A Transactions-Based Model of the Monetary Transmission Mechanism

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What are the effects of open market operations? How do these differ from money falling from heaven? We propose a new explanation of how open market operations can change real and nominal interest rates which emphasizes three often mentioned, but seldom explicitly articulated, features of actual monetary economies: (*i*) going to the bank is costly so that people will tend to bunch cash withdrawals; (*ii*) people don't all go to the bank simultaneously; and, because of these, (*iii*) at any instant of time agents hold different amounts of cash. We show that these considerations imply that an open market purchase of a bond for fiat money will drive down nominal and real interest rates, lead to a delayed positive price response, and have damped persistent effects on both prices and nominal interest rates if agents have logarithmic utility of consumption. We assume output is exogenous, so that the model can shed only indirect light on the relationship between money and aggregate output.

The model is a hybrid of the type suggested by Robert Clower (1967) which assumes that agents require cash in advance of expenditures, and the partial equilibrium inventory theoretic models of James Tobin (1956) and William Baumol (1952) which stress that transaction costs necessitate that money withdrawals be periodic, so that it will not be optimal for agents to go to the bank at each instant.[1]

Formally, we assume that money withdrawals are staggered, so that a fixed fraction of the population makes a withdrawal each period. While it would certainly be preferable to make the time between trips to the bank an endogenous choice variable as in the Tobin-Baumol model, analysis of this consideration outside of steady states is too complicated, so that we choose this simpler formulation. (See Boyan Jovanovic, 1982, for the analysis of the steady-state time between trips to the bank in a model similar in structure to ours.) As we hope will be clear, letting the time between trips to the bank adjust to changes brought about by open market operations would not substantially alter our conclusion regarding the nonneutrality of money.

Although the timing of withdrawals is fixed, the size of these withdrawals is endogenously determined. All income receipts are assumed to accrue as interest earning deposits, so that bank withdrawals are the only source of cash for consumers. The Clower cash-in-advance constraint implies that withdrawals for each agent equal his planned nominal spending over the ensuing time interval before his next withdrawal. Planned nominal spending is determined by the possibility of intertemporal consumption substitution and thus is influenced by expected prices and future nominal interest rates. The model is completely deterministic and we assume that those expectations are in fact realized; we attribute to agents perfect foresight.

We assume that only consumers hold stocks of fiat money. The consumers use money to buy goods from firms. We assume that firms deposit their cash receipts instantly into the interest-bearing accounts of their various claimants and hold no money themselves. Similarly, under certainty, it is difficult to see why banks would hold cash at positive interest rates; we assume they hold none. Under this formulation, the money

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[1]After writing the first draft of this paper, we became aware of Julio Rotemberg's 1982 paper which employs a model of staggered withdrawals to analyze the effects of changes in the money supply. That analysis deals with capital accumulation and endogenous output changes and emphasizes different channels of persistence.

stock is held exclusively by consumers to finance spending before their next withdrawal. Equilibrium requires that the flow of cash into the bank at each period equal agents' desired withdrawals. The flow into the bank consists of the nominal value of firms' receipts (nominal *GNP*) plus any changes in nominal money introduced by open market operations.

An essential feature of our model is the fact that it is optimal for people to take time to run down their cash balances. Thus if there is to be a steady flow of money out of the bank, there will have to be a steady rate at which people run out of money. Thus the cross-sectional distribution of money holdings at a given point in time cannot be degenerate. If everyone holds exactly the same amount of money, then they will exhaust at the same day. On the dates when they don't exhaust, there would be no one to hold the (non-interest) bearing money which flows from the stores to the banks. Thus it is impossible to have everyone exhaust at the same time.

Under this formulation, it is straightforward to see how an open market purchase differs from a transfer to each agent proportional to his existing nominal balances. When the money supply increases through an open market operation, agents at the bank must be induced to hold the whole of the increase and thus a disproportionately larger share of the stock of money. This is because most of the people who are not at the bank (i.e., those people who have not yet exhausted their cash balances) will not find it optimal to go to the bank and withdraw extra cash, until they exhaust their current cash. Thus, the share of nominal spending attributable to agents at the bank must rise. To induce agents at the bank to withdraw and hence consume more, banks must lower the real and nominal interest rates. Since this new money is spent gradually over the interval before the next trip to the bank, the price level rises gradually through time, even though prices are completely flexible. This scenario contrasts with a proportional transfer which would raise all nominal prices by the same percentage and thus have no real effects.

The analysis shows that open market operations have some novel implications for the distribution of wealth. We emphasize that the new money withdrawal is financed by running down other asset holdings of equal nominal value; there is no direct benefit bestowed on the recipients of the new money. Rather, wealth is redistributed through two indirect channels. The first channel involves the asymmetry of existing nominal holdings of money. Since those agents not currently at the bank have more money than those at the bank (who have none), the inflation induced by money creation redistributes wealth from those not at the bank to those at the bank. The second channel is more subtle; the focus is on the redistribution arising from interest rate changes. We assume that all current period receipts accrue as interest earning deposits. Thus those agents not at the bank implicitly lend their current period income to those making withdrawals. A decline in interest rates redistributes wealth from creditors to debtors, which enhances the relative position of those making withdrawals when interest rates decline as a result of open market purchases.

The paper is organized as follows: Section I outlines the model; Section I discusses how to calculate the equilibrium; Section III contains a description of the steady state with constant nominal money supply; and Section IV considers the effects of a once and for all unanticipated increase in the money supply brought about by an open market operation.

I. The Model

We assume a unique perishable consumption good which is exogenously supplied at a rate of one unit per period. This good is purchased by consumers with "money"—a non-interest-bearing nominally denominated asset which is supplied exclusively by the government. All income receipts are assumed to accrue as interest-earning bank deposits. Hence bank withdrawals are the only source of cash for consumers. We assume that each agent goes to the bank every other period and withdraws sufficient cash to finance consumption expenditures over the ensuing two periods. Let superscript a denote an agent who goes to the bank every odd period, and b denote an agent who goes every even period. Each agent must choose bank withdrawals and spending to maximize

$$(1) \qquad \sum_{t=1}^{\infty} \beta^{t-1}u(C_t), \qquad 0<\beta<1,$$

subject to a wealth constraint and a constraint which states that only money can purchase goods. For an agent of type a, the latter constraint is[2]

$$(2) \qquad P_1C_1 = M_0^a$$

$$P_tC_t + P_{t+1}C_{t+1} = M_{t-1}^a, \qquad t \text{ even},$$

and the former

$$(3) \qquad P_1C_1 + \sum_{t \text{ odd}} \frac{M_t^a}{\alpha_t} \leq M_0^a + W_0^a,$$

where M_0^a = agent a's initial money, W_0^a = agent a's initial nonmoney wealth, M_t^a = agent a's money holding at the end of period t, $\alpha_t = R_1 \times R_2 \times \ldots \times R_{t-1}$ = nominal growth factor between 1 and t ($\alpha_1 \equiv 1$), and R_t = one plus the nominal interest rate earned on assets between t and $t+1$.

A consumer's nonmonetary wealth consists of three components. The first is a claim to a fraction of the stores' revenue which is deposited into interest earning accounts every period. Let s^a, s^b denote the fraction of the stores' revenue claimed by a and b, respectively. We also assume that agents hold one-period maturity government bonds. Such bonds are perfect substitutes for interest-earning deposits. Taxes to finance the interest payments on government bonds are assumed to be an equal lump sum levy paid each period from each agents' interest-bearing account. Let B_0^a, B_0^b denote the initial nominal holdings of government bonds by agents a and b, respectively. Let $B_0 = B_0^a + B_0^b$ denote aggregate government debt. Since the present value of future tax levies must equal the current nominal debt, B, per capita taxes in this two-agent economy equal $B/2$. Combining these three terms yields

$$(4) \qquad W_0^a = s^a\left(\sum_{t=1}^{\infty} P_t/\alpha_t\right) + B_0^a - B/2.$$

For equilibrium, we required a sequence of nominal prices P_t, and nominal interest rates R_t, such that both the money market and the goods market clears given the exogenously specified path for nominal money M_t, and the initial distribution of both money and nonmonetary wealth. Assuming one agent of each type, goods-market equilibrium requires

$$(5) \qquad C_t^a + C_t^b = 1, \qquad \text{all } t,$$

and money market equilibrium requires

$$(6) \qquad M_t^a + M_t^b = M_t,$$

where the sequence of M's and C's maximize equation (1), subject to (2) and (3).

II. Derivation of Equilibrium

The calculation of equilibrium prices and interest rates is facilitated by exploiting the recursive nature of the problem. Consider the problems of an agent who has just withdrawn M_t from the bank and must allocate this between expenditure in $t+1$ and $t+2$. Formally, his problem is

$$(7) \qquad \max_{C_1, C_2} u(C_1)+\beta u(C_2)$$

$$\text{subject to} \qquad P_1C_1 + P_2C_2 = M_t.$$

Let $C_2 = \phi(P_1/P_2, M_t/P_2)$ be the solution for second-period consumption. Note that if utility is logarithmic ($u(C) = \ln C$), then second-period consumption is independent of first-period prices $C_2 = (\beta/(1+\beta))M_t/P_2 \equiv \bar{\phi}M_t/P_2$.

Next consider the flow equilibrium of money into and out of the bank implied by

[2]For notational simplicity, we neglect the possibility that agents will not wish to exhaust their money holdings before returning to the bank. In periods $t \geq 3$, it may be verified that the cash-in-advance constraint will always bind, since in a world of certainty with positive nominal rates agents never withdraw more than they spend. We will give a condition below which assures that the initial cash constraint also is binding.

the stock equilibrium condition:

$$M_t^a + M_t^b = M_t.$$

Consider first an odd period in which agent a makes a withdrawal. In such a period, a exhausts his initial money holdings M_{t-1}^a to finance period t consumption

$$(8) \qquad P_t C_t^a = M_{t-1}^a.$$

Since agent b did not go to the bank during this period, his final money holdings equal his initial money holdings less his period t expenditures

$$(9) \qquad M_t^b = M_{t-1}^b - P_t C_t^b.$$

Substituting (8) and (9) into the stock equilibrium condition for $t-1$,

$$(10) \qquad P_t C_t^a + M_t^b + P_t C_t^b = M_{t-1}.$$

Substitute the goods-market equilibrium condition (4) into the above to obtain

$$(11) \qquad M_t^b + P_t = M_{t-1}.$$

Finally, subtracting (11) from (5), we obtain

$$(12) \qquad M_t^a = P_t + (M_t - M_{t-1}) = P_t + \Delta M_t.$$

Equation (12) states that the flow of aggregate money withdrawals from the bank (in the case equal to M_t^a) must equal the flow of money into the bank. The latter consists of two terms. The first is the aggregate value of stores' receipts, P_t, which are assumed to be deposited instantly by stores into the interest-earning accounts. The second term, ΔM_t, is the change in aggregate nominal money engendered by government open market operations.

Given that aggregate money withdrawals in each period $t \geq 2$ equal $P_t + \Delta M_t$, the money-stock equilibrium condition may be expressed as

$$(13a) \quad P_t + \Delta M_t + P_{t+1}\phi(P_t/P_{t+1}, (P_{t-1} + \Delta M_{t-1})/P_{t+1}) = M_t \qquad \text{for } t \geq 2,$$

$$(13b) \quad P_1 + \Delta M_1 + P_2\phi(P_1/P_2, M_0^b/P_2) = M_0^a + M_0^b + \Delta M_1.$$

Equation (13) is a nonlinear second-order difference equation in P_t, with only one initial condition. However, if $u(C)$ is homothetic and $\phi(\ )$ is increasing in its first argument, it may be shown (Grossman, 1982) that, given the initial distribution of money holdings and the time path of aggregate money supply, there exists only one price path satisfying (13).

Considerable simplification is possible for the case of logarithmic utility. In this case, nominal spending depends only on nominal money withdrawals, independent of prices. Equation (13) becomes

$$(13') \quad P_t + \Delta M_t + \bar{\phi}(P_{t-1} + \Delta M_{t-1}) = M_t.$$

This is a first-order difference equation whose solution is given by

$$P_t = (-\bar{\phi})^{t-2} P_2 + \sum_{j=1}^{t-2} \left[M_{j+1}(1-\bar{\phi}) + \bar{\phi} M_j \right] (-\bar{\phi})^{t-2-j}, \qquad \text{for } t > 2,$$

and initial conditions given by

$$P_1 = M_0 - M_0^b \bar{\phi}, \quad P_2 = M_1(1-\bar{\phi}) + \bar{\phi}^2 M_0^b.$$

Two-period nominal interest rates can be derived as a function of the price path and path of nominal money withdrawals as follows. Consider the choice of optimal money withdrawals of an agent at the bank at $t-1$. At an interior optimum the agent is indifferent between withdrawing an extra \$1 and spending it in period t, or letting the \$1 grow to $\$R_{t-1}R_t$ in period $t+1$ and spending this amount in $t+2$. This gives rise to the first-order condition

$$(14)\quad u'(C_t) = \beta^2 u'(C_{t+2}) \frac{P_t}{P_{t+2}} R_{t-1} R_t.$$

Market clearing implies that the consumption at t of someone who withdrew at $t-1$ is

$$(15)\quad P_t C_t = P_{t-1} + \Delta M_{t-1} - P_{t+1}\phi(P_t/P_{t+1}, (P_{t-1} + \Delta M_{t-1})/P_{t+1}).$$

Substituting (15) in (14) allows the calculation of all two-period interest factors. For the case of logarithmic utility, these are given by

$$(16)\quad R_{t-1}R_t = \beta^{-2} \frac{\Delta M_{t+1} + P_{t+1}}{\Delta M_{t-1} + P_{t-1}}.$$

Hence, two-period interest rates are determined by the "shadow prices" necessary to sustain the real allocation previously determined. Thus, given a particular R_1, the entire sequence of one-period interest rates may be found. The equilibrium value of R_1 is determined by the need to satisfy the budget constraint (3). Let $X_t = R_t R_{t+1}$. Then the present value of consumer a expenditure is given by

$$(17)\quad P_1 C_1^a + P_2 C_2^a + P_3 C_3^a + \frac{P_4 C_4^a + P_5 C_5^a}{x_1} + \frac{P_6 C_6^a + P_7 C_7^a}{x_1 x_3} + \dots,$$

which is clearly determined by nominal prices and two-period interest rates. Assume consumer a's wealth emanates from ownership of a fraction s^a of the stores' revenue which is deposited into interest-earning accounts every period. The consumer's wealth is $W_0^a + M_0^a$, where W_0^a is given by

$$(18)\quad W_0^a = s^a\{P_1 + P_2/R_1 + P_3/x_1 + P_4/R_1 x_2 + P_5/x_1 x_3 + P_6/R_1 x_2 x_4 + \cdots\} + B_0^a - B/2.$$

Thus, there exists a unique R_1 such that budget balance occurs for consumer a. Since markets clear at these prices, it is obvious that agent b's budget constraint is also satisfied.

To verify that we have in fact found an equilibrium, it is necessary to show that all nominal interest rates are positive and consumers will want to exhaust their initial money balances. A sufficient condition for the latter is

$$u'(C_1) \geq \beta u'(C_2) P_1/P_2$$

for both consumers. In the next section we will verify that these conditions are satisfied in the steady state with constant money, and hence for "small" perturbations in money supply about this reference path.

III. The Steady State

We first analyze a timeless steady state with constant money supply, prices, and interest rates. Equilibrium condition (13) implies

$$(19)\quad P + P\phi(1,1) = M.$$

Thus, comparing alternative steady states, money is neutral in the sense that a proportional increase in total money raises prices by the same proportion. The cross-sectional distribution of money holdings is found by noting that the agent who has just gone to the bank has P dollars, and the agent who will go next period has $P\phi(1,1)$. If $u'(0) = \infty$, then it is clear that $1 > \phi(1,1) > 0$ so that the agent who has just gone to the bank has more money than one who went last period.

In the steady state with constant prices, the two-period nominal interest rate factors (equation (16)) equal β^{-2}. The one-period interest rate is not determined by any "marginal" conditions. The interest rate R_1 will determine all one-period rates using $R_t R_{t+1} = \beta^{-2}$. The initial interest rate R_1 can be determined from the cross-sectional distribution of wealth. Thus, to avoid a "sawtooth" pattern of one-period interest rate factors, we must choose an initial distribution of wealth which will give rise to a constant one-period interest rate of β^{-1}. It may be verified that for this initial distribution of wealth, the wealth of consumers who are at the bank is constant over time. We now derive consequences of $R_t = \beta^{-1}$ for s^a and B^a.

Consider the present value of expenditures of a type a consumer (one who will exhaust in the first period). This is given by

$$(20)\quad P_1C_1 + \sum_{i\ \text{even}} \frac{P_iC_i + P_{i+1}C_{i+1}}{R^i} = P\phi(1,1) + P\frac{1}{1-\beta^2}.$$

The wealth of a is

$$(21)\quad M_0^a + s^a \sum_{i=1}^{\infty} \frac{P}{R^{i-1}} + B^a - \frac{B}{2} = P\phi(1,1) + s^a P\frac{1}{1-\beta} + B^a - \frac{B}{2}.$$

Equating (20) and (21), budget balancing implies

$$(22)\quad B^a - \frac{B}{2} = P\left[\frac{1}{1-\beta^2} - s^a\frac{1}{1-\beta}\right].$$

For example, if $B^a = B/2$, then $s^a = 1/(1+\beta) > 1/2$. Alternatively if $s^a = s^b = 1/2$, then $B^a - B/2 = P/2(1+\beta) > 0$.

These calculations show that consumers who hold more money at the end of a period will necessarily hold less interest-earning assets than an agent with a symmetric consumption profile and lower money holdings. When agents make money withdrawals, they implicitly borrow wealth from those agents not currently at the bank. Next period, the receipts accruing at the bank are sufficient to repay these borrowings and convert this agent from a net debtor to a net creditor. In this way, the one-period interest rate serves only to redistribute wealth between agents currently making withdrawals and those agents who are not at the bank and have deposits credited to their accounts this period.

IV. An Open Market Operation

We consider the effects of a once-and-for-all increase in the stock of nominal money brought about by an open market operation, given that the initial distribution of money and interest-earning wealth corresponds to the steady state derived in the previous section. For the initial situation to correspond to the steady state, the open market operation must have been unanticipated in previous periods. Hopefully, this experiment captures the qualitative properties of an unanticipated permanent change in the money supply brought about by an open market operation in stochastic environments where agents foresee the possibility of this random event.

An open market operation is an exchange of money for a bond. Consider a central bank purchase of a government bond for fiat money at $t=1$. In the new equilibrium, the agent who goes to the bank during this period (agent a) must be induced to withdraw the whole of increase, which from equation (12) yields

$$(23)\quad M_1^a = P_1 + \Delta M_1.$$

This implies that nominal money withdrawals of an agent at the bank must rise by greater percentage than nominal prices; it is not possible for real magnitudes to remain invariant to monetary changes. The basic reason for this is that people at the bank at the time of the operation must be induced to hold the whole of the monetary injection, and thus a disproportionate share of the stock. Those consumers not at the bank cannot increase total nominal spending before returning to the bank; they respond to increased prices by reducing real consumption.

These points can be made transparent by examining the case of logarithmic utility. The equilibrium price path for the case when $\Delta M_1 = k\overline{M}$ is (from (13′))

$$(24)\quad P_t = (1+k)P + (-\bar{\phi})^{t-2}(P_2 - (1+k)P), \qquad t \geq 3,$$

where $P = (1+\bar{\phi})^{-1}\overline{M}$, the initial steady-state price level, $P_1 = P$, and $P_2 = (1+\bar{\phi})^{-1} \bar{M} + k\bar{M}(1-\bar{\phi}) < (1+k)P$.

From equation (24) it can be seen the initial price is unaffected by open market operations, since with logarithmic utility, nominal spending depends only on the initial distribution of money, independent of current and future prices. The second-period prices rise, but by less than the percentage change in nominal money. This arises because the new money withdrawn by a is spent gradually. By the third period, prices actually rise by more than their long-run value. This arises because period-two money withdrawals have risen by less than k percent. Hence the agent who exhausts in period three must hold more than the steady-state value. Since the agent who exhausts has a higher marginal propensity to spend out of nominal balances (equal to one) than an agent with two periods to exhaust (equal to $1-\bar{\phi}$), total nominal spending and hence, with fixed output, prices must rise by more than the steady-state value. This argument repeats itself with prices more than k percent higher than the old steady state at the odd dates and less than k percent higher on the even dates. These oscillations are damped and the price level converges to the new steady-state level of $(1+k)P$.

The effects of an open market operation on two-period nominal interest rates is given by equation (16). The initial two-period nominal rate is

$$(25)\quad \beta^2 R_1 R_2 = P_3/(P_1+\Delta M) = \frac{P((1+k)(1+\bar{\phi}))-\bar{\phi}P_2}{P(1+k(1+\bar{\phi}))} = \frac{P(1+k(1+\bar{\phi}))+\bar{\phi}(P-P_2)}{P(1+k(1+\bar{\phi}))} < 1,$$

and the initial two-period real rate is

$$(26)\quad \beta^2 R_1 R_2 \frac{P_1}{P_3} = \frac{P_1}{P_1+\Delta M} < 1, \qquad \text{when } k>0.$$

Thus both real and nominal two-period interest rates fall with an increase in money achieved by an open market operation. This fall is necessary to induce agents at the bank to withdraw and spend more money.

The persistent effects of an open market operation on nominal prices will affect the course of nominal interest rates. From (16), two-period nominal interest rates are determined by

$$(27)\quad \beta^2 R_t R_{t+1} = P_{t+2}/P_t, \qquad \text{for } t>1.$$

Substituting from (24), it can be seen that two-period interest rates are higher than the steady-state value on the even dates, and less than the steady-state value on the odd dates. As P_t converges to $(1+k)P$, $\beta^2 R_t R_{t+1}$ converges to its old value of unity.

Finally, the initial interest rate $R_1 - 1$ must fall in response to an expansion in the money supply. As was shown previously, the initial interest rate is chosen so that the wealth constraint holds for both agents. An open market operation permanently increases the welfare of consumers currently at the bank. This will appear as a drop in the interest rate in periods when these agents are borrowers, and a rise in the interest rates when they were lenders.

For the case of logarithmic utility this may be calculated directly. Use (25) and (26) to show

(28)

$$R_t = \frac{P_{t+1}}{P_t} R_1 \frac{P_1+\Delta M}{P_2} \qquad t = 3,5,7,\ldots,$$

$$R_t = \frac{P_{t+1}}{P_t} \frac{1}{\beta^2 R_1} \frac{P_2}{P_1+\Delta M} \qquad t = 2,4,6,\ldots.$$

The present value of consumer a's expenditure in the new equilibrium is given by

$$(29)\quad P_1 C_1^a + \sum_{t \text{ even}} \frac{M_{t-1}^a}{\alpha_{t-1}} = P_1 C_1^a + \Delta M + P_1 + (P_1+\Delta M)\frac{\beta^2}{1-\beta^2}.$$

In the new equilibrium, each person's tax liability is $(B+\Delta B)/2$. The wealth of consumer a is given by

$$(30)\quad M_0^a + B^a - \frac{B+\Delta B}{2} + s^a \sum_{t=1}^{\infty} \frac{P_t}{\alpha_t}$$

$$= M_0^a + \frac{\Delta M}{2} + s^a \left\{ P_1 + \frac{P_2}{R_1}\frac{1}{(1-\beta^2)} + (P_1 + \Delta M)\frac{\beta^2}{1-\beta^2} \right\}.$$

Equating the right-hand sides of equations (29) and (30), and using (24) permits us to compute R_1. If $\Delta M > 0$, it may be shown that $R_1 < \beta^{-1}$, its old steady-state value.[3] Since $P_2 > P_1$, the one-period real rate also falls.

It may be shown (Grossman, 1982) that damped oscillatory price behavior with overshooting after two periods and a fall in the current nominal and real rate in response to an open market purchase is not limited to logarithmic utility. If $u(C)$ is homothetic and $\phi(\ )$ is increasing in its first argument, these properties will also hold.

V. Conclusion

The principal analytic result of this paper is that open market operations can have real effects. We have shown that a monetary expansion will lead to a temporary reduction in both real and nominal interest rates and lead to a gradual increase in prices. The model gives analytic support to the notion that money matters in the short run, but not in the long run when prices adjust proportionally to money changes. These conclusions are similar to those of traditional Keynesian analysis which, unlike our model, assumes some sort of short-run price stickiness. However, in our model the distinction between "short" and "long" run is that the distribution of money holdings is taken as exogenous in the short run, but endogenous in the long run.

The model has emphasized how a change in the money supply affects the spending decision of those agents making withdrawals at the time of an open market operation. Considerations of intertemporal substitution imply that the real rate must decline to induce these agents to consume more. Because this new money is spent gradually, prices will rise slowly and reach their steady-state level long after the interval of time between trips to the bank. A natural question is how long this transition period is likely to be. A "period" in our model corresponds to one-half the length of time between trips to the bank for a representative consumer. In the United States, average money holding is sufficient to purchase about 1/7 (the inverse of income velocity) year's worth of *GNP*. If spending occurs at a constant rate, then the average holding of money is exactly one-half the total expenditures between trips to the bank, so that the representative consumer goes to the bank every 2/7 of a year, or about fifteen weeks. Hence the length of a period is seven and one-half weeks. In our model, prices first reach (and exceed) their steady-state level after two periods, so the model indicates that the transition period during which prices rise in response to a monetary injection is about fifteen weeks. This empirical issue is complicated by the fact that the monetary intermediation channels in the actual economy are more intricate than those of our model. Our estimate is likely to underestimate the duration of the transition period to the extent that firms hold idle balances and do not instantly transmit their proceeds to the bank. However, this estimate is too long if some con-

[3]Assuming $s^a = 1/(1+\beta)$, and $-\Delta B = \Delta M = kM$, equating (29) and (30) yields

$$R_1 = \left[1 + k/(1+\beta^2)\right] \Big/ \left[\beta + k(1+2\beta)\left((1+\beta^2)/2 - \beta^2/(1+\beta)\right)\right].$$

Let $R_1 = \beta^{-1} + e$. Note that

$$\text{sign}(e) = \text{sign}\left(\beta + k\beta/(1+\beta^2) - \beta + k(1+2\beta) \times \left((1+\beta^2)/2 - \beta^2(1+\beta)\right)\right)$$

and $\text{sign}(e) = \text{sign}(k)\text{sign}(2\beta(1+\beta) - (1+2\beta)((1+\beta^2)(1+\beta-\beta^2+\beta^3))$. Hence $0<\beta<1$ implies $\text{sign}(e) = -\text{sign}(k)$.

sumers receive cash payments directly without having to make withdrawals, or if money is used to buy intermediate goods from firms which do not hold cash themselves, or if consumers make withdrawals before exhausting cash balances because of a precautionary demand not explicitly considered in the model.

There are other reasons to suspect that monetary inpulses will have a more delayed response than suggested by the model. If the time between trips to the bank were made endogenous, rather than the fixed interval assumed in the model, then it could be imagined that the decline in nominal interest rates would induce a longer time before the next return, as the cost of holding money goes down. In this case, the new money would be spent more gradually than the case presented in this paper, and the price rise would be slower and longer. Similarly, the rate of spending for the recipients of the new money would not have to rise as much; the decline in real rates would be lower than the model's conclusion.

Some of the model's conclusions are very different from those of earlier theories. The Clower cash-in-advance constraint makes current prices less sensitive to anticipation of future money than suggested by the analysis of Miguel Sidrauski (1967), which assumes that money provides services, much as a consumer durable. This is because in our model, the rise in current spending associated with a rise in anticipated inflation is limited by the cash-in-advance constraint. For example, in the extreme case of logarithmic utility, the current price level is unaffected by anticipated future monetary injections (see equation (13′)). This extreme result is a consequence not only of the assumption that the current money supply puts an upper bound on spending, but also on the assumption that the time between trips to the bank is fixed. A sufficiently large anticipated inflation will cause people to go to the bank sooner, and hence spending will become more sensitive to anticipated inflation.

Our model, where all consumers live forever, and in which bonds can coexist with money, should be contrasted with consumption-loan models of "money." In many of the consumption-loan models, money and bonds cannot coexist and what is called money could as easily be called "bonds" (see Truman Bewley, 1980, for an example of this approach, and references to other work which uses bonds and money interchangeably). This is to be contrasted with the approach of Jean-Michael Grandmont and Yves Younes (1972) which implicitly uses a Clower constraint in a consumption-loan model framework. However, they do not discuss the tradeoffs between bonds and money and the effects of an open market operation. Their model and others which use the Clower constraint such as Robert Lucas (1980), assume (implicitly) that all individuals engage in trade intermediated by money during a "period." They do not analyze what happens during the period. We emphasize that all individuals cannot be decreasing their money holdings at the same time during this period. A model in which bonds and money coexist without the assumption of a Clower constraint appears in John Bryant and Neil Wallace (1979), and Thomas Sargent and Wallace (1982). Jovanovic considers a general equilibrium transaction demand for money model very close to the one we consider. However, he only analyzes steady states, and helicopter monetary injections.

The fact that people hold money for the sole purpose of spending it implies that money will flow through the economy from individuals to stores to banks and then back to individuals. A snapshot of the economy will reveal some consumers who have just made a withdrawal—thus holding a large amount of money, and some customers who are about to make a withdrawal—thus holding a small amount of money. The fact that money flows is the necessary dynamic counterpart of the fact that, at an instant of time, the cross-sectional distribution of money holdings must not be degenerate. This feature distinguishes our model, and is the source of the dynamic effect on prices and interest rates which we show to be a necessary consequence of an open market operation.

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