316-632 International Monetary Economics — Final Exam Answers

1. **The permanent-income hypothesis and the current account** (90 marks).

   (a) (7 marks). Iterating the current account identity forward and imposing a no-Ponzi-game constraint gives

   $$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t = (1+r)B_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t$$

   (b) (7 marks). The standard consumption Euler equation is

   $$u'(C_t) = \beta(1+r)E_t\{u'(C_{t+1})\}$$

   Using quadratic utility and $\beta(1+r) = 1$ gives

   $$1 - a_0 C_t = E_t\{1 - a_0 C_{t+1}\}$$

   Hence

   $$C_t = E_t\{C_{t+1}\}$$

   This means consumption must follow a random walk.

   (c) (10 marks). Take date-zero conditional expectations on both sides of the intertemporal budget constraint and simplify to get

   $$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t E_0\{C_t\} = (1+r)B_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t E_0\{Y_t\}$$

   Now use $C_0 = E_0\{C_1\} = E_0\{E_1\{C_2\}\}$, etc, to get

   $$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_0 = (1+r)B_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t E_0\{Y_t\}$$

   or

   $$C_0 = \frac{r}{1+r}\left[(1+r)B_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t E_0\{Y_t\}\right]$$

   Or at any date $t \geq 0$

   $$C_t = \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} E_t\{Y_s\}\right]$$

   The consumption function is the same as under perfect foresight but with $\{Y_t\}$ replaced by $E_0\{Y_t\}$. Hence certainty equivalence.

(d) (10 marks). Take

$$C_{t+1} = \frac{r}{1+r}\left[(1+r)B_{t+1} + \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)}E_{t+1}\{Y_s\}\right]$$

$$C_t = \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}E_t\{Y_s\}\right]$$

and subtract to get

$$C_{t+1}-C_t = r(B_{t+1}-B_t)+\frac{r}{1+r}\left[\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)}E_{t+1}\{Y_s\} - \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}E_t\{Y_s\}\right]$$

Now use the current account identity to get

$$\begin{aligned}
B_{t+1} - B_t &= rB_t + Y_t - C_t \\
&= rB_t + Y_t - \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}E_t\{Y_s\}\right] \\
&= Y_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}E_t\{Y_s\}
\end{aligned}$$

Plug back in to $C_{t+1} - C_t$ and rearrange to get

$$C_{t+1} - C_t = \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)}(E_{t+1} - E_t)Y_s$$

The term $(E_{t+1} - E_t)Y_s$ for $s \geq t+1$ denotes the revision of the expectation of Y_s between t and $t+1$.

(e) (8 marks). Write the stochastic process as

$$Y_{t+1} = Y_t + \rho(Y_t - Y_{t-1}) + \epsilon_{t+1}, \qquad 0 < \rho < 1$$

Hence

$$\begin{aligned}
E_t\{Y_{t+1}\} &= Y_t + \rho(Y_t - Y_{t-1}) \\
(E_{t+1} - E_t)Y_{t+1} &= Y_{t+1} - E_t\{Y_{t+1}\} = \epsilon_{t+1}
\end{aligned}$$

Similarly

$$\begin{aligned}
(E_{t+1} - E_t)Y_{t+2} &= (1+\rho)\epsilon_{t+1} \\
(E_{t+1} - E_t)Y_{t+3} &= (1+\rho+\rho^2)\epsilon_{t+1}
\end{aligned}$$

$$\vdots$$

and for any $s \geq t+1$,

$$(E_{t+1} - E_t)Y_s = \frac{1-\rho^{s-t}}{1-\rho}\epsilon_{t+1}$$

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(f) (10 marks). Now solving for the change in consumption,

$$
\begin{aligned}
C_{t+1} - C_t &= \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-(t+1)} (E_{t+1} - E_t) Y_s \\
&= \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-(t+1)} \frac{1 - \rho^{s-t}}{1 - \rho} \epsilon_{t+1} \\
&= \frac{r}{1+r} \epsilon_{t+1} \left[\frac{1}{1-\rho} + \frac{1}{1+r}\frac{1-\rho^2}{1-\rho} + \left(\frac{1}{1+r}\right)^2 \frac{1-\rho^3}{1-\rho} + \cdots \right] \\
&= \frac{r}{1+r} \epsilon_{t+1} \left[\frac{1}{1-\rho} \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k - \frac{\rho}{1-\rho} \sum_{k=0}^{\infty} \left(\frac{\rho}{1+r}\right)^k \right] \\
&= \frac{r}{1+r} \epsilon_{t+1} \left[\frac{1}{1-\rho}\frac{1+r}{r} - \frac{\rho}{1-\rho}\frac{1+r}{1+r-\rho} \right] \\
&= \frac{1+r}{1+r-\rho} \epsilon_{t+1}
\end{aligned}
$$

Hence, if $0 < \rho < 1$, the change in consumption is more volatile than the output innovation. This seems paradoxical if the representative consumer is smoothing consumption!

(g) (8 marks). The current account is given by

$$
CA_{t+1} \equiv B_{t+2} - B_{t+1} = r B_{t+1} + Y_{t+1} - C_{t+1}
$$

Now use $Y_{t+1} - E_t\{Y_{t+1}\} = \epsilon_{t+1}$ and $C_{t+1} - E_t\{C_{t+1}\} = C_{t+1} - C_t$ to write the current account innovation as

$$
CA_{t+1} - E_t\{CA_{t+1}\} = (Y_{t+1} - E_t\{Y_{t+1}\}) - (C_{t+1} - C_t)
$$

given that B_{t+1} is predetermined. Hence

$$
\begin{aligned}
(Y_{t+1} - E_t\{Y_{t+1}\}) - (C_{t+1} - C_t) &= \epsilon_{t+1} - \frac{1+r}{1+r-\rho}\epsilon_{t+1} \\
&= \frac{1+r-\rho-1-r}{1+r-\rho}\epsilon_{t+1} \\
&= -\frac{\rho}{1+r-\rho}\epsilon_{t+1}
\end{aligned}
$$

so that a positive output innovation leads to a current account deficit (the CAD covaries negatively with output).

2. **Diversification and risk aversion** (60 marks).

(a) (10 marks). The key first order condition for this problem can be written

$$
u'(C_1) = \beta(1+r)\frac{\pi(s)}{p(s)} u'[C_2(s)]
$$

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with CARA utility, $u'(C) = \exp(-\gamma C)$, so

$$\exp(-\gamma C_1) = \beta(1+r)\frac{\pi(s)}{p(s)}\exp(-\gamma C_2(s))$$

and similarly for foreign.

(b) (10 marks). Take logs of the first order conditions for home and foreign

$$-\gamma C_1 = \log\left[\beta(1+r)\frac{\pi(s)}{p(s)}\right] - \gamma C_2(s)$$

$$-\gamma C_1^* = \log\left[\beta(1+r)\frac{\pi(s)}{p(s)}\right] - \gamma C_2^*(s)$$

Sum over the two countries and clear goods markets to get

$$-\gamma Y_1^w = 2\log\left[\beta(1+r)\frac{\pi(s)}{p(s)}\right] - \gamma Y_2^w(s)$$

Rearranging gives

$$\log\left[\beta(1+r)\frac{\pi(s)}{p(s)}\right] = \frac{\gamma}{2}[Y_2^w(s) - Y_1^w]$$

or

$$\frac{p(s)}{1+r} = \beta\pi(s)\exp\left(-\frac{\gamma}{2}[Y_2^w(s) - Y_1^w]\right)$$

The price of contingent consumption is higher when people are more patient, when the state s is more likely, and is lower the more risk averse they are and the greater is the excess of endowment in state s relative to the initial endowment.

(c) (10 marks). Adding up over the s states gives

$$\frac{1}{1+r}\sum_{s=1}^{S}p(s) = \sum_{s=1}^{S}\beta\pi(s)\exp\left(-\frac{\gamma}{2}[Y_2^w(s) - Y_1^w]\right)$$

$$= \beta\exp\left(\frac{\gamma}{2}Y_1^w\right)\sum_{s=1}^{S}\pi(s)\exp\left(-\frac{\gamma}{2}Y_2^w(s)\right)$$

Hence, given the normalization $\sum_{s=1}^{S}p(s) = 1$,

$$1+r = \frac{1}{\beta}\frac{\exp\left(-\frac{\gamma}{2}Y_1^w\right)}{\sum_{s=1}^{S}\pi(s)\exp\left(-\frac{\gamma}{2}Y_2^w(s)\right)}$$

The real interest rate is the rate of time preference scaled by a constant term that depends on risk aversion, the likelihood of the various states, and the relative scarcity of endowments.

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(d) (15 marks). To show efficiency, first show that the Euler equations hold and that markets clear for the proposed allocations. Market clearing is trivial. The Euler equations at the proposed allocations are, for example,

$$
\begin{aligned}
\exp(-\gamma C_1) &= \exp\left(-\gamma\left[\frac{1}{2}Y_1^w - \mu\right]\right) = \exp\left(-\frac{\gamma}{2}Y_1^w\right)\exp(\gamma\mu) \\
&= \exp\left(+\frac{\gamma}{2}Y_2^w(s)\right)\exp\left(-\frac{\gamma}{2}Y_2^w(s)\right)\exp\left(-\frac{\gamma}{2}Y_1^w\right)\exp(\gamma\mu) \\
&= \exp\left(+\frac{\gamma}{2}[Y_2^w(s) - Y_1^w]\right)\exp\left(-\frac{\gamma}{2}Y_2^w(s)\right)\exp(\gamma\mu) \\
&= \beta(1+r)\frac{\pi(s)}{p(s)}\exp\left(-\frac{\gamma}{2}Y_2^w(s)\right)\exp(\gamma\mu) \\
&= \beta(1+r)\frac{\pi(s)}{p(s)}\exp(-\gamma C_2(s))
\end{aligned}
$$

Checking now that the budget constraints are satisfied:

$$
\begin{aligned}
C_1 + B_2 + xV_1 + (1-x)V_1^* &= Y_1 + V_1 \\
C_2(s) &= (1+r)B_2 + xY_2(s) + (1-x)Y_2^*(s)
\end{aligned}
$$

Under the proposed allocations, $x = -1/2$ and so the first of these becomes

$$
\frac{1}{2}(Y_1 + Y_1^*) - \mu + B_2 + \frac{1}{2}V_1 + \frac{1}{2}V_1^* = Y_1 + V_1
$$

or,

$$
\mu = B_2 - \frac{1}{2}(Y_1 - Y_1^*) - \frac{1}{2}(V_1 - V_1^*)
$$

Similarly, from the second budget constraint,

$$
C_2(s) = \frac{1}{2}(Y_2(s) + Y_2^*(s)) - \mu = (1+r)B_2 + \frac{1}{2}Y_2(s) + \frac{1}{2}Y_2^*(s)
$$

so that

$$
\mu = -(1+r)B_2
$$

Using this to substitute for B_2 gives

$$
\mu = -\frac{1}{1+r}\mu - \frac{1}{2}(Y_1 - Y_1^*) - \frac{1}{2}(V_1 - V_1^*)
$$

or

$$
\mu\left(1 + \frac{1}{1+r}\right) = \frac{1}{2}(Y_1^* - Y_1) + \frac{1}{2}(V_1^* - V_1)
$$

(e) (15 marks). Guess new (market clearing) allocations of the form

$$
C_1 = \frac{\gamma^*}{\gamma + \gamma^*}Y_1^w - \mu, \qquad C_1^* = \frac{\gamma}{\gamma + \gamma^*}Y_1^w + \mu
$$

The Euler equations are satisfied if

$$u'(C_1)V_1 = \beta E_1\{u'(C_2)Y_2\}$$

$$= \beta \sum_{s=1}^{S} u'[C_2(s)]Y_2(s)\pi(s)$$

with CARA utility

$$\exp(-\gamma C_1)V_1 = \beta \sum_{s=1}^{S} \exp(-\gamma C_2(s))Y_2(s)\pi(s)$$

Hence the security price that supports this allocation must be

$$V_1 = \beta \sum_{s=1}^{S} \exp(-\gamma[C_2(s) - C_1])Y_2(s)\pi(s)$$

$$= \beta \sum_{s=1}^{S} \exp\left(-\frac{\gamma\gamma^*}{\gamma + \gamma^*}[Y_2^w(s) - Y_1^w]\right)Y_2(s)\pi(s)$$

Similarly,

$$V_1^* = \beta \sum_{s=1}^{S} \exp\left(-\frac{\gamma\gamma^*}{\gamma + \gamma^*}[Y_2^w(s) - Y_1^w]\right)Y_2^*(s)\pi(s)$$

Following the same steps as in part d) above but with $x = \gamma^*/(\gamma + \gamma^*)$, etc, gives

$$\mu\left(1 + \frac{1}{1+r}\right) = \frac{\gamma^*}{\gamma + \gamma^*}(Y_1^* + V_1^*) - \frac{\gamma}{\gamma + \gamma^*}(Y_1 + V_1)$$

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