316-632 International Monetary Economics — Final Exam

1. **The permanent-income hypothesis and the current account** (60 marks). Consider a dynamic, stochastic model of a small open endowment economy that faces a constant real interest rate $r > 0$ and where an infinitely-lived representative consumer has preferences ordered by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t) \right\}$$

where:

$$u(C_t) \equiv C_t - \frac{a_0}{2} C_t^2, \qquad a_0 > 0$$

$$\beta \equiv \frac{1}{1+r}$$

The current account identity is

$$B_{t+1} - B_t = rB_t + Y_t - C_t$$

where $\{Y_t\}$ follows an exogenous stochastic process, to be discussed below, and B_t denotes net foreign assets. Notice that there is no government expenditure and no investment (no capital accumulation).

(a) (7 marks). Iterate the current account identity forward to obtain an intertemporal budget constraint.

(b) (7 marks). Show optimizing behavior by the representative consumer requires that the following Euler equation be satisfied

$$C_t = E_t\{C_{t+1}\}$$

If so, what kind of stochastic process must consumption follow?

(c) (10 marks). Use the Euler equation and the intertemporal budget constraint to solve for the optimal level of consumption. Explain why optimal consumption must satisfy the *certainty-equivalence principle*.

(d) (10 marks). Explain why the *change* in consumption between any two dates t and $t+1$ must satisfy

$$C_{t+1} - C_t = \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-(t+1)} (E_{t+1} - E_t) Y_s$$

How do you interpret the expression $(E_{t+1} - E_t) Y_s$ for $s \geq t+1$?

(e) (8 marks). Let the stochastic process for $\{Y_t\}$ be

$$Y_{t+1} - Y_t = \rho(Y_t - Y_{t-1}) + \epsilon_{t+1}, \qquad 0 < \rho < 1$$

Show that for $s > t$,

$$(E_{t+1} - E_t) Y_s = \frac{1 - \rho^{s-t}}{1 - \rho} \epsilon_{t+1}$$

(f) (10 marks). Solve for the change in consumption, $C_{t+1} - C_t$, in terms of r, ρ, and ϵ_{t+1}. Explain why consumption innovations $C_{t+1} - E_t\{C_{t+1}\}$ are more volatile than output innovations $Y_{t+1} - E_t\{Y_{t+1}\}$. Why does this seem paradoxical?

(g) (8 marks). Use the current account identity to derive the response of the current account to an output innovation. Does the current account co-vary positively or negatively with output innovations?

2. **Diversification and risk aversion** (60 marks). Consider a two-period, two country model where uncertainty in the second period is given by a probability distribution $\pi(s)$ over S states of nature. Each country has a given first period endowment Y_1 (or Y_1^*) and a random second period endowment $Y_2(s)$ (or $Y_2^*(s)$). Each country is populated by a representative consumer with expected utility

$$u(C_1) + \beta \sum_{s=1}^{S} \pi(s) u[C_2(s)], \qquad 0 < \beta < 1$$

Each consumer has constant absolute risk aversion (CARA) preferences

$$u(C) = -\frac{\exp(-\gamma C)}{\gamma}, \qquad \gamma > 0$$

and faces the intertemporal budget constraint

$$C_1 + \frac{1}{1+r} \sum_{s=1}^{S} p(s) C_2(s) = Y_1 + \frac{1}{1+r} \sum_{s=1}^{S} p(s) Y_2(s)$$

where $p(s)/(1+r)$ denotes the price of a claim to consumption in date $t = 2$ state s. (And similarly for the foreign country).

(a) (10 marks). Show that the Euler equations governing optimal consumption can be written

$$\exp(-\gamma C_1) = \beta(1+r)\frac{\pi(s)}{p(s)} \exp(-\gamma C_2(s)), \qquad s = 1, ..., S$$

$$\exp(-\gamma C_1^*) = \beta(1+r)\frac{\pi(s)}{p(s)} \exp(-\gamma C_2^*(s)), \qquad s = 1, ..., S$$

(b) (10 marks). Let world output be denoted

$$Y_1^w \equiv Y_1 + Y_1^*$$
$$Y_2^w(s) \equiv Y_2(s) + Y_2^*(s), \qquad s = 1, ..., S$$

Solve for the equilibrium prices of contingent consumption, $p(s)/(1+r)$, as a function of β, Y_1^w, and $\{Y_2^w(s), \pi(s)\}_{s=1}^{S}$. Explain how $p(s)/(1+r)$ depends on each of these primitives. Provide economic intuition for these results.

(c) (10 marks). Solve for the equilibrium real interest rate r in terms of the same primitives. Again provide economic intuition for your results.

(d) (15 marks). Suppose that each consumer is restricted to trading in riskless bonds and claims to shares in foreign and home second period output. The budget constraints for the home country then become

$$C_1 + B_2 + xV_1 + (1-x)V_1^* = Y_1 + V_1$$
$$C_2(s) = (1+r)B_2 + xY_2(s) + (1-x)Y_2^*(s)$$

where B_1 denotes the quantity of a riskless bond that pays net return $r > 0$ and x denotes the portfolio share chosen to invest in the home country (which

pays $Y_2(s)$ in the second period if the state is s). The remaining $1 - x$ share is invested in the foreign country and pays $Y_2^*(s)$ in the second period if the state is s. Show that the resulting allocation is still efficient, i.e., is the same as with the complete markets structure assumed in parts a) — c), and that home and foreign consumption allocations are given by

$$C_1 = \frac{1}{2}Y_1^w - \mu, \qquad C_1^* = \frac{1}{2}Y_1^w + \mu$$

$$C_2(s) = \frac{1}{2}Y_2^w(s) - \mu, \qquad C_2^*(s) = \frac{1}{2}Y_2^w(s) + \mu$$

where μ is a time invariant constant. Find a formula for μ. Show that to support this equilibrium, both countries make equal investment in the risky world mutual fund and that one country makes riskless loans to the other at date 1 which are then repaid with interest at date 2.

(e) (15 marks). Redo part (d) but now suppose that the foreign country has a different coefficient of absolute risk aversion, $\gamma^* \neq \gamma$. How do your answers change? How do your answers depend on whether $\gamma^* > \gamma$ or $\gamma^* < \gamma$? Give economic intuition for your results.

<div align="center">END OF EXAM</div>