HW 7

1) Consider (again) the time it takes for a call center to answer its calls. The call center claims that the mean time to answer a call is no more than 3 minutes. In a random sample of 7 calls, the average time for the call center to answer was 191 seconds, with a sample standard deviation of 11.4 seconds.

A) Provide the null and alternative hypotheses. Justify your choice for the alternative hypothesis.
B) Test the call center’s claim, at the 1% level of significance.
C) Would the conclusion change if we somehow knew that the population standard deviation were 11.4 seconds? (In reality, of course we will not know the population standard deviation, but this question helps us to understand the impact of not knowing it.)

2) Recall the study of 80 students who used a private tutor to help them improve their SAT scores. Their score on the mathematical section improved by an average of 11 points, with a sample standard deviation of 65 points.

A) Is there evidence, at the 5% level of significance, that tutoring improves the math score?
B) Compute the p-value corresponding to the hypothesis test. Interpret the p-value. What does it tell us about whether tutoring improves the math score?

3) Use the data in Market to test whether IBM has a lower mean return than the market. To do this, first use Calculator to create a variable called IBMedge, defined as IBMRet − MarketReturn.

A) Formulate the null and alternative hypotheses.
B) Run descriptive statistics for IBMedge to do the hypothesis test, by hand. Use the 5% level of significance.
C) Compute the p-value corresponding to the t-statistic, and interpret it in the context of the question as to whether IBM is beaten by the market.
D) Now, let Minitab do the hypothesis test using Stat → Basic Statistics → 1-Sample t, One or more samples, each in a column: IBMedge, perform hypothesis test. (You need to specify the null hypothesis, and you may need to use options to set the confidence level [100(1−significance level)], and the alternative hypothesis.) Get the p-value from Minitab. Compare with your answer in C).
E) Does the fact that stock returns are not normally distributed have any impact on the validity of the t-test and corresponding p-value? Explain.

4) A study was done in Finland of twins where one twin smoked and the other didn’t. There were 22 pairs of twins where at least one twin had died by the end of the study. In 17 of these 22 cases, the smoker died first. Compute the p-value to determine the strength of the evidence that smoking reduces life expectancy. To do this, first determine the distribution of X, the number of cases where the smoker dies first, under the null hypothesis that smoking has no effect on life expectancy. Then calculate the probability that if the null hypothesis were true, X would be at least 17.
To do this, use Minitab to get \( \text{Prob}(X \leq 16) \) with Calc \( \rightarrow \) Probability Distributions \( \rightarrow \) Binomial, Cumulative Probability, Number of Trials: 22, Event probability: (you figure it out), Input Constant: 16. Once you have your \( p \)-value, interpret it.

5) Recall the data set NormTemp. The first column (Temp) contains the body temperatures of 130 randomly selected subjects. Use Minitab’s one-sample \( t \) to get the \( p \)-value corresponding to the null hypothesis that the mean temperature is 98.6 degrees Fahrenheit. Interpret the \( p \)-value and explain why it is appropriate to use a two-sided alternative hypothesis here. In the end, does the 98.6 “normal temperature” seem to be folklore or fact?

6) In this and the next two problems, we will use Minitab to do hypothesis testing in settings where there are two independent samples. Here, we consider the two-sample \( t \)-test, for the data set NormTemp. The second column (Gender) is 1 for male, 2 for female, and the third column (HeartRate) is measured in beats per minute.

A) Make side-by-side boxplots for the temperatures of males and females. Do there seem to be any differences?

B) Use the two-sample \( t \)-test to formally test for a difference in mean body temperature between males and females. The null hypothesis is that \( \mu_1 - \mu_2 = 0 \), where \( \mu_1 \) is the population mean temperature for males, and \( \mu_2 \) for females. We won’t worry about how to calculate the test statistic or the resulting \( p \)-value. Instead, we will simply let Minitab do it for us. Use the commands Stat \( \rightarrow \) Basic Statistics \( \rightarrow \) 2-Sample \( t \), Both samples are in one column, Samples: Temp, Sample IDs: Gender. Use Options \( \rightarrow \) Alternative hypothesis to select the alternative hypothesis, in this case “Difference \( \neq \) hypothesized difference”. Do not check the “Assume equal variances” box. (In general, I recommend not checking this box, since it’s an overly restrictive assumption). Based on the 95% CI, is there evidence that the mean temperature depends on gender? Explain. For the \( t \)-test, state the null and alternative hypotheses in terms of body temperatures. Based on the Minitab \( t \)-test output, can the null hypothesis be rejected at level .05? At level .01? Give an interpretation of the \( p \)-value.

C) Repeat parts A) and B) for HeartRate.

7) Recall the study by Goetzmann and Ibbotson on performance of mutual funds (Handout 3). Let’s do a hypothesis test to compare two binomial probabilities: \( \text{Prob} (\text{Current Winner} | \text{Past Loser}) \) and \( \text{Prob} (\text{Current Winner} | \text{Past Winner}) \). Use Stat \( \rightarrow \) Basic Statistics \( \rightarrow \) 2 Proportions, Summarized Data, First Events: 285, First Trials: 778, Second Events: 482, Second Trials: 778, Options \( \rightarrow \) Alternative \( \rightarrow \) (Decide what you want here. The null hypothesis is that the first probability minus the second probability is zero, and the three alternatives given by Minitab also refer to the first probability minus the second probability.) For Test Method in Options, use “Estimate the proportions separately.” Provide the output from Minitab on the two sample proportions and the \( p \)-value for the Fisher Exact Test. Based on this, does it seem
plausible that the difference between the two sample proportions is due to chance alone?

8) We will look here at the results of a very large trial of an HIV vaccine, funded by the U.S. National Institutes of Allergy and Infectious Diseases and the U.S. Army Medical Research and Materiel Command. The trial was conducted on 16,400 people in Thailand, all of whom were HIV negative at the start of the trial. Half of the people received a placebo, and half received the vaccine (actually, a combination of six vaccines). Both groups were followed for three years afterwards. Of the 8,200 who received the vaccine, 51 developed HIV. Of the 8,200 who received the placebo, 74 developed HIV. Proceed as in problem 8 above to perform a hypothesis test using Minitab's 2 Proportions.

A) Formulate the null and alternative hypotheses.
B) Based on the \( p \)-value from the Fisher Exact test, was the effect of the vaccine statistically significant at level 0.05? At level 0.01?
C) If the vaccine were actually ineffective, how likely would we be to observe such a big difference in results between the vaccine and placebo as we actually observed in the study?
D) In computing the \( p \)-value above, we are focusing on the given study in isolation. In fact, there have been several similar previous studies. What does that fact do to the probability of finding at least one study with such a small \( p \)-value as we got here, even if the vaccine is actually ineffective?