316-466 Monetary Economics — Homework 2

This problem asks you to analyze a stochastic version of the Sidrauski model. Time is discrete and denoted $t = 0, 1, ...$ There is a representative consumer who faces a random sequence of money growth shocks and endowment shocks (there is no production). Money growth shocks are denoted μ_t and endowment shocks are denoted y_t and both are Markov processes. The state at t is

$$s_t = (\mu_t, y_t)$$

and a finite history is

$$s^t = (s_t, s_{t-1}, \cdots s_0)$$

with s_0 known at date zero. As of date zero, the probability of s^t is $f(s^t \mid s_0)$. The consumer has expected utility preferences over consumption and end of period real balances $m_t(s^t) \equiv M_{t+1}(s^t)/P_t(s^t)$.

$$U(c, m) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u[c_t(s^t), m_t(s^t)] f(s^t \mid s_0) \qquad 0 < \beta < 1$$

The utility function u is assumed to be strictly increasing and strictly concave in both arguments.

The representative consumer trades in money and in a complete set of nominal, one-period, state contingent bonds. She has the flow budget constraint

$$P_t(s^t)c_t(s^t) + M_{t+1}(s^t) + \sum_{s'} q_t(s^t, s')B_{t+1}(s^t, s') \leq P_t(s^t)y_t(s^t) + M_t(s^{t-1}) + B_t(s^{t-1}, s_t) - T_t(s^t)$$

where $T_t(s^t)$ denotes lump-sum taxes to be described below and where $M_0 > 0$ and $B_0 = 0$ are given initial conditions.

1. Explain the notation on the right hand side of this constraint. Why do I write $M_t(s^{t-1})$ and not $M_t(s^t)$? Why do I write $B_t(s^{t-1}, s_t)$, and not $B_t(s^{t-1})$?

2. Formulate a Lagrangian and derive first order conditions that allow you to price state contingent bonds and to characterize the demand for money. Interpret these conditions.

3. Suppose the price of a riskless bond is

$$\frac{1}{1 + i_t(s^t)} = \sum_{s'} q_t(s^t, s')$$

   Derive a formula for this price. Use this formula to construct a "Fisher equation" relating real interest rates, nominal rates, and expected inflation.

4. Suppose that the government's flow budget constraint is

$$\mathcal{M}_{t+1}(s^t) + T_t(s^t) = \mathcal{M}_t(s^{t-1})$$

where $\mathcal{M}_t$ denotes the exogenous money supply. If the growth rate of the money supply is

$$\frac{\mathcal{M}_{t+1}(s^t)}{\mathcal{M}_t(s^{t-1})} = 1 + \mu_t(s^t)$$

what does this imply about taxes? Specifically, if the money growth rate is $\mu_t(s^t)$, what must the tax be? Is the tax positive or negative when money growth is positive?

5. What do $c_t(s^t)$, $B_{t+1}(s^t, s')$, and $M_{t+1}(s^t)$ equal in equilibrium? Express your answers in terms of primitives of the model (parameters, exogenous variables, etc).

6. (You can drop the s^t notation for this problem in order to make the notation more attractive). Suppose now that the money growth rate and endowment are AR(1) processes

$$\begin{aligned}
\mu_{t+1} &= (1-\rho)\bar{\mu} + \rho\mu_t + \epsilon_{t+1} \qquad 0 < \rho < 1 \\
\log(y_{t+1}) &= (1-\phi)\log(\bar{y}) + \phi\log(y_t) + v_{t+1} \qquad 0 < \phi < 1
\end{aligned}$$

where ϵ_{t+1} and v_{t+1} denote mean-zero homoscedastic shocks. Assume that the period utility function is

$$u(c, m) = \gamma\log(c) + (1-\gamma)\log(m) \qquad 0 < \gamma < 1$$

Characterize the non-stochastic steady state of this model (i.e., the steady state where $\mu_t = \bar{\mu}$ and $y_t = \bar{y}$. Make sure you can say what consumption, real balances, inflation, nominal interest rates, real interest rates, the money supply and the price level are in this steady state. Log linearize the model around this non-stochastic steady state. That is find, a set of *stochastic* difference equations that approximate the model local to the non-stochastic steady state. Does this model exhibit monetary neutrality even in the short run (i.e., away from the steady state)? What about superneutrality?

Chris Edmond, 6 September 2003