Risk and Risk-Sharing in Two-Country Models

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Overview

Two paths to **variable Pareto weights**

- Capital market frictions
- **Recursive preferences**

Plan of attack

- Two-country model + recursive preferences
- Home bias in consumption, **stochastic volatility**

Intellectual debts

- Colacito and Croce, Kollmann, Tretvoll
- Anderson; Collin-Dufresne, Johannes, and Lochstoer

Recursive preferences

Recursive preferences

Time aggregator

$$U_{jt} \;=\; V[c_{jt}, \mu_t(U_{jt+1})] \;=\; [(1-\beta)c_{jt}^{\rho} + \beta\mu_t(U_{jt+1})^{\rho}]^{1/\rho}$$

Certainty equivalent function

$$\mu_t(U_{jt+1}) \;=\; \left[E_t(U_{jt+1}^{\alpha})\right]^{1/\alpha}$$

Features

- If $c_{jt} = c$ is constant $\Rightarrow$ $U_{jt} = c$
- V, μ both homogeneous of degree one (hd1)
- Intertemporal substitution: $\text{IES} = 1/(1-\rho) > 0$
- Risk aversion: $\text{RA} = 1 - \alpha > 0$
- Traditional **additive preferences** if $\alpha = \rho$

Recursive preferences (continued)

Intertemporal marginal rate of substitution

$$m_{jt+1} \;=\; \beta \left(\frac{c_{jt+1}}{c_{jt}} \right)^{\rho-1} \left(\frac{U_{jt+1}}{\mu_t(U_{jt+1})} \right)^{\alpha-\rho}$$

Epstein-Zin term is white noise plus risk adjustment

$$\begin{aligned}
\log U_{t+1} &= E_t(\log U_{t+1}) + \big[\log U_{t+1} - E_t(\log U_{t+1}) \big] \\
\log \mu_t(U_{t+1}) &= \alpha^{-1} \log E_t(e^{\alpha \log U_{t+1}}) \\
&= E_t(\log U_{t+1}) \\
&\quad + \alpha^{-1} \big[\log E_t(e^{\alpha \log U_{t+1}}) - E_t(\alpha \log U_{t+1}) \big]
\end{aligned}$$

Two-country model

Two-country model: technology

Production of intermediate goods

$$
\begin{aligned}
y_{jt} &= f(k_{jt}, z_{jt}) = \left[(1-\eta)k_{jt}^{\nu} + \eta z_{jt}^{\nu}\right]^{1/\nu} \\
y_{1t} &= a_{1t} + a_{2t} \\
y_{2t} &= b_{1t} + b_{2t}
\end{aligned}
$$

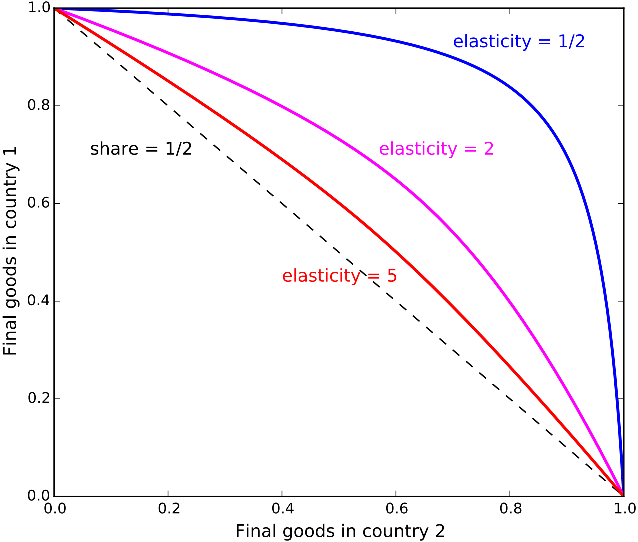
Armington aggregator for final goods

$$
\begin{aligned}
c_{1t} + i_{1t} &= h(a_{1t}, b_{1t}) = \left[(1-\omega)a_{1t}^{\sigma} + \omega b_{1t}^{\sigma}\right]^{1/\sigma} \\
c_{2t} + i_{2t} &= h(b_{2t}, a_{2t})
\end{aligned}
$$

Capital stocks

$$
k_{jt+1} = (1-\delta)k_{jt} + i_{jt}
$$

Armington aggregator: final goods frontier



Two-country model: shocks

Productivities

$$\begin{bmatrix} \log z_{1t+1} \\ \log z_{2t+1} \end{bmatrix} = \begin{bmatrix} 1-\gamma & \gamma \\ \gamma & 1-\gamma \end{bmatrix} \begin{bmatrix} \log z_{1t} \\ \log z_{2t} \end{bmatrix} + \begin{bmatrix} {v_t}^{1/2} w_{1t+1} \\ v^{1/2} w_{2t+1} \end{bmatrix}$$

Conditional variance ("volatility")

$$v_{t+1} = (1-\varphi_v)v + \varphi v_t + \tau w_{3t+1}$$

Innovations $\{w_{1t}, w_{2t}, w_{3t}\}$ independent standard normal

Pareto problem

Pareto problem

Bellman equation $[s_t = (k_{jt}, z_{jt}, v_t)]$

$$\begin{aligned} J(U_t, s_t) &= \max_{\{c_{1t}, U_{t+1}\}} V\left\{c_{1t}, \mu_t[J(U_{t+1}, s_{t+1})]\right\} \\ \text{s.t.} \quad & V\left\{c_{2t}, \mu_t(U_{t+1})\right\} \geq U_t \quad (\lambda_t) \\ & \text{plus resource constraints and shocks} \end{aligned}$$

Notation

- J is agent 1's utility, U is agent 2's utility ("promised utility")
- λ_t is (relative) Pareto weight

Fundamental tradeoff

- Give you more today (c_{2t})
- Give you more in the future ($\mu_t(U_{t+1})$)

Pareto problem: Pareto weight

First-order conditions

$$
\begin{aligned}
c_{1t}^{\rho-1}/p_{1t} &= \lambda_t^* c_{2t}^{\rho-1}/p_{2t} \\
\beta\ [J_{t+1}/\mu_t(J_{t+1})]^{\alpha-\rho}\lambda_{t+1}^* &= \lambda_t^*\ \beta\ [U_{t+1}/\mu_t(U_{t+1})]^{\alpha-\rho}
\end{aligned}
$$

Additive case ($\alpha = \rho$)

$$
\lambda_{t+1}^* = \lambda_t^*
$$

Otherwise

$$
\log \lambda_{t+1}^* - \log \lambda_t^* = (\alpha - \rho)\ [\text{white noise} + \text{risk adjustment}]
$$

Pareto problem: consumption

First-order conditions (repeated)

$$
\begin{aligned}
c_{1t}^{\rho-1}/p_{1t} &= \lambda_t^* c_{2t}^{\rho-1}/p_{2t} \\
\beta\,[J_{t+1}/\mu_t(J_{t+1})]^{\alpha-\rho}\lambda_{t+1}^* &= \lambda_t^*\,\beta\,[U_{t+1}/\mu_t(U_{t+1})]^{\alpha-\rho}
\end{aligned}
$$

Consumption and real exchange rate

$$
e_t \;=\; p_{2t}/p_{1t} \;=\; \lambda_t^*\,(c_{2t}/c_{1t})^{\rho-1}
$$

Numerical examples: exchange economy

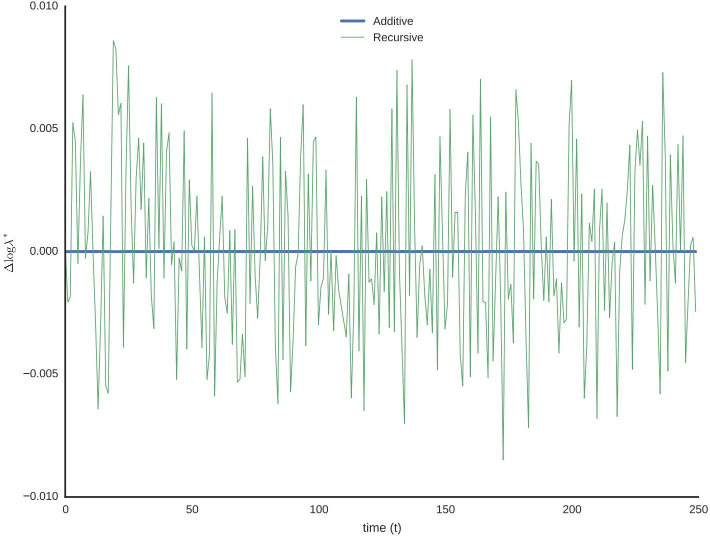
Computation

Method adapted from Collin-Dufresne, Johannes, and Lochstoer

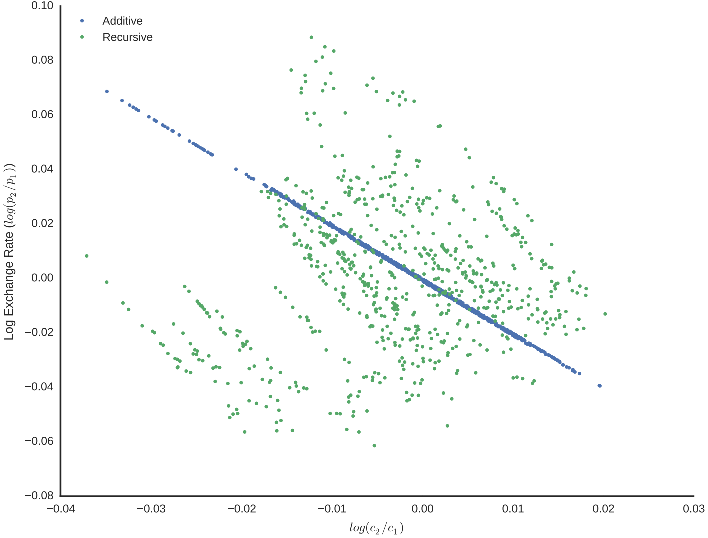
- Global projection method
- Implemented in Julia for speed
- State changed from U_t to $s_{at} = a_{1t}/y_{1t}$ or λ_t^*

P2C2E

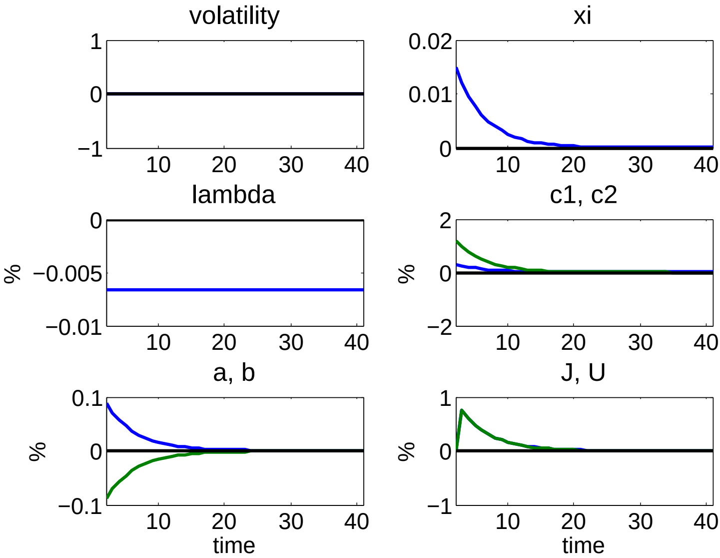
Dynamics of the Pareto weight



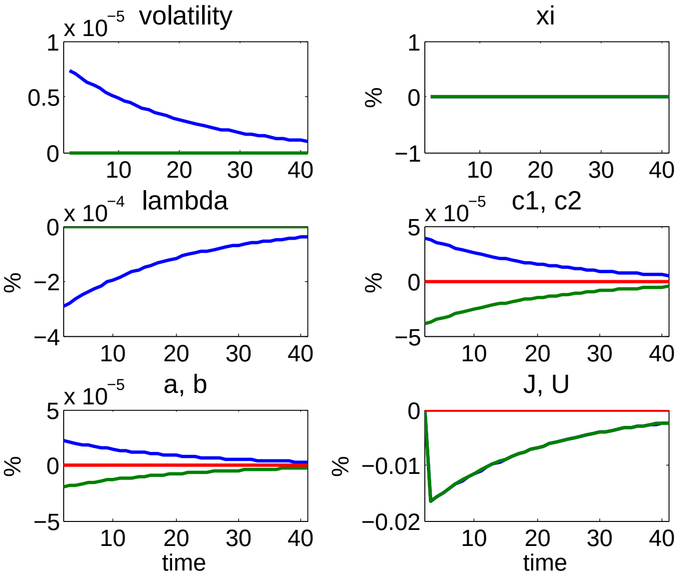
Consumption and real exchange rate



Responses to impulse in productivity in country 2 (blue first)



Responses to impulse in volatility (blue first, red additive)



Numerical examples: production economy

Investment and volatility

[We're working on this, harder than we thought]

Stability

Stability of the Pareto weight

What we know

- Colacito and Croce: If $\rho = \sigma = 0$, stable by theorem
- Colacito and Croce, Tretvoll: With some other parameter values, solutions seem stable

Open question

- What configurations of parameter values generate stability?
- Hint at problem: log Pareto weight close to martingale
- Hint at solution: shape of Pareto frontier (J vs U) reflects final goods frontier

Last thought

What would you do with this material?