316-632 INTERNATIONAL MONETARY ECONOMICS

This exam lasts **120 minutes** and has two questions. Both questions are worth 60 marks. Allocate your time accordingly. Within each question there are a number of parts and the weight given to each part is also indicated. Even if you cannot complete one part of a question, you should be able to move on an answer other parts, so do not spend too much time. If you feel like you are getting stuck, move on to the next part. You also have **15 minutes perusal** before you can start writing answers.

Question 1. *Backus/Smith* (60 marks). Let there be countable dates, $t = 0, 1, 2...$ and let there be Z possible states of nature that may be realized at each date $t \geq 1$. Index the states by $z_t \in \mathcal{Z} = \{1, 2, ..., Z\}$. A history of states z^t is a vector $z^t = (z_0, z_1, ..., z_t)$. The unconditional probability of a history z^t being realized as of date zero is denoted $\pi_t(z^t)$. The initial state z_0 is known as of date zero.

Let there be I countries with $i \in \mathcal{I} = \{1, 2, ..., I\}$. Country i is endowed with the stream $y^i = \{y_t^i(z^t)\}_{t=0}^{\infty}$ of a single internationally traded consumption good. In addition to the single tradable good, each country produces and consumes its own non-tradable good, so that at any date and state there are a total of $I+1$ commodities. Let y^i denote country i's endowment of the single traded good and let $Y^i = \sum_i y^i$ denote the world supply of the traded good. Let X^i denote country i's endowment of its own non-traded good.

If $a_t^i(z^t)$ and $b_t^i(z^t)$ denote consumption of the traded and non-traded goods by country i, the representative consumer in country i has the expected utility function

$$u(a^i, b^i) = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t U[a_t^i(z^t), b_t^i(z^t)] \pi_t(z^t), \qquad 0 < \beta < 1$$

Period utility is given by the CES function

$$U(a, b) = \frac{1}{1-\sigma} \left\{ [\alpha a^\rho + (1-\alpha) b^\rho]^{\frac{1}{\rho}} \right\}^{1-\sigma}$$

with $\sigma > 0$, $0 < \alpha < 1$, and $\rho < 1$. Goods are perfect substitutes if $\rho = 1$ and are perfect complements if $\rho = -\infty$.

Each consumer faces a single intertemporal budget constraint. Let $Q_t(z^t)$ and $Q_t^i(z^t)$ denote the date zero (Arrow-Debreu) prices of the traded and the ith traded good, respectively. Similarly, let $q_t(z^t)$ and $q_t^i(z^t)$ denote the associated spot prices. Then the budget constraint is

$$\sum_{t=0}^{\infty}\sum_{z^t}[Q_t(z^t)a_t^i(z^t)+Q_t^i(z^t)b_t^i(z^t)] \leq \sum_{t=0}^{\infty}\sum_{z^t}[Q_t(z^t)y_t^i(z^t)+Q_t^i(z^t)X_t^i(z^t)]$$

In your answers, you may make use of the following index numbers. Aggregate consumption is

$$c = [\alpha a^{\rho} + (1-\alpha)b^{\rho}]^{\frac{1}{\rho}}$$

The price index associated with consumption bundle (a, b) when the spot prices are (q, q_i) is

$$p_i = \left[\alpha^{\frac{1}{1-\rho}} q^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} q_i^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}$$

(a) (20 marks): Set up the equivalent social planner's problem with welfare weights $\omega_i > 0$. Derive first order conditions that characterize the solution to the planner's problem. Explain how the planner's Lagrange multipliers relate to the market prices in the decentralized model. Provide a formula for the bilateral real exchange rate $e_t^{ij}(z^t)$ between countries i and j. Explain why PPP will or will not hold in this model.

Solution: The equivalent planning problem has the planner choose $\{a_t^i(z^t), b_t^i(z^t)\}_{t=0}^{\infty}$ to maximize

$$\sum_i \sum_{t=0}^{\infty}\sum_{z^t} \omega_i \beta^t U[a_t^i(z^t), b_t^i(z^t)]\pi_t(z^t)$$

subject to the resource constraints

$$\begin{aligned} \sum_i a_t^i(z^t) &\leq Y_t(z^t) \\ b_t^i(z^t) &\leq X_t^i(z^t) \end{aligned}$$

Let the planner's Lagrange multipliers be $Q_t(z^t) = \beta^t \pi_t(z^t) q_t(z^t)$ for the traded good and $Q_t^i(z^t) = \beta^t \pi_t(z^t) q_t^i(z^t)$ for each of the I non-traded goods. (The choice of notation implies the usual relationship between the planner's Lagrange multipliers and market prices). The first order conditions for the planner include

$$\omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial a_t^i(z^t)} = q_t(z^t)$$

and

$$\omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial b_t^i(z^t)} = q_t^i(z^t)$$

In general, absolute PPP will not hold. Different countries have different endowments $X_t^i(z^t)$ of their non-traded goods, so the price $p_t^i(z^t)$ of the consumption aggregate $c_t^i(z^t)$ will not tend to be the same everywhere. Of course, this also means that consumption indices will not necessarily be perfectly correlated across countries. Also, with the assumed utility function the first order conditions of the planner's problem can be written

$$\begin{aligned} q_t(z^t) &= \omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial a_t^i(z^t)} = \alpha \omega_i c_t^i(z^t)^{1-\rho-\sigma} a_t^i(z^t)^{\rho-1} \\ q_t^i(z^t) &= \omega_i \frac{\partial U[a_t^i(z^t), b_t^i(z^t)]}{\partial b_t^i(z^t)} = (1-\alpha)\omega_i c_t^i(z^t)^{1-\rho-\sigma} b_t^i(z^t)^{\rho-1} \end{aligned}$$

We can plug these expressions for the spot prices into the solution for the price index to get

$$p_t^i(z^t) = \left[\alpha^{\frac{1}{1-\rho}} q_t(z^t)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} q_t^i(z^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} = \omega_i c_t^i(z^t)^{-\sigma}$$

Now we can use the definition of the real exchange rate $e_t^{ij}(z^t)$ to write

$$e_t^{ij}(z^t) \equiv \frac{p_t^j(z^t)}{p_t^i(z^t)} = \frac{\omega_j c_t^j(z^t)^{-\sigma}}{\omega_i c_t^i(z^t)^{-\sigma}} = \left(\frac{\omega_j}{\omega_i}\right)\left(\frac{c_t^i(z^t)}{c_t^j(z^t)}\right)^{\sigma}$$

The consumption ratio and real exchange rate are monotonically related. We can test this.

(b) (20 marks): What does the model predict about the relationship between changes in the bilateral real exchange rate and consumption growth ratios across pairs of countries? Explain how the theory places testable restrictions on the mean and standard deviation of real exchange rates and growth in consumption ratios. Are these predictions borne out by cross-country data? Why or why not?

Solution: The bilateral real exchange rate between countries i and j is

$$e_t^{ij}(z^t) = \left(\frac{\omega_j}{\omega_i}\right)\left(\frac{c_t^i(z^t)}{c_t^j(z^t)}\right)^{\sigma}$$

From now on, suppress the z^t notation. Then in logs

$$\log(e_t^{ij}) = \log\left(\frac{\omega_j}{\omega_i}\right) + \sigma \log\left(\frac{c_t^i}{c_t^j}\right)$$

and in growth rates

$$\Delta \log(e_t^{ij}) = \sigma \Delta \log\left(\frac{c_t^i}{c_t^j}\right)$$

So according to our theory, the unconditional moments of bilateral real exchange rate growth should line up with the unconditional moments of growth rates in the corresponding consumption ratios. For example,

$$\mathsf{E}\left\{\Delta \log(e_t^{ij})\right\} = \sigma \mathsf{E}\left\{\Delta \log\left(\frac{c_t^i}{c_t^j}\right)\right\}$$

and

$$\mathsf{Std}\left\{\Delta \log(e_t^{ij})\right\} = \sigma \mathsf{Std}\left\{\Delta \log\left(\frac{c_t^i}{c_t^j}\right)\right\}$$

We can calculate the empirical counterparts to these moments by computing the sample analogs. In the data, it is not easy to detect any positive relationship between real exchange rate growth and growth in consumption ratios.

(c) (20 marks): Consider the special case of the model when $\sigma = 1$ and $\rho = 0$. If so, the consumption index becomes

$$c = a^{\alpha} b^{1-\alpha}$$

with associated price index

$$p_i = \frac{q^{\alpha} q_i^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

Solve for the equilibrium spot prices q and q_i and then use these findings to solve for consumption of each good a, b, the consumption index c and the price index p. Finally, solve for the bilateral real exchange rate. Provide economic intuition for all your solutions.

Solution: The spot price of the traded good is given by

$$\begin{aligned} q_t(z^t) &= \omega_i \frac{\alpha}{a_t^i(z^t)} \\ &\implies q_t(z^t) \sum_i a_t^i(z^t) = \alpha \sum_i \omega_i \equiv \alpha \bar{\omega} \end{aligned}$$

so

$$q_t(z^t) = \alpha\bar{\omega}Y_t(z^t)^{-1}$$

Similarly the spot price of each non-traded good is

$$q_t^i(z^t) = \omega_i \frac{1-\alpha}{b_t^i(z^t)} = (1-\alpha)\omega_i X_t^i(z^t)^{-1}$$

This implies the consumption allocations

$$\begin{aligned} a_t^i(z^t) &= \frac{\omega_i}{\bar{\omega}} Y_t(z^t) \\ b_t^i(z^t) &= X_t^i(z^t) \end{aligned}$$

And so the equilibrium price and quantity indexes are

$$\begin{aligned} c_t^i(z^t) &= \left(\frac{\omega_i}{\bar{\omega}}\right)^\alpha Y_t(z^t)^\alpha X_t^i(z^t)^{1-\alpha} \\ p_t^i(z^t) &= \frac{\bar{\omega}^\alpha \omega_i^{1-\alpha}}{Y_t(z^t)^\alpha X_t^i(z^t)^{1-\alpha}} = \frac{\omega_i}{c_t^i(z^t)} \end{aligned}$$

This implies consumption ratios

$$\frac{c_t^i(z^t)}{c_t^j(z^t)} = \left(\frac{\omega_i}{\omega_j}\right)^\alpha \left(\frac{X_t^i(z^t)}{X_t^i(z^t)}\right)^{1-\alpha}$$

Similarly, the real exchange rate is

$$e_t^{ij}(z^t) \equiv \frac{p_t^j(z^t)}{p_t^i(z^t)} = \left(\frac{\omega_j}{\omega_i}\right) \left(\frac{c_t^i(z^t)}{c_t^j(z^t)}\right) = \left(\frac{\omega_j}{\omega_i}\right)^{1-\alpha} \left(\frac{X_t^i(z^t)}{X_t^j(z^t)}\right)^{1-\alpha}$$

The real exchange rate varies stochastically depending on the relative supplies of each country's non-traded goods. The bilateral real exchange rate does not depend on the world supply of goods. Up to to the multiplicative constant $\left(\frac{\omega_j}{\omega_i}\right)^{-\alpha}$, the real exchange rate and the consumption ratio are the same thing and so they should have the same stochastic properties. Finally notice that if $\alpha = 1$ so that consumers do not care for non-traded goods, then relative prices like the real exchange rate are constant.

Question 2. *Nominal Assets and Exchange Rates* (60 marks). Two-country monetary models frequently give rise to the following formulas. The nominal pricing kernel for home, "dollar," assets is (in standard notation)

$$q_t(z^t, z') = \beta \frac{U'[c_{t+1}(z^t, z')]}{U'[c_t(z^t)]} \frac{P_t(z^t)}{P_{t+1}(z^t, z')} \frac{\varphi_{t+1}(z^t, z')}{\varphi_t(z^t)}$$

while the nominal pricing kernel for foreign, "euro," assets is

$$q_t^*(z^t, z') = q_t(z^t, z') \frac{\mathcal{E}_{t+1}(z^t, z')}{\mathcal{E}_t(z^t)}$$

where $\mathcal{E}_t(z^t)$ denotes the spot nominal exchange rate.

(a) (10 marks): Give a "no-arbitrage" explanation for the relationship between the dollar and euro pricing kernels.

Solution: Suppose that I want to ensure that I have a dollar tomorrow in state z'. I could just buy a bond that pays a dollar in that state, such a bond has price $q_t(z^t, z')$ today. But another way to get a dollar in state z' is to buy just the right number of euro bonds so that when I convert euros to dollars in state z' I get exactly one dollar. A dollar tomorrow will require $\frac{1}{\mathcal{E}_{t+1}(z^t,z')}$ euros at $t+1$ which can be bought for $\frac{q_t^*(z^t,z')}{\mathcal{E}_{t+1}(z^t,z')}$ euros at t. But $\frac{q_t^*(z^t,z')}{\mathcal{E}_{t+1}(z^t,z')}$ euros at t is equal to $q_t^*(z^t, z') \frac{\mathcal{E}_t(z^t)}{\mathcal{E}_{t+1}(z^t,z')}$ dollars at t. So I could lay out this many dollars to make sure that I have a dollar in z' at $t+1$. If there are to be no arbitrage profits, it had better be the case that this is equal to the original dollar price $q_t(z^t, z')$.

(b) (10 marks): Let i_t and i_t^* denote the one-period nominal interest rates on safe dollar and euro bonds. Provide formulas that relate the **price** of a dollar and a euro bond to the marginal rate of substitution of the representative consumer in the home country.

Solution: The safe nominal interest rate on dollar bonds is found from the price of a dollar bond, as follows

$$\begin{aligned} \frac{1}{1+i_t(z^t)} &= \sum_{z'} q_t(z^t, z') \\ &= \sum_{z'} \left\{ \beta \frac{U'[c_{t+1}(z^t, z')]}{U'[c_t(z^t)]} \frac{P_t(z^t)}{P_{t+1}(z^t, z')} \frac{\varphi_{t+1}(z^t, z')}{\varphi_t(z^t)} \right\} \end{aligned}$$

or more informally

$$\frac{1}{1+i_t} = \mathsf{E}_t\left\{\beta\frac{U'(c_{t+1})}{U'(c_t)}\frac{P_t}{P_{t+1}}\right\}$$

The safe nominal interest rate on euro bonds is given by

$$\begin{aligned}\frac{1}{1+i_t^*(z^t)} &= \sum_{z'} q_t^*(z^t,z') \\ &= \sum_{z'}\left\{q_t(z^t,z')\frac{\mathcal{E}_{t+1}(z^t,z')}{\mathcal{E}_t(z^t)}\right\} \\ &= \sum_{z'}\left\{\beta\frac{U'[c_{t+1}(z^t,z')]}{U'[c_t(z^t)]}\frac{P_t(z^t)}{P_{t+1}(z^t,z')}\frac{\mathcal{E}_{t+1}(z^t,z')}{\mathcal{E}_t(z^t)}\right\}\end{aligned}$$

or more informally

$$\frac{1}{1+i_t^*} = \mathsf{E}_t\left\{\beta\frac{U'(c_{t+1})}{U'(c_t)}\frac{P_t}{P_{t+1}}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right\}$$

Notice that this is a relationship between foreign nominal interest rates and the home country nominal pricing kernel.

(c) (15 marks): Give a definition of the one period "forward exchange rate". Let $\mathcal{F}_t$ denote the forward rate. Give a "no-arbitrage" explanation for the relationship between the forward and spot exchange rates and the interest rates on nominal assets.

Solution: The (one-period) forward exchange rate is an agreement to purchase one euro at $t+1$ with a number $\mathcal{F}_t(z^t)$ of dollars at time t. That is, the forward rare is a one-period-ahead contract to lock in the spot rate at which you will trade next period. Given a complete set of state contingent nominal securities, this asset is redundant and we can figure out how to price it given the spot nominal exchange rate and the nominal interest rates in each country. An agent can borrow one dollar, use it to buy $\frac{1}{\mathcal{E}_t(z^t)}$ euros, use those euros to buy bonds that pay $1+i_t^*(z^t)$ each for a total of $\frac{1+i_t^*(z^t)}{\mathcal{E}_t(z^t)}$ at $t+1$. If the forward rate is $\mathcal{F}_t(z^t)$, then this total can be turned into a return of $[1+i_t^*(z^t)]\frac{\mathcal{F}_t(z^t)}{\mathcal{E}_t(z^t)}$ dollars for sure at date $t+1$. But we already know that the price of a dollar for sure at date $t+1$ is $\frac{1}{1+i_t(z^t)}$. If there are to be no arbitrage profits, it had better be the case that these two prices are the same

$$\frac{1}{1+i_t(z^t)} = \frac{1}{1+i_t^*(z^t)}\frac{\mathcal{E}_t(z^t)}{\mathcal{F}_t(z^t)}$$

The term on the left is the price of a bond that pays a dollar for sure at $t+1$, the term on the right is the price of a contract that delivers a dollar for sure via the appropriate euro assets with the spot exchange rate at which the payment is made at $t+1$ locked in forward. Thus, both contracts are riskless. This is often written

$$\frac{\mathcal{F}_t(z^t)}{\mathcal{E}_t(z^t)} = \frac{1+i_t(z^t)}{1+i_t^*(z^t)}$$

and is the so-called covered interest parity condition.

(d) (15 marks): Using your answer from part (b), explain why we might expect to observe a relationship of the form

$$i_t - i_t^* \approx \mathsf{E}_t \left\{ \Delta \log \mathcal{E}_{t+1} \right\}$$

in data. What extra assumptions do you have to make to get this approximation? Give economic intuition for the implied relationship between interest differentials and expected exchange rate depreciations.

Solution: Recall that the safe dollar bond price is

$$\frac{1}{1+i_t} = \mathsf{E}_t \left\{ \beta \frac{U'(c_{t+1})}{U'(c_t)} \frac{P_t}{P_{t+1}} \right\}$$

while the safe euro bond price is

$$\frac{1}{1+i_t^*} = \mathsf{E}_t \left\{ \beta \frac{U'(c_{t+1})}{U'(c_t)} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}$$

Expanding the conditional expectation on the right

$$\begin{aligned} \frac{1}{1+i_t^*} &= \mathsf{E}_t \left\{ \beta \frac{U'(c_{t+1})}{U'(c_t)} \frac{P_t}{P_{t+1}} \right\} \mathsf{E}_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} + \mathsf{Cov}_t \left\{ \beta \frac{U'(c_{t+1})}{U'(c_t)} \frac{P_t}{P_{t+1}}, \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \\ &= \frac{1}{1+i_t} \mathsf{E}_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} + \mathsf{Cov}_t \left\{ \frac{1}{1+i_t}, \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \end{aligned}$$

We do not get the textbook uncovered interest parity relationship. Only if the covariance term is zero and exchange rate changes have small variance do we get

$$i_t - i_t^* \approx \mathsf{E}_t \left\{ \Delta \log \mathcal{E}_{t+1} \right\}$$

This hypothesis says that interest rate differentials merely reflect expected exchange rate movements. For example, if the nominal interest rate in the home country is high, that merely reflects the expected depreciation of the dollar against the euro.

(e) (10 marks): Let the forward premium be $\log\left(\frac{\mathcal{F}_t}{\mathcal{E}_t}\right)$. If the condition in part (d) holds, what empirical properties should the forward premium have? Does the data support these predictions? Why or why not?

Solution: Approximately, we have

$$\log\left(\frac{\mathcal{F}_t}{\mathcal{E}_t}\right) \approx i_t - i_t^*$$

the so-called covered interest parity condition. If the covered and uncovered interest parity conditions both held, we ought to get the relationship

$$\log\left(\frac{\mathcal{F}_t}{\mathcal{E}_t}\right) \approx \mathsf{E}_t\left\{\Delta \log \mathcal{E}_{t+1}\right\}$$

or

$$\log \mathcal{F}_t \approx \mathsf{E}_t\{\log \mathcal{E}_{t+1}\}$$

That is, if both interest parity conditions held, forward rates should be approximately equal to expected spot rates. One of the major puzzles in international macroeconomics is that this relationship fails badly. The so-called forward premium anomaly reflects the fact that countries with relatively high interest rates seem to experience nominal exchange rate appreciations, whereas the covered interest parity condition tells us that relatively high nominal interest rates, $i_t - i_t^* > 0$, should go hand in hand with $\mathsf{E}_t\left\{\Delta \log \mathcal{E}_{t+1}\right\} > 0$, that is, with expected nominal exchange rate depreciations. Regressions of the form

$$\Delta \log \mathcal{E}_{t+1} = \beta_0 + \beta_1 \log\left(\frac{\mathcal{F}_t}{\mathcal{E}_t}\right) + \text{noise}$$

typically estimate $\beta_1 = -0.88$ or thereabouts, not the $\beta_1 \approx 1$ that we expect from our theory. (Not even the sign is "right"!).