# 316-632 INTERNATIONAL MONETARY ECONOMICS NOTE 4a

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## Complete markets

We will briefly consider three representations of the complete markets paradigm: the planning problem, the Arrow-Debreu ("date zero") market economy and the Radner ("sequential") market economy. To begin with, we retain the focus on economies with a single consumption good at each date and state. (Later we will want to discuss international relative prices like the real exchange rate and so we will turn to models with more than one type of consumption good).

### A. One internationally traded good

Let there be countable dates, $t = 0, 1, 2...$ and let there be $Z$ possible states of nature that may be realized at each date $t \geq 1$. Index the states by $z_t \in \mathcal{Z} = \{1, 2, ..., Z\}$. A **history** of states $z^t$ is a vector $z^t = (z_0, z_1, ..., z_t) = (z^{t-1}, z_t)$. The unconditional probability of a history $z^t$ being realized as of date zero is denoted $\pi_t(z^t)$. The initial state $z_0$ is known as of date zero.

Let there be $I$ countries with $i \in \mathcal{I} = \{1, 2, ..., I\}$. The representative consumer in country $i$ has preferences over a consumption stream $c^i = \{c_t^i(z^t)\}_{t=0}^{\infty}$ given by the expected utility function

$$u(c^i) = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t U[c_t^i(z^t)] \pi_t(z^t), \qquad 0 < \beta < 1$$

Country $i$ is endowed with the stream $y^i = \{y_t^i(z^t)\}_{t=0}^{\infty}$ of a single internationally traded consumption good. Countries have identical preferences but different endowments, and this gives rise to an incentive for trade across dates and states of nature.

### Social planner's problem

Before turning to the decentralized problem with complete asset markets, we first study the equivalent social planner's problem. Suppose that the social planner attaches welfare weights $\omega_i > 0$ to country $i$. Then the planner's problem is to choose consumption allocations $c^i$ to maximize

$$\sum_i \omega_i u(c^i) = \sum_i \sum_{t=0}^{\infty} \sum_{z^t} \omega_i \beta^t U[c_t^i(z^t)] \pi_t(z^t)$$

subject to resource constraints for each $t$ and $z^t$

$$\sum_i c_t^i(z^t) \leq Y_t(z^t) \equiv \sum_i y_t^i(z^t)$$

Let the Lagrange multiplier associated with the date $t$ and history $z^t$ resource constraint be $Q_t(z^t) \equiv \beta^t \pi_t(z^t) q_t(z^t)$. (Since $\beta^t \pi_t(z^t) > 0$, there is no harm in introducing this normalization). Then the Lagrangian for the social planner is

$$\begin{aligned}
\mathcal{L} &= \sum_i \sum_{t=0}^{\infty} \sum_{z^t} \omega_i \beta^t U[c_t^i(z^t)] \pi_t(z^t) + \sum_{t=0}^{\infty} \sum_{z^t} Q_t(z^t) \left[ Y_t(z^t) - \sum_i c_t^i(z^t) \right] \\
&= \sum_{t=0}^{\infty} \sum_{z^t} \beta^t \pi_t(z^t) \left[ \sum_i \omega_i U[c_t^i(z^t)] + q_t(z^t) \left[ Y_t(z^t) - c_t^i(z^t) \right] \right]
\end{aligned}$$

The planner's problem breaks down into a **sequence of static problems**, one for each date and state. There is no physical state variable (e.g., capital) linking periods together.

The relevant first order conditions are just

$$\omega_i U'[c_t^i(z^t)] = q_t(z^t)$$

Now let's assume the constant relative risk aversion utility function so that marginal utility is $U'(c) = c^{-\sigma}$. Then we can solve for consumption (as in Note 3a) to get

$$c_t^i(z^t) = \left( \frac{\omega_i}{q_t(z^t)} \right)^{1/\sigma}$$

Aggregating over countries gives solutions for the planner's shadow prices

$$q_t(z^t) = \left( \sum_i \omega_i^{1/\sigma} \right)^{\sigma} Y_t(z^t)^{-\sigma}$$

and from the point of view of date zero

$$Q_t(z^t) = \beta^t \pi_t(z^t) \left( \sum_i \omega_i^{1/\sigma} \right)^{\sigma} Y_t(z^t)^{-\sigma} \tag{1}$$

With the solutions for the shadow prices in hand, we can easily determine the planner's consumption allocations

$$c_t^i(z^t) = \left(\frac{\omega_i^{1/\sigma}}{\sum_i \omega_i^{1/\sigma}}\right) Y_t(z^t)$$

Each country gets a fixed share (proportional to its relative welfare weight) of the world endowment at that date/state.

### Arrow-Debreu

The corresponding Arrow-Debreu economy involves each country owning its stream $y^i = \{y_t^i(z^t)\}_{t=0}^{\infty}$ of the single tradable consumption good and facing date zero prices $Q_t(z^t)$. Country $i$ faces the single budget constraint

$$\sum_{t=0}^{\infty} \sum_{z^t} Q_t(z^t) c_t^i(z^t) \leq \sum_{t=0}^{\infty} \sum_{z^t} Q_t(z^t) y_t^i(z^t)$$

and has first order conditions

$$\beta^t \pi_t(z^t) U'[c_t^i(z^t)] = \lambda_i Q_t(z^t)$$

where $\lambda_i$ is the single Lagrange multiplier on the intertemporal budget constraint. The market clearing conditions are

$$\sum_i c_t^i(z^t) = \sum_i y_t^i(z^t)$$

for each $t$ and $z^t$. Notice that if $\lambda_i = \omega_i^{-1} > 0$, the competitive equilibrium of the Arrow-Debreu economy corresponds to the solution to the social planner's problem.

### Radner (sequence of markets)

An alternative decentralization of the planning problem involves countries trading in a sequence of spot markets, rather than one single market at date zero. At each date $t$, the possible states that might realize in the subsequent period are denoted by some $z' \in \mathcal{Z}$. We suppose that each country can trade in a complete set of Arrow-securities, one for each $z' \in \mathcal{Z}$. (remember, these are securities that pay one unit of consumption if and only if the relevant state $z'$ does in fact realize). Let $v_t(z^t, z')$ denote the price at date $t$ of an Arrow security that pays in state $z'$ in the next period, $t+1$. Similarly, let $B_{t+1}^i(z^t, z')$ denote the quantity of such securities held by country $i$. Each country faces the same financial prices.

The flow budget constraint of country $i$ is then

$$c_t^i(z^t) + \sum_{z'} v_t(z^t, z') B_{t+1}^i(z^t, z') \leq B_t^i(z^t) + y_t^i(z^t)$$

Now let $\eta_t^i(z^t) \geq 0$ denote the Lagrange multiplier corresponding to this flow constraint. The Lagrangian for country $i$ is

$$\mathcal{L}_i = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t U[c_t^i(z^t)]\pi_t(z^t) + \sum_{t=0}^{\infty} \sum_{z^t} \eta_t^i(z^t) \left[ B_t^i(z^t) + y_t^i(z^t) - c_t^i(z^t) - \sum_{z'} v_t(z^t, z') B_{t+1}^i(z^t, z') \right]$$

The first order conditions for this problem include

$$\frac{\partial \mathcal{L}_i}{\partial c_t^i(z^t)} = 0 \Longleftrightarrow \beta^t \pi_t(z^t) U'[c_t^i(z^t)] = \eta_t^i(z^t)$$

and

$$\frac{\partial \mathcal{L}_i}{\partial B_{t+1}^i(z^t, z')} = 0 \Longleftrightarrow v_t(z^t, z') \eta_t^i(z^t) = \eta_{t+1}^i(z^t, z')$$

Hence the individual multipliers $\eta_t^i(z^t)$ on the flow constraints in the Radner economy correspond to the product of individual multipliers and date zero prices $\lambda_i Q_t(z^t)$ in the Arrow-Debreu economy. Moreover, the price of an Arrow security is

$$v_t(z^t, z') = \frac{\eta_{t+1}^i(z^t, z')}{\eta_t^i(z^t)} = \beta \frac{U'[c_{t+1}^i(z^t, z')]}{U'[c_t^i(z^t)]} \frac{\pi_{t+1}(z^t, z')}{\pi_t(z^t)} = \frac{Q_{t+1}(z^t, z')}{Q_t(z^t)}$$

Given the equivalence between the planning problem and the decentralized economies, we can easily solve for the spot market Arrow security prices using equation (1), namely

$$Q_t(z^t) = \beta^t \pi_t(z^t) \left( \sum_i \omega_i^{1/\sigma} \right)^{\sigma} Y_t(z^t)^{-\sigma}$$

Hence

$$v_t(z^t, z') = \beta \left( \frac{Y_{t+1}(z^t, z')}{Y_t(z^t)} \right)^{-\sigma} \frac{\pi_{t+1}(z^t, z')}{\pi_t(z^t)}$$

We can use this solution to price other assets. For example, the equilibrium world real interest rate on safe one-period bonds bought or sold at date $t$ satisfies

$$\frac{1}{1 + r_t(z^t)} = \sum_{z'} v_t(z^t, z') = \sum_{z'} \beta \left( \frac{Y_{t+1}(z^t, z')}{Y_t(z^t)} \right)^{-\sigma} \frac{\pi_{t+1}(z^t, z')}{\pi_t(z^t)} \tag{2}$$

or

$$\frac{1}{1 + r_t} = \mathsf{E}_t \left\{ \beta \left( \frac{Y_{t+1}}{Y_t} \right)^{-\sigma} \right\}$$

### Economic implications

First, absolute purchasing power parity (PPP) holds in this world. There is but a single tradable consumption good available at each date and state and its relative price is the same in every country. In order for PPP to potentially fail, we need to introduce country-specific goods that may not be traded in international markets. Second, consumption in any country $i$ is monotonically related to consumption in any other country $j$. Specifically, the ratio

$$\frac{c_t^i(z^t)}{c_t^j(z^t)} = \left(\frac{\omega_i}{\omega_j}\right)^{1/\sigma} \tag{3}$$

is constant for all dates and states. Consumption in country $i$ is perfectly correlated with consumption in country $j$. Third, real interest rates $r_t(z^t)$ are equalized across countries (there are no cross-country interest rate differentials).

### B. Non-traded goods

Suppose that in addition to the single tradable good, **each country** produces and consumes its own non-tradable good, so that at any date and state there are a total of $I+1$ commodities. Let $y^i$ denote country $i$'s endowment of the single traded good and let $x^i$ denote country $i$'s endowment of its **own** non-traded good. As before, $Y_t(z^t) = \sum_i y_t^i(z^t)$ denotes the world supply of the traded good.

Now let $a_t^i(z^t)$ and $b_t^i(z^t)$ denote consumption of the traded and non-traded goods by country $i$ ("aluminium" is traded but "bricks" are not). Let $Q_t(z^t)$ and $Q_t^i(z^t)$ denote the date zero (Arrow-Debreu) prices of the traded and the $i$th traded good, respectively. Similarly, let $q_t(z^t)$ and $q_t^i(z^t)$ denote the associated spot prices.

The representative consumer in country $i$ has the expected utility function

$$u(a^i, b^i) = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t U[a_t^i(z^t), b_t^i(z^t)]\pi_t(z^t), \qquad 0 < \beta < 1$$

and faces the single intertemporal budget constraint

$$\sum_{t=0}^{\infty} \sum_{z^t} [Q_t(z^t)a_t^i(z^t) + Q_t^i(z^t)b_t^i(z^t)] \leq \sum_{t=0}^{\infty} \sum_{z^t} [Q_t(z^t)y_t^i(z^t) + Q_t^i(z^t)x_t^i(z^t)]$$

The market clearing conditions in this world economy will be

$$\sum_i a_t^i(z^t) = \sum_i y_t^i(z^t)$$

for the single traded good. Similarly, for each country $i$

$$b_t^i(z^t) = x_t^i(z^t)$$

We will look for prices consistent with individual optimization and market clearing.

### Aside on quantity and price indices

Before turning to the solution of this equilibrium problem, I'll introduce a couple of concepts that will be important in what follows. We will ultimately be interested in equilibrium price and quantity indices $p$ and $c$ which have the following properties. The **quantity index** $c$ is a constant returns to scale function of the state dependent consumptions $a, b$ such that utility $U(a, b)$ can be expressed in the form

$$U(a, b) = V(c(a, b))$$

for some monotone increasing function $V$. Similarly, the **price index** $p$ is a constant returns to scale function of the spot prices $q_t(z^t)$ and $q_t^i(z^t)$. At equilibrium, these indices satisfy

$$q_t(z^t)a_t^i(z^t) + q_t^i(z^t)b_t^i(z^t) = p_i[q_t(z^t), q_t^i(z^t)]c_i[a_t^i(z^t), b_t^i(z^t)]$$

See Note 4b for more on these indices and for the algebra needed to derive them in practice. In a mild abuse of notation, I will use the following shorthand in what follows

$$\begin{aligned} p_t^i(z^t) &\equiv p_i[q_t(z^t), q_t^i(z^t)] \\ c_t^i(z^t) &\equiv c_i[a_t^i(z^t), b_t^i(z^t)] \end{aligned}$$

At equilibrium we will have $b_t^i(z^t) = x_t^i(z^t)$, the exogenously given supply of that country's non-traded good. So in general, price and quantity indices will vary across countries. This variability will induce movements in international relative prices. For example, given the cross-country price indices, the bilateral **real exchange rate** between a country $i$ and another country $j$ is defined by

$$e_t^{ij}(z^t) \equiv \frac{p_t^j(z^t)}{p_t^i(z^t)}$$

If we let $P_t^i(z^t)$ denote the date zero price corresponding to the spot price index $p_t^i(z^t)$ we can also define the real interest rate $r_t^i(z^t)$ associated with a sure claim to a unit of the consumption index $c_t^i(z^t)$, specifically

$$\frac{1}{1+r_t^i(z^t)} = \sum_{z'} \frac{P_{t+1}^i(z^t, z')}{P_t^i(z^t)}$$

[compare this to equation (2)]. Again, with non-traded goods real interest rates will typically vary across countries and different pairs of countries will have different real interest differentials.

#### Equivalent planner's problem

We solve for the equilibrium by solving an equivalent planning problem. The planner chooses $a_t^i(z^t), b_t^i(z^t)$ to maximize

$$\sum_i \omega_i u(a^i, b^i) = \sum_i \sum_{t=0}^{\infty} \sum_{z^t} \omega_i \beta^t U[a_t^i(z^t), b_t^i(z^t)] \pi_t(z^t)$$

subject to the resource constraints

$$\begin{aligned} \sum_i a_t^i(z^t) &\leq Y_t(z^t) \\ b_t^i(z^t) &\leq x_t^i(z^t) \end{aligned}$$

Let the planner's Lagrange multipliers be $Q_t(z^t) = \beta^t \pi_t(z^t) q_t(z^t)$ for the traded good and $Q_t^i(z^t) = \beta^t \pi_t(z^t) q_t^i(z^t)$ for each of the $I$ non-traded goods.

The first order conditions include

$$\omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial a_t^i(z^t)} = q_t(z^t)$$

and

$$\omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial b_t^i(z^t)} = q_t^i(z^t)$$

In general, absolute PPP will not hold. Different countries have different endowments $x_t^i(z^t)$ of their non-traded goods, so the price $p_t^i(z^t)$ of the consumption aggregate $c_t^i(z^t)$ will not tend to be the same everywhere. Of course, this also means that consumption indices will not necessarily be perfectly correlated across countries. (As we will see, this latter conclusion depends critically on the degree of separability between traded and non-traded goods in the period utility function $U$).

### Isoelastic examples solved

The usual assumption in applied work is that the period utility function is of the isoelastic form

$$U(a,b) = \frac{1}{1-\sigma}\{[\alpha a^\rho + (1-\alpha)b^\rho]^{\frac{1}{\rho}}\}^{1-\sigma}$$

with $\sigma > 0$, $0 < \alpha < 1$, and $\rho < 1$. Goods are **perfect substitutes** if $\rho = 1$ and are **perfect complements** if $\rho = -\infty$. The elasticity of substitution between goods is measured by $\frac{1}{1-\rho}$.

Notice that utility has the decomposition

$$V(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad c = [\alpha a^\rho + (1-\alpha)b^\rho]^{\frac{1}{\rho}}$$

With this utility function, marginal utilities are

$$\begin{aligned}
\frac{\partial U(a,b)}{\partial a} &= V'(c)\frac{\partial c}{\partial a} \\
&= [\alpha a^\rho + (1-\alpha)b^\rho]^{\frac{-\sigma}{\rho}}\,[\alpha a^\rho + (1-\alpha)b^\rho]^{\frac{1-\rho}{\rho}}\,\alpha a^{\rho-1} \\
&= [\alpha a^\rho + (1-\alpha)b^\rho]^{\frac{1-\sigma-\rho}{\rho}}\,\alpha a^{\rho-1} \\
&= c^{1-\sigma-\rho}\alpha a^{\rho-1}
\end{aligned}$$

and similarly

$$\frac{\partial U(a,b)}{\partial b} = V'(c)\frac{\partial c}{\partial b} = c^{1-\sigma-\rho}(1-\alpha)b^{\rho-1}$$

The consumption index is clearly

$$c_t^i(z^t) = \left[\alpha a_t^i(z^t)^\rho + (1-\alpha)b_t^i(z^t)^\rho\right]^{\frac{1}{\rho}}$$

It takes a considerable amount of algebra (see Note 4b) but it can be shown that the associated price index is

$$p_t^i(z^t) = \left[\alpha^{\frac{1}{1-\rho}}q_t(z^t)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}}q_t^i(z^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{4}$$

Also, with this utility function the first order conditions of the planner's problem can be written

$$
\begin{aligned}
q_t(z^t) &= \omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial a_t^i(z^t)} = \alpha\omega_i c_t^i(z^t)^{1-\rho-\sigma} a_t^i(z^t)^{\rho-1} \\
q_t^i(z^t) &= \omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial b_t^i(z^t)} = (1-\alpha)\omega_i c_t^i(z^t)^{1-\rho-\sigma} b_t^i(z^t)^{\rho-1}
\end{aligned}
$$

We can plug these expressions for the spot prices into the solution for the price index (4) to get

$$
\begin{aligned}
p_t^i(z^t) &= \left[\alpha^{\frac{1}{1-\rho}} q_t(z^t)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} q_t^i(z^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \\
&= \left\{\left[\omega_i c_t^i(z^t)^{1-\rho-\sigma}\right]^{\frac{\rho}{\rho-1}} \left[\alpha a_t^i(z^t)^{\rho} + (1-\alpha) b_t^i(z^t)^{\rho}\right\}^{\frac{\rho-1}{\rho}} \\
&= \omega_i c_t^i(z^t)^{1-\rho-\sigma} c_t^i(z^t)^{\rho-1} \\
&= \omega_i c_t^i(z^t)^{-\sigma}
\end{aligned}
$$

Now we can use the definition of the real exchange rate $e_t^{ij}(z^t)$ to write

$$
e_t^{ij}(z^t) \equiv \frac{p_t^j(z^t)}{p_t^i(z^t)} = \frac{\omega_j c_t^j(z^t)^{-\sigma}}{\omega_i c_t^i(z^t)^{-\sigma}} = \left(\frac{\omega_j}{\omega_i}\right)\left(\frac{c_t^i(z^t)}{c_t^j(z^t)}\right)^{\sigma}
$$

The consumption ratio and real exchange rate are monotonically related. We can test this.

**A further special case: Additive separability**

Let the period utility function be

$$
U(a,b) = \alpha \log(a) + (1-\alpha)\log(b)
$$

which is a special case of the CES example above[1] when $\rho = 0$ (so that there is unit elasticity of substitution) and relative risk aversion is $\sigma = 1$. In this case, the quantity and price indices become

$$
\begin{aligned}
c(a,b) &= a^{\alpha} b^{1-\alpha} \\
p(q,q_i) &= A q^{\alpha} q_i^{1-\alpha}, \qquad A \equiv [\alpha^{\alpha}(1-\alpha)^{1-\alpha}]^{-1}
\end{aligned}
$$

[1] You need to use l'Hôpital's rule to show this.

The model can be solved out as follows. First, the spot price of the traded good is

$$
\begin{aligned}
q_t(z^t) &= \omega_i \frac{\alpha}{a_t^i(z^t)} \\
&\implies q_t(z^t) \sum_i a_t^i(z^t) = \alpha \sum_i \omega_i \equiv \alpha\bar{\omega}
\end{aligned}
$$

so

$$q_t(z^t) = \alpha\bar{\omega} Y_t(z^t)^{-1}$$

Similarly the spot price of each non-traded good is

$$q_t^i(z^t) = \omega_i \frac{1-\alpha}{b_t^i(z^t)} = (1-\alpha)\omega_i x_t^i(z^t)^{-1}$$

This implies the consumption allocations

$$
\begin{aligned}
a_t^i(z^t) &= \frac{\omega_i}{\bar{\omega}} Y_t(z^t) \\
b_t^i(z^t) &= x_t^i(z^t)
\end{aligned}
$$

And so the equilibrium price and quantity indexes are

$$
\begin{aligned}
c_t^i(z^t) &= \left(\frac{\omega_i}{\bar{\omega}}\right)^{\alpha} Y_t(z^t)^{\alpha} x_t^i(z^t)^{1-\alpha} \\
p_t^i(z^t) &= \frac{\bar{\omega}^{\alpha}\omega_i^{1-\alpha}}{Y_t(z^t)^{\alpha} x_t^i(z^t)^{1-\alpha}} = \frac{\omega_i}{c_t^i(z^t)}
\end{aligned}
$$

This implies consumption ratios

$$\frac{c_t^i(z^t)}{c_t^j(z^t)} = \left(\frac{\omega_i}{\omega_j}\right)^{\alpha} \left(\frac{x_t^i(z^t)}{x_t^i(z^t)}\right)^{1-\alpha}$$

By contrast with the single traded good case studied in Section A, the consumption ratio is not constant [compare this with equation (3) above]. Similarly, the real exchange rate is

$$e_t^{ij}(z^t) \equiv \frac{p_t^j(z^t)}{p_t^i(z^t)} = \left(\frac{\omega_j}{\omega_i}\right)\left(\frac{c_t^i(z^t)}{c_t^j(z^t)}\right) = \left(\frac{\omega_j}{\omega_i}\right)^{1-\alpha} \left(\frac{x_t^i(z^t)}{x_t^j(z^t)}\right)^{1-\alpha}$$

The real exchange rate varies stochastically depending on the relative supplies of each country's non-traded goods. The bilateral real exchange rate does not depend on the world supply of goods.

Up to to the multiplicative constant $\left(\frac{\omega_j}{\omega_i}\right)^{-\alpha}$, the real exchange rate and the consumption ratio are the same thing and so they should have the same stochastic properties. Finally notice that if $\alpha = 1$ so that consumers do not care for non-traded goods, we are back in the model of Section A and relative prices like the real exchange rate are constant.

Finally, the price of a risk free bond that pays one unit of the consumption index in country $i$ is

$$\begin{aligned}\frac{1}{1+r_t^i(z^t)} &= \sum_{z'} \frac{P_{t+1}^i(z^t, z')}{P_t^i(z^t)} \\ &= \sum_{z'} \frac{\beta^{t+1}\pi_{t+1}(z^t, z')p_{t+1}^i(z^t, z')}{\beta^t \pi_t(z^t)p_t^i(z^t)}\end{aligned}$$

or

$$\frac{1}{1+r_t^i} = \beta \mathsf{E}_t \left\{ \left(\frac{Y_t}{Y_{t+1}}\right)^{\alpha} \left(\frac{x_t^i}{x_{t+1}^i}\right)^{1-\alpha} \right\}$$

The price and interest rate on this bond depends on both the growth rate of the world supply of the traded good and the growth rate of country $i$'s non-traded good. Roughly speaking (i.e., ignoring the expectation) we have

$$r_t^i \simeq \alpha \log\left(\frac{Y_{t+1}}{Y_t}\right) + (1-\alpha)\log\left(\frac{x_{t+1}^i}{x_t^i}\right)$$

and the real interest differential between countries $i$ and $j$ would be

$$r_t^i - r_t^j \simeq (1-\alpha)\left[\log\left(\frac{x_{t+1}^i}{x_t^i}\right) - \log\left(\frac{x_{t+1}^j}{x_t^i}\right)\right]$$

The real interest rate in country $i$ is relatively high when the growth rate of its supply of non traded goods is relatively high. Notice again that if $\alpha = 1$, we are back to the world with only traded goods and there is no interest differential across countries.

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