316-632 International Monetary Economics

Problem Set #3 *Due 6 October, 2004*

Let there be 2 countries with $i = 1, 2$. Let there be countable dates, $t = 0, 1, 2...$ and let there be Z possible states of nature that may be realized at each date $t \geq 1$. Index the states by $z_t \in \{1, 2, ..., Z\}$. A history is a vector $z^t = (z_0, z_1, ..., z_t)$. The unconditional probability of a history z^t being realized as of date zero is denoted $\varphi_t(z^t)$. The initial state z_0 is known as of date zero.

There is a single internationally traded good. County i has endowment $y_t^i(z^t)$ of this good at date t. Let the world supply of the good at date t be denoted

$$x_t(z^t) = \sum_i y_t^i(z^t)$$

Suppose that each country is populated by a representative consumer with preferences

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t U[c_t(z^t)]\varphi_t(z^t), \qquad 0 < \beta < 1$$

and period utility function

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad \sigma > 0$$

Question 1. Consider a social planner with welfare weights $\omega_i > 0$ each $i = 1, 2$. Formulate the planning problem and solve for the consumption allocations.

Question 2. Now consider a decentralized monetary economy with complete asset markets. Suppose that households are worker/shopper pairs (with timing as in the lecture notes) and that in the home country ($i = 1$) the asset market constraint is

$$\begin{aligned} & M_t^1(z^t) + P_t^1(z^t)\tau_t^1(z^t) + \sum_{z'} q_t^1(z^t, z')B_{t+1}^1(z^t, z') + \mathcal{E}_t(z^t)\sum_{z'} q_t^2(z^t, z')B_{t+1}^{12}(z^t, z') \\ \leq \; & P_{t-1}^1(z^{t-1})y_{t-1}^1(z^{t-1}) + M_{t-1}^1(z^{t-1}) - P_{t-1}^1(z^{t-1})c_{t-1}^1(z^{t-1}) + B_t^1(z^{t-1}, z_t) + \mathcal{E}_t(z^t)B_t^{12}(z^{t-1}, z_t) \end{aligned}$$

(here a superscript 1 indicates country 1, superscript 12 indicates country 1's holdings of bonds denominated in the currency of country 2, etc). The cash-in-advance constraint for goods market purchases is

$$P_t^1(z^t)c_t^1(z^t) \leq M_t^1(z^t)$$

Formulate a Lagrangian and characterize the optimal choices of consumption, money and bond holdings. Symmetric constraints apply for country $i = 2$, so also characterize optimal choices of consumption, money and bond holdings for them. Be clear about the currency denominations that you use when setting up the constraints for country 2. Explain how your Lagrange multipliers in this decentralized problem relate to the welfare weights in the planning problem.

Question 3. Let $\mathcal{M}_t^1(z^t)$ denote the exogenous **supply** of money in the home country with growth rate

$$1 + \mu_t^1(z^t) \equiv \frac{\mathcal{M}_t^1(z^t)}{\mathcal{M}_{t-1}^1(z^{t-1})}$$

The government's budget constraint is, in country 1,

$$\mathcal{B}_t^1(z^{t-1}, z_t) \leq \mathcal{M}_t^1(z^t) - \mathcal{M}_{t-1}^1(z^{t-1}) + P_t^1(z^t)\tau_t^1(z^t) + \sum_{z'} q_t^1(z^t, z')\mathcal{B}_{t+1}^1(z^t, z')$$

(I use script letters like $\mathcal{B}$ and $\mathcal{M}$ to denote the government's holding of bonds and issue of money, to distinguish these from the household's money and bonds). A symmetric constraint applies for the government in country 2. Using the government and household budget constraints for countries 1 and 2, derive the goods, money and bond market clearing conditions for this model.

Question 4. Using your solutions for complete markets consumption allocations from Question 1, solve for equilibrium consumption and money holdings for each country. Use these solutions to derive equilibrium price levels and the equilibrium nominal exchange rate. You may assume that the money growth rates are always such that the nominal interest rate is positive. Show how to solve for the nominal interest rate in each country. Explain whether the real exchange rate is constant or not in this model. Also, explain how nominal interest rates, the exchange rate, and inflation rates in each county depend on the money growth policies in the two countries. Give economic intuition for all your findings.

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