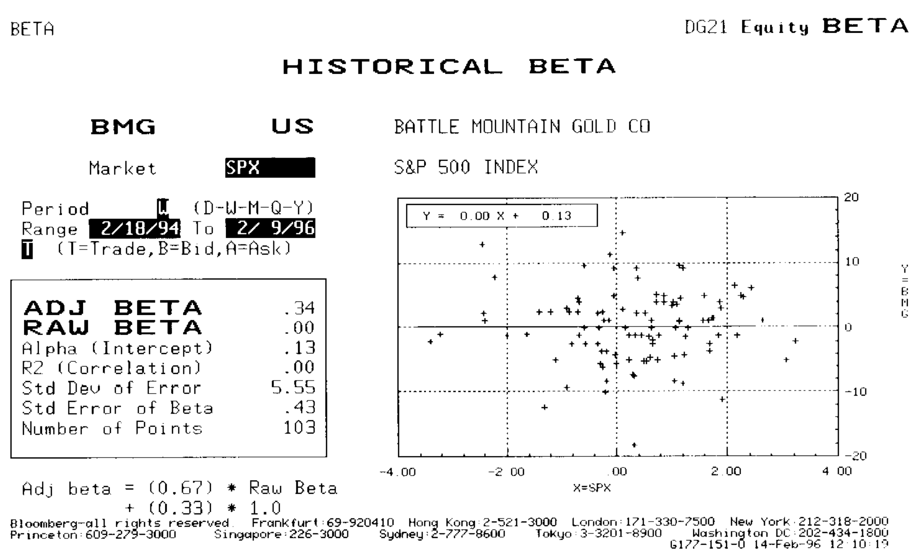
Then the SML implies:

$$Er_{MRK} = 5\% + 1.19(8.5\%) = 15.1\%$$

Example 2

We saw before an example where we estimated
$\beta_{\text{GE}} = 1.44$, $\beta_{\text{MSFT}} = 0.88$.
Empirical evidence suggests that over time the betas of stocks move toward the average beta of 1. For this reason, a raw estimate of beta is often *adjusted* using the following formula: $\beta_{adj} = w\,\beta_{raw} + (1\text{-}w)\,1$, where typically w=0.67.

Example 3



VII. The CML and SML

A. the CML vs. SML

- Both specify a relation between risk and *Er*:

  Expected Return = "*Time Premium*" + "*Risk Premium*"

  where

  "*Risk Premium*" = "*quantity of risk*" × "*price of risk*"

- Measure of risk.
  - In the CML, risk is measured by σ.
  - In the SML, risk is measured by β.

- Applicability:
  - CML is applicable only to an investor's final (combined) portfolio (which is efficiently diversified, with no Unique risk)

    In the CAPM world, everybody holds portfolios which lie on the CML.
  - SML is applicable to *any* security, asset or portfolio (which may contain both components of risk).

    In a CAPM world every asset lies on the SML.

B. All assets (and portfolios) lie on the SML yet only efficient portfolios which are combinations of the market portfolio and the riskless asset lie on the CML

How can this be?

1. First note that since by definition, for any portfolio *p*

$$\sigma[r_p,r_M] = \rho[r_p,r_M]\,\sigma[r_p]\,\sigma[r_M]$$

it follows that

$$\beta_{p,M} = \frac{\sigma[r_p,r_M]}{\sigma^2[r_M]} = \frac{\rho[r_p,r_M]\,\sigma[r_p]\,\sigma[r_M]}{\sigma^2[r_M]} = \frac{\rho[r_p,r_M]\,\sigma[r_p]}{\sigma[r_M]}.$$

2. Thus, the SML can be written as

$$SML:\ E[r_p] = r_f + \frac{E[r_M] - r_f}{\sigma[r_M]}\{\rho[r_p,r_M]\,\sigma[r_p]\}.$$

3. Comparing this equation to the CML, which hold for efficient portfolios or assets (subscript *ef* below)

$$CML:\ E[r_{ef}] = r_f + \frac{E[r_M] - r_f}{\sigma[r_M]}\sigma[r_{ef}]$$

we see that:

an asset *p* lies on the SML and the CML if $\rho[r_p,r_M]=1$.

an asset *p* only lies on the SML and is not a combination of the riskless asset and the market portfolio if $\rho[r_p,r_M]<1$

VIII. "Overpricing"/"Underpricing" and the SML

In the "CAPM world", there is no such thing as overpricing/underpricing.

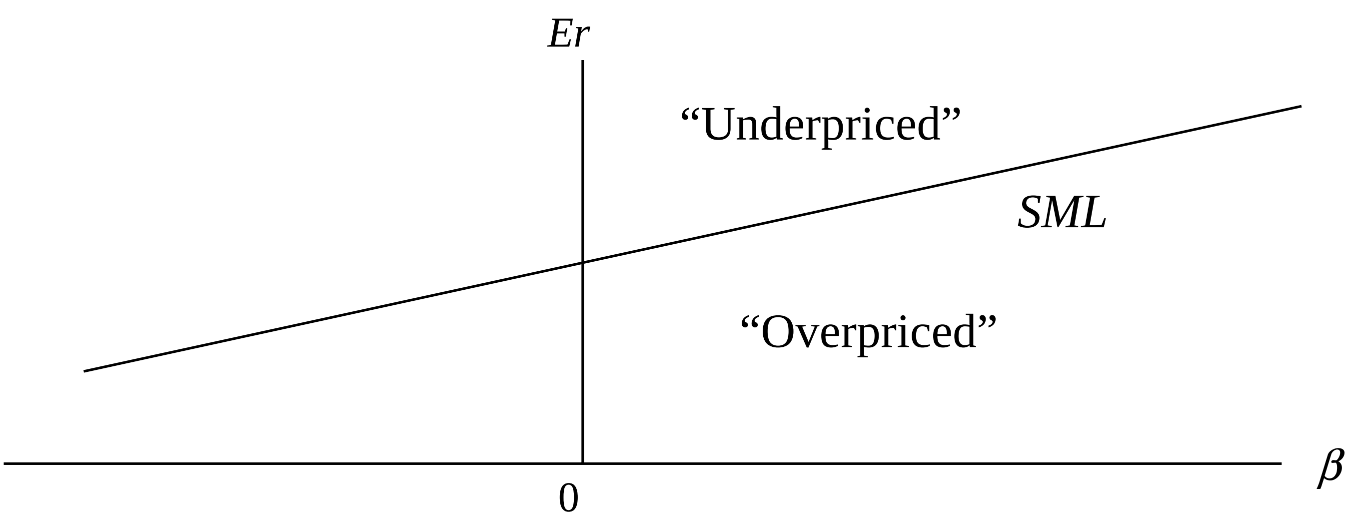
Every asset is correctly priced, and is positioned on the SML.

For practical real-world purposes, however, we can compare an asset's given price or expected return relative to what it should be according to the CAPM, and in that context we talk about over/under pricing.

Then:

1. Assets above the SML are underpriced relative to the CAPM.
   Why? Because the assets' "too" high expected return means their price is "too" low compared to the "fair" CAPM value

2. Assets below the SML are overpriced relative to the CAPM.
   Why? Because the assets' "too" low expected return means their price is "too" high compared to the "fair" CAPM value



IX. Uses of CAPM in Corporate Finance

1. Application to Capital Budgeting: Establishing hurdle rates for a firm's projects

Getting beta from equivalent stand-alone traded firms: e.g., $\beta=1.4$

Calculating expected rate of return (the "hurdle rate"):

e.g., $r_f = 5\%$, $E(r_M)=13\%$

So: $E(r_{\text{project}}) = 0.05 + 1.4\,(0.13-0.05) = 16.2\%$

Project Specification:

- Investment=$5,000
- Expected payoffs: $5,000 in year 3, $5,000 in year 7

Net Present Value Calculation (NPV): Adjusting the time value of money analysis for the risk of the cash flows. (Also see the *Additional Readings*. More on this when we cover equity *Valuation*).

$$NPV = -5{,}000 + 5{,}000/1.162^3 + 5000/1.162^7 = -65.28$$

2. Establishing fair compensation for a regulated monopoly

e.g., $r_f=5\%$, $E(r_M)=13\%$, $\beta=0.7$,

So: $E(r_{\text{project}}) = 0.05 + 0.7\,(0.13-0.05) = 10.6\%$

Regulated Project Specification:

- Investment= $200 million
- Required annual profit: $200 \times 0.106 = \$21.2$ million

Note: 10.6% is called, in this context, the *cost of equity capital* and it affects the prices the monopoly (e.g. power utility) will charge.

X. Additional Readings

The article is by William Sharpe (a Nobel Laureate), one of the creators of what we refer to as the CAPM. He describes in the article the *multi-index models* (which he refers to as *factor models*), the *CAPM*, and the *APT*, and the relation between all these. The APT – the Arbitrage Pricing Theory – derives a general relationship between expected return and risk using only no arbitrage arguments (i.e., relying only on the investors preferring more to less). Although we do not cover the APT in the course (mainly because it applies to well diversified portfolios, but not necessarily to individual stocks), it is worthwhile to be familiar with this alternative approach to asset pricing (see also BKM, chapter 11).

The APT, the CAPM, and the CAPM’s extensions are all called “Asset Pricing” models because they tell us *how assets are priced*. We focused the discussion on the expected return, E[r], because it is exactly this quantity which we need to be able to price assets. Why?
As we discussed in the context of “over”/“under” pricing,
we know that an asset that pays a random cash flow CF next period will have a return: $r = CF/P - 1$, where P is the current price.
Taking expectations, we get $E[r] = E[CF]/P - 1$.
Rearranging, we find the price: $P = E[CF]/(1+E[r])$.

That is, we discount the expected cash flow using an appropriate discount rate, which is provided to us by an asset pricing model in terms of the associated risks (and various models differ in their normative guidance of how exactly to calculate E[r]).