Long-run Economic Growth

Part I: Production and Solow’s Growth Model

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Agenda

- Two classes on ‘long-run economic growth’
  - the aggregate production function, a fundamental tool (today)
  - Robert Solow’s growth model
  - growth accounting (next class)
  - productivity and institutions
- To start with, what do we mean by ‘long-run’ growth?
  - trend versus business cycle
  - a key conceptual distinction in macroeconomics

Long-run growth: US trend

Short-run growth: US business cycle
Long-run economic growth

- Questions
  - why does a country's output per person grow over time?
  - why is there so much cross-country variation in output per person?

- Data
  - Penn World Tables, careful cross-country measurements

Cross-country GDP differences, 2004

<table>
<thead>
<tr>
<th>Country</th>
<th>real GDP per person</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>10,938</td>
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<tr>
<td>Brazil</td>
<td>7,204</td>
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<td>China</td>
<td>5,332</td>
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<td>Egypt</td>
<td>4,759</td>
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<td>France</td>
<td>26,168</td>
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<td>India</td>
<td>2,990</td>
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<td>Ireland</td>
<td>28,956</td>
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<td>Japan</td>
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<td>United States</td>
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<td>Zimbabwe</td>
<td>2,438</td>
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</tbody>
</table>

Units: 2000 US dollars, PPP basis. Source: Penn World Tables, 6.2
Production functions

• Relate output, real GDP, to inputs, e.g., capital and labor

• Mathematical version
\[ Y = AK^\alpha L^{1-\alpha}, \] ‘Cobb-Douglas’ function

• Definitions
\[ Y = \text{quantity of output, real GDP} \]
\[ K = \text{quantity of physical capital used (plant and equipment)} \]
\[ L = \text{quantity of labor used} \]
\[ A = \text{‘total factor productivity’ i.e., everything left out} \]
\[ \alpha = 0.33 \text{ in US data} \]

Properties

• More inputs give more output: positive marginal products
\[ \frac{\partial Y}{\partial K} > 0, \quad \frac{\partial Y}{\partial L} > 0 \]

• Output effect of additional inputs falls: diminishing returns
\[ \frac{\partial^2 Y}{\partial K^2} < 0, \quad \frac{\partial^2 Y}{\partial L^2} < 0 \]

• Double all inputs, double output: constant returns to scale
\[ \lambda Y = AF(\lambda K, \lambda L), \quad \lambda > 0 \]

Capital stock \( K \)

• Meaning: physical capital stock, plant and equipment

• Why does it change?
  – depreciation
  – destruction
  – new investment

• Mathematical version
\[ K_{t+1} - K_t = I_t - \delta K_t \]
  – for simplicity, a constant rate of depreciation, \( \delta \)

• Adjustments for quality? embodied technology?
Capital stock per worker $K/L$

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<th>Year</th>
<th>USA</th>
<th>FRA</th>
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in 2000 dollars at ‘PPP’

Labor supply $L$

- Meaning: units of work effort
- Why does it change?
  - population growth
  - fraction of population employed, hours worked
- Adjustments for quality?
  - skills? education? other factors?
  - if call this $H$ for human capital, then

$$Y = AK^\alpha (HL)^{1-\alpha}$$

$$= (AH^{1-\alpha})K^\alpha L^{1-\alpha}$$

- ‘augmented production function’

Human capital?

Employed as fraction of population

Average years schooling, 15+ population

13 15 14 16
Productivity

• Standard number
  – labor productivity (output per worker), \( Y/L \)

• Our preferred number
  – total factor productivity, \( A \)

• How do we measure it?
  – use the production function to solve for \( A \)
    \[ A = \frac{Y}{K^{\alpha}L^{1-\alpha}} = \frac{Y}{L} \cdot \left( \frac{K}{L} \right)^{\alpha} \]

• US example
  \( \frac{Y}{L} = 67,865 \) and \( \frac{K}{L} = 177,007 \) and \( \alpha = \frac{1}{3} \) so \( A = 1,208 \)

Simple productivity calculations

<table>
<thead>
<tr>
<th>Country</th>
<th>( Y/\text{person} )</th>
<th>( Y/L )</th>
<th>( K/L )</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
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<td>Zimbabwe</td>
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<td>5,416</td>
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</table>

Units: 2000 US dollars, PPP basis. Source: Penn World Tables, 6.2

What have we learned so far?

• Production function links output to inputs and productivity
  \[ Y = AK^{\alpha}L^{1-\alpha} \]

• Capital input, \( K \)
  – plant and equipment
  – a consequence of investment

• Labor input, \( L \)
  – population growth
  – participation and hours
  – skills etc

• TFP, \( A \), can be inferred from data on output and inputs

Rest of this class

• Robert Solow’s growth model
  – pieces of the model
  – dynamics
  – ‘steady state’
  – convergence hypothesis

• Main implication: total factor productivity is key
  – long-run growth due to productivity increases, not capital investment
  – cross-country variation in output per person due to variation in productivity levels
Solow's growth model

- Production function
  \[ Y_t = AK_t^\alpha L^{1-\alpha}, \text{ constant } A, L \text{ for simplicity} \]
- National income accounting
  \[ Y_t = C_t + I_t \]
- Constant savings rate \( s \)
  \[ S_t = I_t = sY_t \]
- Capital accumulation: depreciation rate \( \delta \)
  \[ K_{t+1} - K_t = I_t - \delta K_t \]

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Capital accumulation dynamics

- Put it all together
  \[
  K_{t+1} - K_t = I_t - \delta K_t
  = S_t - \delta K_t
  = sY_t - \delta K_t
  = sAK_t^\alpha L^{1-\alpha} - \delta K_t
  \]
- Simplify notation: capital per worker
  \[ k_t \equiv \frac{K_t}{L} \]
- Then
  \[ k_{t+1} - k_t = sAk_t^\alpha - \delta k_t \]

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Savings versus depreciation

- We have
  \[ k_{t+1} - k_t = sAk_t^\alpha - \delta k_t \]
- Given parameters \( \alpha, A, \delta, s \) and initial capital per worker \( k_0 \)
- How does \( k_t \) change over time?
  - capital increasing if investment is bigger than depreciation
  - capital decreasing if investment is smaller than depreciation

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### Steady state

- If investment equals depreciation, capital per worker is constant

- Call the value of $k$ that makes this happen *steady state* capital per worker $\bar{k}$
  
  $0 = sA\bar{k}^\alpha - \delta \bar{k}$

- Solving this we get
  
  $\bar{k} = \left( \frac{sA}{\delta} \right)^{\frac{1}{1-\alpha}}$

- Properties
  - higher when $s, A$ higher
  - lower when $\delta$ higher
  - higher when $\alpha$ higher if $sA > \delta$

- Capital stock *gravitates* to this steady state value

### Implications for output

- Capital per worker converges to steady state
  
  $\bar{k} = \left( \frac{sA}{\delta} \right)^{\frac{1}{1-\alpha}}$

- Therefore output per worker converges to steady state too
  
  $\bar{y} = A\bar{k}^\alpha = A^{\frac{1}{1-\alpha}} \left( \frac{s}{\delta} \right)^{\frac{\alpha}{1-\alpha}}$

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### Gravitation to steady state

![Graph showing the gravitation to steady state](image)

### Convergence

![Graph showing convergence to steady state](image)
What about sustained, long-run growth?

- Capital investment alone cannot generate long-run growth
  - why? diminishing returns
- What can generate long-run growth in output per worker?
  - growth in total factor productivity $A$
- Suppose $A$ grows at rate $g$
  \[
  A_{t+1} - A_t = gA_t \iff A_t = (1 + g)^t \bar{A}
  \]

Sustained growth

Convergence hypothesis

- Basic idea
  - poor countries should grow faster than rich
  - eventually, poor countries should catch-up to the rich
- Absolute: countries with same $g$ and same $\bar{A}, \delta, s$ should have same output per worker in long run
- Conditional: countries with same $g$ but different $\alpha, \bar{A}, \delta, s$ should grow at same rate but have different long run levels
  - countries with higher $s$ will have higher output per worker
  - but will not grow faster in long run

Convergence across US states?

![Trends in absolute income](image1)
![Trends in relative income](image2)
Why does higher savings increase output?

Main implications restated

- Capital investment does not explain long run growth of output
  - but high savings associated with high level of output
- Productivity growth explains long run growth of output
- Short run boosts to growth are possible
  - but run into diminishing returns

Historical lessons

- Soviet economic growth very impressive in 1950s – early 1960s
  - 8-9% per year, roughly three times faster than US
  - concern about strategic implications of Soviet growth
- But ‘perspiration not inspiration’
  - too much reliance on physical capital accumulation
  - generates growth in short run only
  - cannot generate sustained long run growth
- Other examples
  - Japan in 80s? East Asian NIEs in 90s? China? India?

Level vs. growth effects
Levels of output

• Solow model predicts output per worker

\[ \bar{y} = A^{\frac{1}{1-\alpha}} \left( s \delta \right)^{\frac{\alpha}{1-\alpha}} \]

- cross-country \( \bar{y} \) explained by four parameters \( \alpha, A, \delta, s \)
- we can measure \( \alpha, \delta, s \)
- do not vary enough to explain enormous variation in \( \bar{y} \)
- again leaves TFP as explanation

• What causes TFP to grow, to vary across countries?
  - guess what we will talk about next week

What have we learned today?

• Production
  - production function links output to inputs and productivity
  - ‘total factor productivity’ inferred from data on output and inputs

• Solow model
  - countries with high savings rates have high levels of output
  - but capital investment cannot generate long run growth in output
  - need productivity growth to generate long run growth in output
  - need cross-country variation in productivity levels to explain enormous disparities in data