HW 4

1) Compute the expected value ($\mu$) and the standard deviation ($\sigma$) for the discrete random variable $X$ that takes values 1, 2, 4 with probabilities 0.2, 0.4, 0.4, respectively.

2) **Empirical Rule for Sum of Two Dice.** Suppose you throw two dice. Each die can come up as 1, 2, 3, 4, 5 or 6, and the results from the two dice are independent of each other. We are interested in the random variable $X$, the sum of the two numbers that land face up. The possible values for $X$ are 2, 3, ..., 12.

   A) Make a table giving the probability distribution of $X$. Explain briefly how you did the calculations.

   B) Show that $E[X]=7$, and $\text{Var}[X]=210/36=5.833$.

   C) Although the distribution of $X$ is not a normal distribution, a graph of it would look somewhat bell-shaped. (This is not a coincidence. The more dice you toss, the closer the distribution of the sum comes to a normal distribution. More on this later in the course.) For now, let’s see how well the empirical rule works. Show that the probability that the $Z$-score for $X$ is between $-1$ and $1$ is $24/36=0.667$. Show that the probability that the $Z$-score for $X$ is between $-2$ and $2$ is $34/36=0.944$.

3) **Odds.** Odds are often used to describe the payoff for a bet. Consider horseracing, for example. If the odds for a horse are $a:b$, then the bettor must risk $b$ dollars to make a profit of $a$ dollars. In the 2009 Kentucky Derby, the odds for Mine That Bird were 50.60 to 1, so a $2$ ticket returned $103.20$. This represents twice the odds plus the original $2$ bet. The odds given by casinos racetracks and bookmakers are usually set so that the house has an advantage. For any bet, if the true probability of winning is $b/(a+b)$ then this is a fair bet in the sense that the bettor’s expected profit is zero.

   A) If a fair coin is tossed once, what are the odds for Heads? (That is, the fair odds.)

   B) If a fair die is tossed once, what are the odds of a four?

4) **Odds bet in craps.** In the casino game of craps, two dice are thrown repeatedly and we are interested in the sum for each roll. The fair odds of a six being thrown before a 7 are 6:5. The casino will pay these fair odds if you make an “odds” bet (which I believe makes this the only fair bet in any casino game), as opposed to the “place” bet on the same proposition which pays at the unfair odds of 7:6.

   A) What would an odds bet of $5$ return? (This includes the original $5$ bet.)

   B) What would a place bet of $6$ return?
C) Show directly that the probability of throwing a 6 before a 7 is 5/11. To do this, first note that there are 36 equally likely outcomes in throwing two dice, for example, (1,6) and (6,1). Next, determine the probability distribution of the sum of the two dice. One final hint: the probability of throwing a 6 before a 7 is the same as \( P(A|B) \) where \( A = \{ \text{6 on next roll} \} \) and \( B = \{ \text{6 or 7 on next roll} \} \).

D) Now, use this probability to calculate the expected profits for a $5 “odds” bet and a $6 “place” bet on a 6 being thrown before a 7. Note that for both the “odds” bet and the “place” bet the probability of winning is the same, but the payoffs are different. Why is the “odds” bet fair, and the “place” bet unfair?

5) What is the probability of obtaining 4 of a kind in a 5-card poker hand dealt from a fresh deck? (An example of 4 of a kind is 4 kings, together with one non-king).

6) For the New York 6/51 Lotto discussed in class, show that the probability of winning Fourth Prize (matching 4 winning numbers and the bonus number) is 1/27,287 and the probability of winning Fifth Prize (matching 4 winning numbers but not the bonus number) is 1/1,269.

7) Read the attached Mr. Lotto article on the New York Numbers from the Daily News. In this game, three digits (from 0 to 9) are drawn at random, with replacement. We will verify some of the numbers given by Mr. Lotto.

A) In a “straight” bet, the player predicts the numbers, in order. Show that the probability of winning a straight bet is 1 in 1,000.

B) In a “combo” (combination, also called “boxed play”), you select the three numbers, and you win if these numbers come up in any order. Show that there are 120 no-match combos, that is, combinations where no digit is repeated.

C) Mr. Lotto says that no-match combos are drawn about 72% of the time. Show that in fact the probability that the number drawn will have no repeated digits is exactly 72%.

D) Show that there are 90 double-digit combos, such as 1-2-2, and 10 triple-digit combos, such as 1-1-1. Note that adding the number of no-match, double-digit and triple-digit combos gives 120+90+10=220. This explains Mr. Lotto’s claim that “The game features just 220 unique digit combinations.
8) Suppose a 40-year-old male purchases a $100,000 10-year term life policy from an insurance company, meaning that the insurance company must pay out $100,000 if the insured male dies within the next 10 years.

A) Use the accompanying life table to determine the insurance company’s expected payout on this policy. (Hint: Remember that your universe here is the set of males 40 and older).

The age intervals in the table contain all ages from the lower limit up to (but not including) the upper limit.

B) What would be the expected payout if the same policy were taken out by a 50-year-old male?

<table>
<thead>
<tr>
<th>Age Interval</th>
<th>Number of Deaths</th>
</tr>
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<tbody>
<tr>
<td>0-1</td>
<td>1,527</td>
</tr>
<tr>
<td>1-10</td>
<td>495</td>
</tr>
<tr>
<td>10-20</td>
<td>927</td>
</tr>
<tr>
<td>20-30</td>
<td>1,901</td>
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<tr>
<td>30-40</td>
<td>2,105</td>
</tr>
<tr>
<td>40-50</td>
<td>4,502</td>
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<tr>
<td>50-60</td>
<td>10,330</td>
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<tr>
<td>60-70</td>
<td>19,954</td>
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<tr>
<td>70-80</td>
<td>28,538</td>
</tr>
<tr>
<td>80 and Over</td>
<td>29,721</td>
</tr>
<tr>
<td>Total</td>
<td>100,000</td>
</tr>
</tbody>
</table>

9) A multiple-choice quiz has 20 questions. Each question has five possible answers, of which only one is correct.

A) What is the probability that sheer guesswork will yield at least 10 correct answers?

B) What is the expected number of correct answers by sheer guesswork?

C) Suppose 5 points are awarded for a correctly answered question. How many points should be deducted for an incorrectly answered question, so that for a student guessing randomly, the expected score on a question is zero? (Most standardized tests use this method to set penalties for guessing).

D) If a student is able to correctly eliminate one option as a possible correct answer but is still guessing randomly, what happens to his/her expected score for that question? Use your answer to C) as the number of points being deducted for an incorrect answer.

10) A motel has 15 bedrooms. From past experience, the manager knows that 20% of the people who make room reservations don’t show up. The manager accepts 20 reservations. If a customer with a reservation shows up and the motel has run out of rooms, it is the motel’s policy to pay $100 as compensation to the customer. What is the expected value of the compensation that the motel must pay?

11) Suppose that X and Y are random variables with Var(X)=1, Var(Y)=4 and Cov(X,Y)=1.5.

A) Evaluate the correlation, Corr(X,Y)

B) Evaluate Var(X+Y)

C) Evaluate Var(X−Y).
IF FAVORABLE ODDS is your aim, New York Numbers should be your game.

All you have to do to win is match the three digits drawn each night. The odds of getting all three numbers in the exact order drawn — a straight win — are 1 in 1,000. The payoff is $500 for a $1 bet.

But there are easier alternatives.

The game features just 220 unique digit combinations. Only 120 winning possibilities are no-match combos, such as 1-2-3. No-match combos are drawn about 72% of the time, or nearly three out of four days. If you hit with one of these, you win $80 for a $1 boxed play, a bet where your picks may be drawn in any order.

Twenty-seven percent of all New York Numbers combos are double-digit, such as 1-2-2. The game features 90 of these. They show up on average every fourth drawing and pay a healthy $160 per $1 bet.

Lastly, there are triple-digit combos, such as 1-1-1, which account for 1% of the wins and must be played straight. A triple-digit number should hit about once every 100 drawings.

Now that you know all the odds, you can select a type of bet along with the numbers you want to play.

Doubles may be a good bet if they haven't hit in two weeks.

Triples may be probabilities if they are nearing or past the 100-game mark without a hit.

And no-match combos are good plays most of the time.

Notable Numbers

305: These three digits haven't been drawn together in more than a year. Not since 11/21/86 has 305 been drawn, and 035 hasn't been drawn since 4/29/92.

553: Double 5's haven't been drawn in the last 62 days. 553 hasn't hit in any order — 535, 355 or 533 — since 2/21/95.

Daily News lottery columnist Stephen Allensworth is the publisher of the New York and New Jersey Weekly Lottery News, sold at most lottery agents. Send questions and comments to: Mr. Lotto, c/o New York Daily News, Box 3311, 450 W. 33rd St., New York, N.Y. 10001