HOMEWORK 6

1) For the moving average $X_t = \sum_{j=-\infty}^{\infty} a_j \varepsilon_{t-j}$ show that $var\ X_t = \sigma^2 \sum_{j=-\infty}^{\infty} a_j^2$, where $\sigma^2 = var\ \varepsilon_t$.

2) Show that if $\{\varepsilon_t\}$ has spectral density $f_\varepsilon(\lambda) = \frac{\sigma^2}{2\pi}$ then $\{\varepsilon_t\}$ is a white noise process with $var\ \varepsilon_t = \sigma^2$.

3) Find a moving average representation for the time series $\{X_t\}$ having spectrum $f_X(\lambda) = \frac{\sigma^2}{2\pi}(1+\cos\lambda)^2$. Is your representation one-sided?

4) Show that the autoregression $X_t + .5X_{t-1} = \varepsilon_t$ exists. What is the spectral density of $\{X_t\}$? Find the one-sided moving average representation of $\{X_t\}$.

Problems 5-8 concern the process

$$X_t - 1.1X_{t-1} = \varepsilon_t \quad , \qquad (1)$$

where $\{\varepsilon_t\}$ is zero-mean white noise.

5) Show that the autoregression (1) does not exist. In other words, there is no weakly stationary process $\{X_t\}$ satisfying (1), and

$$E[X_s \varepsilon_t] = 0 \text{ for all } s < t \quad . \qquad (2)$$

6) For any $t > 1$, show that any series $\{X_t\}$ satisfying (1) can be written as $X_t = \sum_{k=0}^{t-1}(1.1)^k \varepsilon_{t-k} + (1.1)^t X_0$. Since the MA weights $(1.1)^k$ increase with k, it appears that X_t depends more strongly on the distant past than on the recent past.

7) Suppose $x_0 = 0$, and that the $\{\varepsilon_t\}$ are *iid* standard normal. Use Equation (1) to simulate $\{x_t\}_{t=1}^{100}$ in Splus. Plot the resulting time series. Because of the appearance of this plot, and the results of Problem 6, this process (and any process with a root inside the unit circle) is usually referred to as *explosive*.

8) Prove that

$$\frac{1}{1-1.1\,e^{-i\lambda}} = \left[\frac{-e^{i\lambda}}{1.1}\right]\left[\frac{1}{1-(1/1.1)e^{i\lambda}}\right] .$$

Then use the spectral representation to show that the process $\{X_t\}$ given by (1) can be written as $X_t = \frac{-1}{1.1}\sum_{k=0}^{\infty}(1.1)^{-k}\varepsilon_{t+k+1}$. Thus, when expressed in terms of *future* innovations, $\{X_t\}$ is not explosive. Show that condition (2) (given in Problem 5) does not hold for this process, however. (Many books refer to a process whose autoregressive polynomial has one or more roots inside the unit circle as nonstationary. We have shown here that this terminology is incorrect.)