HW 2

1) Consider the data on performance of mutual funds, Eg 3 from Handout 3. “Past Loser” indicates that the fund lost value over some period in the past (as specified in Goetzmann and Ibbotson’s study.) “Current Loser” indicates that the fund lost value in a more recent period. Let $A=\{\text{Past Loser}\}$, and $B=\{\text{Current Loser}\}$. Based on these data, calculate the following probabilities, and write in words what these probabilities represent.

A) Find $P(A)$, $P(B)$ and $P(A \cap B)$.
B) Are the events $A$ and $B$ independent?
C) Find $P(B | A)$ and $P(B | \bar{A})$. Are they equal?
D) Show that $P(B) P(B | A) \neq P(\bar{B} | A)$.

2) Based on past data, suppose that the probability of a failure (at least a partial one) of a Falcon 9 rocket for a given launch is $2/124 = 0.016$.

A) What is the probability that at least one of the next six Falcon 9 launches will result in a failure?
B) What is the probability that at least one of the next ten Falcon 9 launches will result in a failure?

3) Suppose the probability that a child born in the US in 2020 will survive past age 80 is 40%, and the probability that he/she will survive past age 90 is 20%. For an 80-year-old who was born in the US in 2020, what is the probability of surviving past age 90?

4) **Empirical Rule for Sum of Two Dice.** Suppose you throw two dice. Each die can come up as 1,2,3,4,5 or 6, and the results from the two dice are independent of each other. We are interested in the random variable $X$, the sum of the two numbers that land face up. The possible values for $X$ are 2,3,...,12.

A) Make a table giving the probability distribution of $X$. Explain briefly how you did the calculations.
B) Show that $E[X]=7$, and $\text{Var}[X]=210/36=5.833$.
C) Although the distribution of $X$ is not a normal distribution, a graph of it would look somewhat bell-shaped. (This is not a coincidence. The more dice you toss, the closer the distribution of the sum comes to a normal distribution. More on this later in the course.) For now, let’s see how well the empirical rule works. Show that the probability that the $Z$-score for $X$ is between $-1$ and 1 is $24/36=0.667$. Show that the probability that the $Z$-score for $X$ is between $-2$ and 2 is $34/36=0.944$.

5) **Odds.** Odds are often used to describe the payoff for a bet. Consider horseracing, for example. If the odds for a horse are $a:b$, then the bettor must risk $b$ dollars to make a
profit of $a$ dollars. In the 2021 Kentucky Derby, the odds for Medina Spirit were 12.10 to 1, so a $2$ ticket returned $26.20. This represents twice the odds plus the original $2$ bet. The odds given by casinos racetracks and bookmakers are usually set so that the house has an advantage. For any bet, if the true probability of winning is $b/(a+b)$ then this is a fair bet in the sense that the bettor’s expected profit is zero.

A) If a fair coin is tossed once, what are the odds for Heads? (That is, the fair odds.)
B) If a fair die is tossed once, what are the odds of a four?

6) **Odds bet in craps.** In the casino game of craps, two dice are thrown repeatedly and we are interested in the sum for each roll. The fair odds of a six being thrown before a 7 are 6:5. The casino will pay these fair odds if you make an “odds” bet (which I believe makes this the only fair bet in any casino game), as opposed to the “place” bet on the same proposition which pays at the unfair odds of 7:6.

A) What would an odds bet of $5$ return? (This includes the original $5$ bet.)
B) What would a place bet of $6$ return?
C) Show directly that the probability of throwing a 6 before a 7 is 5/11. To do this, first note that there are 36 equally likely outcomes in throwing two dice, for example, (1,6) and (6,1). Next, determine the probability distribution of the sum of the two dice. One final hint: the probability of throwing a 6 before a 7 is the same as $P(A|B)$ where $A=\{6 \text{ on next roll}\}$ and $B=\{6 \text{ or 7 on next roll}\}$.
D) Now, use this probability to calculate the expected profits for a $5$ “odds” bet and a $6$ “place” bet on a 6 being thrown before a 7. Note that for both the “odds” bet and the “place” bet the probability of winning is the same, but the payoffs are different. Why is the “odds” bet fair, and the “place” bet unfair?

7) **Roulette Doubling (Martingale) System.** Roulette wheels in casinos in the US have 38 numbers, of which two are green (0 and 00), 18 are black and 18 are red. A bet on black pays at even money, 1:1 odds (though these odds are clearly not fair due to the green numbers.) Each of the 38 numbers is equally likely to occur on any given spin of the wheel, and results from successive spins are independent. (Casinos expend considerable effort to ensure that these properties hold; otherwise, gamblers would have opportunities for arbitrage.) Let’s consider a doubling system, at a table with a $100$ minimum bet, and a $100,000$ maximum bet. In terms of $100$ chips, that’s a $1$-chip minimum and a 1000-chip maximum. To start the system, bet $1$ chip on Black. If you win, you’re up $100$ and that’s the end of the system. If you lose, bet $2$ chips on Black. If you win at this point, you’re once again up $100$ (having lost one chip and then won two chips), and that’s the end of the system. If you lose, bet $4$ chips on Black. Continue doubling if you lose. Once you finally win (no matter how long this takes), you will be up $100$, since successive powers of 2 add up to one less than the next power of two, for example, $1+2+4+8=15=16−1$. So as long as you can keep playing until the first time Black is rolled, you will win your $100$. So, what’s the catch? Unfortunately, the table maximum of 1000 chips eventually becomes a problem, since if you lose 10 times in a row, you will be down $1+2+4+8+16+32+64+128+256+512=1023$ chips, and you will not be allowed to
make the next bet, since a 1024-chip bet would exceed the table limit. Suppose you play this system just once, until either you get your $100 profit, or you spectacularly go bust with a losing streak of 10 non-Black numbers. Compute the expected net winnings (profit minus loss) for the system, in Dollars.

8) Suppose a 40-year-old male purchases a $100,000 10-year term life policy from an insurance company, meaning that the insurance company must pay out $100,000 if the insured male dies within the next 10 years.
A) Use the accompanying life table to determine the insurance company’s expected payout on this policy. (Hint: Remember that your universe here is the set of males 40 and older).
   The age intervals in the table contain all ages from the lower limit up to (but not including) the upper limit.
B) What would be the expected payout if the same policy were taken out by a 50-year-old male?

<table>
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<th>Age Interval</th>
<th>0-1</th>
<th>1-10</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80 and Over</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>Number of Deaths</td>
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<td>495</td>
<td>927</td>
<td>1,901</td>
<td>2,105</td>
<td>4,502</td>
<td>10,330</td>
<td>19,954</td>
<td>28,538</td>
<td>29,721</td>
<td>100,000</td>
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